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INFORMATION ACQUISITION FROM MULTI-AGENT RESOURCES; ABSTRACT

Zhisheng Huang and Peter van Emde Boas

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Institute for Logic, Language and Computation
Plantage Muidergracht 24
1018TV Amsterdam
Telephone 020-525.6051, Fax: 020-525.5101

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Zhisheng Huang
Center for Computer Science in Organization and Mangament (CCSOM)
University of Amsterdam

Peter van Emde Boas
Department of Mathematics and Computer Science
University of Amsterdam

ILLC Prepublications
for Logic, Semantics and Philosophy of Language
ISSN 0928-3307

Coordinating editor: Dick de Jongh

received August 1993

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Zhisheng Huang* and Peter van Emde Boas**

*Center for Computer Science in Organization and Management(CCSOM)
University of Amsterdam
**ILLC, Depts. of Math. and Comp. Sci., FWI
University of Amsterdam
peter@fwi.uva.nl

August 26, 1993

Abstract

Rational agents, information systems and knowledge bases all share the property that they may gain efficiency by combining information from multiple sources. However, as clearly indicated by the notorious "Judge puzzle" proposed by W. Schoenmakers in 1986, combining information from several sources is a dangerous operation. The resulting database may turn out to be inconsistent, but even worse, there are situations where the result is consistent but supports inferences which contradicts the beliefs of each of the contributing agents.

In this paper we investigate the possibilities and impossibilities of strategies for dealing with this problem. A first attempt is to characterize those situations where information can be combined without risking the undesirable situation that a derivable proposition contradicts the beliefs of all agents involved: the relevant notion is called *Absolute safety*. It turns out that in that case only trivial solutions exist. It follows that any non-trivial strategy must use information about the epistemic states of the agents involved.

Subsequently we investigate less restrictive notions of safety. The more interesting of these notions are those which involve not only propositions about the world but also epistemic information relating the knowledge of the various agents involved. For this purpose we use the logic of belief dependence, which has been designed by the first author, and has been used successfully for designing effectively computable belief revision procedures.

The results characterizing the generalized safety notions generalize for this extended logic. We present a notion of almost safety within this framework which describes the safety of combining information under the hypothesis that agents eventually might have exchanged their information among themselves. For this notion of almost safety an explicit solution to the Judge puzzle is given.

1 Introduction

Making models of multi-agent epistemic systems has become one of the most interesting and popular topics in artificial intelligence and in the theory of knowledge based expert systems. Information systems in the real world are loaded by combining information from many informants. However, as is generally known, combining information may lead to inconsistent knowledge bases. An even more subtle risk was indicated by W. J. Schoenmakers [7] when he published his *Judge puzzle*; this puzzle describes the situation where one agent, called the judge, by combining information from two other agents, called the witnesses, consistently obtains a conclusion which contradicts the belief sets of both witnesses:

> Once upon a time a wise but strictly formal judge questioned two witnesses. They spoke to him on separate occasions. Witness W1 honestly stated his conviction that proposition p was true. Witness W2 honestly stated that he believed that the implication $p \rightarrow q$ holds. Nothing else was said or heard. The judge, not noticing any inconsistency accepted both statements and concluded that q had to be true. However, when the two witnesses heard about his conclusion they were shocked because they both were convinced that q was false. But they were too late to prevent the verdict to be executed...

As pointed out by Schoenmakers in the above story, no one could be blamed for this situation to arise. The witnesses, even though formally required to tell everything they knew, are not responsible since neither of them was asked about q and hardly could know at the time of interrogation that the truth of q was at stake. The judge on the other hand had no reason to even consider the possibility that his argument was unsound, since there was no contradiction in the testimony. He might have asked on, and confronted the witnesses with his conclusion that q was true. For the judge this would have been a possibility, but, as Schoenmakers indicates, this possibility is not present in the case of a knowledge base being loaded with information from various sources, since by the time proposition q turns out to be relevant the two informants no longer are available. And therefore Schoenmakers concludes:

> Intelligent database systems may behave perfectly in splendid isolation, operating on one world without inconsistencies, but even when they are consistent they may produce unacceptable results when operating on the information that is accessible in a community of such systems. Their results will be acceptable, most of the time, but nobody knows when.

Consequently it becomes relevant to look for a characterization of situations where combining information from multi sources is *safe*, which informally means that any conclusion drawn from the combined information is not disagreed upon by all informants. At the same time our combination operator should support at least the derivation of one proposition not already supported by one of the contributing agents.

After having formalized this problem we prove a triviality theorem expressing that such a combining operation can't exists. The above form of *absolute safety* can't be achieved. A more refined approach is obtained by considering both the information contributed and the complete belief sets of the agents. In this context the notions of *safety* and *strong safety* are defined, and some characterizations are given. It follows that dangerous situations only arise when there is disagreement between two agents about one of the propositions which is actually communicated. The above results once more indicate that, in a multi agent environment, one should make a strict distinction between information accepted on behalf of an other agent, and information which is incorporated in your own belief set. The resulting process of accepting information followed by incorporating it, has been one of the motivations for the introduction of the *logic of belief dependence* [2] which has been subsequently applied for describing belief revision strategies [3], and for a previous solution to the Judge puzzle [4].

Compared to our previous paper[4] we believe that we presently can make a much stronger case for the "contrived" solution to the judge puzzle presented in the final section of this paper. The triviality theorem shows that there is no simple solution for the problem. The characterization of the less restrictive safety notions shows that danger is caused by disagreement between agents and disagreement between agents is a fact of life we can't get around. The idea of a two stage process for belief incorporation has been argued elsewhere; it is also supported by psychological research. However, when generalizing the safety notions to the case of our epistemic logic of belief dependence, the characterizations for the propositional cases extend. The best we can hope for is a specific belief incorporation strategy for the judge which is approximatively safe.

Therefore we propose the notion of *almost safety*, which characterizes the situation where the conclusion of the judge is not contradicted by all witnesses, provided they would have had access to each other's information. This hypothetical situation can be expressed in terms of sub-beliefs, and therefore an effectively testable condition is obtained for deciding whether a specific belief revision operator for the judge is almost safe or not.

2 Combining information from multiple agents; the triviality result

In the sequel I denotes a finite and non-empty set of the informants and a the receiver, the agent who receives and combines information from the informants I. In this section, we study the case of the language of propositional logic $\mathbf{L_P}$, where information communicated among agents consists only of pure propositional formulas without modal operators. The language $\mathbf{L_P}$ is constructed from a primitive proposition set Φ_0 and the Boolean connectives recursively as usual. Moreover, the logical notions, such as, semantic model, satisfiability relations $\models$, and consequence operation Cn, are defined as usual.

The agent's obtained information, is a mapping ψ from the informants I into the formula set $\mathbf{L_P}$. We use the notation $\{\psi_i\}_{i\in I}$ to denote the set $\{\psi(i) \in \mathbf{L_P} : i \in I\}$. The set $\{\psi_i\}_{i\in I}$ is called the *obtained information set.* Each informant may offer a set of formulas[1]; however, since each informant can only contribute a finite set of formulas, we can just take its corresponding conjunction formula to denote the finite set. Furthermore, the informants' original belief sets are represented by a mapping Ψ from the informant set I into the powerset of the formula set. Similarly, we also use the notation $\{\Psi_i\}_{i\in I}$ to denote the set $\{\Psi(i) \in \mathcal{P}(\mathbf{L_P}) : i \in I\}$, which is called an *original information set.*

In this paper, we only study the cases in which all informants honestly offer information they actually have. This leads to the following definition:

Definition 2.1 (Potential information set) *An original information set $\{\Psi_i\}_{i\in I}$ is said to be a potential information set of an obtained information set $\{\psi_i\}_{i\in I}$ iff it satisfies the following conditions:*
(i) (Honesty Condition) $\Psi(i) \models \psi(i)$, for all $i \in I$, and
(ii) (Consistency Condition) $\Psi(i)$ is consistent, for all $i \in I$.

In the sequel we shall use the phrase set for information set when no confusion can arise.

Definition 2.2 (Danger) *Suppose that an original set $\{\Psi_i\}_{i\in I}$ is a potential set of an obtained $\{\psi_i\}_{i\in I}$. The set $\{\psi_i\}_{i\in I}$ is said to be dangerous with respect to the set $\{\Psi_i\}_{i\in I}$ iff there exists a $\varphi \in \mathbf{L_P}$ such that*
(i) $\{\psi_i\}_{i\in I} \models \varphi$
(ii) $\Psi(i) \models \neg\varphi$ for all $i \in I$.

Remarks: Condition (i) means that the receiver's obtained information implies a fact φ for which Condition (ii) says that all informants originally believe the negation of that fact φ.
In the following, by the notation $\{\psi_i\}_{i\in I}$ and $\{\Psi_i\}_{i\in I}$, we generally mean an obtained set and an original set respectively if it cannot cause any ambiguities.

Definition 2.3 (Absolutely Safety) *A consistent set $\{\psi_i\}_{i\in I}$ is said to be absolutely safe iff it is not the case that $\{\psi_i\}_{i\in I}$ is dangerous with respect to any of its potential sets $\{\Psi_i\}_{i\in I}$.*

[1]Here information, belief, and formula are interchangeable.

Definition 2.4 (Triviality) *A set $\{\psi_i\}_{i \in I}$ is trivial iff for any formula φ, such that $\{\psi_i\}_{i \in I} \models \varphi$, there exists an $i \in I$ such that $\psi(i) \models \varphi$.*

It turns out that absolute safety is a condition which is so strong that it supports only trivial situations:

Theorem 2.1 (Triviality Theorem) *A consistent set $\{\psi_i\}_{i \in I}$ is absolutely safe iff it is trivial.*

The proof for this result is easy. Assuming non-triviality there exists a proposition ϕ such that for no i one has $\{\psi_i\}_{i \in I} \models \varphi$; consequently the potential set $\Psi(i) = \{\psi(i), \neg\varphi\}$, for all $i \in I$ is dangerous with respect to $\{\psi_i\}_{i \in I}$. The converse implication is a direct consequence of the triviality condition. Consequently, the best one can hope for are safety notions which explicitly relate the obtained set and the potential set. Two possible definitions are:

Definition 2.5 (Safety) *If an obtained $\{\psi_i\}_{i \in I}$ is consistent, and an original set $\{\Psi_i\}_{i \in I}$ is a potential set of $\{\psi_i\}_{i \in I}$, then the set $\{\psi_i\}_{i \in I}$ is said to be safe with respect to the set $\{\Psi_i\}_{i \in I}$ iff the set $\{\psi_i\}_{i \in I}$ is not dangerous with respect to the set $\{\Psi_i\}_{i \in I}$.*

Definition 2.6 (Strongly Safety) *If a set $\{\psi_i\}_{i \in I}$ is consistent, and $\{\Psi_i\}_{i \in I}$ is a potential set of $\{\psi_i\}_{i \in I}$, then the set $\{\psi_i\}_{i \in I}$ is said to be strongly safe with respect to the set $\{\Psi_i\}_{i \in I}$ iff for any φ, if $\{\psi_i\}_{i \in I} \models \varphi$, then there exists an $i \in I$ such that $\Psi(i) \models \varphi$.*

The connection between these two notions is illustrated by the following:

Propositions 2.1 *If $\{\psi_i\}_{i \in I}$ is a consistent set, and $\{\Psi_i\}_{i \in I}$ is a potential set of $\{\psi_i\}_{i \in I}$, then the set $\{\psi_i\}_{i \in I}$ is safe with respect to its potential set $\{\Psi_i\}_{i \in I}$ iff
for any φ, if $\{\psi_i\}_{i \in I} \models \varphi$, then it is not the case that for all $i \in I$, $\Psi(i) \models \neg\varphi$.*

It follows that strong safety is a stronger notion than safety; see the third example below.

Example 2.1

- *$\langle p, p \to q \rangle$ is neither strongly safe nor safe with respect to its potential set $\langle \{p, \neg q\}, \{\neg p, \neg q\} \rangle$.(**Judge Puzzle Story**)*
- *$\langle p \to q, q \to p \rangle$ is neither strongly safe nor safe with respect to $\langle \{\neg p, q\}, \{\neg p, \neg q\} \rangle$.*
- *$\langle p, p \to q \rangle$ is safe with respect to $\langle \{p, p \vee q\}, \{p \to q, q \to p\} \rangle$, but not strongly safe with respect to $\langle \{p, p \vee q\}, \{p \to q, q \to p\} \rangle$. (**Distinction between safety and strong safety**)*

One can proceed to give alternative characterizations of these safety notions. Evidently a trivial obtained set is strongly safe with every of its potential sets. Also the safety notions are trivial for the case of a single informant. The two theorems below relate safety to consistency and to disagreement:

Theorem 2.2 (Safety Theorem) *If an obtained set $\{\psi_i\}_{i \in I}$ is consistent, and $\{\Psi_i\}_{i \in I}$ is a potential set of $\{\psi_i\}_{i \in I}$, then the set $\{\psi_i\}_{i \in I}$ is safe with respect to the original set $\{\Psi_i\}_{i \in I}$ iff there exists an $i \in I$ such that $\Psi(i) \cup \{\psi_i\}_{i \in I}$ is consistent.*

Theorem 2.3 (Disagreement Theorem) *If a consistent set $\{\psi_i\}_{i \in I}$ is dangerous with respect to a potential set $\{\Psi_i\}_{i \in I}$, then there exists a formula φ, and there exist an $i \in I$ and a $j \in I$ such that $\psi(i) \models \varphi$ and $\Psi(j) \not\models \varphi$.*

The implication of the disagreement theorem is that in a multi-agent information system in order to guarantee safety, agents must be prohibited to talk about something if they disagree among each other about it! Therefore each of them needs full information about each others propositional attitudes, which clearly represents an unrealistic assumption. Still the result implies that we should focus on the cases where disagreement may arise. As we indicate below, the logic of belief dependence turns out to be a useful tool in this direction.

3 Logic of belief dependence

The logic of belief dependence was introduced [2] in order to model the situation where agents rely on each other with respect to information. It also provides a tool for modeling a two stage process for information acquisition in a multi-agent system, where in the first stage agents include information of other agents in *compartimentalized sub-beliefs*; in the second stages these sub-beliefs are processed and *incorporated* into the agents own beliefs. For further information and motivation we refer to [2].

Our logic involves in the first place the general notions of knowledge and belief, which are the equivalents of those notions in epistemic and doxastic logic. In our logic for belief dependence, we generally use $L_i\varphi$ to represent the fact that agent i knows or believes the formula φ. There exists a second important notion used for reasoning about dependent knowledge and beliefs; this notion is called the *dependent operator*, or alternatively *rely-on relation*, and it is denoted by $D_{i,j}$. Intuitively, we can give $D_{i,j}\varphi$ a number of different interpretations: "agent i relies on agent j about the formula φ", or, "agent j is the credible advisor of agent i about φ".

We furthermore introduce a *compartment operator*, or alternatively called a *sub-belief operator*, written $L_{i,j}$. Intuitively, $L_{i,j}\varphi$ can be read "agent i believes φ due to agent j". From the viewpoint of *minds society*, $L_{i,j}\varphi$ can be more intuitively interpreted as "agent i believes or knows φ on the mind frame indexed j.

The resulting language is sufficiently rich for formalizing both stages in the multi-agent information acquisition process mentioned above: compartmentalized information is modeled by sub-beliefs $L_{i,j}\varphi$ for agent i, whereas incorporated information corresponds to general beliefs of agent i, namely, $L_i\varphi$.

Supposed we have a set $\mathbf{A_n}$ of n agents, and a set Φ_0 of primitive propositions, the language $\mathbf{L_D}$ for belief dependence logics is the minimal set of formulas closed by usual syntactic rules.

Definition 3.1 (D-model) *A belief dependence D-model is a tuple $M = (S, \pi, \mathcal{L}, \mathcal{D})$, where S is a set of states, $\pi(s, .)$ is a truth assignment for each state $s \in S$, and $\mathcal{L} : \mathbf{A_n} \to 2^{S\times S}$, which consists of n binary serial accessibility relations on S, $\mathcal{D} : \mathbf{A_n} \times \mathbf{A_n} \times S \to 2^{\mathbf{L_D}}$.*

The truth relation $\models$ is defined inductively as follows:

$M, s \models p,$	where p is a primitive proposition, iff $\pi(s, p)$ =true,
$M, s \models \neg\varphi$	iff $M, s \not\models \varphi$
$M, s \models \varphi_1 \wedge \varphi_2$	iff $M, s \models \varphi_1 \wedge M, s \models \varphi_2$,
$M, s \models L_i\varphi$	iff $M, t \models \varphi$ for all t such $(s, t) \in \mathcal{L}(i)$
$M, s \models D_{i,j}\varphi$	iff $\varphi \in \mathcal{D}(i, j, s)$.

For D-models, we define sub-beliefs as $L_{i,j}\varphi \stackrel{\text{def}}{=} D_{i,j}\varphi \wedge L_j\varphi$, which implies that system is honest because the honesty axiom $L_{i,j}\varphi \to L_j\varphi$ holds.[2]

Here is a minimal logic system for D-model, called **LD system.**

Axioms:

(BA) All instances of propositional tautologies.

(KL) $L_i\varphi \wedge L_i(\varphi \to \psi) \to L_i\psi$.

(DL) $\neg L_i\bot$.

Rules of Inference:

(MP) $\vdash \varphi, \vdash \varphi \to \psi \Rightarrow \vdash \psi$.

(NECL) $\vdash \varphi \Rightarrow \vdash L_i\varphi$.

Definitions:

(Lijdf) $L_{i,j}\varphi \stackrel{\text{def}}{=} D_{i,j}\varphi \wedge L_j\varphi$

Theorem 3.1 *The logic* **LD** *is sound and complete in the class of D-model.*

[2] In the section 5 we need the notion of reliable sub-beliefs $L^r_{i,j}\varphi$ which is defined as $D_{i,j}\varphi \wedge D_{j,j}\varphi \wedge L_j\varphi$.

For the belief maintenance operation during the second phase of the information assimilation, we introduce the notion of the *belief maintenance model*, which is an ordered couple $< \mathbf{K}, \triangle >$ such that $\mathbf{K}$ is a set of belief sets and $\triangle : \mathbf{K} \times \mathbf{L_D} \to \mathbf{K}$ is a function assigning a belief set $\triangle(K, \varphi)$ to any belief set $K \in \mathbf{K}$ and each formula φ in $\mathbf{L_D}$. We shall write alternatively $K\triangle\varphi$ to represent $\triangle(K, \varphi)$.

4 Information assimilation in a belief dependence framework

In this section, we consider the information assimilation problem in the framework of belief dependence. It is easy to extend the definitions which have appeared in section 2 for the case of belief dependence by simply replacing the propositional formulas into $\mathbf{L_D}$ formulas, the models into D-models, and the relation $\models$ for propositional logic into its counterpart for belief dependence logic. Moreover, whenever we say a formula set K is consistent, we mean that K is consistent with respect to the **LD** system unless stated otherwise.

In the resulting theory the (negative) results from section 2 remain valid, indicating that for a solution of problems like the judge puzzle a translation into the language of belief dependence logic by itself will not suffice.

Definition 4.1 (Configuration) *A configuration C is a four place tuple $\langle a, I, \psi, \Psi \rangle$ where $a \in A_n$ is an agent, called receiver, $I \subseteq A_n$ is a finite and non-empty set of informants, $\psi : I \to \mathbf{L_D}$ is a mapping from I into $\mathbf{L_D}$, called the obtained information, and $\Psi : I \to \mathcal{P}(\mathbf{L_D})$ is a mapping from I into the powerset of $\mathbf{L_D}$, called original information.*

Moreover, we define:
$L^-_{i,j}(K) \stackrel{\text{def}}{=} \{\varphi \in \mathbf{L_D} : K \models L_{i,j}\varphi\}$, which denotes the agent i's sub-belief set indexing j.
$L^-_i(K) \stackrel{\text{def}}{=} \{\varphi \in \mathbf{L_D} : K \models L_i\varphi\}$, which denotes the agent i (incorporated) belief set.
$L^+_{a,I}(\{\psi_i\}_{i\in I}) \stackrel{\text{def}}{=} \{L_{a,i}\psi(i) \in \mathbf{L_D} : i \in I\}$, which is a formula set corresponding the obtained information.

Theorem 4.1 (Triviality Theorem(Restated)) *A consistently obtained set $\{\psi_i\}_{i\in I}$ is absolutely safe iff it is trivial.*

Theorem 4.2 (Disagreement Theorem(Restated)) *Let $C = \langle a, I, \psi, \Psi \rangle$ be a configuration. Suppose that $\{\psi_i\}_{i\in I}$ is consistent, and $\{\Psi_i\}_{i\in I}$ is a potential set of $\{\psi_i\}_{i\in I}$. If $\{\psi_i\}_{i\in I}$ is dangerous with respect to the set $\{\Psi_i\}_{i\in I}$, then there exists a formula φ, and there exists an $i \in I$ and a $j \in I$ such that $\psi(i) \models \varphi$ and $\Psi(j) \not\models \varphi$.*

For a belief set K and an agent a, we want to induce a configuration for a from K. We may realize this by the following construction:

Definition 4.2 (Induced Configuration) *Suppose that K be a belief set, and a be an agent $\in A_n$. A configuration $C = \langle a, I_a, \psi, \Psi \rangle$, called the induced configuration for a from K, is constructed as follows:*
(1) I is the set $\{i \in I : \exists\varphi(L_{a,i}\varphi \in K)\}$.
(2) If I is not empty, then for all $i \in I, \Psi(i) = L^-_i(K)$, otherwise the induced configuration does not exist.
(3) For all $i \in I$, if $L^-_{a,i}(K)$ is finite, then let $\psi(i)$ be $\wedge L^-_{a,i}(K)$, otherwise the induced configuration does not exist.

For a belief set K and an agent $a \in A_n$. K is said to be a *DB set for a* if the induced configuration for a from K exists. For an agent $a \in A_n$ and a DB set K for a, induced configurations $\langle a, I, \psi, \Psi\rangle$ for a from K are unique. Moreover, $\Psi(i)$ is a potential set of ψ for each $i \in I$. Therefore, we can use the notation $C(a, K)$ to denote the induced configuration for a from K.

Definition 4.3 (Safety for a in K) *For an agent $a \in A_n$ and a DB set K for a, if $C(a, K) =< a, I, \psi, \Psi >$ is the induced configuration for a from K, then $\{\psi_i\}_{i\in I}$ is said to be safe for a in K iff $\{\psi_i\}_{i\in I}$ is safe with respect to $\{\Psi_i\}_{i\in I}$.*

Theorem 4.3 (Safety Theorem(Restated)) *Let a and K be an agent and a DB set respectively. Suppose that the induced configuration for a from K, $C(a, K) = \langle a, I, \psi, \Psi\rangle$, then $\{\psi_i\}_{i\in I}$ is safe for a in K iff there exists an $i \in I$ such that $L_i^-(K) \cup \{\psi_i\}_{i\in I}$ is consistent.*

5 Almost safety

In order to evaluate whether obtained information is safe the receiver a needs information on the true belief states of his informants; the translation into the belief dependence logic and the introduction of configurations has not changed this necessity. However, if we take into consideration the mechanisms which might produce sub-beliefs in a multi-agent environment, these mechanisms themselves might provide us with additional structure which supports the introduction of alternative weaker safety notions.

The notion of *almost safety* defined in this section is based on one possible hypothesis concerning the creation of sub-beliefs. This is the so-called *initial role-knowledge assumption*, which states that within a multi agent environment the dependency relations are common knowledge: it is not known who knows or believes what, but for each proposition it is known how the agents depend on each other concerning this proposition.

That this information is relevant is shown by the situation below: assume that agent i believes ϕ and says so to the receiver. The receiver has been told previously that agent j believes $\neg\phi$. Moreover, according to the initial role-knowledge assumption it is common knowledge that $D_{i,j}\phi$, so the receiver knows that as well. In this situation the receiver can conclude that something strange is going on: would the two agents i and j have been given the possibility to exchange their information, agent i would have been convinced by j that his belief concerning ϕ was wrong. It is based on this information that the receiver can disregard the information provided by i substituting it by the opposite information provided by agent j.

The notion of almost safety formalizes safety with respect to the hypothetical scenario which would arise had all informants exchanged their information before sharing their knowledge with the receiver.

Definition 5.1 (Combined Sub-belief)
$L_{i,I}^-(K) \stackrel{\text{def}}{=} \{\varphi \in \mathbf{L_D} : (\exists j \in I)(K \models D_{i,j}\varphi \wedge D_{j,j}\varphi \wedge L_j)\}$.

Definition 5.2 (Almost Safety) *For an agent $a \in A_n$ and a DB set K for a, if $C(a, K) = \langle a, I, \psi, \Psi\rangle$ is the induced configuration for a from K, $\{\psi_i\}_{i\in I}$ is said to be almost safe for a in K iff for any φ, if $\{\psi_i\}_{i\in I} \models \varphi$, then, either (i) there exists $i \in I$ such that $L_i^-(K) \not\models \neg\varphi$, or (ii) there exists an $i \in I$ such that $L_{i,I}^-(K)$ is consistent and $L_{i,I}^-(K) \models \varphi$.*

We illustrate this notion by an example of a possible configuration for the judge puzzle: Consider A DB set $K = \{L_{w1}(p\wedge\neg q),\ L_{w2}((p \to q)\wedge\neg q),\ D_{w1,w1}p,\ D_{w2,w2}(p \to q),\ D_{w2,w1}p, L_{a,w1}p,$ $L_{a,w2}(p \to q)\}$ So, $I = \{w1, w2\}$, $\psi(w1) = p$, $\psi(w2) = p \to q$, $\Psi(w1) = \{p \wedge \neg q\}$, $\Psi(w2) = \{(p \to q)\wedge\neg q\}$. The induced configuration for a from K is $\langle a, I, \psi, \Psi\rangle$, with $\{\psi_i\}_{i\in I} = \{p, p \to q\}$. Moreover, from $K \models L_{w1}p \wedge D_{w1,w1}p \wedge D_{w2,w1}p$, we have $L_{w2,I}^-(K) = \{p, p \to q\}$ so $L_{w2,I}^-(K)$

is consistent. Evidently, for any φ, if $\{\psi_i\}_{i\in I} \models \varphi$, then $L^-_{w2,I}(K) \models \varphi$. Therefore, $\{\psi_i\}_{i\in I}$ is almost safe for a in K.

It is an easy consequence of the definition that almost safety is a weaker notion than safety. The notions turn out to be equivalent in the degenerate case that the informants don't rely on each other concerning any proposition. By a straightforward generalization of previous characterizations we obtain:

Theorem 5.1 (Almost Safety Theorem) *Let $a \in A_n$ be an agent, K be a DB set for a, and $C(a,K) = \langle a, I, \psi, \Psi\rangle$ be the induced configuration for a from K; then $\{\psi_i\}_{i\in I}$ is almost safe for a in K iff there exists an $i \in I$ such that either $L^-_i(K) \cup \{\psi_i\}_{i\in I}$ is consistent, or $L^-_{i,I}(K)$ is consistent and $L^-_{i,I}(K) \models \{\psi_i\}_{i\in I}$.*

This theorem establishes that almost safety is a property which, in principle can be effectively tested for a given configuration. This test is called the almost safety test for $\{\psi_i\}_{i\in I}$.

6 An Almost safe belief revision strategy for the judge

In this section we study the process of belief revision corresponding to the second stage of the two stage information acquisition process. Given a configuration where the receiver has obtained sub-beliefs by hearing statements by his informants, the receiver will subsequently revise his own belief by incorporation part of these sub-beliefs into his own belief. Clearly he should do so in a safe way; we now have the tools available for formalizing this requirement.

Let $\mathbf{K}$ be a set of belief sets. As announced before a belief maintenance operation $\triangle$: $\mathbf{K} \times \mathbf{L_D} \to \mathbf{K}$ is a function assigning a belief set $\triangle(K,\varphi)$ to any belief set $K \in \mathbf{K}$ and each formula φ in $\mathbf{L_D}$. The function $\triangle$ can be defined in many ways. For our application we are interested in belief maintenance operators with a special form where the rational agent checks whether or not a special formula φ_i belongs to the belief set X when she faces the new information ρ'_i. If so, then the result of $\triangle(X,\rho'_i)$ is a new belief set Y_i.[3].

$$\triangle(X,\rho) = \begin{cases} Y_{i_1} & \text{if } \varphi_{i_1} \in X, \rho = \rho'_{i_1} \\ \cdots & \cdots \\ Y_{i_n} & \text{if } \varphi_{i_n} \in X, \rho = \rho'_{i_n} \\ X & \textbf{otherwise} \end{cases}$$

We are considering the belief maintenance operation for the receiver a. Therefore, the belief set X is $L^-_a(K)$. As in our previous paper [3] we use the traditional update operations like revision, contraction and expansion $+$, to define the belief maintenance operation. It is known that such belief revision and contraction functions are not unique, and therefore we select a group of revision functions which satisfies the AGM postulates [1]. Let the selected revision function be $\dot{+}$. We define the contraction function $\dot{-}$ by Harper Identity in terms of the revision function $\dot{+}$. Therefore, we obtain a definition of the operation in question of the following form:[4]:

$$\triangle(L^-_a(K),\rho) = \begin{cases} L^-_a(K)\theta_1\psi_{i_1} & \text{if } \varphi_{i_1} \in L^-_a(K), \rho = \rho'_{i_1} \\ \cdots & \cdots \\ L^-_a(K)\theta_n\psi_{i_n} & \text{if } \varphi_{i_n} \in L^-_a(K), \rho = \rho'_{i_n} \\ K & \textbf{otherwise} \end{cases}$$

where $\theta_i \in \{\dot{+}, \dot{-}, +\}$.

[3]This form is called a *type 3 belief maintenance operation* in [5]

[4]called a *type 4 belief maintenance operation* in[5]

Definition 6.1 (AS Operation) *A belief maintenance operation* $\triangle : \mathbf{K} \times L_D \to \mathbf{K}$ *is said to be an almost safety one for agent* $a \in A_n$ *with respect to* $\{\psi_i\}_{i\in I}$*, iff for any DB set* $K \in \mathbf{K}$ *for* a *such that* $L_a^-(K) \models \wedge L_{a,I}^+(\{\psi_i\}_{i\in I})$ *and* $L_a^-(K) \not\models \wedge\{\psi_i\}_{i\in I}$, $\triangle(L_a^-(K), \wedge L_{a,I}^+(\{\psi_i\}_{i\in I})) \models \wedge\{\psi_i\}_{i\in I} \Rightarrow \{\psi_i\}_{i\in I}$ *is almost safe for* a *in* K.

Definition 6.2 (CAS Operation) *A belief maintenance operation* $\triangle : \mathbf{K}\times L_D \to \mathbf{K}$ *is said to be a complete almost-safety one for agent* $a \in A_n$ *with respect to* $\{\psi_i\}_{i\in I}$*, iff for any DB set* $K \in \mathbf{K}$ *for* a *such that* $L_a^-(K) \models \wedge L_{a,I}^+(\{\psi_i\}_{i\in I})$ *and* $L_a^-(K) \not\models \wedge\{\psi_i\}_{i\in I}$, $\triangle(L_a^-(K), \wedge L_{a,I}^+(\{\psi_i\}_{i\in I})) \models \wedge\{\psi_i\}_{i\in I} \Leftrightarrow \{\psi_i\}_{i\in I}$ *is almost safe for* a *in* K.

Observe that in the above definition it seems that the belief set of agent a gets revised by the combined sub-belief which agent a already supports; however, this support represents a compartimentalized belief state whereas the belief maintenance operation $\triangle$ produces an incorporated belief state. The conditions express that the receiver, supporting a set of compartimentalized sub-beliefs but not yet supporting the contributed information itself, will support it after belief revision only if it is almost safe information; the stronger CAS notion requires also that this last condition is also sufficient. Evidently this condition requires that the belief maintenance operation actually is a revision. Our format for this operator therefore further specializes to:[5]

$$\triangle(L_a^-(K), \wedge L_{a,I}^+(\{\psi_i\}_{i\in I})) = \begin{cases} L_a^-(K)\dot{+}\{\psi_i\}_{i\in I} & \text{if } \varphi_{i_1} \in L_a^-(K) \text{ or ... or } \varphi_{i_n} \in L_a^-(K) \\ L_a^-(K) & \textbf{otherwise} \end{cases}$$

For this type of belief revision operator we now can characterize almost safety:

Theorem 6.1 (AS Operation Theorem) *Consider a belief maintenance operation* $\triangle$ *is of the form:*

$$\triangle(L_a^-(K), \wedge L_{a,I}^+(\{\psi_i\}_{i\in I})) = \begin{cases} L_a^-(K)\dot{+}\{\psi_i\}_{i\in I} & \textit{if } \varphi_{i_1} \in L_a^-(K) \textit{ or ... or } \varphi_{i_n} \in L_a^-(K) \\ L_a^-(K) & \textbf{otherwise} \end{cases}$$

$\triangle$ *is an AS operation for* a *with respect to* $\{\psi_i\}_{i\in I}$ *iff* ($\varphi_{i_1} \in L_a^-(K)$ *or ... or* $\varphi_{i_n} \in L_a^-(K) \Rightarrow$ *the almost-safety test holds for* $\{\psi_i\}_{i\in I}$.)

Theorem 6.2 (CAS Operation Theorem) *Consider a belief maintenance operation* $\triangle$ *is of the form:*

$$\triangle(L_a^-(K), \wedge L_{a,I}^+(\{\psi_i\}_{i\in I})) = \begin{cases} L_a^-(K)\dot{+}\{\psi_i\}_{i\in I} & \textit{if } \varphi_{i_1} \in L_a^-(K) \textit{ or ... or } \varphi_{i_n} \in L_a^-(K) \\ L_a^-(K) & \textbf{otherwise} \end{cases}$$

$\triangle$ *is an CAS operation for* a *with respect to* $\{\psi_i\}_{i\in I}$ *iff* ($\varphi_{i_1} \in L_a^-(K)$ *or ... or* $\varphi_{i_n} \in L_a^-(K) \Leftrightarrow$ *the almost-safety test holds for* $\{\psi_i\}_{i\in I}$.)

For the special case of the judge puzzle where $I = \{w_1, w_2\}$ and $\{\psi_i\}_{i\in I} = \{\psi(w_1) = p, \psi(w_2) = p \to q\}$, where p, q are primitive propositions, we now specify an almost safe belief maintenance operation:[6]

The Definition of Operation $\triangle_{cas1}$ (for Agent a):

(a) $D_{w_1,w_2}(p \to q) \wedge \neg D_{w_1,w_1}(p \to q) \Rightarrow L_a^-(K)\triangle_{cas1}L_{a,w_1}p \wedge L_{a,w_2}(p \to q) = L_a^-(K)\dot{+}p\wedge(p \to q)$.

(b) $D_{w_2,w_1}p \wedge \neg D_{w_2,w_2}p \Rightarrow L_a^-(K)\triangle_{cas1}L_{a,w_1}P \wedge L_{a,w_2}(p \to q) = L_a^-(K)\dot{+}p \wedge (p \to q)$.

(c) $\neg L_{w_1}\neg q \Rightarrow L_a^-(K)\triangle_{cas1}L_{a,w_1}p \wedge L_{a,w_2}(p \to q) = L_a^-(K)\dot{+}p \wedge (p \to q)$.

[5] called a *type 5 belief maintenance operation* in [5]

[6] Which is represented by a simplified type 5 belief maintenance operation[5].

(d) $\neg L_{w_2}\neg p \Rightarrow L_a^-(K)\triangle_{cas1}L_{a,w_1}p \wedge L_{a,w_2}(p \rightarrow q) = L_a^-(K)\dot{+}p \wedge (p \rightarrow q)$.
(e) **Otherwise** $\Rightarrow L_a^-(K)\triangle_{cas1}L_{a,w_1}p \wedge L_{a,w_2}(p \rightarrow q) = L_a^-(K)$.

Of the above four cases, cases (a) and (b) are representative for the problem of Schoenmakers; the puzzle is solved since we need no further information about source agents' beliefs other than the general information about the rely-on relations among agents. Cases (c) and (d), dealing with non-problematic situations are added for obtaining a complete operation.

Theorem 6.3 *The operation $\triangle_{cas1}$ is a CAS operation for a with respect to $\langle p, p \rightarrow q\rangle$.*

Application of the above ideas leads to a new appreciation of the judge puzzle. Assuming that the judge draws his conclusion based on a CAS operation, we find that the unacceptability of the state of affairs as indicated by the story only is a temporary stage in the process of exchanging information and incorporation of beliefs. A possible scenario for the continuation of the story (the part which Schoenmakers did not include in his paper) is presented below:

> When the judge was told that p was true by the witness $w1$ and that the implication $p \rightarrow q$ was true by the witness $w2$, the judge had to figure out whether these assertions could be accepted together. Now the judge had good reasons for not asking the witnesses for more information about their knowledge, since he could base his decision already on his knowledge of the rely-on relation. He knew that witness $w1$ was the only authority concerning the statement p, and that witness $w2$ was the only authority concerning the conditional $p \rightarrow q$. Moreover, this information was common knowledge among both witnesses and himself. Therefore, the judge could safely arrive to the conclusion q was true, and ordered to execute the verdict. Still, both witnesses, $w1$ and $w2$, protested claiming that q was false. The judge patiently told witness $w1$ about the witness $w2'$s belief, helding that $p \rightarrow q$ was true. Because the witness $w1$ accepted that $w2$ was the authority on the implication $p \rightarrow q$, $w1$ accepted this assertion, and had to agree with the judge. A similar thing happened with witness $w2$. The judge told witness $w2$ about $w1$'s belief, that is, that p was true. The witness $w2$ also had to agree with the judge's verdict, since $w2$ accepted that the $w1$ was the authority about p. In the end everybody was satisfied.

7 Conclusions

We have formalized the problem of information acquisition in a multi agent environment. The danger of accepting information from several agents as illustrated in the judge puzzle is an inherent consequence of disagreements among the informants; there exists no absolute safe set of obtained information except trivial sets, and safe or strongly safe sets are defined only relative the full believe state which in general is unknown to the receiver.

Formalizing this problem in a belief dependence framework does not offer a simple way out; however, by assuming the initial role-knowledge assumption and by considering a highly specialized belief maintenance operation an effectively computable almost safe solution for the judge puzzle has been obtained. Notwithstanding the complexity of the solution it is at least based on a general theory supported by psychological evidence; also the tools used in the solution were not developed for this purpose. We consider it highly unlikely that there exist "cleaner" solutions to this problem (aside of simply denying it to be a problem).

For designers of intelligent database systems and expert systems our results suggest the following guideline: When combining expertise from different expert sources, ensure that the contributing agents involved recognize each other to be the expert on their respective contributions.

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