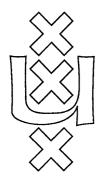
# Institute for Language, Logic and Information

# THE KINEMATICS OF PRESUPPOSITION

David I. Beaver

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# THE KINEMATICS OF PRESUPPOSITION

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# The Kinematics of Presupposition\*

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# 1 Introduction

Russell's analysis of definite descriptions provides us with celebrated propositions like "There is one and only one King of France, and he is bald". Strawson, however, took the Fregean line that sometimes referring expressions could fail to denote, and was not prepared to accept that "The King of France is bald" asserted the existence or uniqueness of a King, preferring to believe that these were necessary conditions for the sentence to have a meaning at all: presuppositions as we would now call them.

The intuition that a Russellian analysis cannot be right is perhaps strongest in the case of imperatives. If the conditions imposed by definite descriptions were part of the propositional content of a sentence, then we should expect the command "Shave the king of France's head!" to be, in part, an instruction to restore the French monarchy, or at least to restore the king's head. (Further still, if it turns out that the restored head is hairless, we should presumably have to restore the hair as well, since this is, arguably, a precondition for removing it.) Yet the desire for such restoration plays no part in any natural interpretation of the sentence.<sup>1</sup>

Suppose we go along with such reasoning, and say that there is more to meaning than what is asserted, there additionally being a component of *presupposition*: then our problems are just beginning. The Russellian analysis involves both a perspicuous notation for the meaning of a sentence, and a well understood logic. Strawson's approach suggests neither. If we want to even begin the task of reasoning about Strawsonian meanings we will have to notate those meanings in such a way that both assertive and presuppositional aspects of meaning are transparent, define a suitable semantics for the resulting structures, and ascertain the relevant (non-standard) logic.

Permit me to make a slightly jarring leap, to an oddity of modern dynamic semantics. In a dynamic logic, formulae are typically given a relational interpretation, and may be thought of as instructions for an agent to move across some space: in this space the formula relates points at which the instructions are valid to points which the agent may end up at by following the instructions. For our purposes the space will be a space of information states, and the agent will tend to move in the direction of increasing information. Yet not all formulae will denote movement: those that do not are what Veltman (1990) has termed tests.

<sup>\*</sup>I am particularly grateful to Robin Cooper, Paul Dekker, Willem Groeneveld, Lex Holt, Laszlo Kalman, Ewan Klein and Henk Zeevat for valuable feedback on earlier versions of these notes.

<sup>&</sup>lt;sup>1</sup>Grice (1989) credits this sort of objection to the Russellian analysis to Searle.

A good example of a test arises with Veltman's might operator, which he uses to model epistemic possibility: the intuition is that you can say "possibly X" just in case at that point in the discourse X has not been ruled out. He defines a dynamic semantics for a simple extension of a propositional logic, in which the semantics for ordinary formulae are naturally viewed in terms of Stalnaker's assertion perspective (Stalnaker 1978). An information state is modelled as a set of worlds, sometimes called the context set, intuitively those worlds which the agent still considers as possible candidates for the real world, and an ordinary formula is an instruction to narrow down that set by throwing out worlds which are not compatible with some new information. A formula of the form might  $\phi$  is a test: it is an instruction to maintain the current context set provided some of its constituent worlds support  $\phi$ , and otherwise move to the absurd information state, here identified with the empty context set.

Let us say that an assertion is a formula that can help us reach a maximal non-contradictory information state, potentially narrowing down the context set to a single world. In that case a formula of the form  $might\ \phi$  is non-assertive: it cannot productively inform the agent. In terms of the spatial analogy it says: stand still, take a look around, and if you cannot see  $\phi$  on any of the paths ahead, commit suicide. After hearing such an instruction, an agent is at best in the same place as before, and at worst nowhere. The conundrum is this: how can formulae which can never productively inform be useful in modelling natural language, where the whole point is to productively inform? Whatever the successes of Veltman's analysis of "might" (which anyway he only provides as an initial example of his framework, the main applications being within default reasoning) it cannot be the final answer, since telling somebody what you regard as possible may be informative.

In fact I believe that whilst a test operator cannot adequately capture the meaning of "might", there is a place for test operators in our analysis of meaning. I will claim that whilst the main purpose of language is to assert information, so that we should not expect the translation of any sentence to be a pure test, there is a non-assertive component to meaning which can be usefully modelled using a test operator within a dynamic logic. We might say that the result is a logic for Strawsonian meanings. Incidentally, thinking about the behavior of presuppositions will also provide us with a natural resolution to the problem of defining an assertive *might* operator, showing that from an appropriate perspective Veltman's test can provide information after all.

I should stress that a dynamic perspective on presupposition is not in itself novel. Most of the hard work is already to be found in papers by Karttunen, Karttunen & Peter's (hence K&P), and Heim. (I will sometimes refer collectively to the model they have developed as the CCP model, from Heim's description of sentence meanings in terms of Context Change Potentials). Indeed, presupposition phenomena formed one of the main motivations for Stalnaker's assertion model. However, the last few years have seen a flowering in the understanding and technical elaboration of all things dynamic, and in the remainder of this paper I will attempt to show how our understanding of the major problems of presupposition theory can benefit from application of the tools of dynamic semantics.

#### A Handy Guide to this Paper

Presupposition being a fairly well circumscribed area, there is a forest of problems of both a philosophical and a linguistic nature that any theory must tackle. I have divided the problems into three groups: the projection of presuppositions from truth-functionally connected compounds, the behavior of presuppositions in quantificational contexts, and the accommodation

of presupposed information. There is a section devoted to each of these topics, each section starting with an empirically oriented subsection concerning the chief problem(s) to be dealt with, followed by further subsections containing the mixture of technicality and speculation that constitutes the theory. The summary below should help to make clear the relationship between the different sections, and provide an overview of the different motivations for the approach adopted.

- Section 2 is devoted to making explicit the links between existing context change models of presupposition and Veltman's *Update Semantics*. This serves to introduce the framework which is used throughout the remainder of the paper.
- Section 3 concerns the interaction between presupposition, quantification and anaphora. K&P have presented a Montague style treatment of presupposition which is heterogeneous in the sense that meanings are split up into two parts, one containing the presupposition, and the other containing the assertion. Unfortunately such an approach cannot adequately account for the projection of presuppositions from quantificational contexts. It is shown how this motivates the sort of approach developed in Section 2, which is essentially homogeneous and uses a unary operator to embed presuppositions within the primary assertion.
- The homogeneous account improves over that presented by K&P through providing a simple mechanism for quantifiers in the assertion to bind variables in the presupposition. Considering some more data makes it clear that it is also advantageous to allow quantifiers in the presupposition to bind variables in the assertion, and this inspires the introduction of a Groenendijk & Stokhof (hence G&S) style dynamic approach to variable binding. Utilising such an approach also makes the model appropriate to the demands of standard cases of discourse and donkey anaphora.
- It is shown how the resulting account can make sensible predictions on a range of examples that have troubled existing CCP theories without invoking any non-compositional mechanism of local or intermediate accommodation. These examples concern the strength of the presupposition that emerges from a quantified context. It also becomes clear that the kinematic account developed in Section 3 can provide some overview regarding what the options are within a CCP theory and where the existing accounts fit in.
- A theory of presupposition that involved no notion of accommodation would be inadequate, since it would fail to account for the informativeness of presuppositions. In Section 4 it is shown how *global accommodation* can be modelled within a CCP account.
- The model developed to account for global accommodation provides an interesting perspective on a range of issues on the border between semantics and pragmatics. Section 4.3 is devoted to a speculative exploration of some of these issues, including the definition of an assertive *might* operator, the strengthening of the typical CCP predictions regarding the presuppositions of the conditional, and the licensing of definite descriptions.
- The final parts of Section 4, are devoted to a comparison with some other types of account. Firstly I consider the relationship between the kinematic model developed in these notes and default accounts of presupposition, showing that presuppositions

can be defeasible within a CCP theory, and showing how some of the classic cases of presupposition cancellation could be interpreted in terms of accommodation. I then turn briefly to van der Sandt-ian accounts involving accommodation of representations into discourse structure, suggesting a way in which the kinematic model could enable a crucial distinction to be made between different types of accommodation.

# 2 Update Semantics and Presupposition Projection

# 2.1 The Projection Problem

As originally conceived by Langendoen and Savin (1971), the projection problem consists of determining the presuppositions of complex sentences in terms of the presuppositions of their parts. To cast the problem in this way is to assume that there is some monolithic class of presuppositional constructions, presumably including clefts, definite descriptions, factive verbs and many more, all of which behave the same way under embedding. Whilst I am by no means committed to that assumption, for the most part I will concentrate on factive constructions like "regret", "know" and "is annoyed that", in the hope that they are representative of the remainder of the class. At any rate, factive constructions certainly form a class worth study in there own right.

To simplify the analysis, I will assume a translation of English sentences into a formal language, and cast the projection problem in terms of inference relations that must hold in a logic for this formalism. Although the translation function itself will not be formally specified in this paper, it should be clear that the function envisaged in some sense preserves the structure of the original English: the translation will at least be compositional in spirit.

For the purposes of this section a propositional language will be used, consisting of some atomic propositions, the normal truth-functional connectives connecting in their normal way, and two additional unary operators  $\diamondsuit$  and  $\partial$ . The  $\diamondsuit$  will play the role of Veltman's might operator, and  $\partial \phi$  will be interpreted as "the presupposition that  $\phi$  holds", and the translation of a simple sentence involving a factive verb, such as "John regrets that Bill is happy" will assume a division of the sentence's meaning into a presupposition and an assertion.<sup>2</sup> Let us say that in this case the presupposition is the atomic proposition that Bill is happy, which will be written BIH, and the assertion is another atomic proposition, which we can imagine is concerned with John's mental state, written R(J,BIH). Then the translation of the sentence will be the conjunction  $\partial(BIH) \land R(J,BIH)$ .

Let me make some observations on this translation. It is important that the apparent complexity of R(J,BIH) is purely a notational convenience: as an atomic proposition, semantically R(J,BIH) has no internal structure, and in particular it has nothing to do with the atomic proposition BIH: this is clearly inadequate. I do not pretend to have given a suitable semantics for "regrets", or given any explanation of the source of its presupposition. A full account of the semantics of "regrets" would demand an understanding of the nature of intensionality and the folk psychological notions implicit in attitude reports: I would not know where to begin with these topics! I will merely say that I find it useful to abstract across what presuppositional constructions have in common by assuming that they have a definite presuppositional component and regarding this component as simply conjoined with the primary content. Following this approach, the logic of presupposition becomes quite transparent.

<sup>&</sup>lt;sup>2</sup>Unlike K&P, I will not be assuming such a simple division for complex sentences.

I note in passing that, so far as I am aware, nobody else has chosen to represent presuppositions using a distinct unary operator. One might say that whilst other notations tend to hide away the presuppositions within the meaning of the atomic propositions, or else keep the presuppositions as some formal aside, explicit but separate from the principal logical form, in the current paper translations wear their presuppositions on their sleeves. Given that there is no commitment to a psychologically real level of logical form here, it is hard to see any empirical significance in this decision. Yet the notation chosen here will simplify some of the technicalities, especially when, in Section 3, we come to the issue of scope and binding relations between presupposition and assertion.

We are now in a position to state the projection problem in terms of inferences we would like to have between formulae of the given propositional language. These inferences are summarised in Table 1. The first three inference patterns express the standard properties of presupposition, namely emergence from atomic contexts, embedding under negation, and embedding in the antecedent of a conditional: this behavior is often used as a test for the presence of a presupposition. The fourth pattern is more contentious, giving a type of restricted emergence of presuppositions from the consequent of a conditional: later we will consider how this pattern can be strengthened. The final pattern represents emergence from recursive embedding within presuppositional contexts. Of course, these five inference patterns do not represent the only desiderata for our logic. These are merely additional inference patterns above and beyond all the classical entailments that we would expect to hold between non-presuppositional formulae.

If we know	Then we know	Inference Pattern	
John regrets that Bill is happy.	Bill is happy	$\partial(A) \wedge B \models A$	
$\partial(\mathtt{Bih})\wedge\mathtt{R}(\mathtt{J},\!\mathtt{Bih})$	ВІН		
John doesn't regret that Bill is happy.	Bill is happy	$\neg(\partial(A) \land B) \models A$	
$\neg(\partial(\mathtt{BiH})\land\mathtt{R}(\mathtt{J},\mathtt{BiH}))$	ВІН		
If John regrets that Mary is singing, then he should buy ear-plugs.	Mary is singing	$(\partial(A) \land B) \Rightarrow C \models A$	
$(\partial(MIS) \wedge R(J,MIS)) \Rightarrow JSBE$	MIS		
If Mary is in the bath, then John regrets that she is singing.	Mary's singing if bathing	$A \Rightarrow (\partial(B) \land C) \models A \Rightarrow B$	
$(MIB \Rightarrow (\partial(MIS) \land R(J,MIS))$	$MIB \Rightarrow MIS$		
Bill regrets that John regrets that Bill is happy	Bill is happy	$\partial(\partial(A) \wedge B) \wedge C \models A$	
$\partial(\partial(\mathtt{BiH})\wedge\mathtt{R}(\mathtt{J},\mathtt{BiH}))\wedge\mathtt{R}(\mathtt{B},\mathtt{R}(\mathtt{J},\mathtt{BiH}))$	він		

Table 1: The Projection Problem

It has been my intention to present in Table (1) an abstract characterisation of the projection problem that is, in so far as this is possible, theory neutral. I would hope that even proponents of defeasible accounts of presupposition (see eg. Gazdar 1979; Mercer 1988) would not object too strongly: after all, the '=' in the table could be interpreted as defeasible entailment. There is some discussion of defeasibility in Section 4.4, below.

# 2.2 The Context-Change-Potential Model

The Context Change Potential model has developed from Robert Stalnaker's work on assertion and pragmatic presupposition (Stalnaker 1974; Stalnaker 1978) and a series of papers by Lauri Karttunen (eg. Karttunen 1973; Karttunen 1974) culminating in a joint paper with

Stanley Peters (K&P 1979), which still represents the only serious attempt at a strictly compositional account of presupposition. After taking some battering in the late seventies and early eighties, their ideas were given a new lease of life by Irene Heim (Heim 1983), and up to now her short paper has remained the definitive statement on what can be achieved in the CCP approach.

In Definition 2, below, I present a renotation of the CCP model in the form of a logic over formulae in the propositional language introduced earlier. The resulting system is strikingly close to the Update Semantics (US) introduced in Veltman (1990), also discussed above, the main difference being my introduction of the  $\partial$  operator, which brings a new type of partiality into the logic. Whereas in Veltman's semantics formulae are interpreted as total functions between partial models of reality (context sets), in the semantics below formulae are interpreted as partial functions between partial models of reality.<sup>3</sup>

## Definition 1 (Symbols)

 $\phi, \psi$  are (possibly atomic) propositions; a world, w, assigns a boolean to every elementary proposition; a context set,  $\sigma$ , is a set of worlds;  $X \doteq Y$  means "X is Y if Y is defined, else X is undefined"; \( \) is set subtraction.

#### Definition 2 (Semantics of Partial Update Logic)

$$\sigma\llbracket\phi\rrbracket \ \stackrel{.}{=} \ \{w \in \sigma \mid w(\phi)\} \quad (for \ atomic \ \phi)$$

$$\sigma\llbracket\phi \land \psi\rrbracket \ \stackrel{.}{=} \ \sigma\llbracket\phi\rrbracket\llbracket\psi\rrbracket$$

$$\sigma\llbracket\neg\phi\rrbracket \ \stackrel{.}{=} \ \sigma \land \sigma\llbracket\phi\rrbracket$$

$$\sigma\llbracket\partial\phi\rrbracket \ \stackrel{.}{=} \ \sigma \ iff \ \sigma\llbracket\phi\rrbracket = \sigma \ else \ undefined$$

$$\sigma\llbracket\Diamond\phi\rrbracket \ \stackrel{.}{=} \ \sigma \ iff \ \sigma\llbracket\phi\rrbracket \neq \emptyset$$

$$\emptyset \ iff \ \sigma\llbracket\phi\rrbracket = \emptyset$$

$$\sigma\llbracket\phi \Rightarrow \psi\rrbracket \ \stackrel{.}{=} \ \sigma\llbracket\neg(\phi \land \neg\psi)\rrbracket$$

In all of these clauses a formula is interpreted as a function from context sets to context sets, that is to say, a CCP. Let us consider some of the clauses individually.

The base case interprets an atomic proposition as an instruction to remove from the context set any worlds that are not compatible with the new proposition, just as was discussed in the introduction to this paper.

The clause for conjunction defines the local context at a given point in a discourse in terms of the initial context of utterance incremented with the information provided by the discourse up to that point. Thus conjunction becomes functional composition: we update with the first conjunct to get to an intermediary state, and then with the second to take us to the final output. But why should conjunction be asymmetric?<sup>4</sup> Here we can find some new motivation beyond that cited by Karttunen or Heim, with Veltman-esque examples like the following:

<sup>&</sup>lt;sup>3</sup>The relation between the CCP model and US is also clear in Zeevat (1991).

<sup>&</sup>lt;sup>4</sup>There is a case to be made for also having a symmetric (ie. static) conjunction in the logic, which could be defined by  $\sigma[\![\phi\&\psi]\!] \doteq \sigma[\![\phi]\!] \cap \sigma[\![\psi]\!]$ . It would be natural to use this, for example, within complex lexical translations, where there is no reason to say that there is any privileged order to the different components of lexical meaning.

- (1) a. Maybe this is Frank Veltman's example. It isn't his example!
  - b.  $\Diamond$ FVE  $\land \neg$ FVE
- (2) a. ? This is not Frank Veltman's example! Maybe it's his example.
  - b.  $\neg FVE \land \Diamond FVE$

As has been indicated, the  $\diamondsuit$  is essentially Veltman's *might* operator, apart from some syntactic restrictions that he places on its occurrence. The first example gives us a function from contexts in which the origin of the example may be uncertain to contexts in which the example is known not to be Frank's. By contrast, the second example gives us a function which only has the empty set (that is, the absurd information state) in its range, this presumably accounting for its oddity. So, the asymmetry of conjunction can be motivated not only by presupposition data, but, at least in some cases, also by independent principles of textual coherence.

Negation is defined as a simple set complement operation: to update a context with  $\neg \phi$ , try updating with  $\phi$ : the resulting subset of worlds contains only those members of the original context that are compatible with  $\phi$ , and since we are seeking the set of worlds that is incompatible with  $\phi$ , we must take the subset away from the original context.

The crucial clause is that for  $\partial$ . A formula of the form  $\partial \phi$  is interpreted as a fixed-point function that is only defined on a context if  $\phi$  is already accepted in that context. The facts of presupposition projection are then explained in terms of the definedness of a formula's subparts within the local contexts where the subparts occur. Consider the projection of presuppositions from within a negation, as in example (3).

- (3) a. John doesn't regret that Bill is happy
  - b.  $\neg(\partial BIH \wedge R(J,BIH))$

The argument of the negation is simply evaluated in the same context as that where the negation itself occurs, so a negated formula will be defined on a context only when the argument is defined — in other words the presuppositions of a negative will be the same as for a corresponding positive.

It might be helpful to consider the sort of partial function associated with the translation of example (3): Figure 1 shows some of the relevant updates. In this example there are only two relevant propositions, concerning Bill's and John's mental states respectively, and so it is only necessary to consider contexts made up of four worlds which differ with respect to the value they assign to these two propositions. It can be seen that the function associated with the formula in (3b) is not defined on contexts where either Bill's happiness is in doubt or Bill is known to be unhappy. On the other hand, in contexts that only contain worlds where Bill is happy, the effect of updating with this formula is simply to remove any worlds where John has regrets.

Finally, we need a notion of semantic entailment for the system above:

Definition 3 (Entailment in Partial Update Logic) 
$$\phi_1, \ldots, \phi_n \models \psi \ iff \llbracket \phi_1 \rrbracket \ldots \llbracket \phi_n \rrbracket = \llbracket \phi_1 \rrbracket \ldots \llbracket \phi_n \rrbracket \llbracket \psi \rrbracket$$

This is the second of the three entailment notions considered by Veltman, and essentially it says that one thing entails another if once you know the first you have nothing to learn from the second. This yields all the entailments in Table 1.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup>Whilst the entailment relation in definition 3 has its merits, it also has its failings. For instance, as Willem

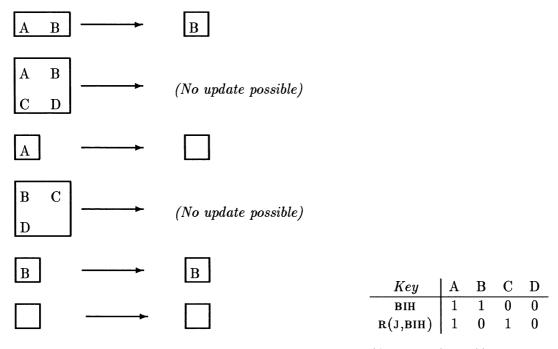


Figure 1: Updates associated with  $\neg(\partial BIH \land R(J,BIH))$ 

# 3 Presupposition, Quantification and Anaphora<sup>6</sup>

# 3.1 The Quantificational Projection Problem

Once again I will simplify by assuming a translation of English sentences into a formal language, this time a first order predicate language augmented with the unary operator  $\partial$ , and expressing the problem in terms of inferences between sentences of this language.

It is fair to say that there is significant disagreement about the facts of the projection of presuppositions from quantified contexts, but by considering the behavior of existing accounts it should be possible at least to gather some minimal constraints on the logic: these can be strengthened later, according to taste. In what follows I have divided the constraints into three dimensions of variation, sensitivity, dynamism and weakness.<sup>7</sup>

Groeneveld has pointed out to me, it does not yield *substitutivity*: two formulae which mutually entail each other may behave differently when embedded within other formulae. In particular we have the unintuitive result that for any formula  $\phi$ ,  $\partial \phi \models \phi$  and  $\phi \models \partial \phi$ , but that formulae containing  $\phi$  may have different entailments to formulae containing  $\partial \phi$ . However, there are alternatives to definition 3. Suppose we were to define a notion of entailment  $\models^*$  as follows:

$$\phi_1, \ldots, \phi_n \models^* \psi \text{ iff } \llbracket \phi_1 \rrbracket \ldots \llbracket \phi_n \rrbracket = \llbracket \phi_1 \rrbracket \ldots \llbracket \phi_n \rrbracket \llbracket \psi \rrbracket \text{ and } \mathcal{D}(\llbracket \phi_1 \rrbracket \ldots \llbracket \phi_n \rrbracket) \supseteq \mathcal{D}(\llbracket \psi \rrbracket)$$

The additional restriction on domains means that one formula entails another if whenever you are in a position to learn something from the second, you could learn the same (and possibly more besides) from the first. Now we have  $\phi \models^* \partial \phi$  but  $\partial \phi \not\models^* \phi$ . On this notion we would in fact lose all the entailments in Table 1, but this is not as serious as it might seem. We would at least retain weaker versions of all the inference patterns where the conclusions are embedded under a  $\partial$  operator. At any rate, I do not offer  $\models^*$  as a final solution, but merely as a pointer for further research.

<sup>&</sup>lt;sup>6</sup>Much of the technical development in this section reflects joint work with Paul Dekker whose reformulation of DPL (Dekker 1992) is crucial to the current paper.

<sup>&</sup>lt;sup>7</sup>The reader should be wary of taking my word on what existing accounts predict, as I have occasionally used some latitude in interpreting what has been said and attempting to apply it to specific examples that may not have been considered by the original authors. Having said that, I do not expect my characterisation

#### Sensitivity

K&P (1979) define a Montague style fragment in which the meaning of a sentence is divided into two components. One of these components contains the primary assertion of the sentence, and the other contains the presuppositions. Each syntactic composition in the grammar is accompanied not just by a rule for combining components of the primary assertion, but also by a rule for putting together the presuppositions. It might seem that this provides a completely general framework in which to state how presuppositions behave, and in fact there have been suggestions that the system is *ad hoc* since there are so few constraints on what projection rules can be defined.

Yet in spite of the apparent generality there is a serious restriction inherent in the paired-meanings approach: there is no obvious way to define scope and binding relations between the presupposition and the assertion. In particular, the presupposition is *insensitive* to variable bindings introduced in the assertion, and as K&P point out in a footnote, this causes problems with cases like Example (4).

- (4) a. ? Somebody managed to succeed George IV
  - b.  $\exists x \text{ succeeded\_g4}(x) \land \partial(\text{difficulty\_succeeding\_g4}(x))^{8}$

This sentence is odd since it suggests that the person who succeeded George IV had some difficulty ascending to the throne when in fact we know that the next in line had no difficulty. However, the K&P system only gives this sentence the presupposition that it was difficult for somebody to succeed George IV. Since this presupposition is obviously satisfied, there being many people for whom it was difficult to succeed George IV, the K&P system does not predict that Example (4) is anomalous.

In the suggested translation of Example (4) above, the presupposition contains a free variable. For the formula to make sense, the presupposition must be *sensitive* to the external binding of that variable. Expressing the need for sensitivity in terms of an inference pattern gives us something like:  $\exists x a(x) \land \partial(b(x)) \models \exists x a(x) \land b(x)$ . (We will attend later to the question of whether there is a stronger universal presupposition associated with the sentence.)

K&P have at least two options in order to cure their system's insensitivity. Firstly they could define the semantics of their system so as to achieve the necessary binding: this simply seems unnatural given their notation. Secondly they could include more of the asserted information in the presupposition, in the case of Example (4) the information that the relevant person with difficulty was in fact the successor to the throne. But following this path would lead to the duplication of all the asserted information in the presupposition, and, at least from a notational point of view, this also seems awkward. It should be clear that I prefer a third option, namely choosing a notation in which the problem does not arise, for in defining a logic over the language used in this section, sensitivity seems not so much a dimension of variation as plain common sense.

#### **Dynamism**

That K&P's system does not allow binding of variables in the presupposition by quantifiers in the assertion is one side of a coin. The other side is that quantifiers in the presupposition

of these accounts to be particularly contentious.

<sup>&</sup>lt;sup>8</sup>Note that in all the translations below I have not explicitly delimited the scope of existential quantification. It will turn out to be useful to assume a non-standard, dynamic notion of scope, in which we have associativity, so that  $(\exists x\phi) \land \psi \equiv \exists x(\phi \land \psi)$  and we can simply ignore the bracketing.

cannot bind variables in the assertion. Consider examples (5) and (6):

- (5) a. I know that a thief broke in. He stole my thesis.
  - b.  $\partial(\exists x \text{ thief}(x) \land \text{broke\_in}(x)) \land \text{know}(me, \exists y \text{thief}(y) \land \text{broke\_in}(y)))$  $\land \partial(\exists y \text{ my\_thesis}(y)) \land \text{stole}(x, y)^9$
- (6) a. If John is married then his wife is very tolerant.
  - b.  $\operatorname{married}(j) \Rightarrow (\partial(\exists x \text{ wife\_of}(j,x)) \wedge \operatorname{tolerant}(x))$

I do not want to make any great claims for the treatment of definite descriptions implicit in the translations above: for a start I have completely left aside the issue of uniqueness. What does seem right to me is that these translations make definites presuppose rather than assert the existence of an object, and that the object thus introduced can later fill argument slots in asserted predications. Whether or not the filling of the slot is mediated by a pronoun, it seems to me that the simplest explanation of the slot filling will be, as usual, in terms of bound variables. The same holds for the coreference of an indefinite introduced within a presupposition (eg. "a thief" in (5)) and a later pronoun: a bound variable solution would be nice. This means that the  $\partial$ -operator must be externally dynamic, in the sense of G&S (1990). In terms of entailment this will yield patterns like:  $\partial(\exists xa(x)) \wedge b(x) \models \exists xa(x) \wedge b(x)$ . We will return to the treatment of definite descriptions in Section 4, where the issue of licensing will be briefly discussed.

Note that making presuppositions dynamic would achieve the same results as van der Sandt's DRT-based model (van der Sandt 1990) achieves non-compositionally, by invoking accommodation. When a presupposition is not satisfied during the construction of a DRS, the DRS is repaired by addition of the presupposed information in some appropriate place. As a default the information is added to the top level of the DRS — so-called global accommodation. This is what would happen in example (5). However, in example (6), where some form of accommodation is necessary to provide the discourse marker for the wife, global accommodation would produce the unwelcome inference that John really does have a wife, whereas it is clear in this example that the existence of a wife is in doubt. Thus van der Sandt is forced to invoke intermediate or local accommodation, where the existence of a wife is only added to the antecedent or consequent of the conditional. Calculating the exact landing site of the presupposition requires quite a complicated chain of pragmatic reasoning in this case: it would be nice if we could predict the right result in a CCP account without local or intermediate accommodation. Ideally the two premises married(j)  $\Rightarrow \exists x \text{ wife\_of}(j,x)$ and married(j)  $\Rightarrow$  ( $\partial(\exists y \text{ wife\_of}(j,y)) \land \text{tolerant}(y)$ ) should not entail  $\exists y \text{ wife\_of}(j,y)$ , and this should be determined in purely semantic and declarative terms.

#### Weakness

Consider examples (7–9):

<sup>&</sup>lt;sup>9</sup>Here I have not stuck to the language of FOPL+ $\partial$ , in order to give a rough and ready translation. Note that the higher order part, the "know" clause is by assumption non-presuppositional, and so does not affect the main point which is to establish the behaviour of presuppositions.

- (7) a. Everyone who serves his king will be rewarded.
  - b.  $\forall x((\partial(\exists y \text{ king\_of}(x,y)) \land \text{serves}(x,y)) \Rightarrow \text{rewarded}(x))$
- (8) a. No nation cherishes its king.
  - b.  $\neg(\exists x \text{ nation}(x) \land \partial(\exists y \text{ king\_of}(x,y)) \land \text{cherishes}(x,y))$
- (9) a. A fat man was pushing his bicycle.
  - b.  $\exists x \text{ fat-man}(x) \land \partial(\exists y \text{ bike\_of}(x,y)) \land \text{pushing}(x,y)$

Heim's raw CCP account (Heim 1983), without the addition of an accommodation mechanism, predicts that these examples have the presuppositions that everyone has a king, every nation has a king, and every fat man has a bike respectively.<sup>10</sup>

Intuitions vary in the case of (7). Neither of the accounts in Cooper (1983) and van der Sandt (1990) predict any presupposition at all for this sentence, both yielding the meaning that everyone who has a king and serves him will be rewarded: deriving such a meaning seems a good minimal constraint on a theory, whether or not there is an additional universal presupposition.

As far as examples (8) and (9) are concerned, intuitions seem much clearer: the presuppositions predicted by Heim's raw CCP account are too strong. Cooper gives example (8) the same universal presupposition as Heim, whereas van der Sandt predicts merely that the sentence means that no nation which has a king cherishes him. Both van der Sandt and Cooper only attribute to example (9) an existential presupposition, namely that some fat man has a bike.

Van der Sandt's theory makes sensible predictions not only with regard to the dimension of strength, where his predictions are uniformly weak, but also regarding sensitivity and dynamism. The question remains as to whether a sensitive, dynamic and weak account of presupposition can be given entirely within the CCP model, or whether, as one might gather from a reading of Heim (1983), a successful account will inevitably depend on a van der Sandt-ian mixture of local, intermediate and global accommodation. In what follows I shall attempt to show that all the relevant desiderata, summed up in Table 2, can be satisfied within a pure CCP model.

# 3.2 Kinematic Predicate Logic

Lewis (1979) refers collectively to all the things we have to keep track of in a conversation as the *conversational score*. In this section we will be concerned with what Lewis calls the *kinematics* of such a multi-component score, although we will be restricting our attention to two components, one keeping track of the propositions that have been established (as in Section 2), and the other keeping track of the individuals that have been mentioned. The resulting *Kinematic Predicate Logic* (KPL) must satisfy not only presuppositional desiderata, but also the demands placed by discourse and donkey anaphora, and all in the spirit of aesthetic compositionality, where the form of the meaning evokes the same feelings as the

<sup>&</sup>lt;sup>10</sup>I refer to the predictions of Heim's "raw", unaccommodating, CCP account because in Heim (1983) she discusses but does not fully explicate the accommodation mechanism she has in mind, so it is difficult to be sure what the predictions would be like. One imagines that she would want to derive more or less the same results as van der Sandt does in his theory, which contains a much more fully developed account of accommodation.

Desiderata	Inference		Examples	
Sensitivity	$\exists x a(x) \land \partial b(x)$	=	$\exists x a(x) \land b(x)$	(4)
Dynamism	$\partial(\exists x a(x)) \wedge b(x)$	F	$\exists x a(x) \land b(x)$	(5)
Weakness	$\forall x(a(x) \land \partial b(x)) \Rightarrow c(x)$	$\not\models$	$\forall x(a(x) \Rightarrow b(x))$	(7)
		=	$\forall x((a(x) \land b(x)) \Rightarrow c(x))$	
	$\neg(\exists x a(x) \land \partial b(x))$	$\not\models$	$\forall x(a(x) \Rightarrow b(x))$	(8)
		<b>=</b>	$\neg(\exists x a(x) \land b(x))$	
	$\exists x a(x) \land \partial b(x)$	¥	$\forall x (a(x) \Rightarrow b(x))$	(9)

Table 2: The Quantificational Projection Problem

form of the utterance.<sup>11</sup>

To achieve this the Propositional Update Logic developed in Section 2 must be extended with the ideas in the Dynamic Predicate Logic (DPL) of G&S (1990): but we must tread carefully. As noted in G&S (1991b) there are some technical problems in effecting such a unification. These can be overcome if DPL is reformulated so that all formulae define updates, instead of the situation in G&S where requantification over a variable causes a downdate, loss of information about the variables previous value. One way of removing downdating is to adopt Paul Dekker's suggestion of switching from total variable assigns as in DPL to the partial variable assigns in his EDPL. This will leave us with more or less what Heim (1983) had in the first place: a context is defined as a set of pairs of worlds and partial variable assignments, and a formula denotes a function from contexts to contexts. However, the combination of recent dynamic ideas will yield a more uniform treatment of worlds and variables, and also give us more of an overview of the options for a CCP treatment of presupposition. Let us begin by defining a logic (definitions 4, 5 and 6) which meets all the given desiderata, including all those in Table 2, and then briefly try to see where it diverges from Heim's account, as well as Cooper's and K&P's.

#### Definition 4 (Symbols for KPL)

A world, w, is a total function from n-ary predicates, R, to sets of n-tuples of individuals from some fixed domain; An assignment, f or g, is a partial function from variables,  $x_i$ , to individuals; A state, S, is a set of pairs of worlds and assignments such that all the assignments have the same domain of variables,  $V_s$ ;  $g <_x f$  means f extends g wrt x but is otherwise identical;  $g \ge f$  means g is an extension of f.

<sup>&</sup>lt;sup>11</sup>Kinematics may be just "dynamics for the upper classes" (an aspersion cast by Johan van Benthem during the colloquium), but at least it avoids terminological conflict with the predominant use of *dynamic semantics* to refer to G&S style treatment of anaphoric information and *update semantics* to refer to Veltman style treatment of worldly information.

<sup>&</sup>lt;sup>12</sup>The interested reader is referred to Dekker's paper in this volume (Dekker 1992).

#### Definition 5 (KPL Semantics)

```
S[R(x_1,\ldots,x_n)] = \{\langle w,f\rangle \in S \mid \langle f(x_1),\ldots,f(x_n)\rangle \in w(R)\}
iff \{x_1,\ldots,x_n\} \subseteq V_s \text{ else undefined}
S[\![\phi \wedge \psi]\!] \doteq S[\![\psi]\!][\![\psi]\!]
S[\![\neg \phi]\!] \doteq S \setminus \{\langle w,f\rangle \in S \mid \exists g \geq f, \langle w,g\rangle \in S[\![\phi]\!]\}
S[\![\exists x \phi]\!] \doteq \{\langle w,f\rangle \mid \exists g <_x f, \langle w,g\rangle \in S\}[\![\phi]\!] \text{ iff } x \notin V_s \text{ else undefined}
S[\![\partial \phi]\!] \doteq S[\![\phi]\!] \text{ iff } \forall \langle w,f\rangle \in S, \exists g, \langle w,g\rangle \in S[\![\phi]\!]
S[\![\phi \Rightarrow \psi]\!] \doteq S[\![\neg(\phi \wedge \neg(\psi))]\!]
S[\![\psi x \phi]\!] \doteq S[\![\neg(\exists x \neg \phi)]\!]
```

#### Definition 6 (Entailment in KPL)

$$\phi_1, \dots, \phi_n \models \psi \text{ iff } \forall S, w, f \quad \text{if} \quad \langle w, f \rangle \in S[\![\phi_1]\!] \dots [\![\phi_n]\!]$$

$$then \quad \exists g \geq f, \langle w, g \rangle \in S[\![\phi_1]\!] \dots [\![\phi_n]\!] [\![\psi]\!]$$

The definitions of predication and conjunction should be straightforward. Note that a predication is only defined on a context in which all of the predicated variables have already been introduced, so now undefinedness can be introduced in two ways: conventional presupposition failure and abuse of variables.

Negation is basically a set complement operation as before, but complicated by the possibility that the formula within the negation introduces new variables. The definition essentially says update the context with the formula, restrict all the assignment functions to their original values, and take the resulting context away from the original.

The existential definition is intuitively molecular, consisting of  $\exists x$ , which increments the context adding to every assignment function all possible x-extensions, composed with  $\phi$ . It would be quite easy to define  $\exists x$  as a formula in its own right, which could be conjoined with other formulae (eg.  $\phi$ ) compositionally. It is important that the domain of the function defined by  $[\exists x \phi]$  contains only states which do not already provide valuations for x: implicit in the definition is the idea that existential quantification is treated as the introduction of a new discourse referent. Given that a discourse will start off without any referents having been introduced, it is natural to think of the state of zero information as the set of all pairs of a worlds and empty assignments.

Finally we consider the clause for the  $\partial$ -operator. Firstly, note that the  $\partial$  is made dy-namic in a very simple way: provided certain conditions are met the update of a context with  $\partial \phi$  is obtained by updating with  $\phi$ . In a sense we always locally accommodate, rather than umm-ing and ah-ing about local accommodation as in van der Sandt or Heim's account. Note that accommodation itself does not challenge aesthetic compositionality: the challenge arises either from arbitrary application of local accommodation, or from intermediate accommodation which allows modification of bits of discourse representation arbitrarily far from the presupposition trigger.

The conditions on the  $\partial$ -operator provide a check that when the context is updated with  $\phi$  no worlds are lost. Consider the interpretation of  $\exists x \text{ fat-man}(x) \land \partial(\exists y \text{ bike\_of}(x,y)) \land \text{pushing}(x,y)$ , from example (9). By the time fat-man(x) has been interpreted, the context may contain several alternative values of x (corresponding to different assignment functions) for each remaining world. The  $\partial$  clause will be undefined if any of the worlds have no value

of x such that x is a bike owner. That is, for each world in the context set, there must be at least one fat man (value of x) who owns a bike. It is this condition that gives the definition its necessary weakness and sensitivity. Note that the "always locally accommodate" principle means that the outgoing context for the entire formula will only contain values of x which are bike owners: not only is it checked that there is at least one suitable fat man, but all the others are discarded.

The conditions on the  $\partial$ -operator provide a check that when the context is updated with  $\phi$  no worlds are lost. Consider the interpretation of  $\exists x a(x) \land \partial b(x)$ , which can be thought of as a translation of "A fat man was riding his bike." By the time a(x) has been interpreted, the context may contain several alternative values of x (corresponding to different assignment functions) for each remaining world. The  $\partial b(x)$  clause will be undefined if any of the worlds have no value of x such that x is in the extension of b. That is, for each world in the context set, there must be at least one fat man (value of x) who owns a bike. It is this condition that gives the definition its necessary weakness and sensitivity. Note that the "always locally accommodate" principle means that the outgoing context for the entire formula will only contain values of x in the extension of b: not only is it checked that there is at least one suitable fat man, but all the others are discarded.

The net effect of this definition is that a presupposition cannot carry any new factual information, which would involve discarding worlds from the context set, but can carry information about which individuals the speaker had in mind, and can introduce new individuals.

Definition 6 is a modified version of the first entailment definition in this paper (definition 3). It says one thing entails another if updating an arbitrary context with the first will take you to a context where updating with the second provides no new factual information and provides no further restrictions on the variables already introduced. The only complication is that the entailed formula can introduce new variables, which allows such inferences as  $\exists xa(x) \models \exists ya(y)$ .

#### 3.3 Choices

What are the options in a CCP based account of the interaction between quantification and presupposition? To answer this question in full generality would obviously be difficult, since the options include not only different accounts of the semantics of presupposing constructions, but also any number of different treatments of quantification and connectives. Let us simplify, by assuming fixed definitions of quantifiers and logical connectives, namely those in the clauses of definition 5 apart from that for " $\partial$ ". Now we can ask the simpler question, what are the options for a definition of the  $\partial$ -operator? In what follows I will sketch roughly where I believe the accounts of K&P, Cooper and Heim fit in to the KPL framework in terms of alternative definitions of  $\partial$ . Regrettably I do not have the space here to explain their work in as much detail as it deserves, and for the moment I must refer the interested reader to the relevant papers in the bibliography.

Heim's account is very close to that presented here: she has essentially identical semantics for the quantifiers and junctions, although the technical elaboration differs somewhat. This makes it very easy to see how her treatment of presupposition fits in to the KPL picture. She defines presupposition not in terms of a unary operator, but as a relation between sentences and the propositions they presuppose: "S presupposes p iff all contexts that admit S entail p", where S is some sentence and p is a proposition. She then goes on to consider the interpretation of variables, and defines a context to be a set of pairs of worlds and partial assignment

functions, as for KPL above. The immediate consequence of this is that a presupposition is only satisfied if updating the context with the presupposition would not remove any world-assignment pairs. Put another way, not only must the hearer know all the relevant facts, but she must also know which individuals the speaker had in mind, a strong condition.

Suppose we make the following definitions of relations between contexts S, S':

```
S \sqsubseteq_W S' \text{ iff } \forall \langle w, f \rangle \in S(\exists g \langle w, g \rangle \in S'),
S \sqsubseteq_S S' \text{ iff } \forall \langle w, f \rangle \in S(\exists g \geq f, \langle w, g \rangle \in S').
```

Then the presupposition operator in definition 5 is given by:  $S[\![\partial \phi]\!] \doteq S[\![\phi]\!]$  iff  $S \sqsubseteq_W S[\![\phi]\!]$ , else undefined. Heim's presupposition relation is equivalent to the definition of a unary operator:  $S[\![\partial_{\text{heim}} \phi]\!] \doteq S$  iff  $S \sqsubseteq_S S[\![\phi]\!]$ , else undefined.  $\partial_{\text{heim}}$ , unlike  $\partial_{\text{heim}}$ , is static as there is no mechanism for a presupposition to introduce new variables. Of course, this fails to reflect the mechanism of accommodation that Heim envisages, which should introduce a sort of dynamism into the system by allowing global accommodation of discourse markers.

Both K&P and Cooper assume a treatment of quantifiers as binary relations rather than as one place predicates. This increases their range of choices for how quantifiers and presuppositions and quantifiers interact, allowing a presupposition to behave differently according to whether it occurs in the restrictor or scope of a quantifier, and making it difficult to capture their accounts as precisely as Heim's within the KPL framework.

Still, defining  $\mathcal{W}(S) = \{\langle w, \emptyset \rangle \mid \exists f, \langle w, f \rangle \ inS \}$ , a function which effectively strips a context of all its information about variables, we can parody the situation in K&P as being loosely equivalent to the definition of a unary presupposition operator:  $S[\partial_K \& P \phi]] \doteq S \ iff \ \mathcal{W}(S)[\![\phi]\!] = \mathcal{W}(S)$  else undefined. This definition, by not allowing any flow of information about the values of variables between the presupposition and the assertion is static and insensitive as we should expect of an account where presuppositions and assertions are kept separate.

One of the main motivations of Cooper's account is to overcome the insensitivity of the K&P account. Strikingly, he observes regarding the George IV example, (4) above: "We need a semantics that will have the effect represented by... $\exists x[x \text{ succeeds George IV} \land PRESUP-POSED$  difficult for x to succeed George IV]". This, of course, amounts to a statement of the programme adopted in the current paper of defining a unary presupposition operator.

However, Cooper did not in fact pursue this line any further, and instead adopted an approach whereby predicates are given *paired intensions* in the model. The meaning of a predicate is defined as a function from worlds to a positive and a negative extension. This effectively yields a four valued logic in which he is able to reproduce the essential CCP inheritance properties while still allowing for a sensible account of binding of variables in

 $<sup>^{13}</sup>$ It can be seen that  $\partial_{\text{heim}}$  is strong in the sense used to define the quantificational projection problem, whilst  $\partial$  is weak. Note that we need not be committed to a unitary account of different presupposing constructions in terms of one or the other sort of operator: conceivably some constructions might have strong and others weak presuppositions, though this poses more questions than it answers. Which constructions should have which presuppositions and why? One can even conceive of single constructions having multiple types of presupposition. For instance, we could attribute to "managed to X" a weak presupposition that some effort had been made to do X, and a strong presupposition that there was difficulty involved in X. In that case "Somebody managed to sit on the bench" would presuppose that it would be difficult for anybody to sit on the bench, and that the person who sat on the bench made some effort in order to do so. I have not given this enough thought to make it a serious analysis of "manage" in its own right, I am merely trying to exemplify the fact that the choices available go beyond simply picking the single analysis of presupposition that produces the most sensible predictions in a majority of cases. The question of whether all this extra choice is actually relevant must await further empirical investigation.

the presupposition by quantifiers in the assertion. The binary quantifiers approach allows him, for example, to define the semantics of a universal "every X Ys" as presupposing that everything in the positive extension of X is either in the positive or negative extension of Y, effectively excessively weak regarding presuppositions of the restrictor, and strong regarding presuppositions of the scope. This has its problems: for instance the weakness of the restrictor means that the sentence "every person who read my PhD thesis laughed for days" does not presuppose that I have a PhD thesis.

We could, again very loosely, view Cooper's account in terms of some unary presupposition operator (although to do it justice we would need to define binary quantifiers):  $S[\partial_{COOper}\phi] \doteq S$  iff  $S \sqsubseteq S[\phi]$ , else undefined. Here I have not committed myself to exactly what  $\sqsubseteq$  is, since it seems to flip between something like  $\sqsubseteq_S$  and something like  $\sqsubseteq_W$  according to the position of the presupposition trigger. At least it should be clear in this definition that  $\partial_{COOper}$  is static, there still being no way for a quantifier in the presupposition to bind a variable in the assertion, and sensitive.

It is important to realise that no claim is being made regarding the overall expressive power of the logics resulting from the alternative definitions of " $\partial$ ", although the logics could conceivably differ. The point is to see which definitions make for the most *natural* treatment of presupposition: more aesthetics!

# 4 Accommodation

# 4.1 The Accommodation Problem

Although it is not normally the primary function of presuppositions to inform, neither is it the case that we are incapable of learning anything from them. It is well known that presuppositions can be used with the obvious intention of providing new information. (If you did not already believe that, then you should do now that you have read the previous sentence.)

More generally, whatever the speaker's actual intentions, when a speaker presupposes something of which a hearer is unaware, the hearer is not necessarily at a dead end. Lewis (1979) suggests that the hearer may make running repairs on his knowledge of the discourse context, by accommodating any missing information needed for interpretation to proceed smoothly. In what follows I will make some suggestions about how to formalise this process, but to simplify I will only consider accommodation at the propositional level, as a development of the Partial Update Logic presented in section 2 rather than the Kinematic Predicate Logic of section 3.

Yet even having allowed that accommodation occurs, there is a difficulty caused by the fact that people do not seem to accommodate the logically weakest proposition needed to make sense of an utterance. To understand this we must go beyond the simple principles of contextual entailment considered so far in this paper, and consider pragmatic issues determining the hearer's model of the speaker.

## 4.2 The Logic of Epistemic Alternatives

#### Accommodation as Filtering

The common ground of the participants in a discourse situation can be seen as a partial model of reality, and we have seen how this can be modelled as a set of worlds which is

gradually emptied. The speaker, unless possessed of extrasensory powers, can never be sure precisely what is the common ground. A certain amount of the common ground is established by the discourse itself, by a process like that described above. But it would be laborious to start every conversation with an assumption of zero common knowledge, and build up from scratch each time. Limitations of mortality and attention span mean that a certain amount of common ground must be assumed by the speaker.

But how does a hearer know what the speaker has assumed on a given occasion of utterance? He does not know, so he is forced to take into account his own uncertainty about the common ground. Thus we must model the hearer's knowledge state not as one single partial model, but as a set of such partial models. The speaker may have assumed the common ground to be any one of these partial models. I will call them *epistemic alternatives*, and refer to propositions entailed by an epistemic alternative over and above those explicitly asserted in a discourse as *presumptions*.<sup>14</sup>

An utterance may or may not provide the hearer with information about the speaker's presumptions. In the simplest case, when the utterance carries no awkward presuppositions, the hearer must update each of the epistemic alternatives with the new proposition. But when there is a presupposition, it may become clear to the hearer that some of the alternatives do not correspond to the speaker's view of the common ground, and these alternatives must be discarded. Thus we arrive at a slightly different perspective from Lewis: accommodation is a form of filtering operation on epistemic alternatives, which perhaps this gives it less of the feeling of a repair strategy, and makes it seem more like part of the orderly process of communication.

In Figure 2 an initial hearer information state (i) is shown updated with the utterance "John doesn't regret that Bill is happy" to produce a new state (ii). State (i) consists of a fairly arbitrary set of epistemic alternatives, the six small boxes, each of which is characterised by a set of worlds. To simplify, the epistemic alternatives are restricted to combinations of four worlds, represented by the letters A-D, which differ only with respect to the two propositions BIH and R(J,BIH.

The topmost alternative, consists only of worlds where Bill is happy, so the hearer believes it possible that the Bill's happiness is being assumed to be common knowledge. This alternative is compatible with the presuppositions of the utterance, and updating with the utterance simply removes from this alternative any worlds where John has a negative state of mind regarding Bill's happiness. The result is an epistemic alternative in the state (ii).

The second alternative contains all the worlds under consideration, so before processing the utterance the hearer thought it plausible that there were no relevant presumptions. However, this alternative is incompatible with the presuppositions of the utterance, and is filtered out. A hearer who was certain that nothing had been taken for granted would soon run out of options.

The third alternative shows that before the utterance the hearer thought it possible that both Bill being happy and John having regrets were believed by the speaker to be common knowledge. This is incompatible with the assertion of the utterance, and after updating yields the empty, or contradictory, alternative. This takes us to a slight quirk in the formalisation I will present: the contradictory alternative is persistent. no matter what you learn, if you

<sup>&</sup>lt;sup>14</sup>Some — the die-hard semanticists — may like to think of these knowledge states as generalised quantifiers over worlds. It is interesting to observe that in G&S (1991a) a knowledge state is a generalised quantifier over assignments, but I shall not attempt to interpret this similarity here.

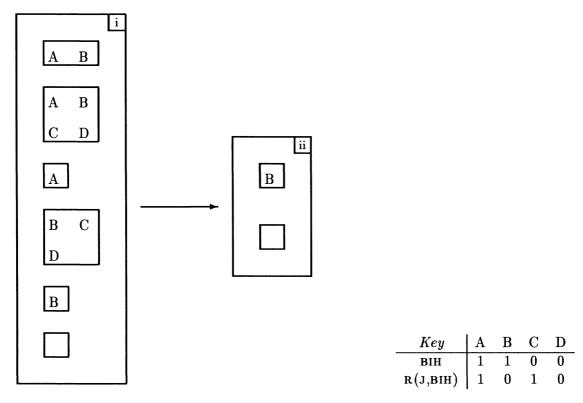


Figure 2: An accommodating update for  $\neg(\partial BIH \land R(J,BIH))$ 

ever wondered whether the speaker thought the common ground was absurd, then you still wonder it. Furthermore it may be that you considered alternatives which in the light of what the speaker later said lead you to believe that maybe the speaker thought that the common ground was contradictory all along.<sup>15</sup>

Rather than going through the remaining epistemic alternatives, here is a summary of what is happening in Figure 2: when the hearer updates with the new proposition, all the alternatives that include worlds where Bill is not happy (C or D worlds) are immediately filtered out, leaving only alternatives where Bill is happy, whilst the main propositional content of the sentence, concerning John's psychological state, removes from these remaining alternatives any worlds where John has regrets about Bill's happiness (that is, the A worlds).

Below a semantics is presented for the same propositional language as in section 2, but with sentences interpreted as functions between sets of epistemic alternatives.

<sup>&</sup>lt;sup>15</sup> It would be possible to alter the model so as to remove the absurd alternative at every step, yet I think that this would miss an important problem which results from some simplifications I am making. The first simplification is one of logical omniscience. Although the common ground is partial, it is defined in terms of worlds which are total and consistent. There is no means to represent a common ground which is flawed in some domains, but consistent in others, characteristics which presumably typify the common ground of resource bounded agents like ourselves. One direction to take in order to remedy this would be to view propositions instead of worlds as the basic objects of the common ground, and not assume completion under logical closure, but from a technical point of view this would considerably complicate matters. The second simplification is that there is no mechanism for down-dating, and it is this that means that the common ground can never be repaired once it has reached a contradictory state. In a real discourse situation it may well be that the speaker recognizes that a hearer has contradictory beliefs and, for the sake of argument, goes along with those beliefs just so as to demonstrate their inconsistency. We will consider such a case later.

#### Definition 7 (Symbols for Lifted Update Logic)

A world, w, assigns a boolean to every elementary proposition; an epistemic alternative,  $\sigma, \tau$ , is a set of worlds; a state  $\Sigma$  is a set of epistemic alternatives;  $\downarrow$  defines a function from a singleton set to its only element, otherwise undefined.

### Definition 8 (Lifted Update Logic)

```
\Sigma[\![\phi]\!] = \{\sigma \mid \exists \tau \in \Sigma, \sigma = \{w \in \tau \mid w(\phi)\}\} \quad (for \ atomic \ \phi)
\Sigma[\![\phi \land \psi]\!] = \Sigma[\![\phi]\!][\![\psi]\!]
\Sigma[\![\neg \phi]\!] = \{\sigma \mid \exists \tau \in \Sigma, \sigma = \tau \setminus \downarrow (\{\tau\}\![\![\phi]\!]\!])\}
\Sigma[\![\partial \phi]\!] = \{\sigma \in \Sigma \mid \{\sigma\}\![\![\phi]\!]\!] = \{\sigma\}\}
\Sigma[\![\phi \Rightarrow \psi]\!] = \Sigma[\![\neg(\phi \land \neg\psi)]\!]
```

Such a semantics can be derived straightforwardly from the earlier semantics in definition labeldefn:pul, using a simple lift operation. For atomic propositions, the update is defined intersectively on each set of worlds which is an epistemic alternative within an incoming state  $\Sigma$ , and the set containing as its elements all the resulting sets of worlds is the output. I have simplified conjunction by defining it as functional composition at the level of sets of alternatives, rather defining it pointwise on each alternative. However, negation is defined as a lift from the earlier definition, becoming a set complement operation on each epistemic alternative.

The definition for  $\partial$  shows how a test at the level of partial models becomes assertive at the level of epistemic alternatives — below this is put to use in the definition of an assertive might operator.  $\partial \phi$  provides an eliminative update, removing all those epistemic alternatives which do not support the proposition  $\phi$ . an immediate consequence of this is that there is no source of partiality in Lifted Update Logic: all formulae define total functions.

Our earlier notion of entailment for update logic would still make sense in this system, but we can also choose Veltman's third type of dynamic entailment. This says that updating the minimal information state (here the power set of the set of worlds) with the antecedent leaves us in a state to which the consequent adds nothing. This definition will preserve all the presupposition projection entailments listed in Table 1.

Definition 9 (Entailment in lifted update logic) 
$$\phi_1, \ldots, \phi_n \models \psi \text{ iff } \mathcal{P}(W)\llbracket \phi \rrbracket \ldots \llbracket \phi_n \rrbracket = \mathcal{P}(W)\llbracket \phi \rrbracket \ldots \llbracket \phi_n \rrbracket \llbracket \psi \rrbracket$$

The lifted semantics provides a new perspective on Veltman's *might* operator. Raising the operator to the new level yields:

Definition 10 (Lifted semantics of might) 
$$\Sigma \llbracket \Diamond \phi \rrbracket = \{ \sigma \in \Sigma \mid \downarrow (\{\sigma\} \llbracket \phi \rrbracket) \neq \emptyset \}$$

When a speaker expresses  $\diamond \phi$  he is telling you (at least) that he doesn't know to the contrary of  $\phi$ . The definition above says that in this situation the hearer is expected to update his knowledge state by discarding all models of the common ground which do not allow for the possibility of  $\phi$ . At the level of partial models, there was simply no way of

capturing the informational content of "epistemic-might", so perhaps the level of sets of sets of worlds is not quite so bizarre as it first seems.<sup>16</sup>

We still have not exhausted the information content of an utterance expressing epistemic possibility, for when a speaker says "Perhaps X", he conversationally implicates that he does not know X. This too may have an effect on the common ground, which could be modelled in the current system:

```
Definition 11 (Pragmatics of might) \Sigma \llbracket \Diamond \phi \rrbracket = \{ \sigma \in \Sigma \mid \sigma \supset \downarrow (\{\sigma\} \llbracket \phi \rrbracket) \supset \emptyset \}
```

This definition says that when the speaker expresses  $\diamond \phi$ , the hearer should discard not only the alternatives which contradict  $\phi$ , but also those alternatives which positively affirm it.

One can imagine a similar treatment for the conversational implicatures associated with disjunction and conditionals. When, for instance, a conditional is uttered, it is implicated that the speaker is not certain about the truth or falsity of either the antecedent or the consequent, so perhaps epistemic alternatives which confirm or deny one or other of these should be discarded. Such a treatment will be assumed in Section 4.3, although in Section 4.4 a more sophisticated alternative will be considered.

# 4.3 What We Really Accommodate

One complaint that has been raised against the CCP theories of presupposition is that they frequently predict presuppositions that are too weak. Consider the contrast between the first and third of the following sentences:

- (10) a. If the walls are thin then it'll be annoying Bill that Mary is singing.
  - b.  $TW \Rightarrow \partial MIS \wedge A(B,MIS)$
- (11) a. If the walls are thin then Mary is singing.
  - b.  $TW \Rightarrow MIS$
- (12) a. If Mary is in the bath then it'll be annoying Bill that she is playing with his rubber duck.
  - b. in\_bath(m)  $\Rightarrow$   $(\partial(\exists x \operatorname{duck}(x) \land of(b,x)) \land \partial(\operatorname{play}(m,x)) \land a(b,\operatorname{play}(m,x)))$

On hearing the first sentence, I think I would normally conclude that Mary is singing. Similarly, on hearing the third sentence I would conclude that Bill had a rubber duck. However, regarding this latter example, I do not think I would come to the conclusion that Mary is definitely playing with Bill's rubber duck, although I would conclude that if she is in the bath then she is playing with his rubber duck. The problem with standard CCP models is that they lead us to expect a presupposition of a weak conditional in all these cases, so that Mary might not be singing and Bill might not have a duck. What is wrong?

<sup>&</sup>lt;sup>16</sup>This is not to claim that the real meaning of "epistemic-might" should be in terms of the common ground: when somebody expresses  $\Diamond \phi$  he is talking not about the common ground but about his own belief state. So the real semantics of  $\Diamond \phi$  should be in terms of the speaker's attitude towards  $\phi$ , not the speaker's attitude towards whether the common ground contains  $\phi$ . None the less, an utterance of  $\Diamond \phi$ , if accepted, will also have the effect of constraining the common ground to contain some  $\phi$ -worlds.

I believe that the CCP predictions are, in a certain sense, correct. Belief in the conditional in the second sentence is necessary and sufficient for interpretation of the first sentence. If we knew that in fact Mary sings whenever she knows that the walls are thin — say because she is an operatic extrovert — then hearing the first sentence might not lead us to conclude that she is singing. Such a logically weakest precondition for interpretation I shall call a linguistic presupposition, "linguistic" because one has the feeling that it can be generated just by looking at the sentences linguistic form, and without knowing too much about what the non-logical parts of the sentence really mean. The real bits of information that people seem to accommodate when faced with a presupposition failure I shall call cognitive presuppositions, "cognitive" because their calculation can require arbitrary amounts of world knowledge. If there is such a thing as a grammar module in the brain, then it does not on its own calculate cognitive presuppositions. Having made this distinction we might, admittedly on somewhat shaky ground, go on to claim that Karttunen consistently predicts the correct linguistic presupposition.<sup>17</sup>

But is it enough that the CCP model gets the right linguistic presuppositions: surely it is the cognitive presuppositions in which we are interested? K&P offer an informal explanation of how extra implicatures can yield the requisite strengthening. I will suggest that the apparatus I have described, in the form of a lifted update semantics, has the potential to provide an adequate and formal version of this explanation.

Figure 3 shows how two states, (i) and (ii) evolve when updated with (10) and (11), where the epistemic alternatives under consideration are characterised in terms of theeight worlds A-H which differ with respect to the three propositions TW, MIS and A(B,MIS). I assume that epistemic alternatives are filtered out either because of presupposition failure or as a result of conversational implicature.<sup>18</sup>

State (i) contains four alternatives (discounting the absurd alternative, which I ignore here), corresponding to there either being no presumptions, or the presumption that Mary is singing if the walls are thin, or the presumption that Mary is singing, or the presumption both that the walls are not thin and that Mary is singing.

The topmost arrow in Figure 3 shows this state being updated with (10) to produce state (iii). This update removes alternatives where nothing is assumed (because of presupposition failure), and where the walls are known not to be thin (because of the implicature that the truth of antecedent of a conditional must be a live option.) This results in a state which is ambiguous between the speaker having assumed that everyone knew that Mary is singing, and having assumed that everyone knew that Mary is singing if the walls are thin.

A second path of two arrows shows state (i) being updated firstly with (11), yielding state (iv), then with (10) to produce state (vi). The only significant new element in this combined update is that the implicature from (11) that Mary's singing is in doubt removes the an alternative where she is known to be singing. Thus the end state, (vi), shows unambiguously that the conditional TW→MIS is in the common ground, and this conclusion is arrived at

<sup>&</sup>lt;sup>17</sup>My cognitive presupposition seems to be just what Stalnaker (1974) has in mind when he uses the term pragmatic presupposition: the reason I have refrained from using his terminology is that it would immediately suggest that what I have called linguistic presupposition was in fact semantic presupposition. Yet this identification might be misleading: the calculation of linguistic presupposition already involves what could be thought of as a partly pragmatic context-change based account of meaning. Having said that, I must confess that I find the identification with pragmatic and semantic presupposition rather appealing.

<sup>&</sup>lt;sup>18</sup> For the case considered the implicature filtering could be achieved by defining the conditional as follows:  $\Sigma[\phi \Rightarrow \psi] = \{\sigma \mid \emptyset \subset \downarrow (\{\sigma\}[\phi]) \land \emptyset \subset \downarrow (\{\sigma\}[\psi]) \land \exists \tau \in \Sigma, \{\tau\}[\neg(\phi \land \neg \psi)] = \{\sigma\}\}$ 

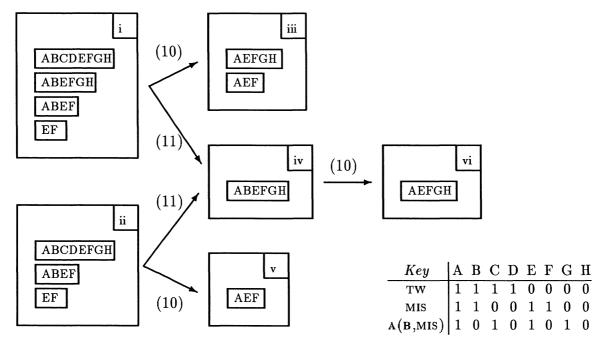


Figure 3: Updates with (10) TW  $\rightarrow \partial$  (MIS)  $\land$  A(B,MIS) and (11) TW  $\rightarrow$  MIS

without any presupposition triggered accommodation.

For a hearer to be in state (i) he must think it reasonable for a speaker to assume that it is common knowledge that Mary is singing if the walls are thin. Yet, for whatever reasons, Mary singing if the walls are thin is an odd thing to take for granted. State (ii) in the figure is the same as state (i) except the speaker is not allowing for the possibility that Mary singing if the walls are thin is being taken for granted. That is not to say that the proposition  $TW \rightarrow MIS$  has been ruled out in state (ii). Updating (ii) with this unlikely fact is a perfectly consistent thing to do, and, as shown, results in state (iv) which, as was seen above, can itself be updated with  $TW \rightarrow \partial(MIS) \land A(B,MIS)$  to produce state (vi). However, if state (ii) is updated directly with  $TW \rightarrow \partial(MIS) \land A(B,MIS)$ , as shown by the lowermost arrow in the diagram, the result is state (v) in which global accommodation of MIS has occurred.

To sum up, the states of lifted update logic can be used to distinguish propositions not only according to whether they are accepted, refuted or open, but also according to whether they are plausible presumptions. Furthermore, I believe it to be essential that plausibility rather than entailment is the central notion of an account of global accommodation, as is shown by (12). It seems clear that a hearer whose previous domain knowledge does not entail the proposition that Mary is playing with Bill's rubber duck if she is in the bath, can accept (12), and accept it without accommodation of the proposition that Mary is playing with Bill's rubber duck. And if the same hearer did not know that Bill had a duck, he can be expected to accommodate that proposition, and do so without even considering the possibility that Bill's duck ownership is conditional on Mary's bathing.<sup>19</sup>

<sup>&</sup>lt;sup>19</sup>It is worth pointing out that doubts have been cast on the factivity of the verb "to annoy", but I do not think that this is relevant. Firstly, even if it were not factive, there would still a problem of explaining the contrast between what is globally accommodated in (10) and in (12). Secondly, the effects discussed above are quite robust, and still seem to hold in a case like "If Mary is in the bath, then her playing with the rubber duck is distracting her from singing." where the playing event is introduced as a definite description.

This, of course, does not tell us much about why one proposition may be plausible as a presumption but another not. In its full generality, that would be a difficult problem to say the least, but perhaps an update approach can contribute at least something.

Veltman (1990) defines a system in which an information state is a pair  $\langle S, \epsilon \rangle$  of a set of worlds and an ordering over those worlds. S encodes factual information in the same way as in his "might" system, or the related Partial Update Logic presented in section 2 of this paper;  $\epsilon$  is used to encode default rules. For instance, if Martin Stokhof has a default rule to the effect that person X's paper will not be finished on time, then worlds in which X finishes on time will be lower in his ordering than worlds where X does not, and in the absence of factual information to the contrary, he presumes that come the deadline X's paper will not be finished.

How does this relate to the presence or absence of epistemic alternatives in the hearer's model of the common ground? Well, imagine that Jeroen is talking to Martin about X. Martin knows that Jeroen has recently seen X, but does not know what Jeroen is assuming Martin knows about X. Martin is entertaining a number of epistemic alternatives concerning Jeroen's picture of the common ground, and given Martin's presumptions about X, he will certainly favour epistemic alternatives in which the paper is not finished. So, even though the truth of this has not yet been established in the discourse, Martin will not bat an eyelid when Jeroen says that its a pity that X's paper will not be ready in time to go in the proceedings. If Martin had maintained any epistemic alternatives where the paper was finished, or where the paper being finished was still in doubt, these would simply be filtered out as a result of processing the factive construction "pity that...". The general principle exemplified by this scenario is that our ordering over worlds will induce an ordering over epistemic alternatives, and so partially determine what information we can accommodate. In effect the hearer projects his own expectations onto his model of the speaker's beliefs.

However, it would be wrong to say that we can only accommodate information that could have been presumed to be the case anyway. Consider examples (13)–(15), the first of which is from Goldberg, Kalman and Szabo (1990), the paper which has inspired the following discussion:

- (13) Joe got married yesterday. The rabbi spoke very harshly.
- (14) Joe got married yesterday. The priest spoke very harshly.
- (15) ? Joe got married yesterday. The rabbi spoke very harshly. So did the priest.

On hearing that some person called Joe got married, I for one would not assume that there was a rabbi present, nor assume that there was a catholic priest. Yet both examples (13) and (14) are perfectly coherent pieces of discourse. In this case my default rules lead me to expect that there was just one person present who was in some way licensed to perform the ceremony, although even this might be over-ridden by an explicit statement to the contrary. But, in Veltman's terms, there will remain a number of optimal worlds, worlds in which there was a rabbi, a catholic priest, a captain of a ship, and so on. On the basis of my default rules alone, I will not be able to choose between these default extensions to the known information. So, I maintain epistemic alternatives corresponding to all the default extensions of the current knowledge state. Note, however, that as soon as I come across the definite description "the rabbi", I filter out all the other epistemic alternatives, and cannot easily process a discourse like that in example (15).

- (16) ? Seamus Murphy and Mary O'Docherty got married yesterday. The rabbi spoke very harshly.
- (17) Seamus Murphy and Mary O'Docherty, who are both Jewish, got married yesterday. The rabbi spoke very harshly.

This sort of reasoning may also explain why some people will find example (16) strange. If you, like me, do find it odd, then that could be because you have a default rule to the effect that people with Irish sounding names are not Jewish, so that after the first sentence you do not maintain an epistemic alternative involving a rabbi, and at the point you meet the definite your processing is interrupted. At this point the sort of accommodation you must perform in order to understand the discourse must be much more like a Lewis-style repair strategy than like the orderly process of narrowing down the assumptions that have been made we have been describing so far. Regarding this example, it is important to realise that the oddity does not arise simply from our being unable to accept that Seamus and Mary are Jewish, for then example (17) would also be an awkward discourse, which it is not. We can accept that Seamus and Mary are Jewish, but, without explicit information to that effect, they will not be Jewish in any of our default extensions, and this may explain why some speakers will find (16) strange but (17) acceptable.

## 4.4 Defeasibility and Changing Perspectives

It has often been suggested that presuppositions are defeasible. This might be motivated by cases like the following, where the presupposition is explicitly cancelled:

(18) John doesn't know that Bill is happy. In fact Bill is not happy.

The linguistic community has to some extent split into two camps on this issue, a Kartunnen influenced group who believe that the major problem of presupposition theory is to account for when presuppositions are projected, and a Gazdar influenced group who think the major problem is to account for when presuppositions are cancelled.

I do not believe that a defeasible treatment of presupposition is incompatible with the general approach developed in this paper. To support this claim, which some may find surprising, I present in definitions 12–14, below, an update logic in which presuppositions introduce default rules, along the lines of the treatment of defaults in Veltman (1990).

**Definition 12 (Symbols for Defeasible Update Logic)** An information state  $\mathcal{I}$  is a pair  $\langle \mathcal{I}_0, \mathcal{I}_1 \rangle$ , where  $\mathcal{I}_0$  is a set of worlds  $w \in W$  and  $\mathcal{I}_1$  is a preordering relation "at least as plausible as" on worlds; the optimal worlds in  $\mathcal{I}$  are defined as the set  $m_{\mathcal{I}} = \{w \in \mathcal{I}_0 \mid \neg \exists w' \in \mathcal{I}_0 \mid \langle w', w \rangle \in \mathcal{I}_1 \land \langle w, w' \rangle \notin \mathcal{I}_1 \}$ ; the minimal information state  $\mathbf{1} = \langle W, W \times W \rangle$ ; the absurd information state  $\mathbf{0} = \langle \emptyset, \{\langle w, w \rangle \mid w \in W \} \rangle$ .

#### Definition 13 (Defeasible Update Logic)

```
\mathcal{I}\llbracket\phi\rrbracket = \langle \{w \in \mathcal{I}_0 \mid w(\phi)\}, \mathcal{I}_1 \rangle \quad (for \ atomic \ \phi)
\mathcal{I}\llbracket\phi \wedge \psi\rrbracket = \mathcal{I}\llbracket\phi\rrbracket\llbracket\psi\rrbracket
\mathcal{I}\llbracket\neg\phi\rrbracket = \langle \mathcal{I}_0 \setminus (\mathcal{I}\llbracket\phi\rrbracket)_0, (\mathcal{I}\llbracket\phi\rrbracket)_1 \rangle
\mathcal{I}\llbracket\partial\phi\rrbracket = \langle (\mathcal{I}\llbracket\phi\rrbracket)_0, \{\langle w, w' \rangle \in \mathcal{I}_1 \mid \phi \in \mathcal{I
```

$$(\langle \{w\}, \mathcal{I}_1 \rangle \llbracket \phi \rrbracket)_0 = \{w\} \to (\langle \{w'\}, \mathcal{I}_1 \rangle \llbracket \phi \rrbracket)_0 = \{w'\}\} \rangle$$

$$\mathcal{I}\llbracket presumably \phi \rrbracket = \mathcal{I} iff (\langle m_{\mathcal{I}}, \mathcal{I}_1 \rangle \llbracket \phi \rrbracket)_0 = m_{\mathcal{I}}$$

$$\text{else } = \mathbf{0}$$

# Definition 14 (Entailment in Defeasible Update Logic) $\phi_1, \ldots, \phi_n \models \psi \ iff \ \mathbb{I}\llbracket \phi \rrbracket \ldots \llbracket \phi_n \rrbracket = \mathbb{I}\llbracket \phi \rrbracket \ldots \llbracket \phi_n \rrbracket \llbracket \psi \rrbracket$

Here an information state consists of a pair the first member of which is a set of worlds, intuitively representing definitely established facts, and the second member of which is an ordering over worlds, encoding default extensions to the definite facts. I shall refer to the two components of an information state as the *established worlds* and the *default ordering* respectively.

The meaning of an atomic presupposition is defined intersectively on the established worlds, as earlier in this paper, and conjunction is defined as functional composition, again as earlier. Negation is a complement operation on the established worlds, combined with inheritance of any new defaults triggered in the argument.

The crucial clauses are those for  $\partial$  and presumably. The  $\partial$  clause, whilst reminiscent of Veltman's normally, is importantly different in that it both asserts the truth of its complement and changes the default ordering so that worlds where the complement is untrue are at least as implausible as worlds where the complement holds. The factual assertion in the  $\partial$  definition reflects the fact that presuppositions occurring in simple positive contexts are not normally defeasible: it is unnatural say "John knows that Bill is happy but Bill is unhappy." Also  $\partial$  is not subject to the syntactic restrictions on its occurrence which hold of Veltman's normally:  $\partial$  can occur arbitrarily deeply embedded within formulae. This reflects the fact that it makes sense for presuppositional defaults to be simply inherited, whereas this intuition would not hold of explicit default rules. The sentence "Bill is not normally happy" should not trigger a default rule to the effect that Bill should be expected to be happy. However, it does make sense for the first sentence in (18) "John doesn't know that Bill is happy" to trigger such a default rule.

The clause for  $presumably \phi$  provides a check that  $\phi$  holds in the subset of the established worlds which are maximally compatible with the known defaults. This yields the following inference patterns:

- 1.  $\phi \wedge \partial \psi \models presumably \phi$
- 2.  $\phi \wedge \partial \psi \models presumably \psi$
- 3.  $\neg(\phi \land \partial \psi) \models presumably \neg \phi$
- 4.  $\neg(\phi \land \partial \psi) \models presumably \psi$
- 5.  $\neg(\phi \land \partial \psi) \land \neg \psi \not\models presumably \psi$

I take it that this gives at least some substance to the claim made earlier in this paper that the inferences in Table 1, which I claimed characterised the projection problem, could be treated as being defeasible. This is reflected most obviously in the difference between the fourth and fifth of the inference patterns immediately above, where we see that addition of a flat contradiction of an earlier presupposition can non-monotonically remove previous entailments. Intuitively, this corresponds directly to the case in (18) above, where the first sentence alone would suggest that Bill was happy, but the combination of the two sentences does not.

It would be nice to present a similar account of the defeasibility of conversational implicatures. Unfortunately, DUL as presented above is not up to the task. Expression of a conditional "if A then B" implicates that neither A nor its negation is known (and similarly for B). However, this implication is cancellable: we can say things like "if A then B, and we all know whether A!", and of course an argument with the form of *Modus Ponens* will crucially involve the antecedent of a conditional being known information. So what is needed is a preference for default extensions which contain some worlds where the antecedent holds and some where it doesn't, and a logic for conversational implicature would involve an ordering not over single worlds but over epistemic alternatives, an idea introduced in Section 4.3, above. I leave such a development for another time.

Should simple cancellation examples like (18) lead us to abandon a logic of presupposition based on contextual entailment, like the earlier PUL, KPL and LUL systems in this paper, in favour of a default logic like DUL? I think we should be wary of jumping to conclusions without much closer scrutiny of the data. Consider the following examples, the first two of which seem to involve presupposition cancellation:

- (19) John doesn't know that Bill is happy, since Bill is not happy.
- (20) John doesn't know that Bill is happy, he merely believes it.
- (21) John doesn't think Bill is happy, he is totally convinced.
- (22) John doesn't believe that Bill is happy, he knows it.

My feeling is that examples like (19) and (20) pattern naturally with (21) and 22). The first thing to note is that all these examples carry "marked" stress, whether contrastive or not, so in principle there might be something to divide them from the cases considered elsewhere in this paper. More importantly, examples (21) and (22) suggest that negation can have some interesting non-truth functional effects. It is clear that John knowing something entails his belief in it, and John being totally convinced about something entails that he thinks it true, so that the negative particles in (21) and (22) must mark something other than the fact that the relevant clauses are untrue. I would suggest that whatever the negative particle is doing in these two examples, it is doing the same in (19) and (20). So perhaps these classic so-called cases of presupposition cancellation are not merely peculiarities for the presupposition theorist to deal with, but part of a much wider problem for truth-conditional accounts of negation. This is not to say that examples (19)–(22) in any way count against a default theory of presupposition, merely that they are not in themselves a sufficient motivation.

The following examples *prima facie* constitute a problem for the non-default CCP model developed in this paper, and have often been taken as motivating a default account like that presented in the current section:

- (23) You say that someone in this room loves Mary. Well maybe so. But it certainly isn't Fred who Loves Mary. And it certainly isn't John.... (after enumerating people in the room.) And that's everybody in the room!
- (24) If I realize later that I have not told the truth, then I will tell confess it to everyone.

(25) I was not aware that you are allowed to do that! (Said eg. by a teacher to a child who is smoking behind the bicycle sheds.)

In the first of these examples (adapted slightly from an example due to Ed Keenan) the cleft sentences all presuppose that someone loves Mary, and this is apparently cancelled by shear weight of logical argument. In the second (due to Gazdar) and third examples conversational implicatures seem to cancel out the presuppositions that I have not told the truth and that you are allowed to do whatever it is respectively.

Yet all three examples could be interpreted in terms of an interesting form of accommodation, which I will call temporary global accommodation. The idea is that in these cases a sort of perspective shift occurs, whereby the speaker temporarily assumes a common ground (normally an apparent belief state of one of the other coversational participants) that he believes to be false. This is clear in (23), which has the form of a reductio ad absurdum argument. The speaker announces with his "well maybe so" that one could take a perspective where somebody loved Mary. He then implicitly presumes the proposition to be true in order to demonstrate its inconsistency with other known facts. The same can occur explicitly, with a statement like "let us assume that somebody loves Mary".

Similarly, it might be claimed that in (24) the speaker is implicitly taking the perspective of somebody who thinks that the speaker is lying, and that in (25) the speaker is taking the perspective of someone who believes that whatever is being done is allowed. All of this is rather tentative, and I do not wish it to be taken as an attack on default accounts of presupposition. On the contrary: I am not attempting to deny that presuppositions are defeasible, but to begin to explain this defeasibility in terms of underlying principles of communication. Yet there is much work to do. Temporary global accommodation is a dangerously powerful mechanism, and a satisfactory account of presupposition would have to involve considerable constraints on its application. My intuition is that all cases of temporary global accommodation will be highly marked. After all, if a speaker could simply change perspective in mid-flow without giving some hint of what was happening, there would be little hope of ever providing a formal model of meaning! At a minimum, one should expect a speaker to maintain a constant perspective unless there is a salient alternative perspective to which he can shift.

#### 4.5 On the Accommodation of Representations

Temporary global accommodation can have much the same effects as a combination of local and intermediate accommodation in a representationalist account like that of van der Sandt. Local accommodation would lead to addition within the representation of each of the negations in (23) of the proposition that somebody loves Mary, and addition within the negation in (25) of the proposition that whatever was being done was allowed, whilst intermediate accommodation would lead to addition within the antecedent of the conditional in (24) of the proposition that the speaker lied.

Indeed, a theory involving a judicious mixture of local, intermediate and global accommodation is a force to be reckoned with, and I think it fair to say that van der Sandt's theory, which crucially involves all three types of accommodation, is the most empirically adequate of any theory of presupposition currently around. Yet even his account is not perfect! Consider the following:

(26) If John's unhappy, then it's a pity that both John and Mary are unhappy.

(27) ?I don't know whether Bush is ill. However, every Republican regrets that Bush is ill.

I think van der Sandt would admit that his theory as it currently stands makes the wrong predictions about both of these. On hearing (26) most people would conclude that Mary is unhappy. Yet van der Sandt would I think predict, after accommodation into the antecedent of the conditional, a meaning something like "If John and Mary are unhappy, then its a pity." The second discourse is intuitively very odd. Yet van der Sandt would predict accommodation of the fact that Bush is ill into the restrictor of the universal, yielding a meaning for the second sentence logically equivalent to "If Bush is ill then every Republican regrets it." This is a perfectly sensible meaning for a sentence to have, but it is not the meaning of the second sentence in (27).

On the other hand, these examples should not be problematic for the CCP model developed in this paper, since in that there are no notions of local or intermediate accommodation. Yet the CCP model developed here is not incompatible with there being a level of discourse representation on which local and intermediate accommodation could operate. But then the processes of local and intermediate accommodation would be quite distinct from the mechanism of global accommodation as I have described it.

Global accommodation, as has been suggested, should be seen as a natural part of the orderly communication process in which conversational participants gradually determine their common ground. On the other hand, there may be times when what is said does not appear compatible with any assumption of common ground. Then the only alternative for a hearer will be to assume that the speaker did not really mean what he said, so the hearer will begin a non-compositional tampering with his representation of the meaning of the speaker's utterance, adding in things that the hearer seems to have omitted to mention but obviously meant. Yet this last process is complicated, and that should explain the observation of both Heim and van der Sandt, which goes unexplained in their work, that global accommodation is always preferred to local accommodation. Incidentally, within the theories of van der Sandt or Heim it would seem considerably less natural to assert that local and intermediate accommodation are fundamentally difficult processes, since in their theories some examples which are obviously very natural and not apparently difficult to process require local or intermediate accommodation — for example (6) above.

# 5 Conclusion

Concerning the interaction between quantification and presupposition, we have seen that a range of examples that had seemed awkward for CCP accounts, but which have been treated successfully in van der Sandt's theory, can in fact also be treated in a pure CCP model without invoking local or intermediate accommodation. However, there is much more data to be looked at, and the simple first-order treatment of quantification assumed is obviously inadequate.

I have contended that global accommodation should be viewed not as a repair strategy on discourse representations, but as addition of information concerning the common ground. This eventually led to the possibility that local and intermediate accommodation of presuppositions, if these types of accommodation occur at all, may be quite different in nature from global accommodation. A common fault of CCP theories is that they predict presuppositions that are too weak. It was suggested that the key to some of these difficulties might lie in a

formalisation of how conversational participants maintain sets of plausible epistemic alternatives. This began to take us in the direction of a formal theory of pragmatics unifying the treatment of presupposition and implicature.

I tried to show that Gazdarian default accounts of presupposition are not so distant from CCP accounts as they might seem. The moral is that the approach of projecting weak non-defeasible presuppositions which are then stengthened using implicature-triggered defaults, is similar in effect to producing a strong but cancellable presupposition in the first place, and either method is compatible with a dynamic view on interpretation.

I hope to have shown that the full potential of CCP based accounts of presupposition has not yet been realised, although after a somewhat dormant decade some might have thought that the CCP paradigm had been exhausted. It seems to me that there are several promising areas requiring both further technical development, and a deeper exploration of relevant data.

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