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p u b l i c a t i o n

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LP-94-16, *received*: September 1994

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When Variables Don't Vary Enough*

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August 1994

In this paper the interpretation of variables in a dynamic semantics will be considered. The main empirical concern will be the interaction of presupposition and quantification, but I will also consider how epistemic modals and quantifiers combine. I will show how these empirical considerations motivate a choice between two styles of quantification. The first of these styles involves treating quantified variables rather like discourse markers, whereas the second style gives variables a more classical interpretation. I will argue for the second, more conservative option.

I will also argue that the solution is empirically superior to some other treatments of quantified presuppositions, including those treatments which, in the current *vogue*, make heavy use of relatively unconstrained notions of *accommodation*. Such an argument of course depends on there being a fact of the matter, a generally accepted picture of just what the data is regarding the interaction of presupposition and quantification. Unfortunately, there is considerable divergence of views across the presupposition literature, and there is not even one paper in which a serious attempt is made to systematically investigate the behavior of different presupposing constructions within different quantifiers and different discourse contexts. Indeed, of the existing discussions, all (that I am aware of) use decontextualised single sentences as the main source of data, and none include any indication that the determination of what various quantified sentences presuppose involved application of standard tests for presupposition. In the following section I will consider the empirical problem, before proceeding with a theoretical analysis. The data is considered in greater detail in [Be93b] and in [Be94b].

1 Presupposition and Quantification: The Data

In what follows I will consider a range of examples involving two presuppositional constructions: possessives and the semi-factive verb *discover*. The reader may well wonder whether other presuppositional constructions will behave the same way. I can only reply that I am interested in the very same question, but that an answer will have to await further investigation. For instance, I strongly suspect that regarding the type of cases I will consider there is considerable variation amongst attitude verbs which presuppose the truth of their propositional argument, but I am not under the impression that this variation correlates with the factive/semi-factive distinction of Karttunen. In [Be94b] I argue that much of the observed variation, possibly including the special first person behavior of semi-factives, is connected with issues of topic-focus articulation and discourse coherence, and not with presupposition *per se*.

It is partly because of the interaction of these other factors that I have tried to give most of the examples to be considered together with some discourse context. However, the reader is

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to be warned that small variation in intonation can still affect the judgements concerning the examples below, though I will attempt to point out cases where this is a serious problem. Also, as an obvious point of methodology, giving too many of these examples to the same informant (or conference audience) in one sitting produces unpredictable results. For instance, salience of a set of car owning individuals in one example is likely to affect the reading of a following example presupposing car ownership. Despite these caveats, most, if not all, of the following data is quite robust, and this will be sufficient to contradict many of the assumptions that have been made elsewhere in the literature[1].

1.1 Indefinites binding a trigger do not carry a universal presupposition.

Although this is not a contraversial point, it is worth making for two reasons. Firstly, because it should get the reader into the swing of the type of example to be considered, and secondly because the basic system introduced in [He83], which provides the jumping off point for the technical developments in this lecture, makes an incorrect prediction for these cases. In some of the following examples there may be several presupposing constructions, and bold emphasis will be used to indicate to the reader which clause contains the presupposition relevant to the discussion. In the following two examples the NP her car is bound by the indefinite a member of the team, and the factive VP discovered that the car's radiator had sprung a leak is bound by the indefinite *a company car*, respectively. It is clear that the italicized part of the first example is compatible with not every team member owning a car, and the italicized part of the second example is compatible with not every company car which someone in sales has having sprung a leak in the radiator.

E1 How will everybody get to the match?

A member of the team came to the last match in her car, and she might again, but all the others will come by bike since they don't own cars.

E2 How many employees had problems with their cars last year?

Hardly any. **There was somebody in sales with a company car who discovered that the car's radiator had sprung a leak**, but that's all.

I should make clear that in deriving the general conclusion that "indefinites binding a trigger do not carry a universal presupposition", I am trying to say that there will not be cases where such a universal inference is made. For instance, there could be uses of the italicized part of the second of these examples, especially with stress on discovered which license an inference that all the sales staff's company cars' radiators developed leaks. What I mean by "presupposition" here is something like "minimal precondition for interpretation". Thus the fact that there are uses of the italicized part of the second example in contexts where the universal does not hold will be sufficient for me to conclude that there is no universal presupposition. Now, to take such a restrictive notion of presupposition is immediately to invite speculation as to what is the link between the notion of presupposition being employed here and the uses of the term

[1] I have relied on a relatively small set of informants (8), half of whom answered an 18 question email questionnaire, and to the other half of whom I presented a smaller selection of the examples verbally. I dare say that to the average card carrying linguist this will seem like a hopelessly small sample. Consider this section, if you will, a pilot study. For, so far as I am aware, nobody has ever conducted larger scale empirical work concerning the interaction of presuppositions and quantification: I cannot even find evidence of a researcher consistently bothering to ask any informants at all for their opinions of the data. Perhaps, it is not the sort of thing one talks about. Perhaps the prevalent use of de-contextualised single sentence examples has meant that it has been all but impossible to find informants who would volunteer clear judgements.

elsewhere in the presupposition literature. For the moment, let us just say that what is at stake here is what has been called "semantic presupposition", and that a fuller theory, by detailing the links between this notion and that of "pragmatic presupposition", should explain certain additional defeasible inferences that may arise with very similar examples to those considered here. A formal theory, very much in the spirit of Stalnaker's proposals [St74], and which can be seen as an attempt to connect semantic and pragmatic notions of presupposition can be found in [Be93b] and [Be:b].

1.2 Triggers bound in the restrictor of a quantifier act as domain restrictors

In the following 4-way example, the presupposition of car ownership occurs in the restrictor of a quantifier. It is clear that the discourse as a whole does not license any inference that every team member has a car. It appears that the presupposition of car ownership serves to restrict the domain of quantification to car owning individuals, so that the italicized clause must have a meaning equivalent to most(/few/all/none) of the team members who own a car and went to the last match in it, will drive to this match.

E3 How many team members will drive to the match?

I don't know whether all of the team members own a car, and I don't know for sure how many will drive. However, apparently **most(/few/all/none) of the team members who went to the last match in their car, will drive to this match.** I would guess that about 5 will drive in total.

A similar comment applies to the following example. The discourse does not license an inference that every employee's car radiator sprang a leak last year, and we must conclude that the semi-factive discover is restricting the domain of quantification to individuals whose car radiators actually did spring a leak.

E4 How many of your employees put antifreeze in their car radiators?

I don't know whether all of the employees own a car, and I don't know for sure how many of them put antifreeze in the radiator. But, if my past experience is anything to go by, **most/few/all/none of the employees who discovered that their car radiators had sprung a leak last winter will remember to put antifreeze in this year.**

Space does not allow a detailed discussion of predictions derived elsewhere in the literature, but I will summarise the relevant properties of some of the more influential treatments. The claim that there is no universal presupposition for these cases is again in conflict with the predictions derivable in the basic system of [He83], although accords with the predictions in [Co83], [KP79] and [vdS92]. However, the Karttunen and Peters treatment would not give any prediction that the presupposition caused domain restriction, and would thus tend to give the wrong truth conditions. For instance, all of the employees who discovered that their car radiators had sprung a leak last winter will remember to put antifreeze in this year would be false if some employees whose radiators did not develop leaks last year forget to put antifreeze in this year.

1.3 Triggers bound in the scope of a quantifier do not act as domain restrictors.

As discussed in the previous lecture, the situation is different when the presupposition trigger is in the scope of the quantifier. The following two example discourses are completely bizarre:

E5 How many team members and cheerleaders will drive to the match?

Few of the 15 team members and none of the 5 cheerleaders can drive, but **every team member will come to the match in her car.** So expect about 4 to drive.

E6 How many of your employees with company cars had problems with their car radiators last year?

Although few of the sales staff had any problems with their cars last year, **all of the management discovered that their car radiators had sprung a leak.** However, most of the management didn't have a single problem with their car radiator the whole year: they are generally quite conscientious about car maintenance.

Why are these examples odd? Regarding the first of the two examples I suppose that the italicized proposition is incompatible with there being team members without a car or team members with a car who do not use it to come to the match, although these requirements are contradicted by the assertion that few of the team members can drive at all, and that only 4 will drive to the match. Similarly, the italicized clause in the second example seems to be incompatible with there being either members of management whose car radiators had no problems, or who failed to discover problems that arose, and the first of these is bluntly denied by the following claim that most of the management had no problem with their car radiator.

Neither [Co83], [KP79] or the basic system of [He83] provide any mechanism whereby a presupposition in the scope could trigger domain restriction. However, the system in [vdS92] would allow "intermediate accommodation" to occur, so that at the level of discourse representation the presupposition triggered in the scope is realised in the quantifier's restrictor. In other words, on van der Sandt's account, the italicized sentence in E5 would become a claim about the car owning team members, and the italicized sentence in E6 would become a claim about those members of management whose car radiators had sprung a leak. One supposes that if the notion of accommodation introduced in [He83] were more fully developed, similar predictions would result[2].

1.4 Triggers bound in the scope of a quantifier do not result in a universal semantic presupposition.

We have seen that presuppositions in the scope of a quantifier cannot trigger domain restriction. It is often claimed (eg. [He83],[Ei93]) that a universal presupposition results in such cases. How can this hypothesis be tested? Firstly, observe that a simple positive assertion with a universal quantifier does lead to a universal inference concerning satisfaction of the presupposition. For instance, every team member turned up to the match in her own car will, in contexts which do not license domain restriction to car owners, lead to an inference that every team member has a car. However, the following examples indicate that the universal negative quantifiers no and none of the do not lead to such a universal inference. In the first of the following two examples, it is plausible that not every team member owns a car, and in the second it is clear that none of the management had any problems with their car radiators:

[2] See [Ze92] for a development of Heim's ideas. But this development is made with hindsight as to how van der Sandt went about solving a similar problem, and it is by no means clear that it is faithful to the original rather vaguely specified conception of Heim. In particular, in Zeevat's development what is accommodated is precisely what is presupposed, whereas Heim tells me that the idea she had in mind involved a more subtle pragmatically driven link between what is presupposed and what is accommodated. However, [He83] is wholly ambiguous on this point — I must admit that I had originally read it with much the same interpretation in mind as Zeevat managed to formalise.

E7 How many team members and cheerleaders will drive to the match?

A few of the 15 team members and none of the 5 cheerleaders can drive, but **no team member will come to the match in her own car.**

E8 How many of your employees with company cars had problems with their car radiators last year?

Although a few of the sales staff discovered problems with their car radiators last year, **none of the management discovered that their car radiators had sprung even a single leak.** The reason that none of the management hade a single problem with their car radiator is that they are generally quite conscientious about car maintenance.

Given that in a simple assertion of a sentence with a universal positive quantifier there is an inference to universal satisfaction of the scopal presuppositions by objects in the domain of quantification, can we say whether this universal inference is itself to be thought of as a presupposition? So far as I know, there has been no serious attempt to answer this question. The obvious approach is to try the standard presupposition tests based on inheritance through embedding contexts. Of the next four examples, the first two involve an embedding in the antecedent of a conditional, and the second two use negation. Of these tests, the conditional test is generally to be preferred, because of the difficulty in saying just what the negation of a universally quantified sentence is. Sentence external negation (it is not the case that...) is troublesome because it is often associated with cases of presupposition denial, and I have settled for an NP-internal negation.

All of the following four examples are judged felicitous. However, in the first and third cases, it is clear from the discourse that some of the team members might not own a car, and in the second and fourth examples, there are managers whose car radiator did not develop a leak[3].

E9 How many team members will drive to the match?

I don't know for sure whether all of the team members own a car, and I don't know for sure how many will drive. But, usually one or two(/only one or two) go by public transport, so **if every(/no) team member turns up to the match in her own car, I'll be surprised.**

E10 Do you expect any of your employees to have problems with their car radiators next year?

Usually there are one or two(/only one or two) people who forget to put antifreeze in their cars. So **if nobody(/everybody) discovers that their radiator has a leak this year, I'll be surprised.**

E11 Will everybody on the team need their parking spaces this season, or do you think I can use one?

Last season the whole team used to drive to matches. But I suspect that this season there might be one or two that don't even own a car. Given also the recent pre-match traffic jams,

[3]I should point out that of all the examples considered here, these four caused me the most trouble. Slight variants tend to send informants into panic mode, and in presenting examples in spoken form, intonation was critical, although I am unable to report any definite conclusions as to the how intonation affects acceptability in these cases. This sensitivity arises partly because of the complexity of applying embedding tests to sentences that are already semantically complex, involving universal quantification. But I have to say that there could well be other factors, and that of all my empirical conclusions, the claim that universals with a presupposition in the scope do not themselves carry a universal presupposition is the one I would most readily drop. Note also that it would be straightforward to adapt the technical system to be presented later so as to accord with a claim of universal presupposition.

I think its safe to say that **not every team member will come to the match in her car**. So I guess we'll be able to find you a spare place.

E12 In the big freeze last year, all the company cars had radiator problems, and many of the sales staff complained about being given cars which were poorly maintained. Do you think the same will happen again?

It's possible. But recently we've been using a new product in all our cars, a chemical which when added to the radiator top-up tank, slowly re-seals any holes which form. So, regardless of whether everybody's car actually does develop radiator problems, I'm just about certain that **not everybody will discover that their car radiator has a leak this year**, and I expect far fewer complaints.

Since embedded versions of universal sentences appear not to carry any of the universal implications of the simple sentences, I conclude that the simple sentences do not carry universal presuppositions. In the later technical presentation the purported universal presuppositions of the simple sentences will be analysed as entailments rather than presuppositions.

1.5 Donkey bound presuppositions do not trigger (modal or quantificational) domain restriction

In van der Sandt's model a presupposition arising in the consequent of a conditional can be accommodated in the antecedent of the conditional. Thus he would predict that a sentence if someone's in the team, she comes to matches in her car has a reading equivalent to if someone who owns a car is in the team, she comes to matches in her car. One would also expect a development of the accommodation model of Heim to lead to this sort of prediction [4]. However, it is clear from the following examples that such this type of accommodation leads to the false prediction that the following two discourses have a felicitous reading, when informants are uniformly unable to make any sense of them at all:

E13 How many team members and cheerleaders will drive to the match?

Few of the 15 team members and none of the 5 cheerleaders can drive, but **if someone's in the team, she comes to matches in her car**. So expect about 4 cars.

E14 How many of your employees had problems with their car radiators last year?

Nearly all the company cars survived the entire year without any problems with the radiator. However, **if an employee's car was inspected regularly, it was eventually discovered that the radiator had sprung a leak**. In these few isolated cases, we sacked the employee.

As with sentences E5 and E6, it is straightforward to show that the reason for the infelicity is not because accommodation leads to an incoherent discourse, but because accommodation does not happen. We simply find a minimal variant on the original example where the material van der Sandt would claim can be accommodated is added as an explicit restrictive clause, as in the following case:

E15 How many of your employees had problems with their car radiators last year?

Nearly all the company cars survived the entire year without any problems with the radiator. However, **if an employee's car was inspected regularly, it was eventually discovered that the radiator had sprung a leak, if indeed it had done so**. In these few isolated cases, we sacked the employee.

[4] As indeed is the case in [Ze92].

Note that it is difficult to test whether the universal inference (to car ownership by team members) arising from sentences like if someone's in the team, she comes to matches in her car is in fact a presupposition. There are no tests for presupposition that can easily be applied to a conditional sentence.

1.6 Unbound presuppositions syntactically within the restrictor or scope of a quantifier which cannot express negation are projected.

I have argued that bound presupposing constructions in the scope of a quantifier do not lead to universal presuppositions. However, presuppositions arising within quantificational constructions do not simply disappear. Consider the case of a presupposed closed proposition occurring in the scope of a quantifier:

E16 Is the company likely to remain solvent this year, and how does this relate to projected staff turn over?

I don't know whether the company finances are healthy or not, but every employee who has discovered that the company is going bankrupt is looking for another job.

This discourse is not felicitous, presumably because the claim that the speaker does not know whether company finances are healthy is in contradiction with the clear inference that the company is going bankrupt. Replacement of the presupposition trigger with a non-factive expression produces an acceptable discourse:

E17 Is the company likely to remain solvent this year, and how does this relate to projected staff turn over?

I don't know whether the company finances are healthy or not, but every employee who has come to believe that the company is going bankrupt is looking for another job.

It is interesting to observe that E16 provides another difficulty for accommodation models. For in such a model (van der Sandt's or Heim's), the proposition that the company is going bankrupt may be accommodated into the (discourse representation of) the restrictor the quantifier. In van der Sandt's model, this would lead to a DRS for the quantification like the following:

[x: going-bankrupt(c),employee(x),discover(x,going-bankrupt(c))] $\Rightarrow$[looking-for-a-job(x)]

On standard DRT semantics, this has a meaning equivalent to if the company is going bankrupt then everybody who has discovered it is looking for another job. This is a perfectly reasonable meaning for a sentence to have, but it is by no means the meaning of everybody who discovered that the company is going bankrupt is looking for another job.. I conclude that either a model of presupposition must be developed not involving presupposition-triggered local accommodation, or else van der Sandt's model must be altered so that local accommodation is much more restricted. I will pursue the first option.

2 Technical Preliminaries

To show how the data from the previous section can be dealt with, I will introduce a family of dynamic logics based on the system presented in [He83], but extending and modifying it in various ways[5]. The models for Heimian logics can be seen as sets of first order models (the elements of $\mathcal{W}$) defined over a constant domain $\mathcal{D}$:

[5]Dekker's EDPL [De93], which combines ideas form [GS91a] and [He83] provides much of the technical inspiration for this reformulation. See also [Be92].

Definition D1 (Models) $\langle \mathcal{D}, \mathcal{W}, \|.\| \rangle$

The use of a fixed domain does not represent any philosophical commitment, but allows technical simplification.

We begin with the notions in [He83] of a sequence, which is a partial variable assignment, and a world-sequence pair. A sequence maps each variable either onto an element of the domain or onto the emptyset, which is assumed not to be a member of the domain. The *domain of a sequence* is the set of all variables not mapped onto the emptyset by that sequence. The *domain of a world-sequence pair* is just the domain of the sequence, and in terms of sequence domains and world-sequence pair domains, a notion of *extension* is introduced. One sequence or world-sequence pair extends ($\geq$) another just in case all the valuations that the first gives to variables in its domain are preserved in the second, although the second may have a larger domain. World-sequence pairs can be extended by *adding* a new assignment: if pair i differs from j by the addition of the assignment of the variable x to d ($i >_{x:d} j$, then i and j agree on the values of all variables apart from x, x is not in the domain of j, and the sequence in i maps x onto d.

Definition D2 (Sequences; World-Sequence Pairs) *(f,g,h; i,j,k)*

$$\begin{array}{rcl}
SEQ & = & (\mathcal{D} \cup \{\emptyset\})^{VAR} \\
domain(f) & = & \{v \in VAR \mid f(v) \neq \emptyset\} \\
domain(\langle w, f\rangle) & = & domain(f) \\
f \geq g & \textit{iff} & \forall v \in domain(g)\ f(v) = g(v) \\
\langle w, f\rangle \geq \langle w', g\rangle & ifof & w = w' \ \wedge \ f \geq g \\
\langle w, f\rangle >_{x:d} \langle w', g\rangle & \textit{iff} & w = w' \wedge \forall v \neq x f(v) = g(v) \wedge g(x) = \emptyset \wedge f(x) = d
\end{array}$$

A Heimian context, or information state, is a set of world-sequence pairs in which all the sequences have the same domain[6]. The *domain of an information state* is defined as the domain of its elements. Also useful later will be an information subtraction operation ($\backslash$) analogous to set subtraction. The result of subtracting one state from another is the subset of world-sequence pairs in the first which do not have extensions in the second.

Definition D3 (Information) *(I,J,K)*

$$\begin{array}{rcl}
INFO & = & \{I \in 2^{\mathcal{W} \times SEQ} \mid \exists \delta \forall i \in I\ domain(i) = \delta\} \\
domain(I) & = & \{v \in VARI \mid \exists \langle w, f\rangle \in I\ v \in domain(f)\} \\
I \backslash\backslash J & = & \{i \in I \mid \neg \exists j \in J\ j \geq i\}
\end{array}$$

It is natural to define a partial ordering of *precedence* ($\preceq$) over information states, such that, intuitively, one state precedes another if the second contains at least as much information as the first. In Stalnaker's model of assertion [St79] information increase corresponds to the elimination of possible worlds. In the current setting, there are two ways to gain information: world-sequence pairs can be eliminated, and new variables can be added to the domain. Thus one state precedes another if the set of world-sequence pairs in the second contains only extensions of world-sequence pairs in the first. I will not discuss in detail the algebraic structure imposed

[6]The possibility of allowing the domains of the component world-sequence pairs presents an interesting vista. For instance, a sentence like a wolf might come to the door could introduce a variable which was only in the domain of those world-sequence pairs where, in the given world, a wolf actually does come to the door. I will not pursue this here.

on the set of information states INFOby the precedence ordering, save to point out that there are unique top and bottom elements. The bottom, $\boxed{:-)}$, is a state of blissful ignorance, that state which informationally precedes all others: it is the set of all pairs of a world and the sequence with null domain. The top $\boxed{:-(}$ is simply the empty set, and can be thought of as *absurdity*, for it represents the state arrived in after accepting contradictory information, so that there no longer remain any possible worlds compatible with all the accepted information.

Definition D4 (The Structure of INFO)

$$\begin{array}{lcl} I \preceq J & \textit{iff} & \forall j \in J\, \exists i \in I\; j \geq i \\ \boxed{:-)} & = & \{\langle w, f\rangle \in \mathit{INFO} \mid \mathit{domain}(f) = \emptyset\} \\ \boxed{:-(} & = & \emptyset \end{array}$$

The following obvious properties, of which the formula $\boxed{:-)} \preceq \boxed{:-(}$ is a memorable corollary, say as much as I intend to about the algebraic structure of INFO:

Fact F1

$$\begin{array}{ll} \forall I \in \mathit{INFO} & \boxed{:-)} \preceq I \\ \forall I \in \mathit{INFO} & I \preceq \boxed{:-(} \end{array}$$

At the heart of the system in [He83] is the notion of a Context Change Potential (CCP). Formulae of Heimian logics will be interpreted as CCP's, which are defined here as binary relations between contexts. If a pair of information states $\langle I, J\rangle$ is in the denotation of a formula, then the formula provides a way of updating an input state I to produce the output J. This definition of CCP allows more generality than will be used in any of the systems presented here, since it is possible for a single input of a CCP to have multiple outputs, when in fact no formula of a Heimian logic will have this property[7].

Definition D5 (Context Change Potentials) $\mathit{CCP} = 2^{\mathit{INFO} \times \mathit{INFO}}$

Having established the semantic realm in which formulae are to be interpreted, it is now possible to introduce some standard logical notions, and also some less standard ones. We begin with the notions of *satisfaction* and *entailment*:

Definition D6 (Satisfaction)

$$I \models \phi \quad \textit{iff} \quad \forall J, i\; (I[\![\phi]\!]J \wedge i \in I) \rightarrow (\exists j\; j \geq i \wedge j \in J)$$

Definition D7 (Entailment)

$$\phi_1, \ldots, \phi_n \models \psi \quad \textit{iff} \quad \forall I, J\; I[\![\phi_1]\!] \circ \ldots \circ [\![\phi_n]\!]J \rightarrow J \models \psi$$

An information state I satisfies a formula ϕ if updating I with ϕ does not eliminate any world-sequence pairs, although it may extend them. Note the use of $I[\![\phi]\!]J$ to mean that the

[7] In [Be93a] the full relationality of CCP's is used to introduce limited non-determinism into the interpretation of language, providing a semantic model of non-deterministic pronoun resolution, as found in [Kam81]. Thus if an input information state contains two variables in its domain which are established to correspond to females, then a sentence containing the pronoun she will have at least two outputs, since the pronoun can be resolved to either variable.

pair $\langle I, J\rangle$ stand in the update relation given by the interpretation of ϕ. A sequence of premise formulae entail a single conclusion if updating any information state sequentially with all the premises yields a state which satisfies the conclusion. Note that since CCP'S are defined as binary relations between inputs and outputs, sequential update is just relational composition ($\circ$), such that an input I can be sequentially updated with with ϕ and then ψ to produce J just in case $I[\![\phi]\!] \circ [\![\psi]\!] J$. See [Ve91] for more on the definition of entailment in a dynamic setting.

Truth tends not to be a central notion in dynamic logics, but the semantics of attitude predicates will be essentially static with respect to their propositional argument. A static notion of truth with respect to a world and an assignment will thus be useful. The following notion of truth, which says that a formula is true with respect to a world-sequence pair if an extension of the pair is contained in some possible output of the formula, is by no means the only one possible[8].

Definition D8 (Truth)

$$\phi \textit{ is } \text{true } \textit{wrt } i \quad \textit{iff} \quad \exists I, J\, I[\![\phi]\!]J \wedge \exists j \geq i \;\; j \in J$$

A formula is tautologous if it is satisfied in every information state. It is easily shown that tautologies are true with respect to any world-sequence pair:

Definition D9 (Tautology)

$$\phi \textit{ is a tautology} \quad \textit{iff} \quad \forall I\, I \models \phi$$

A formula can be persistent, in the sense that if it is satisfied in some state, then it is also satisfied in all more informative states. All of the systems to be presented will be non-monotonic, in the sense that there will be some formulae which are not persistent.

Definition D10 (Persistence)

$$\phi \textit{ is persistent} \quad \textit{iff} \quad \forall I, J\, (I \models \phi \wedge I \preceq J) \rightarrow (J \models \phi)$$

Finally we arrive at the essense of the CCP model of presupposition, the notions of *admission* and of *presupposition* itself. Although CCP's in Heimian logics do not occupy the full space of binary relations between states, they still occupy a larger space than the one-to-one functions. In general, the CCP of a formula will be equivalent to a partial function, with domain, or input, given by the set of states occurring as first coordinates of the relation, and range, similarly, given by the set of states occurring as second coordinates. Crucially, there may be some states which are not in the domain of the *update* function corresponding to a formula. In this case, it is said that these states fail to admit the formula, since the meaning of the formula provides no update from them:

Definition D11 (Admission)

$$I \textit{ admits } \phi \quad \textit{iff} \quad \exists J\, I[\![\phi]\!]J$$

Presuppositions are preconditions for update. The presuppositions of a formula are all those formulae that must be satisfied in order for update to occur. In other words, a formula ϕ presupposes another formula ψ if ψ is satisfied in every state in the domain of the update function given by ϕ. Since the states in the domain of a formula's update function are just those which *admit* the formula, we arrive at the following definition:

[8] The reason for considering extensions of the world-sequence pair has to do with the possibility of the formula introducing new variables. For instance, the formula corresponding to a man walks will be satisfied in a world-sequence pair with an empty domain, but where at least one man walks in the given world, in spite of the fact that all non-absurd outputs of the formula have a non-empty domain, containing a variable established to be a walking man.

Definition D12 **(Presupposition)**

$$\phi \textit{ presupposes } \psi \quad \textit{iff} \quad \forall I\ (I \textit{ admits } \phi) \rightarrow I \models \psi$$

Definition D13 **(Presupposition Failure)**

The presuppositions of ϕ fail in the state I *iff* $\neg(I$ ***admits*** $\phi)$
*(I **is called** a* dead-end.*)*

There has been much controversy in the presupposition literature over the relation between presupposition and entailment. Although I will refrain from entering into the debate here[9], it may still be useful to state the formal relation between presupposition and entailment in the logics to be considered:

Fact F2 (Presupposition and Entailment) *If ϕ presupposes ψ and ψ is persistent, then $\phi \models \psi$.*

3 Heimian Predicate Logic — First Version

The language of HPL is an extension of FOPL with equality, definite descriptions and attitude predicates, the latter two being included as sources of presupposition. Attitude predications in HPL split into two further classes, factive and non-factive, factive attitude predications being syntactically distinguished by underlining, eg. $\underline{\text{regret}^a(x,\phi)}$.

Definition D14 **(Syntax of HPL)** *Given variables V, n-ary predicates $\mathcal{P}^n$ and attitude predicates $\mathcal{P}^a$, the language of HPL is given by recursion over the following rewrite rules:*

$$\begin{aligned}\text{FORM} \implies\ & \mathcal{P}^n(V_1,\ldots,V_n) \mid \mathcal{P}^a(V,\text{FORM}) \mid \underline{\mathcal{P}^a(V,\text{FORM})} \mid (V_i = V_j) \mid \\ & \exists V \mid \forall V\, \text{FORM} \mid (\text{DET}(V,\text{FORM},\text{FORM}) \mid \\ & (\text{FORM} \wedge \text{FORM}) \mid (\text{FORM} \vee \text{FORM}) \mid (\text{FORM} \rightarrow \text{FORM}) \mid (\neg\text{FORM}) \mid\end{aligned}$$

The semantics of HPL formulae is defined recursively in the normal fashion, the base of the recursion being n-ary non-attitudinal predications and equality statements[10]. Consider the case of a unary predication $P(x)$. Which world-sequence pairs are compatible with such a predication? Presumably those pairs for which the sequence maps the variable x onto an object which is in the extension of the predicate P in the world. So, we can say that a pair $\langle w,f\rangle$ is compatible with the predication $P(x)$ just in case $f(x) \in \|P\|_w$. Extending to the case of an n-ary predication, $\langle w,f\rangle$ is compatible with the predication $P(x_1,\ldots,x_n)$ just in case

[9] But see [Be:c].

[10] To get a feeling for how the semantics works it might be useful to consider how the basic idea of Stalnaker's assertion model would translate into the current approach. For Stalnaker, an information state is just a set of worlds, and update of a state with an atomic proposition is just given by removing those worlds from the state which are not compatible with the proposition. If $\|p\|$ is the set of worlds where p holds, the following HPL-like semantic clause results:

$$I[\![p]\!]J \quad \textit{iff} \quad J = I \cap \|p\|$$

This can be read as "I can updated with p to yield J just in case J is the intersection of I and the set of worlds where p holds.

$\langle f(x_1), \ldots, f(x_n)\rangle \in \|P\|_{\mathrm{w}}$. The following clause says that a state I can be updated with an n-ary predication to yield a state J just in case all the predicated variables are defined in I (ie. are in its domain), and J is the subset of world-sequence pairs in I which are compatible with the predication:

Definition D15 (Simple Predication)

$$\begin{array}{lll} I[\![P(x_1,\ldots,x_n)]\!]J & \textit{iff} & \{x_1,\ldots,x_n\} \subseteq \textit{domain}(I) \ \wedge \\ & & J = \{\langle w,f\rangle \in I \mid \langle f(x_1),\ldots,f(x_n)\rangle \in \|P\|_{\mathrm{w}}\} \end{array}$$

Equality statements $x = y$, likewise, only define an update on a state if both variables are in its domain, in which case the output is the subset of world-sequence pairs in the input state for which the sequence maps both variables onto the same object:

Definition D16 (Equality)

$$I[\![x = y]\!]J \quad \textit{iff} \quad \{x,y\} \subseteq \textit{domain}(I) \ \wedge \ J = \{\langle w,f\rangle \in I \mid f(x) = f(y)\}$$

The static extension of an attitude predicate is a set of pairs, where the first element of the pair is an individual, and the second is a set of worlds. For a world-sequence pair $\langle w,f\rangle$ to be compatible with an attitudinal predication $P^a(x,\phi)$, the extension of the predicate at w must include $\langle f(x), \{w' \mid \phi \text{ is } \textit{true} \text{ wrt } \langle w',f\rangle\}\rangle$. Updating with a non-factive attitudinal predication $P^a(x,\phi)$, analogously to the simple predication case, is only possible if the variable x is in the domain of the input state, in which case the output is that subset of the world-sequence pairs of the input which are compatible with the predication:

Definition D17 (Non-factive Attitudes)

$$\begin{array}{lll} I[\![P^a(x,\phi)]\!]J & \textit{iff} & x \in \textit{domain}(I) \ \wedge \\ & & J = \{\langle w,f\rangle \in I \mid \langle f(x), \{w' \mid \phi \textit{ is} \text{ true } \textit{wrt} \ \langle w',f\rangle\}\rangle \in \|P^a\|_{\mathrm{w}}\} \end{array}$$

Factive attitudinal predications are defined in terms of non-factive attitudinal predications, with the sole addition of a constraint only allowing update to occur if the propositional argument is satisfied in the input[11]:

Definition D18 (Factive attitudes)

$$I[\![\underline{P^a(x,\phi)}]\!]J \quad \textit{iff} \quad I \models \phi \ \wedge \ I[\![P^a(x,\phi)]\!]J$$

The connectives offer no surprises to those familiar with other dynamic systems. Conjunction is defined in terms of relational composition, such that updating with $\phi \wedge \psi$ has the same effect as updating with ϕ and then updating with ψ. Updating a formula with the negation of ϕ produces a state containing all those world-sequence pairs in the input which do not have extensions in the state obtained by updating with ϕ itself. Note the use of the state-complement operation $\setminus$ introduced earlier. Implication and dijunction are defined using classical equivalences, although, this being a dynamic logic, it is significant that equivalences are chosen which maintain the order of the arguments.

[11] It is thus assumed that all factive predicates can be defined in terms of underlying non-factive predicates. Whilst I cannot see a clear counter-example to this assumption, I have to say that the only reason for making it here is technical simplicity: it is neither natural nor empirically motivated. In [Be93b], however, a more abstract treatment of attitudinal predications is given which does not necessitate such a commitment.

Definition D19 (Connectives)

$$\begin{array}{rcl}
[\![\phi \wedge \psi]\!] & = & [\![\phi]\!] \circ [\![\psi]\!] \\
I[\![\neg\phi]\!]J & \textit{iff} & \exists K\ I[\![\phi]\!]K \wedge J = I \setminus\!\!\setminus K \\
[\![\phi \rightarrow \psi]\!] & = & [\![\neg(\phi\ \wedge\ \neg\psi)]\!] \\
[\![\phi \vee \psi]\!] & = & [\![\neg(\neg\phi\ \wedge\ \neg\psi)]\!]
\end{array}$$

In [He83], as in [He82] and [Kam81], a radically non-quantificational view of indefinites is espoused. As for quantificational determiners, indefinites introduce variables, and the syntactic restrictor and scope (typically a CN and a VP respectively) are taken, semantically, to predicate properties of this variable, but the indefinite is not seen as having any intrinsic quantificational force. The apparent quantificational force of an indefinite derives entirely from the context in which it is used. In the last five years, particularly through the work of Groenendijk and Stokhof[12], it has become clear that the idea underlying this radical analysis of indefinites can be regarded not as being non-quantificational, but as utilising an alternative approach to quantification. The essence of the approach is *random extension*, the addition to an information state of a variable which is completely underspecified as to its value.

To evaluate a sentence a woman is walking, a variable with unspecified value is added, and then the value of this variable is restricted to be in the extension of woman and walking. This procedure will have two effects. Firstly, it will cause the removal of any world-sequence pairs in the input where the extensions of woman and walking have an empty intersection. Secondly, provided there are at least some world-sequence pairs remaining in the output, they will all be defined for the variable x, and map it onto objects which are walking women. The fact that x is defined in the output state means that it can behave as a discourse marker, with reuse of x in later sentences having the effect of anaphoric reference.

Adding a variable x with unspecified value to a state I produces a new state J (notated $I{+_x}J$) containing any world-sequence pair which adds to one in the original state an assignment to the variable in question:

Definition D20 (Random Extension)

$$I +_x J \quad \textit{iff} \quad x \notin \textit{domain}(I)\ \wedge\ J = \{j \mid \exists d \in \mathcal{D} \exists i \in I\ j >_{x:d} i\}$$

The existential quantification over objects d in the domain means that for any object in the domain, there is some world-sequence pair in J such that the sequence maps x onto that object, and in this sense the value of x could be called unspecified, random, or, perhaps, *arbitrary*. We obtain the following definitions for existential and, using the standard duality, universal quantification:

Definition D21 (Random extension based pseudo-quantifiers)

$$\begin{array}{rcl}
I[\![\exists x\ \phi]\!]J & \textit{iff} & I +_x \circ [\![\phi]\!]\ J \\
[\![\forall x\ \phi]\!] & = & [\![\neg\exists x\ \neg\phi]\!]
\end{array}$$

Are these really quantifiers? Relating the HPL quantifiers to the standard meta-language notions could provide reassurance:

Fact F3 (Pseudo-quantifiers are quantificational)

$$\begin{array}{lll}
I \models \exists x P x & \textit{iff} & (x \notin \textit{domain}(I)\ \wedge\ \forall \langle w, f\rangle \in I\ \exists d \in \mathcal{D}\langle d\rangle \in \|P\|_w \\
I \models \forall x P x & \textit{iff} & (x \notin \textit{domain}(I)\ \wedge\ \forall \langle w, f\rangle \in I\ \forall d \in \mathcal{D}\langle d\rangle \in \|P\|_w
\end{array}$$

[12] See [GS91a], [Ch92], [De93].

3.1 Examples

Let us consider a rather restricted model $\mathcal{M}$ in which the domain $\mathcal{D} = \{a, b, c\}$, and there are two worlds: $\mathcal{W} = w_1, w_2$. Let us say that in both worlds, both a and b are men, but c is a woman, so, using a predicate M for "man", $\|M\|_w = \{a, b\}$ for $w = w_1, w_2$. Let us say further that a is broke (predicate B) in both worlds, but b is only broke in w_1: we will not investigate c's financial situation until later. The only remaining detail we need for noe is that whilst b is fully aware of his financial situation in every world, a has only discovered (predicate D) the seriousness of his financial plight in world w_2, but in w_1 remains blissfully ignorant. States will be represented using the notation $w_1\langle x : a, y : b\rangle$ for the pair consisting of the world w_1 and the sequence mapping x onto a and y onto b. So for $\mathcal{M}$, $\boxed{:\text{-})} = \{w_1\langle\rangle, w_2\langle\rangle\}$.

E18 A man is broke.

HPL Translation: $\exists x M(x) \wedge B(x)$

Denotation: A function from states where x is undefined to states consisting of world-sequence pairs $\langle w, f\rangle$ which extend pairs on the input with by having x defined, and where x is mapped onto an object which is in the extension of Man and Broke, ie. $f(x) \in \|M\|_w \cap \|B\|_w$.

Sample Update: $\boxed{:\text{-})} \;\mapsto\; \{w_1\langle x : a\rangle, w_1\langle x : b\rangle, w_2\langle x : a\rangle\}$

Another Sample Update: $\{w_2\langle\rangle\} \;\mapsto\; \boxed{:\text{-(}}$ (Note that this is *not* a case of presupposition failure. The update is defined, but happens to lead to a state containing no world-sequence pairs.)

Sample Dead-end: $\{w_1\langle x : a\rangle\}$ (because x is in its domain)

E19 He has discovered that he is broke.

HPL Translation: $\underline{D(x, B(x))}$

Denotation: For an individual α to have discovered that (s)he is broke in a world w, we must have that the pair consisting of α and the set of worlds where α is Broke be in the extension of Discover at w, ie $\langle \alpha, \{w' \mid \alpha \in \|D\|_{w'}\rangle \in \|D\|_w$. The formula $\underline{D(x, B(x))}$ denotes a function from states where x is broke to states where x is broke and has discovered it. More formally, $\underline{D(x, B(x))}$ is a function from states which contain only world-sequence pairs $\langle w, f\rangle$ where x is mapped onto an individual in the extension of B (ie. $f(x) \in \|B\|_w$) to that subset of those pairs where $\langle f(x), \{w' \mid f(x) \in \|D\|_{w'}\rangle \in \|D\|_w$.

Sample Update: $\{w_1\langle x : a\rangle, w_1\langle x : b\rangle, w_2\langle x : a\rangle\} \;\mapsto\; \{w_1\langle x : b\rangle, w_2\langle x : a\rangle\}$

Sample Dead-end: $\{w_1\langle x : a\rangle, w_2\langle x : b\rangle, w_2\langle x : a\rangle\}$ (because $B(x)$ is not satisfied in the input)

E20 A man is broke. He has discovered that he is broke.

HPL Translation: $(\exists x M(x) \wedge B(x)) \wedge \underline{D(x, B(x))}$

Denotation: The sequencing of $\exists x M(x) \wedge B(x)$ and $\underline{D(x, B(x))}$, thus a function from states where x is not defined to states where x is mapped to an individual that is a broke man who has discovered that he is broke.

Sample Update: $\boxed{:\text{-})} \;\mapsto\; \{w_1\langle x : b\rangle, w_2\langle x : a\rangle\}$

Sample Dead-end: $\{w_1\langle x : a\rangle\}$ (because x is in its domain)

E21 A man has discovered that he is broke.

HPL Translation: $\exists x M(x) \wedge \underline{D(x, B(x))}$

Denotation: Break this down into two updates, firstly of an initial state with $\exists x M(x)$ to produce an intermediate state, and secondly of this state with $\underline{D(x, B(x))}$ to produce the final output. The first takes a state not having x in its domain to a state containing every x-extension of an input world-sequence pair such that x is mapped onto an object in the extension of Man.

Update with $\underline{D(x, B(x))}$ can proceed if and only if $B(x)$ is satisfied at this point, so that for every world-sequence pair $\langle w, f \rangle$ in the intermediate state, the object $f(x)$ must be in the extension of Broke at w. This will be the case if and only if for every world-sequence pair in the initial state, the extension of Man at the world is a subset of the extension of Broke. Thus the complete formula, $\exists x M(x) \wedge \underline{D(x, B(x))}$, is a function from states not having x in their domain and where for every world, the extension of Broke includes the extension of Man to states where x is established to be a Broke Man who has Discovered that he is Broke.

Sample Update: $\{w_1\langle\rangle\} \mapsto \{w_1\langle x : a\rangle\}$

Sample Dead-end: $\boxed{:-)}$ (because in w_2 there is a man who is not broke, so that the extension of B does not include the extension of M. Thus no update is defined from the intermediate state onwards, and so the update as a whole is undefined.)

3.2 Presupposition Projection in HPL

Although we are concerned here primarily with the interaction of presupposition and quantification, it is as well to note that regarding more general presupposition properties, HPL behaves much as one would expect a Karttunen-derived model of presupposition to behave[13]. Note the occurrence of conditionalised presuppositions when a presupposition is triggered in the second conjunct of a conjunction or in the consequent of an implication, and also observe that (3) demonstrates the relation with a standard semantic definition of presupposition in terms of negation. The following fact will hold for all the systems introduced:

Fact F4 (Karttunen-style Presupposition Projection)

1. *$\underline{P^a(x, \phi)}$ presupposes ϕ*
2. *If ϕ presupposes ψ then:*
   (a) $\neg\phi$, $\phi \wedge \psi$, $\phi \rightarrow \psi$ and $\phi \vee \psi$ all presuppose ψ
   (b) $\chi \wedge \phi$, $\chi \rightarrow \phi$ and $\chi \vee \phi$ all presuppose $\chi \rightarrow \phi$
3. *Given that ψ is persistent,*
   ($\phi \models \psi$ and $\neg\phi \models \psi$) iff ϕ presupposes ψ

More pertinent to current aims is the fact that, as seen in example E21 above, when a presupposition trigger is bound by an indefinite, a universal presupposition results:

[13] Which is not to say that the various models put forward in [Kar73, Kar74, KP79] all have the same projection properties. See [Be:c] for discussion. Also, see [Be93b] for a defense of the properties in F4.

Fact F5 (Heimian projection from quantified contexts) *If ϕ presupposes ψ, then:*[14]

$$\exists x\ \phi \quad \textit{presupposes} \quad \forall x\ \psi$$
$$\forall x\ \phi \quad \textit{presupposes} \quad \forall x\ \psi$$

These results are both in conflict with the conslusions of §8.2, the result that existentially quantified propositions yield universal presuppositions being most clearly at variance with our intuitions.

4 Extending the Problem

Before suggesting a solution to the problem, I will attempt to generalise it in two directions, extending the language of HPL but maintaining, for the moment, the semantics of the HPL fragment of the extensions. Firstly I will show how the random-extension based approach to quantification in HPL can be extended to cover binary quantifier relations. We will see that the binary quantifiers so defined have comparable presupposition projection properties to the unary quantifiers in HPL, and are thus also at variance with the empirical claims of §8.2. Secondly I will introduce modal operators along the lines of the treatment of epistemic modality in [Ve91], and we will see that problems occur when variables are *quantified-in* to modal contexts. These problems, though unconnected with presupposition in that they would still occur in a version of the logic which did not involve presupposition introducing constructions, are analogous to those occurring when presupposition triggers are bound by quantifiers. Put another way, quantifying-in to modal contexts is comparable with quantifying-in to presupposition triggers, and produces similar problems. This will suggest a single solution to both problems.

[14] For proof, suppose ϕ presupposes ψ and $x \notin \mathit{domain}(I)$:

(1)	I admits $\exists x\,\phi$	hypothesis
(2)	$\exists J\ I +_x J \ \wedge\ J$ admits ϕ	defn. of $\exists, x \notin \mathit{domain}(I)$
(3)	$\exists J\ I +_x J \ \wedge\ J \models \psi$	defn. of admission
(4)	$\exists J\ I +_x J \ \wedge\ J[\![\psi]\!]\boxed{:-(}$	defns. of $\models, \neg$
(5)	$I[\![\exists x\,\neg\psi]\!]\boxed{:-(}$	defn. of $\exists$
(6)	$I[\![\neg\exists\neg\psi]\!]I$	defn. of $\neg$
(7)	$I \models \forall x \psi$	defns. of $\models, \forall$

This establishes that $\exists x\ \phi$ presupposes $\forall x\ \psi$. The second proposition can be seen from the fact that the admission conditions of $\exists x\,\phi$ and $\forall x\,\phi$ are identical, all the following steps being reversible (the assumption that $x \notin \mathit{domain}(I)$ still holds):

(1)	I admits $\neg\exists x\,\neg\phi$	hypothesis
(2)	I admits $\exists x\,\neg\phi$	defn. of $\neg$
(3)	$\exists J\ I +_x J \ \wedge\ J$ admits $\neg\phi$	defn. of $\exists$
(4)	$\exists J\ I +_x J \ \wedge\ J$ admits ϕ	defn. of $\neg$
(5)	I admits $\exists x\,\phi$	defn. of $\exists$

4.1 Heimian Quantifier Logic — First Version

Definition D22 (Syntax of HQL) *As for HPL but with the following additional rewrite rules:*

$$\begin{aligned} \text{DET} &\Longrightarrow \text{SOME} \mid \text{EVERY} \mid \text{NO} \mid \text{MOST} \mid \text{FEW} \mid \text{EXACTLY-ONE} \\ \text{FORM} &\Longrightarrow \text{DET}(V, \text{FORM}, \text{FORM}) \end{aligned}$$

To evaluate a quantificational formula $Q(x, \phi, \psi)$, we randomly extend the input with x, then update with the restrictor ϕ to get an intermediate state R, and then update with the scope ψ to get another intermediate state S. The output should be those world-sequence pairs i from the input such that the relevant quantifier relation Q holds between two sets, the set of values x is mapped to after update in the restrictor, and the set of values it is mapped to after update with the scope. First a semi-formal version, and then the definition proper[15]:

$$\begin{array}{lll} I[\![Q(x,\phi,\psi)]\!]J & \textit{iff} & \exists R, S \\ & & I \text{ randomly extended with } x \text{ and updated with } \phi \text{ gives } R \\ & & \text{and } R \text{ updated with } \psi \text{ gives } S \text{, and} \\ & & J = \left\{ i \in I \mid \; Q \begin{pmatrix} \text{values } x \text{ takes in extensions of } i \text{ in R} \\ \text{values } x \text{ takes in extensions of } i \text{ in S} \end{pmatrix} \right\} \end{array}$$

Definition D23 (Externally static binary quantifiers) *For Q one of* EVERY, NO, MOST, FEW*:*

$$\begin{array}{lll} I[\![Q(x,\phi,\psi)]\!]J & \textit{iff} & \exists R, S \; I +_x \circ [\![\phi]\!] R [\![\psi]\!] S \;\wedge \\ & & J = \left\{ i \in I \mid \; Q \begin{pmatrix} \{d \mid \exists \langle w, f\rangle \in R \; \langle w, f\rangle \geq i \;\wedge\; f(x) = d\} \\ \{d \mid \exists \langle w, f\rangle \in S \; \langle w, f\rangle \geq i \;\wedge\; f(x) = d\} \end{pmatrix} \right\} \end{array}$$

This definition produces quantifiers which are *internally dynamic* (cf. [GS91b]), in the sense that discourse markers introduced in the restrictor can become bound in expressions in the scope. However, these dynamic quantifiers are *externally static*: they do not have any net effect of introducing new discourse markers. But it is not too difficult to use the above definition as the basis of one for quantifiers which are both internally and externally dynamic. In the following definition, the subset of world-sequence pairs in the input state which are compatible with the quantification is calculated as before, and *stored* in a variable T. The output J is calculated by taking all those world-sequence pairs from S (the state resulting from random extension and update with both restrictor and scope) which are extesnions of world-sequence pairs in T:

[15] For discussion of dynamic generalised quantifiers the reader is referred to [Ch92]. In common with the definitions he provides, the dynmamic quantifiers defined here will all be *conservative* regardless of the conservativity of the underlying quantifier relation. Also observe that the so called *proportion problem* does not arise with this definition, because we are explicitly counting over objects in the domain rather than assignment functions: see [Roo87] for discussion.

Definition D24 (Externally dynamic binary quantifiers) *For Q one of* SOME, EXACTLY-ONE*:*

$$
\begin{array}{lll}
I[\![Q(x,\phi,\psi)]\!]J & \textit{iff} & \exists R,S,T\ I +_x \circ [\![\phi]\!] R [\![\psi]\!] S \ \wedge \\
& & T = \left\{ i \in I \mid Q \left(\begin{array}{l} \{d \mid \exists \langle w,f\rangle \in R\ \langle w,f\rangle \geq i\ \wedge\ f(x) = d\} \\ \{d \mid \exists \langle w,f\rangle \in S\ \langle w,f\rangle \geq i\ \wedge\ f(x) = d\} \end{array} \right) \right\} \\
& & \wedge\ J = \{s \in S \mid \exists t \in T\ s \geq t\}
\end{array}
$$

Space does not permit me to discuss these quantifier definitions at length, but the interested reader should be able to confirm that they behave as dynamic generalized quantifiers should, supporting donkey anaphora and so forth. The following fact confirms that at least the dynamic generalized quantifiers SOMEand EVERYstand in the expected relation with the unary quantifiers $\exists$ and $\forall$:

Fact F6 (Properties of version 1 quantifiers) *Assuming standard interpretations for* SOME *and* EVERY*:*

$$
\begin{array}{lll}
[\![\text{SOME}(x,\phi,\psi)]\!] & = & [\![\exists x\ \wedge\ \phi\ \wedge\ \psi]\!] \\
[\![\text{EVERY}(x,\phi,\psi)]\!] & = & [\![\forall x\ \phi \rightarrow \psi]\!]
\end{array}
$$

It is relatively easy to see from the above definitions that a quantificational update of a state I with $[\![Q(x,\phi,\psi)]\!]$ will be defined only provided that there are intermediary states R and S such that $I +_x \circ [\![\phi]\!] R [\![\psi]\!] S$. But this will be the case if and only if an update from I with $\exists x\, \phi \wedge \psi$ is defined. Thus all the quantifiers given by the above definitions will tend to yield universal presupppositions just as with the earlier unary quantifiers, and in conflict with the empirical claims made in §8.2.

Fact F7 (Heimian projection from quantified contexts) *If ϕ presupposes ψ, then (for Q a quantifier):*

$$
\begin{array}{ll}
Q(x,\phi,\chi) & \textit{presupposes}\ \ \forall x\ \psi \\
Q(x,\chi,\phi) & \textit{presupposes}\ \ \forall x\ \chi \rightarrow \psi
\end{array}
$$

4.2 Modal Heimian Quantifier Logic

Definition D25 (Syntax of MHQL) *As for HQL but with the following additional rewrite rule:*

$$\text{FORM} \implies (\Diamond \text{FORM}) \mid (\Box \text{FORM})$$

The *might* operator (here $\Diamond$) introduced in the update semantics of [Ve91] is a *consistency test.* Intuitively, $\Diamond\phi$ will be satisfied in an information state just in case ϕ is still an open possibility in that information state. The open possibilities, in this sense, are all the formulae with which the information state can be updated without yieldin absurdity. So, a formula is consistent with a state if updating with the formula produces a non-absurd state, and is inconsistent with a state if updating produces absurdity. A formula is consistent in case there is some state it is consistent with:

Definition D26 (Consistency)

$$\begin{array}{rll} \phi \textit{ is consistent with } I & \textit{iff} & \exists J\, I[\![\phi]\!]J \wedge J \neq \boxed{:-(} \\ \phi \textit{ is inconsistent with } I & \textit{iff} & \exists J\, I[\![\phi]\!]J \wedge J = \boxed{:-(} \\ \phi \textit{ is consistent} & \textit{iff} & \exists I\ \phi \textit{ is consistent with } I \end{array}$$

Veltman's *might* operator is defined in a semantics where formuale are interpreted as functions from sets of worlds to sets of worlds. Thus exactly how the definition should be carried over to a semantics where formulae are interpreted as binary relations between sets of world-sequence pairs is partly a matter of taste. However, having expressed the desired interpretation of the operator in terms of the general notion of consistency rather than in the particular space of semantic denotations used by Veltman, the following definitions for $\Diamond$ and its dual $\Box$ become natural:

Definition D27 (Epistemic Modals)

$$\begin{array}{rll} I[\![\Diamond\phi]\!]J & \textit{iff} & (J = I \wedge \phi \textit{ is consistent with } I) \\ & & \vee \left(J = \boxed{:-(} \wedge \phi \textit{ is inconsistent with } I\right) \\ [\![\Box\phi]\!] & = & [\![\neg\Diamond\neg\phi]\!] \end{array}$$

Above, I called the *might* operator a *consistency test.* In fact *test-hood* is given a technical interpretation in [Ve91], which is easily adapted to the current setting. A formula is a *test* just in case updating a state with that formula is possible, and either has no effect (the output being identical to the input) or produces absurdity. The tautologies form a special class of trivial tests which always *succeed*, and the inconsistent formulae form another trivial class which always *fail.*

Definition D28 (Test)

$$\phi \textit{ is a test} \quad \textit{iff} \quad \forall I, J\, I[\![\phi]\!]J \rightarrow \left(I = J \vee J = \boxed{:-(}\right)$$

The following properties are easily verified, and provide some indication of how the modals behave[16]:

Fact F8 (Properties of modals)

1. *For any ϕ, $\Diamond\phi$ is a test.*
2. *For any non-tautological ϕ admitted by at least one state, $\Diamond\phi$ is not persistent.*
3. *$\Box\phi$ tests satisfaction. ie.*

$$\begin{array}{rll} I[\![\Box\phi]\!]J & \textit{iff} & (J = I \wedge I \models \phi) \\ & & \vee \left(J = \boxed{:-(} \wedge I \not\models \phi \wedge I \textit{ admits } \phi\right) \end{array}$$

[16]Note that the fact that $\Diamond\phi$ is typically not persistent provides an easy way (cf. F2) to construct a formula which does not entail its presuppositions: $\underline{D(x, \Diamond B(y))} \wedge \neg B(y) \not\models \Diamond P(y)$. The premise denotes a function from states where x and y are defined and where y being Broke is an open possibility, to states where x has at some earlier point Discovered that y being broke is an open possibility, but where in fact y is not Broke. Thus $\Diamond P(y)$ is presupposed but not entailed by $\underline{D(x, \Diamond B(y))} \wedge \neg B(y)$. I mention this example just for its technical interest: I cannot say that it reflects any strong empirical intuitions about presupposed statements of modality.

Quantifying-in to modalities has long been a source of philosophical and technical difficulty, and many of the problems that occur when modalities are imported into a dynamic logic are already present in the more familiar static setting of standard modal predicate logic. The following HMQL fact seems to contradict Kripke's arguments about the non-contingency of identity in modal predicate logics:

Fact F9 (Modal identity is non-Kripkean)

$$\Diamond(x = y) \not\models x = y$$
$$\Diamond(\neg(x = y)) \not\models \neg(x = y)$$

I would argue that, contrary to appearances, this property does not actually conflict with Kripke's intuitions, since the modalities in HMQL are intended to be interpreted as epistemic modalities, concerning the epistemic state of an agent gathering information, whereas Kripke's arguments concern metaphysical modality. Briefly, in a metaphysical setting, if the possibility of two individuals being identical remains, then there is no way to distinguish them, and Kripke is just being a good Leibnizian in providing arguments that $\Diamond(x = y) \models x = y$. But in an epistemic setting, such an argument would be disastrous. When we allow for the possibility of individuals we have experienced in different guises being the same, we may also allow for the possibility that they turn out to be different [17]:

E22 a. The first lady is not spying

b. $\neg$spying(x)

E23 a. I can see a woman in the Whitehouse

b. SOME(y, woman(y), i-see-in-w(y))

E24 a. She might be spying.

b. $\Diamond$spying(y)

E25 a. However, she might be the first lady.

b. $\Diamond(y = x)$

Such discourses create problems in the system of Dynamic Modal Predicate Logic of van Eijck and Ceparello [EC92], which, although it has epistemic pretensions, uses an essentially Kripkean notion of modal identity. In DMPL the natural translation of the discourse would be inconsistent, whereas in the current system the sequenced conjunction of the above 4 LFs yields a consistent formula, according with our intuition that the discourse is indeed consistent.

So, HMQL has its good points, but, as the following example demonstrates, it is far from perfect. Recall the financial situation introduced in §8.3: of the Men a and b, a was Broke in worlds w_1 and $w_2$and b was Broke in w_1 but not in w_2. I am now in a position to reveal that the woman c's finances are healthy in both possible worlds. For the following example, assume the extension of Person includes all three individuals in both worlds:

E26 Everybody might be broke. (narrow scope might reading)

HMQL translation: EVERY($x, P(x), \Diamond B(x)$)

[17] In my Rochester presentation, I gave a semantics for definite descriptions, and thus a more realistic translation for this example. The interested reader is referred to [Be93b], where the same example is treated more thoroughly.

Denotation: Given an input I, We begin by finding states R and S such that $I +_x o[\![P(x)]\!]R[\![\Diamond B(x)]\!]S$. In this particular model, all individuals are in the extension of Person in every world, so R contains all the x-extensions of the input.

Since $\Diamond$ is a test and there is no possibility here of presupposition failure, updating with $\Diamond B(x)$ can have only two results: if $B(x)$ is consistent with R then $S = R$, otherwise $S = \boxed{:-(}$. If at least one individual is in the extension of Broke in some world-sequence pair in R (and not otherwise), then $B(x)$ will be consistent with R, so $S = R$. But if $S = R$ then clearly every extension of an input world-sequence pair in R will also be in S. In this case, all input world-sequence pairs will be compatible with the quantifier associated with EVERY, i.e. the superset relation. So, if at least one individual is in the extension of Broke in at least one input world-sequence pair, then all world sequence pairs in the input will be preserved on the output. On the other hand, if no individuals are in the extension of Broke in any input world-sequence pairs, a similar argument shows that the result of updating will be $\boxed{:-(}$: no input world-sequence pairs will be compatible with the quantifier.

So, the output is identical to the input if for some input pair at least one individual in the extension of P is also in the extension of B, and otherwise the output is $\boxed{:-(}$.

Sample Update: $\boxed{:-)} \mapsto \boxed{:-)}$

The update of $\boxed{:-)}$ with EVERY$(x, P(x), \Diamond B(x))$ is not as informative as we would wish, since it leaves us in the same state of blissful ignorance that we started in. Given that there is at least one individual, namely c who is definitely not Broke in $\boxed{:-)}$, it seems that the claim that everybody might be broke contradicts the initial information, and should produce a state of absurdity. Thus the denotation does not accord with intuition.

More generally, a statement EVERY$(x, \phi, \Diamond\psi)$ should state that for each individual such that ϕ holds, we do not know to the contrary of ψ, but in fact, under the current HMQL semantics, it only states something much weaker[18]:

Fact F10 (Improper quantifying-in) [19]
If $I \models SOME(x, \phi, \Diamond\psi)$, *then* $I \models EVERY(x, \phi, \Diamond\psi)$

[18] The problem identified by F10 also arises in other attempts to combine a Groenendijk and Stokhof style analysis of quantification and anaphora with a Veltman style analysis of epistemic modality, such as those in [Ver92, De93]. See [GSV94] for a closely related solution to that presented here, and also a far more elaborate discussion of Veltman's *might* operator.

[19] The proof uses the following lemmas, deriving from the definitions of test-hood, satisfaction, conjunction and negation:

L1 *If* ψ *is a test,* $I \models \phi \wedge \psi$ *and* $I[\![\phi]\!]J$, *then* $J \models \psi$.

L2 *If* $I[\![\phi]\!]J$ *and* $J \not\models \psi$, *then* $I \not\models \phi \wedge \psi$.

L3 *If* $I \not\models \phi$ *and* I *admits* ϕ, *then* $I \models \neg\phi$.

(1)	$I \models \text{SOME}(x, \phi, \Diamond\psi)$	hypothesis
(2)	$I \models (\exists x\, \phi) \wedge \Diamond\psi$	F6, associativity of $\exists$
(3)	If $I[\![\exists x\, \phi]\!]J$ then $J \models \Diamond\psi$	Lemma L1, test-hood of $\Diamond$ (F8)
(4)	If $I[\![\exists x\, \phi]\!]J$ then $J \not\models \Diamond\neg\psi$	defns. of $\models$, $\Diamond$, $\neg$
(5)	$I \not\models (\exists x\, \phi) \wedge \neg\Diamond\psi$	Lemma L2
(6)	$I \models \neg(\exists x\, \phi \wedge \neg\Diamond\psi)$	Lemma L3, admission of $\exists x\, \phi$ guaranteed by (2)
(7)	$I \models (\exists x\, \phi) \rightarrow \Diamond\psi$	defn. of implication
(8)	$I \models \text{EVERY}(x, \phi, \Diamond\psi)$	F6

5 Conclusion

5.1 The Problem

My reading of the problem is as follows: **variables are not varying enough in quantification.** Instead of ranging over alternative members of the domain, the use of random extension means that a variable is interpreted as a single underspecified object. This can be clarified in terms of the notion of a *possible property*:

Definition D29 (Possible Properties)

1. *x has possible value d in I iff* $\exists\langle w, f\rangle \in I \;\; f(x) = d$

2. *P is a possible property of x in I iff* $\exists\langle w, f\rangle \in I \;\; P(x)$ *is true wrt.* $\langle w, f\rangle$

3. *P is a possible property of the x-value d in I iff* $\exists\langle w, f\rangle \in I \;\; f(x) = d$ *and* $P(x)$ *is true wrt.* $\langle w, f\rangle$

Regarding epistemic modality, the fact that variables are interpreted as underspecified objects makes them unsuitable for talking about possible properties of the individual domain members. Since a variable x *might* be any member of the domain amongst its possible values, it has as possible properties the union of all the possible properties of its possible values. Intuitively, if we want to know whether a quantification $Q(x, \phi, \Diamond P(x))$ is satisfied, we should not be looking at whether an underspecified x has P as one of its possible properties, but at whether the different x-values have possible property P.

The other side of the coin, which affects presupposition, concerns what I will call the set of *necessary* values of a variable or of a value that that variable can take:

Definition D30 (Necessary Properties)

1. *P is a necessary property of x in I iff* $\forall\langle w, f\rangle \in I \;\; P(x)$ *is true wrt.* $\langle w, f\rangle$

2. *P is a necessary property of the x-value d in I iff* $\forall\langle w, f\rangle \in I \;\; f(x) = d$ *and* $P(x)$ *is true wrt.* $\langle w, f\rangle$

The only properties that a variable *must* have in an information state, are the properties that its values are established to have. So, just as the set of possible properties of a variable is the union of the possible properties of its values, the set of necessary properties of a variable is the intersection of the necessary properties of its values. When a quantifier binds a presupposition trigger, update is only defined if the presupposition is established to hold. Thus, for a quantification $Q(x, \phi, \underline{D(x, P(x))})$ to define an update, P must be amongst the necessary properties of x in the local context where $\underline{D(x, P(x))}$ is evaluated. But this will only be the case if all of the values of x have P as a necessary property. This is what produces the universal presupposition.

5.2 The Solution

Perhaps I have laboured the point somewhat. But it is an important point. To solve the problems both with presupposition and with epistemic modality, what is needed is a re-interpretation of variables, so that they do not act as underspecified objects within quantificational contexts. So, to make DMQL work properly, we need to change the semantics of quantification such that the values that variables take are considered one at a time instead of all at once.

I will present this alternative semantics as a minor modification of the earlier definitions, since this clarifies exactly what the differences are. To look at the values a variable takes one at a time, I will define a way to *slice up* an information state with respect to the values taken by a particular variable:

Definition D31 (Slicing)

$$I_{x:d} \quad = \quad \{\langle w, f\rangle \in I \mid f(x) = d\}$$

Thus $I_{x:d}$, the d^{th} *x-slice of I*, contains all the world-sequence pairs in I which give x the value d. To update a state with a formula looking at the values of a variable x one at a time, we use the *sliced meaning* of the formula with respect to x:

Definition D32 (Sliced Meanings)

$$\begin{array}{lll} I[\![\phi]\!]_{\mathrm{x}} J & \textit{iff} & \exists d \in \mathcal{D}\, I^x_d \textit{ admits } \phi \ \wedge \\ & & J = \cup\{K \mid \exists d \in \mathcal{D} I^x_d[\![\phi]\!]K\} \end{array}$$

An update from I to J with the sliced meaning of a formula w.r.t. x is obtained by cutting up I into each of it's $I_{x:d}$ slices, updating each of these slices with the formula to get a set of output slices, and then gluing all the slices back together (by taking their union) to produce the combined output J. Additionally, there is a definedness condition $\exists d \in \mathcal{D}\, I^x_d$ admits ϕ which must be satisfied in order for update with the sliced meaning to occur. This condition is what will ultimately lead to quantifiers yielding existential rather than universal presuppositions.

Minor surgery is sufficient to repair the problems with the version 1 semantics. The original semantics for HPL's unary existential quantifier, in D21, was:

$$I[\![\exists x\ \phi]\!] J \quad \textit{iff} \quad I +_x\circ[\![\phi]\!]\ J$$

To modify this definition so that the values of variables are considered one at a time, the occurrence of $[\![\phi]\!]$ is simply replaced with $[\![\phi]\!]_{\mathrm{x}}$ to give:

Definition D33 (HPL Existential) *(Version 2)*

$$I[\![\exists x\ \phi]\!] J \quad \textit{iff} \quad I +_x\circ[\![\phi]\!]_{\mathrm{x}}\ J$$

A similar modification needs to be made to the semantics of binary quantifiers (D23 and D24): the occurences of $[\![\phi]\!]$ and $[\![\psi]\!]$ are replaced with $[\![\phi]\!]_{\mathrm{x}}$ and $[\![\psi]\!]_{\mathrm{x}}$ respectively. This gives the following final definitions:

Definition D34 (Externally static binary quantifiers) *(Version 2)*

$$\begin{array}{lll} I[\![Q(x,\phi,\psi)]\!]J & \textit{iff} & \exists R, S\ I +_x\circ[\![\phi]\!]_{\mathrm{x}} R [\![\psi]\!]_{\mathrm{x}} S\ \wedge \\ & & J = \left\{ i \in I \mid\ \mathcal{Q}\left(\begin{array}{c} \{d \mid \exists\langle w,f\rangle \in R\ \langle w,f\rangle \geq i\ \wedge\ f(x) = d\} \\ \{d \mid \exists\langle w,f\rangle \in S\ \langle w,f\rangle \geq i\ \wedge\ f(x) = d\} \end{array} \right) \right\} \end{array}$$

Definition D35 (Externally dynamic binary quantifiers) *(Version 2)*
For Q one of SOME,
EXACTLY-ONE*:*

$$\begin{array}{lll} I[\![Q(x,\phi,\psi)]\!]J & \textit{iff} & \exists R, S, T\ I +_x\circ[\![\phi]\!]_{\mathrm{x}} R [\![\psi]\!]_{\mathrm{x}} S\ \wedge \\ & & T = \left\{ i \in I \mid\ \mathcal{Q}\left(\begin{array}{c} \{d \mid \exists\langle w,f\rangle \in R\ \langle w,f\rangle \geq i\ \wedge\ f(x) = d\} \\ \{d \mid \exists\langle w,f\rangle \in S\ \langle w,f\rangle \geq i\ \wedge\ f(x) = d\} \end{array} \right) \right\} \\ & & \wedge\ J = \{s \in S \mid \exists t \in T\ s \geq t\} \end{array}$$

5.3 Results

Under the new semantics, F5 and F7, which state that the unary and binary quantifiers yield universal presuppositions, no longer hold. We retain, however, existential presuppositions from quantified contexts[20]:

Fact F11 (Existential projection from quantified contexts) *If ϕ presupposes ψ, then (for Q a quantifier and* true *any tautology):*

$$Q(x, \phi, \chi) \quad \textit{presupposes} \quad \text{SOME}(x, \textit{true}, \psi)$$
$$Q(x, \chi, \phi) \quad \textit{presupposes} \quad \text{SOME}(x, \chi, \psi)$$

The problematic result in F10, showing that quantifying-in to an epistemic modality did not have the desired effects, no longer holds under the version 2 semantics. So, for instance, regarding the financial model discussed earlier, it is now the case that $\boxed{:-)} \models \text{SOME}(x, P(x), \Diamond B(x))$ but $\boxed{:-)} \not\models \text{EVERY}(x, P(x), \Diamond B(x))$. This is as we would expect: there is an individual who might be *B*roke in $\boxed{:-)}$ (in fact there are two, a and b), but it is not the case that everybody might be *B*roke (since c is definitely not *B*roke.) Thus the denotation specified for E26 is completely altered, and in particular the sample update $\boxed{:-)} \mapsto \boxed{:-)}$ will not be in the denotation of $\text{EVERY}(x, P(x), \Diamond B(x))$ under the new semantics.

5.4 Discussion

Since only predicate logics have been defined, with no formal mechanism for relating sentences of natural language to formulae of these logics, I cannot claim to have provided here a fully compositional treatment of the interaction between presupposition, epistemic modality and quantification. However, in [Be93b] it is shown how logics like those considered here can be embedded into a three sorted variant of classical type theory, and how this can be used to define a compositional grammar in the style of Montague.

Apart from providing an improved treatment of epistemic modality, the system offers a treatment of a range of problems from the presupposition literature. For instance, quantifiers can bind presuppositions, unlike in Karttunen and Peters' system [KP79]. Their problems resulted from the decision to split meanings into separate assertive and presuppositional components. There is no such split in the current system: presupposition and assertion are simply different aspects of a single (dynamic) meaning. In addition, all the empirical requirements of §8.2 are met

[20] By adjusting the admission condition in the definition of *sliced meaning* D32, alternative presuppositional behavior would be produced, so I cannot claim that the existential presuppositions are motivated just by having chosen to look at the values of variables one at a time. Rather, I would have to say that interpreting variables in this way gave the flexibility needed to remove the universal presuppositions in the first version of the semantics. It would be possible to use different definitions of slicing for different quantifiers, if there were evidence that the different quantifiers had different presuppositional properties. In §8.2, no evidence was found that the different quantificational determiners varied in this respect, but I suspect that some may find the conclusion that no quantifiers yield universal presuppositions contraversial. Such people are welcome to use the general framework provided here to reformulate the admission conditions as they see fit.

Also the contention that presupposition triggers in the restrictor of a universal do not lead to universal presuppositions is perhaps less contraversial than the claim that presupposition triggers in the scope of a universal do not lead to universal presuppositions. So some might wish to advocate different admission conditions for the slice operation used in the restrictor of a quantifier than in the scope.

However, note that the uncontentious empirical observation in §8.1.6 means that the admission condition cannot be removed altogether. If there were not at least an existential requirement, that for some value of the quantified variable update is defined, then unbound presupposition triggers within the syntactic scope of a quantifier would no longer constrain the input context to the quantifier.

in the modified semantics for HMQL. I leave it to the reader to verify this last claim, perhaps by considering the denotations of formulae corresponding to the various different examples presented there.

Can we draw any general morals? A first moral is that there has been a serious lack of empirical study into the interaction of presupposition and quantification, and that consequently theorists have been largely operating in the dark. The conclusions drawn in §8.2 I hope will prove contraversial, for perhaps that would convince others to perform more extensive surveys of their own. Are there, as I have assumed, even any existential presuppositions? The discourses in §8.2 are organised in such a way as to reveal the minimal semantic constraints needed to avoid presupposition failure. When informants are presented with a single sentence example, and asked about whether they would make various inferences on the basis of the example, they will usually make stronger inferences than those given by the minimal semantic constraint. Why? Are there other presuppositional mechanisms at work apart from the minimal precondition for update that I have discussed? There is clearly much empirical work still to be done.

Secondly, perhaps it has been demonstrated that traditional problems of modal predicate logic, albeit dragged into the barely-Tarskian world of dynamic semantics, are still of relevance. The admittedly rather superficial digression into the semantics of epistemic modality has, I hope, shed additional light on the nature of variables. In particular it has shown that whilst a measure of discourse-marker-ness is useful, it should not be taken too far. Variables within the scope of their introducing quantifier do not appear to behave as discourse markers. They should not be interpreted as single underspecified individuals but rather as place-markers for each of the individuals being quantified over.

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