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PARALLEL QUANTIFICATION

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Parallel Quantification

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Abstract

We investigate various forms of parallel quantification, both from a linguistic and a logical point of view. In particular, we are interested in finding a general branching definition. Our natural language examples suggest strong connections between parallel and collective readings. In fact, many people seem to confuse the two classes of readings. Following the intuitions of [10], we investigate the effects of various notions of maximality. In our logical treatment, starting from basic intuitions of globality and parallelism, we introduce various semantic principles that parallel definition schemes should satisfy. These strongly suggest that the three parallel definition schemes we eventually find are in fact the only ones. In the course of the article we will introduce some notions concerning quantifiers that may be applicable to other parts of generalized quantifier theory as well.

1 Introduction

In standard logical languages all formulas have a linear format. It would seem that this suits the interpretation of natural language, which, after all, is spoken in linear time or written in lines. For instance, in formulas or natural language sentences with a binding two-quantifier prefix, like (1a-b), one of the bindings precedes the other.

(1) a. $\forall x \cdot \exists y \cdot Rxy$
b. Most townsmen hate most villagers.
c. $\mathbf{most}^{[\![\text{townsman}]\!]}x \cdot \mathbf{most}^{[\![\text{villager}]\!]}y \cdot [\![\text{hate}]\!]xy$

On the semantic side, taking the most preferred reading (1c) for (1b), this means that the rightmost variable is bound within the range of the leftmost quantifier. Thus, the y in (1a) depends on our choice for x, and the choice of villagers in (1c) depends on the choice of townsmen.

So logical and natural languages seem to share the principals of syntactic linearity and semantic linear dependence. However, on both sides of the equation, things are not all that clear. Constructions have been proposed in which variable bindings are independent of each other. We will call the ensuing dyadic quantifiers *parallel*.

In logic, the linearity principle is challenged by so-called *branching* quantification, discovered by Henkin ([8]). The idea behind the construction of branching quantifier prefixes is the following. Consider (2a) and its Skolem equivalent (2b).

(2) a. $\forall x_1, x_2 \cdot \exists y_1, y_2 \cdot \phi(x_1, x_2, y_1, y_2)$

b. $\exists f_1, f_2 \cdot \forall x_1, x_2 \cdot \phi(x_1, x_2, f_1(x_1, x_2), f_2(x_1, x_2))$
c. $\exists f_1, f_2 \cdot \forall x_1, x_2 \cdot \phi(x_1, x_2, f_1(x_1), f_2(x_2))$
d. $\begin{matrix} \forall x_1 \cdot \exists y_1 \searrow \\ \\ \forall x_2 \cdot \exists y_2 \nearrow \end{matrix} \phi(x_1, x_2, y_1, y_2)$

In (2a), y_1 and y_2 depend on both x_1 and x_2, as is shown clearly by the Skolem-functions f_1, f_2 in (2b). It seems rather unnatural, that there shouldn't be a formula in some first-order formalism where the dependency-scheme is like in (2c), i.e. y_1 depending only on x_1 and y_2 only on x_2. Therefore, we want something like (2d) to express (2c) in a quasi first-order language, where the branching format signifies that the dependency scheme is non-linear here. Bindings that occur on the same line are interpreted in the usual, linear way. However, there is no dependency between bindings occurring on different lines. Therefore, (2d) has exactly the same interpretation as the Skolem-formula (2c), the formula it was meant to mimic. It is the simplest form of branching standard quantifiers that is genuine, in that it is not reducible to a first order sentence, and in fact, as Ehrenfeucht showed, adding the quantifier-prefix from (2d) to first-order logic would lead to non-axiomatizability ([8], pp. 181-183).

Does branching occur in natural language? Examples of natural language sentences that may express the branching formula (2d) are rather complicated and highly disputed. But the question can be answered positively by introducing branching of generalized quantifiers, cf. [1]. In that case, genuine branching already occurs in the form (3a), prime examples being sentences of the form (3b), like e.g. (3c-d).

(3) a. $\begin{matrix} Q_1^C x \searrow \\ \\ Q_2^D y \nearrow \end{matrix} Rxy$

b. Q_1 C's and Q_2 D's [all] R each other.
c. Most philosophers and most linguists agree with each other about branching quantification.
d. More than half the dots and more than half the stars are all connected by lines.

In the present article, we will only be concerned with this two quantifier format. However, in general, it's not entirely clear what we mean by (3a). One of our tasks in this article will be to find reasonable truth conditions for the interpretation of branching formulas.

The second class of parallel quantification that occurs in natural language is formed by so-called *cumulative* quantification ([9]), graphically represented as (4a). Natural language sentences allowing this reading are in general of one of the forms (4b-c), like e.g. (4d-e). These are easily interpreted, though we still have to be careful not to confuse them with certain forms of collectives, cf. section 2.1.

(4) a. $\begin{matrix} Q_1^C x \mid \\ \\ Q_2^D y \mid \end{matrix} Rxy$

b. Q_1 C's R Q_2 D's [together]
c. Q_2 D's are R-ed by Q_1 C's
d. In the kitchen, four boys were eating three pizzas.
e. Three elephants were chased by a dozen hunters.

My aims in this article are the following:

- To find an acceptable general interpretation for the branching formula (3a). In particular, I will reconcile Westerståhl's and Sher's truth conditions for this formula ([13], [10]), which at first sight seem totally different, by showing that, after some modifications, their basic ideas and intuitions can be used to obtain a single branching scheme **B1**, cf. section 5.3.
- To make a clear distinction between parallel quantification and certain collective readings that have been confused with parallel quantification, and suggest a possible reason behind the apparent connection between the two groups of readings.
- To make plausible that, even from a logical point of view, there can be only three parallel forms of dyadic quantification, one of these being the cumulative reading, and the other two representing two possible interpretations of branching quantification.

Here is how I will go about. I will start linguistically and examine the meaning of sentences that allow parallel readings, in section 2. The analysis will lead to a number of *parallel definition schemes* $\sigma = \sigma(Q_1, Q_2, R)$, where Q_1, Q_2 represent the two quantifiers occurring in the sentence and R is a relation denoting the VP. These will generate parallel unary dyadic quantifiers $\sigma(Q_1, Q_2)$ for specified classes of unary logical quantifiers Q_1, Q_2. These classes may consist of all quantifiers, all upward monotonic ones, all downward monotonic ones, all convex ones, etc. . To make them really work in interpreting natural language, we modify these *unary parallel definition schemes* to binary ones $\sigma^*(Q_1, C, Q_2, D, R)$, Q_1, Q_2 binary logical quantifiers with respective domain restrictors (noun denotations) C, D, in section 3.

In section 4 I will start a logical analysis of parallel quantification, by introducing semantic principles that parallel definition schemes $\sigma = \sigma(Q_1, Q_2, R)$ should satisfy, as well as some that are characteristic for specific classes of parallel quantifiers. As an example, I mention the principle of

symmetry $\sigma(Q_1, Q_2, R) \Leftrightarrow \sigma(Q_2, Q_1, \breve{R})$,

which can be found in [13] and [3]. The relevance of these principles is threefold. First of all, we may eventually be able to use such principles to find elegant mathematical characterizations of specific classes of parallel quantifiers. Also, they allow us to analyze the *concept* of parallel quantification. It is of course impossible to take any reading of a parallel-type natural language sentence to be its parallel reading, as these sentences are usually highly ambiguous. By analyzing readings by their principles we can check whether they are indeed parallel. Finally, we may even be able to use a number of semantic principles to show that the number of parallel definition schemes is limited – my guess is that there are only three. These three applications of the logical approach started in section 4 will be the subject of section 5, where I will present partial answers to the associated questions.

2 Natural Language

In this section we will analyze natural language examples of parallel quantification in order to see if these can help in finding an acceptable interpretation of branching quantification.

In doing so, we have to be aware of the danger of accepting *a* reading of a parallel-type sentence as *the* parallel reading. This means that we have to check possible readings against what our intuition tells us about the *concept* of parallel quantification. It will turn out that collective readings are easily misinterpreted as parallel and that to avoid this we have to apply the intuition that parallel readings ought to be 'global', evaluating the *entire* VP denotation R, not just some product contained in it.

We will set out with the easier form of parallel quantification, the cumulative readings. After that, we will review Barwise's treatment of monotonic branching ([1]), and Sher's generalization of Barwise's upward branching ([10]). The more logically oriented approach towards finding a general branching definition found in [13] will be investigated later, in section 5.

Before we start our discussion of natural language examples of parallel quantification, two technical remarks have to be made.

1. In the present section, we will only consider unary, logical quantifiers, using definition schemes $\sigma = \sigma(Q_1, Q_2, R)$, where Q_1, Q_2 are two unary logical quantifiers and R is a 2-place relation interpreting the VP. From these, we easily obtain the more general definition schemes $\sigma^*(Q_1, C, Q_2, D, R)$, Q_1, Q_2 binary, logical and conservative, with respective domain-restrictors C, D denoting nouns, cf. section 3. Furthermore, considering only logical quantifiers isn't a real restriction, since parallel sentences featuring non-logical quantifiers do not seem to make much sense.

2. Since our definitions will crucially involve cross-products $A \times B$ – the basic independent items in relational algebra –, we have to make a choice regarding the empty relation, which is multi-representable as a cross-product by taking either of A, B empty. Taking into account that the cumulative interpretation

   $\mathbf{C}\ \exists A, B \neq \emptyset \cdot domR = A\ \wedge\ rangeR = B\ \wedge\ A \in Q_1\ \wedge\ B \in Q_2$

   automatically does not allow that exactly one of A, B is empty, and noting the failure of sentences like (5), I propose that we disallow cross-products of the form $A \times \emptyset$, $\emptyset \times B$ if A, B aren't empty themselves.

   (5) *No men and three women hate each other.

   We achieve this by treating the empty relation separately, defining

   $$\sigma(Q_1, Q_2, \emptyset) \quad \Leftrightarrow \quad \emptyset \in Q_1 \wedge \emptyset \in Q_2$$

   for all parallel definition schemes σ, and considering only non-empty relations henceforth. Note that the other uniform choice, which is actually implicit in Barwise's original definitions, and therefore also in Sher's and Westerståhl's definitions,

   $$\sigma(Q_1, Q_2, \emptyset) \quad \Leftrightarrow \quad \emptyset \in Q_1 \vee \emptyset \in Q_2\ ,$$

   would leave (5) satisfiable.

2.1 Cumulatives

Cumulative quantification, graphically represented as (6a), is the simplest form of parallel quantification. The semantics for this class is fixed as the cumulative reading **C** (equivalently **C′**) of cumulative-type sentences like (6b-c). As an example, the cumulative reading of (6c) states that three boys ate a total of four pizzas, and that this assertion describes the whole boys-eating-pizzas situation.

(6) a. $\begin{matrix} Q_1x \\ Q_2y \end{matrix} \Big| Rxy$

b. Three elephants were chased by a dozen hunters.

c. Three boys ate four pizzas.

C $Q_1x \cdot \exists y \cdot Rxy \ \wedge \ Q_2y \cdot \exists x \cdot Rxy$

C′ $dom\ R \in Q_1 \ \wedge \ range\ R \in Q_2$

In this paragraph I will point at a collective reading that seems related to the cumulative interpretation. Consider (7a). One may argue that this should imply (6b). The reading of cumulative-type sentences that allows additions like in (7a) is **T**(ogether): there are groups A ('hunters') in Q_1 ('a dozen') and B ('elephants') in Q_2 ('three') that match up to each other cumulatively in the relation R ('chase'). I will call this the *cumulating collective* interpretation.

(7) a. Three elephants were chased by a dozen hunters, and two others were chased by ten.

b. In the kitchen, three boys ate four pizzas.

c. Last summer in this hotel, 148 beds served 4193 guests.

T $\exists A \in Q_1, B \in Q_2 \cdot (R[A] = B \ \wedge \ \breve{R}[B] = A)$[1]

Restating the cumulative reading **C** in terms of this collective reading, it says that the choices A, B in **T** are *maximal* sets (in any sense of maximality) such that the condition $R[A] = B \wedge \breve{R}[B] = A$ holds; in other words, **T** holds, and provides a description of all of R. This reading comes out more clearly if we modify our examples in such a way that it becomes clear that we are describing the whole situation within a certain context, like in (7b-c).

The idea that parallel quantification should be 'global' will come back in our analysis of branching quantification. It remains an open question how the reading **T** should be captured in Van der Does' framework ([4]), and what its exact relation with **C** is. My guess would be that parallel quantifiers can be obtained as globalized versions of certain collective readings. The deeper reason behind this connection may have something to do with static versus dynamic interpretation contrast.

[1] Where $R[X]$ and $\breve{R}$ are defined by

$$R[X] \ = \ \{d \mid \exists x \in X \cdot Rxy\}$$
$$\breve{R}xy \ \Leftrightarrow \ Ryx$$

2.2 Monotonic branching

A first step towards a truth-definition for branching non-standard quantifiers was made in [1]. In solving the disputed question whether genuine branching occurs in natural language, Barwise argues that the (affirmative) answer is most easily given by considering branching of generalized quantifiers. For this purpose he uses sentences like (8a-c).

(8) a. Quite a few philosophers and quite a few linguists [all] agree with each other about branching quantification.
b. Less than half the dots and less than a third of the stars are all connected by lines.
c. Less than half the villagers and most townsmen [all] hate each other.

In the intended branching reading of (8a) we are only interested in the relation 'agree' between philosophers on the one hand, and linguists on the other, not among the groups of linguists and philosophers themselves. Inserting the predicate quantifier 'all' in these sentences seems to help us obtain the branching reading.

Note that there is no linear dependence between the quantifiers in (8a-b). Furthermore, both quantifiers in (8a) are upward monotonic, and both quantifiers in (8b) are downward monotonic. Both these sentences make perfect sense, but mixed monotonicity examples like (8c) are definitely trickier. In fact, both [1] and [13] claim that this type of sentence is impossible to make sense of. On this issue I am inclined to agree with Sher, who does obtain an acceptable reading for the mixed monotonicity case. Exploiting the monotonicity-behavior of the underlying monadic quantifiers, Barwise proposes the following upward (**BU**) and downward (**CD**) branching definition schemes.[2]

BU $\exists A, B \neq \emptyset \cdot (A \times B \subseteq R \wedge A \in Q_1 \wedge B \in Q_2)$
CD $\exists A, B \neq \emptyset \cdot (A \times B \supseteq R \wedge A \in Q_1 \wedge B \in Q_2)$[3]

It seems obvious that these provide *sufficient* conditions – in fact, they are the minimal monotonic definition schemes, cf. section 5.1. But we will see later that **CD** may be too strict.

Here are some justifications for the truth-definition **BU**, some of which also hold for **CD**:

- Intuitively, **BU** seems to provide the preferred reading of (8a).
- The branching of $\forall, \exists$ is a special case of **BU**.
- Upward branching dyadic quantifiers are upward monotonic under definition **BU**, and the same kind of thing holds for **CD**. In section 4 we argue that this preservation of monotonicity behavior is one of the semantic principles that parallel definition schemes should satisfy.

[2] Actually, unlike us, he doesn't treat the empty relation separately, so he gives $\exists A \in Q_1, B \in Q_2 \cdot A \times B \subseteq R$ in the upward case. The effect of this definition as opposed to **BU**, is that $\emptyset \in Q \Leftrightarrow \emptyset \in Q_1 \vee \emptyset \in Q_2$, rather than $\emptyset \in Q \Leftrightarrow \emptyset \in Q_1 \wedge \emptyset \in Q_2$. As we explained in the second remark preceding this section, we prefer the latter. In the downward case there isn't any difference, but we use this formulation to get uniform definition schemes.

[3] Here is the key to the labeling of these definition schemes: **B**=branching, **C**=cumulative, **U**=upward, **D**=downward. The reason behind the actual names of the present two schemes will become clear in the course of this section.

- $\mathbf{BU}(Q_1, Q_2, R)$ implies both its linear variants $Q_1 x \cdot Q_2 y \cdot Rxy$ and $Q_2 y \cdot Q_1 x \cdot Rxy$.[4]
- A point stressed by Barwise is that **BU**, **CD** are analogous to

$$\begin{array}{lcl} A \in Q & \Leftrightarrow & \exists A' \in Q \cdot A' \subseteq A \\ A \in Q & \Leftrightarrow & \exists A' \in Q \cdot A' \supseteq A \end{array} \tag{1}$$

  which provide valid second-order definitions, for upward and downward monotonic monadic Q, respectively.

In connection with the last remark, it should be noted that there are other obvious second-order equivalents:

$$\begin{array}{lcl} A \in Q & \Leftrightarrow & (\forall A' \cdot A' \supseteq A \ \Rightarrow \ A' \in Q) \\ A \in Q & \Leftrightarrow & (\forall A' \cdot A' \subseteq A \ \Rightarrow \ A' \in Q) \end{array} \tag{2}$$

for upward/downward monadic Q, respectively. So as far as the 'monadic analogue' principle goes, we might just as well have defined upward and downward branching as

CU $\forall A, B \neq \emptyset \cdot (A \times B \supseteq R \ \Rightarrow \ A \in Q_1 \wedge B \in Q_2)$
BD $\forall A, B \neq \emptyset \cdot (A \times B \subseteq R \ \Rightarrow \ A \in Q_1 \wedge B \in Q_2)$

Note that the downward variant **BD** wouldn't have made any sense if we had allowed A, B such that, e.g., $A = \emptyset, B \neq \emptyset$; yet another reason for treating the empty relation separately.

Before we continue, two remarks should be made about the four definition schemes in this section.

1. As is easily checked, **BU** implies **CU** and **CD** implies **BD** for Q_1, Q_2 of the intended monotonicity behavior, but the converses of these implications don't hold.
2. **BU** and **BD** estimate R from below, whereas **CU** and **CD** do the opposite. This discrepancy between the ways of approaching R may be a fundamental difference between branching and cumulative quantification, though at this point this can be no more than a guess.

Definition **CU**, to be honest, doesn't make much sense as a branching definition; it does not even pass the test of coincidence with the branching of $\forall, \exists$:

example
Consider a domain $E = \{1, 2\}$, a formula

(9) $$\begin{array}{l} \forall x \searrow \\ \qquad\quad Rxy \\ \forall y \nearrow \end{array}$$

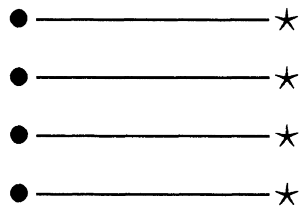
[4]However, there is no connection between $\mathbf{CD}(Q_1, Q_2, R)$ and its linear variants. [13] calls this fact surprising, but here is a simple explanation. If we write $Q_1 = \neg Q'_1$, $Q_2 = \neg Q'_2$ (so Q'_1, Q'_2 are upward monotonic), the first linear variant gets the form $\neg Q'_1 x \cdot \neg Q'_2 y \cdot Rxy$, or, equivalently, $(\neg Q'_1 \neg) x \cdot Q'_2 y \cdot Rxy$. The converse $(\neg Q'_1 \neg)$ of Q'_1 is upward monotonic, whence the whole quantifier prefix is upward monotonic. In contrast, the branching version should be downward monotonic. So there *shouldn't* be an implicational connection here! In fact, it seems to me that downward monotonic branching formulae should imply the *negations* of their linear counterparts, and if we use **CD**, they indeed do.

and a relation $R = \{(1,1),(2,2)\}$. (9) is equivalent to $\forall x \cdot \forall y \cdot Rxy$, so it shouldn't hold for this particular relation. However, $\mathbf{CU}(\forall, \forall, R)$ holds. □

So we reject the proposed definition **CU** as a branching definition. It is in fact, as you have probably guessed by its name, the upward cumulative reading. On the other hand, **BD** seems to make a lot of sense:

example
Consider the sentence

(10) Less than four dots and less than four stars are all connected by lines.



There are 4 groups of dots and stars that are all connected by lines, all consisting of 1 dot and 1 star. If we interpret (10) by **CD**, we would reject this relation of groups of 1 dot and 1 star that are all connected by lines, on the grounds that there are too many dots and stars involved in the connectedness relation. However, as I read (10), it only says something about the number of dots and stars that are all connected to each other,[5] not about how many dots are connected to some star and how many stars are connected to some dot.[6] Therefore, I would accept (10) here. □

The reading described in this example is captured exactly by **BD** above. It is reminiscent of the ideas presented in [10], the terms 'big parties' and 'all' reflecting the concept '*maximal each-all dependence*' discussed there. Sher's approach will be the subject of the next section. Again I stress that the existence of the reading **BD** for downward branching-type sentences does not automatically imply that it is the branching reading. However, since I cannot find another explanation for its existence as a reading, and it is in accordance with my intuition of the branching concept, I accept it as a possible downward branching reading. I called it **BD**, for if it exists, I don't see what it can be but the downward branching definition scheme.

Here are some advantages of the interpretation **BD**:

- **BD** seems to provide the preferred meaning of downward branching-type sentences, as is illustrated by the example above.

[5] But remember that we are not interested in connections amongst the groups of dots and stars.

[6] If you do not obtain this reading, try binding the compound subject together by pronouncing it slightly quicker, and stressing 'all' by hesitating before it and putting it in a slightly higher tone. Also, the **BD** reading can be brought out by paraphrasing (10) as (11):

(11) Less than (four dots and four stars) are all connected by lines.

This suggests a connection between **BD** and *resumptives* – another line of future research. For an overview of various types of dyadic quantification, cf. [3]

- **BD** captures the negative flavor of sentences like (10); in contrast, **CD** is distinctly positive.
- Downward branching quantifiers are downward monotonic under **BD** as well as **CD**.
- As we already mentioned, **BD** has a monadic analogue (2).
- As Sher notes, **CD** is equivalent to the cumulative interpretation **C**.[7] There seems to be no reason why this should be so. If we use **BD** as the downward branching reading, the unwanted equivalence disappears.

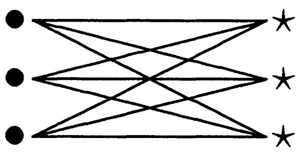
For these reasons, I prefer **BD** as the downward branching reading over **CD**. However, it seems fair to say that **CD** should not be rejected as a branching scheme just like that – after all, no one but Sher has objected to it. Therefore, rather than choosing between **BD** and **CD**, I will investigate both these alternatives in this article. I will combine the four monotonic parallel definition schemes to get more general schemes in section 5.2.

2.3 Branching

A linguistically inspired approach towards finding a general branching definition was made by [10]. The general feeling about the branching formula

$$\begin{array}{l} Q_1 x \searrow \\ \qquad\quad Rxy \\ Q_2 y \nearrow \end{array}$$

is that its interpretation should have something to do with the basic independent objects in relational algebra: Cartesian products $A \times B$. Sher's basic intuition is that of these products only *maximal* ones are relevant. For instance, it would be rather strange to use sentence (13) as a description of the following graph, since there are not two dots and two stars, but three dots and three stars that are all connected by lines.[8]



(13) Two dots and two stars are all connected by lines.

Taking these considerations into account, Sher simply defines

S $\exists A, B \neq \emptyset \cdot (A \times B \subseteq \text{maximal in } R \ \wedge \ A \in Q_1 \ \wedge \ B \in Q_2)$,

[7]This is of course the reason why I called it **CD**: it is certainly the downward cumulative reading, and only one of the alternatives for downward branching.

[8]Admittedly, (13) can be said to hold in this situation, in its reading paraphrased by (12), but this is certainly not the branching reading.

(12) There are two dots and two stars that are all connected by lines.

where ⊆maximal is defined in the obvious way:

$$A \times B \subseteq\text{maximal in } R \;:\equiv\; A \times B \subseteq R \;\wedge$$
$$A \times B \subseteq A' \times B' \subseteq R \Rightarrow A \times B = A' \times B'$$

I don't know how Sher got to this maximality intuition, but it seems sound to me; if we are interested in the validity of a sentence like (13), we look for big cross-products within our relation and check if their size is 2×2. Another thing that can be said in favor of **S** is that it contains Barwise's **BU** as a special case. Also, I do feel that **S** can be obtained as *a* reading of many branching-type sentences, in the way shown by (14a-c):

(14) a. Four boys in my class and three girls in your class have all dated each other.
b. Four boys in my class have dated the same three girls in your class, and three girls in your class have dated the same four boys in your class.
c. Four boys in my class have dated the same three girls in your class.

The 'the same' sentence (14c) can obviously be interpreted with **S**, which by the way is why I called this definition scheme **S** in the first place. The reasoning behind the ⊆maximality condition comes out clearly in the relative failure of (15a) and (15b).

(15) a. ?Four of my friends applied to the same three graduate programs, and to the same four graduate programs.
b. ?Four of my friends applied to the same three graduate programs, and two of my other friends also applied to these.

It may be argued that (15a) and (15b) make some sense, they definitely sound strange. The point is, that the clauses before the comma are not exhaustive. We conclude that the **S**-reading is maximized because of the above pragmatic reasons.

Should branching quantification involve ⊆maximality? Sher sensed that some kind of maximality is involved in branching quantification, and rightly so, witness the example at the beginning of this section as well as the discussion below. I think the reason why out of all possible concepts of maximality she chose ⊆maximality is that she took 'the same'-sentences to be some way of expressing branching quantification – the examples (15a-b) are actually adapted from [10]. This is another confusion between collective readings and parallel quantification. Although parallel quantification and collective readings are probably related, they definitely have different underlying concepts. This should come out as local versus global assessment of the relation R, as well as in validity of certain preservation properties I will present in section 4.3.

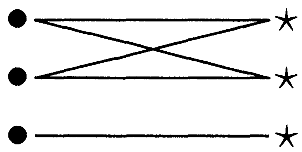
So Sher's maximality intuition makes sense, her definition **S** is compatible with Barwise's **BU**, and it can be obtained as a reading for many branching-type sentences. But I have serious doubts as to whether it is a branching reading. For one thing, my feeling about the whole concept of parallel quantification tells me that the branching definition should be global, evaluating all of the relation R at the same time, whereas **S** only looks at one cross-product contained in R. The situation is similar to what we saw in section 2.1, where we found that cumulative-type sentences allow both a cumulative and a cumulating collective reading. This suggests that we bring out the branching reading more clearly if we paraphrase the branching-type sentences as in (16), where it is obvious that we are interested in all of the relation $[\![\text{agree}]\!]$.

(16) All in all, few linguists and few philosophers all agree with each other about branching quantification.

If you are not convinced by these vague considerations, here is something more tangible. Consider (17a-b)

(17) a. Most townsmen and most villagers hate each other.
b. Less than half the townsmen and less than half the villagers hate each other.
c. Most townsmen hate the same majority of villagers.

These two sentences obviously cannot be true at the same time. However, if we would use Sher's interpretation **S**, they would both be true in the following situation, where the domain contains three townsmen (dots) and three villagers (stars):



So Sher's proposed branching definition **S** fails here. The formal principle of

locatedness $\sigma(Q_1, Q_2, R) \wedge \sigma(Q_1, Q_2', R) \Rightarrow Q_2 \cap Q_2' \neq \emptyset$.

is inspired by the failure of (17a-b), cf. section 4. In fact, the quantifiers in (17a-b) don't allow a 'the same' sentence; if you try, you end up with something like (17c), the description 'majority of' revealing that **S** is in fact a collective reading. All of these considerations lead us to the conclusion that **S** is the 'the same' interpretation, not a branching reading. I will call **S** the *maximal each-all collective* reading.

If we accept Sher's basic maximality intuition as a basis for our branching definition, there has only been one point where we could have gone wrong: in choosing the maximality notion. Looking at the example above, and considering that we want a global measure, that looks at all of the cross-products contained in a relation, we can only come to the conclusion that we need a notion of maximality that is based on cardinality rather than inclusion. Here is a first attempt:

$$\begin{aligned} A \times B \text{ is } (E, \#)\text{maximal in } R \;&:\equiv\; A \times B \subseteq R \;\wedge \\ &\quad A' \times B' \subseteq R \;\Rightarrow\; A' \preceq_E A \wedge B' \preceq_E B\ , \end{aligned}$$

where $\preceq_E$ is a double cardinality notion taking into account the remainder $E - A$, which consists of the elements in the entity domain E that are non-A's:

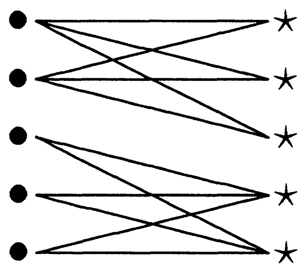
$$A' \preceq_E A \;:\equiv\; |A'| \leq |A| \;\wedge\; |E - A'| \geq |E - A|$$

Using $\preceq_E$ we define $=_E$ and $\prec_E$ in the obvious way. For instance, if $E = I\!N$, $A = E - \{0\}$, $B = E$, then $A \prec_E B$, even though $|A| = |B|$. Of course, when E is finite, $=_E$ comes down to equi-cardinality. It is worth noting that the global behavior of $(E, \#)$maximality, i.e. the fact that it considers all of the products contained in R at the same time, can be ascribed to the fact that the $\preceq_E$-relation is logical in that it is invariant under pairs of permutations. In fact, it is the closure of $\subseteq$ under permutations.

However, the following example shows a subtlety that $(E, \#)$cardinality does not take care of.

example

(18) a. At least two dots and at least two stars are all connected by lines.
b. Two or three dots and three stars are all connected by lines.



The connectedness relation consists of two cross-products between dots and stars, of sizes 2×3 and 3×2, respectively. This means that neither of the two cross-products is $(E, \#)$maximal. However, (18a) should definitely hold here, as is properly modeled by the upward branching scheme **BU**. Therefore, a branching definition based on $(E, \#)$-maximality can never work, because it fails to coincide with **BU** for upward monotonic Q_1, Q_2.

Next, consider (18b). I would say that this sentence is true here. The point is, that we should take into account the actual quantifiers used in the sentence we are interpreting.

First look at the dots. In the first cross-product there are two, in the second one three. So is there any difference? The relevant quantifier **2or3** doesn't make a distinction between two or three, so neither should our notion of maximality.[9] We conclude that on the side of the dots, there is no real difference in size between the two cross-products. The stars then tell us, that the upper cross-product should be considered maximal. Checking its size against 2×3 against **2or3**, **3**, we accept (b) in this situation. □

Here is the definition formalizing the ideas introduced in the example:

$$
\begin{aligned}
&A \times B \text{ is } (E, \#)\text{maximal in } R \text{ relative to } Q_1, Q_2 \;:\equiv \\
&\quad A \times B \subseteq R \;\wedge \\
&\quad A' \times B' \subseteq R \;\Rightarrow\; (A' \preceq_{Q_1,E} A \wedge B' \preceq_{Q_2,E} B) \;,
\end{aligned}
$$

where the ordering relations $\preceq_{Q,E}$ are defined by

$$A' \preceq_{Q,E} A \;:\equiv\; A' \preceq_E A \vee A' \sim_{Q,E} A \;,$$

and the equivalence relations $\sim_{Q,E}$ ('is in the same convex part of Q') are defined as

$$
\begin{aligned}
A' \sim_{Q,E} A \;:\equiv\; & A' \preceq_E A \;\wedge\; (A' \preceq_E A'' \preceq_E A \Rightarrow (A \in Q \Leftrightarrow A'' \in Q)) \;\vee \\
& A \preceq_E A' \;\wedge\; (A \preceq_E A'' \preceq_E A' \Rightarrow (A \in Q \Leftrightarrow A'' \in Q))
\end{aligned}
$$

[9] However, if the quantifier would have been **2or4**, and the numbers of dots 2 and 4, respectively, there would have been a difference: we pass the 'gap' 3 when going from 2 to 4.

It may be revealing to see the accompanying strict ordering relation $\prec_{Q,E}$ as induced by the following ordering of $P(E)/\sim_{Q,E}$:

$$[A']_{\sim_{Q,E}} \prec^*_E [A]_{\sim_{Q,E}} \quad :\equiv \quad A' \prec_E A$$

A closer examination of the ordering relations we introduced in this section may be worthwhile. For example, does a relativization of $\subseteq$ to a quantifier Q lead to an acceptable ordering relation? Can we characterize the orderings by their properties? These questions, however, are not within the scope of the present article.

Our branching definition based on the maximality intuition is simply:

B1 $\exists A, B \neq \emptyset \cdot A \times B$ $(E, \#)$maximal in R relative to $Q_1, Q_2 \;\wedge\; A \in Q_1 \wedge B \in Q_2$

We will see in section 5 that the same definition can be obtained by applying the techniques introduced in [13] to **BU+BD**.

3 Binary Quantifiers

At the start of section 2 we put aside the issue of binary quantifiers and conservativity with a short remark, because it would unnecessarily complicate the definitions of parallel quantification we were after. However, now that we have the unary definitions, we should check if we were right in claiming that we can easily find their extensions to binary quantifiers.

The first question we should answer is whether or not non-conservative binary quantifiers allow parallel quantification. One quantifier that has been claimed to be non-conservative is **many**. Some attempts to obtain parallel readings with this quantifier are shown in (19a-d).

(19) a. Many boys ate four pizzas.
b. *Many townsmen and most villagers all hate each other.
c. Many townsmen and many villagers all hate each other.
d. Many townsmen and villagers all hate each other.

Cumulative-type sentences with the quantifier **many** like (19a) obviously don't allow a cumulative reading. On the branching side, it is impossible to make sense of sentences like (19b); the complex nature of the sentence does not allow us to check the group of townsmen against some reference set. A **many/many** branching-type sentence like (19c) seems to make some sense, but in fact its only possible reading is the resumptive (19d).[10]

Another quantifier that does not satisfy conservativity is **only**. Curiously, it seems to have exactly the same behavior in parallel environments as **many**. The examples (20a-d) behave exactly like (19a-d).

(20) a. Only boys eat four pizzas.
b. *Only townsmen and most villagers all hate each other.
c. Only townsmen and only villagers all hate each other.
d. Only townsmen and villagers all hate each other.

[10]By 'resumptive' we mean pair-quantification, as in 'Most lovers will eventually hate each other.'

That the branching-type sentence (20c) should be read as the resumptive (20d) nicely corresponds to intuitions about (21a) in its meaning (21b), which has been claimed to be the resumptive (21c).

(21) a. No one hates no one.
b. No one hates anyone.
c. $\mathbf{no}^2(x,y) \cdot [\![\text{hate}]\!](x,y)$

We conclude from the above survey that only conservative binary quantifiers allow parallel readings, so we can restrict attention to those. It is obvious that parallel quantifiers defined with conservative binary quantifiers should be conservative themselves. There are various types of conservativity of dyadic quantifiers, but in the parallel case only few come into consideration. If we do not have to take into account the possibility of the empty set messing things up, it is quite easy to choose a type of conservativity that works. Two examples suffice.

(22) a. Last summer in this hotel, 200 beds served 4000 guests.
b. Most townsmen and most villagers all hate each other.

In (22a) it is obvious that we are only interested in guests sleeping in beds, not in guests sleeping on the floor or hotel employees sleeping at their work station. Also, in (22b), we are only interested in the hate relation between townsmen and villagers.

Judging from the above two examples, it is tempting to define binary parallel definition schemes σ^* from their unary counterparts σ by the following rule:

(23) *For binary logical conservative Q_1, Q_2:
$\sigma^*(Q_1, C, Q_2, D, R) \Leftrightarrow \sigma(Q_1(C), Q_2(D), R \cap (C \times D))$

However, if we apply this type of conservativity against parallel-type sentences of which the interpretation does involve the empty set, we get into trouble. Consider the following natural language examples involving the quantifier **no**. It is obvious that (24a-b) fail.

(24) a. *No boys ate three pizzas together.
b. *No men and three women all hate each other.

However, if we use (23) to generate our binary parallel definition schemes, (24a-b) would be left satisfiable. The point is, that although we disallowed cross-products of the form $A \times B$ when exactly one of A, B is empty, we didn't exclude non-empty products $A \times B$ such that exactly one of $A \cap C, B \cap D$ is empty. The best way to solve this problem is to consider all domains at once and assume the principle of extension.[11]

First note that we have been rather sloppy in section 2, where we tacitly assumed a fixed domain $E \times E$. However, this will cause no problems, since the definitions we gave will work uniformly in all non-empty domains $C \times D$. What we want to do now, is to use our unary parallel definition schemes $\sigma(Q_1, Q_2, R)$ to derive the accompanying binary ones $\sigma^*(Q_1, C, Q_2, D, R)$, making sure that for every product $A \times B$ that turns up in the interpretation, we will automatically have $A \cap C = A$, $B \cap D = B$.

[11]For monadic quantifier properties like conservativity, extension, logicality (permutation-invariance), monotonicity, etc., cf. [12] or [2], as always.

To achieve this, we use the notion of *relativization* of a unary quantifier. All binary quantifiers Q that satisfy extension and conservativity, can be had as relativizations Q'^r of unary quantifiers Q', and vice versa – cf. [12], pp. 18,64. This means that

$Q'_C(A \cap C) \quad \Leftrightarrow \quad Q_E(C, A) \quad$ for all $A, C \subseteq E, C \neq \emptyset$

Using this fact, the type of conservativity we want in the parallel case is clear. We state it in the form of a semantic principle that parallel definition schemes should satisfy. If σ is a unary parallel definition scheme, then its corresponding binary parallel definition scheme σ^* should satisfy

conservativity preservation For all unary logical Q_1, Q_2:

$$\sigma^*(Q_1^r, C, Q_2^r, D, R) \;\Leftrightarrow\; \sigma((Q_1)_C, (Q_2)_D, R \cap (C \times D))$$

Note that this type of conservativity removes the problems the empty products caused in the earlier conservativity attempt (23). It corresponds to the P-conservativity (type 2 conservativity) of [4], ch. 5 – yet another connection between parallel quantification and collective readings.

Here are the binary forms of the parallel definition schemes from section 2.

BU* $\exists A, B \neq \emptyset \cdot A \subseteq C \wedge B \subseteq D \wedge A \times B \subseteq R \cap (C \times D) \wedge Q_1(C, A) \wedge Q_2(D, B)$
BD* $\forall A, B \neq \emptyset \cdot (A \subseteq C \wedge B \subseteq D \wedge A \times B \subseteq R \cap (C \times D)) \Rightarrow (Q_1(C, A) \wedge Q_2(D, B))$
CU* $\forall A, B \neq \emptyset \cdot (A \subseteq C \wedge B \subseteq D \wedge A \times B \supseteq R \cap (C \times D)) \Rightarrow (Q_1(C, A) \wedge Q_2(D, B))$
CD* $\exists A, B \neq \emptyset \cdot A \subseteq C \wedge B \subseteq D \wedge A \times B \supseteq R \cap (C \times D) \wedge Q_1(C, A) \wedge Q_2(D, B)$
C* $Q_1(C, dom(R \cap (C \times D))) \wedge Q_2(D, range(R \cap (C \times D)))$
B1* $\exists A, B \neq \emptyset \cdot A \times B$ $(C, D, \#)$maximal in $R \cap (C \times D)$ relative to $Q_1, Q_2 \wedge$ $Q_1(C, A) \wedge Q_2(D, B)$

where $(C, D, \#)$maximality relative to Q_1, Q_2 is defined in the obvious way.

We saw in this section that non-conservative quantifiers do not allow parallel quantification. Also, for conservative quantifiers we have a uniform way of obtaining binary parallel definition schemes from unary ones. From these two facts we can conclude that we were right in considering only unary logical quantifiers on fixed universes. We will continue to do so when we start our logical analysis of parallel quantification, in section 4.

4 Principles

In the previous section we showed that the analysis of binary parallel definition schemes $\sigma^*(Q_1, C, Q_2, D, R)$ can be reduced to the treatment of unary schemes $\sigma(Q_1, Q_2, R)$ by the principle of conservativity preservation. In the present section we will start our logical analysis of these unary schemes, by introducing semantic principles that they should satisfy.

As a first remark, from a technical point of view, we may look at unary parallel quantifiers in two ways. The first one is to ask, which dyadic quantifiers Q qualify as parallel quantifiers. The second one is to determine which definition schemes $\sigma = \sigma(Q_1, Q_2, R)$ lead to classes of parallel quantifiers. These two points of view come down to the same thing, since all parallel quantifiers should obviously be

decomposing $\forall A, A', B, B' \neq \emptyset \cdot (A \times B \in Q \wedge A' \times B' \in Q \;\Rightarrow\; A \times B' \in Q)$[12]

[12]This is a rather general principle that holds for many dyadic quantifiers that are definable by two monadic ones; e.g. positive Fregean ones.

By defining

$$\begin{array}{rcl} A \in Q_1 & :\equiv & \exists B \neq \emptyset \cdot A \times B \in Q \;\; (A \neq \emptyset) \\ B \in Q_2 & :\equiv & \exists A \neq \emptyset \cdot A \times B \in Q \;\; (B \neq \emptyset) \\ \emptyset \in Q_1 & :\equiv & \emptyset \in Q \\ \emptyset \in Q_2 & :\equiv & \emptyset \in Q, \end{array}$$

which is possible exactly because of the decomposing principle, we are back at the level of parallel definition schemes, with the extra principle of

productivity $\forall A, B \neq \emptyset \cdot (\sigma(Q_1, Q_2, A \times B) \;\Leftrightarrow\; A \in Q_1 \wedge B \in Q_2)$
$\sigma(Q_1, Q_2, \emptyset) \;\Leftrightarrow\; \emptyset \in Q_1 \wedge \emptyset \in Q_2$.[13]

In this section we will examine semantic principles that we could impose on scope-independent definition schemes σ, in the way propagated by Van Benthem in [3]. In order to check the 'soundness' of the principles we propose, we can use two perspectives. From a linguistic point of view, we should look at all instances of parallel-type sentences, and see if they satisfy all principles. From a logical point of view, we can check the principles against a couple of basic intuitions.

First of all, we can view parallel quantifiers as two-dimensional forms of monadic ones. Secondly, the two monadic quantifiers used in the parallel one have to be treated as on a par. Finally, in contrast with collective readings, parallel quantification should involve a 'global' condition. The principles of productivity and symmetry model part of the parallelism intuition.

symmetry $\sigma(Q_1, Q_2, R) \;\Leftrightarrow\; \sigma(Q_2, Q_1, \breve{R})$

The latter principle doesn't seem to be of much use in itself; it just ensures that every other principle we propose will have a symmetric meaning. I.e. there is no difference between giving a full symmetric formulation of a principle or a partial one.

The basic insight of global parallelism gives rise to principles of heredity of structural properties, of the form 'if Q_1, Q_2 both have the property X then so does the parallel quantifier $\sigma(Q_1, Q_2)$'. The conservativity preservation from section 3 is an example of this type of principle. Also, this intuition suggests a syntactic format for parallel definition schemes – cf. the discussion in 5.1.

Eventually, a number of such semantic principles may lead to a characterization of the various forms of parallel quantification. However, it may also be that we need some restrictions on the syntactic format of the definition schemes in addition, to take care of the intuition from section 2.2 that they should have monadic analogues. In this article, however, we have to settle for less. What we do achieve is a breakdown of our intuitions about parallel quantification in small, easily manageable parts – and possibly to use these to generate parallel definition schemes automatically. In connection herewith, we can cut back the number of possible parallel definition schemes, by showing that only some schemes satisfy all principles we found.

Here is how we will go about. We will start with a reexamination of monadic quantification, and draw some conclusions from this discussion for parallel quantification. Next, we

[13] Note that this is exactly the empty relation handling from section 2.

will examine two further basic groups of semantic principles, monotonicity principles and preservation properties.[14] Application of these principles will be the subject of section 5.

4.1 Monadic quantifiers

Unary logical monadic quantifiers can be divided in a more or less standard way into four classes of ascending 'complexity'. The simplest quantifiers are the upward monotonic ones. The closure of this class under negation is the class of monotonic quantifiers. Closing these under - arbitrary or simple - conjunctions gives us the class of convex quantifiers.[15] The class of logical monadic quantifiers in general is obtained from this class by closing it under arbitrary disjunction.

In the parallel case, we want to obtain a general definition schemes from the comparatively simple monotonic cases, via an intermediary convex scheme. The step from monotonic to convex schemes goes back to [11] and has been applied to branching quantification in [13]. Convex monadic quantifiers are split into monotonic ones in the following way:

$$\begin{aligned} A \in Q^+ &:\equiv \exists A' \in Q \cdot A' \subseteq A \\ A \in Q^- &:\equiv \exists A' \in Q \cdot A' \supseteq A \end{aligned}$$

This appropriately corresponds to the view of the class of convex quantifiers as the closure of the class of monadic quantifiers under conjunctions, since we have

$$Q = Q^+ \cap Q^- \quad \Leftrightarrow \quad Q \text{ is convex.}$$

The meaning of the + and - operations is probably best explained by some examples.

1. If Q is upward monotonic, then $Q^+ = Q$, and Q^- is the trivially true quantifier T.
2. If Q is downward monotonic, then $Q^- = Q$, and Q^+ is the trivially true quantifier T.
3. If $Q =$**between 3 and 6**, then $Q^+ = \exists_{\geq 3}$, $Q^- = \exists_{\leq 6}$.
4. If $Q =$**4or5**, then $Q^+ = \exists_{\geq 4}$, $Q^- = \exists_{\leq 5}$.

Going from convex to general logical monadic quantifiers, we use the equivalence relation $\sim_{Q,E}$ ('is in the same convex part of Q') we introduced in 2.3:

$$\begin{aligned} A \sim_{Q,E} A' \;:\equiv\; & (A \preceq_E A' \wedge A \preceq_E A'' \preceq_E A' \Rightarrow Q(A) = Q(A'')) \vee \\ & (A' \preceq_E A \wedge A' \preceq_E A'' \preceq_E A \Rightarrow Q(A) = Q(A'')) \end{aligned}$$

Again, this corresponds to the view of the class of logical monadic quantifiers as the closure of the class of convex ones under arbitrary disjunctions, since we have

$$Q = \bigcup_{A \in Q} [A]_{\sim_{Q,E}}$$

where $[A]_{\sim_{Q,E}}$ is the equivalence class of A under $\sim_{Q,E}$. Obviously, for every pair Q, E, all equivalence classes $[A]_{\sim_{Q,E}}$ are convex. Again, we give some examples to clarify the meaning of $\sim_{Q,E}$.

[14] The first group consisted of the two basic principles of symmetry and productivity.

[15] These are often called continuous, but that is a confusing term.

1. If Q is convex, and $A \in Q$, then $Q = [A]_{\sim_{Q,E}}$.
2. If $Q =$ **an even number of**, then the equivalence classes induced by $[\![Q]\!]_{I\!N}$ are exactly the sets $S_n = \{A \subseteq I\!N \mid |A| = n\}$, and $Q = \bigcup_{n \text{ even}} S_n$.
3. If $Q =$ **3or4or7**, then the equivalence classes of Q contained in Q are $\{A \mid |A| \in \{3,4\}\}$ and $\{A \mid |A| = 7\}$.

Of course, there are many other ways of going from monotonic quantifiers via convex ones to general ones. However, the above way seems to reflect intuitions about quantifiers. Also, it leads to a smooth way of generalizing monotonic parallel definition schemes, that corresponds to our intuitions about parallel-type sentences.

4.2 Monotonicity in Q_1, Q_2

It is obvious that all parallel definition schemes should be monotonic in Q_1, Q_2; the more sets Q_1, Q_2 accept, the more relations its parallel combination $\sigma(Q_1, Q_2)$ should accept.

monotonicity in Q_1, Q_2 $\sigma(Q_1, Q_2) \subseteq \sigma(Q_1 \cup Q_1', Q_2 \cup Q_2')$

However, more should be said about the exact way in which σ is monotonic in Q_1, Q_2; for instance, it could be continuous in Q_1, Q_2. To start with, we saw in 2.3 that branching quantification should at least satisfy the principle of

locatedness $\sigma(Q_1, Q_2, R) \wedge \sigma(Q_1', Q_2, R) \;\Rightarrow\; Q_1 \cap Q_1' \neq \emptyset$

It seems that this is a principle that any parallel definition scheme should meet. For one thing, cumulatives satisfy it. More importantly, from our intuitions about the parallel and global nature of parallel definition schemes, we can conclude that parallel quantifiers defined with totally different monadic quantifiers should never accept the same relations.

One aspect of the principle of locatedness is that neither of Q_1, Q_2 can be trivially false if $\sigma(Q_1, Q_2)$ is to accept any relations. This is obviously a sound principle. The formalization reads

non-triviality $\neg\sigma(\emptyset, Q_1, R)\; ; \sigma(T, T, R)$,

where T is the trivially true quantifier. Assuming non-triviality, we may consider the following strengthening of locatedness:

$\wedge$-preservation $\sigma(Q_1, Q_2 \cap Q_2', R) \;\Leftrightarrow\; \sigma(Q_1, Q_2, R) \wedge \sigma(Q_1, Q_2', R)$

This is a very strong principle, and I would prefer to do without it. Still, it makes some sense. On monotonic quantifiers, $\wedge$-preservation corresponds to the view of monadic quantifiers we gave in section 4.1. Also, it goes well with our basic intuition of global parallelity, but then again, so does

$\vee$-preservation $\sigma(Q_1, Q_2 \cup Q_2', R) \;\Leftrightarrow\; \sigma(Q_1, Q_2, R) \vee \sigma(Q_1, Q_2', R)$

and we certainly do not want to impose that on all parallel definition schemes - especially not together with $\wedge$-preservation, since that would lead to a condition that seems much to strong. And if we cannot impose $\vee$-preservation, then from a logical point of view, we cannot impose its counterpart $\wedge$-preservation either on all parallel definition schemes. Of course, both principles may still hold for specific classes of parallel quantifiers.

We may also consider less conservative monotonicity principles. Here is one that describes the parts of the quantifiers Q_1, Q_2 that are relevant to the relation R.

relevance
$$R \supseteq S \wedge \sigma(Q_1, Q_2, S) \;\Rightarrow\; (\sigma(Q_1', Q_2', R) \Leftrightarrow \sigma(Q_1' \cap Q_1^+, Q_2' \cap Q_2^+, R)$$
$$R \subseteq S \wedge \sigma(Q_1, Q_2, S) \;\Rightarrow\; (\sigma(Q_1', Q_2', R) \Leftrightarrow \sigma(Q_1' \cap Q_1^-, Q_2' \cap Q_2^-, R)$$

This is an immensely strong principle, but I believe it makes a lot of sense. For one thing, all our parallel definition schemes satisfy it. But more crucially, it combines orderings on relations and quantifiers in a natural way. Also, it does seem an exact modeling of intuitions about which parts of the quantifiers Q_1, Q_2 are relevant to R. Finally, without proof, we note that relevance implies monotonicity preservation.

Still, we have to be very careful with principles as strong as this one. When we will apply the principles, in section 5, we will always try to avoid using relevance.

The last strengthening of the principle of monotonicity in Q_1, Q_2 we may consider is again based on the observations we made in section 4.1. Just like there, in going from convex to general quantifiers, we use the notion of $\sim_{Q,E}$ to get the principle of continuity in $Q_1/\sim_{Q_1,E}$ and in $Q_2/\sim_{Q_2,E}$:

$$\sigma(Q_1, Q_2, R) \;:\equiv\; \exists A \in Q_1 \cdot \sigma([A]_{\sim_{Q_1,E}}, Q_2, R)$$
$$\sigma(Q_1, Q_2, R) \;:\equiv\; \exists B \in Q_1 \cdot \sigma(Q_1, [B]_{\sim_{Q_2,E}}, R)$$

Continuity in $Q_1/\sim_{Q_1,E}$ states that the truth of $\sigma(Q_1, Q_2, R)$ depends precisely on the convex parts of Q_1. For instance, the truth of a statement like $\sigma(\mathbf{2or4}, Q_2, R)$ should depend on the truth of either of $\sigma(\mathbf{2}, Q_2, R)$, $\sigma(\mathbf{4}, Q_2, R)$. The point is that there is a 'gap' between **2** and **4**, so that these two cannot cooperate in making $\sigma(\mathbf{2or4}, Q_2, R)$ true.

Continuity in $Q_1/\sim_{Q_1,E}$ is a very strong property, but in contrast with $\wedge$-preservation I think we should accept it, as it makes a lot of sense, cf. also [13], where exactly the same thing is done for some simple separate branching cases.

This ends our discussion of the various types of monotonicity in Q_1, Q_2 we could consider. We adopt locatedness and continuity in $Q_1/\sim_{Q_1,E}$, and keep relevance in reserve.

4.3 Preservation principles

From a logical point of view, parallel quantifiers should inherit as many structural properties from their defining monadic ones as possible. A parallel quantifier $\sigma(Q_1, Q_2)$ should inherit the property X exactly if both Q_1 and Q_2 both satisfy X. This type of preservation models part of the 'global parallelity' intuition. An example is the principle of conservativity preservation from section 3, which shows how to obtain binary parallel definition schemes from unary ones.

Here are two further obviously valid preservation principles:

logicality preservation $\sigma(Q_1, Q_2, R) \Leftrightarrow \sigma(Q_1, Q_2, \{(\pi_1 a, \pi_2 b) | (a, b) \in R\}$ for logical Q_1, Q_2 and permutations π_1, π_2.

monotonicity preservation If Q_1, Q_2 are both upward monotonic, downward monotonic or convex, respectively, then so is $\lambda R \cdot \sigma(Q_1, Q_2, R)$.

This ends our overview of likely candidate semantic principles that parallel definition schemes should meet. It seems, however, that more principles (and perhaps syntactic restrictions in addition) are needed to characterize the class of parallel definition schemes. Also, it might be worthwhile to find principles that characterize specific classes of parallel quantification.

5 Definition Schemes

In this section we use the semantic principles from the last section to build general unary parallel definition schemes, starting with the monotonic cases **BU**, **BD**, **CU**, **CD**. We will start by presenting some evidence that suggests that these four are indeed the natural building stones. Next, we will see how these four can be combined to give convex parallel definition schemes. It will turn out that only three of the possible combinations can work. We will also analyze what principles we have to assume in order to build a unique convex scheme out of two monotonic ones. Finally, we will give full generalizations of the three remaining convex definition schemes.

5.1 Monotonic Schemes

In this section we will have a closer look at the monotonic parallel definition schemes we found in section 2.2. It is obvious that the four schemes we found there all satisfy the semantic principles we gave in section 4. However, it is not true that these four are the only ones that satisfy these principles. Still, I am convinced that they are the only monotonic parallel definition schemes, and I will give reasons for this conviction in the present section. To start with what we do have is

proposition 5.1 Under monotonicity in Q_1, Q_2 and productivity:

(25) **BU** is the minimal upward monotonic independent definition scheme.
(26) **CU** is the maximal upward monotonic independent definition scheme.
(27) **BD** is the maximal downward monotonic independent definition scheme.
(28) **CD** is the minimal downward monotonic independent definition scheme.

proof

(25). If **XU** is an upward monotonic definition scheme, then all cross-products $A \times B$, $A, B \neq \emptyset, A \in Q_1, B \in Q_2$ must be accepted by **XU**, by productivity. So by preservation of monotonicity, **XU** accepts all relations accepted by **BU**.

(26). If **YU** is an upward monotonic definition scheme, and R is accepted by **YU**, then for all $A, B \neq \emptyset$ such that $A \times B \supseteq R$, $A \times B$ is accepted by **YU**, since **YU** preserves monotonicity. So by productivity, $A \in Q_1, B \in Q_2$.

(27) and (28) are similar. □

This proposition may be seen as a full characterization of our four monotonic schemes. In the present perspective, however, it means more or less that **BU**, **BD**, **CU**, **CD** are the most natural choices for monotonic parallel definition schemes. Also, we seem to have ruled out schemes with other second-order generalized quantifiers, even monotonic ones. The scheme **XU** below, for instance, fails against productivity.

XU $\exists_{>2} A, B \neq \emptyset \cdot A \times B \subseteq R \wedge A \in Q_1 \wedge B \in Q_2$

However, there is at least one unnatural upward monotonic scheme that satisfies all principles, though we certainly do not want to call it a parallel definition scheme: **YU** (equivalently, for non-empty Q_1, Q_2,**YU**$'$)[16]

YU $\exists A, B, X, Y \neq \emptyset \cdot A \times Y \cup X \times B \subseteq R \wedge A \in Q_1 \wedge B \in Q_2$
YU$'$ $\exists x \cdot Q_2 y \cdot Rxy \wedge \exists y \cdot Q_1 x \cdot Rxy$

The point is, that our semantic principles are only partial implementations of such vague principles as 'Q_1, Q_2 should be treated as on a par' and 'the truth condition of parallel quantifiers should have something to do with the basic independent objects of relational algebra: cross-products'.[17] These vague considerations seem to suggest a syntactic format, rather than a semantic principle. I will presently describe this format. First of all, the relevant parallel notions are

$$R^{\supseteq} := \begin{cases} \{\langle A, B\rangle \mid \emptyset \neq A \times B \subseteq R\} & \text{if } R \neq \emptyset \\ \{\langle \emptyset, \emptyset \rangle\} & \text{otherwise} \end{cases}$$
$$R^{\subseteq} := \{\langle A, B\rangle \mid A \times B \supseteq R \ \wedge \ A = \emptyset \Leftrightarrow B = \emptyset\}$$
$$Q_1 \otimes Q_2 := \{\langle A, B\rangle \mid A \in Q_1 \wedge B \in Q_2 \ \wedge \ A = \emptyset \Leftrightarrow B = \emptyset\}$$

All of the three notations are exactly in line with the vague notions we mentioned above, in that they extract precisely cross-products (more precisely, pairs of sets) out of the three variables R, Q_1, Q_2 in our definition schemes. Furthermore, there seem to be no plausible operations connecting these with products while keeping the order Q_1, Q_2, other than the above three. This means that we should be able to state our definition schemes in terms of algebraic expressions in $Q_1 \otimes Q_2$, $R^{\supseteq}$, $R^{\subseteq}$. So what we want in our definitions is a direct comparison between $Q_1 \otimes Q_2$ on the one hand, and $R^{\supseteq}$ or $R^{\subseteq}$ on the other. Also, these comparisons should be simple, because monotonic quantifiers (and also $\otimes$ products of two quantifiers of the same monotonicity) typically divide D_{et} into two parts with one cardinality border between them. This should be reflected in the syntactic form of the definition scheme by referring exactly once to $Q_1 \otimes Q_2$ and $R^{\supseteq}$ or $R^{\subseteq}$. Taking into account that our schemes have to be monotonic in Q_1, Q_2, we seem to be left with four options:

(29) a. $Q_1 \otimes Q_2 \cap R^{\supseteq} \neq \emptyset$
b. $R^{\subseteq} \subseteq Q_1 \otimes Q_2$
c. $R^{\supseteq} \subseteq Q_1 \otimes Q_2$
d. $Q_1 \otimes Q_2 \cap R^{\subseteq} \neq \emptyset$

Closer examination reveals that (29a-b) give only acceptable truth conditions (i.e. ones that satisfy monotonicity-preservation) for upward monotonic Q_1, Q_2, and (29c-d) will only work for downward Q_1, Q_2.

As you may have guessed, (29a-d) are exactly the four monotonic parallel definition schemes we had before: they are equivalent to **BU**, **CU**, **BD**, **CD**, respectively. I know that the above evidence is not conclusive, but it seems to me that there is a strong case for the conjecture that there are only four monotonic parallel definition schemes; two for each monotonicity behavior.

[16] In fact, it is one of the very few upward monotonic schemes I have found that is not a parallel definition scheme but satisfies all semantic principles.

[17] Alternatively, we may view these as pairs of sets.

5.2 Convex Schemes

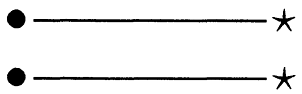
The next step is to combine one of **BU**, **CU** with one of **BD**, **CD**, to find a convex independent definition scheme. One strategy by which combinations can be made can be found in [13], who combines **BU+CD** as in (30a); in general, this method reads (30b).

(30) a. $\mathbf{B2C}(Q_1, Q_2, R) \;:\equiv\; \mathbf{BU}(Q_1^+, Q_2^+, R) \wedge \mathbf{CD}(Q_1^-, Q_2^-, R)$
b. $\mathbf{IC}(Q_1, Q_2, R) \;:\equiv\; \mathbf{IU}(Q_1^+, Q_2^+, R) \wedge \mathbf{ID}(Q_1^-, Q_2^-, R)$

where $\mathbf{IU} \in \{\mathbf{BU},\mathbf{CU}\}$,$\mathbf{ID} \in \{\mathbf{BD},\mathbf{CD}\}$ and **IC** are the upward, downward and convex versions of an eventual general maximized independent definition scheme **I**, and the $^+$ and $^-$ operations are as defined in section 4.1.

To test the adequacy of this method, first of all, we have to check if the resulting convex definition schemes have the monotonic ones as special cases. This follows from the principle of non-triviality, which all four monotonic schemes satisfy. So in order for the method to generate the proper result, it is necessary and sufficient that the union of the two monotonic schemes **IU**, **ID** satisfies locatedness. For three of the possible combinations this requirement holds. However, in the **CU+BD** case, assuming locatedness, we can prove that there is no convex parallel definition scheme that extends the two monotonic ones.

example In the following relation



both $\mathbf{CU}(\exists_{\geq 2}, \exists_{\geq 2}, R)$ and $\mathbf{BD}(\exists_{<2}, \exists_{<2}, R)$ hold, which is in conflict with the principle of locatedness. □

Next, we would like to show that the method is unique. I haven't been able to prove this strong general result – at least, not without the principle of relevance –, but there is quite some evidence that suggests that (30b) is *the* way of combining convex definition schemes. To start with, we can prove that, assuming the principle of monotonicity in Q_1, Q_2, the following holds:

(31) $\mathbf{IC}(Q_1, Q_2, R) \;\Rightarrow\; \mathbf{IU}(Q_1^+, Q_2^+, R) \wedge \mathbf{ID}(Q_1^-, Q_2^-, R)$

This is obvious, since **IU** and **ID** are to be special cases of **IC**, while $Q^+, Q^- \subseteq Q$. What we want to have in (31) is $\Leftrightarrow$ instead of $\Rightarrow$. To prove the $\Leftarrow$ side of the assertion, we need the not very convincing principle of relevance:

proposition 5.2 Under relevance, Westerståhl's method is the only way of combining two monotonic quantifier schemes to find a convex one.

proof Let **XU**, **YD** be monotonic schemes with a convex scheme σ extending. Suppose that both $\mathbf{XU}(Q_1^+, Q_2^+, R)$ and $\mathbf{YD}(Q_1^-, Q_2^-, R)$ hold. Then $\sigma(Q_1^+, Q_2^+)$ and $\sigma(Q_1^-, Q_2^-)$ hold too. So by relevance plus the fact that $Q = Q^+ \cap Q^-$, we have $\sigma(Q_1, Q_2, R)$. □

If we do not adopt relevance, we are left with circumstantial evidence. Still, this evidence is rather strong. For one thing, there are the arguments from [13]. Also, Westerståhl's method corresponds to the view of monadic quantifiers we gave in section 4.1. Finally, we will see in section 5.3 that the eventual generalizations of **CC** and **B1C** below are equivalent to the cumulative scheme **C** and to the intuitive scheme **B1** we found in section 2.3, respectively. This is also the reason why we are allowed to call these convex schemes **CC** and **B1C**.

CC $domR \in Q_1 \wedge rangeR \in Q_2$
B1C $\exists A, B \neq \emptyset \cdot A \times B \subseteq R \wedge (\emptyset \neq A' \times B' \subseteq R \Rightarrow (A \in_1^- \wedge B' \in Q_2^-))$

The last combination, **BU+CD**, in fact allows a stronger result. It is the case that [13] is concerned with. Westerståhl gives only vague considerations to justify his definition, but in view of the semantic principles we introduced in section 4, we can in fact *prove*

proposition 5.3 Assuming monotonicity in Q_1, Q_2 + productivity + convexity-preservation, Westerståhl's method generates the only convex parallel definition scheme that extends **BU+CD**.

proof As Westerståhl notes, his original definition **B2C** is equivalent to **B2C′**

B2C $\exists A, B, A', B' \neq \emptyset \cdot A \times B \subseteq R \subseteq A' \times B' \ \wedge$
$A \in Q_1^+ \wedge B \in Q_2^+ \wedge A' \in Q_1^- \wedge B' \in Q_2^-$

B2C′ $\exists A, B, A', B' \neq \emptyset \cdot A \times B \subseteq R \subseteq A' \times B' \ \wedge \ A, A' \in Q_1 \wedge B, B' \in Q_2$

It suffices to prove that **B2C′** is the strictest parallel definition scheme. But that is obvious, by productivity + convexity-preservation. □

One thing to note about the difference between the two convex branching schemes found in this section is that **B1C** does not contain Van Benthem's proposal **B2N** for branching numerals **n**, **m** as a special case, while **B2C**, which was meant to incorporate this definition, does.

B2N $\exists A, B \cdot |A| = n \ \wedge \ |B| = m \ \wedge \ R = A \times B$

However, **B1C** also seems to provide reasonable readings in the numeral case, so coincidence with **B2N** cannot be used to decide between the two branching readings we are left with.

We conclude that, although Westerståhl's method is probably correct, there is still some work to be done to prove it is the only right one. In particular, we need to find a more elegant and convincing principle than relevance that can deal with the tricky **BU+BD** and **CU+CD** combinations.

5.3 General Schemes

Generalizing from convex to general parallel definition schemes is totally fixed by the principle of continuity in $Q_1/\sim_{Q_1,E}$, $Q_2/\sim_{Q_2,E}$. We cannot do without it, however, although I would like to have partial results here that do not need the continuity principle.

What we can do is check whether the resulting definitions after applying the continuity principle to the three convex schemes from the last section are in line with our intuitions. And indeed they are.

C $\exists A \in Q_1, B \in Q_2 \cdot domR \in [A]_{\sim_{Q_1,E}} \wedge rangeR \in [B]_{\sim_{Q_2,E}}$
B1 $\exists A, B \neq \emptyset \cdot A \times B \subseteq R \wedge$
$\emptyset \neq A' \times B' \subseteq R \Rightarrow A' \in [A]^{-}_{\sim_{Q_1,E}} \wedge B' \in [B]^{-}_{\sim_{Q_2,E}}$
B2 $\exists A, A', B, B' \neq \emptyset \cdot A \times B \subseteq R \subseteq A' \times B' \wedge$
$A \in Q_1 \wedge B \in Q_2 \wedge A \sim_{Q_1,E} A' \wedge B \sim_{Q_2,E} B'$

It is easy to prove that **C** and **B1** are indeed equivalent to the old cumulative definition scheme Cand to the maximality-based **B1** from section 2.3, respectively. Also, **B2** corresponds to the two cases of non-convex branching that are considered in [13]. These facts provide further evidence that our way of building general definition schemes out of the monotonic ones was the right one.

All in all, we found three parallel definition schemes. We presented strong evidence for the conjecture that they are the only ones, though we failed to come up with a full proof of this assertion.

6 Concluding Remarks

We analyzed the concept of parallel quantification both from a linguistic and a logical point of view. Although we did come up with three acceptable definition schemes, and presented evidence to the effect that they are the only ones, there is still a lot of work to be done. In fact, I could think of many questions that need to be answered. Here are some.

Is there a connection between parallel quantification and the collective readings found in [4]? And if so, is there a reason for this connection? Can we fully characterize the class of parallel definition schemes? And if so, do we need syntactic restrictions in addition to semantic principles? Or can we find syntactic formats that correspond exactly to our semantic principles? Can we characterize the notions of maximality from section 2.3? Can they be applied in other fields than parallel quantification?

Finally, I would like to make a broader remark. It seems to me that the Fregean quantification versus parallel quantification contrast is only one of a whole range of parallelism versus sequentiality phenomena. This suggests that 'linear lifting' may not be the only major way of combining two unary objects to get a 2-dimensional one. Parallel lifting may well be a significant alternative.

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Other Prepublications

X-93-01 Paul Dekker — Existential Disclosure, revised version