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# CATEGORIAL GRAMMAR AND NATURAL REASONING

Víctor Sánchez Valencia

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# Categorial Grammar and Natural Reasoning

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# CHAPTER I

# INTRODUCTION

## 1 GENERAL CLAIM

This report describes a Natural Logic i. e. a system of reasoning whose vehicles of inference ('logical forms') are in close accord with natural language. The claim of this report is that an adequate local proof system for natural language is possible. We aim to show that there is no opposition between a semantical and a proof-theoretical approach to natural reasoning: the proof system begins where the semantics leaves off. This assertion has not only a systematical value but also a heuristic one. Fundamental examples of the way in which the semantics of natural language can be exploited in setting up a natural logic are the Generalized Quantifier approach to natural language quantification ([Barwise and Cooper 1981]), the treatment of indefinite descriptions in the framework of Discourse Representation theory ([Kamp 1981]), and the uniform account of natural language coordination due to Partee and Rooth ([Partee and Rooth 1983]).

### 1.1 GENERALIZED QUANTIFIERS

The main idea of the Generalized Quantifiers approach to natural language quantification is this. Determiners D (*every, no, not every, some*) denote a relation between set of objects, namely the denotation of those X, Y for which DXY holds. These denotations of determiners have certain interesting monotone properties:

> If every XY holds and Y is smaller than Z, then every XZ holds as well. (upward monotonicity)

This property is shared by the denotation of *some* in both arguments and by *not every* in its first argument.

> If every XY holds and Z is smaller than X, then every ZY holds as well. (downward monotonicity).

This property is shared by the denotation of *no* in both its arguments and by *not every* in its second argument.
The monotone properties of determiners can be and is used to support global principles of inference. In its general form, monotonicity asserts that

> expressions occurring in (downward) upward monotone positions may be replaced by expressions with a (smaller) larger denotation.

Monotonicity is pivotal for Natural Logic because it is present across all categories. There are monotone verbs, adverbs, prepositions, connectives. Moreover, there are inclusions between adverbs, adjectives, noun phrases. Not surprisingly, inference rules based on the general form of monotonicity are the core of the Natural Logic to be presented in this report.

### 1.2 DISCOURSE REPRESENTATION

Inferences are in fact pieces of discourse. It is a sensible heuristic move to take advantage of insights about discourse processing in the construction of a logic which tries to explain a particular kind of discourse. The model of discourse representation described in [Kamp 1981] is relevant for Natural

Logic because of the way in which indefinite descriptions are represented in this framework. An expression of the form *a CN VP* is processed by introducing a discourse referent $u$ and the sentences *CN(u)* and *u VP*. We shall show that this treatment of indefinite descriptions can be incorporated fruitfully into a natural logic which treats noun phrases as generalized quantifiers. To combine the generalized quantifier perspective and Kamp's proposal we shall exploit a compelling analogy between Kamp's representation strategy and a well-known proof-system: Beth's tableaus method ([Van Benthem and Van Eijck 1982]).
We shall show that the introduction of Kamp's treatment of indefinite descriptions to the monotonicity calculus described in [Sánchez 1991] constitutes a simple extension which deals satisfactorily with scope problems. Of course Kamp's proposal focuses on anaphorical phenomena. Our Natural Logic will presuppose information about anaphoric linking as given in extensions of Kamp's framework, for instance in [Van Eijck 1985].

BETH'S TABLEAUS The method of *semantic tableaus* was introduced by E. W. Beth in [Beth 1955]. The idea behind this method consists in the systematical search for a *counter-example* to a given sequent $K/P$. Here $K$ and $P$ are classes of formulas. A counter-example to $K/P$ is a *model* (*valuation*) in which all members of $K$ are true and all members of $P$ false. The search for the counter-example is guided by the so-called *reduction rules* i. e. rules that reduce the valuation of a complex formula to the valuation of simpler ones. Every constant is provided with two reduction rules which can be applied when the constant is the last one introduced in the formula under consideration. For instance, the reduction rules for the set $\{\neg, \wedge, \exists\}$ are the following:

- if $\neg A$ should be true, then $A$ should be false.
  if $\neg A$ should be false, then $A$ should be true.
- if $(A \wedge B)$ should be true, then $A$ and $B$ should be true.
  if $(A \wedge B)$ should be false, then $A$ should be false or $B$ should be false.
- if $\exists x A(x)$ should be true, then $A(d_i)$ should be true for a new object $d_i$.
  if $\exists x A(x)$ should be false, then $A(d_i)$ should be false for all objects $d_i$ already introduced in the construction of the tableau.

Notice that making $(A \wedge B)$ false leads to the introduction of sub-tableaus.
When a sub-tableau contains an obviously conflicting demand, i. e. to make a formula both true and false, it represents a failed attempt to construct a counter-example. We therefore abandon this sub-tableau and call it *closed* . When all the sub-tableaus are closed we say that the tableau itself is *closed*, and the sequent is called *valid*. If some sub-tableau is not closed and no further reduction rule is applicable then we have a counter-example: this sub-table describes a situation in which all formulas in $K$ are true and all formulas in $P$ are false. As we pointed out, this method of semantic tableaus will be adapted in this report to a fragment of English based on a Categorial Grammar. It is the use of Categorial Grammar which allows us to define reduction rules for natural language sentences: the syntactical analyses of an expression $X$ provided by the grammar contains information about the last expression introduced in the construction of $X$. In our CNL the above reduction rules will take the following form:

- if $Neg(A)$ should be true, then $A$ should be false.
  if $Neg(A)$ should be false, then $A$ should be true.
- if $(A\ and\ B)$ should be true, then $A$ and $B$ should be true.
  if $(A\ and\ B)$ should be false, then $A$ should be false or $B$ should be false.
- if $a\ CN\ VP$ should be true, then $d_i\ is\ a\ CN$ and $d_i\ VP$, should both be true for a new $d_i$.
  if $a\ CN\ VP$ should be false, then $d_i\ VP$, should be false for all $d_i$ such that $d_i\ is\ a\ CN$ is true.

We are convinced that Beth's method is an adequate model for the representation of natural language inference, witness the connexion between Beth's first reduction rule for the existential quantifier and Kamp's treatment of indefinite descriptions. This use of discourse referents in inference has even a traditional antecedent in Aristotles' non-syllogistic proof of the so-called *conversion rule*:

> First, then, let us take a negative universal premiss having the terms A and B. Then if A applies to no B, neither will B apply to any A; for if it applies to some, e. g. c, it will not be true that A applies to no B, because c is B. (See [Aristotle I])

Furthermore, there is an intimate connection between Beth's tableaus and the ***dialogical logic*** in which the logical constants are defined by rules for their use in critical debates (cf. [Barth and Krabbe 1982]).

### 1.3 Generalized Coordination

Several writers have noticed that all major syntactical categories can be conjoined with the Boolean particles *and* and *or*. We have Noun Phrase coordination (*abelard and every logician*), Verb Phrase coordination (*dances or sings*), Adverbial coordination (*clumsily or unimaginatively*), Adjectival coordination (*male and wise*). In modern semantics it has been possible to give a uniform meaning to the Booleans across all the relevant categories. Coordination in the interpretation domain of expressions of category $(a, b)$ is defined in terms of coordination in the domain of the expressions of category $b$. This allows the recursive definition of coordination, starting with sentential coordination and assuming the obvious interpretation of sentential *and* and *of* ([Partee and Rooth 1983]).
This treatment suggests that a uniform explanation of Booleans as they occur in inference is possible. For instance, the denotation of $X$ *and* $Y$ is smaller that the denotation of $X$ in the sense that whatever is $X$ *and* $Y$ is also $X$. Thus by using monotonicity we shall be able to pass from $F(X \textit{ and } Y)$ to $F(X)$. But monotonicity does not exhaust the logic of *and*. We also need an inference rule which regulate the introduction of this particle. This can be done by generalizing the second reduction rule for sentential *and*:

- if $F(X \textit{ and } Y)$ should be false, then $F(X)$ should be false or $F(Y)$ should be false, where $(X \textit{ and } Y)$ occurs in $F$ in upward monotone position.

This simple extension of Beth's treatment of sentential conjunction allows us to explain inferences such as

- every man dances, every man sings/every man dances and sings.
- a man dances or sings/ a man dances or sings.
- every man and abelard loves heloise/abelard loves heloise.

## 2 Sources of Natural Logic

The insight that natural reasoning can be represented by a natural logic has several sources. A few examples are [Geach 1962], [Suppes 1979], and [Sommers 1982]. Geach notices that several non-trivial inferences in natural language can be explained directly by using the traditional Dictum de Omni principle —a principle that corresponds closely to upward monotonicity. Suppes presents a system of inference for English whose logical forms are the analysis trees provided by a context-free grammar. In Suppes' logic for English, linguistic form and grammatical form coincide: there is no need for the construction of a special apparatus generating logical forms in the account of inference. Finally, Sommers' concern is both the construction of a formal language which would reflect his view on the syntax of natural language and to make inference a matter of monotone substitutions. To this end, Sommers devised a formalism in which the monotone properties of traditional quantifiers and connectives are made explicit.

## 3 Categorial Natural Logic

In this report we shall construct a Categorial Natural Logic (thereafter CNL) which can be described as a strengthened monotonicity calculus. CNL reflects the idea expressed in [Van Benthem 1987] that natural reasoning displays various different mechanisms of inference using different levels of representation. CNL can be characterized by

- The use of syntactical analyses provided by Categorial Grammar as vehicles of inference (cf. Suppes).
- The use of monotonicity to warrant the sound application of inference rules (cf. Geach).
- The use of monotonicity marking of linguistic forms to indicate which expressions occur in monotone sensitive positions (cf. Sommers).
- The use of discourse referents in order to exploit the inferential properties of indefinite descriptions (cf. Kamp).
- The use of scope marking of linguistic forms to indicate which expression may introduce discourse referents.
- The use of generalized coordination to explain Boolean non-sentential reasoning (cf. Partee and Rooth).

In this report we are concerned primarily with showing the empirical adequacy of CNL. On another opportunity we shall address the theoretical questions which CNL poses, for instance its completeness with regard to an adequate semantics. This report is structured as follows. In the second chapter we describe the specific version of Categorial Grammar on which CNL is based. In the third chapter we turn to the description of the machinery involved in the construction of CNL. We introduce the mechanism of monotonicity and scope marking. Subsequent definitions introduce the operations of substitution and negation. The fourth chapter contains the description of CNL. We shall introduce first the monotonicity part of CNL. This logic can explain all the syllogistic inferences and something more. For instance, we are able to explain natural inferences in which the relevant noun phrases occur in object position. Afterwards we shall extend the system to obtain a more realistic model of natural reasoning. To this end we use Kamp's treatment of indefinite descriptions. The resulting system is a CNL in Beth's tableaus format. Finally, we use the generalized account of coordination to deal with Boolean inference and we sketch a way in which reasoning based on anaphoric relations can be treated within CNL.

# Chapter II

# Categorial Grammar

## 1 Classical Categorial Grammar

Classical Categorial Grammar is a language recognition device first described in [Adjukiewicz 1935] and [Bar-Hillel 1953]. The basic idea of CG is to define a set of categories built from the two basic ones **e**, **t** and a recursive procedure,

if a and b are categories, then so are (b/a) $(a\backslash b)$.

Here the complex category (b/a) searches an argument to its right, whereas $(a\backslash b)$ searches an argument to its left.

In this framework we categorize lexical elements by assigning them to a category, as in the following cases:

$$\begin{array}{l} abelard \in e \\ heloise \in e \\ cries \in (e\backslash t) \\ likes \in (e\backslash t)/e \\ bitterly \in (e\backslash t)\backslash(e\backslash t) \end{array}$$

(In these examples a categorization is represented by $x \in a$, where $x$ is an English expression and $a$ a category. This practice will be continued). To determine the category of a complex expression we first write the categories of its elements. Thereafter, we read the string of categories from left to right. When we find the first substring of the form '(b/a) a' or 'a $(a\backslash b)$' we apply the so-called cancellation rules

Cancellation Rules The adjacent expressions (b/a) a cancel to b.

The adjacent expressions a $(a\backslash b)$ cancel to b.

The result of cancellation is a new string of categories. We rescan the new string, applying the above rules whenever possible. The complex expression belongs to the category $c$ only if the successive applications of the procedure finally lead to the string $c$. Examples 1 a. and 1 b. below illustrate the procedure.

**Example 1**

a. *abelard cries bitterly* $\in t$

$$\begin{array}{ccc} abelard & cries & bitterly \\ e & \underline{(e\backslash t) \qquad (e\backslash t)\backslash(e\backslash t)} & \\ \underline{e \qquad\qquad} & \underline{(e\backslash t)} & \\ & t & \end{array}$$

b. *abelard loves heloise* $\in t$

$$\begin{array}{ccc} abelard & loves & heloise \\ e & \underline{((e\backslash t)/e) \qquad\qquad e} & \\ \underline{e \qquad\qquad} & \underline{(e\backslash t)} & \\ & t & \end{array}$$

## 2 Geach's Categorial Grammar

Expressions of natural language do not need to be categorized once and for all. For instance, *not* can occur as noun phrase negation (*Not every man)* or as predicate negation (*does not cry*). Similarly, *and* can occur as sentence conjunction (*Abelard cries and Heloise wanders*) or as predicate conjunction (*Abelard cries and wanders*). The main observation in [Geach 1972] is that categorial grammar can be extended in order to capture the essential unity of meaning of the items *not, and* across categories.
To represent complex categories, Geach uses a prefix functor. For convenience, in the rest of this report we shall make use of the infix functor ',' for the representation of complex categories, i. e.

If a and b are categories, then so is (b, a).

A complex category does no longer carry the information on which side it takes its argument. The cancellation rules take the form:

Undirected Cancellation Rules

The adjacent expressions (b, a) b cancel to a.

The adjacent expressions b (b, a) cancel to a.

Geach captures the ubiquity of the Booelean particles by adding to the calculus the following rules of type change,

**G1** if x belongs to the category (a, b), then also to the category ((c, a), (c, b)).

**G2** If x belongs to the category (a, (b, c)), then also to the category ((d, a), ((d, b), (d, c))).

For instance, starting from the categorization *not* $\in (t, t)$, the categorization of *not* as predicate negation i. e. *not* $\in ((e, t), (e, t))$ becomes available.
In addition to the above rules, Geach also considers a more general recursive procedure for establishing derived categorizations of expressions,

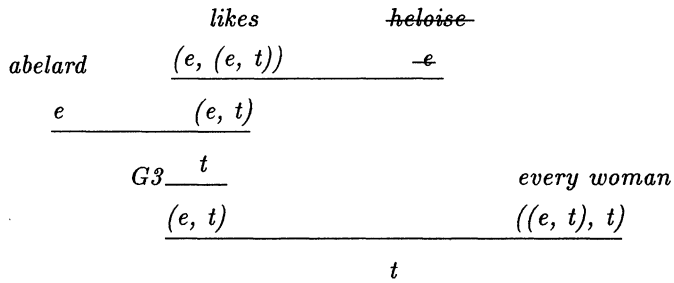
**G3** Let $F(Y) \in t$, and $Y \in e$. Then $F(\,) \in (e,t)$ ;

where $Y$ is a proper sub-expression of $F(Y)$ and $F(\,)$ is the result of deleting $Y$ from $X(Y)$.

Example 2. below, shows that G3 can establish that if *abelard likes heloise* ∈ t, then *abelard likes every woman* ∈ t; modulo the following basic categorization:

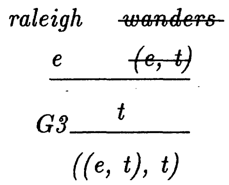
$$abelard\ , heloise \in e$$
$$likes \in (e,(e,t))$$
$$every\ woman \in ((e,t),t)$$

**Example 2** *abelard likes every woman* $\in t$




What kind of procedure is involved here? Notice that the use of the cancellation rule corresponds to a use of Modus Ponens in a Natural Deduction tree. Similarly, Geach's recursive procedure hints at the introduction of the conditionalization rule into categorial grammar —although restricted to the basic type e. If we drop this restriction, several principles of category change become available. For instance, as Example 3. shows, Montague's categorization of proper names can be obtained by using G3.

**Example 3** *raleigh* $\in ((e,t),t)$




Thus [Geach 1972] may be seen as proposing a non-directional Categorial Grammar where the cancellation rule is supplemented with additional principles of category forming —restricted conditionalization being one of them.

## 3 Lambek cum Permutation

In fact, [Lambek 1958] already describes a categorial calculus in which conditionalization supplements the cancellation rule, although this calculus is of the directed variety. CNL, on the other hand, is based on the undirected categorial calculus first introduced in [Van Benthem 1986], the so-called *Lambek cum Permutation Calculus* (thereafter LP). LP may be seen as an intuitionistic implicational logic, with additional restrictions on the book-keeping of assumptions used in derivations. This calculus gives an explanation of G1 by producing a derivation of ((c, a), (c, b)) from (a, b):

$$\dfrac{\dfrac{\dfrac{\dfrac{\cancel{(c, a)} \qquad \cancel{c}}{a} \qquad (a, b)}{b}}{(c, b)}}{((c, a), (c, b))}$$

As a matter of fact, LP does not give an explanation of G2. The reason is that in LP each use of conditionalization withdraws exactly one occurrence of a formula. The following tree shows that the transition from (a, (b, c)) to ((d, a) , ((d, b), (d, c))) is not available in LP:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\cancel{(d, b)} \qquad \cancel{d}}{b} \qquad \dfrac{(a, (b, c)) \qquad \dfrac{\cancel{(d, a)} \qquad \cancel{d}}{a}}{(b, c)}}{c}}{(d, c)}}{((d, b), (d, c))}}{((d, a), ((d, b), (d, c)))}$$

Hence, the transition from sentential *and* to predicate conjunction is not LP derivable. (There are several ways of coping with the ubiquity of Boolean particles. See, for instance, [Moortgat 1989] and [Partee and Rooth 1983]).

## 3.1 Meaning of Derivations

There is a systematic connection between derivations in intuitionistic implicational logic and expressions of the typed lambda calculus. These expressions are constructed from an infinite supply of variables $X_a, Y_a, \ldots$, for each type a, via the following formation rules:

- if N is of type (a, b), and M is of type a, then N(M) is a term of type b (application)
- if N is of type b and X is a variable of type a, then $\lambda$ X.N is a term of type (a, b) (abstraction)

[Van Benthem 1986] shows a systematic connection between derivations in LP and a class of terms –here suggestively called *Lambek terms*– with restrictions on occurrences of free and bound variables:

> N is a Lambek term iff N has at least one free variable and each lambda in N binds exactly one variable.

Van Benthem gives an effective procedure which applied to a derivation in LP yields a Lambek term. Conversely, there is an effective procedure which applied to a Lambek term yields a derivation in LP. The basic idea consists in linking assumptions with variables, Modus Ponens with application and conditionalization with lambda abstraction. For instance, the above proof of G1 corresponds to the Lambek term:

$$\lambda X_{(c,a)}.\lambda Y_c.Z_{(a,b)}(X_{(c,a)}Y_c)$$

Such terms are called the meaning of the corresponding derivation. Sometimes it is more convenient to represent the categorial analysis of an expression by a Lambek term which encodes linearly the same information as a LP derivation. In the construction of CNL we shall use derivation trees for the representation of categorial analyses, but the reader should keep in mind that a more compact representation is available.
For further details about LP we refer the reader to [Van Benthem 1991].

## 3.2 Monotonicity in the Typed Calculus

The typed lambda calculus has an interpretation in the Fregean universe, constructed from $D_e$ (domain of individuals), $D_t$ (truth-values) and the stipulation that $D_{(a,b)}$ is the set of set-theoretical functions from $D_a$ into $D_b$. The sets $D_a$ are partially ordered by a relation $\leq_a$ as follows:

- *If* $c,\ d \in D_e$ *then* $c \leq_e d$ *iff* $c = d$.
- *If* $c,\ d \in D_t$ *then* $c \leq_t d$ *iff* $c = 0$ *or* $d = 1$.
- *If* $c,\ d \in D_{(a,b)}$ *then* $c \leq_{(a,b)} d$ *iff for each* $a \in D_a,\ c(a) \leq_b d(a)$.

By using this ordering relation, we can define the notions of *monotone functions* and of *monotone occurrences* in this setting.

**monotone functions**

- $f \in D_{(a,\,b)}$ *is upward monotone iff for all* $x,\ y \in D_a$ *holds that* $x \leq_a y$ *entails* $f(x) \leq_b f(y)$.
- $f \in D_{(a,\,b)}$ *is downward monotone iff for all* $x,\ y \in D_a$ *holds that* $x \leq_a y$ *entails* $f(y) \leq_b f(x)$.

**monotone occurrences**

Assume that $N'_a$ is like $N_a$ except for containing an occurrence of $M'_b$ where $N_a$ contains a specified occurrence of $M_b$. We shall refer to this specified occurrence of M in N by **M**. We have the following definitions:

- $N_a$ *is upward monotone in* $\mathbf{M}_b$ *iff for all models* $\mathcal{M}$ *and assignments* $f$, $I_f^{\mathcal{M}}(M) \leq I_f^{\mathcal{M}}(M')$ *entails* $I_f^{\mathcal{N}}(N) \leq I_f^{\mathcal{M}}(N')$ .
- $N_a$ *is downward monotone in* $\mathbf{M}_b$ *iff for all models* $\mathcal{M}$ *and assignments* $f$, $I_f^{\mathcal{M}}(M') \leq I_f^{\mathcal{M}}(M)$ *entails* $I_f^{\mathcal{N}}(N') \leq I_f^{\mathcal{M}}(N)$.

In standard logic one uses a syntactical criterion for monotonicity: expressions which occur in syntactical positive position may be replaced by (semantically) stronger expressions; expressions which occur in syntactical negative positions may be replaced by (semantically) weaker expressions. For Natural Logic it is important that in the lambda calculus too, monotonicity is tied up with properly defined positive and negative positions. One may prove

**syntactic monotonicity**

- *If* $N_b$ *is positive (negative) in* $M_a$, *then* $N_b$ *is upward (downward) monotone in* $M_a$.
- *If* $X$ *is positive (negative) in* $N$, *then* $\lambda X.N$ *is upward (downward) monotone in* $M_a$.
- $NM$ *is upward monotone in* $N$ .

See [Sánchez 1991] for a proof of this assertion. The above mentioned correlation between LP derivations and Lambek terms allows us to transfer the syntactical criterion to the derivations themselves. One can show that a suitable marked position in a derivation corresponds with a monotone position in the associated term. This warrants the soundness of monotone substitution at the level of grammatical form.

## 4 Lambek Grammar

In this part we shall describe a variant of LP which we shall call *Lambek Grammar*, hereafter LG. [1] The LG derivations are the linguistic support for the model of natural reasoning developed in this report. LG generates natural deduction trees whose assumptions may be indexed by English expressions or by numerals. We speak then of *numerical* or *lexical* assumptions. LG is characterized by the following properties of its derivations:

- In an application of Modus Ponens the numerical indices of the mayor premiss and the minor premiss should be disjoint.
- In an application of conditionalization exactly one *numerical* assumption is withdrawn.

## 5 A Fragment of English

In this section we introduce a fragment of English for which we shall define a CNL. The expressions of the fragment are the derivations which LG generates by using the initial statements as assumptions. The choice of this first-order fragment is motivated by the following consideration. We want to show that one may give an explanation of simple items of natural reasoning by using grammatical form as vehicle of inference. It seems to us that this aim can be achieved most convincingly by explaining reasonings for which we have a well understood theory, namely first-order logic.

Vocabulary and Initial Assignments

**Determiners** *every, not every, some, a, no* $\in ((e,t),((e,t),t))$

**Proper Names** *abelard, heloise, john, mary, etc* $\in e$

**Discourse Referents** $u_1, \ldots, u_n, \ldots \in e$

**Pronouns** *she, he, herself, he, him, himself, it* $\in e$

**Intransitive Verbs** *dance, wander, etc* $\in (e,t)$

**Transitive Verbs** *love, like, is, etc* $\in ((e,(e,t))$

**Common Nouns** *woman, man, philosopher, etc* $\in (e,t)$

**Extensional Adverbs** *loudly, passionately, beautifully, etc* $\in ((e,t),(e,t)$

[1] A formal definition of LG is possible and already given in [Sánchez 1991]. In this report we skip the formal details.

**Absolute Adjectives** *male*, *female*, *human*, *etc* $\in ((e,t),(e,t))$

**Booleans**

- $if \in (t,(t,t))$
- *and*, *or* $\in (a,(a,a))$, where $a$ is any category ending in $t$
- *not* $\in (t,t)$
- *doesn't* $\in ((e,t),(e,t))$

EXPRESSIONS OF THE FRAGMENT In the following definition we assume the notions of *derivation* and *open assumptions* used in natural deduction calculi.

a. The vocabulary is part of the expressions of the fragment.

b. If each of $X_1 \in a, \ldots, X_n \in a$ is an expression of the fragment, and there is a LG derivation $D$ ending in $b$ containing these assignments among its open assumptions, then $X_1, \ldots, X_n \in b$ is also in the fragment. This derivation is called the analysis of the string $X_1, \ldots, X_n$ .

**Example 4**

*a. every man dances* $\in t$

$$\dfrac{\dfrac{\overset{every}{((e,t),((e,t),t))} \quad \overset{man}{(e,t)}}{((e,t),t)} \qquad \overset{dances}{(e,t)}}{t}$$

*b.* every man loves heloise $\in$ t

$$\dfrac{\overset{every\ man}{((e,t),t)} \qquad \dfrac{\overset{loves}{(e,(e,t))} \quad \overset{heloise}{e}}{((e,t),t)}}{t}$$

*c.* every man loves a woman $\in$ t

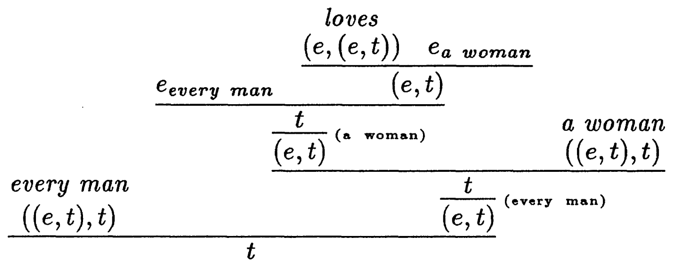
$$\dfrac{\dfrac{\dfrac{\overset{every\ man}{((e,t),t)} \qquad \dfrac{\overset{loves}{(e,(e,t))} \quad e^1}{(e,t)}}{t}}{(e,t)}{\scriptstyle(1)} \qquad \overset{a\ woman}{((e,t),t)}}{t}$$

In Example 4 c. *loves* is not combined with a lexical hypothesis, but with the numerical assumption $e^1$. The withdrawal of this assumption is followed immediately by the application of the lexical hypothesis *a woman*. We will define the numerical assumptions related with lexical assumptions in the described way as:

**shadow assumptions** The numerical assumption $e^i$ is a shadow assumption of the lexical assumption z iff the withdrawal of $e^i$ is followed by the use of z as the major of a Modus Ponens application.

In Example 5. below we show an analysis in which we use a self-explanatory notation for the shadow assumptions:

**Example 5** every man loves a woman $\in$ t




The expression *every man loves a woman* corresponds to two non-equivalent Lambek terms, namely:

- $Z_{a\ woman}\lambda X_e.\ V_{every\ man}(lovesX)$ .
- $V_{every\ man}\lambda Y_e.\ Z_{a\ woman}\lambda X_e.\ (lovesXY)$ .

We then see that the Lambek Grammar can represent the different readings of the above string at the level of the syntax: the two readings indeed have two different derivations.

# Chapter III

# Monotonicity and Scope Marking, Substitution and Negation

## 1 Monotonicity Marking in LG

We have noticed that in the lambda calculus positive occurrences are monotone sensitive positions. In this section we describe a mechanism that transfers the monotonicity information to LG objects. Let us give the motivation behind the mechanism to be described presently. Through the correspondence with the Lambek terms, we know that the major of a Modus Ponens application corresponds to the head of a Lambek terms. This head is upward monotone. The argument of an upward monotone function occurs in an upward monotone sensitive position. Similarly, the argument of a downward monotone function occurs in a downward monotone sensitive position. The introduction of abstraction does not alter the monotonicity of the terms occurring in the body of the abstraction.

### 1.1 Internal Monotonicity Marking

Before presenting the mechanism of monotonicity marking, we introduce a notation which allows us to use the assignments of expressions to categories to produce the intended interpretations of natural language expressions:

- If $a$ and $b$ are categories, then $(a^+, b)$, $(a^-, b)$ and $(a, b)$ are categories.
- If $A \in (a^+, b)$ then the interpretation of the meaning of $A$ is an upward monotone function in $D_{(a,b)}$.
- If $A \in (a^-, b)$ then the interpretation of the meaning of $A$ is a downward monotone function in $D_{(a,b)}$.
- If $A \in (a, b)$ then the interpretation of the meaning of $A$ is an arbitrary function in $D_{(a,b)}$.

Lexical Monotoncity As we observed in section 1.1 we know that the denotations of determiners may have monotone properties. We shall make this information available at the level of the Lambek derivations. Since, for instance, *every* is downward monotone in its first argument and upward monotone in its second argument, we shall not always say that its category is $((e,t),((e,t),t))$. Instead, we shall sometimes use a category with internal monotonicity marking: $((e,t)^-,((e,t)^+,t))$. This example will be extended to cover expressions which play an important role in the construction of CNL:

**Determiners**

- *every* $\in ((e,t)^-,((e,t)^+,t))$ .
- *a*, *some* $\in ((e,t)^+,((e,t)^+,t))$ .
- *no* $\in ((e,t)^-,((e,t)^-,t))$ .
- *not every* $\in ((e,t)^+,((e,t)^-,t))$ .

**Verbs**

- $X \in (e^+, t)$ where $X$ is an intransitive verb.
- $X \in (e^+, (e^+, t))$ where $X$ is an extensional transitive verb.

**Example 6**

*a.*

$$\dfrac{\begin{array}{c} \textit{every} \\ ((e,t)^-,((e,t)^+,t)) \\ + \end{array} \quad \begin{array}{c} \textit{woman} \\ (e,t) \\ - \end{array}}{((e,t)^+,t)}$$

*b.*

$$\dfrac{\begin{array}{c} \textit{a} \\ ((e,t)^+,((e,t)^+,t)) \\ + \end{array} \quad \begin{array}{c} \textit{woman} \\ (e,t) \\ + \end{array}}{((e,t)^+,t)}$$

*c.*

$$\dfrac{\begin{array}{c} \textit{no} \\ ((e,t)^-,((e,t)^-,t)) \\ + \end{array} \quad \begin{array}{c} \textit{woman} \\ (e,t) \\ - \end{array}}{((e,t)^-,t)}$$

*d.*

$$\dfrac{\begin{array}{c} \textit{not every} \\ ((e,t)^+,((e,t)^-,t)) \\ + \end{array} \quad \begin{array}{c} \textit{woman} \\ (e,t) \\ + \end{array}}{((e,t)^-,t)}$$

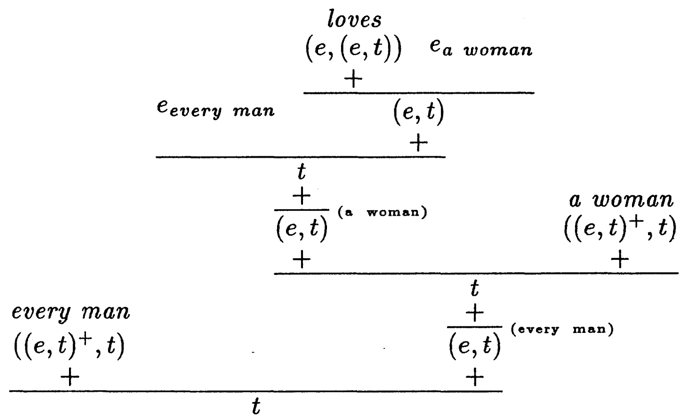
*e.*

$$\dfrac{\begin{array}{c} \textit{every man} \\ ((e,t)^+,t) \\ + \end{array} \quad \dfrac{\begin{array}{c} \textit{loves} \\ (e,(e,t)) \\ + \end{array} \quad \begin{array}{c} \textit{heloise} \\ e \end{array}}{((e,t)^+,t)}}{t}$$

*f.*

$$\dfrac{\begin{array}{c} \dfrac{\begin{array}{c} t \\ + \end{array}}{(e,t)}\,^{(1)} \\ + \end{array} \quad \begin{array}{c} \textit{a woman} \\ ((e,t)^+,t) \\ + \end{array}}{t}
\qquad\text{where}\qquad
t = \dfrac{\begin{array}{c} \textit{every man} \\ ((e,t)^+,t) \\ + \end{array} \quad \begin{array}{c} \dfrac{\begin{array}{c} \textit{loves} \\ (e,(e,t)) \\ + \end{array} \quad e^1}{(e,t)} \\ + \end{array}}{t}$$

*g.*




## 1.2 Polarity of Assumptions and Indices

In the previous subsection we have developed a mechanism that allows us to transmit monotonicity information through the Lambek derivations. As a result lexical items and constituents may occur with monotonicity marking according to the conventions to be introduced presently. Let $D$ be a derivation with conclusion $a$ . Then,

- A node $b$ has polarity iff all nodes in the path from $b$ to $a$ are marked.
- A node $b$ is positive iff $b$ has polarity, and the number of nodes marked by $-$ is even.
- A node $b$ is negative iff $b$ has polarity, and the number of nodes marked by $-$ is odd.
- A lexical index $X$ is positive(negative) iff the node of which it is an index is positive (negative).
- A subderivation $D_1$ with conclusion $c$ is positive (negative) if $c$ itself is positive (negative).
- A string $F$ of lexical indices is positive (negative) iff the subderivation which depends exactly on the members of $F$ is positive (negative).

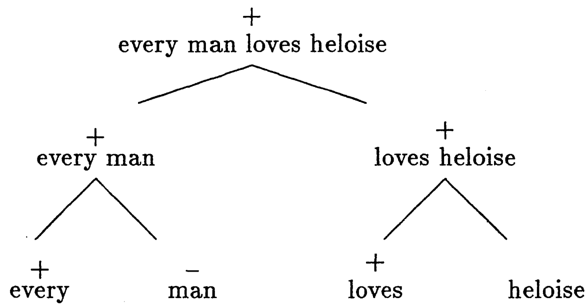
The definition of polarity of subderivations is intended to warrant the following proposition proven in [Sánchez 1991]:

> Let $D$ be a derivation with conclusion $a$ and let $N_a$ be its its meaning. Furthermore, let $D_1$ be a subderivation of $D$ with conclusion $b$ and with $M_b$ as its meaning. Then,
>
> - If $D_1$ is positive (negative) in D, then $N_a$ is upward (downward) monotone in $M_b$.

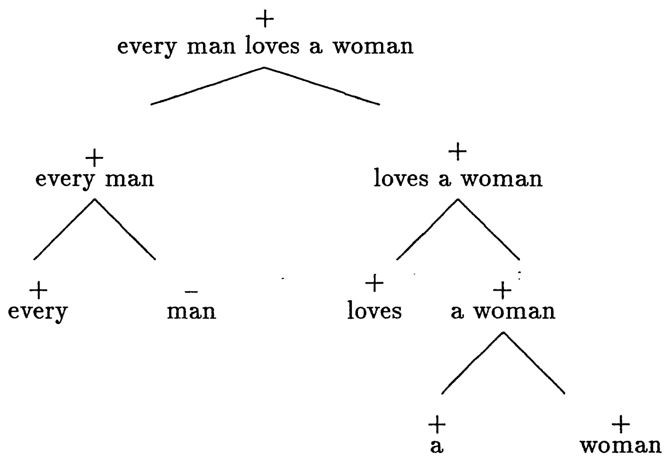
The monotonicity marking of English expressions made available by the above conventions, can be visualized in the following way:

Illustration

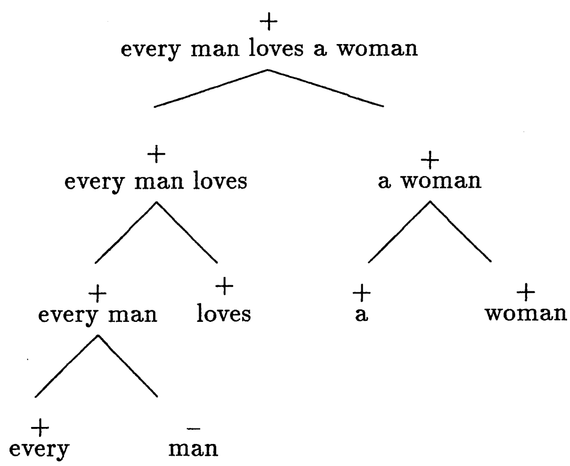
a.




b.




c.



Sometimes we are interested only in partial monotonicity information. For instance, given the string *every man loves a woman* and the marking represented by Example 6 f. we may be interested in representing the fact that *woman* is positive. This will be done by means of the following predictable notation:

$$every\ man\ loves\ a\ woman^{+}.$$

## 2 Scope Marking

As the Examples 6 f. and g. show, monotonicity marking does not distinguish between the two prominent readings of the ambiguous sentence *every man loves a woman.* However, the Lambek derivations we have associated with this sentence are plainly different. In Example 6 f. the assumption corresponding to *every man* has been processed before the assumption corresponding to *a woman.* In the other derivation the order has been reversed. The importance of this difference in processing order has not only semantical importance. In the construction of CNL we shall make use of this difference in formulating inference rules which apply only to the expression which has been last introduced. To this end we introduce the notion of

**wide scope**

Let $D$ be a Lambek derivation of a string $F$ containing the substring $Y$. We shall say that $Y$ has wide scope in $F$ if the last rule applied in $D$ is Modus Ponens and the mayor of this application of Modus Ponens depends exactly on the members of $Y$.
Notation: $F(Y^{\#})$.

**Example 7**

a.

The sentence *every man loves a woman* associated with Example 4 c. has as a scope representation:

$$every\ man\ loves\ (a\ woman)^{\#}\ .$$

b.

The sentence *every man loves a woman* associated with derivation Example 5. has as a scope representation:

$$(every\ man)^{\#}\ loves\ a\ woman\ .$$

Notice that according to the definitions, if an expression has wide scope then it occurs positively, i. e. $F(Y^{\#})$ entails $F(Y^{+})$ .

## 3 Substitution

**Substitution of (e, t)-expressions**

Let $X, Y \in (e,t)$ and assume that $F(X)$ is an expression associated with a derivation $D$. Then $F(Y)$ will be the expression associated with a derivation $D$ which results from $D$ by substituting the left derivation by the right one:

$$\begin{array}{ccc}[X] & \qquad & [Y]\\ D_1 & & D_2\\ (e,t) & & (e,t)\end{array}$$

**Example 8**

a.

Let $F(man)$ be the string *every man dances* associated with Example 4 a. Then $F(small\ man)$ is the string *every small man dances* with derivation:

$$\dfrac{\dfrac{\overset{\text{every}}{((e,t),((e,t),t))} \quad \dfrac{\overset{\text{small}}{((e,t),(e,t))} \quad \overset{\text{man}}{(e,t)}}{(e,t)}}{((e,t),t)} \qquad \overset{\text{dances}}{(e,t)}}{t}$$

b.

Let $F(dances)$ be the string *every man dances* associated with Example 4 a. Then $F(dances\ beautifully)$ is the string *every small man dances* with derivation:

$$\dfrac{\dfrac{\overset{\text{every}}{((e,t),((e,t),t))} \quad \overset{\text{man}}{(e,t)}}{((e,t),t)} \quad \dfrac{\overset{\text{dances}}{(e,t)} \quad \overset{\text{beautifully}}{((e,t),(e,t))}}{(e,t)}}{t}$$

Observe that given $F(X)$ and its Lambek derivation, the derivation associated with $F(Y)$ is unambiguously determined.

**Substitution of ((e,t), t)–expressions**

Let $u \in e$, $X \in ((e,t),t)$. Assume further that $F(X)$ is an expression associated with derivation $D$. Then $F(u)$ will be the expression associated with derivation $D'$ which results from $D$ by

- deleting the assumption indexed by $X$.
- using the assumption indexed by $u$ instead of the shadow of $X$ if it has one or,
- using the assumption indexed by $u$ instead of $X$ itself if it has no shadow.

**Example 9**

a.

Let $F(a\ woman)$ be the string *every man loves a woman* associated with Example 4 c. Then $F(heloise)$ is the string *every man loves heloise* associated with Example 4 b.

b.

Let $F(every\ man)$ be the string *every man loves a woman* associated with Example 4 c. Then $F(abelard)$ is the string *abelard loves* woman associated with the derivation below:

$$\dfrac{\dfrac{\dfrac{\overset{\text{abelard}}{e} \quad \dfrac{\overset{\text{loves}}{(e,(e,t))} \quad e_{a\ woman}}{(e,t)}}{t}}{(e,t)}\ {\scriptstyle(\text{a woman})} \qquad \overset{\text{a woman}}{((e,t)^{+},t)}}{t}$$

# 4 Negation

In this section we extend the definition of negation used in the so-called *square of opposition.*

a.

Let $F[(every\ Y)^{\#}]$ be an expression associated with a derivation $D$.
By $Neg(F[(every\ Y)^{\#}])$ we denote the expression associated with the derivation $D'$ which results from replacing the left derivation in $D$ by the right one:

| | |
|---|---|
| [every Y] | [not every Y] |
| $D_1$ | $D_2$ |
| $((e,t),t)$ | $((e,t),t)$ |

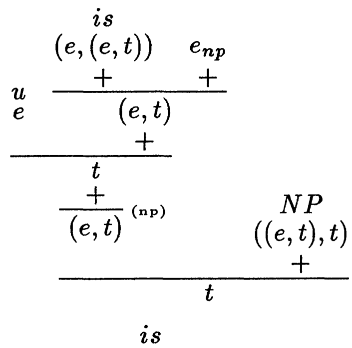
b.

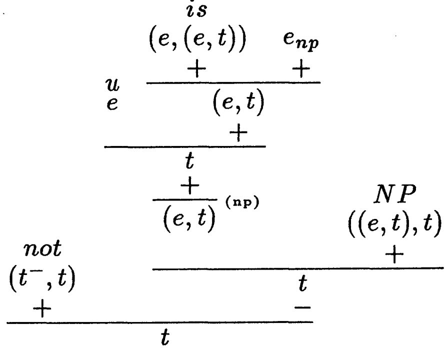
if $F[(someY)^{\#}]$ is an expression associated with a derivation $D$, then by $Neg(F[(someY)^{\#}])$ we denote the expression associated with the derivation $D'$ which results from replacing in the left derivation in $D$ by the right one:

| | |
|---|---|
| [some Y] | [no Y] |
| $D_1$ | $D_2$ |
| $((e,t),t)$ | $((e,t),t)$ |

c.

Let $u\ is\ NP \in t$ be associated with the first derivation below. Then $Neg(u\ is\ a\ NP)$ will be the string $u\ is\ not\ NP$. This string will be associated with the second derivation below:




d.

Let $u\ VP \in t$ be associated with the first derivation below. Then $Neg(u\ X)$ will be the string $u\ doesn't\ VP$. This string will be associated with the second derivation below:

$$\frac{\overset{u}{e} \quad \underset{+}{\overset{VP}{(e,t)}}}{t}$$

$$\frac{\overset{u}{e} \quad \underset{+}{\dfrac{\underset{+}{\overset{doesn't}{((e,t)^{-},(e,t))}} \quad \underset{-}{\overset{VP}{(e,t)}}}{(e,t)}}}{t}$$

**Example 10**

a.

Given *every man loves (a woman)*# with the derivation of Example 4 c. *Neg(every man loves (a woman)*#*)* is the expression *every man loves (no woman)*# associated with the tree below:

$$\dfrac{\dfrac{\dfrac{\overset{\text{every man}}{((e,t),t)} \quad \dfrac{\overset{\text{loves}}{(e,(e,t))} \quad e^{1}}{(e,t)}}{t}}{(e,t)}{\scriptstyle (1)} \qquad \overset{\text{no woman}}{((e,t),t)}}{t}$$

b.

Given *(every man)*# *loves a woman* with the derivation of Example 5. *Neg((every man)*# *loves a woman)* is the expression *(not every man)*# *loves a woman* associated with the tree below:

$$\dfrac{\overset{\text{not every man}}{((e,t)^{+},t)} \qquad \dfrac{\dfrac{\dfrac{\dfrac{e_{every\ man} \quad \dfrac{\overset{\text{loves}}{(e,(e,t))} \quad e_{a\ woman}}{(e,t)}}{t}}{(e,t)}{\scriptstyle (\text{a woman})} \qquad \overset{\text{a woman}}{((e,t),t)}}{t}}{(e,t)}{\scriptstyle (\text{not every man})}}{t}$$

# Chapter IV

# A Categorial Natural Logic

## 1 Natural Reasoning with Generalized Quantifiers

In this section we present the monotonicity part of our CLG. The basic idea consist in providing a proof of the sequent $A_1, \ldots, A_n/B$ by using substitution of expressions of type $(e,t)$. The members of the sequent enter in the proof provided with a Lambek analysis indicating monotonicity and scope marking. We then make use of this information in the use of the inference rules. After the application of an inference rule, the conclusion will also be associated with a Lambek derivation providing monotonicity and scope marking information. The proof system for natural logic developed in this section is similar to Beth's deductive tableaus. A reasoning will be represented by the sequent $A_1, \ldots, A_n/B$. This sequent poses the problem of finding a derivation of the conclusion $B$ from the premises $A_1, \ldots, A_n$. The first step in the construction of the derivation consists in the construction of a Lambek analysis, $D$ , for the premises and the conclusion. This analysis is fixed during the derivation. Then we put $A_1, \ldots, A_n \bullet B$ as the top node of a finite tree of points such that for each node holds that it is either closed or not closed and it has exactly one successor according to the following rules:

RULES

**Marking**

$$\begin{array}{lcl} X & \bullet & A,\ Y \\ X & \bullet & A^*,\ Y \end{array} \qquad \begin{array}{lcl} A,\ X & \bullet & Y \\ A^*,\ X & \bullet & Y \end{array}$$

where $A^*$ is a monotonicity or scope marking of A provided by the fixed Lambek analysis of A.

**Negation**

$$\begin{array}{lcl} X\ Neg(A) & \bullet & Y \\ X & \bullet & A,\ Y \end{array} \qquad \begin{array}{lcl} X & \bullet & Neg(A),\ Y \\ A,\ X & \bullet & Y \end{array}$$

**Monotonicity**

$$\begin{array}{lcl} (every\ x)^{\#}\ is\ a\ y,\ F(x^+),\ X & \bullet & Y \\ (every\ x)^{\#}\ is\ a\ y,\ F(y),\ X & \bullet & Y \end{array}$$

**Conversion**

$$\begin{array}{lcl} (some\ y)^{\#}\ is\ a\ x,\ X & \bullet & Y \\ (some\ x)^{\#}\ is\ a\ y,\ X & \bullet & Y \end{array}$$

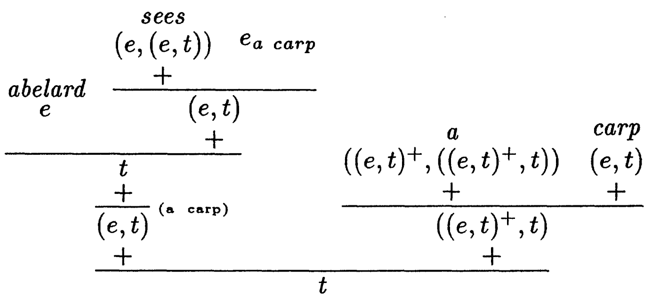
We assume that the X, Y in these rules are sets of expressions. The result of the application of a rule other than the marking rule, is a sentence unambiguously associated with a Lambek derivation D. This new derivation can be used as new input to the marking rule. A node is closed if the same formula with identical scope marking occurs to its left and right. Notation: $A,\ X\ \underset{=}{\bullet}\ A,\ Y$ .
A closed tableau is a tableau in which all nodes without successor are closed.
A proof for the sequent $A_1, \ldots, A_n/B$ is a closed tableau with $A_1, \ldots, A_n \ \bullet B$ as top node.
If we have a proof for the sequent $A_1, \ldots, A_n/B$ then we say that it is valid modulo the Lambek analyses provided in the first step of the derivation.

## 1.1 Examples of Reasoning with Generalized Quantifiers

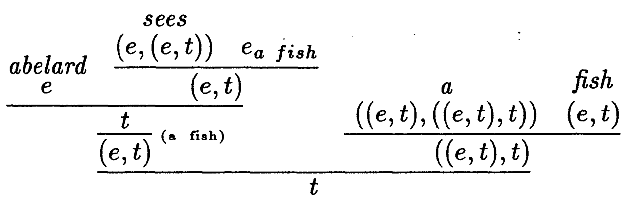
**Inference 1** *abelard sees a carp, every carp is a fish / abelard sees a fish* .

**Analyses 1**

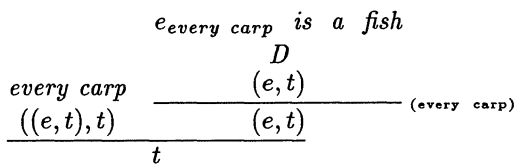
*a.*



*b.*



*c.*



**Proof 1**

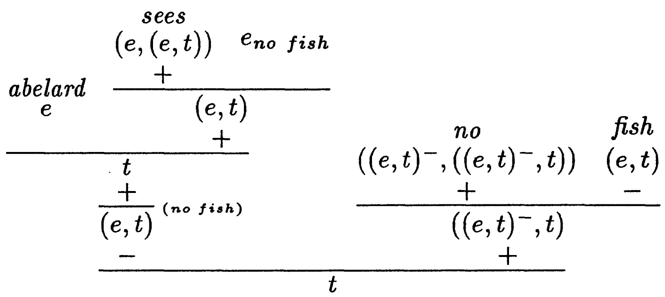
| | | |
|---|---|---|
| *abelard sees a carp, every carp is a fish* | $\bullet$ *abelard sees a fish* | |
| *abelard sees a carp, every carp is a fish* | $\bullet$ *abelard sees* $(a\ fish)^{\#}$ | marking |
| *abelard sees* $(a\ carp^{+})^{\#}$, *every carp is a fish* | $\bullet$ *abelard sees* $(a\ fish)^{\#}$ | marking |
| *abelard sees* $(a\ carp^{+})^{\#}$, $(every\ carp)^{\#}$ *is a fish* | $\bullet$ *abelard sees* $(a\ fish)^{\#}$ | marking |
| *abelard sees* $(a\ fish)^{\#}$, $(every\ carp)^{\#}$ *is a fish* | $\underset{=}{\bullet}$ *abelard sees* $(a\ fish)^{\#}$ | monotonicity |

Tableaus like the above can be simplified in several ways. Self-evident abbreviations will be introduced without explicit warning.

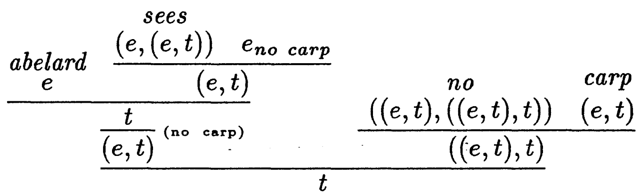
**Inference 2** *abelard sees no fish, every carp is a fish / abelard sees no carp.*

**Analyses 2**

*a.*



*b.*



*c. Analysis 1 c.*

**Proof 2**

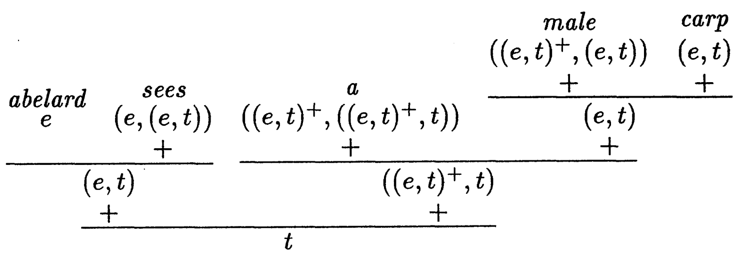
| | | |
|---|---|---|
| *abelard sees no fish, every carp is a fish* | *abelard sees no carp* | |
| *abelard sees (no fish)*$^{\#}$*, every carp is a fish* | *abelard sees (no carp)*$^{\#}$ | marking |
| *abelard sees (a carp)*$^{\#}$*, (every carp)*$^{\#}$ *is a fish* | *abelard sees (a fish)*$^{\#}$ | negation twice |

COMMENT Notice that given the derivation of $NEG[F(a\ X)^{\#}]$, the derivation of $F(a\ X)^{\#}$ is unambiguously determined. In the present case, we obtain the Analyses 1 a. and b. of the previous inference. Hence we have reduced the initial problem to the problem resolved in the previous example. That is the reason why we may close the tableau at this stage.

**Inference 3** *abelard sees a male carp, every carp is a fish/abelard sees a male fish*

**Analyses 3**

*a.*



*b.*

$$\dfrac{\dfrac{\overset{abelard}{e}\quad \overset{sees}{(e,(e,t))}}{(e,t)}\qquad \dfrac{\overset{a}{((e,t),((e,t),t))}\quad \dfrac{\overset{male}{((e,t),(e,t))}\quad \overset{fish}{(e,t)}}{(e,t)}}{((e,t),t)}}{t}$$

*c. Analysis 1 c.*

**Proof 3**

| | |
|---|---|
| *abelard sees a male carp, every carp is a fish* | *abelard sees a male fish* |
| *abelard sees a* $(male\ carp^{+})^{\#}$, $(every\ carp)^{\#}$ *is a fish* | *abelard sees a* $(male\ fish)^{\#}$ |
| *abelard sees a* $(male\ fish)^{\#}$ | = |

REMARK CNL in its present form can not cope with elementary syllogistic inferences in which slightly different grammatical forms are involved, for instance

every fish swims, bobo is a fish/ bobo swims

is a valid syllogism which CNL can not recognize yet. There are several alternative solutions to this problem. The one we advocate here consists of using a new formulation of upward monotonicity:

| | |
|---|---|
| $(every\ x)^{\#}\ VP,\ F[(is\ a\ x)^{+}],\ X$ | $Y$ |
| $(every\ x)^{\#}\ VP,\ F(VP),\ X$ | $Y$ |

The explanation of the reasoning mentioned now takes the following form:

**Inference 4**

**Analyses 4**

*a.*

$$\dfrac{\underset{+}{\dfrac{\underset{+}{\dfrac{\overset{bobo}{e}\quad \dfrac{\underset{+}{\overset{is}{(e,(e,t))}}\quad e_{a\ fish}}{\underset{+}{(e,t)}}}{t}}}{(e,t)}}\ {\scriptstyle (a\ fish)}\qquad \dfrac{\overset{a\ fish}{((e,t)^{+},t)}}{+}}{t}$$

*b.*

$$\dfrac{\overset{every\ fish}{((e,t),t)}\quad \overset{swims}{(e,t)}}{t}$$

*c.*

$$\frac{\begin{matrix} bobo \\ e \end{matrix} \quad \begin{matrix} swims \\ (e,t) \end{matrix}}{t}$$

**Proof 4**

| | |
|---|---|
| *every fish swims, bobo is a fish* | *bobo swims* |
| *(every fish)# swims, bobo (is a fish)+* | *bobo (swims)#* |
| *bobo swims* | |
| *bobo (swims)#* | |

TRADITIONAL SYLLOGISTIC The above examples of natural reasoning should have illuminated the way in which the monotonicity part of CNL works. Next we show that the defined CNL can explain traditional syllogistic inference. We shall follow Aristotles' own strategy. We show that all the syllogisms of the first figure are derivable from monotonicity, negation and conversion. Since all the other syllogisms are reducible to the syllogisms of the first figure, we have proven that CNL generates all traditional syllogisms.

**Inference 5**

**Analyses 5**

*a.*

$$\frac{\dfrac{\begin{matrix} every \\ ((e,t)^-,((e,t)^+,t)) \\ + \end{matrix} \quad \begin{matrix} x \\ (e,t) \\ - \end{matrix}}{\begin{matrix} ((e,t)^+,t) \\ + \end{matrix}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\begin{matrix} [e_{every\ x}\ is\ e_{a\ y}] \\ D \\ t \end{matrix}}{(e,t)}\,{\scriptstyle (a\ y)} \quad \dfrac{\begin{matrix} a \\ ((e,t)^+,((e,t)^+,t)) \\ + \end{matrix} \quad \begin{matrix} y \\ (e,t) \\ + \end{matrix}}{\begin{matrix} ((e,t)^+,t) \\ + \end{matrix}}}{\begin{matrix} t \\ + \end{matrix}}}{\begin{matrix} (e,t) \\ + \end{matrix}}\,{\scriptstyle (every\ x)}}{}}{t}$$

*b.*

$$\frac{\dfrac{\begin{matrix} some \\ ((e,t)^+,((e,t)^+,t)) \\ + \end{matrix} \quad \begin{matrix} x \\ (e,t) \\ + \end{matrix}}{\begin{matrix} ((e,t)^+,t) \\ + \end{matrix}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\begin{matrix} [e_{some\ x}\ is\ e_{a\ y}] \\ D \\ t \end{matrix}}{(e,t)}\,{\scriptstyle (a\ y)} \quad \dfrac{\begin{matrix} a \\ ((e,t)^+,((e,t)^+,t)) \\ + \end{matrix} \quad \begin{matrix} y \\ (e,t) \\ + \end{matrix}}{\begin{matrix} ((e,t)^+,t) \\ + \end{matrix}}}{\begin{matrix} t \\ + \end{matrix}}}{\begin{matrix} (e,t) \\ + \end{matrix}}\,{\scriptstyle (some\ x)}}{}}{t}$$

*c.*

$$\dfrac{\dfrac{\overset{no}{((e,t),((e,t),t))} \quad \overset{x}{(e,t)}}{((e,t),t)} \qquad \dfrac{\dfrac{\dfrac{\dfrac{[e_{no\ x}\ is\ e_{a\ y}]}{\overset{D}{t}}}{(e,t)}{\scriptstyle (a\ y)} \quad \dfrac{\overset{a}{((e,t),((e,t),t))} \quad \overset{y}{(e,t)}}{((e,t),t)}}{t}}{(e,t)}{\scriptstyle (no\ x)}}{t}$$

*d.*

$$\dfrac{\dfrac{\overset{not\ every}{((e,t),((e,t),t))} \quad \overset{x}{(e,t)}}{((e,t),t)} \qquad \dfrac{\dfrac{\dfrac{\dfrac{[e_{no\ x}\ is\ e_{a\ y}]}{\overset{D}{t}}}{(e,t)}{\scriptstyle (a\ y)} \quad \dfrac{\overset{a}{((e,t),((e,t),t))} \quad \overset{y}{(e,t)}}{((e,t),t)}}{t}}{(e,t)}{\scriptstyle (not\ every\ x)}}{t}$$

**Proof 5** *every M is a P, every S is a M/ every S is a P (barbara).*

| | | |
|---|---|---|
| *every M is a P, every S is a P* | *every S is a P* | |
| *(every M)*$^{\#}$ *is a P, (every S)*$^{\#}$ *is a* $M^{+}$ | *(every S)*$^{\#}$ *is a P* | marking |
| *(every M)*$^{\#}$ *is a P, (every S)*$^{\#}$ *is a P* | *(every S)*$^{\#}$ *is a P* | monotonicity |

**Proof 6** *every M is a P, some S is a M/ some S is a P (darii).*

| | | |
|---|---|---|
| *every M is a P, some S is a P* | *some S is a P* | |
| *(every M)*$^{\#}$ *is a P, (some S)*$^{\#}$ *is a* $M^{+}$ | *(some S)*$^{\#}$ *is a P* | marking |
| *(every M)*$^{\#}$ *is a P, (some S)*$^{\#}$ *is a P* | *(some S)*$^{\#}$ *is a P* | monotonicity |

**Proof 7** *no M is a P, every S is a M/ no S is a P (celarent).*

| | | |
|---|---|---|
| *no M is a P, every S is a M* | *no S is a P* | |
| *(no M)*$^{\#}$ *is a P, (every S)*$^{\#}$ *is a M* | *(no S)*$^{\#}$ *is a P* | marking |
| *(every S)*$^{\#}$ *is a M, (some S)*$^{\#}$ *is a P* | *(some M)*$^{\#}$ *is a P* | negation |
| *(every S)*$^{\#}$ *is a M, (some* $S^{+}$*)*$^{\#}$ *is a P* | *(some M)*$^{\#}$ *is a P* | marking |
| *(every S)*$^{\#}$ *is a M, (some M)*$^{\#}$ *is a P* | *(some M)*$^{\#}$ *is a P* | monotonicity |

**Proof 8** *no M is a P, some S is a M/ not every S is a P (ferio).*

| | | |
|---|---|---|
| *no M is a P, some S is a M* | *not every S is a P* | |
| *(no M)*$^{\#}$ *is a P, (some S)*$^{\#}$ *is a M* | *(not every S)*$^{\#}$ *is a P* | marking |
| *(some S)*$^{\#}$ *is a M, (every S)*$^{\#}$ *is a P* | *(some M)*$^{\#}$ *is a P* | negation |
| *(some* $S^{+}$*)*$^{\#}$ *is a M, (every S)*$^{\#}$ *is a P* | *(some M)*$^{\#}$ *is a P* | marking |
| *(some P)*$^{\#}$ *is a M, (every S)*$^{\#}$ *is a P* | *(some M)*$^{\#}$ *is a P* | monotonicity |
| *(some M)*$^{\#}$ *is a P, (every S)*$^{\#}$ *is a P* | *(some M)*$^{\#}$ *is a P* | conversion |

# 2 Reasoning with Discourse Referents

Until now we have been working with a Natural Logic in which proper names do not play a relevant role in the derivations. This situation will be changed in the present section. We shall show that CNL can be used to explain inferences between different readings of one and the same string. We claim that the incorporation of Kamp's treatment of indefinite descriptions to CNL allows us to model this kind of inference. Moreover, we shall also show that by accommodating Kamp's proposal into CNL we are able to give a natural explanation of several other inferences as well. As a matter of fact, CNL now takes the form in which one gives intuitive explanations of Beth's tableaus. Not surprisingly the generative strength of CNL is enhanced considerably. We assume the marking and negation rules introduced in Section 1. Next, we add the following monotonicity and discourse referent rules to the system:

Monotonicity Rules

| | | | |
|---|---|---|---|
| $F(every\ x)^+,\ u\ is\ a\ x,\ X$ | $Y$ | $u\ is\ a\ x,\ X$ | $F(a\ x)^+$ |
| $F(every\ x)^+,\ F(u),\ X$ | $Y$ | $X$ | $F(a\ x)^+,\ F(u)$ |

Discourse Referent Rules

| | | | |
|---|---|---|---|
| $F(a\ x)^\#,\ X$ | $Y$ | $X$ | $F(every\ x)^\#$ |
| $u\ is\ a\ x,\ F(u),\ X$ | $Y$ | $X,\ u\ is\ a\ x$ | $F(u)$ |

comment Remember that if $F(NP)$ is associated with a derivation $D$, then the tree associated with $F(u)$ can be determined unambiguously. We shall assume, clumsily, that discourse referent rules do not apply to expressions of the form *u is an X* when $X$ is a lexical common noun. Otherwise we would be captured in an infinite tableau.

## 2.1 Examples of Reasoning with Discourse Referents

**Inference 6** *bobo swims, bobo is a fish / a fish swims*

**Analyses 6**

*a.*

$$\frac{\underset{((e,t),t)}{a\ fish}\quad \underset{(e,t)}{swims}}{t}\ +$$

*b. Analyses 4 a. and c.*

**Proof 9**

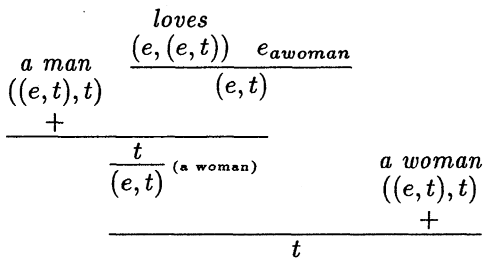
| | | |
|---|---|---|
| *bobo is a fish, bobo swims* | *a fish swims* | marking |
| | $(a\ fish)^+$ *swims* | marking |
| | *bobo swims* ═ | monotonicity |

**Inference 7** *a man loves a woman/a man loves a woman.*

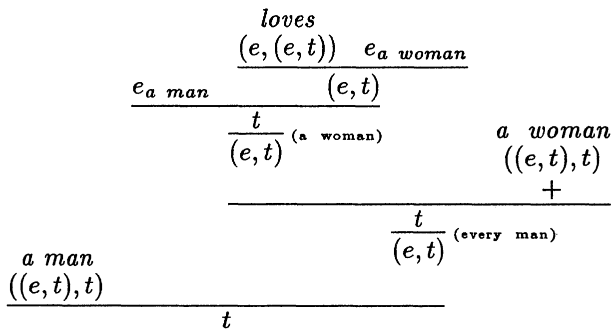
This sequent has a trivial proof: provide the premiss and the conclusion with the same analysis. But it has also a less trivial proof when we associate premiss and conclusion with different analyses. The use of the discourse referents allows us to show that CNL can give a proof of the less trivial case.

**Analyses 7**

*a.*




*b.*




We shall associate the premiss with Analysis 7 a. and the conclusion with Analysis 7 b.

**Proof 10**

| | | |
|---|---|---|
| *a man loves a woman* | *a man loves a woman* | |
| *a man loves (a woman)*$^{\#}$ | *(a man)*$^{+}$ *loves a woman* | marking |
| *u is a woman, a man loves u* | | discourse referent |
| *(a man)*$^{\#}$ *loves u* | | marking |
| *v is a man, v loves u* | | discourse referent |
| | *v loves a woman* | monotonicity |
| | *v loves (a woman)*$^{+}$ | marking |
| | *v loves u* | monotonicity |

COMMENT The introduction of the first discourse referent yields the string *a man loves u*. This string is associated with Analysis 8 a. below. This derivation gives us new scope information which is used in the subsequent marking. The new application of the discourse referent rule yields the string *v loves u* associated with Analysis 8 b. Similarly, the first application of the monotonicity rule yields the string *v loves a woman* associated with Analysis 8 c. below. From this last derivation we extract the new monotonicity marking used in the continuation of the proof. The result is the string *v loves u* associated with Analysis 8 b.

**Analyses 8**

*a.*

$$\dfrac{\underset{+}{\overset{a\ man}{((e,t),t)}} \qquad \dfrac{\overset{loves}{(e,(e,t))} \quad \overset{u}{e}}{(e,t)}}{t}$$

*b.*

$$\dfrac{\overset{v}{e} \qquad \dfrac{\overset{loves}{(e,(e,t))} \quad \overset{u}{e}}{(e,t)}}{t}$$

*c.*

$$\dfrac{\dfrac{\dfrac{\overset{v}{e} \qquad \dfrac{\overset{loves}{(e,(e,t))} \quad e_{a\ woman}}{(e,t)}}{t}}{(e,t)}\,{\scriptstyle (\text{a woman})} \qquad \underset{+}{\overset{a\ woman}{((e,t),t)}}}{t}$$

**Inference 8** *every man loves a woman/every man loves a woman*

**Analyses 9**
*Examples 6 f. and g.*

**Proof 11**

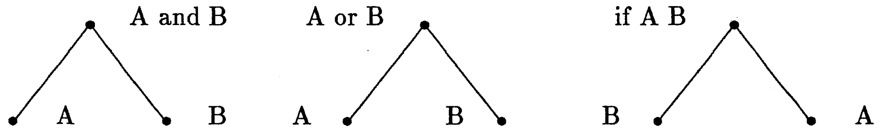
| | | |
|---|---|---|
| *every man loves a woman* | *every man loves a woman* | |
| *every man loves (a woman)*$^{\#}$ | *(every man)*$^{\#}$ *loves a woman* | marking |
| *v is a woman, every man loves v* | | discourse referent |
| *u is a man* | *u loves a woman* | discourse referent |
| | *u loves (a woman)*$^{+}$ | marking |
| | *u loves v* | monotonicity |
| *(every man)*$^{+}$ *loves v* | | marking |
| *u loves v* | | monotonicity |
| = | | |

We think that the previous examples should have convinced the reader of the viability of CNL as a model of natural reasoning. In the next section we shall consider an extension which makes CNL more flexible.

## 3 Reasoning with Boolean Connectives

A natural extension of CNL consists in the incorporation of Boolean connectives to the fragment. The strategy we follow is a combination of the generalized conjunction and disjunction approach and a monotonicity approach to propositional reasoning advanced by Peirce (see [Sánchez 1991]) adapted here to the method of semantic tableaus. We assume the tableau rules introduced so far and add the following ones:

PROPOSITIONAL RULES




$A$ *and* $B$ • $Y$
$A,\ B$ • $Y$

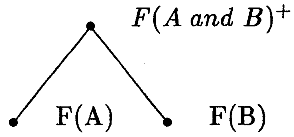
$X$ • $A$ *or* $B$
$X$ • $A,\ B$

$X$ • *if* $A\ B$
$A$ • $B$

Conjunctions and disjunction of other than sentential categories are regulated by the following rules. Let $A,\ B \in a$, $and \in (a,(a,a)$, where a is any complex category ending in $t$. Then

GENERALIZED CONJUNCTION RULES

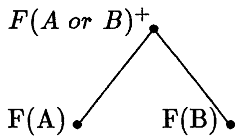
$F(A \text{ and } B)^{+}$ • $Y$
$F(A), F(B)$ • $Y$




Similarly, let $A,\ B \in a$, $or \in (a,(a,a)$, where a is any complex category ending in $t$. Then

GENERALIZED DISJUNCTION RULES

$Y$ • $F(A \text{ or } B)^{+}$
$Y$ • $F(A), F(B)$




## 3.1 EXAMPLES OF REASONING WITH GENERALIZED BOOLEAN CONNECTIVES

**Inference 9** *heloise dances and sings/heloise dances*

**Analyses 10**

*a.*

$$\frac{\underset{e}{heloise} \quad \dfrac{\underset{(e,t)}{dances} \quad \dfrac{\underset{((e,t),((e,t),(e,t)))}{and} \quad \underset{(e,t)}{sings}}{((e,t),(e,t))}}{\underset{+}{(e,t)}}}{t}$$

*b.*

$$\frac{\underset{e}{heloise} \quad \underset{(e,t)}{dances}}{t}$$

**Proof 12**

*heloise dances and sings* • *heloise dances*
*heloise (dances and sings)*$^{+}$ • marking
*heloise dances* • $=$ generalized conjunction

**Inference 10** *heloise dances/heloise dances or sings*

**Analyses 11**

*a.*

$$\dfrac{\underset{e}{heloise} \qquad \underset{+}{\dfrac{\underset{(e,t)}{dances} \qquad \dfrac{\underset{((e,t),((e,t),(e,t)))}{or} \qquad \underset{(e,t)}{sings}}{((e,t),(e,t))}}{(e,t)}}}{t}$$

*b.*

$$\dfrac{\underset{e}{heloise} \qquad \underset{(e,t)}{dances}}{t}$$

**Proof 13**

*heloise dances* • *heloise dances or sings*
• *heloise (dances or sings)*$^{+}$ marking
• *heloise dances* $=$ generalized disjunction

**Inference 11** *every man dances, every man sings/every man dances and sings*

**Analyses 12**

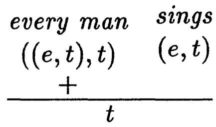
*a.*

$$\dfrac{\underset{\underset{+}{((e,t),t)}}{every\ man} \qquad \dfrac{\underset{(e,t)}{dances} \qquad \dfrac{\underset{((e,t),((e,t),(e,t)))}{and} \qquad \underset{(e,t)}{sings}}{((e,t),(e,t))}}{(e,t)}}{t}$$

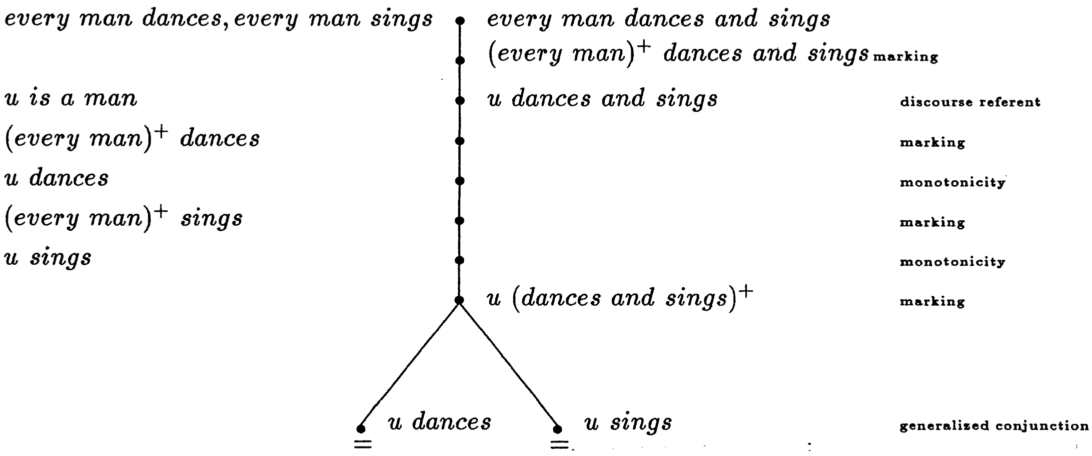
*b.*

$$\dfrac{\underset{\underset{+}{((e,t),t)}}{every\ man} \qquad \underset{(e,t)}{dances}}{t}$$

*c.*



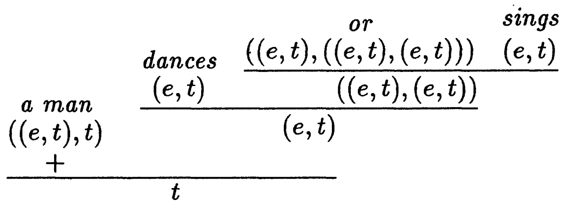

**Proof 14**



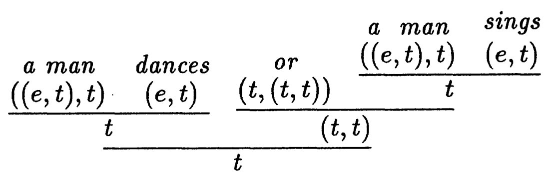

**Inference 12** *a man dances or sings/a man dances or a man sings*

**Analyses 13**

*a.*




*b.*



**Proof 15**

| | | |
|---|---|---|
| *a man dances or sings* | *a man dances or a man sings* | |
| *(a man)$^+$ dances or sings* | | marking |
| *u is a man, u dances or sings* | | discourse referent |
| *u (dances or sings)$^+$* | | marking |
| | *a man dances, a man sings* | propositional logic |
| | *(a man)$^+$ dances, (a man)$^+$ sings* | marking |
| | *u dances, u sings* | monotonicity |

*u dances* =      *u sings* =      generalised disjunction

# 4 Prospects: Anaphoric Reasoning

The system described in the previous pages does not consider natural reasoning in which anaphorical phenomena are prominent. This is a serious drawback because in any kind of discourse, anaphora seem to play an important role. With regard to such inferences, CNL can only be used when we presuppose that the derivations produced by the Lambek Grammar are supplemented with an anaphorical analysis. Let us assume that the sentences analysed by the grammar are provided with anaphoric information, for instance linking pronoun and antecedens by means of indices. We then assume the rules introduced so far with the following modification of the marking rule and the closure condition on nodes:

Extended Marking

$$\begin{array}{ccc} X & \bullet & A, Y \\ X & \bullet & A^*, Y \end{array} \qquad \begin{array}{ccc} A, X & \bullet & Y \\ A^*, X & \bullet & Y \end{array}$$

where $A^*$ is a monotonicity, scope or anaphoric marking of $A$ provided by the fixed Lambek analysis and the anaphoric indexing of $A$ . A node will be closed when the same expression with the same scope and anaphoric marking occurs at both sides. Next, we add a general rule for the treatment of anaphora:

Pronoun Rules

$$\begin{array}{ccc} X & \bullet & F(Pron_k), Y \\ X & \bullet & F(u), Y \end{array} \qquad \begin{array}{ccc} F(u), X & \bullet & Y \\ F(Pron_k), X & \bullet & Y \end{array}$$

provided that expression $u$ of type $e$ has been introduced in place of the marked antecedent of $Pron_k$ at a previous stage in the construction of the tableau.

Illustration

| | |
|---|---|
| *(a man)$_i^{\#}$ admires a woman who respects him$_i$* | |
| *u is a man, u admires a woman who respects him$_i$* | discourse referent |
| *u admires a woman who respects u* | pronoun rule |

Our general pronouns rule allows us to postpone the elimination of the pronoun. For instance the previous sentence could be processed as follows:

ILLUSTRATION

| | |
|---|---|
| $(a\ man)_i^{\#}$ *admires a woman who respects* $him_i$ | |
| *u is a man, u admires a woman who respects* $him_i$ | discourse referent |
| *u admires* $(a\ woman\ who\ respects\ him_i)^{\#}$ | marking |
| *v is a woman who respects* $him_i$ *u admires v* | discourse referent |
| *v is a woman who respects u* | pronoun rule |

We shall give examples of inferences in which anaphoric marking is important. We would want these examples to be seen the following way: given the premiss and both their Lambek analyses and their markings, the system shows that the conclusion follows. But we do not provide a mechanism for obtaining the anaphoric marking from the Lambek derivations. We only want to show the empirical strength of CNL when the anaphoric marking is given.

## 4.1 EXAMPLES OF REASONING WITH ANAPHORA

**Inference 13** *a man dances or sings/a man dances or he sings*

**Analyses 14**

*a. Analysis 13 a.*

*b.*

$$\dfrac{\dfrac{\underset{((e,t),t)}{a\ man} \quad \underset{(e,t)}{dances}}{t} \quad \dfrac{\underset{(t,(t,t))}{or} \quad \dfrac{\underset{e}{he} \quad \underset{(e,t)}{sings}}{t}}{(t,t)}}{t}$$

ANAPHORIC MARKING $(a\ man)_i$ *dances or* $he_i$ *sings*

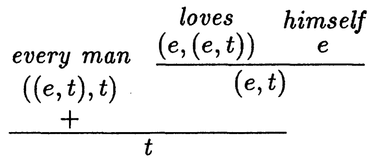
**Proof 16**

| | | |
|---|---|---|
| *a man dances or sings* | *a man dances or he sings* | |
| $(a\ man)^{\#}$ *dances or sings* | $(a\ man)_i^{+}$ *dances or* $he_i$ *sings* | marking |
| *u is a man, u dances or sings* | | discourse referent |
| *u* $(dances\ or\ sings)^{+}$ | | marking |
| | $(a\ man)_i^{+}$ *dances,* $he_i$ *sings* | prop.logic |
| | *u dances* | monotonicity |
| | *u sings* | pronoun rule |
| *u dances* = | *u sings* = | generalized disjunction |

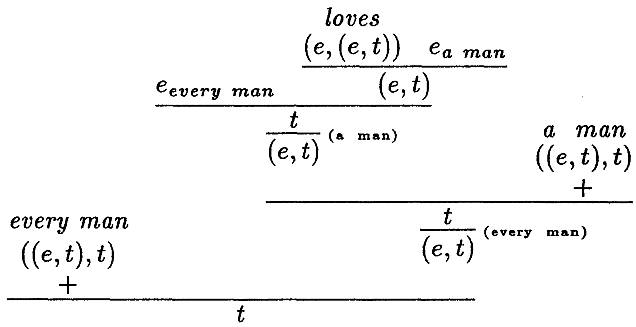
**Inference 14** *every man loves himself/every man loves a man*

**Analyses 15**

*a.*



*b.*



ANAPHORIC MARKING $(every\ man)_i$ loves $himself_i$

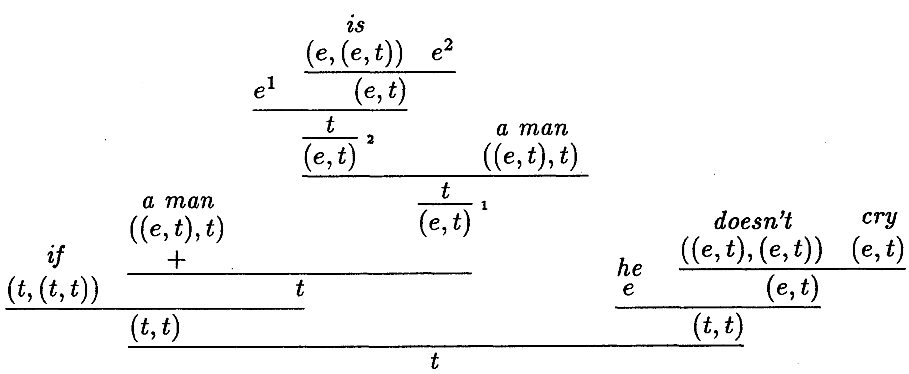
**Proof 17**

| | | |
|---|---|---|
| *every man loves himself* | *every man loves a man* | |
| $(every\ man)_i^+$ *loves* $himself_i$ | $(every\ man)^+$ *loves a man* | marking |
| *u is a man* | *u loves a man* | discourse referent |
| | *u loves* $(a\ man)^+$ | marking |
| | *u loves u* | monotonicity |
| *u loves* $himself_i$ | | monotonicity |
| *u loves u* | | pronoun rule |

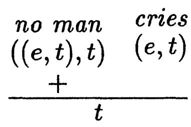
**Inference 15** *if a man is a man he doesn't cry/no man cries*

**Analyses 16**

*a.*

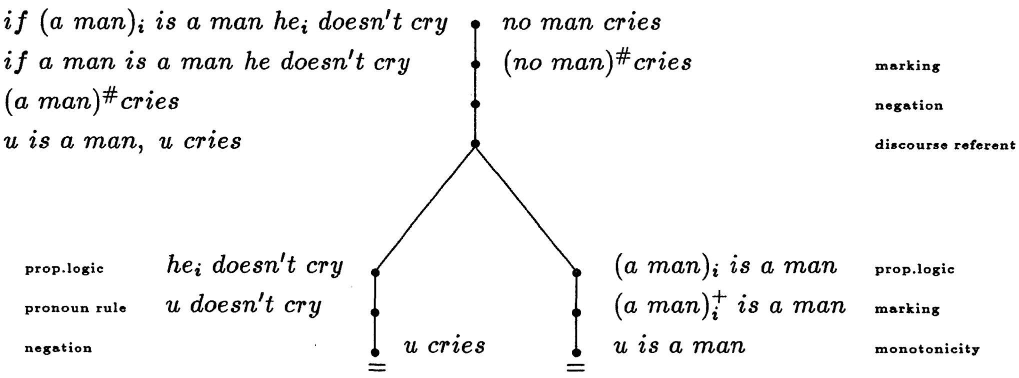


*b.*




ANAPHORIC MARKING *if* $(a\ man)_i$ *is a man* $he_i$ *doesn't cry*

**Proof 18**

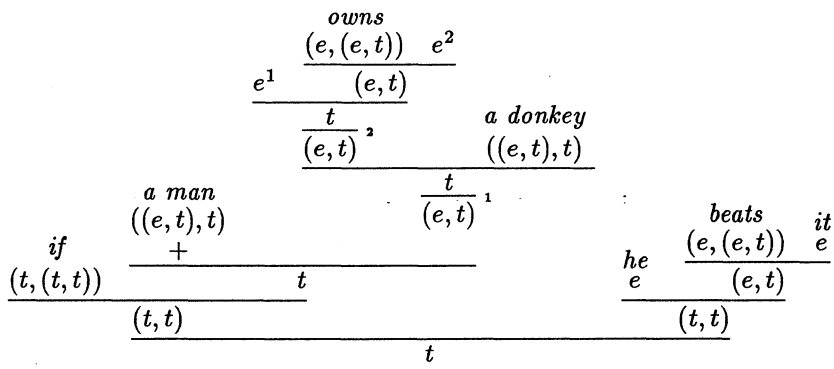



(In the following inference we assume that *u is a X who VP* is equivalent to *u is a X and u VP*).

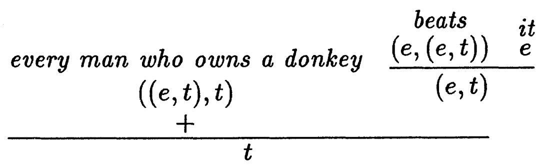
**Inference 16** *if a man owns a donkey he beats it/every man who owns a donkey beats it.*

**Analyses 17**

*a.*




*b.*

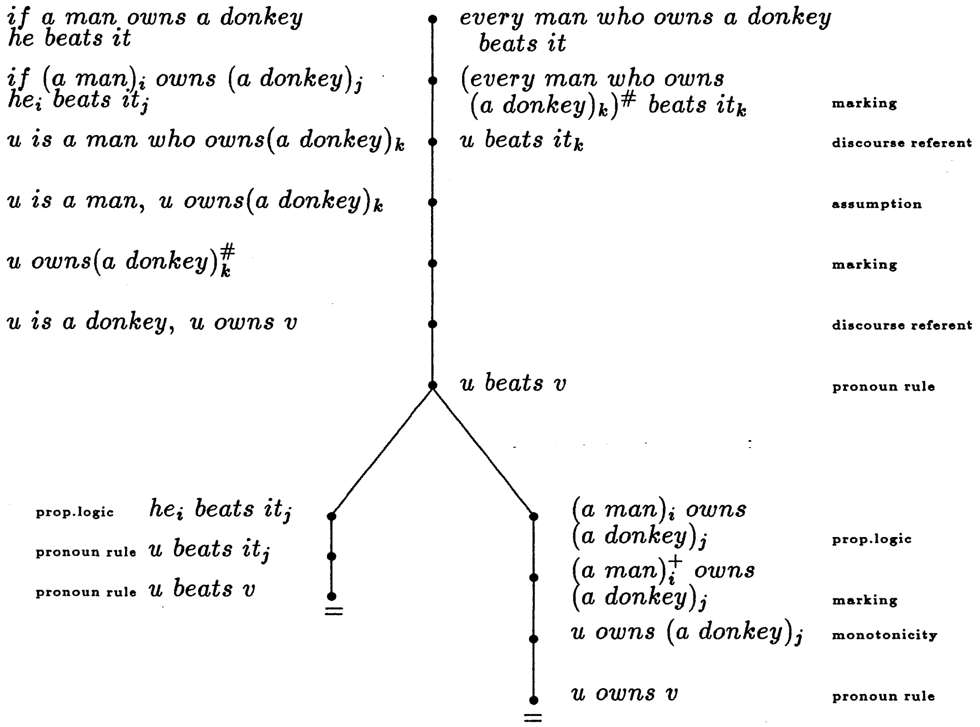



ANAPHORIC MARKING

*if* $(a\ man)_i$ *owns*$(a\ donkey)_j$ $he_i$ *beats* $it_j$

*every man who owns*$(a\ donkey)_k$ *beats* $it_k$

**Proof 19**




COMMENT Geach's Donkey sentences were introduced in [Geach 1962] to show the limitations of a naive monotonicity calculus. [Van Benthem 1987] illustrates the problem with this reasoning:

$$\frac{\begin{array}{c}\textit{a (man who owns a garden)}^{+}\textit{ sprinkles it}\\ \underline{\textit{every man who owns a garden owns a house}}\end{array}}{\textit{a man who owns a house sprinkles it}}$$

This example can be used to show the inadequacy of a monotonicity calculus vis a vis anaphoric phenomena. Geach's proposal for the treatment of this counter-example consisted in denying that the expression X who TV a Y occurs as a logical unity in donkey sentences. This is a radical solution which we do not need have to accept. As Van Benthem himself observes, the problem here is one of anaphoric annotation and it affects only a naive monotonicity calculus. In the present form of CNL the above example is not derivable.

Let us conclude this report with an assessment of the situation. The intended anaphoric reading of the above sentences is, of course,

- *a man who owns (agarden)$_i$ sprinkles it$_i$*

- *a man who owns (ahouse)$_j$ sprinkles it$_i$*

In our CNL Van Benthem's example takes the form

| | |
|---|---|
| *a man who owns a garden sprinkles it,*<br>*every man who owns a garden owns a house* | *a man who owns a house sprinkles it* |
| *a(man who owns (a garden)*$_k$*)*$^+$<br>*sprinkles it*$_k$ | *a man who owns (a house)*$_i$ *sprinkles it*$_j$ |
| *every man who owns a garden*<br>*is man who owns a house* | |
| *a man who owns a house sprinkles it*$_k$ | |

But the string *a man who owns a house sprinkles it* does not show the same pattern of anaphoric marking in both its occurrences. Consequently, the tableau will not close. Notice that we can still provide a proof of the following inferences:

- a man who owns a small garden sprinkles it/ a man who owns a garden sprinkles it.
- a small man who owns a garden sprinkles it/ a man who owns a garden sprinkles it.
- a man who owns a garden sprinkles it / a man who owns or rents a garden sprinkles it.

The point is that these new cases do not disturb the anaphoric marking of the premiss.

# Chapter V

# Concluding remarks

In this report we have been engaged in the construction of a realistic model for natural reasoning. The empirical strength of CNL has been demonstrated by applying it to several standard examples of natural reasoning. The architecture of CNL described in this report involves the principles of

- monotonicity
- discourse referent introduction
- substitution of anaphora
- generalized Boolean coordination

We have shown that prerequisites for these principles are:

- a system of monotonicity marking
- a system of wide scope marking
- a system of anaphoric marking
- a general definition of coordination.

In a certain sense, the monotonicity rule of section 1 becomes superfluous as soon as we introduce the monotonicity rules of section 2. However, a simple extension of the English fragment, containing non-first order determiners like *most* or *few* makes them indispensable. The point is that we still lack reduction rules based on discourse referents for Noun Phrases built up from such determiners. This report gives a system of monotonicity and wide scope marking and uses a common definition of coordination across all Boolean categories. But, it presupposes a theory of anaphoric marking. Further research on the Lambek Grammar itself should (and can) result in such a theory.

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