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Belief Dependence, Revision and Persistence

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Abstract

A rational agent, when confronted with information inconsistent with his beliefs will either revise his beliefs in order to incorporate this new information, or persist in his beliefs and reject this information. A less rational agent even may revise his beliefs without there being a real conflict with the new information. In this paper, we propose a rational balance strategy between belief revision and belief persistence in a multiple agent environment. Our proposal is based on the framework of logic of belief dependence. The proposed strategy, called the *confidence priority strategy*, has the impact that the more confidently a formula φ is believed by an informant, the more credible the belief is. The strategy is plausible for cooperative multiple agent environments. The distinct roles played by the agents are determined in order to capture an intuitive understanding of the strategy.

1 Introduction

Belief revision is one of major problems for belief management techniques and knowledge representation systems [3, 4, 10, 11]. *Belief revision* refers to the process in which a new sentence that is inconsistent with an agent's beliefs or knowledge is added. *Belief persistence* refers to the process in which agent's belief or knowledge are preserved when confronted with new information. Belief revision and belief persistence are very important features for flexible and intelligent knowledge based systems. However, as far as we know, there exists no existing formalism or approach to study how to balance rationally between those two important processes.

We are looking for an intuitive approach by which the choice between belief revision and belief persistence can be determined by artificial rational agents. In reality the problem seems to largely depend on pragmatics and psychological notions such as intention, goal, and attitudes of agents. In other words, we are looking for a formalism where one can mechanically compute, given a description of the state of the world, whether an agent will revise his beliefs or persist when confronted with new incompatible information.

In this paper we present a rational formalism which makes the choice between belief revision and belief persistence computable. Our proposal uses the notions and formalisms of logic of belief dependence as introduced by the first author at the previous Amsterdam Colloquium [6].

2 Belief Dependence and Belief Revision

2.1 Logic for Belief Dependence

In multiple agent environments, it is frequently beneficial to enable agents to communicate their knowledge or beliefs among each other. Under such circumstances some agents may rely on others for their beliefs or knowledge. We called the phenomenon *belief dependence.* In [6, 8], a formal theory for belief dependence in a multiple agent environment is described, which is designed to serve as a foundation for understanding rational behavior of artificial agents. In this subsection, we give an overview of the main notions in this approach.

Our logic involves in the first place the general notions of knowledge and belief, which are the equivalents of those notions in epistemic and doxastic logic. In our logic for belief dependence, we generally use $L_i\varphi$ to represent the fact that agent i knows or believes the formula φ. As is well known, the modal operator L represents an epistemic operator, when the logic is an **S5** system, whereas L is a doxastic operator if the logic is a weak **S5** system.

There exists a second important notion used for reasoning about dependent knowledge and beliefs; this notion is called the *dependent operator*, or alternatively *rely-on relation*, and it is denoted by $D_{i,j}$. Intuitively, we can give $D_{i,j}\varphi$ a number of different interpretations: "agent i relies on agent j about the formula φ", "agent i depends on agent j about believing φ", or even more specifically, "agent j is the credible advisor of agent i about φ".

In the communication of knowledge and belief among agents, agents do not necessarily view knowledge and belief accepted from other agents as their own knowledge, even though they may originally have asked for such information. In terms of cognitive psychology, these beliefs are compartmentalized [12]. In logics for belief dependence, we therefore introduce a *compartment operator*, or alternatively called a *sub-belief operator*, written $L_{i,j}$. Intuitively, $L_{i,j}\varphi$ can be read "agent i believes φ due to agent j". From the viewpoint of *minds society*, $L_{i,j}\varphi$ can be more intuitively interpreted as "agent i believes or knows φ on the mind frame indexed j". Also, as argued for in [8], we claim that an appropriate procedure for formalizing information assimilation should involve two phases: *compartmentalization* and *incorporation* of information. Compartmentalized information consists of those fragments of information which are accepted and remembered as isolated beliefs and which are treated somewhat different from the beliefs that are completely believed, whereas incorporated information consists of the beliefs that are completely believed by the agents. In the logic for belief dependence, compartmentalized information is modeled by sub-beliefs $L_{i,j}\varphi$ for agent i, whereas incorporated information corresponds to general beliefs of agent i, namely, $L_i\varphi$. Note that in this interpretation the law that $L_i\varphi \equiv L_{i,j}\varphi \wedge D_{i,j}\varphi$ is not universally valid.

In [8], several semantic models for logics of belief dependence are presented. In this paper, we select the **D**-model as the basic model. Formally, *a belief dependence D-model* is a tuple $M = (S, \pi, \mathcal{L}, \mathcal{D})$, where S is a set of states, $\pi(s, .)$ is a truth assignment for each state $s \in S$, $\mathcal{L} : \mathbf{A_n} \rightarrow 2^{S \times S}$, is a collection of n binary accessibility relations on S, and $D : \mathbf{A_n} \times \mathbf{A_n} \times S \rightarrow 2^{\mathbf{L_D}}$ is the mapping describing the belief dependence: $D(i, j, s)$ is the set of formulas φ for which agent i depends on agent j in state s. Here $\mathbf{A_n}$ is a set of agent names, and $\mathbf{L_D}$ is the formal language for the logic of belief dependence.

2.2 Belief Revision

There exist three kinds of update operations used for belief maintenance [3, 4]. They are: *Expansion, Revision*, and *Contraction*

Expansion: A new proposition φ is added to a given knowledge set K. Formally, the knowledge set that results from expanding K by a proposition φ is denoted $K + \varphi$.

Revision: A new proposition which is inconsistent with a knowledge set K is added, but in order to keep the resulting set consistent some of the old propositions in K must be removed. The resulting of revising K by a proposition φ is denoted $K \dot{+} \varphi$.

Contraction: A proposition in K is retracted without adding any new proposition. The resulting knowledge set of contracting K with respect to the formula φ is denoted $K \dot{-} \varphi$.

Suppose the added proposition is φ, the knowledge set K, and the base of the knowledge set be B. A possible definition for the expansion operation reads:
$Cn(B) + \varphi \stackrel{\text{def}}{=} Cn(Cn(B) \cup \{\varphi\}) = Cn(B \cup \{\varphi\})$
where Cn is the *consequence operation*, a closure operator which maps sets of propositions to set of propositions and which has the following properties:

inclusion $A \subseteq Cn(A)$

iteration $Cn(A) = Cn(Cn(A))$

monotonicity $Cn(A) \subseteq Cn(B)$ whenever $A \subseteq B$

For revision and contraction, the following condition, called *Levi Identity*, is generally required to be satisfied:
$K\dot{+}\varphi = Cn((K\dot{-}\neg\varphi) \cup \{\varphi\})$

Although the the revision and contraction operations to a large extent are a matter of pragmatics, and seem to be outside of the scope of logical analysis, there have been proposed a number of intuitive plausible constraints in the work of Alchourrón, Gärdenfors, and Makinson; see [3, 4]. Gärdenfors presents a set of constraints on the contraction operator which are called the *Gärdenfors Postulates*:

Closure $A\dot{-}\varphi$ is a theory

Inclusion $A\dot{-}\varphi \subseteq A$

Vacuity If $\varphi \notin A$, then $A\dot{-}\varphi = A$

Success If $\varphi \notin Cn(\vee)$, then $\varphi \notin A\dot{-}\varphi$

Preservation If $Cn(\varphi) = Cn(\psi)$, then $A\dot{-}\varphi = A\dot{-}\psi$

Recovery $A \subseteq Cn((A\dot{-}\varphi) \cup \{\varphi\})$

$(A\dot{-}\varphi) \cap (A\dot{-}\psi) \subseteq A\dot{-}(\varphi \wedge \psi)$

If $\varphi \notin A\dot{-}(\varphi \wedge \psi)$, then $A\dot{-}(\varphi \wedge \psi) \subseteq A\dot{-}\varphi$.

3 Belief Maintenance in a Logic of Belief Dependence

As argued for in [8], we believe that an appropriate procedure for formalizing information assimilation should pass two phases: compartmentalization and incorporation of information.

For a multiple agent environment, we assume that some primitive rely-on relations about some propositions among those agents have been determined at the metalevel. We call this assumption the *initial role-knowledge assumption.* We believe that the assumption is appropriate and intuitive, because, in a multiple agent environment, some agents have to possess a minimal amount of knowledge about some other agents, in order to guarantee that they will start to communicate at all. Also, in a reliable communication network, assuming that agents are honest, no-doubt and something more [8], primitive rely-on relations often collapse into primitive communication relations, and this turns them into observable entities.

In [8], the formalism describing the first phase of information assimilation is described. This paper involves the problem of determining how the complete sub-belief and the complete rely-on relations can be captured, using the primitive rely-on relations. This paper also introduces some role-appraisal axioms such as "fool believer", and "stubborn believer".

In this paper, we would like to treat the second phase of information assimilation in a multiple agent environment. Based on the role-appraisal information mentioned above, it turns out to be possible to determine some rational belief maintenance strategies, which enable us to figure out whether and how compartmentalized beliefs can be assimilated into the incorporated beliefs for the agents. In other words, this role-appraisal information yields rational strategies for agents which determine how and whether their beliefs are revised or persisted.

More formally the process can be described as follows: if an agent i relies on another agent j about believing formula φ, and agent j believes φ, then agent i will accept the belief φ. However, in the first stage of assimilation of knowledge and beliefs, agent i only accepts φ as a sub-belief $L_{i,j}\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi$. Agent i does not necessarily accept the belief as his own incorporated belief.

In the second stage of assimilation of knowledge and belief agent i is supposed to change his compartmentalized belief into incorporated belief, i.e., change $L_{i,j}\varphi$ into $L_i\varphi$. However, if we just transform the sub-belief in this way for any agent and any formula φ, then we will find that the resulting beliefs may turn out to be inconsistent. In order to avoid such conflicts, some old beliefs in $L_i\varphi$ should be removed. Alternatively, the agent may as well reject the new information in order to avoid the inconsistency. Thus, under those circumstances, we must use the belief update operations revision $\dot{+}$ and contraction $\dot{-}$ to describe the process precisely, and we will need further information in order to decide which operator will be invoked under which circumstances.

For a further formalization we need some further notations. If K is a knowledge set, we define $L_{i,j}^-(K) \stackrel{def}{=} \{\psi | L_{i,j}\psi \in K\}$, denoting the set of agent i's sub-belief indexed j. Moreover, $L_i^-(K) \stackrel{def}{=} \{\varphi | L_i\varphi \in K\}$, denotes agent i's belief set. The belief maintenance operator is denoted by $\triangle$.

Formally, a *belief maintenance model*, or *BMM*, is an ordered couple $< \mathbf{K}, \triangle >$ such that $\mathbf{K}$ is a set of belief sets and $\triangle : \mathbf{K} \times \mathrm{Sent}(L) \rightarrow \mathbf{K}$ is a function assigning a maintenance operation $\triangle(K, A)$ to any belief set $K \in \mathbf{K}$ and any L-sentence A. We shall write $K\triangle\varphi$ as an alternative representation for $\triangle(K,\varphi)$. In particular, $L_i^-(K)\triangle L_{i,j}\varphi$ denotes the belief maintenance result on the formula set $L_i^-(K)$ if the new information is $L_{i,j}\varphi$.

In order to deal with different kinds of belief conflicts, we will use the following five types of update strategies[1]:

(Positive-revision)	$L_i^-(K)\triangle L_{i,j}\varphi = L_i^-(K)\dot{+}\varphi$
(Negative-revision)	$L_i^-(K)\triangle L_{i,j}\varphi = L_i^-(K)\dot{+}\neg\varphi$
(Persistence)	$L_i^-(K)\triangle L_{i,j}\varphi = L_i^-(K)$
(Positive-contraction)	$L_i^-(K)\triangle L_{i,j}\varphi = L_i^-(K)\dot{-}\varphi$
(Negative-contraction)	$L_i^-(K)\triangle L_{i,j}\varphi = L_i^-(K)\dot{-}\neg\varphi$

The update strategies of negative-revision and positive-contraction seem to be rather counterintuitive at first sight. However, they describe the behavior of an agent i which doesn't trust his informant j at all; if j believes something this is taken to be a good reason for not accepting it as an incorporated belief, so agent i will rather retract it from his own belief or even add its negation.

For the belief conflict mentioned above, where $\varphi \in L_{i,j}^-(K)$ and $\neg\varphi \in L_i^-(K)$, we have three reasonable and plausible choices:

1. $\varphi \in L_{i,j}^-(K), \neg\varphi \in L_i^-(K) \Rightarrow L_i^-(K)\triangle L_{i,j}\varphi = L_i^-(K)\dot{+}\varphi$.
   (Agent i accepts the new belief φ on behalf of agent j's believing φ, although agent i originally believes that φ is false.)

[1] In the following, we make no distinction between $\dot{+}$ and $+$.

2. $\varphi \in L^-_{i,j}(K), \neg\varphi \in L^-_i(K) \Rightarrow L^-_i(K)\triangle L_{i,j}\varphi = L^-_i(K)\dot{-}\neg\varphi$.
(Agent i does not believe that φ is false on behalf of agent j's believing φ, although he originally believes that φ is false.)

3. $\varphi \in L^-_{i,j}(K), \neg\varphi \in L^-_i(K) \Rightarrow L^-_i(K)\triangle L_{i,j}\varphi = L^-_i(K)$.
(Even though agent j believes φ while agent i believes that φ is false, agent i persists his belief.)

Note that for this special case the alternative of negative-revision, $L^-_i(K)\triangle L_{i,j}\varphi = L^-_i(K)\dot{+}\neg\varphi$, is useless, because $\neg\varphi \in L^-_i(K)$ implies $L^-_i(K)\dot{+}\neg\varphi = L^-_i(K)$. In other words, the strategy coincides with persistence. The same holds for the strategy of positive contraction.

The above analysis shows that there exist three different plausible choices for the belief conflict situation under consideration. In order to disambiguate this conflict situation we refine the notion of sub-belief and we introduce three new credibility operators in the formalism: *High-credibility sub-belief, Neutral-credibility sub-belief* and *Low-credibility sub-belief*, denoted $HL_{i,j}\varphi$, $NL_{i,j}\varphi$ and $LL_{i,j}\varphi$ respectively.

The formula $HL_{i,j}\varphi$ means that agent i views agent j as an agent with high credibility on φ, and $NL_{i,j}\varphi$ and $LL_{i,j}\varphi$ mean respectively that with neutral credibility and that with low credibility.

Based on those credibility operators, the three possible result of the above belief conflict are determined as follows:

SH-Pr $\varphi \in L^-_{i,j}(K) \wedge \neg\varphi \in L^-_i(K) \wedge HL_{i,j}\varphi \Rightarrow L^-_i(K)\triangle L_{i,j}\varphi = L^-_i(K)\dot{+}\varphi$.
(Because agent j believes φ and agent j is viewed as an agent with high credibility about φ, agent i accepts the new belief φ, although he originally believes that φ is false.)

SN-Nc $\varphi \in L^-_{i,j}(K), \neg\varphi \in L^-_i(K) \wedge NL_{i,j}\varphi \Rightarrow L^-_i(K)\triangle L_{i,j}\varphi = L^-_i(K)\dot{-}\neg\varphi$.
(Because agent j believes φ and agent j is viewed as an agent with neutral credibility about φ, agent i retracts his original belief that φ is false)

SL-Pe $\varphi \in L^-_{i,j}(K), \neg\varphi \in L^-_i(K) \wedge LL_{i,j}\varphi \Rightarrow L^-_i(K)\triangle L_{i,j}\varphi = L^-_i(K)$.
(Although agent j believes φ while agent i believes that φ is false, agent i persists in his belief, because of agent j's low credibility.)

4 Role Analysis

In the previous section we have solved the problem of determining which of the three meaningful belief revision strategies will be invoked in the case of a conflict between the new information and the agent's previous belief by refining the notion of sub-belief into three new notions. This means, however that we now must cope with a new problem involving these credibility operators; we must invent a strategy which decides which credibility operators will be the result of the first stage of the process of information assimilation under which circumstances. This choice must be based on the logic of belief dependence and not invoke other information outside the formalism.

Below are some plausible axioms for credibility operators. The credibility operator $HL_{i,j}\varphi$ is interpreted as "agent i views agent j as an agent with high credibility about φ". If we suppose that each agent's perspective on credibility is correct and that these viewpoints are common knowledge among agents, then we can consider those notions concerning highness, neutralness, and lowness as an order relation. In other words, the axioms which we are going to describe are intended to describe the situation where credibility is directly linked to the true observable expertise of the agents. This expertise moreover should be common knowledge among agents. For this scenario, we have the following axioms for our credibility operators:

Axioms:

Irreflexivity $\neg HL_{i,i}\varphi$.

Asymmetry $HL_{i,j}\varphi \rightarrow \neg HL_{j,i}\varphi$.

Transitivity $HL_{i,j}\varphi \wedge HL_{j,k}\varphi \rightarrow HL_{i,k}\varphi$.

Reflexivity $NL_{i,i}\varphi$.

Symmetry $NL_{i,j}\varphi \rightarrow NL_{j,i}\varphi$.

Transitivity $NL_{i,j}\varphi \wedge NL_{j,k}\varphi \rightarrow NL_{i,k}\varphi$.

Definition $LL_{i,j}\varphi \stackrel{def}{=} HL_{j,i}\varphi$.

A little reflection shows that the above axioms can easily be violated in situations where agents are misinformed about each other's expertise, or where there exist disagreements among the agents.

Next we make some analysis on possible configurations among the relied-on relations. In order to obtain an intuitive understanding on this approach, we will use the notion of *role* to refer to different types of agents characterized in terms of being relied on in different configurations; this relates to role theory in social science[1, 9].

For a single agent, there exist the following different roles which the agent can perform (for the formula φ):

Isolated-A The agent relies on nobody including himself and is relied on by nobody: $(\neg\exists j)(D_{i,j}\varphi \vee D_{j,i}\varphi)$

Isolated-B The agent relies on only himself, while nobody relies on him: $D_{i,i}\varphi \wedge (\neg\exists j \neq i)(D_{j,i}\varphi \vee D_{i,j}\varphi)$

Learner The agent only relies on someone else: $(\exists j \neq i)D_{i,j}\varphi \wedge \neg D_{i,i}\varphi$.

Expert The agent relies on both himself and someone else: $D_{i,i}\varphi \wedge (\exists j \neq i)D_{i,j}\varphi$

Authority The agent is relied on both by himself and by someone else, but he relies on nobody else: $D_{i,i}\varphi \wedge (\exists j \neq i)D_{j,i}\varphi \wedge \neg(\exists j \neq i)D_{i,k}\varphi$.

Diffident agent The agent relies on nobody including himself, but is relied on by someone else: $(\exists j \neq i)D_{i,j}\varphi \wedge \neg(\exists k)D_{i,k}\varphi$

Among the above roles, Role Isolated-A and Role Isolated-B are isolated ones where agents rely on nobody else; in the studies of belief dependence these are trivial roles. The remaining roles are the fundamental roles which are worthy to be named and investigated in depth. For these roles we introduce the notations given below:

$Learner_i\varphi \stackrel{def}{=} \neg D_{i,i}\varphi \wedge (\exists j \neq i)D_{i,j}\varphi$

$Expert_i\varphi \stackrel{def}{=} D_{i,i}\varphi \wedge (\exists j \neq i)D_{i,j}\varphi$

$Authority_i \stackrel{def}{=} D_{i,i}\varphi \wedge (\exists j \neq i)D_{j,i}\varphi \wedge \neg(\exists k \neq i)D_{i,k}\varphi$.

$Diffident-agent_i\varphi \stackrel{def}{=} (\exists j \neq i)D_{j,i}\varphi \wedge \neg(\exists k)D_{i,k}\varphi$.

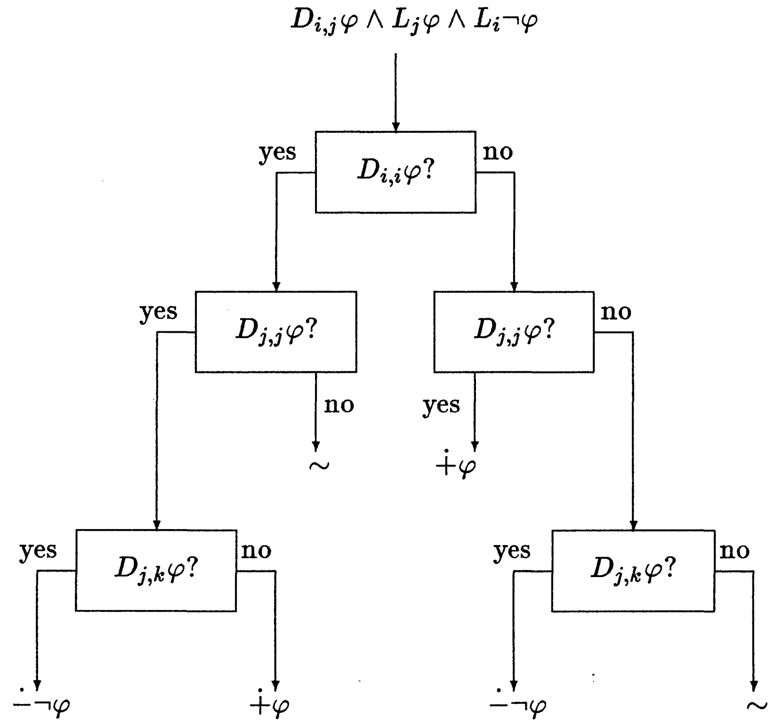

Figure 1: Decision Tree

5 Roles and Credibilities

Using the fundamental roles described above, we now propose a strategy, called the *confidence priority strategy*, yielding an intuitive mechanism for choosing between the three credibility operators. This strategy enforces that the more confidently a formula φ is believed by an informer, the more credible the belief is. We believe that the confidence priority strategy is reasonable and acceptable for cooperative multiple agent environments. If an agent firmly believes a fact φ, to the extent that others rely on him then this indicates that the agent must have strong evidence or a convincing justification for his belief. The other agents therefore should view the agent's belief as a belief with higher credibility.

According to the confidence priority strategy, the fundamental roles can be arranged intuitively in ascendent order as diffident-agent, learner, expert, and authority. Note that in our formalism the notion of credibility is a relative one. For instances, an agents who is a learner will view agents that are expert or authority as agents with high credibility about φ. Whereas neutral credibility will be granted to peers, i.e. in the case where both agents are learners, experts, or authorities. The relative credibility relations can be formalized as follows:

Def-High-credibility $HL_{i,j}\varphi \stackrel{def}{=} \mathit{Diffident-agent}_i\varphi \wedge (\mathit{Learner}_j\varphi \vee \mathit{Expert}_j\varphi \vee \mathit{Authority}_j\varphi) \vee \mathit{Learner}_i\varphi \wedge (\mathit{Expert}_j\varphi \vee \mathit{Authority}_j\varphi) \vee \mathit{Expert}_i\varphi \wedge \mathit{Authority}_j\varphi$.

Def-Neutral-credibility $NL_{i,j}\varphi \stackrel{def}{=} \mathit{Diffident-agent}_i\varphi \wedge \mathit{Diffident-agent}_j\varphi \vee \mathit{Learner}_i\varphi \wedge \mathit{Learner}_j\varphi \vee \mathit{Expert}_i \wedge \mathit{Expert}_j \vee \mathit{Authority}_i \wedge \mathit{Authority}_j\varphi$

Def-Low-credibility $LL_{j,i}\varphi \stackrel{def}{=} HL_{i,j}\varphi$

If we combine our proposals in this section and the previous one we obtain a computational strategy for the standard case of the belief conflict under consideration, i.e. the situation where $\varphi \in L^{-}_{i,j}(K) \wedge \neg\varphi \in L_i(K)$. For the three meaningful revision operators we will use the notations

Positive-revision, **Persistence**, and **Negative-contraction** to denoted the corresponding processes.

If we suppose that axiom (Ldf) $L_{i,j}\varphi \stackrel{def}{=} D_{i,j}\varphi \wedge L_j\varphi$ and the neutral axiom (D¬) $D_{i,j}\varphi \equiv D_{i,j}\neg\varphi$ hold, we have, for the case of normal belief conflict that $\varphi \in L^-_{i,j}(K) \wedge \neg\varphi \in L_i(K)$ implies $D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi$ Therefore, we also call the case in which $D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi$ a normal belief conflict.

For this case of normal belief conflict, we have:
$D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi \Rightarrow D_{i,j}\varphi \Rightarrow (D_{i,j}\varphi \vee D_{i,i}\varphi) \wedge (D_{i,j}\varphi \vee \neg D_{i,i}\varphi) \Rightarrow Expert_i\varphi \vee Learner_i\varphi$

In the case that agent i is an expert about φ, and if $\neg D_{j,j}\varphi$ holds, it follows that agent j is a learner or a diffident-agent, and we have $LL_{i,j}\varphi$. Therefore, the outcome will be persistence, denoted by $\sim$ for short. On the other hand, if $D_{j,j}\varphi$ holds, and if there exists an agent k such that $D_{j,k}\varphi$, the agents j and i are both experts and therefore peers and the outcome will be negative-contraction. Otherwise the process would be positive-revision because agent j is an authority about φ.

For the other case that agent i is a learner about φ, we investigate once more whether $D_{j,j}\varphi$ holds. If this is the case that means that agent j at least is an expert and we have $HL_{i,j}\varphi$ and the process will be positive-revision. On the other hand, if $\neg D_{j,j}\varphi \wedge (\exists k \neq j)D_{j,k}\varphi$ holds, then agent j is a learner, and the outcome will be negative-contraction. In the remaining case the outcome is persistence.

Therefore, the strategy can be expressed as a set of logical sentences as follows:

$D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi \wedge D_{i,i}\varphi \wedge \neg D_{j,j}\varphi \Rightarrow$**Persistence**.

$D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi \wedge D_{i,i}\varphi \wedge D_{j,j}\varphi \wedge (\exists k \neq j)D_{j,k}\varphi \Rightarrow$**Negative-contraction**.

$D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi \wedge D_{j,j}\varphi \wedge \neg(\exists j \neq k)D_{j,k}\varphi \Rightarrow$**Positive-revision**.

$D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi \wedge \neg D_{i,i}\varphi \wedge D_{j,j}\varphi \Rightarrow$**Positive-revision**.

$D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi \wedge \neg D_{i,i}\varphi \wedge D_{j,j}\varphi \wedge (\exists k \neq j)D_{j,k}\varphi \Rightarrow$**Negative-contraction**.

$D_{i,j}\varphi \wedge L_j\varphi \wedge L_i\neg\varphi \wedge \neg D_{i,i}\varphi \wedge \neg D_{j,j}\varphi \wedge \neg(\exists k \neq j)D_{j,k}\varphi \Rightarrow$**Persistence**.
Moreover, the strategy can be expressed as algorithm as it is shown in the figure.

6 Concluding Remarks

Based on the framework of logic for belief dependence, we have proposed a rational balance strategy between belief revision and belief persistence in a multiple agent environment. The confidence priority strategy provides an intuitive, plausible and flexible approach to formalize the relationship between the fundamental roles and credibility operators.

On the other hand, we are convinced that there exist many other strategies which are also plausible and acceptable for multiple agent environments; such alternatives could be based on different analysis and perspectives on the rely-on relations in the models. To capture the other plausible strategies is an interesting topic for further research.

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