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Paul Dekker

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Updates in dynamic semantics*

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Abstract
In this paper I reformulate Groenendijk and Stokhof's system of dynamic predicate logic (*DPL*) as an update semantics. The basic semantic notion is that of update of information about the values of variables. It is shown that the dynamics of interpretation involved, and the validity of certain classical logical equivalences, can be more precisely characterized in the update formulation of *DPL*. I show, furthermore, that the system is easily extended with a perspicuous and uniform account of adnominal and adverbial quantification (symmetric as well as asymmetric).

Contents

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1 Two theories of dynamic semantics

A dynamic semantics formalizes the insight that the meaning of a sentence is its potential to change information, rather than to express it. The dynamic notion of meaning dates back to Stalnaker [1974] and is adopted by quite a number of authors. Two examples of such a dynamic semantics are update semantics (*US*, Veltman [1990]) and dynamic predicate logic (*DPL*, Groenendijk and Stokhof [1991]). These two systems formalize different aspects of the dynamics of discourse. Veltman's *US* is a dynamic semantics for the language of modal (or, rather, epistemic) propositional logic in which interpretation involves update of information about the world. The system of *DPL* gives a dynamic semantic interpretation of the language of first order predicate logic, which keeps stock of the possible values of variables introduced in the course of a piece of text. In this paper, I present an update formulation of the semantics of *DPL*, which can be conceived of as constituting a crucial step towards the integration of the two systems.

I will proceed as follows. In this first section I sketch the two theories and argue for an update formulation of the semantics of *DPL*. In the second section I present the system of *EDPL*, in which the meanings of (predicate logic) formulas are defined as genuine update functions that update information about the values of variables. In the third section the logical properties of the system are studied and the system is compared with *DPL*. The reformulation of *DPL* as an update semantics turns out to enable a more precise delineation of the dynamics of the system. In the fourth section I show that the update formulation of *DPL* also enables a uniform and perspicuous treatment of adnominal and adverbial quantification.

1.1 Update semantics

Veltman's [1990] update semantics is a propositional logic with an additional epistemic operator $\Diamond$ (*might*). *US* deals with information about the world, and the meaning of a *US* formula is defined in terms of update of information about the world. Only formulas fronted by the operator $\Diamond$ do not express factual information about the world. A formula $\Diamond\phi$ expresses that one's information about the world is compatible with ϕ, that is, that it is still possible to update one's information with ϕ. A formula $\Diamond\phi$ says something like: "for as far as my information is concerned, it might, but need not, be the case that ϕ".[1]

In *US*, information about the world is modeled in terms of possible worlds, worlds, which, according to your information might be the actual one. An information state is a subset of the set of possible worlds, W, and the set of all information states is the powerset $\mathcal{P}(W)$ of all possible worlds.[2] The minimal information state is the set of all possibilities, W itself, a state in which one has no idea about what the world is like. The absurd state is the empty set, a state in which all possibilities are excluded. A state of maximal information is any singleton subset $\{i\}$ of W. In such a state, one has excluded all alternatives to i, and one has the information that the

1. It must be pointed out here that the part of the system sketched here only constitutes the rudiment of much more interesting extensions. However, for the purposes of this paper, it suffices to stick to this rudimentary system.
2. Veltman doesn't really use worlds, but sets of atomic sentences which "might give a correct picture of reality". This difference is neglected in what follows.

actual world is i.

Update of information about the world consists in eliminating possibilities. For instance, the interpretation of an atomic sentence p in a state s involves the update of s brought about by eliminating the worlds from s which are inconsistent with p. The resulting state only contains possible worlds in which p is true.

US interpretation is defined as an update function $[\![\ ]\!]$ on the domain of information states. It is defined with respect to a model $M = \langle W, V \rangle$ consisting of a set of worlds W and an interpretation function V that assigns sets of worlds to proposition letters. In what follows, $s[\![\phi]\!]_M$ indicates the result of updating an information state s with ϕ with respect to a model M, that is, the result of applying the function $[\![\phi]\!]_M$ to s. Reference to M is omitted whenever this does not lead to confusion. Interpretation is defined as follows:

Definition 1.1 (Update semantics)

- $s[\![p]\!] \quad = \{i \in s \mid i \in V(p)\}$
- $s[\![\neg\phi]\!] \quad = s - s[\![\phi]\!]$
- $s[\![\phi \wedge \psi]\!] \quad = s[\![\phi]\!][\![\psi]\!]$
- $s[\![\Diamond\phi]\!] \quad = \{i \in s \mid s[\![\phi]\!] \neq \emptyset\}$

Proposition letters are assigned an information content that intersects with the input information state. Negation is associated with set or state subtraction and sentence conjunction is associated with function composition. If we interpret the conjunction of two formulas ϕ and ψ in a state s, we first interpret ϕ in s and next interpret ψ in the state that results from the update of s with ϕ.

The interesting bit comes in with the operator $\Diamond$. In an information state s, $\Diamond\phi$ as it were reflects upon the current stage in the process of information growth and tests whether s can be consistently updated with ϕ. If ϕ is acceptable in a state s, then $\Diamond\phi$ is true in that state and the interpretation of $\Diamond\phi$ in s returns s. However, if you already have the information that ϕ is false, then $\Diamond\phi$ is rejected and its interpretation returns the absurd state.

As is evident from the semantics of *US*, the result of interpreting a formula in a state s is always a subset of s. Interpretation can only *eliminate* possibilities:

Fact 1.1 (Eliminativity)

- $s[\![\phi]\!] \subseteq s$

The eliminativity of *US* interpretation implies that interpretation guarantees update of information. In *US*, the interpretation of any formula in a state s always produces a state which contains at least as specific information about the world as s.

Since *US* has an eliminative semantics, all 'factual' formulas (i.e., formulas without occurrences of $\Diamond$) are stable in the sense that if they are true in a state s, they are true in any update of s. However, a formula $\Diamond\phi$ can be 'unstable'. In the gradual update of information, at some stage $\Diamond\phi$ may be true (if ϕ is not excluded at that stage), whereas at a later stage it is false (if the possibility that ϕ has been excluded in the meantime).

The unstability of $\Diamond\phi$ causes conjunction in *US* to be non-commutative (in what follows, the arrow $\Leftrightarrow$ indicates identity of meaning, and $\not\Leftrightarrow$ indicates the possibility of non-identity):

Fact 1.2 (Non-commutativity)

- $\phi \wedge \psi \not\Leftrightarrow \psi \wedge \phi$

An example of a non-commutative conjunction is $\Diamond p \wedge \neg p$. This conjunction, with this order of conjuncts, is consistent. For instance, let s be a state which is undecided about the truth or falsity of p, i.e., a state in which both p and $\neg p$ are acceptable. In that case, the update of s with $\Diamond p$ is s, since p is acceptable in s, and further update with $\neg p$ is not unacceptable. On the other hand, the commuted conjunction $\neg p \wedge \Diamond p$ is inconsistent. The interpretation of the formula $\neg p$ produces an information state in which p is false and, consequently, $\Diamond p$ unacceptable. So, we see that $\neg p \wedge \Diamond p$, which is inconsistent, is not equivalent with $\Diamond p \wedge \neg p$, which is consistent.

The following pair of examples exemplifies this pattern (granted that we know that John is not Mary):

(1) Somebody is knocking at the door. .... It might be John. .... It's Mary.

(2) Somebody is knocking at the door. .... It's Mary. .... *It might be John.

If somebody hears someone knocking at the door, he may of course be curious who it is and not exclude the possibility that it is, say, John. Still, in that situation, it is perfectly possible for him to find out that it is Mary who is knocking, not John. On the other hand, once he has found out that Mary is knocking on the door, it is excluded that it is John, and then it is quite absurd to say that, as far as he knows, it might be John who is knocking at that door. (Counterfactuals like *But it might have been John* are not discussed in this paper.)

Another property of *US* interpretation worth pointing out is that it is not distributive. A formula $\Diamond\phi$ tests a global property of a state s, viz., its consistency with ϕ, which does not need to hold of all singleton subsets of s. For instance, if $s = \{i, j\}$, and $\{i\}[\![\phi]\!] = \{i\}$ and $\{j\}[\![\phi]\!] = \emptyset$, then $s[\![\Diamond\phi]\!] = s$. However, $\{i\}[\![\Diamond\phi]\!] = \{i\}$ and $\{j\}[\![\Diamond\phi]\!] = \emptyset$, so $\bigcup_{i \in s}\{i\}[\![\Diamond\phi]\!] = \{i\}$.

I now turn to truth and entailment in *US*[3]:

Definition 1.2 (Truth and entailment in update semantics)

- ϕ is true in s with respect to M, $s \models_M \phi$, iff $s \subseteq s[\![\phi]\!]_M$
- $\phi_1, \ldots, \phi_n \models \psi$ iff $\forall M, s$: $s[\![\phi_1]\!]_M \ldots [\![\phi_n]\!]_M \models_M \psi$

A formula ϕ is true in s if after updating s with ϕ we still envisage the possibilities we envisaged in state s, i.e., if $s[\![\phi]\!]$ doesn't contain more information than s. A sequence of premises $\phi_1, \ldots, \phi_n$ entail a conclusion ψ if always, if you update your information with $\phi_1 \ldots \phi_n$, in that order, you arrive at a state of information to which update with ψ adds no more information.

US licenses the deduction theorem:

Fact 1.3

- $\phi_1, \ldots, \phi_n \models \psi$ iff $\phi_1, \ldots, \phi_{n-1} \models \phi_n \rightarrow \psi$

I leave it at this exposition of basic properties of rudimentary *US*. For more details, and for more interesting extensions of *US*, the reader is referred to Veltman [1990].

3. In fact, Veltman does not speak of truth but of acceptance. If, as I will say it, ϕ is true in a state s, Veltman would say that ϕ is accepted in s. Furthermore, Veltman discusses two alternative notions of entailment, but these can be neglected here.

1.2 Dynamic predicate logic

DPL gives a dynamic interpretation of the language of first order predicate logic that accounts, among other things, for intersentential anaphoric relationships as found in *A cowgirl meets a boy. She slaps him.* Like in discourse representation theory and file change semantics, in *DPL* natural language noun phrases are associated with variables, or discourse markers, and information states determine what values they can have given the conditions imposed on them in the course of a discourse.

For instance, if we interpret *A cowgirl meets a boy* and associate *a cowgirl* with a variable x and *a boy* with a variable y, then we end up in a state which contains the information that the value of x is a cowgirl who meets a boy who is the value of y (if there is such a cowgirl, otherwise the sentence is just false). This state is precisely the kind of state we need to interpret a continuation with *She slaps him*, where *she* is associated with x again, and *him* with y. This second sentence then adds the information that the value of x slaps the value of y, and the state that results from interpreting the sequence of two sentences contains the information that the value of x is a cowgirl who meets and slaps a boy who is the value of y.

Since variable assignments are useful ways to carry around information about variables and their reference, information states in *DPL* are thought of as sets of variable assignments. So, if D is a domain of individuals, and V the set of variables we use, then any subset $s \subseteq D^V$ of the set of variable assignments is an information state, and S, the set of all information states, is the powerset of the set of variable assignments: $S = \mathcal{P}(D^V)$. As in *US*, the set of information states contains a minimal information state $s = D^V$, in which all variable assignments are still possible, an absurd state $s' = \emptyset$, which excludes all possibilities, and maximal information states $\{i\}$, for $i \in D^V$, which completely determine the values of all variables.

The language of *DPL* is that of predicate logic, but for ease of exposition I disregard individual constants. The semantics is defined with respect to a model $M = \langle D, F\rangle$ consisting of a non-empty set of individuals D and an interpretation function F that assigns sets of n-tuples of objects to n-ary relation expressions. (Again, reference to M is omitted whenever this does not lead to confusion.) The interpretation of formulas is a function on the domain of information states (as usual, $i[x/d]$ is the assignment j such that j agrees with i on the values of all the variables except, possibly, x and such that $j(x) = d$):

Definition 1.3 (Semantics of DPL)

- $s[\![Rx_1 \ldots x_n]\!] = \{i \in s \mid \langle i(x_1), \ldots, i(x_n)\rangle \in F(R)\}$
- $s[\![x = y]\!] = \{i \in s \mid i(x) = i(y)\}$
- $s[\![\neg\phi]\!] = s - {\downarrow}[\![\phi]\!]$
- $s[\![\phi \wedge \psi]\!] = s[\![\phi]\!][\![\psi]\!]$
- $s[\![\exists x\phi]\!] = s[x][\![\phi]\!]$

where

$${\downarrow}[\![\phi]\!] = \{i \mid \{i\}[\![\phi]\!] \neq \emptyset\}$$
$$s[x] = \{i[x/d] \mid i \in s \ \&\ d \in D\}$$

The interpretation of an atomic formula in a state s involves the intersection of s with the set of assignments with respect to which the formula is true in a classical sense. The negation of ϕ subtracts those i in s which constitute a context $\{i\}$ with respect to which the interpretation of ϕ does not produce the absurd state. Putting it the other way around, the negation of ϕ preserves those i in s with respect to which ϕ produces the absurd information state. Again, conjunction is interpreted as function composition.

The characteristic clause concerns the interpretation of the existential quantifier. If we interpret a formula $\exists x\phi$ in a state s, we take into consideration all values for x and then interpret ϕ. The mediating state $s[x]$ contains the same information as s about the values of all variables except x. About the value of x, $s[x]$ has no information whatsoever: for each i in s (and for each d in D) there is an assignment j in $s[x]$ that is like i except possibly with respect to the value assigned to x (and such that $j(x) = d$).

The interpretation of an existentially quantified formula $\exists x\phi$ can be conceived of as involving the composition of two operations on states: $[x]$ and $[\![\phi]\!]$, respectively. As we saw, also conjunction amounts to function composition. Since composition is an associative operation, the following equivalences hold in *DPL*. The first one is a classical conjunction, but the second one distinguishes *DPL* from static theories:

Fact 1.4 (Donkey equivalences (1))

- $((\phi \wedge \psi) \wedge \chi) \Leftrightarrow (\phi \wedge (\psi \wedge \chi))$
- $(\exists x\phi \wedge \psi) \Leftrightarrow \exists x(\phi \wedge \psi)$

It is typical of *DPL* that the second equivalence also holds if the variable x occurs free in ψ. These equivalences therefore allow *DPL* to deal with the following textbook example (which explains the equivalences' label):

(3) A farmer owns a donkey. He beats it.
$(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \wedge Bxy) \Leftrightarrow$
$\exists x(Fx \wedge \exists y(Dy \wedge (Oxy \wedge Bxy)))$

This sequence of sentences turns out to be equivalent with the sentence *A farmer beats a donkey he owns*. So, although the two sentences *A farmer owns a donkey* and *He beats it* are assigned an interpretation of their own, the intersentential anaphoric relationships (between *a farmer* and *he*, and between *a donkey* and *it*) get established when we combine the two in a conjunction.

The donkey equivalences have a nice corollary. Given the usual definitions of universal quantification and implication in terms of existential quantification, negation and disjunction, we also have the following facts (again, the first one is a classical equivalence and the second one a typical *DPL* equivalence):

Fact 1.5 (Donkey equivalences (2))

- $((\phi \wedge \psi) \rightarrow \chi) = \neg((\phi \wedge \psi) \wedge \neg\chi) \Leftrightarrow$
$\neg(\phi \wedge (\psi \wedge \neg\chi)) = (\phi \rightarrow (\psi \rightarrow \chi))$
- $(\exists x\phi \rightarrow \psi) = \neg(\exists x\phi \wedge \neg\psi) \Leftrightarrow$
$\neg\exists x(\phi \wedge \neg\psi) = \forall x(\phi \rightarrow \psi)$

This fact enables *DPL* to deal with the museum piece donkey sentences:

(4) If a farmer owns a donkey he beats it.
$(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bxy) \Leftrightarrow$
$\forall x(Fx \rightarrow \forall y((Dy \wedge Oxy) \rightarrow Bxy))$

(5) Every farmer who owns a donkey beats it.
$\forall x((Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bxy) \Leftrightarrow$
$\forall x(Fx \rightarrow \forall y((Dy \wedge Oxy) \rightarrow Bxy))$

These sentences are assigned their so called 'strong' readings. Both sentences state that every farmer beats every donkey he owns.

Another typical property of *DPL* is that it has a non-eliminative semantics. It is not generally true that $s[\![\phi]\!] \subseteq s$. Interpretation in *DPL* does not merely involve the elimination of possibilities, but it may also involve the introduction of new possibilities. For instance, the interpretation of a formula $\exists x\phi$ in a state s may contain assignments that assign a cowgirl to x, whereas all assignments in s assign a man to x. The existential quantifier $\exists x$ sets up a new discourse referent, so to speak, by resetting the value of a variable x, and it is precisely this sort of 'act' in *DPL*, together with the fact that these acts affect the interpretation of variables in successive formulas, that makes *DPL* distinct from static theories.

Related to its non-eliminativity is the fact that *DPL* conjunction is not commutative:

Fact 1.6 (Non-commutativity)
- $\phi \wedge \psi \not\Leftrightarrow \psi \wedge \phi$

An example of a non-commutative conjunction is $\exists xMx \wedge Wx$, a *DPL* translation of the sequence *A man walks in the park. He whistles.* In *DPL*, and also intuitively, the meaning of this formula differs from that of $Wx \wedge \exists xMx$, a translation of *He whistles. A man walks in the park.*

DPL interpretation is distributive:

Fact 1.7 (Distributivity)
- $s[\![\phi]\!] = \bigcup_{i \in s}\{i\}[\![\phi]\!]$

This fact tells us that in the update of a state s with a formula ϕ only properties of the individual elements of s count, and not global properties of the state as a whole. This implies that *DPL* interpretation can be given a definition in a lower type, viz., as a relation between assignments instead of as a function on sets of assignments. In fact, the original formulation of *DPL* in [1991] is the relational one.

Notice that distributivity is, as it were, assumed in the clause that defines the interpretation of the negation of a formula ϕ. Since the interpretation of ϕ in a state s is completely characterized by the update of singleton subsets of s with ϕ, the negation of ϕ in s can be defined appropriately in terms of the update with ϕ of each of these singleton subsets.

I now turn to truth and entailment in *DPL*:

Definition 1.4 (Truth and entailment in DPL)
- ϕ is true in s with respect to M, $s \models_M \phi$, iff $s \subseteq \downarrow[\![\phi]\!]_M$
- $\phi_1, \ldots, \phi_n \models \psi$ iff $\forall M, s$: $s[\![\phi_1]\!]_M \ldots [\![\phi_n]\!]_M \models_M \psi$

A formula ϕ is true in a state s iff all possible variable assignments $i \in s$ consitute a state $\{i\}$ which can be succesfully updated with ϕ (i.e., in which the interpretation of ϕ does not produce the absurd state). A conclusion ψ follows from a sequence of premises $\phi_1, \ldots \phi_n$ if the update of any information state s with $\phi_1, \ldots, \phi_n$, successively, produces a state in which ψ is true.

DPL, too, licenses the deduction theorem:

Fact 1.8

- $\phi_1, \ldots, \phi_n \models \psi$ iff $\phi_1, \ldots, \phi_{n-1} \models \phi_n \rightarrow \psi$

From the deduction theorem and the donkey equivalences it follows that entailment in *DPL* is dynamic. For instance, $\exists x Fx$ entails Fx. This fact corresponds to the following line of elementary reasoning:

(6) If a man comes from Rhodes, he likes pineapple-juice. A man I met yesterday comes from Rhodes. So, he likes pineapple-juice.
$\exists x(Mx \wedge Rx) \rightarrow Lx, \exists x(Mx \wedge Rx) \models Lx$

1.3 US and DPL

As Groenendijk and Stokhof [1990] point out, a dynamic semantics in which all sentences are interpreted as eliminative and distributive updates is not really dynamic after all.[4] However, as we have seen above, *DPL* is non-eliminative, and *US* is non-distributive. The respective properties of non-eliminativity and non-distributivity distinguish *DPL* and *US* from static theories of interpretation. Notice that distinct properties distinguish *DPL* and *US* from static theories. The two different ways in which *DPL* and *US* depart from the static paradigm are reflected by a difference in the respective notions of truth (and of entailment). In order to determine whether ϕ is true in a state s in *DPL*, the information contained in s must not be compared with the information contained in the update of s with ϕ. For ϕ to be true in a state s in *DPL*, *each* singleton subset s must allow non-absurd update with ψ. On the other hand, the truth of ϕ in a state s in *US* not solely depends on properties of each of the individual members of s. In *US*, the truth of ϕ in a state s may depend on inherently global properties of s.

The two different notions of truth and entailment in *DPL* and *US* should not be substituted for one another.[5] In fact, *DPL* and *US* are two different systems of dynamic interpretation with conflicting characteristic properties. This is not to say that the two are incompatible. As Groenendijk and Stokhof suggest, the two systems can be combined within a system that preserves the characteristic features of both and that gives a separate treatment of the two different kinds of (update or change of) information that the two systems deal with. See Van Eijck and Cepparello [1992] for an example of such a combination.

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4. If a function τ on a domain of sets is distributive and eliminative, then for all sets s, $\tau(s) = s \cap \downarrow\tau$, where $\downarrow\tau$ is the characteristic set $\{x \mid \tau(\{x\}) = \{x\}\}$ (cf., Groenendijk and Stokhof [1990, p. 57] and van Benthem [1986, p. 62] and [1991, p. 137]). So, if all updates are eliminative and distributive, conjunction (interpreted as function composition) amounts to (commutative) intersection, and, hence, is commutative itself.

5. If we adopt the *US* notion of truth in *DPL*, then the *DPL*-valid entailment $\exists x Px \models \exists y Py$ would no longer be valid. On the *US* notion of truth, $\exists y Py$ is true in s iff Py is true in s, and, clearly, $\exists x Px \not\models Py$ on any of the two notions of entailment. On the other hand, if we adopt the *DPL* notion of truth in *US*, then the *US*-valid entailment $\Diamond p \models \Diamond p$ would no longer go through. On the *DPL* notion of truth, $\Diamond p$ is true in s iff p is true in s, and $\Diamond p \not\models p$ on any of the two notions of entailment.

However, a more tight combination between the two systems can be made, if we remove one of these adversive features (in particular, non-eliminativity) and properly adapt the logic of one of the two (*DPL*) to the format of that of the other (*US*). Doing so, a combination of the two theories results which is more of an integration, since it allows us to employ one elementary notion of truth (and entailment) instead of the product of two, so to speak. To this end, this paper presents a system, labeled *EDPL*, which reformulates the semantics of *DPL* as an update semantics. Before I turn to the exposition of *EDPL*, I give some more intuitive motivation for such an update semantics in the remainder of this section. (Notice that the term 'update' is here taken to indicate *addition* of information, not arbitrary *revision* of information.)

Basically, *US* models update of information about the world brought about by processing sentences. Interpretation in *US* involves a process of information growth. Interpretation in *DPL* is of a rather different nature. "It [*DPL*, PD] (...) restricts the dynamics of interpretation to that aspect of the meaning of sentences that concerns their potential to 'pass on' possible antecedents for subsequent anaphors (...)." (Groenendijk and Stokhof, [1991, p. 43–4]). The dynamics of *DPL* serves to give a compositional account of (the semantics of) anaphoric links between existential quantifiers (indefinite noun phrases) and variables (pronouns) occurring outside their proper scope. The information employed in *DPL* is a means to give a proper interpretation of pronouns, and, when these have been interpreted correctly, it serves no further substantial semantic purpose.[6] From this perspective it is a relatively harmless fact that the occurrence of an existential quantifier $\exists x$ in *DPL* discards all information one might have had about x at that moment. This fact merely implies that further free occurrences of x are interpreted as being bound by this occurrence of the quantifier, not by one before.[7]

However, there is a different, attractive perspective upon the phenomenon of anaphoric relationships, from which the effect of existential quantification in *DPL* is rather counterintuitive. Anaphoric relationships have also been characterized as involving a pronoun's reference to some kind of objects introduced in the universe of discourse by a previous indefinite description. This idea can be traced back, in rudimentary form, to Karttunen's seminal papers [1968a, 1968b]. Karttunen interprets indefinites as establishing so-called discourse referents. The idea is further developed within the frameworks of discourse representation theory (*DRT* Kamp [1981]) and file change semantics (*FCS*, Heim [1982, 1983]). There, indefinite noun phrases induce a genuine update of the discourse representation (or file, respectively) that constitutes the context of interpretation by the addition of a novel discourse referent to its domain.

True, *DPL* does give a, compositional, reformulation of the *DRT* account of intersentential anaphoric relationships. However, with this reformulation a genuine notion of (a domain of) discourse referents is lost. The fact that an existential quantifier $\exists x$ in *DPL* discards previously established information about the value of x, is at odds with the (implicit) idea that the interpretation of a discourse in general involves the update of information about the objects introduced in the discourse. If

6. This fact also shows from *DPL*'s notion of truth. In order to assess the truth of formula ϕ in a state s, the specific information contained in the update of s with ϕ is completely irrelevant.

7. However, see section 4.3 for some of the complications that this may give.

information about the values of variables were to be interpreted as information about objects introduced in a discourse, then an existential quantifier occurrence in *DPL* involves an unmotivated expelling of a specific discourse referent from the universe of discourse in which it has been introduced. For, the question which discourse referent is expelled depends upon the arbitrary choice for the variable with which an indefinite gets associated.

So, from the latter point of view upon anaphoric relationships, the *DPL* notion of information, and of information change, is unsatisfactory.[8] This situation can be improved upon by extending the *DPL* notion of information with the notion of a domain of objects (variables) one has information about, and by excluding, with reference to such a domain, downdate of information about the values of variables in that domain. Crucial to such a reformulation of *DPL*, into the system called *EDPL*, is the notion of information about the values of growing sets of variables.

Like in *DPL*, in the update formulation *EDPL* information about the values of variables is encoded by sets of variable assignments. However, in *EDPL* sets of *partial* variable assignments are used.[9] For any subset of variables X, an information state about the values of X is a set of assignments of objects to the variables in X. An information state in *EDPL*, then, determines the two aspects of information addressed above. In the first place, such a state s determines a domain of variables whose values are at issue, viz., the joint domain of the assignments in s. In the second place, s contains information about the values of variables in that domain.

Since information states in *EDPL* model two aspects of information about variables, also two basic kinds of update of information can be distinguished. Update of information consists either in getting more information about the values of variables, by the elimination of partial variable assignments, or in extending the domain of partial variable assignments (or, of course, in a mixture of both). So, a state t is considered a possible update of a state s, written as $s \leq t$, if the domain of t comprises that of s and if the assignments in t satisfy the restrictions s imposes on the values of variables in the domain of s. Clearly, on such a notion of update, t is an update of s if t at least contains the information that s has about the values of the variables in the domain of s. But, clearly, t may also contain information about more variables.

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8. Cf., also, Hans Kamp, who noticed some kind of a mismatch between the ("ingenious") use to which assignments are put in *DPL*, and the ("attractive") conception of a sentence's meaning as transforming information states: "(...) there are all sorts of linguistic objects (...) whose semantics can naturally be described in terms of input–output relations, but where the relation between the object and its inputs and outputs is very different from that which obtains between a sentence and the information states which it transduces into each other." and "The question (...) to what extent it is possible to see these objects [assignments, PD] as 'information states'. The answer to this question is of course, hardly.", Kamp [1990, p. 111].

9. It may be noticed that Kamp [1981] and Heim [1983], among many others, also employ partial variable assignments. Furthermore, Kamp [1990] proposes to use partial variable assignments in order to give more substance to the *DPL* notion of information. In fact, the semantics of *EDPL* is close in spirit with the version of *FCS* in Heim [1983], as well as with a reformulation of *DPL* suggested by Kamp [1990].

2 An update semantics for DPL

This section presents the system of *EDPL*, an update semantics for dynamic predicate logic. In section 2.1, I first introduce the notions of information and of information update employed in *EDPL*. In section 2.2 I give the semantics of *EDPL*. In section 2.3, *EDPL* interpretation is discussed in more detail, and illustrated by means of some examples. In the next section, section 3, the logical properties of *EDPL* are studied.

2.1 EDPL information states

Following *DPL* I will define the interpretation of the language of predicate logic as a function on a domain of information states. Like in *DPL*, in *EDPL* these information states encode information about the values of variables in a discourse, but contrary to what is the case in *DPL*, in *EDPL* these states are sets of partial variable assignments.

If D is a domain of individuals and V the set of variables, then S^X, the set of information states about the values of $X \subseteq V$, and S, the set of all information states, are defined as follows:

Definition 2.1 (Information states)

- $S^X = \mathcal{P}(D^X)$
- $S = \bigcup_{X \subseteq V} S^X$

An information state about the values of a set of variables X is a set of assignments of individuals to the variables in X. Given such a state $s \in S^X$, I will refer to X as the domain of s, indicated by $D(s)$. Information states contain information about the values of the variables in their domain by restricting their valuations. An information state s is a set of valuations of the variables in the domain of s which are conceived possible in s, and, hence, excludes all other valuations to these variables. So, if x and y are in the domain of s, then s contains the information that the value of x is a man iff no i in s maps x on an individual that is not a man, and s contains the information that the value of x sees the value of y, iff for all i in s, $i(x)$ sees $i(y)$.

With respect to a fixed domain of variables, the notions of minimal and maximal information states are as in *DPL* and *US*. For any domain of variables X, the minimal information state about the values of X is D^X, referred to as $\top^X$. A minimal information state has no information about the values of the variables in its domain, all valuations of the variables are considered possible. A maximal information state about the values of X is $\{i\}$ for any $i \in D^X$. A maximal state has total information about the values of the variables in its domain: only one valuation of them is conceived possible. Furthermore, for any domain X, the absurd information state is $\emptyset$, referred to as $\bot^X$. An absurd information state excludes all assignments to the variables in its domain.

A special set of information states is $S^\emptyset$, the possible states of information about the values of no variables. There are only two such states: the set containing the empty assignment, and the empty set. (In fact, this is just the domain of truth values on its set-theoretic definition.) Interestingly, with respect to the empty domain the minimal information state and the maximal information state coincide. This reflects the fact that one can have no substantial information about the values of no variables. Notice, moreover, that S^V is the set of *(D)PL* information states, the set of sets of assignments to all variables. So, the set of states S encompasses the states of propositional logic ($\{1, 0\}$) and those of (dynamic) predicate logic ($\mathcal{P}(D^V)$).

Since we will often be dealing with assignments and states with different domains, it is useful to have at our disposal some notions relating them. I first define the usual notion of an assigment extension ($D(i)$ indicates the domain of i: if $i \in D^X$, then $D(i) = X$):

Definition 2.2 (Assignment extension)

- j is an extension of i, $i \leq j$, iff $\forall x \in D(i)$: $x \in D(j)$ and $i(x) = j(x)$

Using this notion of assignment extension, we can define two notions which can be conceived to be generalizations of the notion of set membership. The first, $\gtrdot$, generalizes this notion to assignments with larger domains, the second, $\lessdot$, generalizes it to assignments with a smaller domain:

Definition 2.3 (Restriction and extension)

- i is restricted in s, $i \gtrdot s$, iff $D(s) \subseteq D(i)$ and $\exists j \in s$: $j \leq i$
- i is extended in t, $i \lessdot t$, iff $D(i) \subseteq D(t)$ and $\exists j \in t$: $i \leq j$

Assignment i is restricted in s if i is an extension of some element of s. Assignment i is extended in t if i is a restriction of some element of t. In the latter case, I will also say that i survives in t. Clearly, if the domain of s equals the domain of i, then $i \gtrdot s$ iff $i \in s$ iff $i \lessdot s$.

The two notions of restriction and extension give rise to two generalizations of the subset relation, the update relation and the substate relation. The update relation obtains between two states s and t iff t contains more information than s:

Definition 2.4 (Update)

- t is an update of s, $s \leq t$, iff $D(s) \subseteq D(t)$ and $\forall i \in t$: $i \gtrdot s$

An update t of s only considers possible valuations of the variables in $D(s)$ which are possible in s. Hence, t contains at least the information that s contains about the variables in $D(s)$. Moreover, t may contain information about variables which s is silent about. The assignments in t may be proper extensions of assignments in s. So, the definition of $\leq$ precisely captures the notion of update informally described in section 1.3.

The second generalization of the subset relation is the substate relation. The substate relation plays a part, chiefly, in the semantics of *EDPL*, since it acts in the *EDPL* definition of the *DPL*-style (and *DRT*-style) notions of truth and entailment. The relation is defined as follows:

Definition 2.5 (Substate)

- s is a substate of t, $s \sqsubseteq t$, iff $D(s) \subseteq D(t)$ and $\forall i \in s$: $i \lessdot t$

If state s is a substate of state t, then t does not reject valuations of the variables in $D(s)$ which are conceived possible in s. So, according to this definition, if s is a substate of an update of s, then the update of s contains no more information than s about the variables in the domain of s. However, such an update of s may involve the introduction of new variables to the domain of s.

The notions of update and substate are related to the notion of subset in the following way:

Fact 2.1

- If $D(s) = D(t)$, then $s \leq t$ iff $t \sqsubseteq s$ iff $t \subseteq s$
- $s \leq t$ and $t \sqsubseteq s$ iff $t \subseteq s$

2.2 Semantics of EDPL

We are now ready to turn to the semantics of *EDPL*. I will use the following auxiliary notions ($i \leq_X j$ iff $j \in D^{D(i) \cup X}$ and $i \leq j$):

Definition 2.6 (State subtraction and domain extension)

- $s - t = \{i \in s \mid i \not< t\}$
- $s[x] \;= \{j \mid \exists i \in s \colon i \leq_{\{x\}} j\}$

where, if $s \in S^X$ then $s - t \in S^X$ and $s[x] \in S^{X \cup \{x\}}$

Subtracting state t from state s gives a state which contains all those assignments in s which are not extended in t. If the domain of s is a subset of the domain of t, which it is in all cases of state subtraction below, then $s - t$ is the set of assignments in s to the variables in the domain of s which are excluded in t. So, state $s - t$ contains the information that s has about the variables in $D(s)$ supplemented with the information excluded by t.

The state $s[x]$ at most differs from s in that it is defined for x. About the values of variables in the domain of s the new state $s[x]$ contains precisely the same information as s, and, if x is not in the domain of s, $s[x]$ is completely impartial about the value of x. In that case, for each i in s and for each z in D, there is an extension j of i in $s[x]$ that assigns z to x. What is added, one might say, is the information that x has a value. Clearly, if x is already in the domain of s, then $s[x] = s$. In the semantics of *EDPL*, however, it is excluded that $[x]$ operates on states s which are already defined for x.

As in *DPL*, *EDPL* interpretation is defined with respect to a model $M = \langle D, F \rangle$ consisting of a non-empty set of individuals D and an interpretation function F that assigns sets of n-tuples of objects to n-ary relation expressions. (Again, I will omit reference to M whenever this does not lead to confusion.) Interpretation is defined as a *partial* update function on the domain of information states. In the following definition I only indicate the possibility of undefinedness in the source cases:

Definition 2.7 (Semantics of EDPL)

- $s[\![Rx_1 \dots x_n]\!] = \{i \in s \mid \langle i(x_1), \dots, i(x_n) \rangle \in F(R)\}$ if $x_1, \dots, x_n \in D(s)$
- $s[\![x = y]\!] = \{i \in s \mid i(x) = i(y)\}$ if $x, y \in D(s)$
- $s[\![\neg\phi]\!] = s - s[\![\phi]\!]$
- $s[\![\phi \wedge \psi]\!] = s[\![\phi]\!][\![\psi]\!]$
- $s[\![\exists x \phi]\!] = s[x][\![\phi]\!]$ if $x \notin D(s)$

where, if $s \in S^X$, then $s[\![Rx_1 \dots x_n]\!], s[\![x = y]\!] \in S^X$.

The clauses in this definition closely correspond to those in the definition of the semantics of *DPL* and *US*. The interpretation of atomic formulas is the same as in *DPL*, except for the use of partial, instead of total assignments. The interpretation of

$Rx_1 \dots x_n$ in s preserves the assignments in s that map the variables $x_1, \dots, x_n$ onto individuals $z_1, \dots, z_n$ that stand in the relation R, in that order. *EDPL* negation combines features of negation in *DPL* and in *US*. The requirement that assignments i in $s[\![\neg\phi]\!]$ do not have an extension in $s[\![\phi]\!]$ in fact corresponds to the *DPL* requirement that $\{i\}[\![\phi]\!]$ be empty (cf., fact 2.4 below). On the other hand, the result of interpreting $\neg\phi$ in s is defined solely in terms of s and $s[\![\phi]\!]$, like in *US*. Like in *DPL* and *US*, conjunction is interpreted as function composition. Furthermore, like in *DPL*, the existential quantifier $\exists x$ introduces arbitrary valuations of x, the difference being that domain extension is used, instead of reinstantiation. In section 2.3 it will be demonstrated in more detail how these clauses work out in practice.

A major difference with *DPL*, and *US*, is the presence of side conditions, or definedness conditions, on *EDPL* interpretation. Undefinedness is generated by occurrences of free variable and quantifiers. If a formula contains a free variable for which a state is undefined, then the interpretation of the formula is undefined for that state. Conversely, the interpretation of an existentially quantified formula $\exists x\phi$ is undefined for a state s which is already defined for x. This last requirement, in particular, expels the possibility of reinstantiation. Undefinedness persists in the following way. If (the interpretation of) ϕ is undefined for s, then $\neg\phi$ and $\phi \wedge \psi$ are undefined for s. Furthermore, if ϕ is undefined for $s[x]$, then $\exists x\phi$ is undefined for s, and if ψ is undefined for $s[\![\phi]\!]$, then $\phi \wedge \psi$ is undefined for s. Below, in section 3.2, definedness phenomena in *EDPL* will be studied in more detail.

The most important property of *EDPL* is that it is a genuine update semantics. Interpretation in *EDPL* always yields update of information:

Fact 2.2 (Update)

- $s \leq s[\![\phi]\!]$, if defined

(Fact 2.2 is proved by a simple induction on the complexity of ϕ.) Although the assignments in the state that results from updating a state s with ϕ are not themselves elements of the original state s, as is the case in a real eliminative semantics, all these assignments are at least extensions of assignments in s. In other words, the information contained in an initial state of interpretation is properly preserved in the process of interpretation, and this fact distinguishes *EDPL* from *DPL*. Attempts to initialize variables which one has already information about are encountered by an error message.

Since interpretation in *EDPL* has the update property, truth can be adequately defined as follows (the definition of entailment is postponed to section 3.2):

Definition 2.8 (Truth in EDPL)

- ϕ is true in s with respect to M, $s \models_M \phi$, iff $s \sqsubseteq s[\![\phi]\!]_M$
  ϕ is false in s with respect to M, $s \dashv_M \phi$, iff $s[\![\phi]\!]_M = \emptyset$

A formula ϕ is true in a state s iff s is a substate of the update of s with ϕ, i.e., iff the state $s[\![\phi]\!]$ does not contain *more* information than s about the variables in the domain of s. Obviously, we want to allow ϕ to be true in a state s, also if in the update of s with ϕ new variables are introduced. If s contains the information that the value of x sees a donkey, then a sentence *She sees an animal*, $\exists y(Ay \wedge Sxy)$, must be true in s.

Clearly, a formula is true in a state s iff its negation is false in s, and vice versa. However, also if a formula is defined for a state s, it need not be either true or false in s. A state s may have partial information about the values of certain variables, and, therefore, formulas with free variables may be neither true nor false in s. On the other hand, if a formula is defined for it, then it is either true or false in a maximal state of information $\{i\}$. If we have that maximal state $\{i\} \models \phi$, then I will also say that i satisfies ϕ.

As in *DPL*, interpretation in *EDPL* is distributive:

Fact 2.3 (Distributivity)

- $s[\![\phi]\!] = \bigcup_{i \in s}\{i\}[\![\phi]\!]$, if defined

(The fact is again proved by a straightforward induction on the complexity of ϕ.) Update and distributivity jointly entail the following fact:

Fact 2.4

For all s, $i \in D^{D(s)}$:

- $i \in s$ & $\{i\} \models \phi$ iff $i < s[\![\phi]\!]$

Fact 2.4 tells us that an assignment i in s satisfies ϕ iff i survives in the update of s with ϕ. So, the update of a state s with ϕ contains (only) assignments that register, i.e., extend, the assignments in s that satisfy ϕ. This fact shows a crucial difference with interpretation in *DPL*, where we can not in general conclude, considering the assignments in $s[\![\phi]\!]$, which assignments in s satisfy ϕ. In *EDPL* we can.

The facts 2.2 and 2.4 guarantee that the *US*-style notions of negation and truth in *EDPL* correspond to their *DPL* counterparts.[10] Given these facts, the interpretation of $\neg\phi$ in a state s, and the truth of ϕ in s are properly defined in *EDPL* in terms of s and $s[\![\phi]\!]$ only. As we will see in section 4, the above facts also allow us to give a proper, uniform definition of the interpretation of adverbial and adnominal quantifiers.

2.3 EDPL interpretation

I now turn to some examples of *EDPL* interpretation. In this section I simply assume definedness.

Existential quantification

Since existential quantification and conjunction involve the composition of two operations on states, *DPL*'s characteristic donkey equivalences are retained:

Fact 2.5 (Donkey equivalences (1))

- $(\exists x\phi \wedge \psi) \Leftrightarrow (\exists x(\phi \wedge \psi))$
- $((\phi \wedge \psi) \wedge \chi) \Leftrightarrow (\phi \wedge (\psi \wedge \chi))$

Like *DPL*, *EDPL* accounts for the fact that indefinite noun phrases (existential quantifiers) in one sentence may bind pronouns (free variables) in a successive sentence. Again, this is achieved in a compositional way.

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10. For instance, in *EDPL* ϕ is true in s, $s \models \phi$, iff (definition of $\models$) $s \sqsubseteq s[\![\phi]\!]$ iff (definition of $\sqsubseteq$) $\forall i \in s$: $i < s[\![\phi]\!]$ iff (fact 2.4) $\forall i \in s$: $\{i\} \models \phi$ iff (using fact 2.2) $\forall i \in s$: $\{i\}[\![\phi]\!] \neq \emptyset$, which is the *DPL* requirement for ϕ to be true in s.

It may be elucidating to inspect the donkey example in a little more detail. Consider the interpretation of *A man owns a donkey*, $\exists x(Mx \wedge \exists y(Dy \wedge Oxy))$, in a (non-absurd) state s. This gives us $s[\![\exists x(Mx \wedge \exists y(Dy \wedge Oxy))]\!] = s[x][\![Mx]\!][y][\![Dy]\!][\![Oxy]\!]$. If X is the domain of s, this state is the following set of assignments:

(7) $\{j \in D^{X\cup\{x,y\}} \mid \exists i \in s\colon i \leq_{\{x,y\}} j \;\&\; j(x) \in F(M) \;\&\; j(y) \in F(D)$
$\&\; \langle j(x), j(y)\rangle \in F(O)\}$

This state, which will be referred to as state t for the time being, contains the information that the value of x is a man and the value of y is a donkey and that the value of x owns the value of y. For any $i \in s$ and for any man z and donkey z' owned by z, there is an extension j of i in t such that $j(x) = z$ and $j(y) = z'$.

The formula $\exists x(Mx \wedge \exists y(Dy \wedge Oxy))$ is true in s iff $s \sqsubseteq t$, which is the case iff, in the model, there is a man who owns a donkey. Notice that the formula $\exists x(Mx \wedge \exists y(Dy \wedge Oxy))$ can only be true or false in a state s. The formula contains no free variables, and, hence, conveys no information about the values of variables in the domain of s.

The update of t with the sentence *He beats it*, Bxy, gives us $t[\![Bxy]\!]$, which is the following state:

(8) $\{j \in D^{X\cup\{x,y\}} \mid \exists i \in s\colon i \leq_{\{x,y\}} j \;\&\; j(x) \in F(M) \;\&\; j(y) \in F(D)$
$\&\; \langle j(x), j(y)\rangle \in F(O) \;\&\; \langle j(x), j(y)\rangle \in F(B)\}$

The state $t[\![Bxy]\!]$ contains the information that the value of x is a man, that the value of y is a donkey and that the value of x owns and beats the value of y. The little discourse $\exists x(Mx \wedge \exists y(Dy \wedge Oxy)) \wedge Bxy$ then is true in s iff $s \sqsubseteq t[\![Bxy]\!]$, which is the case iff, in the model, there is a man who beats a donkey he owns.

Negation

Like in *US*, the interpretation of $\neg\phi$ in a state s involves the subtraction of $s[\![\phi]\!]$ from s. The subtraction in fact involves the elimination of assignments in s that satisfy ϕ. This is like in *DPL*. In *DPL*, $s[\![\neg\phi]\!]$ is the set $\{i \in s \mid \{i\} \not\models \phi\}$, and in *EDPL* $\{i \in s \mid \{i\} \not\models \phi\}$ (by fact 2.4) equals $\{i \in s \mid i \not< s[\![\phi]\!]\} = s[\![\neg\phi]\!]$. The difference with *DPL* is that $s[\![\neg\phi]\!]$ in *EDPL* is defined in terms of s and the update of s with ϕ solely, whereas in *DPL* $s[\![\neg\phi]\!]$ is defined in terms of s and the distribution of $[\![\phi]\!]$ over the singleton subsets of s.

Let us consider a simple example:

(9) No man sees her.
$\neg\exists y(My \wedge Syx)$

The interpretation of this sentence in a state s yields the state $s[\![\neg\exists y(My \wedge Syx)]\!]$, which is $s - s[y][\![My]\!][\![Syx]\!]$. The state $s[y][\![My]\!][\![Syx]\!]$ that is subtracted from the original state s is the following set of assignments (where X is the domain of s):

(10) $\{j \in D^{X\cup\{y\}} \mid \exists i \in s\colon i \leq_{\{y\}} j \;\&\; j(y) \in F(M) \;\&\; \langle j(y), j(x)\rangle \in F(S)\}$

The subtraction of this state from s, $s - s[y][\![My]\!][\![Syx]\!]$, produces a state that consists of the assignments in s that do not survive in $s[y][\![My]\!][\![Syx]\!]$:

(11) $\{i \in s \mid \neg\exists j\colon i \leq_{\{y\}} j \;\&\; j(y) \in F(M) \;\&\; \langle j(y), j(x)\rangle \in F(S)\}$

In other words, the result of interpreting $\neg\exists y(My \wedge Syx)$ in s is a state that contains the information that the value of x is an individual such that no other individual can be found that is a man and sees her.

The interpretation of a formula $\neg\phi$ in a state s is purely eliminative, i.e., it is unable to extend the domain of s. This corresponds to the fact that, usually, if a noun phrase stands in the scope of a negation it can not serve as an antecedent for pronouns that occur outside the negation's scope (but, for exceptions, see Groenendijk and Stokhof [1991] and Dekker [1993, Ch. 2]). As a consequence, the law of double negation doesn't hold in *EDPL*. The interpretation of the double negation of ϕ in s preserves the assignments in s which satisfy ϕ, i.e., the ones that are extended in the update of s with ϕ, but not the extensions themselves:

Fact 2.6

- $s[\![\neg\neg\phi]\!] = s - (s - s[\![\phi]\!]) = \{i \in s \mid i < s[\![\phi]\!]\}$

So, the double negation of ϕ is equivalent with ϕ for as far as truth conditions are concerned, but it cancels possible extensions of the domain of s induced by ϕ. Adopting the terminology of Groenendijk and Stokhof, we may call $\neg\neg\phi$ the static closure of ϕ and write it as $\downarrow\phi$.

Universal quantification and implication

In *EDPL* universal quantification, and implication are defined, as usual, in terms of existential quantification, negation and conjunction:

Definition 2.9

- $\phi \rightarrow \psi = \neg(\phi \wedge \neg\psi)$
- $\forall x\phi = \neg\exists x\neg\phi$

As in *DPL*, also the second donkey equivalences are valid in *EDPL*:

Fact 2.7 (Donkey equivalences (2))

- $(\exists x\phi \rightarrow \psi) \Leftrightarrow (\forall x(\phi \rightarrow \psi))$
- $((\phi \wedge \psi) \rightarrow \chi) \Leftrightarrow (\phi \rightarrow (\psi \rightarrow \chi))$

The interpretation of universally quantified formulas and of implications can, equivalently, be stated as follows:

Fact 2.8

- $s[\![\forall x\phi]\!] = s - (s[x] - s[x][\![\phi]\!]) = \{i \in s \mid \forall j \in s[x]\text{: if } i \leq j \text{ then } j < s[x][\![\phi]\!]\}$
- $s[\![\phi \rightarrow \psi]\!] = s - (s[\![\phi]\!] - s[\![\phi]\!][\![\psi]\!]) = \{i \in s \mid \forall j \in s[\![\phi]\!]\text{: if } i \leq j \text{ then } j < s[\![\phi]\!][\![\psi]\!]\}$
- $s[\![\forall x(\phi \rightarrow \psi)]\!] = s - (s[x][\![\phi]\!] - s[x][\![\phi]\!][\![\psi]\!]) = \{i \in s \mid \forall j \in s[x][\![\phi]\!]\text{: if } i \leq j \text{ then } j < s[x][\![\phi]\!][\![\psi]\!]\}$

So, for instance, the interpretation of $\forall x(\phi \rightarrow \psi)$ in a state s preserves the assignments i in s every extension of which in $s[x][\![\phi]\!]$ is extended in $s[x][\![\phi]\!][\![\psi]\!]$. For any assignment i in $s[\![\forall x(\phi \rightarrow \psi)]\!]$, every extension of i in $s[x][\![\phi]\!]$ satisfies ψ.

Let us briefly consider an example in which a free variable occurs:

(12) Every man sees her.
$\forall y(My \rightarrow Syx)$

The interpretation of this sentence in a state s yields the state $s[\![\forall y(My \rightarrow Syx)]\!]$, which is $s - (s[y][\![My]\!] - s[y][\![My]\!][\![Syx]\!])$. This state consists of the assignments i in s such that on every extension of i to y such that the value of y is a man, the value of y sees the value of x. So, the resulting state consists of those assignments i in s which assign an individual to x which every man sees.

The second example is the universally quantified donkey sentence:

(13) Every man who owns a donkey beats it.
$\forall x((Mx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bxy)$

The interpretation of this sentence in a state s yields the set of assignments i in s such that every extension of i in $s[x][\![Fx]\!][y][\![Dy]\!][\![Oxy]\!]$ satisfies Bxy. The resulting state contains an assignment i in s iff on every extension j of i to x and y it holds that if the value of x is a man and the value of y is a donkey which the value of x owns, then the value of x beats the value of y. Clearly, this simply requires that every man beats every donkey he owns.

Digression

It has been argued, for instance by Schubert and Pelletier [1988], that the strong readings of donkey sentences are misguided, or, at least, are not the only readings these sentences have. A convincing example is the following sentence, labeled the 'dime implication' here:

(14) If I have a dime in my pocket, I'll put it in the parking meter.

On its most natural reading the dime implication says that if I have one or more dimes in my pocket, then I will throw one in the meter. However, if we interpret the dime implication, like the donkey implication, as one of strong implication, then the sentence would imply that I throw *all* the dimes I have in my pocket in the meter. As concerns this example, the strong reading is quite odd.

It is possible to define a notion of weak implication that assigns conditional sentences the weak truth conditions that Schubert and Pelletier argue for, and that preserves the internal dynamics of the implication. This is, in fact, Chierchia's [1992] definition of implication. I use $\hookrightarrow$ to indicate the weak notion of implication:

Definition 2.10 (Weak implication)

- $\phi \hookrightarrow \psi = \downarrow\phi \rightarrow (\phi \wedge \psi)$

The interpretation of a weak implication $\phi \hookrightarrow \psi$ in a state s gives us those $i \in s$ such that if i has an extension in $s[\![\phi]\!]$, then i has an extension in $s[\![\phi]\!][\![\psi]\!]$. If we interpret the dime implication employing this weak notion of implication, then the sentence is true in a state s if I throw a dime I have in my pocket in the meter, if I have dimes in my pocket at all.

However, it may be worthwhile to point out here that it would be misguided to entirely reject the strong implication in favour of the weak one. Consider the following variant of the dime implication:

(15) If I have a dime in my pocket, I won't throw it in the meter.

Interpreting this sentence as one of weak implication seems to give just as odd results as the strong interpretation of the original dime sentence. On the weak reading of the present sentence, its truth conditions only require that if I have one or more dimes in my pocket, I keep at least one dime but maybe throw all other dimes I have in the parking meter. On the other hand, on the strong reading it is required that I don't throw any dime I might have in my pocket in the meter, and that is as intuition would have it.

What is interesting about the alternative dime sentence is that it completely corresponds to the original dime sentence, that it can be uttered in precisely the same contexts as the original dime sentence, but that it, nevertheless, seems to require the strong interpretation of the conditional, whereas the original dime sentence appears to require the weak interpretation. In the fourth section of this paper, I will come back to the issue of strong and weak implication, and I will show there that the weak and strong readings of conditionals in fact fit in a more general scheme of (universal) adverbial quantification. This concludes the digression.

3 Logical properties of EDPL

This section is confined to the study of the logical properties of *EDPL*. First, in section 3.1, the structure of information states is studied in more detail. I show that it is a (complete) lattice. In section 3.2, definedness phenomena in *EDPL* are given a more precise characterization, and the *EDPL* notion of entailment is presented. In section 3.3, I compare *DPL* and *EDPL* and show that the dynamics of *EDPL* resides solely in its definedness conditions, i.e., in the (order of the) introduction of discourse referents.

3.1 The information lattice

In section 2.1, two relations on the domain of information states were defined, the update and the substate relation. The two relations are two natural generalizations of the subset relation. Both induce a partial order on the domain of information states (the definitions of $\leq$ and $\sqsubseteq$ are repeated here for convenience):

Definition 2.4 (Update)

- $s \leq t$, iff $D(s) \subseteq D(t)$ and $\forall i \in t$: $i > s$

Fact 3.1

$\langle S, \leq \rangle$ is a partial order, i.e., $\leq$ is

- reflexive ($s \leq s$)
- transitive (if $s \leq s'$ and $s' \leq s''$ then $s \leq s''$)
- antisymmetric (if $s \leq s'$ and $s' \leq s$ then $s = s'$)

Definition 2.5 (Substate)

- $s \sqsubseteq t$, iff $D(s) \subseteq D(t)$ and $\forall i \in s$: $i < t$

Fact 3.2

$\langle S, \sqsubseteq \rangle$ is a partial order

Of course, it is a rather intuitive fact that a state is an update of itself (reflexivity), that an update of an update is an update (transitivity) and that if a state s is an update of an update of s, then the update is identical to s itself (antisymmetry). The fact that the substate relation also induces a partial order may be more surprising. The entailment relation (cf., section 3.2), which is stated in terms of the substate relation, is *not* in general reflexive and transitive.

The orders of S induced by $\sqsubseteq$ and $\leq$ have unique weakest and strongest elements. The weakest information state in the sense of $\sqsubseteq$ (i.e., the state t such that $\forall s$: $s \sqsubseteq t$) is $\top^V$, the state of no information about all variables. The strongest state in $\sqsubseteq$ is $\bot^{\emptyset}$, the absurd state of information about no variables ($\forall s$: $\bot^{\emptyset} \sqsubseteq s$). The strongest state in the sense of $\leq$ is $\bot^V$, the absurd state of information about all variables ($\forall s$: $s \leq \bot^V$). The weakest information state in $\leq$ (i.e., the state t such that $\forall s$: $s \leq t$) is $\top^{\emptyset}$, the state of no information about no variables. These observations can be pictured as follows (the ordering relations $\sqsubseteq$ and $\leq$ should be read as if rotated 90^o anticlockwise):

$$\begin{array}{cc@{\qquad\qquad}cc} & \top^V & & \bot^V \\ \sqsubseteq & \vdots & \leq & \vdots \\ & \bot^{\emptyset} & & \top^{\emptyset} \end{array}$$

Notice that $\sqsubseteq$ ($\leq$) ranges from the bottom (top) of propositional logic to the top (bottom) of predicate logic.

The structure $\langle S, \leq \rangle$, moreover, is a (complete) lattice, that is, for every two states s and t there is a unique weakest state t' such that $s \leq t'$ and $t \leq t'$ and a unique strongest state s' such that $s' \leq s$ and $s' \leq t$. These two states are called the information, or state, product and the common ground. They are defined as follows:

Definition 3.1 (State product and common ground)

- $s \wedge t = \{i \in D^{D(s) \cup D(t)} \mid i > s \text{ and } i > t\}$
- $s \vee t = \{i \in D^{D(s) \cap D(t)} \mid i < s \text{ or } i < t\}$

The product of two states s and t contains the combined information contained in s and t. It contains information about the values of all the variables s or t contain information about, and it excludes valuations of these variables which are excluded by s or t. So, $s \wedge t$ contains the information that s or t has about the values of the variables in their combined domain. The common ground of s and t contains just the information s and t agree upon. It contains only information about the values of variables both s and t contain information about, and it only excludes assignments which are excluded by s and t. So, $s \vee t$ contains the information that both s and t have about the values of the variables in their shared domain.

The following fact shows that the product and the common ground of s and t are indeed the weakest common update and the strongest common 'downdate' of s and t, respectively:

Fact 3.3

1. $(s \vee t) \leq s \leq (s \wedge t)$
   $(s \vee t) \leq t \leq (s \wedge t)$

2. $\forall s'$: if $s' \leq s$ and $s' \leq t$ then $s' \leq (s \vee t)$
   $\forall s'$: if $s \leq s'$ and $t \leq s'$ then $(s \wedge t) \leq s'$[11]

So, the information product of s and t precisely indicates the joint information of s and t, the weakest state they might both arrive at by information update. Similarly, the common ground of s and t contains precisely the information s and t agree upon. The product and ground of two states can be depicted by the following snapshot of the lattice of information states:

```
   s ∧ t
   /   \
 s       t
   \   /
   s ∨ t
```

The lattice $\langle S, \leq \rangle$ is not distributive, that is, $s \wedge (t \vee t')$ and $(s \wedge t) \vee (s \wedge t')$ are not in general the same, nor are $s \vee (t \wedge t')$ and $(s \vee t) \wedge (s \vee t')$.[12] However, the *EDPL* interpretation function does distribute over $\wedge$ and $\vee$:

Fact 3.4 (Distributivity of $[\![\ ]\!]$ in $\langle S, \leq \rangle$)
If defined,

- $s[\![\phi]\!] \wedge t[\![\phi]\!] = (s \wedge t)[\![\phi]\!]$
- $s[\![\phi]\!] \vee t[\![\phi]\!] = (s \vee t)[\![\phi]\!]$[13]

In a snapshot:

```
      (s ∧ t)[[φ]]
        /    \
  s[[φ]]      t[[φ]]   if defined
        \    /
      (s ∨ t)[[φ]]
```

The distributivity of $[\![\ ]\!]$ in $\langle S, \leq \rangle$ indicates that formulas in *EDPL* have state independent contents. In section 3.3 these contents will be independently defined.

3.2 Definedness and entailment

The question whether the interpretation of an *EDPL* formula is defined for a state s depends on the domain of s. *EDPL* formulas carry presuppositions that restrict the domains of states with respect to which their interpretation is defined. These presuppositions can be calculated by the following function $|\ |$, which, for any formula

11. Proof of the first clause: By the definition of $\vee$, $\leq$ and $\wedge$. Proof of the second clause: Let $s' \leq s$ and $s' \leq t$. Then $D(s') \subseteq (D(s) \cap D(t)) = D(s \vee t)$. For any $i \in (s \vee t)$: $D(i) = (D(s) \cap D(t))$ and $i \lessdot s$ or $i \lessdot t$. Since $s' \leq s$ and $D(s') \subseteq D(i)$, if $i \lessdot s$, then $i \gtrdot s'$, and, since also $s' \leq t$, if $i \lessdot t$, then $i \gtrdot s'$. So, for any $i \in (s \vee t)$: $i \gtrdot s'$, and, hence, $s' \leq (s \vee t)$. For $\wedge$, similarly.

12. Let $s, t \in S^{\{x\}}$, $s = \{i\}$, $t = \{j\}$, $i(x) \neq j(x)$ and $t' = \bot^{\emptyset}$. Then $(s \wedge t) \vee (s \wedge t') = \bot^{\{x\}} \vee \bot^{\{x\}} = \bot^{\{x\}}$, while $s \wedge (t \vee t') = \{i\} \wedge \top^{\emptyset} = \{i\}$. Furthermore, in that case $(s \vee t) \wedge (s \vee t') = \{i, j\} \wedge \top^{\emptyset} = \{i, j\}$, while $s \vee (t \wedge t') = \{i\} \vee \bot^{\{x\}} = \{i\}$.

13. To prove this fact, use the fact that $s[\![\phi]\!]$ and $t[\![\phi]\!]$ are defined iff $D(s)$ and $D(t)$ are in $\backslash\phi\backslash$ iff (since $\backslash\phi\backslash$ is continuous and closed under $\cap$ and $\cup$) $D(s \wedge t)$ and $D(s \vee t)$ are in $\backslash\phi\backslash$ iff $(s \wedge t)[\![\phi]\!]$ and $(s \vee t)[\![\phi]\!]$ are defined.

ϕ, simultaneously defines the domains of information states for which ϕ is defined and the extension of domain that results from interpreting ϕ:

Definition 3.2 (Domain update)

- $|Rx_1 \ldots x_n| = \{\langle X, Y\rangle \mid X = Y \text{ and } x_1, \ldots, x_n \in X\}$
- $|\exists x\phi| \quad = \{\langle X, Y\rangle \mid x \notin X \text{ and } \langle X \cup \{x\}, Y\rangle \in |\phi|\}$
- $|\neg\phi| \quad = \{\langle X, Y\rangle \mid X = Y \text{ and } \exists Z\colon \langle X, Z\rangle \in |\phi|\}$
- $|\phi \wedge \psi| \quad = \{\langle X, Y\rangle \mid \exists Z\colon \langle X, Z\rangle \in |\phi| \text{ and } \langle Z, Y\rangle \in |\psi|\}$

An easy induction proves the following fact:

Fact 3.5

- ϕ is defined iff $|\phi| \neq \emptyset$

In terms of $|\ |$, we can define what may be called the possible domains $\backslash\phi\backslash$ and range $/\phi/$ of a formula ϕ:

Definition 3.3 (Domains and range)

- $\backslash\phi\backslash = \{X \mid \exists Z\colon \langle X, Z\rangle \in |\phi|\}$
- $/\phi/ = \{Y \mid \exists Z\colon \langle Z, Y\rangle \in |\phi|\}$

Notice the following fact:

Fact 3.6

- $s[\![\phi]\!]$ is defined iff $D(s) \in \backslash\phi\backslash$

So, on the basis of syntactic properties of ϕ we can determine the states for which ϕ is defined.

For any formula ϕ, $\backslash\phi\backslash$ and $/\phi/$ are sets of sets of variables, i.e., they are generalized quantifier over variables, which, moreover, have the following properties:

Fact 3.7

- $\backslash\phi\backslash$ and $/\phi/$ are continuous quantifiers closed under $\cap$ and $\cup$

(A quantifier Q is continuous iff for any X and Y, if $X \in Q$ and $Y \in Q$, then for any Z, if $X \subseteq Z \subseteq Y$ then $Z \in Q$.) Given fact 3.7, there are sets of variables X and Y such that $\backslash\phi\backslash$ equals $\{Z \mid X \subseteq Z \subseteq Y\}$. So, if $\backslash\phi\backslash \neq \emptyset$, it has a smallest element X and a greatest element Y and all elements which are both bigger than X and smaller than Y. The same holds for $/\phi/$.

If $|\phi|$ is not empty, then the smallest element X in the domains $\backslash\phi\backslash$ of ϕ is the set of free variables in ϕ, indicated by $FV(\phi)$. The smallest set in $/\phi/$ can be called the set of variables discussed by ϕ, indicated by $DV(\phi)$. The discussed variables which are not free in ϕ, $DV(\phi)\backslash FV(\phi)$, is the set of variables introduced by ϕ, indicated by $IV(\phi)$. The set of introduced variables $IV(\phi)$ can be used, in its turn, to characterize the domain update $|\phi|$:

Fact 3.8

- $|\phi| = \{\langle X, (X \cup IV(\phi))\rangle \mid X \in \backslash\phi\backslash\}$
  $\quad = \{\langle (Y\backslash IV(\phi)), Y\rangle \mid Y \in /\phi/\}$

We may now turn to the definition of entailment. *EDPL* entailments are relativized to domains of variables. A sequence of premises is defined to entail a conclusion with respect to a domain X, if the update with the premises of any state of information about the values of the variables in X yields a state in which the conclusion is true:

Definition 3.4 (Entailment in EDPL)

- $\phi_1, \ldots, \phi_n \models_X \psi$ iff $\forall M, s \in S^X$: $s[\![\phi_1]\!]_M \ldots [\![\phi_n]\!]_M \models_M \psi$

Like *US*, *EDPL* employs an update notion of entailment. Relative to a certain domain of variables, an inference is valid iff the update with the premises of a state of information always produces a state of information about the values of variables about which subsequent update with the conclusion adds no more information. Furthermore, *EDPL* entailment has the dynamics of *DPL* entailment. Like in *DPL*, free variables in the conclusion of an inference may refer back to objects introduced in the premises.

Entailment is defined relative to a domain of variables because we should not exclude the possibility of undefinedness. Many valid inferences may be undefined for certain states of information and, also if the premisses of a valid inference are defined for some domain X, the conclusion need not necessarily be defined for the update of a state in S^X with the premises.[14] For this reason, we disregard undefinedness in some domains, and judge an inference valid iff (i) there is at least a domain in which the update with the premises and the conclusion of the inference is defined and (ii) the update with the premises of any state with such a domain produces a state in which the conclusion is true.[15]

It must be noticed that the relativization of the entailment relation only discards occurring *undefinedness* of inferences. It certainly does not corrupt the notion of entailment by discarding *counterexamples* to them. Consider the following fact:

Fact 3.9

- If $\phi_1, \ldots \phi_n \models_X \psi$, then for all s: $s[\![\phi_1]\!] \ldots [\![\phi_n]\!] \models \psi$, if defined

(This fact is proved in the appendix.) Fact 3.9 tells us that if we have that $\phi \models_X \psi$, then there is no state s such that $s[\![\phi]\!][\![\psi]\!]$ is defined and such that $s[\![\phi]\!] \not\models \psi$. This fact implies that if $\phi_1, \ldots, \phi_n$ entail ψ with respect to a domain X, and if, for any domain Y, $\phi_1, \ldots, \phi_n, \psi$ is defined for states with domain Y, then $\phi_1, \ldots, \phi_n$ entail ψ with respect to Y. For this reason it is proper to say that $\phi_1, \ldots, \phi_n \models \psi$ iff there is a domain X such that $\phi_1, \ldots, \phi_n \models_X \psi$, and I will do so below.

The deduction theorem holds in *EDPL* :

Fact 3.10

- $\phi_1, \ldots, \phi_n \models \psi$ iff $\phi_1, \ldots, \phi_{n-1} \models \phi_n \rightarrow \psi$

---

14. For instance, for states with a domain Y such that $x \notin Y$, the interpretation of $x = x$ is undefined. However, we would want $x = x$ to be valid, with respect to any domain X such that $x \in X$, and this indeed falls out of the present definition of entailment. Similarly, for all states $s \in S^Y$, if $x \notin Y$, then $\exists y Fy$ is not necessarily defined in state $s[\![\exists x Fx]\!]$. However, the entailment $\exists x Fx \models_X \exists y Fy$ should come out valid for any domain X such that $x, y \notin X$, and indeed it does.

15. In order to allow inferences to be undefined in some domains, we might, alternatively, have required that every update with the premises, if defined, licenses the conclusion, if defined. However, such a (weaker) notion of entailment would generate inferences that owe their validity to being necessarily undefined. For instance, in that case $x = x$ would entail $\exists x(x \neq x)$, which is objectionable.

(For the proof, cf., the appendix.)

3.3 Static and dynamic interpretation

In this section *EDPL* and *DPL* are compared, and the difference between *EDPL* and a static system of interpretation is precisely characterized.

Under the assumption of definedness, *DPL* and *EDPL* are equivalent systems. Let s^V be the total extension of state s, i.e., $s^V = \{g \in D^V \mid g \geqslant s\}$, and for state $t \in S^V$, let t^X be the restriction of t to X, i.e., $t^X = \{i \in D^X \mid i \lessdot t\}$. Then:

Fact 3.11
If $t \in S^V$ and $X, D(s) \in \backslash\phi\backslash$, then
1. $t \models_{M,dpl} \phi$ iff $t^X \models_{M,edpl} \phi$
2. $s \models_{M,edpl} \phi$ iff $s^V \models_{M,dpl} \phi$

(Again, the proof can be found in the appendix.) This fact tells us that, if ϕ is true in a state t in *DPL*, then ϕ is true in *EDPL* in the restriction of t to a domain for which ϕ is defined. Furthermore, if ϕ is true in s in *EDPL*, then ϕ is true in the total extension of s in *DPL*. Given the deduction theorem and fact 3.9, fact 3.11 entails that, under the assumption of definedness, the *DPL* and *EDPL* notions of entailment coincide.

We see that what really distinguishes *EDPL* from *DPL* is the possibility of undefinedness, or, put differently, the insurance against information downdate. I will now show that (un-)definedness also serves to expel complications which obtain at the syntax/semantics interface in *DPL*. First, consider Groenendijk and Stokhof's notions of a test and a condition, which can be defined as follows in *EDPL*:

Definition 3.5 (Tests)
- ϕ is a test iff ϕ is defined and $\forall s$: $D(s) = D(s[\![\phi]\!])$ if defined

A test does not introduce new referents in the universe of discourse, it only tests whether input assignments satisfy certain conditions. According to the above definition, if ϕ is a test then $\forall i, j$: if $j \in \{i\}[\![\phi]\!]$ then $i = j$ if defined, which corresponds to the *DPL* definition of a test. However, on the *DPL* definition also contradictions are a test, which they need not be in *EDPL*.

The notion of a condition is the syntactic counterpart of the (semantic) notion of a test:

Definition 3.6 (Conditions)
The set of conditions is the smallest set of formulas such that:
- if ϕ is an atomic formula or a negation then ϕ is a condition
- if ϕ and ψ are conditions, then $\phi \wedge \psi$ is a condition

The two notions are related in the following way:

Fact 3.12
- If ϕ is defined, then ϕ is a condition iff ϕ is a test

Restricting ourselves to defined formulas, the notions of a condition and a test coincide. This is different in *DPL*. In *DPL* we don't find undefinedness, and, as Groenendijk and Stokhof observe, contradictions which are no condition are also tests. Furthermore, in *DPL*, there are formulas which are no condition or contradiction, but yet are tests, by reidentification after reinstantiation. A simple example is $(x = z) \wedge \exists x(x = z)$. Clearly, being undefined, such a formula, which is no condition, is no test in *EDPL* either.

Next, consider some classical equivalences which are also discussed in Groenendijk and Stokhof [1991, pp. 64 ff]. Definedness precisely delineates the validity of idempotence (it is assumed here that if a or b is undefined, then so is $a = b$):

Fact 3.13 (Restricted idempotence)

- $[\![\phi]\!] = [\![\phi \wedge \phi]\!]$ iff defined

The same goes for commutativity:

Fact 3.14 (Restricted commutativity)

- $[\![\phi \wedge \psi]\!] = [\![\psi \wedge \phi]\!]$ iff defined

The restriction to definedness here covers all three conditions (that $FV(\phi) \cap IV(\psi) = FV(\psi) \cap IV(\phi) = IV(\phi) \cap IV(\psi) = \emptyset$) under which conjuncts can be commuted in *DPL*. We also find the following equivalences:

Fact 3.15

- $\phi \vee (\psi \wedge \chi) \Leftrightarrow (\phi \vee \psi) \wedge (\phi \vee \chi)$ iff defined
- $\phi \rightarrow \psi \Leftrightarrow \neg\psi \rightarrow \neg\phi$ iff defined
- $\phi \wedge \exists x\psi \Leftrightarrow \exists x\phi \wedge \psi$ iff defined

And, iff defined, entailment is reflexive in *EDPL*:

Fact 3.16 (Restricted reflexivity)

- $\phi \models \phi$ iff defined

We see that *EDPL* definedness effectively characterizes the cases in which these classical equivalences hold. This is an improvement in comparison with the situation in *DPL*. In Groenendijk and Stokhof [1991], only sufficient, not necessary conditions for these equivalences are given.

Let us finally consider the (in-)transitivity of the entailment relation. As in *DPL*, entailment in *EDPL* is not (in general) transitive. Groenendijk and Stokhof's counterexample is also a counterexample in *EDPL*. Whereas $\exists xFx$ (as usual) entails $\exists yFy$ and $\exists yFy$ (dynamically) entails Fy, $\exists xFx$ does not entail Fy. Transitivity fails in this example because the goal conclusion Fy critically refers back to the object introduced in the mediating formula $\exists yFy$. However, if we restrict ourselves to inferences defined on a shared domain, then transitivity does hold:

Fact 3.17 (Restricted transitivity)

If $/\phi/ \cap \backslash\psi\backslash \cap \backslash\chi\backslash \neq \emptyset$ then:

- If $\phi \models \psi$ and $\psi \models \chi$, then $\phi \models \chi$

(The proof can be found in the appendix.) Transitivity holds if after ϕ has been processed, a domain of subjects is spoken about, which is a proper domain for the interpretation of both ψ and χ. The restriction on transitivity effectively excludes cases where the mediating formula ψ introduces variables which are free in χ. The conditions under which transitivity holds in *EDPL* are different from those in *DPL*. In [1991], Groenendijk and Stokhof show that $\phi \models \chi$ in *DPL* if, apart from $\phi \models \psi$

and $\psi \models \chi$, ϕ also entails the information expressed by ψ about the variables which are introduced in ψ and which are free in χ. However, if $\phi \models \psi$ in *EDPL*, then it is impossible that ϕ conveys information about variables introduced by ψ.

As we have seen, the possibility of undefinedness positively distinguishes *EDPL* from *DPL*. In order to conclude this section, I will now show that it is also definedness i.e., the (order of the) introduction of discourse referents, that sets *EDPL* apart from a static system of interpretation. This claim is substantiated by showing that, under the assumption of definedness, *EDPL* interpretation can be stated in terms of the product and ground operator defined in section 3.1. Given that these operators are the (commutative) meet and join of the lattice of *EDPL* information states, it appears that it is in the definedness conditions where *EDPL*'s dynamics resides.

For the sake of convenience, I will now conceive of an existentially quantified formula $\exists x\phi$ as the conjunction of $\exists x \wedge \phi$, where $s[\![\exists x]\!] = s[x]$ (if $x \notin D(s)$). This allows us to conceive of both atomic formulas and such 'wild' quantifiers as atoms, and we can associate with the atoms states which can be conceived of as their (static) denotations. Furthermore, in terms of the denotations of these atoms, the denotations of complex formulas can be defined. Let, for any $s \in S^X$, the complement $\overline{s}$ of s be $\{i \in D^X \mid i \notin s\}$. Then $[\phi]$, the denotation of ϕ, is defined as follows:

Definition 3.7 (EDPL denotations)

- $[Rx_1 \dots x_n] = \{i \in D^{\{x_1,\dots,x_n\}} \mid \langle i(x_1), \dots, i(x_n)\rangle \in F(R)\}$
- $[\exists x] \quad = \top^{\{x\}}$
- $[\neg\phi] \quad = \overline{\bot^{FV(\phi)} \vee [\phi]}$
- $[\phi \wedge \psi] \quad = [\phi] \wedge [\psi]$

An atomic formula $Rx_1 \dots x_n$ denotes a state with a domain consisting of $x_1, \dots, x_n$, and which consists of assignments i such that $i(x_1), \dots, i(x_n)$ stand in the relation R. The quantifier $\exists x$ simply denotes the state of no information about the value of x.[16] A negation $\neg\phi$ denotes a state of information about the free variables in ϕ. The common ground of $\bot^{FV(\phi)}$ and $[\phi]$ restricts the assignments in $[\phi]$ to the assignments to the free variables in ϕ which have an extension in $[\phi]$. The complement of this set is the set of assignments to the free variables in ϕ which have no extension in $[\phi]$. Conjunction involves the product of the denotations of the conjuncts.

The following fact is easily verified:

Fact 3.18
If ϕ is defined,

- $D([\phi]) = DV(\phi)$

This fact shows that the static denotation of a (defined) formula ϕ does not distinguish between the free variables in ϕ and the variables introduced by ϕ. More interestingly, we have the following fact:

16. We can give similar definitions of *PL* and *DPL* existential quantification. Let s be the interpretation $[\![\phi]\!]$ of a formula ϕ in *PL*, i.e., the set of (total) assignments verifying ϕ in *PL*. Then $[\![\exists x\phi]\!]$ in *PL* is $(\bot^{V\setminus\{x\}} \vee s) \wedge \top^{\{x\}}$, which is also the *DPL* result $s[x]$ of reinstantiating x in s. This result is obtained by, first, removing x from the domain of s by means of $\bot^{V\setminus\{x\}} \vee s$, and, next, adding x again by taking the product with $\top^{\{x\}}$. Thus conceived, the *PL* and *DPL* quantifier $\exists x$ is associated with the instruction "forget about x, and take x".

Fact 3.19
If defined,

- $s[\![\phi]\!] = s \wedge [\phi]$

(For the proof, cf., the appendix.) In fact, this shows that it is definedness that prevents *EDPL* from collapsing into a static system of interpreation. Definedness ensures that variables are introduced *before* they are referred back to.

It may be notivced that, since the product operation is defined for all pairs of states, and since all *EDPL* atoms have a defined denotation, it is possible to speculate about adopting the definition of $[\![\ ]\!]$ in terms of $[\ ]$ and $\wedge$ and dropping the definedness conditions. If we did so, $\exists x_j Rx_1 \ldots x_j \ldots x_n$ and $Rx_1 \ldots x_j \ldots x_n$ would turn out to be fully equivalent (if $x \in D([\phi])$, $[\exists x\phi] = \top^{\{x\}} \wedge [\phi] = [\phi]$). So, in that case indefinites would really *be* free variables, which is reminiscent of "Lewis' philosophy".

However, it does not seem a viable alternative to adopt such a total (i.e., non-partial) semantics. Notice, first, that it is impossible to determine on semantic grounds what are the free variables of ϕ when the definedness conditions are dropped. Therefore, the denotation of $\neg\phi$ is not 'properly' defined in terms of that of ϕ, since this definition critically refers to syntactic properties of ϕ, i.e., to the free variables of ϕ. Notice, next, that it still is crucial for a proper analysis of negation to distinguish embedded free variable occurrences from existentially bound ones, just in order to properly distinguish the meaning of, for instance, $\neg\exists xFx$ and $\neg Fx$. So, if the definedness conditions are dropped and, hence, $\exists xFx$ is fully equivalent with Fx, then there is no hope of giving a proper, compositional definition of negation.

4 Quantification in EDPL

In this section I introduce quantifiers in *EDPL* and show that the update property of *EDPL* interpretation enables a perspicuous and uniform account of adnominal and adverbial quantification, symmetric as well as asymmetric. The quantifiers are assigned so-called 'internally dynamic' interpretations, those that accounts for anaphoric relationships between antecedent noun phrases in the restriction of the quantifiers and anaphoric pronouns in their nuclear scope. I do not discuss 'externally dynamic' readings, in which antecedent noun phrases in the restriction or in the scope of a quantifier license anaphora beyond the scope of the quantifier. I start with the adverbs.

4.1 Unselective quantification

Lewis [1975] argues that in many cases adverbs of quantification (like *always*, *sometimes*, *usually*) unselectively quantify over the values of 'free variables' (often stemming from indefinite noun phrases) in their restrictive clause. The examples Lewis discusses are of the form *Sometimes/usually/always if x is a man, if y is a donkey, and if x owns y, x beats y* and, clearly, these are paraphrases in the logicians idiom of sentences like:

(16) Sometimes/usually/always if a man owns a donkey, he beats it.

Lewis points out that the quantifying adverbs *sometimes/usually/always* in fact quantify over the 'cases' that verify the restrictive clause. These cases are the admissible assignments of values to the variables that are free in the restriction, or, equivalently, the tuples of individuals that are possible values of these variables. So, the cases that verify the restriction *x is a man, y is a donkey and x owns y* are the maps from x to a farmer and from y to a donkey the farmer owns (i.e., the pairs consisting of a farmer and a donkey he owns). The adverbial quantifier quantifies over these cases, i.e., pairs of individuals in the examples above. If the head is *sometimes*, as in *Sometimes, if a farmer owns a donkey, he beats it*, the sentence says that some pairs consisting of a farmer and a donkey owned, are pairs of which the first element beats the second. If *always* is the head, it is said that every pair of a farmer and a donkey he owns is a pair that stands in the beat relation. And with *usually*, we get that most pairs that consist of a farmer and a donkey owned, stand in the beat relation. (Clearly, if in a quantified construction $A(\phi)(\psi)$ the restriction ϕ contains three (or n) free variables or indefinites, the unselective adverbs quantify over triples (or n-tuples) of individuals.)

This part of the story about adverbs of quantification is formalized quite elegantly in file change semantics, discourse representation theory and *DPL*. In these systems, unselective quantification about the values of any number of free variables is cast in terms of quantification about verifying (output) assignments. In *DPL*, for instance, the formula $Always(\phi)(\psi)$ tests, given an initial assignment g, whether all assignments that verify ϕ with respect to g also satisfy ψ. Similarly, the formula $Sometimes(\phi)(\psi)$ tests whether some assignments that verify ϕ with respect to g satisfy ψ, and the formula $Never(\phi)(\psi)$ tests whether no assignment that verifies ϕ with respect to g satisfies ψ.

EDPL, too, allows a straightforward interpretation of unselectively quantifying adverbs. We restrict ourselves to the adverbs (and determiners, cf., section 4.3) that satisfy the constraints of extension, quantity and conservativity.[17] For any such adverb of quantification A, with its usual set-theoretic interpretation $[A]$, the interpretation is defined as follows:

Definition 4.1 (Adverbs of quantification (symmetric))

- $s[\![A(\phi)(\psi)]\!] = \{i \in s \mid [A](\{j \mid i \leq j \;\&\; j \in s[\![\phi]\!]\})(\{j \mid j \in s[\![\phi]\!][\![\psi]\!]\})\}$

In a state s, for each assignment i in s, a symmetric adverbial quantifier A tests whether $[A]$ applies, first, to the set of extensions of i in the update of s with the restriction of the adverb, and, second, to the set of assignments that also verify the nuclear scope of the adverb. In effect, the adverb quantifies over the values of the variables introduced in the restriction, which corresponds to the set of tuples of individuals satisfying it.

17. Let Q be a quantifier that assigns any domain of individuals E a binary relation Q_E between subsets of E. Then Q satisfies extension iff for all E, E' and for all $A, B \subseteq E \subseteq E'$: $Q_E(A)(B)$ iff $Q_{E'}(A)(B)$. Q satisfies quantity iff for all E, E', if π is a bijection from E to E', then for all $A, B \subseteq E$: $Q_E(A)(B)$ iff $Q_{E'}(\{\pi(a) \mid a \in A\})(\{\pi(b) \mid b \in B\})$. Q is conservative iff for all E and $A, B \subseteq E$, $Q_E(A)(B)$ iff $Q_E(A)(A \cap B)$. Van Benthem [1986, Ch. 1,2] uses these constraints to single out the determiners that qualify as '(logical) quantifiers'. Adverbs and determiners that do not observe all three constraints can be treated within the *EDPL* framework, but they deserve a special treatment.

So, if we interpret *If a farmer owns a donkey he always beats it* in a state s in *EDPL*, we get $s[\![Always(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)))(Bxy)]\!]$. This is the set of assignments i in s such that on every extension of i to x and y, if the value of x is a farmer who owns a donkey which is the value of y, then the value of x beats the value of y. In other words, this formula tests whether all pairs of a farmer and a donkey he owns are pairs of which the first element beats the second element.

A second example is *If a man gives her a present, she usually thanks him for it*, which is translated as $Usually(\exists y(My \wedge \exists z(Pz \wedge Gyzx)))(Txyz)$. The interpretation of this formula in a state s yields a state consisting of all those assignments i in s that assign an individual to x that renders thanks in most cases in which a man gives her a present.

There are some interesting correspondences between the sentential connectives of *EDPL* and adverbs of quantification (as before, $\downarrow\phi$ indicates the static closure $\neg\neg\phi$ of ϕ):

Fact 4.1

- $Sometimes(\phi)(\psi) \Leftrightarrow \downarrow(\phi \wedge \psi)$
- $Always(\phi)(\psi) \Leftrightarrow (\phi \rightarrow \psi)$
- $Never(\phi)(\psi) \Leftrightarrow \neg(\phi \wedge \psi)$

Conjunction (disregarding its external dynamics) and implication appear to fit in the more general scheme of adverbial quantification. A sentence *Sometimes, if a farmer owns a donkey he beats it* has the same truth conditions as the conjunction *A farmer owns a donkey. He beats it.*[18] Furthermore, the sentence *Always, if a farmer owns a donkey he beats it* turns out equivalent with the sentence *Every farmer beats every donkey he owns*, and the sentence *If a farmer owns a donkey he never beats it* turns out equivalent with the sentence *No farmer who owns a donkey beats it.*

4.2 Asymmetric quantification

Adverbs of quantification do not always unselectively quantify over the values of *all* variables introduced in their restriction. Several authors (Bäuerle and Egli [1985], Root [1986], Rooth [1987] and Kadmon [1987, 1990], see also Heim [1990] and Chierchia [1992]) have discussed examples in which adverbial quantifiers involve quantification over the values of a proper *subset* of the introduced variables. Following Rooth and Kadmon, I call this kind of quantification *asymmetric*. We find it in the following sentences:

(17) If a farmer owns a donkey, he is usually rich.

(18) If a DRUMMER lives in an apartment complex, it is usually half empty.

(19) If a drummer lives in an APARTMENT COMPLEX, it is usually half empty.

On its most natural reading, the adverb *usually* in the first example quantifies over farmers who own a donkey and not over farmer-donkey pairs. The sentence says

18. Some features of the adverbially quantified sentence and of the indefinite donkey conjunction are neglected here. Intuitively, an adverbially quantified sentence $Sometimes(\phi)(\psi)$ requires more than one of the cases that verify ϕ to satisfy ψ, a plurality condition that is absent from the (singular) donkey conjunction. On the other hand, such a singular donkey conjunction may have a 'specific' flavour, since it may be taken to talk about a specific farmer and donkey known to the speaker. The adverbially quantified sentence lacks such a reading. See Dekker [1993] for some suggestions for dealing with specificty in the *EDPL* framework. Plurals fall beyond the scope of this paper.

that most farmers who own a donkey are rich. (If the adverb is taken to quantify unselectively, we get the different reading that for most pairs consisting of a farmer and a donkey he owns, it holds that the farmer is rich.) The second and third example are different for a similar reason. In the second example, with focal stress on *drummer*, we (may) find quantification over apartment complexes in which a drummer houses. The example then states that most apartment complexes where a drummer lives are usually half-empty. In the third example, where we find focal stress on *apartment complex*, the adverb may be taken to quantify over drummers. On this reading, the sentence says that most drummers that live in an apartment complex live in an half empty apartment complex.

Examples like these pose a problem for the *DRT* and *DPL* analysis of adverbs of quantification, a problem dubbed the 'proportion problem' by Kadmon [1987]. Of course, it is easy to *annul* the introduction of certain variables in a restriction and to quantify over the values of the remaining variables. So, the sentence *If a man owns a donkey he is usually rich* can be (successfully) interpreted by means of the translation $Usually(\exists x(Mx \wedge \downarrow\exists y(Dy \wedge Oxy)))(Rx)$. However, this approach runs into problems when a pronoun in the nuclear scope is anaphorically related to an indefinite description in the restrictive clause that does not participate in the adverbial quantification. For instance, if we want the example *If a drummer lives in an* APARTMENT COMPLEX, *it is usually half empty* to quantify over drummers, and if, therefore, the interpretation of the restriction only yields assignments varying with respect to the drummers, then the pronoun *it* in the nuclear scope remains unbound.

Of course, in case of asymmetric quantification we don't really want to *annul* the introduction of certain variables in the restriction of a quantifying adverb. When the values of other variables are selected for quantification, we just want the adverb to *neglect* the (possibly different) values of the other ones. For this reason, Root [1986] suggests that asymmetric adverbial quantifiers do not discriminate between assignments that only differ from one another in their assignment to unselected variables. This implies that asymmetric quantifiers in fact quantify over equivalence classes of assignments, each one of which consists of assignments that agree on the values of the selected variables. For instance, in the example *If a drummer lives in an* APARTMENT COMPLEX, *it is usually half empty*, the adverb *usually* selects (the variable associated with) *a drummer*, and in effect quantifies over a set of sets of cases, each element of which is a set of pairs any first element of which is one and the same drummer and any second element any apartment complex the drummer lives in.

In *EDPL*, asymmetric adverbs more naturally fit into the general scheme of adverbial quantification. As we have seen unselective adverbial quantification is satisfactorily dealt with in *EDPL* in terms of quantification over the extensions of an assignment i in an input state s which are an element of the update of s with the restriction ϕ of an adverb. Clearly, the extensions in the update with the restriction are extensions of i to all variables introduced in the restriction and therefore unselective quantification involves quantification over the values of all these variables. Now, in order to account for asymmetric quantification, we do not consider the full extensions of i which are an element of the update with the restriction, but partial extensions which *survive* in the update with the restriction. An asymmetric quantifying adverb quantifies over extensions of i to the selected variables, viz., those that survive the update with the restriction of the adverb, and it relates this set of extensions to the set of extensions that also survive in further update with the nuclear scope of the adverb.

For instance, consider the asymmetric interpretation of *If a man owns a donkey he is usually rich* in a state s, $s[\![Usually_{\{x\}}(\exists x(Mx \wedge \exists y(Dy \wedge Oxy)))(Bxy)]\!]$. The adverb *usually* is here taken to select the values of x, which are donkey owning men, for quantification. The interpretation of this formula preserves an assignment i in the input state s iff most extensions of i to x that survive in $s[\![\exists x(Mx \wedge \exists y(Dy \wedge Oxy))]\!]$ also survive in $s[\![\exists x(Mx \wedge \exists y(Dy \wedge Oxy))]\!][\![Rx]\!]$. In fact, this preserves all assignments $i \in s$ iff most donkey owning man are rich. Notice that in the present example the nuclear scope Rx of the adverb is interpreted with respect to a state that is not only defined for x, the values of which are quantified over, but also for y, the possibly different values of which do not interfere with the quantification. In other words, we can quantify in *EDPL* over the values of *some* variables introduced in the restriction of an adverb, neglect the valuation of others, without denying the others the ability to serve as antecedents for anaphoric pronouns in the nuclear scope.

So, asymmetric adverbs of quantification are assumed to come with a set of selection indices X the values of which the adverb quantifies over. It is required, again, on pain of undefinedness, that an asymmetric adverbial quantifier A_X effectively quantifies over the values of all the variables in X, so X must be a subset of the variables introduced in the restriction. The interpretation then is defined as follows:

Definition 4.2 (Adverbs of quantification (asymmetric))
If $X \subseteq IV(\phi)$

- $s[\![A_X(\phi)(\psi)]\!] = \{i \in s \mid [A](\{j \mid i \leq_X j \ \&\ j < s[\![\phi]\!]\})(\{j \mid j < s[\![\phi]\!][\![\psi]\!]\})\}$

As for an illustration, let us briefly consider two mutually related examples.

(20) If a man gives her a PRESENT, she usually thanks him for it
$Usually_{\{y\}}(\exists y(My \wedge \exists z(Pz \wedge Gyzx)))(Txyz)$

This sentence requires of an assignment i in a state of evaluation s that most extensions j of i with a valuation for y such that $j(y)$ gives $i(x)$ a present, are also valuations such that $i(x)$ thanks $j(y)$ for a present $j(y)$ gives to her. So, interpreted in a state s, this example preserves all those i in s that assign x an individual that renders thanks to most men that give her a present, irrespective of the number of presents given.

(21) If a MAN gives her a present, she usually thanks him for it
$Usually_{\{z\}}(\exists y(My \wedge \exists z(Pz \wedge Gyzx)))(Txyz)$

When interpreted in a state s, this example preserves all those i in s that assign x an individual that renders thanks for most presents given by a man, irrespective of the number of men that give it.

As is to be expected, unselective quantification is a borderline case of asymmetric quantification:

Fact 4.2

- $A_{IV(\phi)}(\phi)(\psi) \Leftrightarrow A(\phi)(\psi)$

Furthermore, we find the following equivalences:

Fact 4.3
If $X \subseteq IV(\phi)$

- $Sometimes_X(\phi)(\psi) \Leftrightarrow Sometimes(\phi)(\psi)$
- $Never_X(\phi)(\psi) \quad \Leftrightarrow Never(\phi)(\psi)$

and

- $Always_\emptyset(\phi)(\psi) \quad \Leftrightarrow (\phi \hookrightarrow \psi)$

So, for the adverbs *sometimes* and *never* it makes no difference whether or not they quantify over a selection of the variables introduced in their restriction. This is as it should be, since there seems to be no evidence that there are distinct asymmetric readings of these adverbs.

On the other hand, for the adverbs *usually* and *always*, it does make a difference whether or not they select variables for asymmetric quantification, and which variables they select. Furthermore, we see that the weak implication ($\hookrightarrow$) addressed in the digression of section 2.3, now appears to be a borderline case of asymmetric adverbial quantification, i.e., universal quantification over the values of an empty set of selection indices. It appears that this weak implication does not constitute a really different notion of implication of its own, but that it fits into the more general scheme of adverbial quantification as one of the many forms of asymmetric quantification.

The claim about weak implication can be further substantiated by slightly varying the dime implication again:

(22) If I have a dime in my pocket, I throw it in the parking meter.
$(\exists y(Dy \wedge Piy) \hookrightarrow Tiy) \Leftrightarrow Always_\emptyset(\exists y(Dy \wedge Piy))(Tiy)$

(23) If a man has a dime in his pocket, he throws it in the parking meter.
$Always_{\{x\}}(\exists x(Mx \wedge \exists y(Dy \wedge Pxy)))(Txy)$

On its most natural reading, the first example was argued to state that if I have a dime in my pocket, then I throw one in the meter, and this reading is captured by interpreting the conditional sentence as one of weak implication (or of universal quantification over the values of an empty set of selection indices). However, the second example, which is a minor variation of the first, is most likely interpreted as stating that every man who has a dime, throws one in the meter.[19] Neither the weak, nor the strong, reading of the implication gives us this. In fact, this example has a mixed strong and weak reading: strong with respect to the men who are faced with a parking meter and who have a dime in their pocket; weak with respect to the number of dimes they throw in. Now, if the weak implication is supposed to constitute a form of implication of its own and if it is used to account for the first example, we still need an explanation of this second, related, example, which, on the preferred reading, does not exemplify a purely weak or strong implication. If, on the other hand, we fit both these sentences into the scheme of asymmetric adverbial

19. Of course, these sentences will have to be understood as restricted to men faced with a parking meter they are obliged to throw a dime in. Furthermore, the analysis will have to be supplemented with a proper interpretation of the definite noun phrase *the parking meter*. However, these two issues do not concern us here. Relevant in the present discussion is the different quantificational behaviour of the two indefinites in the dime implications.

quantification, both get assigned proper readings in a uniform way.

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We have seen that symmetric readings of adverbially quantified sentences are easily accounted for in terms of assignments, and Root has proposed an effective way to deal with asymmetric readings by shifting from a domain of assignments to a domain of equivalence classes of assignments.[20] In the update approach of *EDPL* it appears, furthermore, that asymmetric quantifiers do not even require us to resort to a different level of objects (classes of assignments instead of assignments simpliciter). The Rootian equivalence classes of assignments can be taken to be partial assignments, which are the kind of entities in terms of which the semantics of *EDPL* is defined from the start.

4.3 Adnominal quantification

EDPL is also easily extended with adnominal (binary) quantifiers. Basically, the treatment of these quantifiers offered here is that of Chierchia [1992], also proposed in van Eijck and de Vries [1992]. As is the case with the adverbial quantifiers, the adnominal quantifiers are interpreted as 'internally dynamic', i.e., their analysis accounts for anaphoric relationships between indefinites occurring in the restriction of such quantifiers and anaphoric pronouns in their nuclear scope. I will not present an analysis of the external dynamics of such quantified structures, that is, the anaphoric relationships that may obtain between the noun phrases in quantified structures and pronouns in successive sentences. For a treatment of the external dynamics of adnominal quantifiers we need to extend *EDPL* with plural noun phrases and plural pronouns, and such an enterprise falls beyond the scope of the present paper. (Cf., for instance, van den Berg [1990] for a *DPL*-style treatment of the dynamics of plurals.)

Internally dynamic adnominal quantifiers neatly fit in the general scheme of quantification in *EDPL*. Let D be a binary quantifier which has $[D]$ as its usual set-theoretic interpretation, then:

Definition 4.3 (Binary quantifiers)
If $x \notin D(s)$
- $s[\![Dx(\phi)(\psi)]\!] = \{i \in s \mid [D](\{j \mid i \leq_{\{x\}} j \;\&\; j \lessdot s[x][\![\phi]\!]\})(\{j \mid j \lessdot s[x][\![\phi]\!][\![\psi]\!]\})\}$

On the present definition of adnominal quantification, a binary quantifier Dx quantifies over the possible valuations of a single variable x and, hence, over the individuals in D (that is, if we again assume that the quantifier satisfies the constraints of extension, quantity and conservativity, cf., the remarks in section 4.1).

Let us consider one example:

(24) Most men who gave her a present had packed it up
$Most\, y(\exists z(Pz \wedge Gyzx)(Uyz)$

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20. Instead of Lewisian cases, 'situations' (or 'occasions', or 'events') have been argued to be needed in order to deal with symmetric and asymmetric readings of sentences of the donkey and dime variety (Berman, Kadmon, Chierchia, Heim, van Eijck and de Vries, among others). In [1993, Ch. 5] (section 3.4) I have argued that these situation based analyses neither refute, nor improve upon the Lewisian analysis adopted above.

The interpretation of this example in a state s yields a state consisting of assignments $i \in s$ such that most extensions of i to y under which the value of y is a man who gave a present to the value of x, are extensions under which the value of y has packed up a present he gave to the value of x. Put more simple, the interpretation of this example in a state s preserves those assignments i in s which assign an individual z to x such that most men who gave a present to z gave her a present they had packed up.

Like unselective quantification, adnominal quantification is a form of asymmetric quantification. Adnominal quantification corresponds to asymmetric quantification over a singleton set of selection indicies. So, suppose that adverb A and determiner D have the same set-theoretical interpretation. Then:

Fact 4.4

- $A_{\{x\}}(\exists x\phi)(\psi) \Leftrightarrow \; Dx(\phi)(\psi)$

For instance, consider the sentence *If a man owns a donkey, he is usually rich* on its asymmetric construal in which *usually* selects donkey owning men for quantification. The translation of the sentence is $(Usually_{\{x\}}(\exists x(Mx \wedge \exists y(Dy \wedge Oxy)))(Rx))$, and, under this translation, the sentence is equivalent with the sentence *Most men that own a donkey are rich*, translated as $Most\, x(Mx \wedge \exists y(Dy \wedge Oxy))(Rx)$.

We find the following correspondences between binary and unary quantifiers in *EDPL*:

Fact 4.5

- $An\, x(\phi)(\psi) \quad \Leftrightarrow \downarrow\exists x(\phi \wedge \psi)$
- $No\, x(\phi)(\psi) \quad \Leftrightarrow \neg\exists x(\phi \wedge \psi)$
- $Every\, x(\phi)(\psi) \Leftrightarrow \forall x(\phi \hookrightarrow \psi)$

We see that the binary determiners *a(n)* and *no* have the same truth-conditional content as their usual first order paraphrases. Moreover, observe that *EDPL* licenses a weak and a strong reading of the quantifier *every*, both of which are intuitively motivated. If we treat *every* as a binary quantifier, the weak reading results. This reading is appropriate for the sentence *Every man who has a dime puts in the parking meter*. On the other hand, if we translate *every* by means of the first order universal quantifier, as $\forall x(\phi \rightarrow \psi)$, then the strong reading results, and this gives the proper reading of the (strong) donkey sentence.

Of course, the ambivalence of *every* also shows in its negation *not_every*. In case *every* is read strongly, a quantified structure *Not every A B* translates as $\neg\forall x(A'x \rightarrow B'x) \Leftrightarrow \downarrow\exists x(A'x \wedge \neg B'x)$. This seems appropriate for the negation of the strong donkey sentence. The sentence *Not every man who owns a donkey beats it* has the truth conditions that there is a man who owns a donkey which he does not beat. On the other hand, if *every* is read weakly, as a binary determiner, then *Not every A B* translates as $Not\_every\, x(A'x)(B'x)$, which is equivalent with $\downarrow\exists x(\downarrow A'x \wedge (A'x \rightarrow \neg B'x))$. This seems appropriate for the negation of the weak dime sentence. The sentence *Not every man who has a dime throws it in the meter* then has the truth conditions that there is a man who has a dime and who does not throw any dime he has in the meter.

Infelicitous indices

As was stated at the start of this section, the present analysis of adnominal quantification is, basically, that of Chierchia and van Eijck and de Vries. The most significant difference with these two approaches is that *EDPL* explicitly expels the reinstantiation of variables which are already in use. It can be argued that, in particular, the treatment of quantifiers in Chierchia and van Eijck and de Vries calls for such a novelty constraint on indices (which, by the way, is also Chierchia's conclusion).

Consider the following example, in which an indefinite in the restriction of a determiner is coindexed with the determiner:

(25) Every$_x$ man that has a$_y$ kid who owns a donkey$_x$ is happy.
$Every\,x(Mx \wedge \exists y(Ky \wedge \exists x(Dx \wedge Oyx) \wedge Hxy))(Hx)$

Under this translation, the head determiner *every* has a restriction $Mx \wedge \exists y(Ky \wedge \exists x(Dx \wedge Oyx) \wedge Hxy)$ in which, in a *DPL*-style system, the quantifier $\exists x$ associated with the noun phrase *a donkey* binds the variable x in Hxy. So, if reinstantiation is allowed, the head determiner *every* quantifies, not over men that have a kid who owns a donkey, but over men such that there is a kid who owns a donkey which has the kid, i.e., over men such that there is a donkey which has a kid who owns it.

Furthermore, also the occurrence of x in the nuclear scope Hx gets bound by the quantifier $\exists x$. The reading that results in a non-eliminative system is that, for every man, if there is a donkey which has a kid that owns it, then there is a *happy* donkey which has a kid that owns it. Clearly, this is not at all a sensible reading of the original example *Every man that has a kid who owns a donkey is happy*, but if we assume a standard translation procedure and a free indexing mechanism it is predicted to be one.

Unwanted effects of reinstantiation can also be illustrated with the following example from van Eijck and Cepparello [1992]

(26) Someone hasn't escaped from the fire. *Everyone may have escaped.

Van Eijck and Cepparello observe that this example is strange since, when one has heard that someone did not escape from the fire, she is definitely someone of which it does not hold that she might have escaped. So, on their analysis, the first sentence of example 26 introduces a discourse referent, indicated by a variable x, who has not escaped from the fire and this (partial) object serves to refute the claim that everyone may have escaped, since x might not.

However, if reinstantiation is allowed, this observation does not hold unconditionally. Without going into details, it is easily seen that if *everyone* carries the same index (x) as *someone*, then the evaluation of the second sentence takes place after reinstantiating the value of x, i.e., after throwing away all information we have about the value of x. As a consequence, the witness (x) which should serve to refute the second sentence, has vanished, and the example turns out alright.

The point here is, of course, that if the values of variables introduced in a discourse should count as (partial) objects, relevant for the evaluation of epistemic statements as in the example above, then it should not be allowed that they are discarded, by a blind indexing process. *EDPL* merely restricts this process, by not allowing such reinstantiation to take place, that is, at least, not without motivation.

5 Further developments

In this paper I have argued for the reformulation of dynamic predicate logic into an update semantics. Such a reformulated system, *EDPL*, has been presented and its logical properties have been studied. I have shown that the update formulation not only allows for a fairly perspicuous definition of negation and truth and a uniform definition of (adverbial and adnominal) quantification. With the reformulated system we can also give a more precise characterization of the dynamics at issue, since, in *EDPL*, the dynamics resides in the order and introduction of discourse referents, i.e., in the definedness conditions. To conclude this paper, I will shortly address some issues related to the integration of *EDPL* with Veltman's update semantics. These issues have been discussed more thoroughly in Dekker [1993, Ch. 5].

The notion of information employed in *EDPL* is rather 'flat'. It is concerned with the possible values of variables in ordinary extensional models, and it does not model information about the world. However, this limitation can easily be removed. In Dekker [1993] I have shown how *EDPL* can be turned into a modal dynamic predicate logic, *MDPL*. In *MDPL*, (update of) information about the values of variables and (update of) information about the world go hand in hand. Thus, the notion of information about the values of variables is intensionalized. The information one has about the values of variables in *MDPL* is that they have certain properties (functions from possible worlds to sets of individuals), and that they stand in certain 'relations in intension' (functions from possible worlds to relation extensions).

Having introduced information about the world in *MDPL*, Veltman's epistemic operator $\Diamond$ (*might*) can be properly introduced. The adoption of Veltman's definition of this operator allows us to express the partiality of our information about the values of variables. Consider the following examples, which have already been discussed above:

(1) Somebody is knocking at the door. ... It might be John. ... It's Mary.
$\exists x Kx \wedge \Diamond(x = j) \wedge x = m$

(2) Somebody is knocking at the door. ... It's Mary. ... *It might be John.
$\exists x Kx \wedge x = m \wedge \Diamond(x = j)$

In *MDPL*, the first sentence of both examples presents an object which is associated with the property of knocking at the door. Without information to the contrary, we can record that, as far as we know, this object might be John, even though we later may get informed that it is Mary. On the other hand, as soon as we are informed that it is Mary, we can not say that it might be John, of course (that is, assuming that we know that Mary is not John). The two examples are more fully analyzed in *MDPL* then in *US*, since *MDPL* also accounts for the anaphoric relationships involved.

It must be pointed out, however, that the introduction of the $\Diamond$ operator is not unproblematic. In the first place, and despite first appearances, it is not clear what sense the operator makes, intuitively speaking. Update semantics, and *EDPL*, model the update of information that results from interpreting sentences, and, apparently, both are entirely hearer, or recipient, oriented. Under this perspective, an utterance of $\Diamond\phi$ tells the hearer that *his* state of information can be consistently updated with ϕ and the hearer can only accept this (if true) or reject it (if false). But this does not seem to correspond to any intuitive meaning of *might*. A sentence *It might be the case that such and so* may be used to express partiality of information, but then it refers to the speaker's information in the first place. The utterance of such a sentence does not involve a statement about the state of the hearer. (Except, of course, if the two states coincide; for this reason, the two examples above can be quite naturally interpreted as involving someone's private reflection about (incoming) information.)

In Dekker [1993] I have addressed this issue, and I have argued that one can make more intuitive sense of the *might* operator by embedding the update semantic framework within a more general model of information exchange. There, two restrictions are imposed upon what is called 'proper information exchange'. Roughly speaking, an exchange of information by the utterance of ϕ is called proper if ϕ is true in the speaker's state (i.e., if the speaker has the information that ϕ), and if the update of the hearer's state with ϕ does not produce an absurd information state (i.e., if ϕ is not false in the hearer's state). The first restricton, apparently, restates Grice's maxims of quality. The second restriction is a side condition motivated by the observation that (the resolution of) conflicting information can not properly be dealt with as long as we stick to the exchange of pure, factual information about the world and about the values of variables.

Now, adopting these two restrictions, it is easily shown that an utterance of $\Diamond\phi$ (or $\Box\phi$, *It must be the case that* ϕ) implies a symmetric test of the speaker's and hearer's state. In the exchange model sketched, a proper use of $\Diamond\phi$ ($\Box\phi$) turns out to test whether both speaker and hearer agree on the possibility (truth) of the proposition expressed by ϕ.[21] In other words, in a situation of information exchange, the operators $\Diamond$ and $\Box$ test the common ground of speaker and hearer. Obviously, this does more justice to the intuitive interpretation of the operators.

Still, there are more questions to be answered when we introduce epistemic operators like $\Diamond$, also when they are phrased within a more general theory of information exchange. A main issue is the occurrence of epistemic operators in embedded positions, that is, as operators which are not the main operator of a sentence in a sequence of sentences constituting a discourse. In natural language such occurrences seem to be allowed, as, for instance, in *If John comes, then Mary might come* and *Everybody might have robbed the bank*. (Of course, the correct analysis of such sentences may constitute an issue. It must be noticed, furthermore, that the embedded use of modals is explicitly excluded in Veltman's system, because they threaten idempotence. The point to be made here is that there are other reasons for, at least, being cautious with allowing embedded modals.)

Assume, again, the assumptions made above about proper information exchange, i.e., that the speaker has the information exchanged, and that the hearer has no information to the contrary. Then it can be shown that, as long as modals do not occur in embedded positions, information exchange never allows 'jumping to conclusions', i.e., that, by exchanging information, the speaker and hearer can not obtain information which they together did not have before the exchange. If the modals are allowed to occur in embedded positions then, both in Veltman's system and in mine, speaker and hearer can obtain information which is not licensed by the

21. It must be noted, however, that it is assumed here that ϕ does not contain embedded occurences of epistemic operators, cf., below.

information they had before the exchange.

Consider the following example (the question whether the below translation gives a proper analysis is left aside):

(27) If John comes then Mary might come.
$p \rightarrow \Diamond q$

Suppose that in the information product of the speaker's and hearer's state it is still possible that John comes, that the speaker conceives it possible that John and Mary come, and that the hearer has the information that if John comes, then Mary doesn't. In that case, the speaker is allowed to utter sentence 27, since $p \rightarrow \Diamond q$ is true in his state, and the hearer has no information falsifying the sentence. So, according to the two restrictions stated, the utterance of 27 by the speaker constitutes an example of proper information exchange. However, the update of the hearer's state with example 27 excludes the possibility that John comes. Given his information that not both Mary and John come, he learns from this example that John doesn't come. The troubling fact about this example is that, without further restrictions, it calls this exchange of information proper, although it allows the hearer to jump to a conclusion which is not licensed by information he and the speaker had before. Clearly, if embedded modals are allowed, the framework has to be modified in order to give a proper account of examples like the one above. Dekker [1993] contains a suggestion to improve upon the situation.

A final issue, which I will only mention here, concerns the interaction between modal operators and quantifiers, one of the 'harder' issues also in (static) modal predicate logic. Apparently, one wants to be able to express partial information about (some, all, most, ...) objects in a certain domain of discussion, as with *Everybody might have robbed the bank*. Examples like this, too, raise the question whether we have to take the *might* operator here to reflect upon the speaker's state, the hearer's state, upon both, or upon their common ground. However, they furthermore raise the question what domain of discussion such sentences must be taken to reflect upon.

Of course, we can understand such sentences as quantifying over the domain of objects in the model, possibly restricted by contextual factors. Proceeding along such a line would even more severely force us to take a definite stand in the issues of the (non-)rigidity of proper names, the (in-)dependence of domains on worlds, cross-world identity, and so on. On the other hand, and more in line with the approach taken in this paper, we can also interpret sentences like the one above as quantifying over objects introduced in the course of a discourse. An approach along the latter line can be promising, since it may shed new light on context dependent quantification, and since it may enable a dynamic formulation of the, seemingly similar, notion of situation based quantification of Kratzer [1989]. Since it would go way beyond the purposes of this paper to further reflect upon such elaborations, this subject is left for further research.

Appendix

In the proof of some facts in this paper, the following lemma plays a central role ($i \approx j$ says that for all x such that $i(x)$ and $j(x)$ are defined $i(x) = j(x)$):

Lemma 1

- If $i \approx j$ then $\forall k \in \{i\}[\![\phi]\!]\ \exists! l \in \{j\}[\![\phi]\!]$: $k \approx l$, if defined

This lemma is proved by induction on the complexity of ϕ. The proof is simplified by treating existentially quantified formulas as conjunctions: $\exists x\phi = \exists x \wedge \phi$ where $s[\![\exists x]\!]$ is $s[x]$ if $x \notin D(s)$. Atomic formulas and existential quantifiers then constitute the basic cases of the induction.

1. For atomic formulas the proof of the lemma is straightforward.
2. Suppose that $i \approx j$, that $\{i\}[\![\exists x]\!]$, $\{j\}[\![\exists x]\!]$ are defined and that $k \in \{i\}[\![\exists x]\!]$, i.e., $i \leq_{\{x\}} k$. Clearly, there is an assignment l in $\{j\}[\![\exists x]\!]$ (i.e., $j \leq_{\{x\}} l$) such that $k(x) = l(x)$. Since $i \approx j$, $i \leq_{\{x\}} k$, $j \leq_{\{x\}} l$ and $k(x) = l(x)$, we find that $k \approx l$. Now suppose that an assignment l' is in $\{j\}[\![\exists x]\!]$ and $k \approx l'$. Then $j \leq_{\{x\}} l'$ and $k(x) = l'(x)$ and, hence, $l' = l$.
3. Suppose that $i \approx j$, that $\{i\}[\![\neg\phi]\!], \{j\}[\![\neg\phi]\!]$ are defined and that $k \in \{i\}[\![\neg\phi]\!]$, i.e., $k = i$ and $\{i\}[\![\phi]\!] = \emptyset$. Using the induction hypothesis, $\{j\}[\![\phi]\!] = \emptyset$. So, $j \in \{j\}[\![\neg\phi]\!]$, and, since $i \approx j$ and $k = i$, $k \approx j$. Now suppose $j' \in \{j\}[\![\neg\phi]\!]$, then, by definition, $j' = j$.
4. Suppose that $i \approx j$, that $\{i\}[\![\phi \wedge \psi]\!], \{j\}[\![\phi \wedge \psi]\!]$ are defined and that $k \in \{i\}[\![\phi \wedge \psi]\!] = \{i\}[\![\phi]\!][\![\psi]\!]$. By distributivity there is an assignment f in $\{i\}[\![\phi]\!]$ such that $k \in \{f\}[\![\psi]\!]$. By induction there is an assignment g in $\{j\}[\![\phi]\!]$ such that $g \approx f$, and, again by induction, there is an assignment $l \in \{g\}[\![\psi]\!]$ such that $k \approx l$. By distributivity again, such an $l \approx k$ is in $\{j\}[\![\phi]\!][\![\psi]\!] = \{j\}[\![\phi \wedge \psi]\!]$. Now suppose that an assignment $l' \approx k$ is in $\{j\}[\![\phi]\!][\![\psi]\!]$. Then there is an assignment $g' \in \{j\}[\![\phi]\!]$ such that $l' \in \{g'\}[\![\psi]\!]$. Since (by update) $f \leq k$, $g \leq l$ and $g' \leq l'$, and since $l \approx k$ and $l' \approx k$, we find that $f \approx g$ and $f \approx g'$. Since $i \approx j$, by induction on ϕ, $g = g'$, and, by induction on ψ, $l = l'$.

A fifth clause, dealing with asymmetric adverbs, has to be added in order to prove lemma 1 in the system of *EDPL* with quantifiers. (Unselective adverbial quantification and adnominal quantification can be subsumed under asymmetric quantification.) I employ the following abbreviatons:

$s_i = \{k \mid i \leq_X k\ \&\ k < \{i\}[\![\phi]\!]\}$,
$t_i = \{k \mid i \leq_X k\ \&\ k < \{i\}[\![\phi]\!][\![\psi]\!]\}$,
$s_j = \{l \mid j \leq_X l\ \&\ l < \{j\}[\![\phi]\!]\}$ and
$t_j = \{l \mid j \leq_X l\ \&\ l < \{j\}[\![\phi]\!][\![\psi]\!]\}$.

Notice that if, by the induction hypothesis, lemma 1 holds for $[\![\phi]\!]$ and $[\![\psi]\!]$, then there is a bijection from s_i to s_j, the restriction of which to t_i is a bijection to t_j.

5. Suppose that $i \approx j$, that $\{i\}[\![A_X(\phi)(\psi)]\!]$ and $\{j\}[\![A_X(\phi)(\psi)]\!]$ are defined and that $f \in \{i\}[\![A_X(\phi)(\psi)]\!]$, i.e., $f = i$ and $i \in \{i\}[\![A_X(\phi)(\psi)]\!]$. Since $[A]$ is conservative $[A](s_i)(t_i)$ is true, and since $[A]$ satisfies extension $[A]_{s_i}(s_i)(t_i)$ is true. By induction, there is a bijection from s_i (and t_i) to s_j (and t_j), so, since $[A]$ satisfies quantity, $[A]_{s_j}(s_j)(t_j)$ is true. By extension again we have that $[A](s_j)(t_j)$ is true, and, using conservativity, $j \in \{j\}[\![A_X(\phi)(\psi)]\!]$. Since $f = i$, $f \approx j$. Furthermore, for any $j' \in \{j\}[\![A_X(\phi)(\psi)]\!]$, by definition, $j' = j$.

Fact 3.9

- If $\phi_1, \ldots \phi_n \models_X \psi$, then for all s: $s[\![\phi_1]\!] \ldots [\![\phi_n]\!] \models \psi$, if defined

For expository reasons I show the proof of this fact for single premise entailments only. Suppose (i) $\phi \models_X \psi$, (ii) $s[\![\phi]\!][\![\psi]\!]$ is defined, and (iii) $s[\![\phi]\!] \not\models \psi$. By assumptions (ii) and (iii) there is an assignment j in $s[\![\phi]\!]$ such that $j \not< s[\![\phi]\!][\![\psi]\!]$ and (by update and distributivity) there is an assignment i in s such that $j \in \{i\}[\![\phi]\!]$ and $\{j\}[\![\psi]\!] = \emptyset$. Now take an assignment i' in D^X such that $i \approx i'$. By assumption (i), $\{i'\}[\![\phi]\!][\![\psi]\!]$ is defined. Using lemma 1 we find that there is an assignment j' in $\{i'\}[\![\phi]\!]$ such that $j \approx j'$, and, using lemma 1 again, that $\{j'\}[\![\psi]\!] = \emptyset$. But then $\{i'\}[\![\phi]\!] \not\models \psi$, which contradicts assumption (i). So, by contraposition, if $\phi \models_X \psi$ and $s[\![\phi]\!][\![\psi]\!]$ is defined, then $s[\![\phi]\!] \models \psi$.

Fact 3.10

- $\phi_1, \ldots, \phi_n \models \psi$ iff $\phi_1, \ldots, \phi_{n-1} \models \phi_n \rightarrow \psi$

This fact is proved in two steps.
(i) For any s, $s[\![\phi_1]\!] \ldots [\![\phi_n]\!] \models \psi$ iff (by fact 2.4) $\forall j \in s[\![\phi_1]\!] \ldots [\![\phi_n]\!]$: $\{j\} \models \psi$ iff (fact 2.3) $\forall i \in s[\![\phi_1]\!] \ldots [\![\phi_{n-1}]\!]$ $\forall j \in \{i\}[\![\phi_n]\!]$: $\{j\} \models \psi$ iff (by definition of $\rightarrow$) $\forall i \in s[\![\phi_1]\!] \ldots [\![\phi_{n-1}]\!]$: $\{i\} \models (\phi_n \rightarrow \psi)$ iff (fact 2.4) $s[\![\phi_1]\!] \ldots [\![\phi_{n-1}]\!] \models (\phi_n \rightarrow \psi)$.
(ii) $\phi_1, \ldots, \phi_n \models \psi$ iff $\exists X$: $\phi_1, \ldots, \phi_n \models_X \psi$ iff $\exists X$: $\forall s \in S^X$: $s[\![\phi_1]\!] \ldots [\![\phi_n]\!] \models \psi$ iff (by (i)) $\exists X$: $\forall s \in S^X$: $s[\![\phi_1]\!] \ldots [\![\phi_{n-1}]\!] \models (\phi_n \rightarrow \psi)$ iff $\exists X$: $\phi_1, \ldots, \phi_{n-1} \models_X (\phi_n \rightarrow \psi)$ iff $\phi_1, \ldots, \phi_{n-1} \models (\phi_n \rightarrow \psi)$.

Fact 3.11 is proved by employing the following lemma:

Lemma 2
If $\{i\}[\![\phi]\!]$ is defined and g is a total extension of i, then

- $\{i\} \models_{M,edpl} \phi$ iff $\{g\} \models_{M,dpl} \phi$

This lemma is proved by induction on the complexitity of the normal binding form of ϕ (the normal binding form of a formula ϕ is obtained by substituting all occurences of subformulas of the form $(\exists x\phi_1 \wedge \phi_2)$ and $((\phi_1 \wedge \phi_2) \wedge \phi_3)$ in ϕ by $(\exists x(\phi_1 \wedge \phi_2))$ and $(\phi_1 \wedge (\phi_2 \wedge \phi_3))$ respectively):

1. If $\{i\}[\![Rx_1 \ldots x_n]\!]$ is defined and $g \geq_t i$, $\{i\} \models_{M,edpl} Rx_1 \ldots x_n$ iff $\{g\} \models_{M,dpl} Rx_1 \ldots x_n$, since $i(x_1) = g(x_1)$, ..., $i(x_n) = g(x_n)$ and $F_{edpl}(R) = F_{dpl}(R)$.
2. If $\{i\}[\![\neg\phi]\!]$ is defined and $g \geq_t i$, then $\{i\}[\![\phi]\!]$ is defined and $\{i\} \models_{edpl} \neg\phi$ iff $\{i\} \dashv_{edpl} \phi$ iff (by induction) $\{g\} \dashv_{dpl} \phi$ iff $\{g\} \models_{dpl} \neg\phi$.
3. If $\{i\}[\![\exists x\phi]\!]$ is defined and $g \geq_t i$, then $\forall j \geq_{\{x\}} i$: $\{j\}[\![\phi]\!]$ is defined. Hence, $\{i\} \models_{edpl} \exists x\phi$ iff $\exists j \geq_{\{x\}} i$: $\{j\} \models_{edpl} \phi$ iff $\exists h[x]g$ $\exists j$: $h \geq_t j \geq_{\{x\}} i$ and $\{j\} \models_{edpl} \phi$ iff (by induction) $\exists h[x]g$: $\{h\} \models_{dpl} \phi$ iff $\{g\} \models_{dpl} \exists x\phi$.
4. If $\{i\}[\![\phi \wedge \psi]\!]$ is defined and $g \geq_t i$, then (since ϕ is a test) $\{i\}[\![\phi]\!]$ and $\{i\}[\![\psi]\!]$ are defined and $\{i\} \models_{edpl} \phi \wedge \psi$ iff $\{i\} \models_{edpl} \phi$ and $\{i\} \models_{edpl} \psi$ iff (by induction) $\{g\} \models_{dpl} \phi$ and $\{g\} \models_{dpl} \psi$ iff $\{g\} \models_{dpl} \phi \wedge \psi$.

Fact 3.11
If $t \in S^V$ and $X, D(s) \in \backslash\phi\backslash$, then

1. $t \models_{M,dpl} \phi$ iff $t^X \models_{M,edpl} \phi$
2. $s \models_{M,edpl} \phi$ iff $s^V \models_{M,dpl} \phi$

Suppose $t \in S^V$ and $X \in \backslash\phi\backslash$, i.e., $t^X[\![\phi]\!]$ is defined. Then $t \models_{dpl} \phi$ iff (definition of $\models_{dpl}$) $\{g\} \models_{dpl} \phi$ for all g in t iff (lemma 2) $\{i\} \models_{edpl} \phi$ for all i in t^X iff (distributivity) $t^X \models_{edpl} \phi$. Next, suppose $D(s) \in \backslash\phi\backslash$, i.e., $s[\![\phi]\!]$ is defined. Then $s \models_{edpl} \phi$ iff (distributivity) $\{i\} \models_{edpl} \phi$ for all i in s iff (lemma 2) $\{g\} \models_{dpl} \phi$ for all $g \in s^V$ iff (definition of $\models_{dpl}$) $s^V \models_{dpl} \phi$.

Fact 3.17
If $/\phi/ \cap \backslash\psi\backslash \cap \backslash\chi\backslash \neq \emptyset$ then:

- If $\phi \models \psi$ and $\psi \models \chi$, then $\phi \models \chi$

Suppose (i) $Z \in /\phi/ \cap \backslash\psi\backslash \cap \backslash\chi\backslash$, (ii) $\phi \models \psi$ and (iii) $\psi \models_Y \chi$. Using assumptions (i) and (ii) and fact 3.9, $\phi \models_X \psi$, where X is such that $\langle X, Z\rangle \in |\phi|$. Now consider an assignment k in $s[\![\phi]\!]$, for any $s \in S^X$. Since $\phi \models_X \psi$, $\{k\} \models \psi$. Next consider an assignment k' in D^Y such that $k \approx k'$. By lemma 1 and assumption (iii), $\{k'\} \models \psi$ and for any assignment l in $\{k'\}[\![\psi]\!]$: $\{l\} \models \chi$. Using lemma 1, for any such l: $k \approx l$, hence (by lemma 1) $\{k\} \models \chi$, if defined. Since, by assumption (i), $\{k\}[\![\chi]\!]$ is defined, $\{k\} \models \chi$. So, for any $s \in S^X$ and for any $k \in s[\![\phi]\!]$: $\{k\} \models \chi$, and, hence: $s[\![\phi]\!] \models \chi$. So, $\phi \models_X \chi$, and, consequently, $\phi \models \chi$.

Fact 3.19
If defined,

- $s[\![\phi]\!] = s \wedge [\phi]$

Proof: by induction on the complexity of ϕ. The only non-trivial step in the proof concerns negation. If $\neg\phi$ is defined, by fact 3.18, $FV(\phi) \subseteq D([\phi])$, and $[\neg\phi] = \{i \in D^{FV(\phi)} \mid i \not> [\phi]\} = \{i \in D^{FV(\phi)} \mid i \not> (\{i\} \wedge [\phi])\} =_{ih} \{i \in D^{FV(\phi)} \mid i \not> \{i\}[\![\phi]\!]\} = \{i \in D^{FV(\phi)} \mid \{i\} \dashv \phi\}$. Since, by definedness, $FV(\phi) \subseteq D(s)$, $s \wedge [\neg\phi] = \{i \in s \mid i > [\neg\phi]\} = \{i \in s \mid \exists j \in D^{FV(\phi)}: j \leq i \;\&\; \{j\} \dashv \phi\}$, which, by lemma 1, is $\{i \in s \mid \{i\} \dashv \phi\}$, which, by fact 2.4, is $s[\![\neg\phi]\!]$.

References

Bäuerle, R. & Egli, U., 1985, *Anapher, Nominalphrase und Eselsätze*, Sonderforschungsbericht 99, 105, Konstanz: Universität Konstanz

Benthem, J. van, 1986, *Essays in logical semantics*, Dordrecht: Reidel

Benthem, J. van, 1991, *Language in action*, Amsterdam: North-Holland

Berg, M. van den, 1990, 'A dynamic predicate logic for plurals', in: Stokhof, M. & Torenvliet, L. (eds.), *Proceedings of the seventh Amsterdam colloquium, December 19–22, 1989*, Amsterdam: ITLI, pp. 29–52

Chierchia, G., 1992, 'Anaphora and dynamic binding', *Linguistics and Philosophy* 15, pp. 111–183

Dekker, P., 1993, *Transsentential meditations*, dissertation, University of Amsterdam

Eijck, J. van & Cepparello, G., 1992, *Dynamic modal predicate logic*, Report CS-R9237, Amsterdam: CWI

Eijck, J. van & de Vries, F.-J., 1992, 'Dynamic interpretation and Hoare deduction', *Journal of Logic, Language and Information* 1, pp. 1–44

Groenendijk, J. & Stokhof, M., 1990b, 'Two theories of dynamic semantics', in: Eijck, J. van (ed.), *Logics in AI, European Workshop JELIA '90*, Berlin: Springer Verlag, pp. 55–64

Groenendijk, J. & Stokhof, M., 1991, 'Dynamic predicate logic', *Linguistics and Philosophy* 14, pp. 39–100

Heim, I., 1982, *The semantics of definite and indefinite noun phrases*, dissertation, University of Massachusetts, Amherst

Heim, I., 1983, 'File change semantics and the familiarity theory of definiteness', in: Bäuerle, R., Schwarze, Ch. & Stechow, A. von (eds.), *Meaning, use and interpretation of language*, Berlin: De Gruyter, pp. 164–189

Heim, I., 1990, 'E-type pronouns and donkey anaphora', *Linguistics and Philosophy* 13, pp. 137–178

Kadmon, N., 1987, *On unique and non-unique reference and asymmetric quantification*, dissertation, University of Massachusetts, Amherst

Kadmon, N., 1990, 'Uniqueness', *Linguistics and Philosophy* 13, pp. 273–324

Kamp, H., 1981, 'A theory of truth and semantic representation', in: Groenendijk, J., Janssen, T. & Stokhof, M. (eds.), *Formal methods in the study of language*, Amsterdam: Mathematical Centre, pp. 277–322 [reprinted in: Groenendijk, J., Janssen, T. & Stokhof, M. (eds.), 1984, *Truth, interpretation and information*, Dordrecht: Foris, pp. 2–41]

Kamp, H., 1990, 'Comments on: J. Groenendijk and M. Stokhof: Dynamic Predicate Logic', in: van Benthem, J. (ed.), *Partial and Dynamic Semantics I*, DYANA report R2.1.A, Edinburgh: Centre for Cognitive Science, University of Edinburgh, pp. 109–131]

Karttunen, L., 1968a, *What do referential indices refer to?*, paper P-3854, Santa Monica, California: The RAND Corporation

Karttunen, L., 1968b, *What makes definite noun phrases definite?*, paper P-3871, Santa Monica, California: The RAND Corporation

Kratzer, A., 1989, 'An investigation of the lumps of thought', *Linguistics and Philosophy* 12, pp. 607–653

Lewis, D., 1975, 'Adverbs of quantification', in: Keenan, E. (ed.), *Formal semantics of natural language*, Cambridge: UP, pp. 3–15

Pelletier, F.J. & Schubert, L., 1988, 'Generically speaking', in: Chierchia, G., Partee, B. & Turner, R. (eds.), *Properties, types and meaning II*, Dordrecht: Reidel, pp. 193–268

Root, R., 1986, *The semantics of anaphora in discourse*, dissertation, University of Texas, Austin

Rooth, M., 1987, 'Noun phrase interpretation in Montague grammar, file change semantics, and situation semantics', in: Gärdenfors, P. (ed.), *Generalized quantifiers*, Dordrecht: Reidel, pp. 237–269

Stalnaker, R., 1974, 'Pragmatic presuppositions', in: Munitz, M. & Unger, P. (eds.), *Semantics and philosophy*, New York: University Press, pp. 197–213

Veltman, F., 1990, 'Defaults in update semantics', in: Kamp, H. (ed.), *Conditionals, defaults and belief revision*, DYANA report R2.5.A, Edinburgh: Centre for Cognitive Science, University of Edinburgh, pp. 28–63

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