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ON COMPLEX PLURAL NOUN PHRASES

Jaap van der Does

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Institute for Logic, Language and Computation
Plantage Muidergracht 24
1018TV Amsterdam
Telephone 020-525.6051, Fax: 020-525.5101

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Jaap van der Does
Department of Philosophy
University of Amsterdam

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On Complex Plural Noun Phrases*

Jaap van der Does

ILLC Department of Philosophy
University of Amsterdam

1 Introduction

In the analysis of natural language, linguists and philosophers have been led to postulate ever richer ontology's which enable one to quantify over points or periods of times, situations, events, information states, among other things. A reasonable test for such proposals is to determine whether they can handle some major types of expression in a principled way. From the point of view of quantification theory, in particular, the question arises of whether or not they allow systematic generalizations of the knowledge obtained under the simplifying assumption that only individuals exist.

Van der Does 1993 gives a positive answer to this question for the case where collections of individuals are available. In such a context one is able to treat the collective reading of sentences like:

(1) Three musicians played a trio

However, the NPs considered in this article were all of a simple form. I now aim to show that the results obtained generalize to the cases where an NP has multiple Ns as heads, is a complex definite, or is a partitive.

What kind of question presents itself here? In regard to multiply-headed NPs, the first question is whether the phenomenon is present at all. Are there any other readings than the familiar distributive one? And if so, which ones? We should also look for new variants of constraints such as conservativity, etc. (cf. Westerståhl 1989 for an overview). This topic is addressed in section 2. As we shall see, this more general setting forces us to reconsider the treatment of collective and other readings.

In section 3, I give three liftings to embed a multiple-headed determiner from the 'standard' atomic semantics into a collective one (along the lines of Van Benthem 1991, 67–68). The liftings correspond to the distributive, the collective and the neutral reading of a determiner. In contrast with Van der

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Does 1993 these liftings are strongly contextualized. The collective reading, in particular, now gets a referential rather than a quantificational treatment (cf. Lønning 1987). Using such notions as general applicability, maximality, and logical strength, I argue that the referential collective reading has much to recommend it. It is also shown that the distributive and the neutral reading of an NP cannot be reduced to its collective reading by means of VP modification.

Complex definite and partitive NPs are another source of challenges. In section 4, I first aim to develop the semantics of these NPs compositionally within the familiar atomic setting. Next I show how the insights obtained in section 3 can be applied here as well.

In section 5, the neutral reading of definite NPs will play a special rôle. As is well-known, even simple sentences containing definites may differ widely in logical form, although their syntactic structure is identical (cf. Langendoen 1978, Scha 1981, Roberts 1987, among others). I discuss these issues in some detail. It will appear that the neutral cumulative reading—defined in Van der Does 1993 for independent reasons—gives a satisfactory semantics for some problematic cases.

The appendix discusses basic assumptions and notational conventions, and has a few remarks on the framework in which this research is carried out.

In sum, these are the main themes in my study of the semantics of complex plural NPs:

- Which readings do sentences involving complex plural NPs have in a collective environment?
- Are there general recipes to model these readings by embedding their 'standard' denotations into this richer environment?

In Van der Does 1993 some general recipes are given, so that if an NP has a particular reading it can be obtained via the recipe corresponding to that reading. In this article it is shown that the same is true in regard to complex plural NPs. As a bonus, the following questions are made precise: what is the class of quantifiers associated with a reading, and can these classes be characterized in terms of quantifier properties? Answers to these questions have to be left for future research.

2 Multiply-headed Noun Phrases

In this section, I discuss the collective and other readings of sentences with multiply-headed noun phrases. By way of introduction, I first consider a few sentences with simple NPs. Some observations on the distributive and neutral readings of sentences with multiply-headed NPs follow, and then some on the collective readings of such sentences. It is shown that such NPs do not allow a straightforward generalization of the quantificational treatment of the collective reading. This is interesting, for it has been proposed that the neutral reading could be reduced to this reading. In general, this is not so. Alternatively, it suggests that another treatment of the collective reading is more proficient.

This issue is developed further in section 3, where the information given here is used to arrive at a uniform method to generate the readings.

2.1 Simple Noun Phrases: a First Step

Sentences like (2a–c), which only involve single-headed NPs, have been studied extensively in Van der Does 1993. I limit myself here to stating their most prominent readings and introducing some minor variations in the way they are formalized.

(2) a. Some musicians played a solo
 b. Four musicians played a quartet
 c. Most pianists played a quartet somewhere in Salzburg

Abbreviating nouns and verb phrases in a self-explanatory fashion, (3a–c) shows the most plausible readings of (2a–c), i.e., a distributive, collective, and neutral reading respectively:

(3) a. $|[\![\mathrm{M}]\!] \cap \mathbf{at}([\![\mathrm{Pc}]\!])| \geq 2$
 b. $\exists Z[|Z| = 4 \wedge Z \in [\![\mathrm{Pq}]\!] \cap \wp([\![\mathrm{M}]\!])]$
 c. $|\bigcup([\![\mathrm{Pq}]\!]) \cap [\![\mathrm{P}]\!]| > |\overline{\bigcup([\![\mathrm{Pq}]\!])} \cap [\![\mathrm{P}]\!]|$

Here $\overline{X}$ is the complement of X. In stating the truthconditions, some care is taken to use the determiner denotations in type *et.ett* as much as possible. To do so, one has to employ operations that transform the VP-denotation, possibly restricted to an N-extension, from type *ett* to *et*. In particular, the distributive and the neutral reading of (2a,c), as given in (3a,c), use the operations **at** and union for this purpose. The operation **at**, the extensional core of Montague's *to be*, is defined by:[1]

$$\mathbf{at} := \lambda \mathbf{X} \lambda y.\mathbf{X}(\lambda z.z = y)$$

Given a property of collections $\mathbf{P}$, $\mathbf{at}(\mathbf{P})$ is the set of individuals which occur in a singleton that has $\mathbf{P}$. Similarly, $\bigcup\mathbf{P}$ yields the set of individuals that are members of a collection C, of whatever size, that has $\mathbf{P}$. Note that the collective reading applies a converse to the transformations used in the distributive and neutral readings. The powerset operator, here of type *et.ett*, lifts the N-extension so that it may restrict the VP. It might appear as if the collective reading in (3b) cannot be formulated in terms of the standard determiner denotation. In section 3 I will show that this is in fact possible.

Now that the notions used in stating the truthconditions are explained, they speak for themselves. This is not to say that there is consensus as to the intuitions they try to capture. In particular, the neutral reading has been the topic of some debate (cf. Gillon 1987, 1990; Lasersohn 1989; Lønning 1991; Van

[1] In Van der Does 1993, I opt for shifting the N denotation from *et* to *ett* in order to make it fit the VP extension. To this end I used Scha's AT operator defined by:

$$\mathrm{AT} := \lambda X \lambda Y.\ Y \subseteq X \wedge |Y| = 1$$

One has: $\mathbf{at} \circ \mathrm{AT} = id_{et}$ and $\mathrm{AT} \circ \mathbf{at} = \lambda \mathbf{Y}.\mathbf{Y} \cap \mathrm{AT}(\mathbf{D}_e)$.

der Does 1992, 1993; Verkuyl 1992). I do not repeat the arguments pro and con, but refer to Van der Does (1992, 49–55, 81–83) for an overview.

Next, we have a look at the different readings of multiple-headed NPs.

2.2 Distributive and Neutral Readings

The DETs occurring in (2a–c) are single-headed ones which ask for an N to form an NP. The NPs in (4a–c) are more complex. They can be analysed as formed from a discontinuous DET applied to two Ns.

(4) a. Twelve students but fewer teachers played guitar
b. Most boys and girls bought a parasol
c. More students than teachers walked

On the multiple-headed view, the denotations of these DETs are:

(5) Twelve ... but fewer ...
$\lambda N \lambda N' \lambda VP.\ |N \cap VP| = 12 \wedge |N' \cap VP| < 12$
Most ... and ...
$\lambda N \lambda N' \lambda VP.\ |(N \cup N') \cap VP| > |(N \cup N') \cap \overline{VP}|$
More ... than ...
$\lambda N \lambda N' \lambda VP.\ |N \cap VP| > |N' \cap VP|$

The analyses (5a–c) are used in this article but they are not the only ones possible. For instance, the DET *twelve ... but fewer ...* could also be analysed as a conjunction of the DETs *twelve* and *fewer*, where the latter is linked anaphorically to the former. And the DET *most ... and ...* could also be seen as compounded from the single-headed DET *most* applied to the complex noun N *and* N′ conjoined in a non-Boolean fashion. Even the DET *more ... than ...* has alternative analyses, involving a coordination of some sort (cf. Hendriks 1992).

Keenan (1987) has arguments in favour of the multiple head analysis of the NPs in (4). He observes that the selectional restrictions of these NPs and the readings induced by nominal modifiers accords best with this analysis. The same holds for their conservativity behaviour. In particular, the NP in (4c) cannot be obtained from the DET *More students than* and the N *teachers*, or from the DET *More ... than teachers* and the N *students*. Conservativity would then require that (4c) is equivalent to either (6a) or (6b), which it is not:

(6) a. More students than teachers are teachers who walked
b. More students than teachers are students who walked

Along these lines, some further support for the multiple head analysis can be given by looking at the semantical behaviour of the NPs (5a–c) with regard to collective VPs.

Sentence (7) shows that non-distributive readings are not confined to sentences with simple NPs. Now, if the NP *twelve students but fewer teachers* were a conjunction of two single-headed NPs, one would expect the Ns to restrict the VP each for themselves. Whereas if it were a genuine double-headed NP, a simultaneous restriction by both Ns should also be possible. Although such differences cannot be detected in case of the distributive reading (7a) of (7), they do show up in case of the alternatives for its neutral reading, given in (7b–c).

(7) Twelve students but fewer teachers assembled some cars
distributive reading
a. $|\mathbf{at}([\![\mathrm{Ac}]\!]) \cap [\![\mathrm{S}]\!]| = 12 \wedge |\mathbf{at}([\![\mathrm{Ac}]\!]) \cap [\![\mathrm{T}]\!]| < 12$
options for neutral reading
b. $|\bigcup([\![\mathrm{Ac}]\!] \cap \wp([\![\mathrm{S}]\!]))| = 12 \wedge |\bigcup([\![\mathrm{Ac}]\!] \cap \wp([\![\mathrm{T}]\!]))| < 12$
c. $|\bigcup([\![\mathrm{Ac}]\!] \cap \wp([\![\mathrm{S}]\!] \cup [\![\mathrm{T}]\!])) \cap [\![\mathrm{S}]\!]| = 12 \wedge$
$|\bigcup([\![\mathrm{Ac}]\!] \cap \wp([\![\mathrm{S}]\!] \cup [\![\mathrm{T}]\!])) \cap [\![\mathrm{T}]\!])| < 12$
d. $|\bigcup([\![\mathrm{Ac}]\!]) \cap [\![\mathrm{S}]\!]| = 12 \wedge |\bigcup([\![\mathrm{Ac}]\!]) \cap [\![\mathrm{T}]\!]| < 12$

On the neutral reading of (7), one is interested in the students, teachers, and perhaps others who partake in assembling some cars, and then to compare the 'relevant' students with the 'relevant' teachers. In making this precise, one has to ask whether attention is restricted to collections of only students and only teachers, as in (7b), or to collections of only students and teachers, as in (7c), or to collections that contain other things besides, as in (7d). An answer to this question will point to ways in which the familiar notion of conservativity may be generalized.

$$\text{CONS} \quad D\mathrm{N}_1 \ldots \mathrm{N}_n\mathrm{VP} \Leftrightarrow D\mathrm{N}_1 \ldots \mathrm{N}_n\mathrm{VP} \cap \textstyle\bigcup_i \mathrm{N}_i$$

Although the data resist to be stretched as we please, they are, as always, elastic rather than rigid. Yet I would say that when overtly modified, as in (8), the most generous option (7d) is to be preferred.

(8) Twelve students but fewer teachers compiled some cars,
alone or with others

Without such modification it seems more natural to have (7c), where the VP is restricted to collections that consist only of students and teachers. But on the neutral use of (7) I think the restriction to collections of only students and only teachers in (7b) does not occur. However, in case of (9) the judgments are the other way around (Scha, p.c.).

(9) More nightwalkers than daytrippers joined forces
distributive reading
a. $|\mathbf{at}([\![\mathrm{Wb}]\!]) \cap [\![\mathrm{N}]\!]| > |\mathbf{at}([\![\mathrm{Wb}]\!]) \cap [\![\mathrm{D}]\!]|$
options for neutral reading
b. $|\bigcup([\![\mathrm{Wb}]\!] \cap \wp([\![\mathrm{N}]\!]))| > |\bigcup([\![\mathrm{Wb}]\!] \cap \wp([\![\mathrm{D}]\!]))|$
c. $|\bigcup([\![\mathrm{Wb}]\!] \cap \wp([\![\mathrm{N}]\!] \cup [\![\mathrm{D}]\!])) \cap [\![\mathrm{N}]\!]|$
$> |\bigcup([\![\mathrm{Wb}]\!] \cap \wp([\![\mathrm{N}]\!] \cup [\![\mathrm{D}]\!])) \cap [\![\mathrm{D}]\!])|$
d. $|\bigcup([\![\mathrm{Wb}]\!]) \cap [\![\mathrm{N}]\!]| > |\bigcup([\![\mathrm{Wb}]\!]) \cap [\![\mathrm{D}]\!]|$

Here, the restriction to collections of just nightwalkers and just daytrippers in (9b) is preferred. Still the question is whether (7b,c) and (9b,c) should be seen as different readings of (7) and (9), respectively. I would not favour this. If temporal or contextual information is taken into account, the 'c'-reading could in both cases be opted for. Then, the predicate *to join forces* in (9) does not contain mixed collections of nightwalkers and daytrippers, which renders (9b,c) equivalent. (The status of the 'd'-readings is discussed in section 3.)

What (7–9) suggest is that there are at least four forms of conservativity for multiply-headed NPs. Depending on the reading of the determiner, $D\mathrm{N}_1 \ldots \mathrm{N}_n\mathrm{VP}$ could be equivalent to either of (10a–d).

(10) a. $D\mathrm{N}_1 \ldots \mathrm{N}_n \bigcup(\mathbf{VP} \cap \mathrm{AT}(\bigcup_i(\mathrm{N}_i)))$
b. $D\mathrm{N}_1 \ldots \mathrm{N}_n \bigcup(\mathbf{VP} \cap \bigcup_i \wp(\mathrm{N}_i))$
c. $D\mathrm{N}_1 \ldots \mathrm{N}_n \bigcup(\mathbf{VP} \cap \wp(\bigcup_i \mathrm{N}_i))$
d. $D\mathrm{N}_1 \ldots \mathrm{N}_n \{Y \mid \exists X \in \mathbf{VP} : Y = X \cap \bigcup_i \mathrm{N}_i\}$

If (9b) is reduced to (9c) in the way indicated above, conservativity (10b) would not occur separately.

It should be noted that the restriction of groups in conservativity (10c) is not easily available if the NPs in (7–9) are coordinations of some sort. Then the noun extensions are no longer available when the NPs are combined to form a complex NP. But the restriction is natural if the NPs are multiple-headed.

2.3 Collective Readings

A collective reading attributes a property to a collection which cannot be reduced to properties of individuals (atoms). The predication in (2b) as given by (3b) is of that kind. Now the question is whether the collective reading occurs for sentences with multiple-headed NPs. Here I argue that it does, but apparently not in the quantified form of (3b). To make my point, I use (11) as a prototypical case. In (11a–d), I have listed the four options for its collective reading which seem most reasonable. For different reasons they are all inadequate.

(11) More students than teachers went to the bar together
a. $\exists X \subseteq [\![\mathrm{S}]\!] \exists Y \subseteq [\![\mathrm{T}]\!][|X| > |Y| \wedge [\![\mathrm{Wb}]\!](X) \wedge [\![\mathrm{Wb}]\!](Y)]$
b. $\exists X \subseteq [\![\mathrm{S}]\!] \cup [\![\mathrm{T}]\!][|X \cap [\![\mathrm{S}]\!]| > |X \cap [\![\mathrm{T}]\!]| \wedge [\![\mathrm{Wb}]\!](X)]$
c. $\forall X \subseteq [\![\mathrm{S}]\!] \forall Y \subseteq [\![\mathrm{T}]\!][[\![\mathrm{Wb}]\!](X) \wedge [\![\mathrm{Wb}]\!](Y) \Rightarrow |X| > |Y|]$
d. $\forall X \subseteq [\![\mathrm{S}]\!] \cup [\![\mathrm{T}]\!][[\![\mathrm{Wb}]\!](X) \Rightarrow |X \cap [\![\mathrm{S}]\!]| > |X \cap [\![\mathrm{T}]\!]|]$

To see why these proposals will not do, I sketch some minimal situations with respect to which they yield counterintuitive results.

Ad. (11a) Suppose there are five students and five teachers, and that these students and teachers can both be partitioned into a pair and a triple so that all four collections went to the bar together. Then (11a) would be true. But on the same reading so would (12).

(12) More teachers than students went to the bar together

This is counterintuitive, for (11a) and (12) are contraries. In fact, the context sketched should make them false. As intuition would have it, there are as many students as teachers who went to the bar together in this situation.

Ad. (11b) Imagine a situation in which there are three students—s_1, s_2, and s_3—and three teachers—t_1, t_2, and t_3—such that s_1, s_2, and t_3 went to the bar together, and so did t_1, t_2, and s_3. Then (11b) would make this situation verify the two contraries (11) and (12), which it should not.

Ad. (11c) Suppose that there are three students who went to the bar together, and four teachers forming two pairs each going to the bar together. Now (11c) would be true. However, it is more natural here to hold that more teachers than students went to the bar together.

Ad. (11d) This formula would be false in the circumstance that there is at least one collection of just teachers that went to the bar together. Of course, this requirement is much too strict.

What these counterexamples suggest is that for the truth of (11) one is interested in the *maximal* number of students and teachers of the relevant kind. I see three ways in which maximality can be taken into account.

The first option is to quantify over the maximal elements in a predicate and is discussed in section 3.

A second option would be to use a neutral reading combined with the modifier τ (i.e., a simplified denotation of *together*). This modifier yields the set of non-atomic collections satisfying a predicate.

$$\tau := \lambda \mathbf{X} \lambda Y(\mathbf{X}(Y) \wedge |Y| > 1)$$

Then one would have, for instance:

(13) a. More students than teachers went to the bar together

b. $|\bigcup(\tau([\![\mathrm{Wb}]\!]) \cap \wp([\![\mathrm{S}]\!] \cup [\![\mathrm{T}]\!])) \cap [\![\mathrm{S}]\!]|$
$> |\bigcup(\tau([\![\mathrm{Wb}]\!])) \cap \wp([\![\mathrm{S}]\!] \cup [\![\mathrm{T}]\!])) \cap [\![\mathrm{T}]\!]|$

In words, the students which occurred in a (non-atomic) collection of just students and teachers that went to the bar together outnumber the teachers which occurred in such a collection. Maximality is now taken care of by taking the union of the relevant sets. Note, too, that the truthconditions given by (13b) accord with the intuitions given in the discussion of (11a–d).

A third option would be to introduce a referential collective reading. This has been suggested by Van Benthem (1986, 53), among others. The idea is that (11) describes contextually given sets of students and teachers. It could be formalized as follows:

(14) $|[\![\mathrm{S}]\!] \cap C| > |[\![\mathrm{T}]\!] \cap C| \wedge [\![\mathrm{Wb}]\!]([\![\mathrm{S}]\!] \cap C) \wedge [\![\mathrm{Wb}]\!]([\![\mathrm{T}]\!] \cap C)$

Again we have a form of maximality. In the context C, $[\![S]\!] \cap C$ and $[\![T]\!] \cap C$ are the maximal sets of students and teachers, respectively.[2]

This finishes my discussion of multiply-headed NPs in a collective environment. The observations are summarized as follows:

1. In a collective setting, distributive, collective, and neutral readings of sentences with multiply-headed NPs occur.
2. The conservativity behaviour of the NPs accords better with the multiply-headed analysis than with the 'conjunctive' alternatives.
3. None of the putative collective readings which quantify over collections are available.

As in the single-headed case, not all multiple head NPs allow all readings. Still one is left with the impression that each reading can be obtained in a systematic manner for the multiply-headed NPs that have it. Using the techniques promoted in Van Benthem 1991, the next section shows this impression is justified.

3 Three Readings, Three Liftings

In this section the observations made previously are incorporated into three liftings: one lifting for the distributive reading, one for the collective reading, and one for the neutral reading. The definition of the liftings is followed by a discussion of some alternatives.

The strategy opted for here is to let the liftings work on determiners. For intuitive and other reasons, semanticists have also used what might be called a VP strategy (Link 1984, Roberts 1987, Lønning 1987, among others). Then the collective reading of a determiner is taken as the basic case from which other readings result by means of VP modification. Using such notions as general applicability, maximality, and logical strength, I argue that the truthconditions which result within this strategy are not always the ones we are after.

3.1 Contextualized Liftings

The discussion in section 2 leads to the following liftings for determiners with n heads (the initial string of abstractions $\lambda C_1 \ldots C_n \lambda D \lambda X_1 \ldots X_n.$ is omitted).

(15) D $DX_1 \cap C_1 \ldots X_n \cap C_n \mathbf{at}(\mathbf{Y})$
C $DX_1 \cap C_1 \ldots X_n \cap C_n \bigcup_i C_i \wedge \bigwedge_i \mathbf{Y}(X_i \cap C_i)$
N $DX_1 \cap C_1 \ldots X_n \cap C_n \bigcup(\mathbf{Y} \cap \wp(\bigcup_i(X_i \cap C_i)))$

[2]In case the non-Boolean conjunction of NPs is seen as involving a discontinuous multiple-headed determiner, there would also be a 'mixed' collective option. In particular, one could define, e.g., *Two ... and three ...* as denoting:

$$\lambda XYZ.\ |X \cap Z| = 2 \ \wedge |Y \cap Z| = 3$$

Then the lifting: $DXYC \wedge \mathbf{Z}((X \cup Y) \cap C)$ with C a context set would give *Two students and three teachers went to the bar together* its appropriate meaning:
$|[\![S]\!] \cap C| = 2 \wedge |[\![T]\!] \cap C| = 3 \wedge [\![Wb]\!](([\![S]\!] \cup [\![T]\!]) \cap C)$.

At a conceptual level, the liftings are based on three positions which we seem to take in describing collections. In general, we either describe 'plain' individuals (D), or genuine collections (C), or—less frequently—individuals which take part in collections (N).

The liftings given here differ from those in Van der Does 1993 in the use of context sets (cf. Westerståhl 1984). As presented, the liftings are much like Kaplanian characters: they yield denotations as soon as the contextual information has been supplied. In (15), the most general form is given, where the context set varies per noun argument. Below, I often assume for simplicity that the context is uniform over all arguments.

Another difference with Van der Does 1993 is that the collective reading is referential, not quantificational. This models a suggestion by Van Benthem (1986, 53). It is also reminiscent of the way in which Lønning (1987) uses terms to give NP denotations. The impact of the difference between the referential and the quantificational treatment of collective readings is discussed shortly. Since most of the differences are already present in case of simple single-headed NPs, I mainly restrict myself to those.

3.2 Some Alternative Options

Now I aim to show that the liftings in (15) give much what is desired by comparing them with some alternatives. The options are evaluated by means of three interconnected criteria:

- whether or not their use should be restricted to a particular kind of determiner;
- whether they make use of maximal or minimal elements in the VP whenever required;
- whether they have the proper logical strength.

First I discuss the collective reading, then the distributive reading, then the neutral reading. A summary of the findings follows.

Collective readings The collective reading C is crucially different from its quantificational variant, which is considered most often in the literature. In case of single-headed determiners, the liftings corresponding to the quantificational readings are:

(16) E $\exists Z \subseteq X[\mathbf{Y}(Z) \wedge DXZ]$
U $\forall Z \subseteq X[\mathbf{Y}(Z) \Rightarrow DXZ]$

There are several observations which tell against the liftings in (16). Firstly, it is observed in section 2.3 that there is no straightforward generalization of the quantificational variant to the multiple head case. Of course, a more involved variant might exist. But since the simple lifting C works as well, it will be preferred. Secondly, the use of the liftings E and U should be restricted to determiners of a particular kind (cf. Van Benthem 1986, 52; Lønning 1987, 205; Van der Does 1993). To be precise, E only works as required in case of MON$\uparrow$

determiners, and its dual U only in case of MON$\downarrow$ determiners. Non-monotonic determiners are often treated by means of E. This is not entirely satisfactory, for then a sentence like (17a) comes close to meaning (17b).[3]

(17) a. Exactly three men joined forces
b. At least three men joined forces

The reason is that E allows the existence of several collections of three men who joined forces. Thirdly, E and U do not capture an intuition of maximality: the collection which is said to exist need not be the largest one of the relevant kind. It is not clear whether such maximality is required in case of collective readings (but see, e.g., the notion of kolkhoz-collectivity argued for by Verkuyl, this volume). The distributive and the neutral reading, however, do require a kind of maximizing. And it has been suggested that these readings should be obtained from E and U.

The issue of maximization and quantification is an interesting one.[4] For our present purposes, a first option to consider would be to restrict the quantification in E over the maximal elements in the VP. To this end one introduces the operator MAX:

(18) $\mathrm{MAX} \ := \ \lambda \mathbf{X} \lambda Y(\mathbf{X}(Y) \wedge \neg\exists Z \supseteq Y[\mathbf{X}(Z) \wedge Z \neq Y])$

The maximized version E^+ of E is now given by (19):

(19) $\mathrm{E}^+ \quad \exists Z \subseteq X[\mathrm{MAX}(\mathbf{Y})(Z) \wedge DXZ]$

In my opinion, E^+ does not offer much over and above the logically weaker E. Since the collective E^+ still leaves room for multiple maximal sets, the undesirable meaning shift from (17a) towards (17b) remains. To exclude multiple maximal sets, one could opt for an even stronger variant of E^+, which requires there to be exactly one such maximal set. In effect, this is what the referential collective reading does. E.g., the use of C makes (17a) mean:

(20) $|[\![\mathrm{M}]\!] \cap C| = 3 \wedge [\![\mathrm{Jf}]\!]([\![\mathrm{M}]\!] \cap C)$

Clearly, $[\![\mathrm{M}]\!] \cap C$ is the unique maximal set of men in context C.

Another intuition concerning collective sentences is that they should preclude subcollections of a collection described to have the property given by the VP. In case of (17a) this amounts to requiring that there should be no subsets of the set of contextually given men which joined forces in the same context. In order to obtain this reading, one should use MIN, i.e., the dual of MAX:[5]

(21) $\mathrm{MIN} \ := \ \lambda \mathbf{X} \lambda Y(\mathbf{X}(Y) \wedge \neg\exists Z \subseteq Y[\mathbf{X}(Z) \wedge Z \neq Y])$

The minimized version C^- of C is defined as follows:

[3]They are equivalent if the VP is closed under subsets.

[4]This issue is also raised by Szabolcsi 1981. Sher 1990 and Spaan 1993 discuss the topic with respect to branching quantification. Cf. also Verkuyl's discussion on totalization in this volume.

[5]MIN is an extensional version of the exhaustivization operator used by Groenendijk and Stokhof (1984, 299) in their analysis of wh-complements. They apply the operator to quantifiers rather than to verb phrases.

(22) $\mathrm{C}^- \;:=\; \lambda D\lambda X\lambda \mathbf{Y}.\mathrm{C}(D)(X)(\mathrm{MIN}(\mathbf{Y}))$
$\qquad\qquad = \;\lambda D\lambda X\lambda \mathbf{Y}.D(X\cap C)(C)\wedge \mathrm{MIN}(\mathbf{Y})(X\cap C)$

The lifting C^- gives (17a) the meaning:

(23) $|[\![\mathrm{M}]\!]\cap C| = 3 \wedge \mathrm{MIN}([\![\mathrm{Jf}]\!])([\![\mathrm{M}]\!]\cap C)$

In words, the set of men in context C is the unique set of men in that context which joined forces. This models Verkuyl's kolkhoz-collectivity (cf. this volume).

It should be observed that C, like E, still makes a lifted determiner MON$\uparrow$ in its second argument. Perhaps this is as it should be as soon as a particular collection referred to is described. Still, it seems odd to be able to derive *Exactly three men joined forces or drank tea together* from *Exactly three men joined forces*. Even in case of the collective reading one is inclined to assume that the monotonicity behaviour of the determiner is quite like that on its distributive use. The lifting C^- blocks the inference, since it destroys the monotonicity pattern of a determiner. Which treatment is to be preferred, if any, has to be left for further study.

Distributive readings Rephrased in terms of the present system, Link 1984 suggests to obtain the distributive reading of a sentence by means of the lifting E combined with the distribution operator δ. When viewed as a determiner lifting, this yields E_δ:

(24) $\delta \quad \lambda \mathbf{X}\lambda Y.\mathrm{AT}(Y)\subseteq \mathbf{X}$
$\qquad \mathrm{E}_\delta \quad \lambda DX\mathbf{Y}.\mathrm{E}(D)(X)(\delta(\mathbf{Y}))$

In Van der Does 1993 I argued against this strategy. On a distributive reading, it is expected that the determiner has its familiar monotonicity behaviour. Since E creates right upward monotonicity, its use is inadequate when applied to MON$\downarrow$ determiners. But such determiners often favour a distributive reading. Besides, on the distributive reading one is interested in determining the elements of the maximal set of noun atoms in the VP. The lifting D captures this maximality, while E_δ does not. The maximality of D can be seen most clearly in (25), which is equivalent to D on the conservative determiners.

(25) $DX_1 \ldots X_n \bigcup(\mathbf{Y}\cap \mathrm{AT}(\bigcup_i X_i))$

The second argument of (25) contains the sum of all relevant atoms, and this is the maximal set we are after. That the required maximality is absent in case of E_δ can be seen by means of (26):

(26) Exactly six pianists bought a keyboard each
$\quad \mathrm{D} \quad |\mathbf{at}([\![\mathrm{Bk}]\!])\cap [\![\mathrm{P}]\!]\cap C| = 6$
$\quad \mathrm{E}_\delta \quad \exists Z\subseteq [\![\mathrm{P}]\!]\cap C[\delta([\![\mathrm{Bk}]\!])(Z)\wedge |Z| = 6]$
$\qquad\quad \Leftrightarrow \exists Z\subseteq \mathbf{at}([\![\mathrm{Bk}]\!])\cap [\![\mathrm{P}]\!]\cap C[|Z| = 6]$

In the context reported on, the number of pianists who bought a keyboard should be exactly six—as for D—but E_δ allows there to be more.

In a reaction to an earlier version of Van der Does 1993, Lønning (1991, 50) suggests that the problems noted for E_δ might be overcome if we take the concepts of topic and focus seriously. The issue is taken up by Scha (1991, 56), who holds that a combination of maximization and distribution would solve the problems. Following up on Szabolcsi 1981, he reserves $E_\delta(\mathbf{six})$ for the unfocused use of *six* and M_6 for its focused use:

(27) M_m $\lambda X\mathbf{Y}.\max(\{n \mid E(n)(X)(\delta(\mathbf{Y}))\}) = m$
M_6 $\max(\{n \mid E(n)([\![P]\!])(\delta([\![Bk]\!]))\}) = 6$
$\Leftrightarrow D(\mathbf{six})([\![P]\!])([\![Bk]\!])$

Scha's proposal is attractive. In fact, M_n is equivalent to $D(\mathbf{n})$ for all n. It is not entirely clear, though, how his strategy generalizes to include non-numerical determiners. Retaining the idea to use maximization together with distribution, an alternative would be to maximize the distributed VP rather than the numeral. Indeed, if MAX is used in this way an equivalent of the lifting D results:

(28) E^+_δ $\exists Z \subseteq X[\mathrm{MAX}(\delta(\mathbf{Y}))(Z) \wedge DXZ]$
$\Leftrightarrow DX\mathbf{at}(\mathbf{Y})$

This is because $\mathbf{at}(\mathbf{Y})$ is the unique maximal element in $\delta(\mathbf{Y})$.[6] The observation also has a bearing on the question whether or not a distributive reading can be obtained via a VP strategy. The equivalence in (28) shows that this reading can result from VP modification as soon as the collective reading of an NP is given by E^+.[7] We shall see shortly that such a reduction is still unavailable for the neutral reading.

The above reduction works, since the quantificational E^+ is used to model collective readings. The situation is different for the referential variant C. E.g., the referential distributive reading of (29a) is (29b), while (29c) is that of (29d):

(29) a. Some men walk
b. $|[\![M]\!] \cap C| > 2 \wedge [\![M]\!] \cap C \subseteq \mathbf{at}([\![W]\!])$
c. All men walk
d. $[\![M]\!] \cap C \subseteq C \wedge [\![M]\!] \cap C \subseteq \mathbf{at}([\![W]\!])$
$\Leftrightarrow [\![M]\!] \cap C \subseteq \mathbf{at}([\![W]\!])$

Note that (29a) implies (29c). For *some* as for other determiners this implication is undesirable. In fact, one normally uses (29a) whenever the information to use (29c) is lacking. That is, in using (29a) there is often a scalar implicature that (29c) is not the case. Consequently, if the referential collective reading is preferred to the quantificational one, the distributive reading is better treated by means of D. On that reading the implication is absent.

[6] Using a different notion of maximization Martin van den Berg (p.c.) makes a similar point with respect to the lifting D_1 and N_2 in Van der Does 1993. Other than the present ones, these liftings presume determiners to be conservative.

[7] In Van der Does 1993, where I discussed the problems for the VP-strategy, I did not consider this possibility.

Neutral readings The neutral reading of a sentence should determine precisely which noun individuals occur in a collection having VP. E.g., in using (30) one is interested in the exact number of drums which took part in the joint production of a warsignal.

(30) Exactly six drums signalled 'War!' together

N $\mathrm{N}([\![\mathrm{six}]\!])([\![\mathrm{D}]\!])([\![\mathrm{Sw}]\!])$
$\Leftrightarrow |\bigcup([\![\mathrm{Sw}]\!] \cap \wp([\![\mathrm{D}]\!]))| = 6$

The most prominent situation associated with (30) is that in which one group of six drums jointly signalled 'War!' But there is another less prominent use of this sentence which reports on the total number of drums involved in producing such a signal. Then, the predicate *to signal 'War!' together* need not be assigned to just one group. Different groups may be involved, which do not form a unity at all from the perspective from which they are described. The precision required in such a description is given by N.

The lifting N is applied to determiners, not to verb phrases. So the semanticists who prefer the VP strategy will ask whether this reading can be obtained by means of VP modification. To this end, one could consider to introduce the operation π^* of type *et*(*ett.ett*).

(31)
$$\begin{aligned} \pi^* &:= \lambda X\mathbf{Y}Z.Z \subseteq \bigcup(\mathbf{Y} \cap \wp(X)) \\ \mathrm{E}^+_{\pi^*} &:= \lambda DX\mathbf{Y}.\mathrm{E}^+(D)(X)(\pi^*_X(\mathbf{Y})) \end{aligned}$$

The composition of π^* with the lifting E^+ defined in (19) yields the lifting $\mathrm{E}^+_{\pi^*}$. $\mathrm{E}^+_{\pi^*}$ is equivalent to N, since $\bigcup(\mathbf{Y} \cap \wp(X))$ is the only maximal element in $\pi^*_X(\mathbf{Y})$. Still, for the VP strategy this attempt is not fully successful. The operation π^* gives a VP modifier whenever it is supplied an N extension from the NP in [$_\mathrm{S}$ NP VP]. But in a compositional semantics this extension is not yet available when the VP is processed. By contrast, if the VP modification is made part of the determiner denotation the problem does not arise.

The VP strategy could try another route by introducing a VP modifier which is as independent of noun extensions as δ is. E.g., it could use the operator π of type *ett.ett*.

(32)
$$\begin{aligned} \pi &:= \lambda\mathbf{X}Y.Y \subseteq \bigcup\mathbf{X} \\ \mathrm{E}^+_\pi &:= \lambda DX\mathbf{Y}.\mathrm{E}^+(D)(X)(\pi(\mathbf{Y})) \\ \mathrm{N}^+ &:= \lambda DX\mathbf{Y}.D(X)(\bigcup\mathbf{Y}) \end{aligned}$$

Clearly, if E^+_π yields a defensible neutral reading the VP strategy works for that readings as well. To see whether it does, we first observe that $\bigcup\mathbf{Y}$ is the unique element in $\mathrm{MAX}(\pi(\mathbf{Y}))$ so that E^+_π is equivalent to N^+. Secondly, we note that N^+ has an attractive feature which N lacks; namely, it also takes account of collections with elements that do not belong to the noun extension. More in particular, if N(**six**) is used in the meaning of (30), it cannot describe the drums which made the signals together with other instruments. When N^+(**six**) is used, such a description is possible.

(33) Exactly six drums signalled 'War!' together

N^+ $\mathrm{N}^+([\![\mathrm{six}]\!])([\![\mathrm{D}]\!])([\![\mathrm{Sw}]\!])$
$\Leftrightarrow |\bigcup([\![\mathrm{Sw}]\!]) \cap [\![\mathrm{D}]\!]| = 6$

Unfortunately, N^+ is too weak and hence offers no solution for the VP strategy. To be precise, (34) should be false if each drum forms the modest rhythm section of six overwhelming groups of piccolos. But only N makes it false then:

(34) Exactly six drums produced a lot of noise
N $|\bigcup([\![Pn]\!] \cap \wp([\![D]\!]))| = 6$
N^+ $|\bigcup([\![Pn]\!]) \cap [\![D]\!]| = 6$

A way to circumvent the issue is to assume that relative to a description such mixed collections are not relevant at all. Although (34) should not preclude mixed groups of instruments, it can be maintained that for its truth only drums matter. Whenever the predicate *to produce a lot of noise* holds of mixed collections of drums and piccolos, there should also be a subgroup of just drums with this property if (34) is to be true. Then, N gives what is required after all.

To summarize, there are three sensible embeddings of multiple-headed determiners into a collective environment. They are based on the ways in which collections figure in our descriptions: as atoms (D), as genuine collections (C), or as atoms partaking in a collection (N). I have considered some alternatives of these readings. But they are either not general enough (E, E_δ), or they do not capture a relevant notion of maximality (E, E_δ), or they are logically to weak (N^+). In the course of this discussion it emerged that if the maximized E^+ is used for the collective reading of single-headed NPs, the distributive reading D could be obtained via VP modification. This is not so for the neutral reading N. VP modification does not give the appropriate distributive and neutral reading if the collective reading is treated referentially. But only this collective reading seems to generalize to multiple headed NPs in a straightforward way. I conclude that for the sentences considered the NP is the best place to generate the readings. That is, if an NP allows for a particular reading it can be obtained via the corresponding lifting.

This ends the discussion on readings. Among other things, the next section shows that the liftings can also be used to give the semantics of complex definite NPs and partitive NPs in a collective setting.

4 Definite and Partitive Noun Phrases

Until now, the only complex NPs considered are the multiply-headed ones. But of course there are also definite NPs besides NPs of different kinds of complexity. These too may occur in sentences with collective and other readings. To indicate the power of the present approach, I show in this section that the insights obtained thus far can be extended to complex definite NPs and to partitives.

4.1 Complex Definites

Now, I pay attention to the semantics of NPs such as *the boys* and *these many proofs* within the standard 'atomic' models. Following up on Verkuyl and Van der Does (1991), Verkuyl (1993b), Westerståhl (1984, 1989), I try to account for these NPs in a compositional manner. I aim to give denotations which (partly) explain the distribution within these phrases. The techniques used here are familiar within generalized quantifier theory. This section ends with showing how the denotations proposed can de embedded within a collective environment.

In generalized quantifier theory definite NPs are prime examples of context dependent expressions. For instance, the differences between (35a–c) are captured in terms of Westerståhl's contexts sets.

(35) a. $[_{\text{np}}\ [_{\text{det}}\ \text{the}]\ [_{\text{n}}\ \text{proof}]]$
b. $[_{\text{np}}\ [_{\text{det}}\ \text{this}]\ [_{\text{n}}\ \text{proof}]]$
c. $[_{\text{np}}\ [_{\text{det}}\ \text{those}]\ [_{\text{n}}\ \text{proof}]]$

Disregarding number, the denotations of *the*, *this*, *that*, *these*, and *those* are all of the same form.[8]

(36) $\lambda CXY.X \cap C \subseteq Y$

These determiners differ at most with respect to the set C provided by the context.

It is a distinctive feature of definite determiners that they may be followed by a numeral such as *many*, *few*, *one*, ...

(37) a. these many proofs
b. $[_{\text{np}}\ [_{\text{det}}\ \text{these}]\ [_{\text{n}}\ [_{\text{adj}}\ \text{many}]\ [_{\text{n}}\ \text{proofs}]]]$
c. $[_{\text{np}}\ [_{\text{det}}\ [_{\text{det}}\ \text{these}]\ [_{\text{mod}}\ \text{many}]]\ [_{\text{n}}\ \text{proofs}]]$

There has been some debate on the precise structure of these noun phrases. Sometimes the numerals are treated as adjectives which form complex nouns (cf., 37b). But it has also been suggested that the numeral modifies the determiner so as to yield a complex determiner (cf., 37c). In both cases it seems natural to see the modifiers as derived from the denotation of *two* in type *et.ett*, if only to highlight the close ties of this use of numerals with their use as determiners. The operations in (38) accomplish this.

(38) $\text{A} := \lambda D\lambda X.X \cap \lambda x.D(X)(\top)$
$\text{M} := \lambda DD'\lambda XY.D'(X \cap \lambda x.D(\bigcap D'(X))(\top))(Y)$

Here $\top$ is a constant of type *et* which denotes the domain $\mathbf{D}_e$. Note that the operation A of type $(et.ett)(et.et)$ transforms a determiner denotation into an intersective adjective (i.e., a noun modifier). By contrast, the more complex M of type $et.ett((et.ett)(et.ett))$ turns a determiner into a modifier of determiners, which is a function from determiners to determiners. A few examples should clarify how A and M work. In the course of this discussion some reasons emerge to prefer the structure in (37c) over that in (37b).

The examples use the numeral *many*. When this numeral is used as a determiner it could have the denotation (39a) (cf., Westerståhl 1989 for a discussion of other alternatives).

[8] Cf., Verkuyl and Van der Does (1991). Westerståhl (1984) holds that definites are no determiners (type *et.ett*). Semantically they rather function like nouns (type *et*) in that they denote context sets. On this account their occurrence in phrases such as (35a) should be seen as the tacit partitive *all of the*. I prefer a more direct way of interpretation.

(39) a. $\lambda XY.|X \cap Y| \geq \mu(X)$

b. Many boys dated many girls

Here, μ is a contextually given function from sets to numbers which determines for each noun extension what is to be counted as many. This captures the observation that in Partee's (39b) the measure involved in *many boys* may be different from that in *many girls*. The denotation of the adjective *many* is (40a), which reduces to (40b).

(40) a. $\mathrm{A}([\![\mathrm{many}]\!])$

b. $\lambda X(X \cap \lambda x.|X| \geq \mu(X))$

The set $\lambda x.|X| \geq \mu(X)$ equals $\top$ iff there are many X's, i.e., iff $|X| \geq \mu(X)$ is true. Otherwise this set equals $\emptyset$. Therefore, *many proofs* denotes just *proofs* iff there are many proofs. It denotes the empty set iff there are not many proofs (similarly for the other numerals).

This treatment of the numeral gives the NP *these many proofs* the semantics (41b).

(41) a. $[_{\mathrm{det}}\mathrm{these}][_{\mathrm{n}}[_{\mathrm{adj}}\mathrm{many}][_{\mathrm{n}}\mathrm{proofs}]]$

b. $[\![\mathrm{these}]\!](\mathrm{A}([\![\mathrm{many}]\!])([\![\mathrm{proofs}]\!]))$

c. $|[\![\mathrm{P}]\!]| \geq \mu([\![\mathrm{P}]\!])$

d. $\lambda Y.[\![\mathrm{proof}]\!] \cap C_{these} \subseteq Y$

e. $\lambda Y.\emptyset \subseteq Y$

Accordingly, *these many proofs* denotes (41d) if there are many proofs ((41c) is true). But if not, it is the trivial quantifier (41e), which holds of all properties. It could be argued that this semantics for the complex NPs is incorrect. In particular, *these many proofs* requires that there be many proofs in the context delineated by the demonstrative. But in (41c) the noun extension is not restricted by that context at all. So (41b) fails to capture that the NP *these many proofs* involves many contextually given proofs rather than just many proofs.

A more sophisticated semantics of numerals as adjectives may overcome this difficulty. It is also absent in case the numeral modifies the determiner as in (37c). The semantics for this structure is based on the idea that M generates the denotation (42b).

(42) a. $\mathrm{M}([\![\mathrm{many}]\!])$

b. $\lambda D'\lambda XY.D'(X \cap \lambda x[|\bigcap D'(X)| \geq \mu(\bigcap D'(X))])(Y)$

The lifting M makes the phrase (43a) denote (43b).

(43) a. $[_{\mathrm{np}}\ [_{\mathrm{det}}\ [_{\mathrm{det}}\ \mathrm{these}][_{\mathrm{mod}}\ \mathrm{many}]][_{\mathrm{n}}\ \mathrm{proofs}]]$

b. $\mathrm{M}([\![\mathrm{many}]\!])([\![\mathrm{these}]\!])([\![\mathrm{proofs}]\!])$

c. $|[\![\mathrm{P}]\!] \cap \mathrm{C}_{these}| \geq \mu([\![\mathrm{P}]\!] \cap \mathrm{C}_{these})$

d. $\lambda Y.[\![\mathrm{P}]\!] \cap \mathrm{C}_{these} \subseteq Y$

e. $\lambda Y.\emptyset \subseteq Y$

As before, (43b) reduces to (43d) if (43c) is true; otherwise, (43b) denotes the trivial (43e). Notice that in contrast with (41c), (43c) does take the contextual restriction of the demonstrative into account. It does so by means of a device which is borrowed from Barwise and Cooper (1981, 184), namely to take the intersection $\bigcap Q$ of a quantifier Q. In general this operation yields a non-trivial restriction $\bigcap D'(X)$ whenever D' is a definite determiner. E.g., this set is $[\![\mathrm{P}]\!] \cap \mathrm{C}_{these}$ in case of (43d).

Apart from capturing the appropriate contextual restriction in (37), the use of M also enables us to explain some distribution facts concerning these phrases. As we shall see, the operation $\bigcap$ restricts the first position in a phrase such as *these many boys* to the definite determiners. And M excludes expressions at the second position which reduces to the existential or the universal determiner. The reason is that these expressions always make trivial contributions to the denotation of the entire phrase. So they might as well be left out. (In GQT, and elsewhere, this is the familiar reasoning to show that the use of an expression is analomous.) Now, I give the details of how this triviality comes about.

Barwise and Cooper propose that a determiner is definite if it always denotes a principal filter. That is, a determiner D is definite iff for all domains E and for all $X \subseteq E$ for which $D_E(X)$ is non-trivial, there is a non-empty $Y \subseteq E$ with $D_E(X) = \{Z \subseteq E : Y \subseteq Z\}$. Cf., Barwise and Cooper, 1981, 184. Clearly, if D is definite and non-trivial for $X \subseteq E$: $\bigcap D_E(X) = Y$. But in quite a few other cases $\bigcap D_E(X)$ will invariably be empty. Then D cannot make an informative contribution to the denotation of the complex phrase, which is a sign of markedness. Similarly, the D' in the restriction $\lambda x.D'(\bigcap D(X))(\top)$ may always induce triviality, even if $\bigcap D(X)$ is not itself trivial. This is the case, e.g., if D is *some*, *all*, *no*, and *not all*. These observations lead to the following constraint.

Constraint 4.1 *In a configuration* $[_{\mathrm{det}}\ [_{\mathrm{det}}\ \mathrm{DET}^1]\ [_{\mathrm{mod}}\ \mathrm{DET}^2]]$ *only those* DET^1*s are allowed for which* $\bigcap [\![\mathrm{DET}^1]\!]_E(X)$ *is non-empty in case* $[\![\mathrm{DET}^1]\!]_E(X)$ *is non-trivial. And* DET^2 *is restricted to those D such that $D_E(X)(E)$ does not give the same truth value for all E and all non-empty $X \subseteq E$.*

There are some apparent counterexamples to constraint 4.1 which are discussed shortly. But first I give an impression of the DETs which are allowed in these positions.

As to DET^1, the DETs which occupy this position are the maximal ones defined by:

Definition 4.2 (maximality) *A determiner D is maximal iff for all E and for all $X \subseteq E$ such that $D_E(X)$ is non-trivial, there is a non-empty $Y \subseteq E$ with $D_E(X) \subseteq \mathbf{all}_E(Y)$.*

The definites as defined by Barwise and Cooper are the special case of maximal determiners where $D_E(X) = \mathbf{all}_E(Y)$. Almost by definition we have that D is maximal on E iff for all X such that $D_E(X)$ is non-trivial: $\bigcap D_E(X) \neq \emptyset$. Combined with the fact that natural language seems to lack simple maximal determiners which are not definite, we see that a simple DET^1 in the structure

$[_{\text{det}}\ [_{\text{det}}\ \text{DET}^1]\ [_{\text{mod}}\ \text{DET}^2]]$ can only be definite. Indeed, the NPs (44a–c), in which DET^1 is not maximal, are all ungrammatical.

(44) a. *some many boys
b. *two many boys
c. *most many boys

As to DET^2, prime examples of determiners which violate this constraint are definites (i.e., basically the determiner *all*) and the determiners from the square of opposition (*some*, *all*, *no*, *not all*). This complies with that fact that (45a–e) are unacceptable.

(45) a. *the all boys
b. *the those boys
c. *the some boys
d. *the most boys
e. *the no boys

The phrases (45a,b) are ruled out since $\mathbf{all}_E(X)(E)$ is true for all E and $X \subseteq E$. And a similar explanation can be given for *those* and other definites (cf. Westerståhl 1989). In case of (45c–e) the explanation depends on the fact that on the Russellian view a definite of the form DEF DET *boys* may only be used if the set of contextually given boys is non-empty. But for non-empty X $\mathbf{most}_E(X)(E)$ is equivalent to $\mathbf{some}_E(X)(E)$. And the latter is true for all E and all non-empty $X \subseteq E$. Also, $\mathbf{no}_E(X)(E)$ is false for all such E and X. (Query: what precisely are the D for which $\lambda X.D_E(X)(E)$ is constant for all E?)

Counterexamples to constraint 4.1 are the phrases (46a,b).

(46) a. all the boys
b. some fifteen boys
c. these or those two boys
d. all or some boys
e. *all or some the boys
f. *all or some fifteen boys

It could be argued that in (46a,b) *all* and *some* function as modifiers with a limited distribution. Cf., De Jong 1991, 61, 64. In particular, *all* only modifies definites and *some* only non-vague numerals. This is underpinned by the observation that coordination, which is possible between the determiners in (46c,d), is unacceptable in case of the modifiers in (46e,f). Also problematic for constraint 4.1 are (47a,b).

(47) a. *the at least two boys
b. *those at most five boys

Generalized quantifier theory treats the phrase *at least two* as a non-compound determiner. Since it does not lead to triviality, constraint 4.1 incorrectly fails to rule it out. Again a solution might be to take phrases such as *at least* as modifiers. But here as in the previous case I have nothing principled to offer.

It is time to see how complex definite NPs can be treated within a collective environment. I argued that the analysis of compound determiners by means of M is to be preferred to that by means of A. The use of A fails to give the appropriate contextualized argument of DET^2. Still, it is useful to show for both kinds of approaches how they could be used with in a collective semantics. We shall see that in this respect the treatments are the same: no new differences arise within the richer setting.

Recall that in the worst case the liftings used in section 3 were of the form (48), with F a function of type *et*(*ett.ett*)

(48) $\mathrm{L} := \lambda D X \mathbf{Y}.D(X)(\mathrm{F}_X(\mathbf{Y}))$

(I write $\mathrm{F}_X(\mathbf{Y})$ instead of $\mathrm{F}(X)(\mathbf{Y})$.) Depending on whether the second determiner in a complex definite is analysed as an adjective or a modifier, such a lifting L functions as in (49a) or as in (49b).

(49) a. $\mathrm{L}(D)(\mathrm{A}(D')(\mathrm{X}))(\mathbf{Y})$
$\Leftrightarrow D(\mathrm{A}(D')(\mathrm{X}))(\mathrm{F}_{\mathrm{A}(D')(\mathrm{X})}(\mathbf{Y}))$
b. $\mathrm{L}(\mathrm{M}(D)(D'))(\mathrm{X})(\mathbf{Y})$
$\Leftrightarrow D(X \cap \lambda x.D'(\bigcap D(X))(\top))(\mathrm{F}_{X\cap\lambda x.D'(\bigcap D(X)(\top))}(\mathbf{Y}))$
c $D(X)(\mathrm{F}_X(\mathbf{Y}))$
$\Leftrightarrow \mathrm{L}(D)(X)(\mathbf{Y})$

The denotations (49a,b) look rather different. But in case D and D' comply with constraint 4.1, they both result in (49c).

This ends my discussion of the denotations of complex definite NPs. In section 5, I discuss their problematic behaviour within some transitive sentences. But first I pay attention to the semantics of partitives.

4.2 Partitives

Westerståhl 1984 has a semantics for partitives such as *two of the three*. His idea is that the definite *the* in this partitive does not denote a determiner, type *et.ett*, but a context set, type *et*. The numeral in the partitive—or other DETs which may occur in its stead—is still taken to be of type *et.ett*. Since in the standard atomic semantics, the partitive is of type *et.ett*, one may wonder about the options available for the type of *of*. Westerståhl does not concern himself with this issue and gives the semantics of partitives in one go:

(50) SYN DET_1 of DEF DET_2
SEM $\lambda X \lambda Y.[\![\mathrm{DET}_1]\!](X \cap [\![\mathrm{DEF}]\!])(Y)$
$\wedge\ [\![\mathrm{DET}_2]\!](X \cap [\![\mathrm{DEF}]\!])(\top)$

Here, the partitive is a determiner complex whose grammatical structure is left unanalysed. This semantics gives for instance:

(51) a. Two of the three men whistle

b. $|[\![the]\!] \cap [\![\mathrm{M}]\!] \cap [\![\mathrm{W}]\!]| = 2 \wedge |[\![the]\!] \cap [\![\mathrm{M}]\!]| = 3$

In words, (51a) is true iff there are three men in the context, two of which walk. This is correct. But some might prefer to capture the presupposition that there are three man in the context in another way. For our present purposes we may ignore such difficulties.

Partitives can also induce readings other than the distributive one, as in (52a) meaning (52b). So the question presents itself how to make the transformations introduced in section 3 available for these more complex determiners. Which components of a partitive are to be lifted, and why these?

(52) a. Two of these four men wrote a book together

b. $\exists X \subseteq [\![\mathrm{M}]\!] \cap [\![these]\!]$
$[|X| = 2 \wedge [\![\mathrm{Wb}]\!](X) \wedge |[\![\mathrm{M}]\!] \cap [\![these]\!]| = 4]$

As it turns out, (52) can still be had in a uniform way from Westerståhl's semantics by means of the lift E introduced before. In general, one can use a lift L of type $(et.ett)(et.ettt)$ to transform the partitive denotations to a collective setting, as follows:

(53) $\mathrm{L}(\mathrm{DET}_1 \text{ of } \mathrm{DEF}\ \mathrm{DET}_2)$
$:= \lambda X \lambda \mathbf{Y}.\mathrm{L}(\mathrm{DET}_1)(X \cap \mathrm{DEF})(\mathbf{Y}) \wedge \mathrm{DET}_2(X \cap \mathrm{DEF})(\top)$

Note that in (53) only DET_1 is affected by L. However, if *of* is a three-place relation between two DETs and a DEF it is not evident why this should be so. A more important shortcoming of (50) and (53) is that they cannot account for the examples in (54) (cf. Stockwell, Schachter and Partee as cited in Hoeksema 1984, 20):

(54) a. Each one of us
b. One of the last of the Mohicans
c. Two cooks of those we hired last summer
d. More of these students than of those teachers
e. Most of the professors and of the managers
f. The brightest of the professors but not of the managers
g. Some of this club or of that society
h. The members of this {and,or,but not} of that society

The examples (54a–c) are from Hoeksema (1984, 20) who argues that a partitive contains an *of*-phrase which functions as a postnominal modifier. These modifiers often take the noun *one* or an empty noun as argument, as in (54a,b,d–g). But (54c,h) are among the rare examples where a common noun is modified. The NPs in (54d–h) give some further evidence that partitive *of*-phrases form a constituent. In (54d) they occur in both arguments of a two-ary determiner, and in (54e–h) they are coordinated in different ways.

This is not the place to argue for a particular syntactic analysis of the partitive. Nevertheless, Westerståhl's semantics has to be altered if we are to account for (54).[9] It would be quite unattractive to see all these NPs as obtained from different schemes by filling their slots with DETs, DEFs and Ns. Fortunately, it is not hard to give a semantics for partitives involving *of*-phrases as postnominal modifiers. To do so, one takes *of* to be of type *ett*(*et.et*) so that it yields a modifier when given a noun phrase. Borrowing once more from Barwise and Cooper (1981) its semantics would be:

(55) $\lambda Q_{ett}\lambda X_{et}.X \cap \bigcap Q$

Note that the use of $\bigcap$ requires the NP occurring in the *of*-phrase to be a definite (cf. the discussion in section 4.1). And this seems as it should be.

Here is an example of how this semantics works. On the assumption that (56a) is true, i.e., that four men are referred to by means of *these four*, the denotation of the definite *these four men* is (56b).

(56) a. $|[\![\mathrm{M}]\!] \cap [\![these]\!]| = 4$
b. $\lambda Y.[\![\mathrm{M}]\!] \cap [\![these]\!] \subseteq Y$

The intersection of this NP is the set $[\![\mathrm{M}]\!] \cap [\![these]\!]$. Therefore, the modifier (57a) denotes (57b) whenever (56a) is true.

(57) a. $[_{\mathrm{pp}}$ of $[_{\mathrm{np}}$ these four men$]]$
b. $\lambda X_{et}.X \cap [\![\mathrm{M}]\!] \cap [\![these]\!]$
c. $\lambda X.\emptyset$

In case (56a) is false this phrase denotes the trivial (57c).

Within an $\overline{\mathrm{X}}$-theory, a PP such as (57a) would transform an N into an $\overline{\mathrm{N}}$. When applied to the empty noun $\emptyset_{\mathrm{n}}$, which denotes the domain $\mathbf{D}_e$, one gets (58a) meaning (58b):

(58) a. $[_{\overline{\mathrm{n}}}\ \emptyset_{\mathrm{n}}\ [_{\mathrm{pp}}$ of these four men$]]$
b. $[\![\mathrm{M}]\!] \cap [\![these]\!]$
c. $[_{\mathrm{np}}$ two$_{\mathrm{det}}$ $[_{\overline{\mathrm{n}}}\ \emptyset_{\mathrm{n}}\ [_{\mathrm{pp}}$ of these four men$]]]$
d. $\lambda Y.|Y \cap [\![\mathrm{M}]\!] \cap [\![these]\!]| = 2$

This $\overline{\mathrm{N}}$, in turn, can combine with a DET to form an NP in the familiar way. For instance, the NP *two of these four men* has the structure in (58c) and means (58d). Clearly, all the examples in (54a–h) can be had along these lines. Moreover, the only way to obtain collective and other readings for the NP-structure in (58c) is to lift the main DET in this NP, leaving its *of*-phrase outside the scope of the transformation. This means that the way in which liftings are applied in (53) is now better accounted for than when partitives are seen as complex determiners.

To recapitulate, I have discussed Westerståhl's (1984) treatment of partitives by means of context sets. It was noted that this analysis cannot account for the partitive constructions discussed in Hoeksema 1984 and the references therein. But I have shown that Westerståhl's proposal can be accommodated to the case

[9] Cf. De Jong 1991 for an overview of the issues at stake. Like Keenan and Stavi 1986, she argues that partitives are NPs with a complex determiner structure.

where partitives are analysed as involving a PP. As a result, we now have the semantics for a vast supply of complex NPs. And what is more, the uniformity in obtaining collective and other readings for the simple NPs can be retained unaltered.

Now I return to the semantics of definite NPs. I will focus, in particular, on their semantic behaviour in some problematic transitive sentences.

5 Sentences with Definites

Langendoen (1978) and Scha (1981) are among the first to note that sentences with definite NPs are a challenge for a compositional semantics. In this article, I concentrate on Scha. First, I recall the kind of examples which led to his position that definite NPs are univocally collective. This is followed by an overview of some problems for this analysis. Next, I give an alternative analysis in terms of the semantics developed in section 4. It retains Scha's strategy to combine a particular denotation for definites with the use of meaning postulates. But I show that the neutral cumulative reading presented in Van der Does 1993 gives a more satisfactory semantics of the sentences considered. Distributive uses of definites should be treated separately.

5.1 Scha's Treatment of Definites

Scha took his proposal concerning definite NPs to be an important contribution of his 1981 paper. This is clear from its opening lines:

> Theories which relate English sentences to logical formulas representing their truthconditions usually assume that for every noun phrase in the surface structure there is a quantifier in the logical formula, and that it depends on the determiner of the noun phrase what quantifier that is. The present paper points out some hitherto neglected phenomena which give rise to significant departures from this approach. Scha (1981, 483)

The 'neglected phenomena' show that sentences with definite NPs differ vastly in logical form, even though their syntactic structure remains the same. This led Scha to argue that definites refer 'to the set of all entities in the extension of the noun' (Scha 1981, 495). That is, viewed as a quantifier a simple definite always has the form $\lambda \mathbf{X}.\mathbf{X}([\![\mathrm{N}]\!])$. In terms of the semantics developed in the previous sections, this would mean that *the* in type *ettt* only has its collective reading C. This has the important consequence that definite NPs, like proper names, are scopeless.

Scha's arguments for taking this position centre around (59a–c). To begin with, he states that (59a,b) are true of figure 1 and that (59c) is true of figure 2.

(59) a. The sides of R1 run parallel to the sides of R2
b. The sides of R1 cross the sides of R2
c. The squares contain the circles

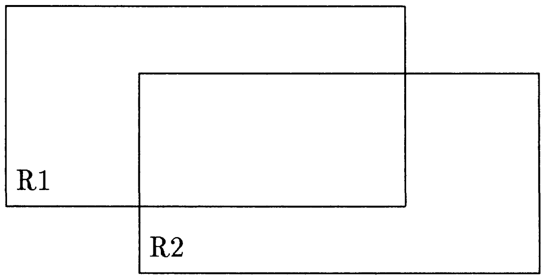

Figure 1: *The sides of R1 run parallel to/cross the sides of R2*

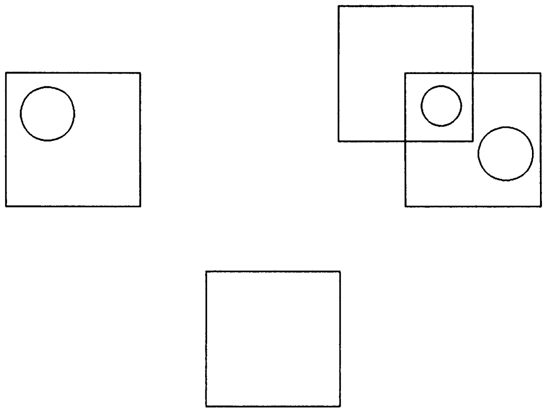

Figure 2: *The squares contain the circles*

This would be due to the fact that (59a–c) respectively mean (60a–c):

(60) a. $\forall x \in \llbracket \text{R1} \rrbracket \exists y \in \llbracket \text{R2} \rrbracket [x \parallel y] \wedge \forall v \in \llbracket \text{R2} \rrbracket \exists w \in \llbracket \text{R1} \rrbracket [v \parallel w]$

b. $\exists x \in \llbracket \text{R1} \rrbracket \exists y \in \llbracket \text{R2} \rrbracket [\llbracket \text{crosses} \rrbracket (x, y)]$

c. $\forall x \in \llbracket \text{C} \rrbracket \exists y \in \llbracket \text{Sq} \rrbracket [\llbracket \text{contains} \rrbracket (x, y)]$

To the extent that these judgments are viable, the problem is of course to account for these differences in a compositional way. Scha's solution is to attribute the semantic variance to the verb. In particular, the verbs satisfy the meaning postulates (61a–c) whenever their arguments are collections.

(61) a. $X \parallel Y$
$\Leftrightarrow \forall x \in X \exists y \in Y [x \parallel y] \wedge \forall v \in Y \exists w \in X [v \parallel w]$

b. $\llbracket \text{crosses} \rrbracket (X, Y)$
$\Leftrightarrow \exists x \in X \exists y \in Y [\llbracket \text{crosses} \rrbracket (x, y)]$

c. $\llbracket \text{contains} \rrbracket (X, Y)$
$\Leftrightarrow \forall x \in Y \exists y \in X [\llbracket \text{contains} \rrbracket (x, y)]$

As I said above, definite NPs should only have the collective reading given by $\lambda \mathbf{Y}.\mathbf{Y}(\llbracket \text{N} \rrbracket)$. It is clear that such NP denotations combined with the verb meanings in (61a–c) give (60a–c).

5.2 Some Problems for Scha's Proposal

I do not object to the overall strategy proposed by Scha (below I basically use the same strategy). Nevertheless, one cannot ignore that Scha's proposal is not fully satisfactory. Here, one may note three problems ordered by increasing urgency:

1. The problem of vagueness and context dependency in predicating a property of a collection;
2. The problem that one reading for definite NPs will not do;
3. The problem that meaning postulates for verbs which work well for definites should also work well for different kinds of NP.

I discuss the problems in this order.

Vagueness and context dependency How many elements of a collection have to be involved before a collective predicate can be attributed to it? This is often a vague and context dependent matter, where at first sight no logical or other structure seems to be present. Therefore, an attempt to capture the required involvement of the elements by means of meaning postulates quickly leads to empirical inadequacy (cf. Roberts 1987, Lønning 1987). This is true, for example, of the meaning postulate (61b). To see this, imagine two impressive arrays of horizontal and vertical lines. As soon as one vertical line crosses one horizontal line the postulate (61b) will make (62) true.

(62) The horizontal lines cross the vertical lines

However, many people think that just one crossing of lines is insufficient for its truth: (61b) is too weak. Meaning postulate (61c) is also a bit troublesome. There is a sense in which figure (3) verifies (59c). But (61c) would make it false,

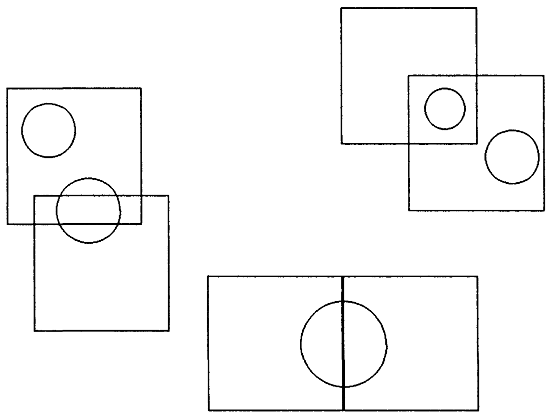

Figure 3: *The squares contain the circles*

since there are circles not contained in a single square.[10]

[10]This is reminiscent of Langendoen's (1978, 188) reasons to adopt his second-order principle (K) to give the truthconditions of simple sentences with definites. Lønning (1987, 223) has similar examples concerning boys eating cakes.

Univocal definites Quite a few sentences—(63a) among them—contain a definite but are nevertheless distributive. How to account for this if the definite has nothing but a collective reading? As Roberts (1987) points out, the distributive reading (63b) of (63a) cannot be obtained via a meaning postulate of *to lift*. The entire VP *to lift a piano* is distributed over the NP *the men*.

(63) a. The men lifted a piano
b. $[\![\mathrm{M}]\!] \subseteq \mathbf{at}([\![\mathrm{Lp}]\!])$

This phenomenon, combined with the intuition that distributivity is a lexical matter of the verb, made Link use the distribution operator δ.

(64) $\delta([\![\mathrm{Lp}]\!])([\![\mathrm{M}]\!])$

The distributive reading of (63a) is now given by (64), where δ is taken to be used covertly. For Scha this route is unattractive. In case of indefinite NPs he holds that distributivity comes from the quantifier. And how to defend this apparent lack of uniformity across the different kinds of NPs? By contrast, I propose to make definites equivocal, just as most other NPs are. The semantics in section 4.2 shows how this can be modelled.

Meaning postulates and indefinites Perhaps the most urgent problem for Scha's meaning postulates is that they are also in use when the verbs combine with indefinite NPs. Often the result of such a combination is undesirable. On the collective reading of the indefinites, (65a) means (65b).

(65) a. Two red lines cross three green lines
b. $\exists X \subseteq [\![\mathrm{rL}]\!] \exists Y \subseteq [\![\mathrm{gL}]\!]$
$[|X| = 2 \wedge |Y| = 3 \;\wedge\; \exists x \in X \exists y \in Y[[\![\mathrm{C}]\!](x, y)]]$

In words, there are two red and three green lines, and at least one of these red lines crosses one or more of those green lines. The truthconditions in (65b) are too weak, and similar unwanted side-effects occur with the other meaning postulates.

In the next section I shall show that the neutral reading of a definite in combination with the use of other meaning postulates leads to more desirable results.

5.3 An Alternative Proposal

The problems in stating the truthconditions of (59a–c) led Roberts (1987, 145) to hold that we should refrain from specifying the relation between collections in terms of their subsets. Instead, we should use the fact that the transitive verb denotes a relation between collections, and stipulate that the sentences are true iff the collections referred to by the definite NPs stand in that relation.

Surely, the simplicity of this position is attractive. But it has the drawback that it makes the treatment of distributive readings a bit arbitrary. These readings *are* specified in terms of the members of a collection. But why make an exception here? Moreover, Roberts employs the distribution operator for this purpose, and in general the use of this operator generates non-existent readings (see Van der Does 1992, 31–33). A way to circumvent such non-readings is to let the determiners do all the work. This yields distributive readings along the same lines as in case of other NPs.

I will show that the neutral cumulative reading rather than the doubly collective reading can be used to solve the problems noted for (59a–c). Instead of giving the general form of the cumulative reading, an example may suffice to show how it works. E.g., the cumulative reading of (66a) is (66b):

(66) a. These 5 Dutch firms own those 60 American computers

b. $\mathrm{N}([\![\text{these 5}]\!])([\![\mathrm{dF}]\!])(\mathrm{dom}([\![\mathrm{O}]\!] \cap \mathrm{RES})) \wedge$
$\mathrm{N}([\![\text{those 60}]\!])([\![\mathrm{aC}]\!])(\mathrm{rg}([\![\mathrm{O}]\!] \cap \mathrm{RES}))$

In (66) the following notions are used:

$$\begin{array}{lcl} \mathrm{RES} & := & \wp(\bigcap [\![\text{these 5}]\!]([\![\mathrm{dF}]\!])) \times \wp(\bigcap [\![\text{those 60}]\!]([\![\mathrm{aC}]\!])) \\ \mathrm{dom} & := & \lambda \mathbf{R} \lambda X \exists Y.\mathbf{R}(X)(Y) \\ \mathrm{rg} & := & \lambda \mathbf{R} \lambda Y \exists X.\mathbf{R}(X)(Y) \end{array}$$

The content of (66) can be paraphrased by:

> Each of these contextually given Dutch firms occurs in a collection which owns some of those contextually given American computers, and, conversely, each of those contextually given American computers occurs in a collection that is owned by some of these contextually given Dutch firms. Moreover, the context referred to by *these* contains 5 Dutch firms, and that referred to by *those* 60 American firms.

One of the reasons why I prefer the more involved neutral cumulative reading to the doubly collective one is that sentences such as (66) are not true in virtue of two collections being related to each other, but rather because some of their subcollections are. In modelling this, one should take care that the definites do not take scope over each other. But this is the case for the cumulative reading. It should be noted that in the paraphrase of (66b) the numerals are used to specify the size of the contextually given collections. They have no verbal material within their scope. Indeed, this cumulative reading is not a total-number-of reading, but a collectivized form of the cumulative reading where both NPs have *all* as their main determiner. Schematically, the truthconditions are:

(67) $\forall x \in X \exists U \exists V[\mathbf{R}(U, V) \wedge x \in U] \wedge$
$\forall y \in Y \exists W \exists Z[\mathbf{R}(Z, W) \wedge y \in W]$
$+$ *numeral conditions*

It is this move to second-order quantification which enables the neutral cumulative reading to solve the problems noted earlier for Scha's (59a–c). To see how the solutions go, I discuss each of them in turn.

(59a) *The sides of R1 run parallel to the sides of R2* This is the one sentence which has not been under attack. Apart from the use of context sets, which I ignore in what follows, its truthconditions in (60) are those of the cumulative reading if *the* denotes universal quantification. As it stands, the logical form of (60) is quite unlike that of (67). There we have (68a) in the first conjunct, where we have (68b) here (similarly for the second conjunct).

(68) a. $\exists V[\mathbf{R}(\{x\}, V)]$
 b. $\exists U \exists V[\mathbf{R}(U, V) \wedge x \in U]$

Yet, as soon as U varies over singletons (68a,b) are equivalent. For the case at hand this can be enforced by adopting (69), which say that only single lines run parallel to each other.

(69) $\forall X, Y[X \parallel Y \Rightarrow |X| = 1 = |Y|]$

In combination with (69), the neutral cumulative reading of (59a) reduces to the distributive (60a), as required.

(59b) *The sides of R1 cross the sides of R2* According to Scha, (59b) has the same meaning as *some side of R1 crosses some side of R2*. But by means of (62) I have shown that in general this is too liberal. I would rather resort to Robert's proposal to leave the 'involvement' of the lines in making (59b) true, vague.

Using the neutral cumulative reading, a shallow analysis of (59b) runs:

(70) $[\![\text{R1}]\!] \subseteq \bigcup(\lambda X.X \text{ crosses sides of R2}) \wedge$
 $[\![\text{R2}]\!] \subseteq \bigcup(\lambda Y.\text{sides of R1 cross } Y)$

That is, each side of R1 is in a collection that crosses sides of R2, and conversely. As soon as we realize that a side can be in a collection that crosses lines without itself crossing a line, we see that (70) is what we are after.

(59c) *The squares contain the circles* For a start, it is worthwhile to compare the neutral cumulative reading (71) of (59c) with its truthconditions as given in (60c):

(71) $[\![\text{S}]\!] \subseteq \bigcup(\lambda X.X \text{ contains circles}) \wedge$
 $[\![\text{C}]\!] \subseteq \bigcup(\lambda Y.\text{squares contain } Y)$

There is a great difference. Recall, though, that I have objected to (60c) since it required each circle to be contained in a square. Instead, (59c) should rather be true whenever each circle is contained in a set of squares. It can be shown that these are the truthconditions we get when adopting the meaning postulate (72).

(72) $\neg \exists X[X \neq \emptyset \wedge [\![\text{Con}]\!](\emptyset, X)]$

In words, the empty collection does not contain a thing. Or, equivalently, if a collection X contains a non-empty collection, it is itself non-empty. As in case of (67), we see that (71) is the conjunction of (73a,b).

(73) a. $\forall x \in [\![\mathrm{S}]\!] \exists X \subseteq [\![\mathrm{S}]\!] \exists Y \subseteq [\![\mathrm{C}]\!] [[\![\mathrm{Con}]\!](X, Y) \wedge x \in X]$

b. $\forall y' \in [\![\mathrm{C}]\!] \exists Y' \subseteq [\![\mathrm{C}]\!] \exists X' \subseteq [\![\mathrm{S}]\!] [[\![\mathrm{Con}]\!](X', Y') \wedge y' \in Y']$

Since (72) excludes X' to be empty, (73b) states the required truthconditions. But the fact that some collections of squares may contain nothing (i.e., Y may be the empty set) turns (73a) into a tautology. So, (71) is equivalent to (73b).

This concludes my discussion of (59a–c). It suggests that the non-distributive readings of transitive sentences with definite NPs can be given by means of their neutral cumulative reading. In this manner, we get a semantics that does justice to the intuitions which Langendoen (1978) and Scha (1981) tried to formalize. The distributive readings of definites should take care of the sentences in which such NPs have scope over each other.

A Background Assumptions

The present research is based on the identification of collections with sets (sums). This identification is not unproblematic, as we know from the work of Hoeksema (1983) and in particular that of Landman (1989). However, their proposal to model collections, or 'groups' as they call them, with sets in the universe of heriditarily finite sets has also been under attack (cf. Schwartzschild 1992). Since there seems to be no serious obstacles in incorporating the more involved notion of collections (see Van der Does 1992, 12–14) and since it simplifies the exposition, I have opted for collections as sets.

I work in an extensional type theory with basic types e and t, i.e. individuals and truth-values respectively, and complex types (α, β), i.e. functions from objects of type α to objects of type β. To reduce the number of parentheses, I write for instance et and $e.et$ instead of (et) and $(e(et))$; and also ett and $et.ett$ instead of $((et)t)$ and $((et)((et)t))$. So, I omit as many parentheses as possible and use association to the left.

The decision to treat collections as sets induces some shifts in the types normally assigned to categories. In particular, intransitive VPs are now extensional properties of collections, and transitive ones relations between them. The common types of these expressions shifts from et and $e.et$ to ett and $et.ett$. In turn this requires NPs, which are properties of intransitive VPs, to be of type $ettt$. Since I restrict myself to Ns that hold of individuals only, their type remains et. Simple DETs, which are relations between Ns and VPs, are therefore of type $et.ettt$, and similarly for more complex DETs. Table 1 summarizes the category-to-type assignment used.
The explicit typing of variables is kept at a minimum by use of some notational conventions:

variables	**type**
$x, y, z, \ldots$	e
$X, Y, Z, \ldots$	et
$R^n, \ldots$	e^n
$\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \ldots$	ett
$\mathbf{R}^n, \ldots$	$(et)^n$
$D, \ldots$	$et.ett$
$\Delta, \ldots$	$et.ettt$

Category	Type	Remarks
Individuals	e, et	both d and $\{d\}$ are individuals
Collections	et	
Ns	et	for simplicity
IVs, VPs	ett	
TVs	$et.ett$	
NPs	$ettt$	
DETs	$et.ettt$	
n-ary DETs	$\underbrace{(et \ldots (et}_{n\ times}.ettt) \ldots)$	

Table 1: Category-to-type assignments

Where for a type α: $\alpha^0 = t$, and $\alpha^{n+1} = (\alpha\alpha^n)$. Variables may also occur with primes and subscripts as usual.

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