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ABSOLUTE TIME, SPECIAL RELATIVITY AND ML^{ν}

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ABSTRACT

A modal concept of absolute attribute is introduced in the modal language ML^{ν} by A. Bressan. In this paper I treat the concept of an Instant as an ML^{ν} absolute attribute to show one way of introducing the relativistic notions of temporal relations for events in the absolute theory of time.

1 INTRODUCTION

The controversy between the absolute and the relational theories of time and space has been famous since the Leibniz-Clark correspondence.[1] According to the absolute theory there is an actual time existing independently of the existence of objects, events and processes. Temporal relations between events depend on temporal relations between instants of time at which these events occur. According to the relational theory there is no time in itself - all assertions about time can be reduced to assertions about events and relations between events.

In some authors' opinion the Special Theory of Relativity (STR) counts in favour of the relational theory. For example, A. Grünbaum argues that the STR opposes the absolutistic conception of time ingredient in the Newtonian theory and that Einstein in fact repudiates philosophical assumptions made by the Newtonian theory.[2] On the other hand, some authors, for example Newton-Smith, argue that the STR is neutral with regard to the absolutist-relationalist controversy.[3] In the STR, Einstein held that time is a parameter depending on a particular reference system. He claimed that it is not the case that every pair of events is unambiguously ordered with respect to one of the relations "earlier than", "later than" or "simultaneous with", -i.e.- that temporal relations between events need not be the same in every reference system, and that those relations depend on reference systems moving relative to one another. Absolute space and time are not presupposed in the STR.

I will discuss the following question: what is the impact of the STR on the absolute theory of time? More precisely, once we have been forced to adopt the new relativistic notion of temporal relations for events, should we take ourselves to be committed to drop the theory of the independent existence and structure of "container" time? I will use the modal language ML^{ν}, in particular the formal definition of a modal concept of absolute attributes introduced in ML^{ν}, to show a possible way out for an absolutist.[4]

Why a modal language in the first place? In special relativity the notion of temporal order of events becomes a relative notion, relative to an inertial frame. That in fact supplies us with a variety of possible temporal orders of events the world can possess. Each perspective or each referential frame determines a particular ordering of the events, a possible history of the world. So, it seems to me quite natural to think about referential frames as coexistent possible worlds.

2 SOME SEMANTICAL FEATURES OF ML^{ν}

Bressan's ML^{ν} is a ν–sorted modal language (ν=1,2,...) based on a type system. The semantic analysis of ML^{ν} is based on the concept of a set of possible cases and on Carnap's method of extension and intension. *Necessity* means "true in every possible case" and *possibility* "true in some possible case". In Bressan's notation N is the necessity operator while $\Diamond$ is the possibility operator introduced in the usual way by $\Diamond \equiv_D \neg N \neg$.

The *extension* of an expression is always relativized to possible cases: the extension of the expression Δ in the possible case γ is what Δ denotes in γ. The semantical rules for ML^{ν} assign an *intension* ("quasi intension" in Bressan's teminology) to every typed expression Δ. The *intension of an expression of the type of individuals* is a function from the possible cases to its extensions, that is, to objects. The function takes each possible case γ into the extension for that expression in γ. *The intension of an n-ary attribute* is a relation between its extensions and possible cases. Bressan's definition of extensions of attributes differs from the usual first-order one: the extension of an n-ary attribute is a set of n-tuples of intensions of the arguments, not a set of n-tuples of extensions of the arguments. Hence, the intension of an n-ary attribute is a relation between n-tuples of intensions of the arguments and possible cases. *The intension of a sentence* p is the class of the possible cases where p holds. For a fixed domain D *the range of the variables* is the set of all intensions of the appropriate type (value assigments for ML^{ν} are functions from the variables to intensions). The above choice for the intensions of the attributes makes it possible to deal within ML^{ν} with nonextensional attributes as well as with extensional ones. In order to illustrate the difference I shall give one of Bressan's examples.

But first the feature of ML^{ν} which is of great importance here: as is usual in intensonal logics, identity in ML^{ν} is always contingent. "$\Delta_1 = \Delta_2$ holds in γ" means that Δ_1 and Δ_2 have the same extension in γ. In order to express that Δ_1 and Δ_2 have the same intension, the necessity operator is needed: $N(\Delta_1 = \Delta_2)$.

Now, for Bressan's example, let M_1 and M_2 be two individual constants denoting the rockets *Mida*$_1$ and *Mida*$_2$ respectively in every possible case. Let m denote that one of the above rockets which at the final instant t_{20} of the twentieth century is at a larger distance from the earth than the other. (If both are at equal distance from the earth, then let m denote *Mida*$_1$.) Then

$$N(M_1 \neq M_2),$$

$$\Diamond p_1 \wedge \Diamond \neg p_1 \quad \text{where } p_1 \equiv_D (m=M_1).$$

Let

$$F_1(x) \equiv_D (p_1 \wedge x=M_1) \vee (\neg p_1 \wedge x=M_2)$$

$$F_2(x) \equiv_D (p_1 \wedge N(x=M_1)) \vee (\neg p_1 \wedge N(x=M_2)).$$

The attribute F_1 is extensional in the sense that the truth value of $F_1(x)$ in γ depends just on the extension of x in γ (try $F_1(m)$), while the truth value of $F_2(x)$ in γ depends essentially on the intension of x. The extensional attributes are defined in ML^V by an appropriate use of identity:

$$Ext(F) \equiv_D (\forall x_1,y_1,\dots,x_n,y_n)(F(x_1,\dots,x_n) \wedge \bigwedge_{i=1}^{n} x_i=y_i \supset F(y_1,\dots,y_n)).$$

F_1 satisfies the condition from the above definition, but F_2 doesn't. In case p_1 holds in γ, both $F_2(M_1)$ and $m=M_1$ hold in γ, but $F_2(m)$ does not. This also shows that Leibniz' Law

$$(x=y \supset \forall F(F(x) \equiv F(y)))$$

does not hold in ML^V, since some attributes are not extensional. But it does hold that

$$(x=y \equiv \forall F(Ext(F) \supset (F(x) \equiv F(y)))) \quad \text{and}$$

$$(N(x=y) \equiv \forall F(F(x) \equiv F(y))).$$

In ML^V the concept of an absolute attribute is introduced using the concepts of modally constant and modally separated attributes.

$$MConst(F) \equiv_D (\forall x_1,\dots x_n)(\Diamond F(x_1,\dots,x_n) \equiv NF(x_1,\dots,x_n))$$

An attribute is said to be *modally constant* if and only if it applies to the same intensions in every possible case.

$$MSep(F) \equiv_D (\forall x_1,y_1,\dots,x_n,y_n)(F(x_1,\dots,x_n) \wedge F(y_1,\dots,y_n) \wedge \Diamond \bigwedge_{i=1}^{n} x_i=y_i \supset \bigwedge_{i=1}^{n} N(x_i=y_i))$$

An attribute is said to be *modally separated* if and only if whenever x and y both fall under F, then x and y are either necessarily identical or necessarily distinct.
An attribute is said to be *absolute* if it is both modally constant and modally separated. Formally:

$$Abs(F) \equiv_D MConst(F) \wedge MSep(F) .$$

Take the concept of body, intuitively characterized as follows: if b is a body then it is the same body, i. e. the same bearer of (possible) properties, in all possible cases. We may assert that *Abs*(*Body*). Now, take the concept of a heavy body. Obviously, ¬ *Abs*(*Heavybody*).
Every absolute attribute F determines its corresponding extensional attribute, in that the latter is precisely the extensionalisation of the former. The *extensionalisation* of an attribute is introduced as follows:

$$F^{(e)}(x_1,\ldots,x_n) \equiv_D (\exists y_1,\ldots,y_n)(F(y_1,\ldots,y_n) \wedge \bigwedge_{i=1}^{n} y_i=x_i)$$

Consider again the example with the rockets and assume that *Body* ($Mida_1$) and *Body*($Mida_2$). Since ◊(m=$Mida_1$) and ◊(m≠$Mida_1$), the definition of *MSep*(*Body*) implies that ¬ *Body*(m). But, because it holds that N(m=$Mida_1$ ∨ m=$Mida_2$), the definition of $Body^{(e)}$(x) implies $Body^{(e)}$(m).
Since natural language does not distinguish between absolute and extensional predication, there are double uses of certain common nouns in natural language. For example, the concept of real number is treated in ML^V as an absolute attribute (*Abs*(*Real*)) with the corresponding extensionalisation (*Ext*($Real^{(e)}$)). So the noun "real number" may have an extensional as well as an absolute meaning depending on the context. Let n be the number of planets. Then n=9, ◊(n≠9), *Real*(9), *MSep*(*Real*) and the defininition of $Real^{(e)}$(x) imply ¬ *Real*(n) and N($Real^{(e)}$(n)). Now, take the sentence "n is a real number", for the n mentioned above. An extensional reading of the common noun "real number" makes it true but an intensional reading of the noun makes it false. The case of the common noun "body" and the sentence

"m is a body", for the m from the example with the rockets, is analogous.

In order to deal with descriptions Bressan defines "there is exactly one x such that $\phi(x)$ holds" by

$$(\exists_1 x)\phi(x) \ \equiv_D (\exists x)(\phi(x) \wedge (\forall y)(\phi(y) \supset y=x))$$

where y is a variable of the same type as x that does not occur in $\phi(x)$ and $\phi(y)$ is obtained from $\phi(x)$ by replacing the free occurrences of the variable x in $\phi(x)$ by occurrences of y. Notice the difference between the above ML^{V}-definition and the usual predicate-logical one:

$$(\exists_1 x)\phi(x) \ \equiv_D (\exists x)(\forall y)(\phi(y) \equiv y=x).$$

To see the point, take the example with the rockets. By the usual definition, $(\exists_1 x)(F_2(x))$ would not hold in γ where p_1 holds, since $m=M_1$, $F_2(M)$ but $\neg F_2(m)$. In other words, for any F, $(\exists_1 x)(F(x))$ defined in the usual way would make F an extensional attribute.
The intension of the term $(\iota x)\phi(x)$ is the pattern of its extensions as γ varies. In every possible case γ where $(\exists_1 x)\phi(x)$ does not hold, the extension of $(\iota x)\phi(x)$ in γ is a Fregean "nonexisting" object identified with an empty class, and in every possible case γ where $(\exists_1 x)\phi(x)$ holds, the extension of $(\iota x)\phi(x)$ in γ is the unique object which is the extension in γ of any y such that $\phi(y)$ holds in γ.
Bressan introduces also an intensional description operator ι_c depending on the parameter c which ranges over the set of possible cases. He defines "there is at most one strictly unique x such that $\phi(x)$ holds" by

$$(\exists^{(1)\cap} x)\phi(x) \ \equiv_D (\forall xy)(\phi(x) \wedge \phi(y) \supset N(x=y))$$

where y is a variable of the same type as x that does not occur in $\phi(x)$ and $\phi(y)$ is obtained from $\phi(x)$ by replacing the free occurrences of the variable x in $\phi(x)$ by occurrences of y. Compare $(\exists_1 x)\phi(x)$ and $(\exists^{(1)\cap} x)\phi(x)$. Due to the use of simple identity in the definition of $(\exists_1 x)\phi(x)$, "$(\exists_1 x)F(x)$ holds in γ" means that there is an intension that falls under F in γ, and that all intensions that fall under F in γ, have the same extension in γ. Due to the use of strict identity in the definition of $(\exists^{(1)\cap}x)\phi(x)$, "$(\exists^{(1)\cap}x)F(x)$ holds in γ" means that all intensions that fall under F in γ, are the same intension. Taking I_C to mean "C is the case that actually holds" the intensional description operator is defined as follows:

$$(\iota_C x)\phi(x) \;=_D\; (\iota x)\Diamond(I_C \wedge \phi(x) \wedge (\exists^{(1)\cap}x)\phi(x)).$$

For example, take n to be the number of planets and take $(\iota x)\phi(x)$ to be $(\iota x)(\mathit{Real}(x) \wedge x{=}n)$. Consider the real case ρ where 9=n ($\mathit{Real}(9)$, $\mathit{Real}^{(e)}(9)$, $\mathit{Real}^{(e)}(n)$, $\neg\mathit{Real}(n)$). Take 9 to be the value of x. Then $(\exists_1 x)\phi(x)$ holds in ρ by the transitivity of identity, and $(\iota x)\phi(x)$ denotes the number nine in ρ. Consider the possible case C where 10=n. Take 10 to be the value of x. Then $(\exists_1 x)\phi(x)$ holds in C and $(\iota x)\phi(x)$ denotes the number ten in C. On the other hand, take $(\iota_\rho x)\phi(x)$ to be $(\iota_\rho x)(\mathit{Real}(x) \wedge x{=}n)$. For 9 as the value of x, $(\exists^{(1)\cap}x)\phi(x)$ holds in ρ since $\mathit{MSep}(\mathit{Real})$. $(\iota_\rho x)\phi(x)$ denotes the number nine in every possible case.

Absolute attributes and extensional attributes mirror Aristotle's distinction between secondary substances and qualities.[5] Absolute attributes can be used to denote things as (primary) substances, while extensional ones can be used for attributing qualities to substances.

We accept the following common abbreviating definition for all appropriate types:

$$x_1,\ldots,x_n \in F \;\equiv_D\; F(x_1) \wedge F(x_2) \wedge \ldots \wedge F(x_n)$$

3 ABSOLUTE TIME AND RELATIVISTIC SIMULTANEITY

In order to capture the concept of time in the absolute theory of time, let us take the concept of Instant (*Inst*) to be an ML^V absolute attribute:

$$t_1,t_2,\ldots \in \mathit{Inst}\ ;\ \mathit{Inst} \in \mathit{Abs}\ .$$

In this way absolute time is considered to consist of objects (primary substances) whose essence (secondary substance) is the absolute property of being an Instant. Now, let $t_{e_1}(t_{e_2},...)$ be defined as the instants at which the event $e_1(e_2,...)$ occurs. The question is whether the noun "instant" employed in the preceding sentence should be used in an absolute or in an extensional way. In order to reach an answer, consider the following example which illustrates the relativistic notion of temporal relations among events.

Three murders occurred on a railway near a small town T. A very interesting case since three guys — Smith, Jones and Brown were the murderers as well as the victims. Jones, when his body was found in a tree just near the railway, had in his hands the latest model of a laser gun. Smith, whose body was found on a train, also had a laser gun. Brown was found dead just near the tracks at some distance from the above mentioned tree and he had, believe it or not, two such guns. A local police inspector P.I., an ex-professor of physics, was supposed to solve this case. After searching the place of the crime and talking to the witnesses, P.I. came to the following facts.
1) Jones, who was in the tree, and Smith, who was on the train, simultaneously fired their laser guns at Brown who was just near the tracks at a certain distance from the tree (Figure 1). Jones and Smith shot at Brown at the moment when the train with Smith was just level with Jones. This was claimed by a witness who was just near Jones and Smith when they shot. The train was moving at the constant velocity in the direction from Brown toward Jones, that is to say, after shooting it was moving in the direction away from Brown.

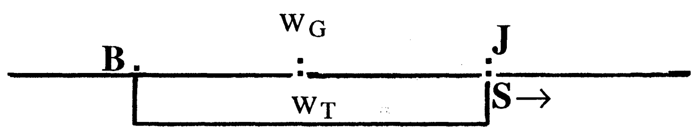

Figure 1. Shooting scenario

2) Brown simultaneously fired his two laser guns. He shot at Jones with the one gun and at Smith with the other one. This was claimed by a witness who was just near Brown when Brown shot.
3) A witness w_G, who was resting on a bench equidistant from Brown and Jones, claimed that "the bullet" that Brown fired off at Jones and "the bullet" that Jones fired off at Brown had passed him at the same time.
4) A witness w_T, who was on the train and who just coincided with the witness w_G precisely when Brown and Jones fired their lazer guns as judged from the ground, claimed that "the bullet" that Brown fired off at Smith and "the bullet" that Smith fired off at Brown had passed him at different moments - first "the bullet" that Smith fired off at Brown and then "the bullet" that Brown fired off at Smith.
These facts were not enough for a complete report since P.I. had to discover who, if anybody, shot in self-defence. He had to establish the order of the shooting. A local insurance company was also waiting for the report because the company was supposed to pay out the life-insurance to the family of a guy who shot in self-defence. Here are the results of P.I.'s further investigations.
Since (non)simultaneity of spatially coincident events holds for every reference system, 1) and 2) imply that Brown simultaneously fired his two laser guns relative to both the ground reference system G and the train reference system T and that Jones and Smith shot simultaneously relative to both G and T.
Each laser "bullet" was travelling at the same speed since the velocity of light is constant and independent of the velocity of its source. Since w_G was equidistant from Brown and from Jones, w_G's testimony implies that Brown and Jones shot simultaneously within the ground reference system G. By the transitivity of simultaneity it follows that within G everybody shot at the same time.
Since the train, with the witness w_T on it, was moving relative to the ground, after the shooting w_T moved away from the witness w_G. So, it is clear that w_T said the truth. At the time w_G sees the two "bullets" in front of him, w_T is to the right of him (Figure 2) - this means that "the

bullet" that Smith fired off has already passed the witness w_T (since Smith shot from the same place as Jones and at the same time as Jones) and that "the bullet" that Brown fired of at Smith will pass w_T only later.

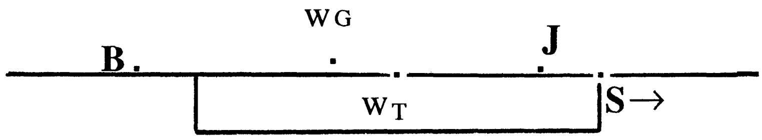

Fig. 2. After the shooting

But, w_T was also at the middle point between two shooting places. So, w_T's testimony implies that Smith shot before Brown within the train reference system T. Hence, within T Smith and Jones had simultaneously shot but before Brown shot.

P.I. did his job perfectly, but the chief inspector started to feel dizzy when he read the report, while the menager of the insurance company was desperate. The question "Who shot in self-defence" could not be answered: relative to G nobody shot in self-defence, but relative to T Brown did!

As I indicated earlier, I am going to interpret possible cases as referential frames. If the instants defined via events were treated as belonging to Inst, then, by modal separation, if two such instants are equal in one possible case, they would be equal in all possible cases. In other words, two events, (non)simultaneous in one referential frame, would be (non)simultaneous in every other referential frame. But such a consequence is not in accordance with the STR. Take two spatially separated events B (Brown shoots) and S (Smith shoots): they are simultaneous in the referential system G but not in the referential system T. So, I will treat instants at which particular events occur as necessarily belonging to $Inst^{(e)}$ and not belonging to *Inst* ("necessarilly" since all referntial frames deal with the same set of events). In this way we may make the notion of simultaneity between events relative to a particular referential system as well as dependent upon relations among the instants of absolute time. We are going to see that the possibility of this "relativization" follows immediately from the features of $ML^{\vee}$, in particular from $ML^{\vee}$'s identity and its distinction between *Inst* and $Inst^{(e)}$.

Taking t_B, t_S, $t_J \in Inst^{(e)}$, consider G where B and S are simultaneous events, that is where $t_B = t_S$ holds. By the definition of the extensionalisation of an absolute attribute, there exist $t_1, t_2 \in Inst$ such that it holds in G that $t_B = t_1$ and $t_S = t_2$. By the transitivity of identity it follows that $t_1 = t_2$, which by modal separation implies $N(t_1 = t_2)$. We have seen that the principle of identity from extensional logic ($x=y \supset (\phi(x) \equiv \phi(y))$ does not hold in $ML^{\vee}$. Hence we cannot infer that $N(t_B = t_S)$.

On the other hand, B and S are not simultaneous events in T, that is $t_B \neq t_S$. This implies that there exist $t_3, t_4 \in Inst$ such that in T holds that $t_B = t_3$ and $t_S = t_4$. In the same way as above, we may infer that $N(t_3 \neq t_4)$ but we may not infer that $N(t_B \neq t_S)$.

Hence, the presence of $t_1, t_2, \ldots$ belonging to *Inst* forces neither "$t_B = t_S$ holds in G" nor "$t_B \neq t_S$ holds in T" to be strenghtened to $N(t_B = t_S)$ or $N(t_B \neq t_S)$ respectively.

We have seen that relative to G Brown and Jones shot simultaneuosly, while relative to T, although Jones and Smith shot simultaneously, Smith and Brown didn't. Relativistically, the notion of simultaneity is no longer an equivalence relation. Within any particular inertial frame simultaneity with respect to that frame is still an equivalence relation. But if event E_1 is simultaneous with event E_2 relative to one referential system and event E_2 is simultaneous with event E_3 relative to another one, then there is no general reason to conclude that E_1 is simultaneous with E_3 relative to either system. Treating the instants defined via events as belonging to $Inst^{(e)}$ captures these requirements. Since identity is contingent, only within a particular possible case identity between the instants belonging to $Inst^{(e)}$ is an equivalence relation. Hence,

$$N(te_1=te_2 \wedge te_2=te_3 \supset te_1=te_3)$$

gives us the notion of transitivity that we really need. On the other hand, a simple identity between the instants belonging to *Inst* can always be replaced by a strict identity. This gives us the notion of "transitivity across possible cases" of identity between instants of absolute time, just what we really need. Now, it should be clear that the truth of $t_B = t_J$ in the case G and the truth of $t_J = t_S$ in the case T do not imply that $t_B = t_S$ holds in either G or T. The presence of t, t_1, t_2,... belonging to *Inst* does not make these inferences valid. That can be seen from the following translation in ML^V of the facts from our example:

(a) $N((\iota_G t)(t\in Inst \wedge t=t_B) = (\iota_G t)(t\in Inst \wedge t=t_J))$
(b) $N((\iota_T t)(t\in Inst \wedge t=t_J) = (\iota_T t)(t\in Inst \wedge t=t_S))$.

If we want to conclude from(a) and (b) that, for example in T, it holds that $t_B = t_S$, we have to be able to infer

$N((\iota_T t)(t\in Inst \wedge t=t_B) = (\iota_T t)(t\in Inst \wedge t=t_S))$;

for that we need:

$N((\iota_T t)(t\in Inst \wedge t=t_B) = (\iota_T t)(t\in Inst \wedge t=t_J))$

which is different from what (a) says and cannot be infered from (a). Since $Inst^{(e)} \notin MSep$, if in one possible case t coincides with t_B, it does not follow that that is so in any other case. Hence, if in one possible case the instants t_1 and t_2 are equal and t_1 coincides with t_B while t_2 coincides with t_J, although t_1 and t_2 have to be necessarily equal, it does not follow that the instants t_3 and t_4, with in some other possible case t_3 coinciding with t_B and t_4 with t_J, have to be equal to t_1 and t_2 respectively. On the other hand, from

(a) $N((\iota_G t)(t\in Inst \wedge t=t_B) = (\iota_G t)(t\in Inst \wedge t=t_J))$
(c) $N((\iota_G t)(t\in Inst \wedge t=t_J) = (\iota_G t)(t\in Inst \wedge t=t_S))$

it follows that
(d) $N((\iota_G t)(t \in Inst \wedge t=t_B) = (\iota_G t)(t \in Inst \wedge t=t_S))$.

That is, from the facts that relative to G events B and J are simultaneous as well as events J and S, it follows that relative to G events B and S are simultaneous, too. As we have seen, (d) does not imply $N(t_B=t_S)$.

Note that if we were dealing only with the extensional concept of Instant which is not modally separated, we would not be able to use the intensional description operator, since the condition $(\exists^{(1)\frown}x)\phi(x)$ would never hold.

4 THE STRUCTURE OF ABSOLUTE TIME

Having seen that our framework captures the relativization of simultaneity, we are now going to deal with the relation "earlier than" between instants. This relation is also going to be relativized, but in a way that it will still appropriatelly structure absolute time.

The space-time appropriate for the STR is Minkowski space-time. Minkowski space-time has the topological structure of E^4, the Euclidean four-dimensional space. But, in contrast to Newtonian space-time, it "splits up" into three-dimensional space and one-dimensional time only relative to a given reference frame. Relative to a given reference frame, the relation of simultaneity between events is an equivalence relation and hence partitions the set of all events into equivalence classes. Relative to a given reference frame, space-time can be viewed as an infinite set of these equivalence classes, one for each instant of time, that is, it can be viewed as Euclidean three-space persisting through time. These facts give us the direction in which we should proceed.

We shall introduce the notion of "isomorphism up to identity" to see more clearly the distinction between *Inst* and $Inst^{(e)}$.
Let $y =_\gamma x$ and R_γ stand for "y=x holds in γ" and "the restriction of the n-ary relation R to γ". Let $\langle A, S \rangle$ be a structure and let $F_\gamma = \{x \mid F(x) \text{ in } \gamma\}$. (The set of intensions F_γ is the extension of F in γ .) A function f

from F_γ to A is an *isomorphism of* $\langle F_\gamma, R_\gamma \rangle$ *onto* $\langle A, S \rangle$ *up to identity in* γ if:

(a) if f(x)=f(x'), then $x =_\gamma x'$;

(b) $f[F_\gamma] = A$;

(c) $R_\gamma(x_1,..,x_n)$ iff $S(f(x_1),..,f(x_n))$.

If there is an isomorphism of $\langle F_\gamma, R_\gamma \rangle$ onto $\langle A, S \rangle$ up to identity in γ, we say that the *structures* $\langle F_\gamma, R_\gamma \rangle$ *and* $\langle A, S \rangle$ *are isomorphic up to identity in* γ and write:

$$\langle F_\gamma, R_\gamma \rangle \cong_{/=\gamma} \langle A, S \rangle.$$

PROPOSITION 1.

Let $F \in Abs$, $F_\gamma = \{y \mid F(y) \text{ in } \gamma\}$, $F^{(e)}{}_\gamma = \{x \mid F^{(e)}(x) \text{ in } \gamma\}$ and $R \in Ext$. Then:

$$\langle F^{(e)}{}_\gamma, R_\gamma \rangle \cong_{/=\gamma} \langle F_\gamma, R_\gamma \rangle.$$

Proof :

Let f be a function from $F^{(e)}{}_\gamma$ to F_γ such that $f(x) = (\iota y)(y =_\gamma x)$.

(a) Suppose f(x)=f(x'). f(x)=f(x') implies $f(x) =_\gamma f(x')$. By def. of f, $x =_\gamma f(x)$ and $x' =_\gamma f(x')$. Hence, $x =_\gamma x'$.

(b) By def. of f, if $y \in f[F^{(e)}{}_\gamma]$, then $y \in F_\gamma$.

Suppose $y \in F_\gamma$. By def. of F_γ, F(y) in γ. By reflexivity of =, $y =_\gamma y$. F(y) and $y =_\gamma y$ imply that $F^{(e)}(y)$ in γ, hence $y \in F^{(e)}{}_\gamma$. Now, from $y =_\gamma y$ and $y \in F^{(e)}{}_\gamma$ it follows that f(y)=y. Hence, $y \in f[F^{(e)}{}_\gamma]$.

(c) Since $R \in Ext$ and $x_i =_\gamma f(x_i)$, for $1 \leq i \leq n$, it follows that

$R_\gamma(x_1,..,x_n) \equiv R_\gamma(x_1,..,x_n)[x_1/f(x_1),..,x_n/f(x_n)]$ holds in γ.

Hence, f is an isomorphism of $\langle F^{(e)}{}_\gamma, R_\gamma \rangle$ onto $\langle F_\gamma, R_\gamma \rangle$ up to identity in γ.⊗

PROPOSITION 2.

Let f be the isomorphism of $\langle F^{(e)}{}_\gamma, R_\gamma \rangle$ onto $\langle F_\gamma, R_\gamma \rangle$ up to identity in γ, with $R \in Ext$. Let $\mathcal{L}$ be a ML^V– sublanguage containing variables $v_1,...,v_m$, n-ary relational symbol R and all logical symbols from ML^V except N. Let V be a $\mathcal{L}$-value assigment, if it is a function from variables to intensions such that $V(v_i) \in F^{(e)}{}_\gamma$ and $V(R) \in \{(x_1,...,x_n; \gamma) \mid x_1,...,x_n \in F^{(e)}{}_\gamma\}$. Then, for any $\mathcal{L}$-formula φ, $\mathcal{L}$-value assigment V and possible case γ

$\varphi[\boldsymbol{\nu}]$ holds in γ iff $\varphi[fV]$ holds in γ.

Proof : by $\mathfrak{L}$-formula induction. ⊗

Here comes our first postulate. The symbol $<$ is going to be interpreted as an extensional binary relation.

POSTULATE 1.
$< \in Ext$.

The motiovation for Postulate 1 is the following: we want for instants belonging to $Inst^{(e)}$ to be possible to hold both $\Diamond(t_E<t_{E'})$ and $\Diamond\neg(t_E<t_{E'})$. Let $\mathfrak{R}$ and $\langle \mathfrak{R}, < \rangle$ stand for the set of real numbers and the real line respectively.

PROPOSITION 3.
$\langle \mathrm{Inst}_\gamma, <_\gamma \rangle \cong \langle \mathfrak{R}, < \rangle$ iff $\langle \mathrm{Inst}^{(e)}{}_\gamma, <_\gamma \rangle \cong_{/=\gamma} \langle \mathfrak{R}, < \rangle$.

Proof : by Proposition 1. ⊗

By the following postulate we start describing the structure of absolute time.

POSTULATE 2.
For every γ: $\langle \mathrm{Inst}_\gamma, <_\gamma \rangle \cong \langle \mathfrak{R}, < \rangle$.

PROPOSITION 4.
For every γ: $\langle \mathrm{Inst}^{(e)}{}_\gamma, <_\gamma \rangle \cong_{/=\gamma} \langle \mathfrak{R}, < \rangle$.

Proof : by Proposition 3 and Postulate 2. ⊗

Relative to a given possible case γ, identitiy between instants belonging to $Inst^{(e)}$ is an equivalence relation and hence partitions the set $\mathrm{Inst}^{(e)}{}_\gamma$ into equivalence classes. Proposition 4 says that, relative to γ, $\mathrm{Inst}^{(e)}{}_\gamma$ can be viewed as a set of these equivalence classes, one for each real number.

PROPOSITION 5.
For every γ, γ^*: $\mathrm{Inst}_\gamma = \mathrm{Inst}_{\gamma^*}$.

Proof : Suppose $t \in Inst_\gamma$, for arbitrary γ. By def. of $Inst_\gamma$, *Inst*(t) holds in γ. Since $Inst \in MConst$, *Inst*(t) holds in every γ. Hence, $t \in Inst_{\gamma^*}$. The other direction is analogous. ⊗

We say that n-ary relation R is *modally constant with respect to attribute F* if and only if whenever for every element of n-tuple $(x_1,...,x_n)$ falling under F, either necessarily $R(x_1,...,x_n)$ or necessarily $\neg R(x_1,...,x_n)$. Formally:

$$R \in MConst_F \equiv_D (\forall x_1,...,x_n)(x_1,...,x_n \in F \wedge \Diamond R(x_1,...,x_n) \supset NR(x_1,...,x_n)).$$

POSTULATE 3.
$< \in MConst_{Inst}$

Postulate 3 says that in every possible case, the relation $<$ holds for the same pairs of instants belonging to *Inst* . Postulate 1 and Postulate 3 make the relation $<$ analogous to the identity relation in the following sense: holding of the relation $<$ between instants belonging to *Inst* is independent of possible cases, while its holding between instants belonging to $Inst^{(e)}$ depends on a particular γ.

PROPOSITION 6.
For every γ, γ^*: $\langle Inst_\gamma, <_\gamma \rangle = \langle Inst_{\gamma^*}, <_{\gamma^*} \rangle$.
Proof : by Proposition 5 and Postulate 3. ⊗

Proposition 6 enables us to omit index γ in $\langle Inst_\gamma, <_\gamma \rangle$. From now on, by an isomorphisim from $\langle Inst , < \rangle$ onto $\langle \mathfrak{R}, < \rangle$, where Inst $=\{t \mid t \in Inst\}$, we mean an isomorphism which is the same function f for every γ: for every γ, γ' and t, f(t) in γ equals f(t) in γ'.

Once an observer in a particular inertial state of motion is selected, the three-dimensional space and the one-dimensional time can be Cartesian coordinatized. But, in contrast to Newtonian space-time, the always nonnegative distance in time between two events is not a quantity invariant for all observers. Hence, we introduce a ternary relation *dist* and take it to be extensional. The intended interpretation of "*dist* $(t_E,t_{E'},n)$ holds in γ", where $t_E,t_{E'} \in Inst^{(e)}$, $t_E,t_{E'} \notin Inst$ and n is a real number, is that $n = |k - m|$, where k and m are temporal coordinates of events E and E' respectively in the reference frame which is the interpretation of γ. Note that, under this interpretation, for every γ and every $t_E, t_{E'}$ such that $t_E, t_{E'} \in Inst^{(e)}$ and $t_E, t_{E'} \notin Inst$, there is exactly one real number n such that $dist(t_E, t_{E'}, n)$ holds in γ.

POSTULATE 4.
$dist \in Ext$.

PROPOSITION 7.
$\forall t_E, t_{E'}, t, t'(t_E, t_{E'} \in Inst^{(e)} \wedge t, t' \in Inst \wedge t_E = t \wedge t_{E'} = t' \supset dist(t_E, t_{E'}, n) \equiv dist(t, t', n))$.
Proof : by Postulate 4. ⊗

POSTULATE 5.
$\forall t, t' \, (t, t' \in Inst \wedge \Diamond\, dist(t, t', n) \supset N\, dist(t, t', n))$.

Postulates 4 and 5 say that the distance between two instants belonging to $Inst^{(e)}$ is dependent on a possible case and need not be the same in every possible case, while the distance between two instants belonging to *Inst* is the same in every possible case.

PROPOSITION 8.
Assume: $t_{E1}, t_{E2}, t_{E3}, t_{E4} \in Inst^{(e)}$; $t, t' \in Inst$; $dist(t_{E1}, t_{E2}, n)$ holds in γ, $dist(t_{E3}, t_{E4}, n)$ holds in γ^*; $t_{E1} < t_{E2}$ holds in γ iff $t_{E3} < t_{E4}$ holds in γ^*.
Then:
$((\iota_\gamma t)(t = t_{E1}) = (\iota_{\gamma^*} t)(t = t_{E3}) \supset (\iota_\gamma t')(t' = t_{E2}) = (\iota_{\gamma^*} t')(t' = t_{E4}))$.
Proof :
Since the distance between t_{E1} and t_{E2} in γ is the same as the distance between t_{E3} and t_{E4} in γ^*, only the following three cases can occur:
Case 1: $t_{E1} < t_{E2}$ holds in γ and $t_{E3} < t_{E4}$ holds in γ^*.
Then, by postulates 1 and 3 it follows that $N((\iota_\gamma t)(t = t_{E1}) < (\iota_\gamma t')(t' = t_{E2}))$ and $N((\iota_{\gamma^*} t)(t = t_{E3}) < (\iota_{\gamma^*} t')(t' = t_{E4}))$. Since $dist(t_{E1}, t_{E2}, n)$ holds in γ, by Postulates 4 and 5 it follows that $N dist((\iota_\gamma t)(t = t_{E1}), (\iota_\gamma t')(t' = t_{E2}), n)$. Since $dist(t_{E3}, t_{E4}, n)$ holds in γ^*, by Postulates 4 and 5 it follows that $N dist((\iota_{\gamma^*} t)(t = t_{E3}), (\iota_{\gamma^*} t')(t' = t_{E4}), n)$. Now, take an arbitrary γ' and suppose

that $(\iota\gamma t)(t=t_{E1}) = (\iota\gamma{*}t)(t=t_{E3})$ and $(\iota\gamma t')(t'=t_{E2}) \neq (\iota\gamma{*}t')(t'=t_{E4})$ hold in γ'. $dist((\iota\gamma t)(t=t_{E1}),(\iota\gamma t')(t'=t_{E2}),n)$ holds in γ'. Since $N((\iota\gamma{*}t)(t=t_{E3}) < (\iota\gamma{*}t')(t'=t_{E4}))$ and $(\iota\gamma t)(t=t_{E1}) = (\iota\gamma{*}t)(t=t_{E3})$, it follows that $(\iota\gamma t)(t=t_{E1})<(\iota\gamma{*}t')(t'=t_{E4})$ holds in γ'. Hence, $\neg dist((\iota\gamma t)(t=t_{E1}),(\iota\gamma{*}t')(t'=t_{E4}),n)$ holds in γ'. But, $N dist((\iota\gamma{*}t)(t=t_{E3}),(\iota\gamma{*}t')(t'=t_{E4}),n)$ and $(\iota\gamma t)(t=t_{E1}) = (\iota\gamma{*}t)(t=t_{E3})$ imply $N dist((\iota\gamma t)(t=t_{E1}),(\iota\gamma{*}t')(t'=t_{E4}),n)$. Contradiction. So, $((\iota\gamma t)(t=t_{E1}) = (\iota\gamma{*}t)(t=t_{E3}) \supset (\iota\gamma t')(t'=t_{E2}) = (\iota\gamma{*}t')(t'=t_{E4}))$ holds in γ'.
Case 2: $t_{E1}= t_{E2}$ holds in γ and $t_{E3}= t_{E4}$ holds in γ^*.
Then $(\iota\gamma t)(t=t_{E1}) = (\iota\gamma t')(t'=t_{E2}))$ and $(\iota\gamma{*}t)(t=t_{E3}) = (\iota\gamma{*}t')(t'=t_{E4})$. Suppose $(\iota\gamma t)(t=t_{E1}) = (\iota\gamma{*}t)(t=t_{E3})$. Then, $(\iota\gamma t')(t'=t_{E2}) = (\iota\gamma{*}t')(t'=t_{E4})$.
Case 3: $t_{E2}< t_{E1}$ holds in γ and $t_{E4}< t_{E3}$ holds in γ^*.
This case is analogous to Case 1. ⊗

The Lorentz transformations give the transformation between the set of space coordinates x,y,z and time τ of an event, as measured in a given reference system S, and a corresponding set x',y',z',τ' belonging to the same event as measured in another reference system S', moving uniformly relative to S. Since the structures $\langle$ Inst , < $\rangle$ and $\langle \mathfrak{R}, < \rangle$ are isomorphic, we may think of the instants belonging to *Inst* as if they are "temporal coordinates", through possible cases, of the instants belonging to $Inst^{(e)}$. We shall show this by an example.
Take two referential frames S and S', moving uniformly relative to one another, that coincide at event E, and let coordinates of E in S and S' be $x=y=z=\tau=0$ and $x'=y'=z'=\tau'=0$ respectively. Two cases can occur:
(i) $(\iota_S t)(t\in Inst \wedge t=t_E) = (\iota_{S'} t')(t'\in Inst \wedge t'=t_E)$;
(ii) $(\iota_S t)(t\in Inst \wedge t=t_E) \neq (\iota_{S'} t')(t'\in Inst \wedge t'=t_E)$.
Suppose (i). Take the isomorphism f from $\langle$ Inst, < $\rangle$ onto $\langle \mathfrak{R}, < \rangle$ such that:
(a) $f((\iota_S t)(t=t_E)) = 0$;
(b) for every t': if $(\iota_S t)(t\in Inst \wedge t=t_E) < t'$, then f(t') is positive;
(c) for every t': if $t' < (\iota_S t)(t\in Inst \wedge t=t_E)$, then f(t') is negative;
(d) for every t, t': if $dist(t,t',n)$, then $|f(t) - f(t')| = n$.

It can be shown that for any event E', $f((\iota_S t')(t'=t_{E'})) = k$ and $f((\iota_{S'} t'')(t''=t_{E'})) = m$ if and only if (x,y,z,k) and (x',y',z',m) are coordinates, related by the Lorentz transformations, of event E' in S and S' respectively.

Let k, m be positive. For one direction, suppose k and m are temporal coordinates of event E' in S and S' respectively. Since k is positive, $t_E < t_{E'}$ holds in S. By Postulate 1 it follows that $(\iota_S t)(t \in Inst \wedge t=t_E) < (\iota_S t')(t' \in Inst \wedge t'=t_{E'})$. Since $|0 - k| = k$, $dist(t_E,t_{E'},k)$ holds in S. By Postulate 4 it follows that $dist(t,t',k)$ holds. From (d) it follows that $|0 - f(t')| = k$. Finaly, since $t < t'$, $f(t') = k$. In the similar way we get that $f((\iota_{S'} t'')(t''=t_{E'})) = m$.

For the other direction, suppose $f((\iota_S t')(t'=t_{E'})) = k$ and $f((\iota_{S'} t'')(t''=t_{E'})) = m$. Then, $|f(t) - f(t')| = |0 - k| = k$. Since there are $t_E,t_{E'}$, belonging to $Inst^{(e)}$ and not belonging to *Inst* , such that $t=t_E$ and $t'=t_{E'}$ hold in S, instants t and t' are in the relation *dist* with some real number. To show that $dist(t,t',k)$ holds, suppose $dist(t,t',j)$ and $k \neq j$. By (d) it follows that $|f(t) - f(t')| = j$. Since $k \neq j$, we get the contradiction: $|f(t) - f(t')| \neq |f(t) - f(t')|$. Hence, $dist(t,t',k)$ holds. Since k is positive, $t < t'$. $dist(t,t',k)$ and Postulate 4 imply that $dist(t_E,t_{E'},k)$ holds in S, which means that $|0 - \tau| = k$, where τ is the time coordinate of event E' in S. Since $t < t'$ implies that $t_E < t_{E'}$ holds in S, from $|0 - \tau| = k$ it follows that $\tau = k$. In the similar way, we get that m is the temporal coordinate of event E' in S'.

If (ii) is the case, take isomorphisms f_S and $f_{S'}$ such that:

(a) $f_S((\iota_S t)(t=t_E)) = 0 = f_{S'}((\iota_{S'} t)(t=t_E))$;

(b) for every t': if $(\iota_S t)(t \in Inst \wedge t=t_E) < t'$, then both $f_S(t')$ and $f_{S'}(t')$ are positive;

(c) for every t': if $t' < (\iota_S t)(t \in Inst \wedge t=t_E)$, then both $f_S(t')$ and $f_{S'}(t')$ are negative;

(d) for every t, t': if $dist(t,t',n)$, then $|f_S(t) - f_S(t')| = n = |f_{S'}(t) - f_{S'}(t')|$.

Then, for any event E', $f_S((\iota_S t)(t=t_{E'})) = n$ and $f_{S'}((\iota_{S'} t)(t=t_{E'})) = m$ if and only if (x,y,z,n) and (x',y',z',m) are coordinates, related by the Lorentz transformations, of event E' in S and S' respectively.

To see that both (i) and (ii) can occur, consider the following three systems S, S' and S" (Figure 3). S and S' are in uniform relative motion, while S" is at rest with respect to S. Systems S and S' are spatially coincident at A, while systems S' and S" are spatially coincident at B.

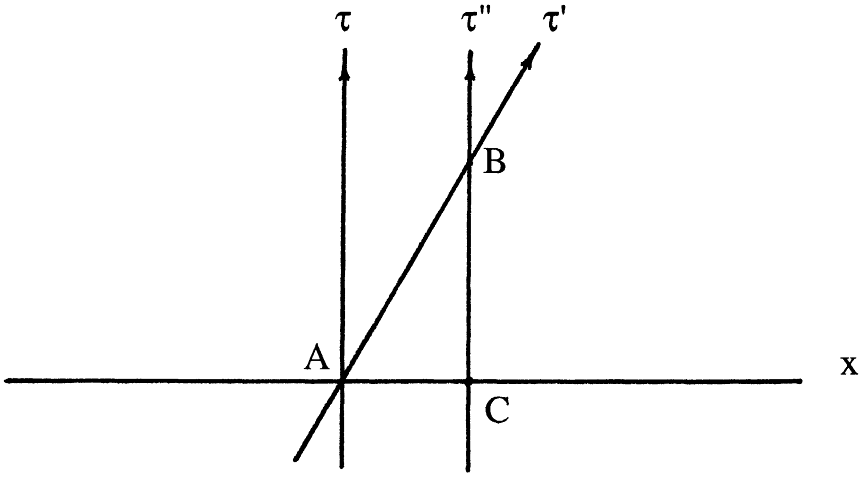

Fig. 3.

Suppose that the temporal coordinate of event A is 0 in all three systems and suppose $(\iota_S t)(t\in \mathit{Inst} \wedge t=t_A) = (\iota_{S'} t)(t\in \mathit{Inst} \wedge t=t_A) = (\iota_{S''} t)(t\in \mathit{Inst} \wedge t=t_C)$. Event B has different time coordinates in S (S") and in S'. Hence, for some n, $\mathit{dist}(t_A,t_B,n)$ holds in S' and $\neg \mathit{dist}(t_A,t_B,n)$ holds in S". From postulate 4 it follows that

$\mathit{dist}((\iota_{S'} t)(t\in \mathit{Inst} \wedge t=t_A),(\iota_{S'} t)(t\in \mathit{Inst} \wedge t=t_B),n)$ and

$\neg \mathit{dist}((\iota_{S''} t)(t\in \mathit{Inst} \wedge t=t_A),(\iota_{S''} t)(t\in \mathit{Inst} \wedge t=t_B),n)$.

From Postulate 5 it follows that

$(\iota_{S'} t)(t\in \mathit{Inst} \wedge t=t_B) \neq (\iota_{S''} t)(t\in \mathit{Inst} \wedge t =t_B)$.

5 CLOCK RETARDATION

Another aspect of the STR which is famous by its discrepancy with what common sense would suppose is the phenomenon of clock retardation. Consider again the situation where G and T are frames of reference moving relative one to another. Suppose that Jones' clock and Smith's clock were synchronized and set at zero when they were spatially together; that is to say Jones' clock reading 0 is an event (0_J) coincident with Smith's clock reading 0 (0_S). Suppose also that there was no shooting at all and let 1_J be the event that is the reading 'one hour' on the clock of Jones and let 1_S be the event that is the reading 'one hour' on the clock of Smith. Relative to Jones' system the event 1_J will not be simultaneous with the event 1_S but with a reading on Smith's clock (say event X_S) which is earlier than 1_S in Smith's system. Let Y_J be an event in Jones' system simultaneous, relative to Jones' system, to 1_S. Relative to Jones' system, 1_J is earlier than Y_J. Relative to Jones' system the clock of Smith reads "one hour" only after more than one hour has passed. Smith's clock is slow relative to Jones' clock. On the other hand, relative to Smith's system the event 1_S will be simultaneous to a reading on Jones' clock (event X_J), which is earlier than the event 1_J in Jones' system. Let Y_S be an event in Smith's system simultaneous, relative to Smith's system, with 1_J. Relative to Smith's system 1_S is earlier than Y_S. Relative to Smith's system, the clock of Jones reads "one hour" only after more than one hour has passed. Jones' clock is slow relative to Smith's clock. Actually, the dilation factor is equal in the two cases - the factor by which the clock of Smith is retarded relative to Jones' clock is the same as the factor by which the clock of Jones is retarded relative to Smith's clock.

How do our absolute instants fit in this phenomenon? Here follows the translation in ML^{V} of the above facts.

Assume that $t_{0_J}, t_{0_S}, t_{1_J}, t_{1_S}, t_{X_J}, t_{X_S}, t_{Y_J}, t_{Y_S} \in$ *Inst*$^{(e)}$ and t,t',t",t'"$\in$ *Inst* . For the sake of convinience we take that (i) holds.

(i) $(\iota_G t)(t=t_{0_J}=t_{0_S}) = (\iota_T t)(t=t_{0_J}=t_{0_S})$

(ii) $(\iota_G t)(t=t_{0_J}=t_{0_S}) < (\iota_G t')(t'=t_{X_J}) < (\iota_G t'')(t''=t_{1_J}=t_{X_S}) <$

$(\iota_G t''')(t'''=t_{Y_J}=t_{1_S})$

(iii) $(\iota_T t)(t=t_{0_J}=t_{0_S}) < (\iota_T t')(t'=t_{X_S}) < (\iota_T t'')(t''=t_{1_S}=t_{X_J}) <$

$(\iota_T t''')(t'''=t_{Y_S}=t_{1_J})$.

Since

dist(t_{0_J},t_{1_J},m) holds in G,

dist(t_{0_S},t_{1_S},m) holds in T,

dist(t_{0_J},t_{X_J},k) holds in G,

dist(t_{0_S},t_{X_S},k) holds in T,

dist(t_{0_J},t_{Y_J},n) holds in G and

dist(t_{0_S},t_{Y_S},n) holds in T,

by Propositions 7 and 8 it follows that

(iv) $(\iota_G t')(t'=t_{X_J}) = (\iota_T t')(t'=t_{X_S})$

(v) $(\iota_G t'')(t''=t_{1_J}) = (\iota_T t'')(t''=t_{1_S})$

(vi) $(\iota_G t''')(t'''=t_{Y_J}) = (\iota_T t''')(t'''=t_{Y_S})$.

The translation shows that everything is as it should be: although coordinate time between two events is relative to a given frame of reference, "one hour" is "one hour" in both cases and the retardation factor is the same in both cases.

6 WHAT TO GIVE UP

We shall now explicate some of the implications of the STR for our framework.

Let us see what the STR tells about the ordering of events. The function called the interval between events is invariant under Lorentz transformations. Consider two events e_1 and e_2 with coordinates (x_1,y_1,z_1,τ_1) and (x_2,y_2,z_2,τ_2) respectively, in a system S. The *interval between the two events* is

$$I^2(e_1, e_2) = c^2(\tau_2 - \tau_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2.$$

$I^2(e_1, e_2) \geq 0$ ($I^2(e_1, e_2) < 0$) means that event e_1 is (is not) causally connectible to event e_2, that is, that a signal can (can not) be send from e_1 to e_2, or conversly. Since $I^2(e_1, e_2)$ is invariant, the property that one event is connectible to another one is the same in every frame of reference. Furthermore, when event e_1 is connectible to event e_2, by measuring the coordinates of event e_2 along the direction of the light rays through event e_1 in the Minkowsi diagram (Figure 4), we can determine whether e_2 is in the future of e_1 (later than e_1) or in the past of e_1 (earlier than e_1).

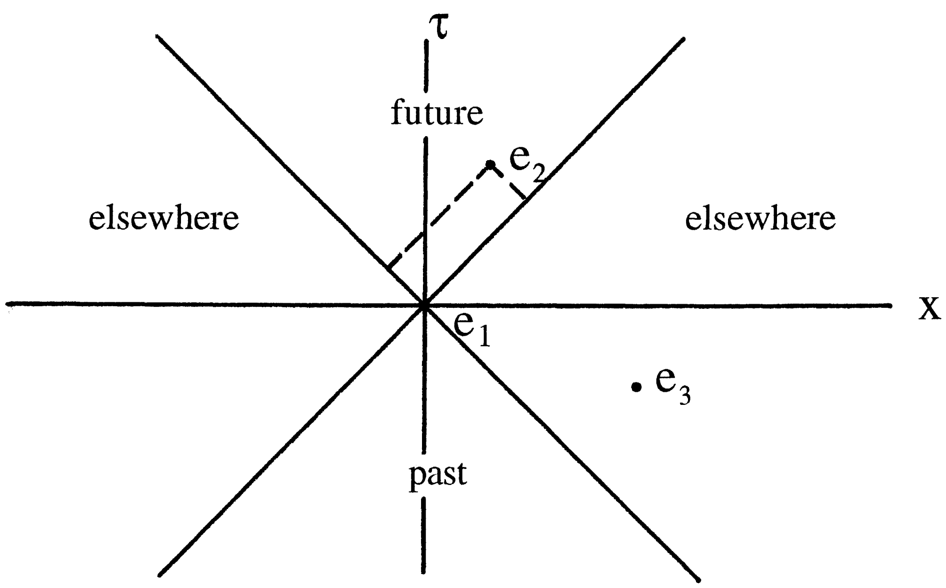

Fig. 4. The Minkowski diagram for an observer

The property that events connectible to e_1 are in the future of e_1 or in the past of e_1 is also invariant. On the other hand, the property of being numerically earlier (later) than e_1 is not invariant for an event which is not connectible to e_1. The relation of connectibility is both reflexive and symetrical, but it is not transitive since for any two events e_1, e_3, which are not connectible, there will be an event e_2 connectible to both e_1 and e_3, and in the future of both e_1 and e_3.

We say that two instants t_{e_1} and t_{e_2} belonging to $Inst^{(e)}$ are *absolutely temporally ordered* if and only if their temporal ordering does not vary through possible cases:

$$t_{e_1} \sim t_{e_2} \equiv_D N(t_{e_1} < t_{e_2}) \vee N(t_{e_1} = t_{e_2}) \vee N(t_{e_2} < t_{e_1}).$$

It is easy to see that $\sim \in MConst_{Inst^{(e)}}$. From this definition and from the reflexivity and symmetry of identity we may deduce that the relation $\sim$ is reflexive and symmetric. However, it does not follow from this definition and from the properties of the relation $<$ that the relation $\sim$ is transitive. Suppose $N(t_{e_1} < t_{e_2})$ and $N(t_{e_3} < t_{e_2})$. Then, $t_{e_1} \sim t_{e_2}$ and $t_{e_2} \sim t_{e_3}$ and $N(t_{e_1} < t_{e_3} \vee t_{e_1} = t_{e_3} \vee t_{e_3} < t_{e_1})$. But, we can not deduce anything about $t_{e_1} \sim t_{e_3}$.
Absolute time order of instants belonging to $Inst^{(e)}$ within our framework corresponds to a unique time order of connectible events. Since events that are not connectible do not have a unique time order, the notion of "absolute temporal ordering of all instants belonging to $Inst^{(e)}$", appropriate for Newtonian physics, has to be given up.

We have seen (Figure 3) that for two systems S' and S", in uniform relative motion and in spatial proximity at some event B, it can hold that

$$(\iota_{S'}t)(t \in Inst \wedge t = t_B) \neq (\iota_{S''}t)(t \in Inst \wedge t = t_B).$$

Thus, two inertial observers can "meet" each other without identifying the instant of the meeting with the same absolute instant. To make the example more intuitive, suppose that observers O' and O", in systems S' and S" respectively, both say "Now" when they meet at B. Then,

$$(\iota_{S'}t)(t \in Inst \wedge t = t_{O'n}) \neq (\iota_{S''}t)(t \in Inst \wedge t = t_{O''n}).$$

Clearly, the notion of a universal absolute instant "now", the same for all coexistent observers, makes no sense.
Keeping that in mind, the well-known twins "paradox", to which the theory of relativity leads, does not seem paradoxical any more.

Consider a three-nonaccelerating-system version of the twin "paradox"[6].

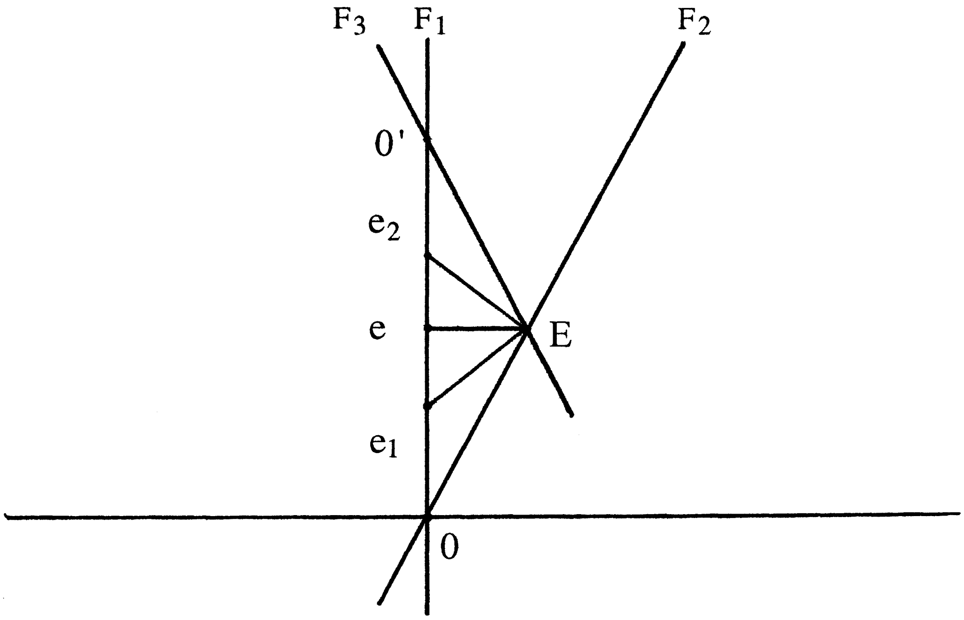

Fig. 5. The paradox of the twins

Let F_1 and F_2 be two non-accelerating frames of reference moving relative to one another (Figure 5) and let C_1 and C_2 be digital clocks associated with F_1 and F_2 respectively. Assume that I am travelling with F_1 and that my twin is traveling with F_2. Suppose that the F_1-path intersects the F_2-path and that me and my twin are the same age when F_1 and F_2 are spatially coincident or, in other words, suppose that the clocks were synchronized and were set to read zero when F_1 and F_2 were in spatial proximity (event 0). Suppose also that a third non-accelerating frame F_3 intersects the paths of F_2 (event E) and F_1 (event 0'). The velocities of F_2 and F_3 relative to F_1 are +v and -v respectively. When F_2 and F_3 are spatially coincident, my twin transfers from F_2 to F_3, that is, the clock C_3 of F_3 is synchronized with C_2 and is set to read what C_2 then reads. This means that my twin has a kind of composite clock: between the departure and E this clock is the clock associated with F_2

and between E and the meeting this clock is the clock associated with F_3. My twin transfers to my frame when F_1 and F_3 coincide. Let C be the composite clock and let F be the "reference system" of my twin. Let 2t be the time elapsed relative to F_1 between 0 and 0'. Let e be an event at F_1 such that for F_1, e is simultaneous with E, that is, with spatial coincidence of F_2 and F_3 and such that t is the time elapsed relative to F_1 between 0 and e. Let T be the time elapsed relative to F_2 between 0 and E as well as the time elapsed relative to F_3 between E and 0'. If, applying the STR, I and my twin compare the time elapsed between our parting from each other and our meeting with each other, his clock will read less than mine. Let e_1 and e_2 be events at F_1 such that e_1 is simultaneous with E for F_2, and e_2 is simultaneous with E for F_3. It is the passage of time between e_1 and e_2 which makes the clock C_1 read a later time at 0', since the composite clock C fails to record it. C regards E to be simultaneous with e_1, since this is so for C_2, and C regards E as simultaneous with e_2, since this is so for C_3. Hence, C regards e_1 to be simultaneous with e_2. So, although we both agree that we part from each other simultaneuosly and that we meet each other simultaneously, at the meeting point more time has elapsed for me than for him! Actually, from the STR point of view, there is nothing paradoxical in this relativistic conclusion since the jump occurs only in the life of one of the twins which means that the situation is not symmetrical for both twins.

Here follows what we get within our framework. Assume that $t \in$ Inst, $t_0, t_E, t_{e_1}, t_e, t_{e_2}, t_{0'} \in Inst^{(e)}$. The assumption that me and my twin are the same age at our parting from each other becomes:

$$(\iota_{F_1}t)(t=t_0) = (\iota_F t)(t=t_0).$$

It follows that

$$(\iota_{F_1}t)(t=t_0) < (\iota_{F_1}t)(t=t_{e_1}) < (\iota_F t)(t=t_E=t_{e_1}=t_{e_2}) < (\iota_{F_1}t)(t=t_e) <$$

$$(\iota_{F_1}t)(t=t_{e_2}) < (\iota_F t)(t=t_{0'}) < (\iota_{F_1}t)(t=t_{0'}).$$

7 CONCLUSION

As the above discussion indicates, the STR does not force an absolutist to give up his classical absolutist conception of time. The relativistic notion of temporal relations for events is compatible with the ontology of the absolute theory of time. So, the introduction of the relativistic notion of simultaneity, with all its consequences, can be understood as a modification of some aspects of the absolute theory.

NOTES

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1 Leibniz (1956).

2 Grünbaum (1964), p. 345.

3 Newton-Smith (1980), p. 195-200.

4 ML^{V} was introduced in Bressan (1972).

5 Bressan (1972), p. 86-91

6 This version of the "paradox" is considered in Newton-Smith (1980), p. 190-195

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