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1 Aims

The present article is on the semantics of plural noun phrases. Perhaps the semantics of no category is studied as thoroughly as that of noun phrases. Yet, the resulting theory of generalised quantifiers is mainly developed by disregarding the fact that most noun phrases are plural. Since it restricts itself to properties of 'plain' individuals rather than of sums or groups, it does not elucidate the typical phenomena of statements about such collections. Conversely, the familiar proposals concerning the semantics of plurals normally concentrate on a small number of noun phrases and leave unclear which options there are in collectivising the theory of generalised quantification.

Here I study systematic ways to combine the standard theory of quantification with the sum theory of collections. The research is carried out within an extensional type theory. In such a framework the theories can be connected by giving ways to transform determiners in type $((et)((et)t))$, i.e., relations between sets, to ones in type $((et)(((et)t)t))$, i.e., relations between sets and sets of sets. I assume for simplicity that common nouns only hold of individuals (denote sets) and that only verb phrases are collective (denote sets of sets). Given that collections viewed as sums may be identified with sets, such 'lifted' determiner denotations are suitable to be used in a collective setting.

There are two reasons why I do not treat the alternative theory of collections as groups. Firstly, the sum approach is easier to handle, and there is no obstacle to transfer the results obtained here to the more involved group setting (cf. Van der Does [1992, 12–14]). Secondly, Schwarzschild's [1992] arguments, which leave little room for groups, are rather convincing.

1.1 Two sources of readings

Whatever the preferred theory of collections may be, the literature has basically two strategies to obtain collective and other readings. On the first and oldest strategy, called the NP strategy here, the noun phrase is the main locus to generate the readings (Bartsch [1973], Bennett [1974], Scha [1981], Verkuyl [1981]). In contrast, a more recent strategy works on the assumption that the readings should be generated within the VP by means of (c)overt modification. This so-called VP strategy is introduced by Link [1983, 1991] and refined by Lønning [1987, 1989] and Roberts [1987], among others. The third option, where the readings of complex sentences depend functionally on the readings of both categories (and perhaps on that of others), is, I think, the correct one (cf. Van der Does [1992, ch. 4]). Roberts comes close to this view where she writes that 'distributivity is a property of predications, combinations of a subject and an object' (Roberts [1987, 100]). However, for the simple transitive sentences studied here it is enough to discern the above two strategies.

It may seem that the VP strategy, with its emphasis on modification, is less interesting when the connection between quantification and collectives is at stake. This is not so, for the problem still remains of how to deal with arbitrary noun phrases. On this view each noun phrase or class of noun phrases should be treated in a single way. But which ways are used in the literature and do they allow a satisfactory generalisation? Moreover, both the NP and VP strategy should be studied since we want to know how they compare, logically as well as empirically.

To enable such a comparison the differences between the two approaches, though real, should be kept at a minimum. I will use the fact that noun phrases often denote sets of verb phrase extensions. So, the VP strategy can to a large extent be mimicked within the NP strategy by combining the different verb phrase modifications with the one treatment of noun phrases. In this way different readings of noun phrases result which may be contrasted with the ones proposed by the NP strategy. (It so happens that conversely the readings given by the NP strategy can be obtained systematically by use of exactly the same modifiers as used within the VP strategy.)

1.2 Three readings, six lifts

Closely related to the question of where the readings come from is the more empirical question of which kind of readings occur and how they should be modeled. Most semanticists would grant that at least a collective and a distributive or 'atomic' reading exist. Less attention has been paid to what I call a neutral reading, but I study this reading separately.

One of the reasons why neutral readings need special care is that it is not entirely clear how to model them. Scha [1981] and Gillon [1987, 1990] take some statements about collections to be neither distributive nor collective but kind of intermediate. E.g., (1) cannot be distributive, for single individuals do not gather.

(1) Five thousand people gathered near Amsterdam

But (1) does have a neutral reading, which is used to describe one or more gatherings with a view to the precise number people involved in them. As we shall see, this precision is not available on its collective reading.

However, the most straightforward way to formalise the neutral reading quickly leads to unwelcome truth conditions. For instance, on its neutral reading a noun phrases cannot combine with a complex verb phrase in the usual way. I try to remedy this by considering some alternatives involving partitions, minimal covers, and so-called pseudo-partitions. I shall argue that none of these alternatives are appropriate, and hence that on its neutral reading a noun phrase does not take scope over complex verb phrases. On the other hand, this reading is the likely candidate to be used in non-iterative polyadic quantification, such as the cumulative or branching variants. By way of example I show how to deal with cumulative readings in a collective setting.

Three readings for each of the two strategies gives a sum total of six lifts to generate them from the familiar denotations. These lifts capture most of the semantical observations found in the literature. To be sure, I do not claim that each determiner allows all the readings. However, I do hold that the readings can be acquired uniformly for all the determiners that do allow them. It is this uniformity which enables us to compare the readings by looking at logical behaviour, scopal behaviour, and quantificational force.

1.3 Main results

Using the results obtained along these lines, I argue that in case of simple sentences the NP strategy is empirically more adequate than the VP strategy. In its purest form the VP strategy takes the collective reading as basic and generates the other readings by means of verb phrase modification. The collective readings which have been proposed in the literature treat the noun phrase so that it leaves the verb phrase outside of its scope. Here the determiner in a noun phrase, if present, rather functions in an adjectival way. For example, numerals select the collections from a noun of a particular size, and similarly for the other determiners. In contrast, the NP strategy allows for more variation in handling the noun phrase. Most of the times it has the verb phrase within its scope (on the distributive and neutral readings), but sometimes it does not (on the collective reading). It is precisely this difference which makes the distributive and neutral reading given by the NP strategy superior to those of the VP strategy. On these readings one is interested respectively in the individuals that have a property *simpliciter* or which partake in a collection having a property. But the relevant individuals are only determined as required if the verb phrase is within the scope of the noun phrase. The conclusion seems to be that without further ado the VP strategy in its purest form is not feasible.

It is held against the NP strategy that not all readings result from noun phrase ambiguities. The problem arises with some conjoined verb phrases, which may only be partly marked for distributivity. Since I do not concern myself with conjunction in the following, I make some remarks on this matter here.

Dowty [1986] observes that if modification is included in a NP denotation, the distributivity of a verb phrase becomes an all-or-nothing issue. Sentence (2), which resembles the examples given by Dowty, is adapted from Lasersohn [1989]:

(2) *a.* Four men met in a bar and had a beer
b. Four men_1 are such that they_1 met in a bar and each of them_1 had a beer

In (2a) the first conjunct of *to meet in a bar and have a beer* is collective and the second distributive. Does this force us to hold that the readings come from the modified VP conjuncts? Not necessarily, for in the common analysis (2b) of (2a) the collectivity or distributivity of the conjuncts may be due to the noun phrases 'they' and 'each of them' (cf. Van der Does [1992, 83-85]). The anaphoric link between these noun phrases and the main subject could be established by means of quantifying in or a similar such device. At any rate, the differences between this and the modification approach are negligible, as the modifiers result from the different readings of the quantifier 'all' by swapping its noun and verb argument.

Note that on both approaches there is a threat of overgeneration. In case of the NP strategy the reading of the main subject may not combine well with the conjoined VP denotations. Whereas the VP strategy should preclude further modification of the complex VP. A convenient way to handle this problem is the use of a feature system as is given in Van der Does [1992, ch. 4]. In such a system, whether a complex expression is distributive, collective or neutral depends functionally on its constituting categories. Mainly to preclude overgeneration, I hold that the reading of a complex sentences should not be attributed to a single category but be determined compositionally.

1.4 Overview

In section 2, I give a quick overview of the models used in the sum theory of collections and show how they are represented within the extensional type theory opted for here. Even at this point I stress that the application of simple type shifts which embed objects of lower level into a collective environment is beneficial. Section 3 studies the treatments of numerals by Scha and Link, the two main proponents of the NP and the VP strategy. With a view to handling arbitrary noun phrases, I analyse their proposals in terms of the familiar numeral denotations from generalised quantifier theory. Section 4 is on determiners. I introduce the six lifts to make determiners suitable for a collective VP, show that they can be generated from two basic lifts by means of modification, and give some examples of lifted determiners. The examples are used to show which lift gives the appropriate collective reading. Section 5 maps the logical relations between the different readings for different kinds of determiner. It also makes a choice among the two treatments of distributive and neutral readings by comparing their quantificational force. Yet, the neutral reading receives further attention in section 6 with a view to its rather special scopal behaviour. As we go along, we also obtain information to what extent neutral readings could be used to reduce the number of readings of a sentence (cf. Verkuyl and Van der Does [1991]).

In this article, the focus is mainly on the empirical side of simple plural noun phrases. Definite as well as more complex plural noun phrases are studied in Van der Does [1992]. One could also take a logical stance, where arbitrary lifts from type $((et)((et)t))$ to type $((et)(((et)t)t))$ are studied in order to find constraints to single out the reasonable ones. Here one would like to know which alternatives there are for such familiar notions as conservativity and the like. Moreover, one would like to characterise the results of the lifts as the unique determiners having certain properties. The readers interested in these issues may consult Van der Does [1992, ch. 5]. There one also finds the proofs of propositions, which are just stated here.

1.5 Tools and notational conventions

In this article I often interchange set theory and type theory without much notice. That one may do so, follows from the well-known equivalence of sets and their characteristic functions. I keep the explicit typing of variables at a minimum by use of some notation:

variables	type
$x, y, z, \ldots$	e
$X, Y, Z, \ldots$	(et)
$R^n, \ldots$	e^n
$\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \ldots$	$((et)t)$
$\mathbf{R}^n, \ldots$	$(et)^n$
$D, \ldots$	$((et)((et)t))$
$\Delta, \ldots$	$((et)(((et)t)t))$

Here α: $\alpha^0 = t$ and $\alpha^{n+1} = (\alpha\alpha^n)$. The variables may have primes or subscripts as usual.

2 The logical framework

The core insight which made people work on the semantics of plurals is captured in the following principle:

> Some properties of collections cannot be reduced to properties between their individual members.

An example of such a property is *to play the 'Große Fuge'*, as in:

(3) The musicians played the '*Große Fuge*'

Plainly, the musicians were able to perform the composition in virtue of their individual achievements and quality. Still, (3) records a fact over and above the complex relationships between the musicians during the performance. It is the fact that to play this fugue is a joint venture.

2.1 CASJ-models

Several authors have addressed the question of how to model collective readings. It is felt that the collections should comply with three principles that 'incorporate all the intuitions about the behaviour of plural objects in natural language' (Link [1991]):[1]

Atomicity Each collection must be the unique combination of all individuals constituting it.

Completeness It should be possible to combine collections into a single new one.

Atoms Individuals have to reappear as a limiting case; as those collections which consist of just one item (the *atoms* of a domain).

Among the structures which satisfy these requirements are the complete atomic join semilattices – CAJS's for short. Link [1983] introduces the idea to replace the familiar domains of discourse, which consist of just individuals, by CAJSs.

[1] The date of this publication is misleading. It has been available as a typescript since 1984.

I do not give definitions, but the main advantage of using algebraic domains is that they enable to capitalise on 'a striking similarity between collective predication and predication involving mass nouns' (Link [1983, 302]).

In the present setting, where mass terms are disregarded, one may as well use a particular kind of set-theoretic CAJS, namely those with a domain of the form $\wp^+(X)$, X a set.[2] The elements of $\wp^+(X)$, the non-empty subsets of X, are to model collections; individuals d appear as singletons $\{d\}$, the atoms of $\wp^+(X)$; the operation assembling sets of collections into a new one is union; and as to atomicity, indeed, for each set Y one has:

$$(4) \quad Y = \bigcup\{\{d\} : d \in Y\}$$

In (4) it is assumed that union is of arbitrary sets of sets and not just of pairs of sets, otherwise atomicity would only be guaranteed for finite sets. Given this much, one defines CASJ-models, whose use essentially goes back to Scha [1981]:

Definition 2.1 (CAJS-**models**) *Let X be a non-empty set. A* CAJS-*model $\mathcal{M}$ is an ordered tuple:*

$$\langle \wp^+(X), \mathrm{AT}(X), \bigcup, \subseteq, [\![-]\!] \rangle$$

The set $\mathrm{AT}(X)$ $(= \{\{d\} : d \in X\})$ *contains the atoms of* $\wp^+(X)$*. The interpretation function* $[\![-]\!]$ *assigns a collection:* $[\![\mathrm{c}]\!] \in \wp^+(X)$*, to each constant c, and an n-place relation among collections:* $[\![\mathrm{R}^n]\!] \subseteq \wp^+(X)^n$*, to each n-ary relation sign* R^n*.*

Models come with formal languages, but I shall not give one in full detail. Rather, I introduce relations, constants and the like as we go along, and deal with them directly in terms of their interpretations. Given that the noun 'musicians' denotes a set of atoms, (3) is formalised by:

$$[\![\textit{play the 'Große Fuge'}]\!](\bigcup [\![\textit{musicians}]\!])$$

As is done more often, *the* is taken to return the collection obtain from the noun. Note that the property *to play the 'Große Fuge'* is attributed to the set of musicians instead of to its members. It is often assumed that this is enough to model the collective reading of (3).

2.2 Type theory

As said, the explicit use of algebraic domains for the purpose of modeling collective readings, originated with Link [1983], and presently this technique is very popular. But there also has been an alternative tradition that works within the extensional type theory with basic types e (entities) and t (truth values), and compound types $(\alpha\beta)$ (functions mapping type α objects onto type β objects). The tradition can be traced back to Bartsch [1973] and Bennett [1974], among others. For reasons given shortly, type theory will in this article be my main

[2]See Landman [1989, 568–571] for convincing arguments why one should use this kind of CAJS.

instrument too. Therefore, I shall now discuss some strategies showing that all that can be done with CAJS-models can be done as well within type theory.

In a CAJS-model, relation signs are interpreted according to the most complex case: without exception relations are between collections. Using a strategy of collectivisation in analogy with Van Benthem [1991, ch. 12], this can also be simulated within type theory. For example, let R be a two place relation sign of type $(e(et))$. In a CAJS-model its interpretation will be a relation between elements of $\wp^{+}(X)$, which are sets. Due to the fact that this domain is, so to speak, of type (et), one could therefore say that the type of R's interpretation $[\![R]\!]$ is $((et)((et)t))$ rather than $(e(et))$. But then a uniform change in the types assigned to expressions achieves the same effect, as follows:

- convert all e's in the types of expressions into (et);
- interpret the resulting expressions as is usual for type theory.

This strategy capitalises on the main characteristic of CAJS-models: that the distinction between individuals and collections is blurred. The reason for wanting this identification is that some expressions, e.g. the verb *to make music*, pertain to individuals and collections alike so that a type distinction between individuals and collections combined with a rigid category-to-type assignment enforces one to give such verbs two lexical entries: one interpreted in type (et) another in type $((et)t)$. Similarly, proper names, which intuitively take their denotation in the set of entities $\mathbf{D}_e$, can be conjoined in a non-Boolean way with NPs such as *the boys*, which may denote the set of boys in $\mathbf{D}_{(et)}$. Again, multiple lexical entries seem to be called for. Some feel that the ambiguities imposed on these expressions are only motived by the logical apparatus.

Changing the semantics to a richer environment, the question comes to the fore of how the new semantics relates to the old, and particularly whether there are systematic ways to connect them. In case of the CASJ-models this is indeed so, as can be show within type theory by means of type changes. The definition of these models can be seen to involve the following shift in types:

$$(et) \Rightarrow ((et)t)$$

It tacitly uses the type shift operators:

$$\begin{array}{lll} \wp^{+} & := & \lambda X \lambda Y.\ Y \subseteq X \wedge Y \neq \emptyset \\ \text{AT} & := & \lambda X \lambda Y.\ Y \subseteq X \wedge |Y| = 1 \end{array}$$

which transform sets – objects of type (et) – into sets of sets – objects of type $((et)t)$. In fact, these operators play a prominent role in the literature on plurals: they are Link's pluralisation operator and Scha's atomiser, respectively.[3] Similarly, the new denotation of proper names results from a use of 'Quine's function,' which embodies the type change: $e \Rightarrow (et)$:

$$I := \lambda x \lambda y.\ x = y$$

This function transforms an individual into the corresponding singleton, so that the we get for proper names c the denotation $\lambda \mathbf{X}.\mathbf{X}(I([\![c]\!]))$ (proper names have no other readings).

[3]As it happens, these authors both use '*' to denote their operator.

Even these simple examples suggest that type theory is well-suited to study collectivity. For our present purposes it is particularly useful, since it gives a framework to transform the standard semantics to the richer collective setting in a uniform way.

3 Numerals

Numerals are a prime source of collective readings, and indeed their semantics figures prominently in the literature. Here I shall focus on the semantics given by Scha [1981] and Link [1983, 1991] together with one inspired by Gillon [1987]. These semantics leave opaque how the numeral denotations in type $((et)(((et)t)t))$ relate to the ones of type $((et)((et)t))$ given in generalised quantifier theory. I shall remedy this by formulating the systems within type theory in terms of the usual denotations:

(5) $\mathbf{n}_{((et)((et)t))} := \lambda X \lambda Y.\ |X \cap Y| = n$

This, in turn, suggests six lifts from type $((et)((et)t))$ to type $((et)(((et)t)t))$ by means of which an arbitrary determiner may receive different readings on the basis of its standard denotation. This way of proceeding has the important advantage that the collective semantics of determiners gets connected with generalised quantifier theory.

To analyse the treatment of numerals in Scha [1981] and Link [1983, 1991], much ground can be covered by studying the simple sentence:

(6) Four men lifted two tables

I discuss which readings of (6) can be detected. As we go along, we learn where Scha and Link locate the quintessence of collectivity and whether they live up to their own opinions.

3.1 Scha

The title 'Distributive, Collective and Cumulative Quantification' of Scha [1981] makes plain that he thinks distributivity and collectivity to reside in the NP. An inspection of his semantics shows that complex NPs have these properties in a derivative sense: it is the determiners which receive different readings in this respect. A numeral (**exactly**) **n**, in particular, has three denotations: a distributive, a collective and a neutral one. Using my own notation and names they are:[4]

$\mathrm{D}_1 \quad \lambda X \lambda \mathbf{Y}.\ |\{d \in X : \mathbf{Y}(\{d\})\}| = n$
$\mathrm{C}_2^a \quad \lambda X \lambda \mathbf{Y}.\ \exists Y \subseteq X\, [|Y| = n \wedge \mathbf{Y}(Y)]$
$\mathrm{N}_2 \quad \lambda X \lambda \mathbf{Y}.\ |\bigcup\{Y \subseteq X : \mathbf{Y}(Y)\}| = n$

[4]The idea behind the labeling is explained in section 4.1. For now, it suffices to say the 'D' stands for distributive, 'C' for collective, and 'N' for neutral. Scha speaks of two collective readings. In section 4.2 it will appear that this terminology is misleading: the C_2 numerals (here called N_2) do not give rise to collective readings.

These readings can be rewritten in terms of the numeral denotation in (5):

$$\begin{array}{ll} \mathrm{D}_1 & \lambda X \lambda \mathbf{Y}.\ \mathbf{n}(X)(\bigcup(\mathbf{Y} \cap \mathrm{AT}(X))) \\ \mathrm{C}_2^a & \lambda X \lambda \mathbf{Y}.\ \exists Z \subseteq X\, [\mathbf{n}(X)(Z) \wedge Z \in \mathbf{Y}] \\ \mathrm{N}_2 & \lambda X \lambda \mathbf{Y}.\ \mathbf{n}(X)(\bigcup(\mathbf{Y} \cap \wp(X))) \end{array}$$

Depending on how the NUM in an NP is marked, a sentence:

$$[_{\mathrm{s}}\ [_{\mathrm{np}_1}\ \mathrm{NUM\ N}]\ [_{\mathrm{vp}}\ \mathrm{V}\ [_{\mathrm{np}_2}\ \mathrm{NUM\ N}]]]$$

could have the nine readings in table 1 (entry form: $\mathrm{NP}_1\mathrm{NP}_2$).

$\mathrm{D}_1\mathrm{D}_1$	$\mathrm{D}_1\mathrm{N}_2$	$\mathrm{D}_1\mathrm{C}_2^a$
$\mathrm{N}_2\mathrm{D}_1$	$\mathrm{N}_2\mathrm{N}_2$	$\mathrm{N}_2\mathrm{C}_2^a$
$\mathrm{C}_2^a\mathrm{D}_1$	$\mathrm{C}_2^a\mathrm{N}_2$	$\mathrm{C}_2^a\mathrm{C}_2^a$

Table 1 *nine readings*

In section 6 we shall see that some of these readings are unwanted. To give a first indication of what they amount to, we have a closer look at three of them: $\mathrm{D}_1\mathrm{D}_1$, $\mathrm{N}_2\mathrm{N}_2$, and $\mathrm{C}_2^a\mathrm{C}_2^a$.

(7) *a.* $|\{d \in [\![man]\!] : |\{d' \in [\![table]\!] : [\![lift]\!](\{d\})(\{d'\})\}| = 2\}| = 4$
b. $|\bigcup\{X \subseteq [\![man]\!] : |\bigcup\{Y \subseteq [\![table]\!] : [\![lift]\!](X)(Y)\}| = 2\}| = 4$
c. $\exists X \in [\![four\ man]\!] \exists Y \in [\![two\ table]\!] : [\![lift]\!](X)(Y)$

On the $\mathrm{D}_1\mathrm{D}_1$ reading of (6) one has (7a): four men each lifted each of two tables; which speaks for itself. The $\mathrm{N}_2\mathrm{N}_2$ reading of (6) gives (7b), which says that there are four men forming collections M and that for each such M there are two tables from which the collections of tables are formed which M lifted. Link [1991] calls these readings 'partitional'. But in a partition the collections do not overlap, whereas N_2 does not require this. The $\mathrm{C}_2^a\mathrm{C}_2^a$ reading of (6), finally, is of a different nature altogether. Shortening, e.g., $\exists X[X \subseteq [\![man]\!] \wedge |X| = 4 \wedge \phi]$ to: $\exists X \in [\![four\ man]\!][\phi]$, one gets (7c). Now (6) states that a collection of four men lifted a collection of two tables.

Table 1 has nine readings, but in this count I disregarded scope ambiguities of NPs. As in Scha's 'strict version' – i.e., his formal system without the rule F4, – the order of the quantifiers is that of the corresponding NPs in (6). The 'loose version' of Scha does allow scope ambiguities and in that system (6) has eighteen readings. Apart from these, there is the so-called cumulative reading in which the NPs are independent of each other. It states that the number of table lifting men is four and the number of man lifted tables is two. In case of numerals some of these readings are equivalent, but for arbitrary determiners this need not be so.

3.2 Link

In his 'Plural' [1991] Link is concerned, among many other things, with showing that sentences like (6) are less ambiguous than Scha would have it.[5] He counts eight readings, some of which are equivalent due to the particular determiners used.

How does Link go about this? He stresses that distributivity is a lexical feature which appears primarily in the head noun of an NP and the head verb of a VP, an observation which can also be found in Link [1983, 310]. This is significant, because it disallows distributivity to occur, say, just at the level of a VP. If distributivity is a lexical feature, a complex VP can only be distributive in a derivative sense, namely in as far as this property is passed on to the VP by its immediate constituents and the way they are combined.

In contrast with Scha, Link does not see distributivity and collectivity as lexical features of determiners. Yet his calculation of the ambiguity of (6) is based on this assumption:

> ...an indefinite PNP gives rise to three different readings, one distributive and two collective. [$\cdots$] Now I have already expressed doubts as to whether I should really distinguish the simple collective [C_2^a] and the partitional reading [N_2]. Be this as it may, I am going to ignore the latter one here. This brings us back to eight different cases... Link [1991]

The calculation is in conflict with the supposition that distributivity is a lexical feature of verbs, but one can make sense of it by calling an NP *distributive* if it binds a distributive argument and *collective* otherwise. Then Scha and Link differ in the number of readings they attribute to an NP. Link's razor leisurely cuts down Scha's distinction between a neutral and a collective reading to leave us C_2^a only.[6] As a result (6) gets eight readings, but one may wonder whether the choice made can be argued for empirically.

To model that the distributivity of an NP depends on the distributivity of the argument it binds, Link introduces the verb modifier δ besides the other operators in (8) for two-place relations (Link [1991]):[7]

$$
\text{(8)} \quad
\begin{array}{lll}
\delta & := & \lambda \mathbf{X} \lambda Y.\ \textsc{at}(Y) \subseteq \mathbf{X} \\
\bullet\bullet & := & \lambda \mathbf{R}^2.\ \mathbf{R} \\
\delta\bullet & := & \lambda \mathbf{R}^2 \lambda X \lambda Y.\ \forall x \in X\ \mathbf{R}(\{x\})(Y) \\
\bullet\delta & := & \lambda \mathbf{R}^2 \lambda X \lambda Y.\ \forall y \in Y\ \mathbf{R}(X)(\{y\}) \\
\delta\delta & := & \lambda \mathbf{R}^2 \lambda X \lambda Y.\ \forall x \in X\ \forall y \in Y\ \mathbf{R}(\{x\})(\{y\})
\end{array}
$$

As to the relational operators, $\delta\bullet$ makes a two-place relation distributive in its

[5] The main aim of Link [1983] is to formalise the similarities between mass terms and plurals.

[6] Link [1991] does not discern among C_2^a and N_2 'for methodological reasons'. The important differences in their logical behaviour does not justify this decision.

[7] For technical convenience I have omitted the requirement that Y be non-empty. In introducing the relational modifiers separately I have followed Link, but they can also be defined from δ (cf. Van der Does [1992, 70]).

first argument and leaves the other one unaffected, and similarly for the other operators. Now, the general scheme of a simple transitive sentence is:

(9) $\mathrm{NP}_1[\lambda X.\mathrm{NP}_2[\lambda Y.\mathrm{DO}(\mathbf{R})(X)(Y)]]$

In (9), DO varies over the relational operators $\bullet\bullet, \delta\bullet, \bullet\delta, \delta\delta$. So, the scheme justifies (6) to have eight readings: there are four instantiations of DO for both orders of the NPs.

The story does not end here. A closer look at Link [1991] shows that his formal system allows (6) to have more readings than the eight listed. Lønning [1987, 1989] and Roberts [1987] have observed that the conviction that distributivity is a matter of the verb is countered by some of the logical forms given. Using a type theoretical analogue, the subject wide-scope doubly distributive reading of (6) is formalised in (10):

(10) $\exists X \in [\![\mathit{four\ man}]\!][\delta(\lambda Y.\exists Z \in [\![\mathit{two\ table}]\!][\bullet\delta([\![\mathit{lift}]\!])(Y)(Z)])(X)]$

On this reading, (6) states that there is a collection of four men and that for each member m in this collection there is a collection of two tables each of which are lifted by m. In terms of verb categories, the distributivity in (10) is non-lexical: it is a property of the VP *to lift two tables*.[8] This makes plain that Link selects his readings from the scheme (11) rather than from (9):

(11) $\mathrm{NP}_1[\lambda X.\mathrm{DO}_1(\lambda Y.\mathrm{NP}_2[\lambda Z.\mathrm{DO}_2(\mathbf{V})(Y)(Z)])(X)]$

Here DO_2 is as before, while DO_1 can be either δ or $\lambda\mathbf{X}.\mathbf{X}$. What has gone unnoticed is that scheme (11) gives a sum total of sixteen possible readings; the previous eight for each of δ and $\lambda\mathbf{X}.\mathbf{X}$. Some of these readings are undesirable, like those in which DO_2 is set to $\delta\delta$ and DO_1 to δ with the effect that an argument place is marked for distributivity twice. It is not clear, though, how they can be precluded when working with verb modifiers.

One way to circumvent the overgeneration in Link's system is Scha's proposal: distributivity and collectivity are lexical features of determiners.[9] To this end, a denotation for $\mathbf{n}$ could be defined with a built-in use of δ, as in (12a) and its equivalent (12b):

(12) *a.* $\lambda X \lambda \mathbf{Y}.\, \exists Z \subseteq X\,[|Z| = n \wedge \delta(\mathbf{Y})(Z)]$
b. $\lambda X \lambda \mathbf{Y}.\, \exists Z \subseteq X\,[|Z| = n \wedge \mathrm{AT}(Z) \subseteq \mathbf{Y}]$

Recall that this is how the VP strategy is mimicked within the NP strategy. Formulated in terms of the type $((et)((et)t))$ denotation of $\mathbf{n}$ this becomes:

$$\mathrm{D}_1^a \quad \lambda X \lambda \mathbf{Y}.\, \exists Z \subseteq X\,[\mathbf{n}(X)(Z) \wedge \mathrm{AT}(Z) \subseteq \mathbf{Y}]$$

With D_1^a numerals at hand, the $\mathrm{D}_1^a\mathrm{D}_1^a$ reading of (6) still reduces to (10b) while the infelicitous readings are blocked. In D_1' the marking of an argument place for distributivity is connected with binding it. Since double bindings are impossible, double distributivity markings are too.

On this view, Link's treatment of numerals is seen as:

[8] Roberts [1987] argues that the distributivity of the VP is lexical: it results from a (c)overt use of *each*. But how to justify the use of $\bullet\delta$ along these lines?

[9] The formalisation of so-called variety agreement in Van der Does [1992, ch. 4] is another system to preclude double markings.

- opting for Scha's collective C_2^a only;
- interchanging Scha's D_1 for the newly distributive D_1^a.

There is a reading generated by the scheme 11 which cannot be obtained by use of the distributive NP denotation.[10] It is the reading where DO_1 is the identity and $DO_2\ \delta\delta$. It makes (6) equivalent to (13):

(13) $\exists X \in [\![\mathit{four\ man}]\!] \exists Z \in [\![\mathit{two\ table}]\!] : \text{AT}(X) \times \text{AT}(Z) \subseteq [\![\mathit{lift}]\!]$

How serious a defect is it that (13) cannot be generated in this way? I'm inclined to think that it is no defect at all. In fact (13) is the branching reading of (6) as proposed by Barwise [1979] for the case of MON$\uparrow$ quantifiers (cf. also Hoeksema [1983, appendix]). But recent research has shown that the branching reading is much more complex. In particular, it is argued by Sher [1990] and Spaan [1993] that some notion of maximality is involved; a notion which is absent in case of collective readings. For this reason I think that we better treat the non-iterative forms of quantification separately. Also, within a collective framework I surmise that these forms of quantification use the neutral rather than the distributive readings of the quantifiers. In the next section I give a first indication of what these neutral readings look like.

3.3 Getting mixed-up

Sentences with large numerals often have an intermediate or neutral reading, which is neither distributive nor collective. Sentence (14) is an example adapted from Link [1991]:[11]

(14) Half a million children gathered all over the country

This sentence will be true even if the collection of children did not gather as a whole; there could be many subgatherings that may or may not overlap. The N_2 numerals of Scha come close to modelling this reading. However, it requires that only groups of children gathered, and this seems too strict. I do not think (14) is false, if some adults join the children.

Gillon [1987] has similar intuitions. His observations imply that (15) should be valid:

(15) Hammerstein and Rodgers wrote a musical together
Rodgers and Hart wrote a musical together
$\Rightarrow$ At least three composers wrote some musicals

Here, too, the most natural explanation is in terms of neutral readings.

I propose to model the neutral reading by means of the 'partaking in' operator π (cf. Link [1987]):

$$\pi := \lambda \mathbf{X} \lambda Y.\ Y \subseteq \bigcup \mathbf{X}$$

[10] I am indebted to one of the referees here.

[11] Since Link [1991] does not discern among C_2^a and N_2, he presumably takes (14) to be collective.

Using either a C_2^a or a N_2 numeral with a π-marked VP, the conclusion of (15) is formalised by (16a-b), which are synonymous to their primed neighbours:

(16) *a.* $\exists X \in [\![\textit{at least three composers}]\!][\pi([\![\textit{write some musicals}]\!])(X)]$
a'. $\exists X \in [\![\textit{at least three composers}]\!][X \subseteq \bigcup[\![\textit{write some musicals}]\!]]$
b. $|\bigcup\{X \subseteq [\![\textit{composer}]\!] : \pi([\![\textit{write some musicals}]\!])(X)\}| \geq 3$
b'. $|\bigcup\{X \subseteq [\![\textit{composer}]\!] : X \subseteq \bigcup[\![\textit{write some musicals}]\!]\}| \geq 3$

Note that neither of the denotations restricts the VP to collections of composers. On both readings composers may have collaborated with others to write a musical.

Applied to transitive verbs, the π-operator leads to the problem of double marking noted earlier for the δ-operator. Again it can be solved by embedding the VP strategy into the NP strategy: incorporate the use of the operator into the meaning of a numeral. As a result, two other denotations are introduced:[12]

$$\begin{array}{lcl} \mathrm{N}_3 & := & \lambda X \lambda \mathbf{Y}.\ |\bigcup\{Z \subseteq X : \pi(\mathbf{Y})(Z)\}| = n \\ & = & \lambda X \lambda \mathbf{Y}.\ |\bigcup\{Z \subseteq X : Z \subseteq \bigcup \mathbf{Y}\}| = n \\ \mathrm{N}_3^a & := & \lambda X \lambda \mathbf{Y}.\ \exists Z \subseteq X[|Z| = n \wedge \pi(\mathbf{Y})(Z)] \\ & = & \lambda X \lambda \mathbf{Y}.\ \exists Z \subseteq X[|Z| = n \wedge Z \subseteq \bigcup \mathbf{Y}] \end{array}$$

This ends my discussion of numerals. In the next section I show how the insights obtained here can be generalised to give a semantics for arbitrary plural noun phrases.

4 Determiners

This following question is central to this section:

> Do systematic and empirically satisfying ways exists in which the determiners of type $((et)((et)t))$ can be related to their readings in type $((et)(((et)t)t))$?

As a first answer, I propose six lifts suggested by the treatment of numerals, three for the NP and three for the VP approach. Although there are other options – we shall encounter some in section 6, – these seem the most reasonable ones. Note that if both strategies were feasible, excessive ambiguity would result. An intransitive sentence such as

(17) Most of the men made music

[12]According to Lønning [1991] both these readings are wrong. They incorrectly validate the inference:

John and Harry ate three pizzas
$\not\Rightarrow$ John ate three pizzas

This observation is correct, I think, for the VP strategy. Then there is no good reason why the modification of the VP in premise and conclusion should differ. The case is different for the NP strategy, where the observation presumes that both NPs should have the same reading. But since the NPs are different this need not be so. The NP in the conclusion may even lack the reading assumed for the premise. Proper names, in particular, do not have the neutral reading: $\lambda \mathbf{X}.[\![\textit{john}]\!] \in \bigcup \mathbf{X}$. And their standard denotation $\lambda \mathbf{X}.\mathbf{X}(\{[\![\textit{john}]\!]\})$ does invalidate the inference.

would have an upperbound of six readings (one for each lift). And for transitive sentences the situation is even worse. The sentence

(18) All men lifted some tables

would get at most seventy-three readings !:

> (*six* $\times$ *six* NP readings $\times$ *two* scope orderings)
> + *one* cumulative reading

It is as of old, when people were taught differences.

Here this embarrassment of riches is to some extent eliminated; on each strategy a determiner has at most three readings. Yet one may find the number of readings still too high (at most nine for a transitive sentence if the neutral reading is confined to the cumulative reading). In trying to attain a further reduction of readings, one could address an underlying methodological issue, namely:

> Where exactly does the line of demarcation run between proper readings and mere models realizing a reading? Link [1991]

An answer to this question cannot result from a mere inspection of one's idiolect. Instead, one should use both logical results and empirical arguments to reduce the readings to the few, if any, which somehow encompass the others. The other 'readings' could then be seen as types of verifying situations to be promoted to explicit readings only if lexical items demand it.

My agenda looks as follows. The lifts are introduced in section 4.1, and section 4.2 indicates their usefulness by means of some examples. In section 5, the strengths and weaknesses of the lifts are compared. I investigate the logical relationships between the readings for arbitrary and for monotonic determiners, besides their quantificational force. The important issue of neutral readings and scope is discussed in section 6. This section also addresses the question whether the neutral readings can be used to effect a reduction in ambiguity.

4.1 Type lifting operators

A first step towards a general semantics for plural noun phrases consists in rewriting the numeral denotations in terms of their standard denotation in type $((et)((et)t))$. In section 3, the following denotations were given:[13]

$$\begin{array}{ll}
\mathrm{D}_1 & \lambda X \lambda \mathbf{Y}.\ \mathbf{n}(X)(\bigcup(\mathbf{Y} \cap \textsc{at}(X))) \\
\mathrm{N}_2 & \lambda X \lambda \mathbf{Y}.\ \mathbf{n}(X)(\bigcup(\mathbf{Y} \cap \wp(X))) \\
\mathrm{N}_3 & \lambda X \lambda \mathbf{Y}.\ \mathbf{n}(X)(X \cap \bigcup \mathbf{Y}) \\
\mathrm{D}_1^a & \lambda X \lambda \mathbf{Y}.\ \exists Z \subseteq X\, [\mathbf{n}(X)(Z) \wedge \textsc{at}(Z) \subseteq \mathbf{Y}] \\
\mathrm{C}_2^a & \lambda X \lambda \mathbf{Y}.\ \exists Z \subseteq X\, [\mathbf{n}(X)(Z) \wedge Z \in \mathbf{Y}] \\
\mathrm{N}_3^a & \lambda X \lambda \mathbf{Y}.\ \exists Z \subseteq X\, [\mathbf{n}(X)(Z) \wedge Z \subseteq \bigcup \mathbf{Y}]
\end{array}$$

[13]In case of N_3, I gave an equivalent form.

This manner of presenting the numerals suggests uniform procedures for turning a determiner of type $((et)((et)t))$ into one of type $((et)(((et)t)t))$, as are captured by the lambda-abstracts in (19):

(19)
$$
\begin{array}{ll}
\mathrm{D}_1 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(\bigcup(\mathbf{Y}\cap \mathrm{AT}(X))) \\
\mathrm{N}_2 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(\bigcup(\mathbf{Y}\cap \wp(X))) \\
\mathrm{N}_3 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(X\cap \bigcup\mathbf{Y}) \\
\mathrm{D}_1^a & \lambda D\lambda X\lambda \mathbf{Y}.\ \exists Z\subseteq X\,[D(X)(Z)\wedge \mathrm{AT}(Z)\subseteq \mathbf{Y}] \\
\mathrm{C}_2^a & \lambda D\lambda X\lambda \mathbf{Y}.\ \exists Z\subseteq X\,[D(X)(Z)\wedge Z\in \mathbf{Y}] \\
\mathrm{N}_3^a & \lambda D\lambda X\lambda \mathbf{Y}.\ \exists Z\subseteq X\,[D(X)(Z)\wedge Z\subseteq \bigcup\mathbf{Y}]
\end{array}
$$

The lifts N_2 and N_3 are also in Van Benthem [1991, 67–68], which made me aware of the virtues of using lifts for this purpose.

The names of the lifts carry information on how they function. The letter 'D' stands for distributive, 'C' for collective, and 'N' for neutral. The superscript 'a' indicates that the 'old' determiner leaves the VP argument outside of its scope. These lifts are called the a lifts, and the other ones the non-a lifts. The subscripts, finally, point to the way in which the noun and the verb phrase extension are related to each other. 1: only the atoms formed out of the noun extension matter; 2: only the collections formed out of the noun extension matter; 3: all members in the noun extension matter that occur in a collection in the verb phrase extension. The determiner lifts are distributed over the two approaches as follows. On the NP strategy we have D_1, C_2^a, and as yet two options for the neutral reading: N_2 and N_3. On the VP strategy we have D_1^a, C_2^a, and N_3^a. So the strategies coincide in their treatment of collective readings.

The system behind the determiner names can be brought to the fore by considering some equivalent forms. The lifts D_1^a and N_3^a can also be obtained by combining C_2^a with the application of a modifier δ or π to its VP argument. This is particularly plain when using the format:

$$
\begin{array}{ll}
\mathrm{D}_1^a & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)\cap \wp(X)\cap \delta(\mathbf{Y})\neq\emptyset \\
\mathrm{C}_2^a & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)\cap \wp(X)\cap \mathbf{Y}\neq\emptyset \\
\mathrm{N}_3^a & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)\cap \wp(X)\cap \pi(\mathbf{Y})\neq\emptyset
\end{array}
$$

Analogously, the non-a lifts can be presented in a uniform manner, now obtaining D_1 and N_3 from N_2 by means of δ or π:[14]

$$
\begin{array}{ll}
\mathrm{D}_1 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(\bigcup(\delta(\mathbf{Y})\cap \wp(X))) \\
\mathrm{N}_2 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(\bigcup(\mathbf{Y}\cap \wp(X))) \\
\mathrm{N}_3 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(\bigcup(\pi(\mathbf{Y})\cap \wp(X)))
\end{array}
$$

Another way to present the non-a lifts stresses that they mainly differ in the

[14] There is an interesting contextdependent variant of N_2 in which the power set operator is changed for a function F with for all X: $F(X)\subseteq\wp(X)$. $F(X)$ is a set of contextually given collections formed out of the members of the noun extension. Whenever $F(X)\subseteq \mathrm{AT}(X)$ we have a distributive reading. And if $F(X)$ is a singleton we have the 'witness' version of the collective reading describing a contextually given collection. As we shall see in section 6, the remaining possibility for $F(X)$ is problematic in case of transitive sentences.

ways the type $((et)t)$ VP is restricted to the type (et) noun:

$$\begin{array}{ll} \mathrm{D}_1 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(\bigcup(\mathbf{Y}|^1_X)) \\ \mathrm{N}_2 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(\bigcup(\mathbf{Y}|^2_X)) \\ \mathrm{N}_3 & \lambda D\lambda X\lambda \mathbf{Y}.\ D(X)(\bigcup(\mathbf{Y}|^3_X)) \end{array}$$

Here the functions $|^i$ $(1 \leq i \leq 3)$ of type $(\,(et)(((et)t)((et)t))\,)$ are defined by:

$$\begin{array}{lll} |^1 & := & \lambda X\lambda \mathbf{Y}.\ \mathbf{Y} \cap \mathrm{AT}(X) \\ |^2 & := & \lambda X\lambda \mathbf{Y}.\ \mathbf{Y} \cap \wp(X) \\ |^3 & := & \lambda X\lambda \mathbf{Y}.\ \lambda Y \exists Z[\mathbf{Y}(Z) \wedge Y = Z \cap X] \end{array}$$

In set notation $|^3$ can be written as $\lambda X\lambda \mathbf{Y}.\{X \cap Z : Z \in \mathbf{Y}\}$. The notation: $\mathbf{Y}|^i_X$ instead of $|^i(X)(\mathbf{Y})$ derives from the fact that the functions are used to restrict a set $\mathbf{Y}$ to a set X.

Proposing determiner lifts is one thing, but it remains to be argued whether they make sense, or if not generally, whether their application should be restricted to particular occasions. This question is addressed in section 5 and 6. For now, I give an impression of how the lifts work by inspecting the lifted forms of a few determiners.

4.2 Some examples

In this section I have a closer look at the lifted variants of **all**, **some**$_{pl}$, **not all**, **at most four** and of the higher-order determiner **most**. I use **some**$_{pl}$ to pay special attention to the logical behaviour of the collective and neutral readings. The lifted singular determiners, like **every** and **some**$_{sg}$, are discussed separately.

All, the$_{pl}$ The plural determiners **all** and **the**$_{pl}$ of type $((et)((et)t))$ denote the relation: $\lambda X\lambda Y.X \subseteq Y$.[15] As a consequence, their a lifts are of the form (20a), or equivalently (20b):

(20) *a.* $\lambda Y\lambda \mathbf{Z}.\ \exists X\,[Y = X \wedge \mathrm{OP}(\mathbf{Z})(X)]$
b. $\lambda Y\lambda \mathbf{Z}.\ \mathrm{OP}(\mathbf{Z})(Y)$

Here 'OP' varies over δ, π and $\lambda\mathbf{X}.\mathbf{X}$. Spelling out the details, the lifts of **all** can be summed up in a table:

(21)
$$\begin{array}{lllll} \mathrm{D}_1(\mathbf{all}) & = & \lambda X\lambda \mathbf{Y}.\ \mathrm{AT}(X) \subseteq \mathbf{Y} & = & \mathrm{D}^a_1(\mathbf{all}) \\ \mathrm{N}_2(\mathbf{all}) & = & \lambda X\lambda \mathbf{Y}.\ X \subseteq \bigcup(\mathbf{Y} \cap \wp(X)) & & \\ \mathrm{C}^a_2(\mathbf{all}) & = & \lambda X\lambda \mathbf{Y}.\ \mathbf{Y}(X) & & \\ \mathrm{N}_3(\mathbf{all}) & = & \lambda X\lambda \mathbf{Y}.\ X \subseteq \bigcup \mathbf{Y} & = & \mathrm{N}^a_3(\mathbf{all}) \end{array}$$

Note that swapping the arguments of the D_1 and the N_3 reading respectively gives δ and π. This simple observation shows that conjoined VPs which are

[15] One may wonder whether an explicit marking for plurality in the semantics of NPs should be used; e.g., as in $\lambda X\lambda Y.X \subseteq Y \wedge |X| > 1$. I choose not to, for often such markings do not give correct truth conditions under negation. In using a plural NP it is rather presupposed that there is a plurality of the required kind; a presupposition which may disappear when forming complexes.

only partly marked for distributivity or neutrality can be handled in essentially the same way on the VP as on the NP strategy (cf. section 1.3).

Some of the denotations in (21) are also familiar from Scha [1981, 491]. He grants **all** a distributive and a collective reading, which are respectively captured by $\mathrm{D}_1(\mathbf{all})$ and $\mathrm{C}_2^a(\mathbf{all})$. The C_2^a reading of **all** also gives Scha's interpretation of **the**; i.e. the $\in$-relation which leaves noun extensions unaffected. For example, (22a) is formalised by (22b-c):

(22) *a.* The sheep flocked
b. $\mathrm{C}_2^a(\mathbf{all})([\![sheep]\!])([\![flock]\!])$
c. $[\![flock]\!]([\![sheep]\!])$

It is nice to see that this use of **the**, which at first appeared so different, can be seen as a lifted form of 'low-level' **all**, thus accounting for the feeling that these determiners are closely related. This is not to say that the determiners function alike in all circumstances. E.g., only (23a) has a collective reading (Dowty [1986]):

(23) *a.* The trees get thinner in the middle
b. All trees get thinner in the middle

Some$_{pl}$ As is usual, I take **some**$_{pl}$ in type $((et)((et)t))$ to be equivalent with **at least two**. Its lifts are:

(24)
$$\begin{array}{llll}
\mathrm{D}_1(\mathbf{some}_{pl}) & = & \lambda X\lambda\mathbf{Y}.\ |\mathrm{AT}(X)\cap\mathbf{Y}|\geq 2 & = \mathrm{D}_1^a(\mathbf{some}_{pl})\\
\mathrm{N}_2(\mathbf{some}_{pl}) & = & \lambda X\lambda\mathbf{Y}.\ |\bigcup(\mathbf{Y}\cap\wp(X))|\geq 2 & \\
\mathrm{C}_2^a(\mathbf{some}_{pl}) & = & \lambda X\lambda\mathbf{Y}.\ \exists Z\subseteq X\,[|Z|\geq 2\wedge Z\in\mathbf{Y}] & \\
\mathrm{N}_3(\mathbf{some}_{pl}) & = & \lambda X\lambda\mathbf{Y}.\ |X\cap\bigcup\mathbf{Y}|\geq 2 & = \mathrm{N}_3^a(\mathbf{some}_{pl})
\end{array}$$

As a result, the C_2^a reading of sentence (25) may be compared with two neutral readings: N_2 and N_3.

(25) Some trumpet players jammed
$$\begin{array}{ll}
\exists Z\subseteq[\![trumpet\ player]\!]\,[|Z|\geq 2\wedge Z\in[\![jam]\!]] & \mathrm{C}_2^a\\
|\bigcup([\![jam]\!]\cap\wp([\![trumpet\ player]\!]))|\geq 2 & \mathrm{N}_2\\
|\bigcup([\![jam]\!])\cap[\![trumpet\ player]\!]|\geq 2 & \mathrm{N}_3
\end{array}$$

In this comparison I shall mainly concentrate on the logical behaviour of the readings. This will show that the neutral readings cannot do duty as collective readings.

On the collective reading of a sentence, one would expect some inferences to be invalid. For instance, assuming that Miles and Chet are trumpet players, while Wayne and Stan are saxophonists, the inference (26) should fail:

(26) Miles and Wayne jammed together
Chet and Stan jammed together
$\not\Rightarrow$ Some trumpet players jammed together

Consequently, N_3, which makes the inference valid, does not give a collective reading. That N_2 does invalidate (26), is due to its sensitivity for 'CN-pure' collections. Relative to the first argument of a determiner, only the collections formed from its extension are counted relevant. However, in as far as the pure CN-collections of a VP are concerned, $N_2([\![DET]\!])([\![CN]\!])$ and $N_3([\![DET]\!])([\![CN]\!])$ behave alike. This explains why both make (27) valid:

(27) Miles made music and Chet made music
$\not\Rightarrow$ Some trumpet players made music together

So, N_2 cannot be used for collective readings either. This leaves C_2^a the one determiner lift that models these readings.

Of course, I do not conclude that N_2 and N_3 are useless. Although plural NPs are a necessary ingredient for sentences to have collective readings, their use is in no way sufficient. As we have seen in section 3.3, plural NPs have a neutral use which exhibits a certain insensitivity towards the structure of collections. And it is here where N_2 and N_3 should be put to work.

Not all, at most four It is sometimes felt that MON↓ determiners are intrinsically distributive but this is not so. Sentence (28a) has a collective reading in that it might mean (28b):

(28) *a.* Not all heroines came together
b. Some heroines came together but not all

An inspection of the lifts shows that (28b) can be had via (29a), i.e., C_2^a(**not all**), which makes it mean (29b):

(29) *a.* $\lambda X \lambda \mathbf{Y}.\, \exists Z \subseteq X[X \cap \overline{Z} \neq \emptyset \wedge Z \in \mathbf{Y}]$
b. $\exists Z \subseteq [\![\mathit{heroine}]\!][[\![\mathit{heroine}]\!] \cap \overline{Z} \neq \emptyset \wedge Z \in [\![\mathit{come\ together}]\!]]$

As is shown by (30), the MON↓ **at most four** has a collective reading, too, which is paraphrased by (30b).

(30) *a.* At most four heroines came together
b. All collections of heroines which came together, contained at most four heroines

The paraphrase (30b) of (30a) is obtained by means of the dual of C_2^a defined by:

$$\begin{aligned} \widetilde{C_2^a} &:= \lambda D \lambda X \lambda \mathbf{Y}.\, {\sim} C_2^a({\sim} D) \\ &= \lambda D \lambda X \lambda \mathbf{Y}.\, \forall Z \in \wp(X) \cap \mathbf{Y}[D(X)(Z)] \end{aligned}$$

Applied to **at most four** this lift gives (30a) the meaning (31):

(31) $\forall Z \in \wp([\![\mathit{heroine}]\!]) \cap [\![\mathit{come\ together}]\!] : |[\![\mathit{heroine}]\!] \cap Z| \leq 4$

Sentence (30a) is one of the simplest which asks for universal quantification over collections (see Link [1987] for a discussion).

Most It has been observed that **most** only has distributive uses in which it quantifies over atoms or individuals (Roberts [1987]). Do (32a,b) have collective readings?

(32) *a.* Most boys came together
b. Most of the boys came together

I agree with Roberts that (32a) is a bit queer. If it should be granted a meaning at all, one should use the neutral N_3:

$$\mathrm{N}_3(\mathbf{most}) \;=\; \lambda X \lambda \mathbf{Y}.\, |X \cap \bigcup \mathbf{Y}| > |X \cap \overline{\bigcup \mathbf{Y}}|$$

But a collective reading of (32b), with its partitive construction, is perfectly in order and can be obtained via a C_2^a lift to get the truth conditions:

$$\exists Z \subseteq [\![boy]\!] \left[|Z| > |[\![boy]\!] \cap \overline{Z}| \wedge Z \in [\![come\ together]\!] \right]$$

Atomic and intrinsic distributive determiners Let us call a DET *atomic* iff it satisfies the following form of conservativity in type $((et)(((et)t)t))$:

$\textsc{Cons}_1$ In type $((et)(((et)t)t))$ a determiner Δ is $\textsc{cons}_1$, iff for all X, $\mathbf{Y}$:

$$\Delta(X)(\mathbf{Y}) \Leftrightarrow \Delta(X)(\mathbf{Y} \cap \textsc{at}(X))$$

NPs are atomic, iff they are generated from an atom, or iff they are formed from an atomic determiner. Examples are proper names like 'Woody' or NPs with the determiners **every** and $\mathbf{some}_{sg}$. Atomic determiners do not permit collective readings because they do not combine with collective predicates.[16] The sentences in (33) are all senseless:

(33) *a.* *Woody flocked
b. *Every bird flocked
c. *Some bird flocked

In order to account for this, one could make uninterpretability correspond to a categorial misfit. Then one uses the fact: if atomic NPs take their denotation in $\mathbf{D}_{((et)t)}$ type theory prohibits them to combine with collective predicates, since these take their denotation in $\mathbf{D}_{((et)t)}$ too. The same would hold for proper names. This move has been made in the literature, e.g. Bennett [1974], but there are good reasons why people tend to reject it nowadays:

1. Some verbs, like *to play chess*, combine with atomic and non-atomic NPs alike.
2. Atomicity is not preserved under non-Boolean coordination; e.g.:

$$\left.\begin{array}{l}\text{Tony and Chick}\\ \text{A drummer and a pianist}\end{array}\right\}\text{jammed together}$$

[16] Atomicity and collectivity have strong ties with syntactic number. Since the fit is not perfect, I prefer to use the semantical terminology.

It is hard to see how 1 or 2 can be obtained, using the common typings of NPs without further ado. Here I shall discuss only 1, since 2 involves non-Boolean coordination which is not treated in this article.

To solve 1, Scha [1981], Link [1983], and others have used the fact that collections as sets always enforce a worst case: type $((et)t)$. They interpret all VPs at this level. The task is now to choose a determiner lift which enable atomic NPs to combine with mixed predicates. But this lift should preclude the NPs to associate with collective predicates to form contingent statements.[17] All this is achieved by allowing proper names to be lifted by the function $\lambda x \lambda y.x = y$, and to obtain other atomic NPs via D_1. Given that *to flock* is collective, the non-contingent (34a-c) result:

(34) *a.* $\{[\![veronica]\!]\} \in [\![flock]\!]$
b. $\text{AT}([\![sheep]\!]) \subseteq [\![flock]\!]$
c. $\text{AT}([\![sheep]\!]) \cap [\![flock]\!] \neq \emptyset$

Since the predication in D_1^a is always of the form (34b), this lift will result in non-contingency too.

As it happens, on using D_1 the techniques of Russell/Bennett and of Scha yield the same truth conditions for sentences with conservative determiners, for then:

$$\mathbf{D}(X)(Y) \Leftrightarrow \mathrm{D}_1(\mathbf{D})(X)(\text{AT}(Y))$$

This is a point in favour of D_1, since D_1^a has the equivalence for conservative MON$\uparrow$ determiners only.

In my opinion, the strategy to make unwell-formed sentences logical validities or contradictories is conceptually not very appealing. Another way to deal with this problem is to use variety features that can mark expressions for distributivity, collectivity and so forth. Sentences require agreement, since an NP and a VP of different variety cannot combine with each other. Then, (33a-c) cannot even be formed, since atomic NP are distributive while the VP *flocked* is collective. Van der Does [1992, ch. 4] introduces such a feature system, also to solve the problems of overgeneration noted in section 1.3.

To summarise this section, the examples have shown that only C_2^a yields collective readings. As yet we have no sufficient ground to choose among the two distributive and the three neutral readings given by the strategies. This issue will occupy the next sections. Besides, the first part of section 5 studies the logical relationships between determiner readings in $((et)(((et)t)t))$.

5 Comparisons

5.1 Maps of readings

The question of Link [1991] – 'Where exactly does the line of demarcation run between proper readings and mere models realizing a reading?' – suggests a

[17] In three- or four-valued semantics, unwell-formed sentences could be taken as un- or overdefined. But in a two-valued semantics, as is used here, the usual option is to make them uninformative; i.e., to let their interpretation be either universally valid or unsatisfiable.

logician's route to reduce ambiguity: go and search for readings which encompass the others and use these unless lexical items force you to do otherwise. This will involve an investigation on how the lifted forms of a determiner relate logically (a topic which is of interest regardless). Here, I adopt the global view on determiners, which makes them functors associating with each domain E the determiner $\mathbf{D}_E$ of type $((et)((et)t))$. The notion of the relative strength of lifted determiners is made precise in the standard manner:

Definition 5.1 (relative strength) *Let L and L' be operators of type*

$$(((et)((et)t))\ ((et)(((et)t)t)))$$

By definition a determiner $\mathbf{D}$ satisfies: $L \longrightarrow L'$, if for all domains E:

$$L(\mathbf{D}_E) \subseteq L'(\mathbf{D}_E)$$

The L' reading of $\mathbf{D}$ is at most as strong as its L reading, if $\mathbf{D}$ satisfies $L \longrightarrow L'$. I write $L \equiv L'$, just in case $L \longrightarrow L'$ and $L' \longrightarrow L$.

Using the arrow-notation, we can draw the maps of readings for arbitrary and for monotone determiners. Proposition 5.2 shows that for arbitrary determiners a weakest reading can always be found.[18]

Proposition 5.2 *Every determiner satisfies the arrows in figure 1.* □

$$\begin{tikzcd}
\mathrm{D}_1 \arrow[r] & \mathrm{D}_1^a \arrow[r] & \mathrm{N}_3^a & \\
& \mathrm{N}_2 \arrow[ur] & \mathrm{N}_3 \arrow[u] & \mathrm{C}_2^a \arrow[ul]
\end{tikzcd}$$

Figure 1: *arbitrary determiners*

Of course, the situation may be different for determiners of a particular kind. The map of readings for MON↑ determiners, for example, has a nice symmetry about it.

Proposition 5.3 *Each* MON↑ *determiner $\mathbf{D}$ satisfies the map in figure 2. Besides this, one has: $\mathrm{D}_1^a \equiv \mathrm{D}_1$ and $\mathrm{N}_3^a \equiv \mathrm{N}_3$.* □

The map for MON↓ determiners is asymmetric again.

Proposition 5.4 *Each* MON↓ *determiner $\mathbf{D}$ satisfies the map in figure 3, and also: $\mathrm{D}_1^a \equiv \mathrm{N}_3^a$.* □

[18] Recall that the proofs are in Van der Does [1992, ch. 5].

$$\begin{tikzcd}
 & \mathrm{N}_3 & \\
 & \mathrm{N}_2 \arrow[u] & \\
\mathrm{D}_1 \arrow[ur] & & \mathrm{C}_2^a \arrow[ul]
\end{tikzcd}$$

Figure 2: MON↑ *determiners*

$$\begin{tikzcd}
\mathrm{N}_3 \arrow[r] & \mathrm{N}_2 \arrow[r] & \mathrm{D}_1 \arrow[r] & \mathrm{N}_3^a \\
 & & & \mathrm{C}_2^a \arrow[u]
\end{tikzcd}$$

Figure 3 MON↓ *determiners*

From a logical point of view it is natural to ask whether there are converses to proposition 5.3 and 5.4. The answer is given in Van der Does [1992, ch. 5], where it is shown that the maps in figure 2 and 3 are typical of conservative MON↑ and MON↓ determiners, in that order.

For now a more important question is: what do the maps tell us with respect to the strategy suggested by Link? A weakest reading is available; namely, N_3^a which is equivalent to N_3 on the MON↑ determiners and to D_1^a on the MON↓ ones. It remains to be seen, though, whether this reading is empirically defensible. If so, it will be *the* candidate to model neutral readings. Otherwise, we are left in general with five readings: D_1, D_1^a, N_2, C_2^a and N_3. This, again, would require us to show which lift gives the best way to capture the distributive reading, D_1 or D_1^a. We should also ask whether it is necessary to discern two neutral readings, N_2 and N_3.

In sum, we have to make observations which give more insight into the behaviour of the distributive and the neutral readings as they are proposed by the NP and the VP strategy. A choice is made in the ensuing section on the quantificational force of determiners.

5.2 Quantificational force

It is quite clear that the lifts induce a semantical change in the determiner denotations. So, how does a determiner function when it is lifted to $((et)(((et)t)t))$? More precisely, does it still determine quantities in an accurate way? And if not, is this modification of the standard meaning desirable?

That the numerals need not behave alike on different readings, is shown by example (35) adapted from Lønning [1987, 205]:

(35) *a.* Yesterday, exactly five boys bought a boat together in the shop
b. Yesterday, exactly five boys each bought a boat in the shop
c. Yesterday, 836 people bought a boat at the trade fair

In case of (35a,b) the total number of boys that bought a boat in the shop during the day may well be more than five. But for (35c), where the number of boat buying clients is an issue, this is impossible.

The explanation is simple. The collective reading in (35a) is about an unspecified collection of exactly five boys, while the distributive (35b) restricts the count to boys who bought a boat all by themselves. Both statements leave the possibility of other groups of boys that bought a boat. In contrast, (35c) is sort of neutral. Here one is not so much interested in whether the people bought a boat by themselves or with others, the sum total of people that bought a boat is at stake. On this reading one discards the structure of collections and just counts the relevant people.

To see which lifts function as required in this respect, I have listed the six readings of (36) as they are obtained from the lifts of **exactly three**:

(36) Exactly three brothers are gossiping

$$
\begin{array}{ll}
|\bigcup([\![gossip]\!] \cap \mathrm{AT}([\![brothers]\!]))| = 3 & \mathrm{D}_1 \\
|\bigcup([\![gossip]\!] \cap \wp([\![brother]\!]))| = 3 & \mathrm{N}_2 \\
|\bigcup([\![gossip]\!]) \cap [\![brother]\!]| = 3 & \mathrm{N}_3 \\
\exists X \subseteq [\![brother]\!][\,|X| = 3 \wedge \mathrm{AT}(X) \subseteq [\![gossip]\!]] & \mathrm{D}_1^a \\
\exists X \subseteq [\![brother]\!][\,|X| = 3 \wedge X \in [\![gossip]\!]] & \mathrm{C}_2^a \\
\exists X \subseteq [\![brother]\!][\,|X| = 3 \wedge X \subseteq \bigcup([\![gossip]\!])] & \mathrm{N}_3^a
\end{array}
$$

Examining the formulae, one would expect none of the *a* readings to give a correct total count: on these readings the numerals leave the verb phrase extension outside their scope. And indeed, focussing on quantities the *a* readings in (36) rather mean:

(37) At least three brothers are gossiping

The reason is that they allow the existence of other collections of gossiping brothers, whose members are left uncounted (cf. Van Benthem [1986, 52], Lønning [1987, 205]). However, this change in meaning should not just be attributed to the *a*-lifted determiners, for a similar change takes place in case of D_1 and N_2: $\mathrm{D}_1(\mathbf{n})$ counts the members of $\mathrm{AT}(X)$ in $\mathbf{Y}$ discarding Xs in larger $\mathbf{Y}$-collections, while $\mathrm{N}_2(\mathbf{n})$ fails to count the Xs in mixed $\mathbf{Y}$-collections. Only $\mathrm{N}_3(\mathbf{n})$, which 'X-rays' the collections, does not miss any of the relevant Xs.

Note, by way of digression, that there is a more surprising reason why an explanation in terms of the *a*-use of a determiner fails: all lifts are equivalent to lifts in which the determiners are treated in this way. This is shown using the uniform formats of D_1, N_2 and N_3 in terms of the restricting functions given in section 4.1 (for convenience the lifts are temporally renamed L_i with $i \in \{1, 2, 3\}$):

$$\mathrm{L}_i \quad \lambda D \lambda X \lambda \mathbf{Y}.\ D(X)(\bigcup(\mathbf{Y}\,|^i_X))$$

Three alternative lifts using the restriction functions are:

$$\mathrm{M}_i \quad \lambda D \lambda X \lambda \mathbf{Y}.\ \exists Z \subseteq X[D(X)(Z) \wedge \exists \mathbf{Z} \text{ cv } Z[\mathbf{Z} = \mathbf{Y}\,|^i_X]]$$

Here, $\mathbf{Z}$ cv Z means that $\mathbf{Z}$ *covers* Z; that is: $\bigcup \mathbf{Z} = Z$. As it happens, for each i the lifts are equivalent to each other:

Proposition 5.5 *All determiners satisfy:* $L_i \equiv M_i$. □

Proposition 5.5 reminds us of the fact that although in a lift a determiner may leave the VP outside its scope, it could still be equivalent to a lift where this is not so. As always, the relevant factor for a determiner is the relation that obtains between its N and VP argument.

Let us return to the main argument. We have seen that for the collective (35a) the change in meaning is just as it should be. The determiner is set to work to specify the size of an unspecified collection of boys which has the property expressed by the VP. All this is realised by C^a_2. On the distributive reading (35b), the predication should be restricted to the relevant individuals (atoms), and the count of these individuals should be correct: exactly five, no more no less. This accuracy is given by D_1 but is beyond reach of D^a_1. In fact, all a lifts turn an arbitrary determiner $\mathbf{D}$ into a determiner $L(\mathbf{D})$ which is MON↑ in type $((et)(((et)t)t))$, so that for all X, $\mathbf{X}$ and $\mathbf{Y}$:

$$L(\mathbf{D})(X)(\mathbf{X}) \textit{ and } \mathbf{X} \subseteq \mathbf{Y} \Rightarrow L(\mathbf{D})(X)(\mathbf{Y})$$

Often such a transformation is undesirable.[19] The conclusion to draw is that Scha's distributive reading of numerals does generalise to arbitrary determiners, whereas Link's treatment does not. In other words, the NP strategy treats the distributive reading empirically more adequate than the VP strategy.

One may hope to save D^a_1 by claiming that distributive readings of sentences result from the combination of a distributive NP with a distributive VP; i.e. one of the form $\delta(\llbracket \mathrm{VP} \rrbracket)$. Then one should compare the lifts on such VPs and see if they fare better in this limited area. But they do not:

Proposition 5.6 *Restricting attention to distributive VPs, one has for arbitrary and for* MON↓ *determiners:*

$$\mathrm{D}_1 \equiv \mathrm{N}_2 \equiv \mathrm{N}_3 \textit{ and } \mathrm{D}^a_1 \equiv \mathrm{C}^a_2 \equiv \mathrm{N}^a_3$$

But for MON↑ *determiners total equivalence results:*

$$\mathrm{D}_1 \equiv \mathrm{N}_2 \equiv \mathrm{N}_3 \equiv \mathrm{D}^a_1 \equiv \mathrm{C}^a_2 \equiv \mathrm{N}^a_3$$

□

[19]This observation is related to the fact that defining a determiner $\mathbf{D}'$ in terms of a determiner $\mathbf{D}$ by means of $\mathbf{D}'(X)(Y) \Leftrightarrow_{def} \exists Z \subseteq Y : \mathbf{D}(X)(Z)$ makes $\mathbf{D}'$ MON↑. The a lifts are higher typed variants of this scheme.

Even though distributive VPs give a massive collapse of readings, proposition 5.6 shows that they still leave D_1^a non-equivalent to the properly counting non-a lifts in the crucial non-MON↑ cases. Hence the earlier observation is left unaffected. In my opinion this is a serious defect of the approach to distributivity as it is found, e.g., in Link's treatment of non-monotonic numerals and in Lønning [1987]. Non-MON↑ determiners are used distributively when the focus is on the precise number of individuals involved in a predication. But as it stands this precision cannot be had within this framework.

Exactly the same argument shows that the neutral readings N_2 and N_3 given on the NP approach are to be preferred to N_3^a which mimics the neutral reading of the VP approach. But can we make a further choice among N_2 and N_3? This section leaves the impression that N_3 gives all that is required, since it is the only one properly counting all the relevant members. However, this very fact makes the resulting reading too weak. Sentence (14), here repeated as (38), should be false if there are half a million families gathering with their one and only child, and not with any of the other families:

(38) Half a million children gathered all over the country

But N_3 would make it true. For this reason I opt for N_2 which only counts the members of gatherings consisting of just children. This does not mean that (38) would be false in a situation where there are also mixed gatherings of children with others. Though, for the children to be relevant for the truth of (38) they should form a subgathering of just children.

For all we know now, distributive readings should be obtained from D_1, collective readings from C_2^a, and neutral readings from N_2. These are precisely the readings proposed by the NP strategy. However, the article is not yet finished: there are some subtleties having to do with neutral readings and scope.

6 Neutral readings and scope

In the literature based on Link, there is consensus on which readings of (39) exists:

(39) Four men lifted two tables

Either the four men lifted the tables all alone or they did it all together; intermediate or neutral readings are often disregarded. In case they are considered, one tends to be in favour of reducing them to the collective reading (cf. Lasersohn [1989], Lønning [1991]). But in the foregoing section I have argued that the reductions as they are proposed in the literature are infelicitous. The reason is that neutral readings of (39) occur when the precise number of table lifting men is at stake, regardless of whether they have acted alone or with others. And for (39) as for other sentences with neutral readings this precision is beyond the given reductions to collective readings.

The observation that neutral readings occur is not new. As we have seen, Scha's N_2 reading of numerals derives from a similar intuition, and such readings formed the core of the debate of Gillon against Lasersohn. What is absent, however, is a logical semantics which models them satisfactorily.

If on neutral readings quantification is the issue, why can't we use N_2, which counts all the relevant members? Whatever the truth conditions of (39) are, it is commonly understood that on its subject wide scope reading ('sws' for short) at most eight tables could be involved. Hence the subject can never have a N_2 reading. In the previous section I have shown that on this reading – using N_2 in its M_2 guise – (39) is equivalent to:

(40) $\exists X \subseteq [\![\textit{four man}]\!]\ \exists \mathbf{Y}\ \mathrm{cv}\ X : \mathbf{Y} = [\![\textit{lifts two tables}]\!] \cap \wp([\![\textit{man}]\!])$

But then the number of tables may vary from two (= two $\times$ the cardinality of the poorest cover $\{\{X\}\}$ of a four element set X) to thirty-two (= two $\times$ the cardinality of its richest cover: $\wp(X)$)! Plainly, N_2 is an inappropriate reading of the subject in a transitive sentence.

A simple way to deal with this embarrassment is to take it as a knock-down argument against the intermediate reading N_2 intends to model. In my opinion this would be too crude, if only because neutral readings of intransitive sentences abound. Moreover, the argument against a N_2 reading of the subject having wide scope does not extend to the object having narrow scope. On the contrary, (41a, sws) even seems to favour it:

(41) *a.* Four men lifted at most two tables together
b. Thesa lifted exactly two tables

Sentence (41) can be used to describe a situation in which a collection of four men lifted one table, then another, then both of them, but no more. And such truth conditions of (41a) are best captured by means of its $C_2^a N_2$ reading. Also in case the subject is a proper name, as in (41b), this suggestion works well. Then, the neutral object reading should be equivalent to its distributive reading. As it stands this is not so since the meaning of *to lift* is unconstrained. But adding the meaning postulate (42a) or even the weaker (42b):

(42) *a.* $\forall x, Y[[\![\textit{lift}]\!](\{x\})(Y) \Rightarrow \forall Z \subseteq Y : [\![\textit{lift}]\!](\{x\})(Z)]$
b. $\forall x, Y[[\![\textit{lift}]\!](\{x\})(Y) \Rightarrow \forall y \in Y : [\![\textit{lift}]\!](\{x\})(\{y\})]$

the set $\lambda Y.\ [\![\textit{lift}]\!](\{[\![\textit{Thesa}]\!]\})(Y)$ will be distributive. Proposition 5.6 states that for such predicates the N_2 and D_1 reading are identical.[20] Given (42), (41b) has the meaning required. In all such cases, the rule seems to be: NPs which allow neutral readings are neutral when they have narrow scope.

[20] Lønning [1991] discusses the possibility of always giving the NP with narrow scope a neutral reading, but rejects it. His main objection is that the neutral reading of (41b), as induced by the object NP, must be equivalent to the distributive one. I have shown how this can be had by means of meaning postulates.

In his discussion, Lønning [1991] proposes to obtain the neutral object reading via verb modification combined with a C_2^a reading of an NP. His solution is as problematic as N_3^a is in that its fails to determine quantities correctly in the crucial non-MON$\uparrow$ cases. Lønning is well aware of this and suggests to try to settle the matter by means of topic/focus articulation. Instead, I prefer using N_2, which gives a simple and fully rigorous solution.

The observations lead us to consider alternatives of N_2 which circumvent the scope problem noted. Using the fact that the N_2 reading can be written as in (43):[21]

(43) $\lambda D \lambda X \lambda \mathbf{Y}.\ \exists Z \subseteq X[D(X)(Z) \wedge \exists \mathbf{Z}\ cover\ Z[\mathbf{Z} = \mathbf{Y} \cap \wp(X)]]$

I discuss three variants by strengthening the notion of cover to that of partition, minimal cover, and pseudo-partition, respectively. Although the pseudo-partitional reading of an NP comes close to what we want, it has unattractive properties too. For this reason I hold that there is no satisfactory neutral reading which may take scope over complex VPs. Instead, the neutral readings of NPs should be used in case of non-iterative quantification. In particular the cumulative reading is well-suited to be treated in this way and gives a defensible stance in the debate between Gillon and Lasersohn.

6.1 Alternative neutral readings

Looking closer at the above counter-example to the use of N_2, we see that the problem is this: the cardinality of an arbitrary cover may exceed that of its underlying set and the restriction function used may allow this distortion of quantificational information. The problem can be solved in one go if we can restrict the quantification in (40) to a kind of cover, call it cover*, with the property:

(44) *If* $\mathbf{Y}$ *covers* X, then* $|\mathbf{Y}| \leq |X|$

Here I shall consider three such covers, which relate as follows:

partitions $\subseteq$ *minimal covers* $\subseteq$ *pseudo-partitions*

Partitions Partitions are covers whose members, all non-empty, do not overlap. The two extreme partitions of a set X are $\text{AT}(X)$ and $\{\{X\}\}$, respectively corresponding to the distributive 'all Xs by themselves' and the collective 'all Xs together'. Since partitions may be thought of as identifying members of X with each other in a particular way, they satisfy (44). And indeed, restricting the quantification over covers in N_2 to quantification over partitions, (39, sws) will again be about two up to eight tables.

However, the restriction to partitions seems too strong. For suppose, following Gillon [1987], that the composers are Hammerstein, Rodgers and Hart. Suppose also that Rodgers collaborated with Hammerstein and with Hart to write musicals, but that the composers did not write a musical together nor all by themselves. Then the partitional reading of NPs makes the sentence (45) false:

(45) Three composers wrote some musicals

The reason is that the set

(46) $\{\{Hammerstein,\ Rodgers\}, \{Rodgers,\ Hart\}\}$

[21] Recall that $\mathbf{Z}$ covers Z iff $\bigcup \mathbf{Z} = Z$.

does not partition the set $\{Hammerstein,\ Rodgers,\ Hart\}$. This indicates that the neutral reading of an NP should not preclude overlapping collections, and that a weaker alternative to the use of partitions should be sought.

Minimal covers Adapting Gillon's suggestion to deal with set denoting NPs, one could restrict the quantification over covers to the so-called minimal covers. By definition, a minimal cover is a cover which does not have covers as real parts.[22]

Definition 6.1 (minimal covers) *A set* $\mathbf{Y}$ *minimally covers a set* X *– notation:* $\mathbf{Y}$ mc X, *– iff:*

- $\mathbf{Y}$ cv X
- $\forall \mathbf{Z}$ cv $X[\mathbf{Z} \subseteq \mathbf{Y} \Rightarrow \mathbf{Z} = \mathbf{Y}]$

Note that minimal covers, which have partitions as a special case, may contain overlapping collections. In particular, on the minimal cover reading of NPs (45) will be true, since the set (46) does minimally cover the set

$$\{Hammerstein,\ Rodgers,\ Hart\}$$

Van der Does [1992, ch. 5] shows that minimal covers satisfy (44). Altering N_2 as indicated, (39a, sws) will be about two to eight tables – as common intuition has it, – while at the same time collections are allowed to overlap.

What we have seen above, is that the partitional reading of NPs solves the scope problem of neutral readings of transitive sentences. But it does not give a satisfactory semantics to intransitive sentences, where the use of N_2 was unproblematic. The minimal cover reading of an NP shows a similar pattern. For instance, the minimal cover reading of *the managers*, as it derives from $N_2(\mathbf{all})$, will give (47a) the truth conditions (47b):

(47) *a.* The managers came together
b. $\exists \mathbf{Y}$ mc $[\![manager]\!] : \mathbf{Y} = [\![come\ together]\!] \cap \wp([\![manager]\!])$

It is conceivable, though, that the gathering of managers was used by some efficient ones, say Ploeger and Timmer, to have a subgathering. Hence it should be possibly that:

$$[\![come\ together]\!] \cap \wp([\![manager]\!]) = \{[\![manager]\!], \{ploeger, timmer\}\}$$

But this is not minimal, since $\{[\![manager]\!]\}$ will do to cover it. So, (47b) wrongly predicts (47a) to be false then (a referee of Lasersohn [1989] gave a similar counterexample to Gillon [1987]).

[22] With regard to definite NPs, Gillon argues that neutral readings abound: each minimal cover of such an NP gives a new reading. In the present set up, where we existentially quantify over cover*, they are used to give but one reading of the NPs which allow neutrality.

Pseudo-partitions Since minimal covers are not satisfactory either, let us finally have a look at the weakest kind of cover which satisfy (44). They are the pseudo-partitions of Verkuyl and Van der Does [1991]:

Definition 6.2 (pseudo-partitions) *A set* $\mathbf{Y}$ *pseudo-partitions a set* X *if and only if* $\mathbf{Y}$ *covers* X *and* $|\mathbf{Y}| \leq |X|$.

Pseudo-partitions point to the main reason why stronger notions of covers are considered. Linguistically, their use makes the interesting claim that in allowing the determiner to leave the VP outside its scope, as is common to do in this area, we have to ensure that its quantificational information is preserved when passing to the part where the VP is treated.[23]

Again, it is unclear whether pseudo-partitional NPs are entirely felicitous in case of intransitive sentences. Consider the variation (48a) of (47a) whose truth conditions on the pseudo-partitional reading is given by (48b):

(48) *a.* Four managers came together
b. $\mathbf{four}([\![manager]\!])([\![come\ together]\!] \cap \wp([\![manager]\!]))$
$\wedge\ |[\![come\ together]\!] \cap \wp([\![manager]\!])| \leq 4$

Allowing for subgatherings of the gathering reported on by (48a), one should argue why there may be at most four of those, as is stipulated by the second conjunct of (48b). In this connection Verkuyl [t.a.] defends a 'once counted, always counted' principle. The idea is that in disregarding multiple occurrences of one and the same manager, we have to discern at most four gatherings to establish the truth of (48). This means that the number of gatherings is strongly dependent on the perspective under which we regard them. However, as soon as the existence of gatherings is not entirely dependent on us, the principle cannot be used to explain the bound. Then the number of gatherings and the number of people involved in these gatherings may vastly differ, just because we count persons not their occurrences.

The behaviour of the pseudo-partitional reading of an NP as the subject of a transitive sentence also leaves room for debate. On such a use the claim would be that (49c) could describe the minimal situation sketched by (49a-b):

(49) *a.* Richard and Harry each lifted two tables
b. Richard and Ellen lifted two tables together
c. Three people lifted two tables

Semanticists tend to have varying judgments on whether (49c) can be so used. According to Verkuyl (Verkuyl and Van der Does [1991, 27]) this use is acceptable, and Gillon [1990] has similar judgments. The reason would be that (49c)

[23] Note that the observations made below with respect to pseudo-partitions also hold for partitions and minimal covers. Also, there is a contextdependent variant of the pseudo-partitional reading:

$$\lambda D \lambda X \lambda \mathbf{Y}.\ D(X)(\bigcup(\mathbf{Y} \cap F(X))) \wedge |\mathbf{Y} \cap F(X)| \leq |\bigcup(\mathbf{Y} \cap F(X))|$$

This is the contextdependent version of N_2 with the extra requirement that the contextually given set of collections $F(X)$ with property $\mathbf{Y}$ has at most the size of the set of the members of such collections.

could report on situations where it is checked which people have fulfilled the minimal duty of lifting two tables (e.g., the paymaster of a removal company may have such qualms).[24] In contrast, Lasersohn [1989] and Lønning [1991, 43] hold that a sentence like (49c) could not be so used, and I agree (cf. Verkuyl and Van der Does, ibidem). On the other hand, Hoeksema (p.c.) observes that this kind of verifying situation is less troublesome when sentences are explicitly modified as in (50a), and the same is true for (50b):

(50) *a.* Three people lifted two tables alone or with others
b. Three people lifted two tables; namely, Richard and Harry alone, and Richard together with Ellen.

These observations focus on the main advantage of using pseudo-partitional NPs. In section 4.1, I have shown that wherever the readings come from, they can always be taken to result from a (tacit) use of modifiers like δ. But (50) reminds us of the fact that natural language has many more ways to modify a VP. Should each of these modifiers give rise to a different reading? But if not, why restrict attention to the familiar 'each' and 'together'?[25] Because of the different kinds of modification, it is desirable to have an NP denotation which is compatible with each of them. Without overt modification this denotation must determine the relevant quantities, but it should leave us underinformed as to what situation is described.

Could we go one step further and hold that such a denotation could be used to give a reduction of ambiguity? Then the NPs which are not intrinsically distributive would just have this one reading, since it encompasses the situations described by means of the distributive and the collective reading. From section 5 we know that this position, defended in Verkuyl and Van der Does [1991], cannot be modeled by means of the weakest reading N_3^a. But the denotation considered here cannot fulfill this purpose either. Apart from the objectionable truth conditions exemplified by means of (48) and (49), this proposal makes sentences too adaptive to the situations they describe. The following dialogue, for instance, is coherent:

(51) A : Four men lifted at most two tables. To be precise, John, Gustav, Larry, and Tom did so all by themselves.
B : No that's not true. They lifted at most two tables together.

Suppose that B is right. It is reasonable to hold then that A's first sentence is false, even before he used the second sentences to be more informative. But if the sentence is unambiguous this cannot be accounted for, since the collective lifting of the pianos would make it true. For me this road to a vague but less ambiguous paradise is closed.[26]

Let us recapitulate what we have achieved up till now. I have argued for a neutral reading of an NP which is used when the precise determination of quantities

[24] Verkuyl [1992] has some other examples where these intermediate situations are easier to get.

[25] I am indebted here to Henk Verkuyl.

[26] The contextdependent version of the pseudo-partitional reading defined in footnote 23 is better of. It allows us to say that A took $F(X)$ to be different from what it really was.

is at stake. This neutral reading should be obtained by means of a general principle from the common denotation of a determiner in type $((et)((et)t))$. In case of intransitive sentences the neutral reading can be had without pain by use of N_2. But the N_2 reading of the subject of a transitive sentence, when widely scoped, will embarrass even the most tolerant semanticist. It has been shown in the previous section that N_2 can be seen to involve an existential quantification over covers, and I have discussed restrictions to stronger kinds of cover in the hope that they will yield a defensible neutral reading of plural NPs. The weakest such restriction is given by the pseudo-partitions of Verkuyl and Van der Does [1991]. Their use comes close to what is required but remains debatable. For (i) they put a restriction on the number of subevents of the event described, and (ii) the resulting truth conditions seem too liberal.

Nevertheless, we should not conclude that the neutral readings of NPs, with their generous attitude towards non-distributive and non-collective predication, should be excluded. In the above, I have tried to stay close to the N_2 reading of an NP under the assumption that it should be able to take scope over other NPs. But it is precisely this assumption which is to be challenged. On the neutral reading of a sentence with more than one NP, the NPs cannot have scope over each other. Yet, this allows the cumulative reading to occur on the basis of relations between sets. (Scha only considers distributive variants of this reading.) Indeed, in the next section I argue that N_2 should be used in this scope independent way to model neutral readings.[27]

6.2 Cumulative readings

On the cumulative reading, (52a) gets the truth conditions given by (52b):

(52) *a.* Three men lifted two tables

b. $\mathbf{three}([\![people]\!])(\bigcup \textsc{dom}([\![lift]\!] \cap \wp([\![thing]\!]) \times \wp([\![table]\!]))) \wedge$
$\mathbf{two}([\![table]\!])(\bigcup \textsc{rg}([\![lift]\!] \cap \wp([\![people]\!]) \times \wp([\![thing]\!])))$

Here the following notions are used:

$$\begin{array}{lll} [\![thing]\!] & := & \mathbf{D}_e \\ \textsc{dom} & := & \lambda \mathbf{R} \lambda X \exists Y.\ \mathbf{R}(X)(Y) \\ \textsc{rg} & := & \lambda \mathbf{R} \lambda Y \exists X.\ \mathbf{R}(X)(Y) \end{array}$$

The content of (52) may be paraphrased by:

> The collections of people which lift a collection of tables are made up of three people, and the collections of tables which are lifted by a collection of people are made up of two tables.

[27] There are other forms of polyadic quantification which leave the NPs independent of each other. For instance, resumptive quantification – which I take to be a special case of cumulative quantification, – or branching quantification. Since cumulative quantification is primarily used to focus on quantities, I restrict myself to this form here.

It has been suggested in the literature that the use of the cumulative reading is superfluous, since it should be reducible to a doubly collective reading (Partee, Link, Roberts (cf. Roberts [1987, 148-149]), Lønning [1991]). This is not so, for such a proposal does not work in case of non-MON↑ determiners.

That is, one restricts attention to the collections of people which lifted tables. Given this part of *to lift*, the first conjunct of (52) merges the collections of people in its domain and sees if the sum total of their members is three, and similarly for the second conjunct.

It remains to be shown how the cumulative reading for arbitrary determiners can be obtained via the mechanisms of lifting. Moreover, there are some well-known problems with this kind of reading, if one wants to comply with a principle of compositionality and at the same time give sentences a reasonable syntactic structure (Scha [1981] does the former but not the latter). A proposal to deal with these matters is in Van der Does [1992, ch. 4]. For now, I shall use my claim to take a stance in the Gillon/Lasersohn debate.

In Gillon [1987] neutral readings are argued for by means of the sentence (53):

(53) Hammerstein, Rodgers and Hart wrote musicals

Gillon observes that (53) could be true if, as in fact, Rodgers collaborated with one of the others, even in the circumstance that none of the composers wrote a musical all by himself.

According to Lønning [1991], (53) does not sustain the claim that neutral readings are called for. In contrast to the predicate *to write a musical*, he takes *to write musicals* to be distributive; that is, from (53), e.g., (54) follows:

(54) Rodgers wrote musicals

This may be correct, but it only shows that Gillon's example is not well-chosen. Some semanticists hold that the bare plural in the predicate of (53) does not carry quantificational information. If this is so, the predicate *to write musicals* is distributive and hence unsuitable to sustain Gillon's claim. But what about (55)?:

(55) Hammerstein, Rodgers and Hart wrote some musicals

This would be true, or so I think, even if Hammerstein and Rodgers wrote exactly one musical together, as did Rodgers and Hart, while none of them wrote any other musical. Then we neither have a distributive nor a collective predication.

I have already rejected Gillon's solution in terms of minimal covers, so it must be shown how the neutral reading of (55) comes about. I claim that this reading is the cumulative one based on relations between sets. Sentence (55) states that the people involved in writing musicals were Hammerstein, Rodgers and Hart and that the total number of the musicals they wrote, collectively or otherwise, is at least two. Note that on this reading (56) is no consequence of (55), as it should not.

(56) Rodgers wrote some musicals

Since proper names denote atoms, this non-inference is a general phenomenon of cumulative readings.

7 Conclusions

In this article I studied different ways in which the semantics of simple plural noun phrases could be developed. Some of the main questions are: which readings does a statement about collections have, where do these readings come from, how are they modeled? For the sentences studied here two strategies are distinguished. One locates the source of the readings in the NP, the other in the VP. For each of these strategies I discerned three readings: a distributive, a collective, and a neutral reading. The strategies and their readings are studied in a systematic way by means of six lifts instantiating the type change:

$$((et)((et)t)) \Rightarrow ((et)(((et)t)t))$$

In this way the different semantics for plural noun phrases are obtained from the standard denotations of their determiners in type $((et)((et)t))$. The lifts are distiled from the treatments of numerals by Scha and Link, the main proponents of the NP and the VP strategy. The present set up connects the sum theory of collections with generalised quantifiers theory in a uniform way. This has the important virtue of allowing a simple comparison between the different proposals. I have obtained the following insights.

Most semanticists grant the existence of a collective and a distributive reading. *Pace* the treatment of groups, the collective reading is adequately captured by C_2^a. This reading derives from Scha's collective denotation of numerals rewritten in terms of their $((et)((et)t))$ denotation. As to the distributive and neutral reading, I argued that their main function is respectively to determine with precision the individuals which have a certain property, or which occur in a collection with that property. In case of the distributive reading this accuracy is given by D_1, which derives from Scha's distributive reading of numerals, but not by D_1^a, which generalises Link. The lift D_1^a fails to do this duty when applied to non-MON$\uparrow$ determiners, which are often the crucial cases. Similarly, the precision required on the neutral reading is captured by N_2 but not by N_3^a. The D_1, C_2^a, and N_2 reading are generalisations of the numeral readings proposed on the NP strategy. I conclude that for the sentences considered in this article the NP strategy is superior to the VP strategy. However, for more complex sentences I think we have to compute the reading of a complex expression compositionally from the readings of its constituting expressions (cf. section 1, or Van der Does [1992, ch. 4]).

The neutral reading of an NP is used to determine the individuals partaking in a relation, independent of whether they do so on their own or as a member of a collection. Although the use of an N_2 reading is unproblematic for simple intransitive sentences, it is problematic in case of transitive sentences. In particular, it does not comply with the common intuition that 'four men lifted two tables' is about eight tables at most. It is not straightforward to find a logical semantics where neutral readings of NPs allow complex VPs within their scope. I have considered some alternatives involving an existential quantification over partitions, minimal covers and pseudo-partitions. The conclusion was that the partitional and the minimal cover reading of an NP are too strict. The pseudo-partitional reading comes close to what is required but it still has some troublesome aspects. I conclude that the NP of an intransitive sentence and the NP of a transitive sentence with narrow scope may have a neutral reading, but not so the NP of a transitive sentence which has wide scope. Both NPs in a transitive sentence can only be neutral in case of non-iterative polyadic quantification. For example, in a collective setting the cumulative reading generalises N_2 to transitive sentences in a way which leaves the scope of the NPs independent of each other.

All in all we have the following situation. Disregarding scope ambiguities, the noun phrases in a transitive sentence $[_s\ NP_1\ [_{vp}\ TV\ NP_2]]$ induce the seven readings (entry form: NP_1NP_2):

$$D_1D_1,\ D_1C_2^a,\ D_1N_2,\ C_2^aD_1,\ C_2^aC_2^a,\ C_2^aN_2,\ \text{the cumulative one}$$

If scope is allowed, the number is twelve (twice the first six plus the cumulative reading). It appeared however that the NP with narrow scope favours a neutral reading. If so, there are three readings without scope,

$$D_1N_2,\ C_2^aN_2,\ \text{the cumulative one}$$

With scope the number of readings is five.

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