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PAST, PRESENT, FUTURE AND SPECIAL RELATIVITY

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1. INTRODUCTION

Adopting the view that *the Special Theory of Relativity (STR)* has implications for the issues of philosophy of the time, I shall discuss the issue of temporal becoming.
According to the *A-theory (tensed theory, dynamic theory)* of time, there is temporal becoming - events are first future, then become present, and finally become past. Temporal properties of presentness, pastness and futurity are called *A-properties*. Events with their A-properties are said to be located in the *A-series*, "the series of positions which runs from the far past through the near past to the present, and then from the present through the near future to the far future"[1]. According to the *B-theory (tenseless theory, static theory)* of time, all successively ordered events have the same ontological status - they exist without becoming. The only temporal properties, according to the B-theory, are so called *B-relations* of earlier than or later than. B-relations among events generate the *B-series*, "the series of positions which runs from earlier to later"[2]. The debate between the A-theory and B-theory is not decisively resolved by the STR, but a number of philosophers such as Grünbaum, Mellor, Smart, Quine and Putnam, claim that the STR favours the B-theory.
Recently, a *third theory of time* was proposed by D. Zeilicovici.[3] He develops a kind of dualistic dynamic theory that recognizes both A-properties and B-relations as truly

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[1] McTaggart (1927), p. 10.

[2] McTaggart (1927), p. 10.

[3] D. Zeilicovici (1989).

temporal. According to Zeilicovici, the B-series is temporal but it does not distinguish between existinig and predicting moments. An A-series is defined as the part of the B-series which contains only existing moments. Each A-series emerges by the addition of a new moment to the existing moments.

The view that I shall present has evolved from an attempt to combine temporal becoming with the STR. It is inspired, just as Zeilicovici's view, by C. D. Broad's theory of time from *Scientific Thought*. The result of combining Broad's idea that "the sum total of existence is always increasing"[4] with the acceptance of the STR as the theory of time will yield a special version of the third theory of time.

2. ARGUMENT FROM STR

Introduction

According to many authors the STR provides a strong argument against McTaggart's A-series.[5] The heart of the argument is the claim that the STR implies the relativization of the notion of A-series to a reference frame, while the relativized A-series implies a nonacceptable frame-dependent concept of existence. I shall first introduce the notion of *"ontological separation of space-time"* based on the intuition that the future is ontologically different from the present and the past. Afterwards, I shall reformulate the STR-argument against the A-series as the STR-argument against the ontological separation of space-time.

Ontological separation of space-time

Minkowki space-time is ontologically separated with respect to an event e if, taking that e is here and now, every other event e' is either

(a) in the past or in the present - which I take to mean that e' is realized, or

(b) in the future - which I take to mean that e' is not realized.

Minkowki space-time is ontologically separated if space-time is ontologically separated with respect to every e.

Argument from STR

The argument against the ontological separation of space-time on the basis of the STR runs as follows.

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[4] Broad (1923), p. 66-67.

[5] See, for ex., Mellor (1974).

To conclude that the STR implies that the notion of "ontological separation of space-time" becomes relativized to a frame of reference, consider the following situation. Let O_1 and O_2 be two observers in uniform relative motion. Suppose O_1 and O_2 agree that event e_1 occurs "here and now" - take e_1 to be their meeting. (See Fig. 1.) Let e_2 be an event spatially separated from e_1 - say, the birth of a star - simultaneous with e_1 for O_1. According to the STR, e_2 is not simultaneous with e_1 for O_2. Assume that O_1 and O_2 are in such relative motion that e_1 is earlier than e_2 for O_2. (See Fig. 1.) Then, only relative to O_1 is the star birth occuring "now", and each of the two observers has its own separation of the space-time with respect to e_1. In general, any change of the velocity of O_2 will result in a different separation of space-time with respect to e_1 for O_2. Since there are no preferred frames of reference, the separation of space-time depends on a frame of reference. Assuming ontological separation of space-time, this implies that some events will be realized as well as not realized! Relative to O_1 event e_2 is realized, ontologically fixed and definite. Relative to O_2 the star birth is an event not (yet) realized. Furthermore, reality itself will be different depending on the reference frame! Relative to observer O_1 the star exists "now", while relative to observer O_2 the star does not (yet) exist.

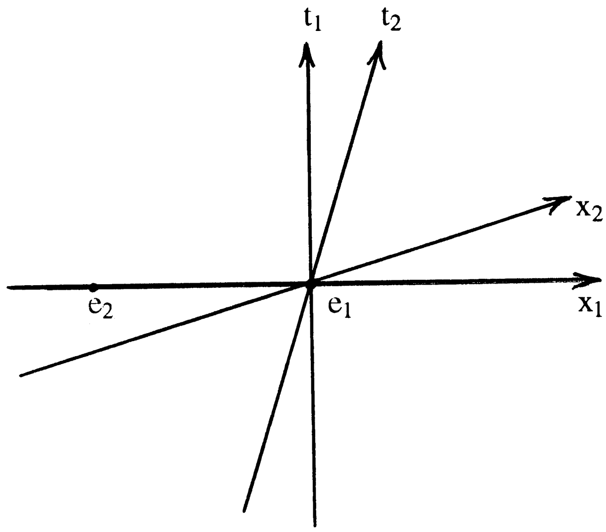

Fig. 1.

Analysis of the argument

The argument is based on the acceptance of the following three assumptions:

1) the following two statements are equivalent:

(i) With respect to e, that is, taking e to be here and now, e' is in the present.

(i') e is simultaneous with e'.

2) the following two statements are equivalent:
(ii) With respect to e, e' is in the past.
(ii') e' is earlier than e.

3) the following two statements are equivalent:
(iii) With respect to e, e' is in the future.
(iii') e is earlier than e'.

Since according to the STR statements (i') - (iii') are true only relative to a frame of reference, the acceptance of assumptions 1) - 3) implies that statements (i) - (iii) are true only relative to a frame of reference.[6]

A way to argue against the argument from the STR is to argue against the consequences of assumptions 1) - 3). Here are two possibilities:
(a) to argue that the STR is false;
(b) to argue that the STR is not a theory about the nature of time, but about the observable behaviour of light rays, rigid bodies and the like.[7] According to this view, there is a difference between the truly temporal relations "simultaneous with", "earlier than", "later than" and the STR relations "STR-simultaneous with", "STR-earlier than", "STR-later than". The statement "e is STR-simultaneous with e'", for example, neither means, nor implies, nor is implied by (i'). The same holds for the statement "e is STR-earlier than e'" and (ii'). Assumptions 1) - 3) are acceptable only if the notions of "simultaneous with", "earlier than", "later than", used in (i') - (iii'), are used in their temporal meaning. Hence, the argument from the STR has no force.
These are not the approaches I want to investigate in this paper. My approach might be called *"let's face it!"- approach.* Namely, I assume that the STR is a well-confirmed physical theory about time. In the next section I shall briefly explain what I find to be the core of this assumption. Developing the "let's face it"- approach, we shall end up claiming that the three assumptions of the argument from the STR (statements **1)** - **3)** in this section) do not hold.

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[6] Putnam argued (1967) that all events in special relativistic space-time are real. His argument is also based on the acceptance of 1) - 3).

[7] See, for ex., Quentin Smith (1993).

3. STR AS A THEORY OF TIME

Event e_1 is in *the STR causal relation* $<$ to event e_2 ($e_1 < e_2$), if event e_1 can causally influence e_2. In the standard model of the space-time geometry of the STR (the set of 4-tuples of real numbers together with its natural vector space structure and the Minkowski metric) "$e_1 < e_2$" is defined by the condition that the vector from e_1 to e_2 is a timelike or lightlike vector oriented towards the future. The causal relation is frame-invariant. The mathematician A. Robb has shown that the space-time geometry of the STR is axiomatizable using the causal relation as the only primitive one.[8]

I find that the acceptance of the STR as a theory of time comes, in first instance, down to the acceptance of the causal relation as the only primitive temporal relation. Taking the relation $<$ to be the only primitive temporal relation implies that a necessary condition for any relation to be temporal is that the relation is definable in terms of $<$. Robb's result ensures that the STR frame-relative relations "simultaneous with", "earlier than", "later than" are indeed temporal.[9]

4. ONTOLOGICAL SEPARATION AND CAUSALITY

Introduction

The lesson we have learned from "the argument from the STR" is that the ontological separation of space-time has to be frame-independent. A way to ensure this is to define the ontological separation in terms of the (invariant) geometry of Minkowski space-time. I shall discuss the question whether such a definition is possible. Eventually, I shall conclude that the ontological separation of space-time is not definable in terms of the invariant geometrical framework.

Preliminaries

Let us assume that space-time is time-oriented, that is, that the set of timelike and lightlike vectors of space-time is consistently partitioned into the set of "future-oriented" vectors and the set of "past-oriented" ones. Let us define for any event e

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[8] Robb (1914).

[9] A consequence of Robb's result is also the fact that the STR frame-relative relation of simultaneity is not conventional. The only STR simultaneity relation definable in terms of $<$ is the standard ($\varepsilon = 1/2$) simultaneity relation. See Malament (1977).

- *"the causal past of e"* as the set of all events e' such that the vector from e to e' is a timelike or a lightlike vector oriented towards the past (that is, as the set of all e' such that e'<e holds);
- *"the causal future of e"* as the set of all events e' such that the vector from e to e' is a timelike or a light-like vector oriented towards the future (that is, as the set of all e' such that e<e' holds); and
- *"the elsewhere of e"* as the set of all events e' such that the vector from e to e' is spacelike (that is, as the set of all e' such that neither e'<e nor e<e' nor e=e' hold). (See Fig. 2.)

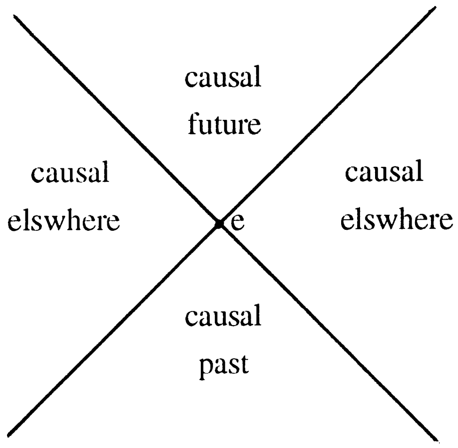

Fig.2.

Up to now, I have introduced two groups of notions:
1) the notions of "causal past of e", "causal future of e" and "elsewhere of e";
2) the notions of "past and present with respect to e" and "future with respect to e", where "with respect to e" means "taking e to be here and now".
For the sake of clarity, let me state once more that I define
- "the past and the present with respect to e" as the set of all events e' such that with respect to e, e' is realized;
- "the future with respect to e" as the set of all events e' such that with respect of e, e' is not realized.

Let eRe' stand for "with respect to e, e' is realized". I shall simply assume that the relation R is reflexive and transitive without giving arguments for this assumption since it is generally considered to be uncontroversial.[10]

[10] See Christensen (1981), Maxwell (1984), Putnam (1967), Stein (1991), Weingard (1972).

Definition of ontological separation

Here comes a part of the definition of the ontological separation of space-time with respect to an event e.

Clause a) With respect to e, all events from the causal future of e are not realized.

The reasoning behind the clause a) is the following. If e' is in the causal future of e, then e could be the cause of e'. Since only future events can be affected, e' is in the future of e. Hence, e' is not realized with respect to e.

Clause b) With respect to e, e itself is realized.

The argument for clause b) is the following. We have already assumed that e is here and now. So, e belongs to the present of e. Hence, e is realized with respect to e.

Clause c) With respect to e, all events from the causal past of e are realized.

The reasoning behind the clause a) is the following. If e' is in the causal past of e, then e' could be the cause of e. Since only the past can influence the present, e' belongs to the past of e. Hence, e' is realized with respect to e.

What about the events that are neither in the causal future of e nor in the causal past of e, that is, what about the events that are in the elsewhere of e? To answer this question I shall use the following theorem, proved by H. Stein[11].

Theorem 1. If

(i) $\mathfrak{R}$ is a reflexive and transitive relation on a Minkowski space-time structure,

(ii) $\mathfrak{R}$ is definable in terms of the geometric structure of space-time, and

(iii) for some events e and e' such that e' is in the causal past of e it holds that $e\mathfrak{R}e'$,

then

(iv) for any two events e and e' it holds that: $e\mathfrak{R}e'$ iff either e=e' or e' is in the causal past of e.

The assumption that all events from the causal past of an event are realized with respect to the event, and the geometrical fact that for every event e of Minkowski space-time there is an event in the causal past of e, imply that, taking our R to be $\mathfrak{R}$, condition (iii) in Theorem 1 holds. Further, I have accepted the assumption that the relation R is reflexive and transitive. Hence, in this framework the following theorem holds.

Theorem 1'.

If R is definable in terms of the geometric structure of Minkowski space-time, then for any two events e and e' it holds that: eRe' iff either e=e' or e' is in the causal past of e.

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[11] Stein (1991).

Theorem 1' has left us with only one possible way to answer the question "What about the events that are in the elswhere of e?" and to complete the definition of the ontological separation:

Clause d) With respect to e, all events from the causal elsewhere of e are not realized.

Argument against clause d)

The absence of any possible causal influence between two events e and e' cannot be (either the reason for e' to be realized with respect to e, or) the reason for e' to be not realized with respect to e.

Imagine the following situation. The occurrence or the nonoccurrence of a seafight depended solely on the "free" decision of an admiral. The admiral actually decided - event e_2 in Fig. 3 - that there was going to be the seafight - event e_3 in Fig. 3. The event e_2 is very "far" from event e_1, but in its causal past. The seafight is in the causal future of the admiral's decision, but the seafight is in the elsewhere of e_1. At e_1 we know about the admiral's decision. I do not see any reason why it is impossible that e_3 has already become, i.e. is realized with respect to e_1, although we still have to wait for a while to receive the message saying that the seafight did occur and that the admiral won the seafight. The fact that we have not yet received the message does not imply that the seafight is not realized. Furthermore, I think that, assuming that event e_2 is distant enough from us - so that when we receive the message about admiral's decision we cannot possibly think that the admiral is still alive - we have to say that e_3 is realized with respect to e_1.

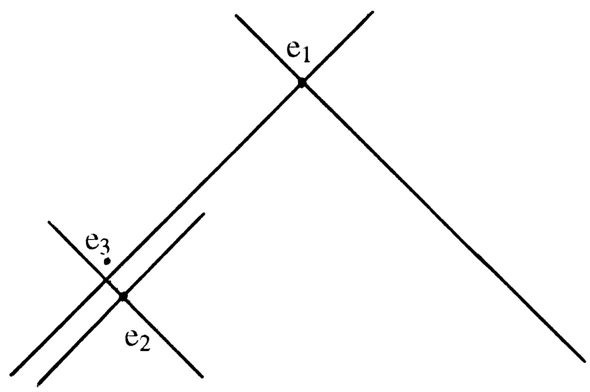

Fig. 3.

Conclusion

Taking that the seafight example shows that we accept the idea of the realization of some "elswhere"- events, I conclude, applying Theorem 1', that the relation R is not definable in terms of the geometry of Minkowski space-time, that is, that the ontological separation of Minkowski space-time due to R is not definable in terms of the geometry of space-time. Since the STR causal relation $<$ is definable in the standard model of Minkowski geometry, relation R is not definable in terms of the causality relation, that is, the ontological separation of Minkowski space-time is not definable in terms of $<$.

5. EXTENDED SPACE-TIME

Introduction

Having shown that the relation R is not definable in terms of the STR-causal relation, I am going to introduce the relation R in the structure of space-time as a new primitive relation on events. For this purpose, I shall represent oriented Minkowski space-time as a non-empty set W of (point) events subject to the STR-causal relation $\leq$ which is reflexive (Postulate 1(i)), transitive (Postulate 1(ii)), directed (Postulate 1(iii) - (iv)) and antisymmetric (Postulate 1(v)). Defining the relations of "being in the elswhere of" and "being in the causal future of" in terms of the relation $\leq$, I shall add, as postulates, the following two geometrical facts about space-time: 1) the elswhere of any event is not empty (Postulate 1(vi)); 2) for any two events e_1 and e_2, if e_2 is in the elswhere of e_1, there is a third event e_3 which is in the causal future of e_1 and in the elswhere of e_2 (Postulate 1(vii)). After that, I shall state a number of new postulates on the relations $\leq$ and R that will lead to the desired separation of space-time. The relation of "being in the present of" will be defined in a way that captures C. D. Broad's view about the present in his *Scientific Thought.*

Extended space-time frame

An *extended space-time frame* is an ordered triple $\langle W, \leq, R\rangle$ in which W is the set of point events, and $\leq$ and R are binary relations on W. Before stating the postulates on the STR-causal relation, I shall define "e_2 is in the causal future of e_1" and "e_2 is in the elsewhere of e_1" in terms of the relation $\leq$ in the following way:

Definition 1. $e_1 < e_2$ iff $(e_1 \leq e_2 \wedge \neg(e_1 = e_2))$.
Definition 2. e_1 ELS e_2 iff $(\neg(e_1 \leq e_2) \wedge \neg(e_2 \leq e_1))$.

Postulate 1.

(i) $\forall e(e \leq e)$

(ii) $\forall e_1,e_2,e_3(e_1 \leq e_2 \wedge e_2 \leq e_3 \rightarrow e_1 \leq e_3)$

(iii) $\forall e_1,e_2\exists e_3(e_1 \leq e_3 \wedge e_2 \leq e_3)$

(iv) $\forall e_1,e_2\exists e_3(e_3 \le e_1 \wedge e_3 \le e_2)$

(v) $\forall e_1,e_2(e_1 \le e_2 \wedge e_2 \le e_1 \rightarrow e_1 = e_2)$

(vi) $\forall e_1\exists e_2(e_1 \text{ ELS } e_2)$.

(vii) $\forall e_1,e_2(e_1 \text{ ELS } e_2 \rightarrow \exists e_3(e_1 < e_3 \wedge e_2 \text{ ELS } e_3))$.

I could have taken as Postulate 1 a full Robb-style axiomatization of Minkowski space-time geometry, but there is no need for that. Postulate 1 in the given form is a description of Minkowski space-time sufficient for our purpose. Here follow some facts on the connections between the relations ≤, < and ELS.

Fact 1.

(i) ELS is irreflexive and symmetric.

(ii) $\forall e_1,e_2(e_1 \le e_2 \vee e_2 \le e_1 \vee e_1 \text{ ELS } e_2)$.

(iii) $\forall e_1,e_2(e_1 \text{ ELS } e_2 \rightarrow \exists e_3(e_1 < e_3 \wedge e_2 < e_3))$.

(iii') $\forall e_1,e_2(e_1 \text{ ELS } e_2 \rightarrow \exists e_3(e_3 < e_1 \wedge e_3 < e_2))$.

(iv) $\forall e_1\exists e_2(e_1 < e_2)$.

(v) $\forall e_1\exists e_2(e_2 < e_1)$.

I proceed by stating the postulates connecting the relation ≤ and the new primitive relation R. The next two postulates reflect the clauses a) - c) in section 4.

Postulate 2. $\forall e_1,e_2(e_1 \le e_2 \rightarrow e_2 \text{ R } e_1)$.

Postulate 2 says that if e_1 is in the causal past of e_2 or e_1=e_2, then with respect to (from now on: wrt) e_2, e_1 is realized.

Fact 2.

(i) $\forall e(e\text{R}e)$.

(ii) $\forall e_1,e_2,e_3(e_1 \le e_2 \wedge e_2 \le e_3 \rightarrow e_3 \text{ R } e_1)$.

(iii) $\forall e_1,e_2\exists e_3(e_3 \text{ R } e_1 \wedge e_3 \text{ R } e_2)$.

(iii') $\forall e_1,e_2\exists e_3(e_1 \text{ R } e_3 \wedge e_2 \text{ R } e_3)$.

(iv) $\forall e\exists e'(e\text{R}e' \wedge e \neq e')$.

Postulate 3. $\forall e_1,e_2(e_1 < e_2 \rightarrow \neg(e_1 \text{ R } e_2))$.

Postulate 3 says that if e_2 is in the causal future of e_1, then wrt e_1, e_2 is not realized.

Fact 3.

(i) R is not symmetric.

(ii) $\forall e_1,e_2,e_3(e_1 < e_2 \wedge e_2 \le e_3 \rightarrow \neg(e_1 \text{ R } e_3)$.

(iii) $\forall e_1,e_2(e_1 \text{ ELS } e_2 \rightarrow \exists e_3(\neg(e_1 \text{ R } e_3) \wedge \neg(e_2 \text{ R } e_3))$.

(iii') $\forall e_1,e_2(e_1 \text{ ELS } e_2 \rightarrow \exists e_3(\neg(e_3 \text{ R } e_1) \wedge \neg(e_3 \text{ R } e_2))$.

(iv) $\forall e_1 \exists e_2 \neg(e_1 R e_2)$.

At this point I want to introduce the notion of "a present event wrt an event e". The leading idea behind the definition that follows is C.D. Broad's conception of the present developed in his *Scientific Thought*: "... the essence of a present event is, not that it precedes future events, but that there is quite literally *nothing* to which it has the relation of precedence." I take this to mean that every present event precedes no realized events. Following such an interpretation of Broad's conception of present, I define a present event wrt e as a realized event wrt e such that nothing from its causal future is realized wrt e. I write "e PRES e'" for "wrt e, e' is present".

Definition 3. e PRES e' iff $(eRe' \wedge \forall e''(e' < e'' \rightarrow \neg(e\ R\ e''))$

Fact 4.
(i) $\forall e(e \text{ PRES } e)$.
(ii) $\forall e,e'(e' < e \rightarrow \neg(e \text{ PRES } e'))$.
(iii) $\forall e,e'(e < e' \rightarrow \neg(e \text{ PRES } e'))$.
(iv) $\forall e,e'(e \text{ PRES } e' \rightarrow (e' = e \text{ or } e \text{ ELS } e'))$.
(v) $\forall e,e_1,e_2(e_1 \neq e_2 \wedge e \text{ PRES } e_1 \wedge e \text{ PRES } e_2 \rightarrow e_1 \text{ ELS } e_2)$.

The next postulate reflects the fact that our primitive temporal relation is actually a causal relation. It is needed to ensure that the notions of past, present and future wrt an event, based on Broad's ideas, fit well with the notions of causal past, causal future and elsewhere of the event. Consider the following situation. (See Fig. 4.) Let e be an event that is here and now, and let events e' and e" be in the elswhere of e. Suppose further that e'< e", eRe', ¬(eRe") and ¬(ePRESe'). Then, since e" is not in the causal

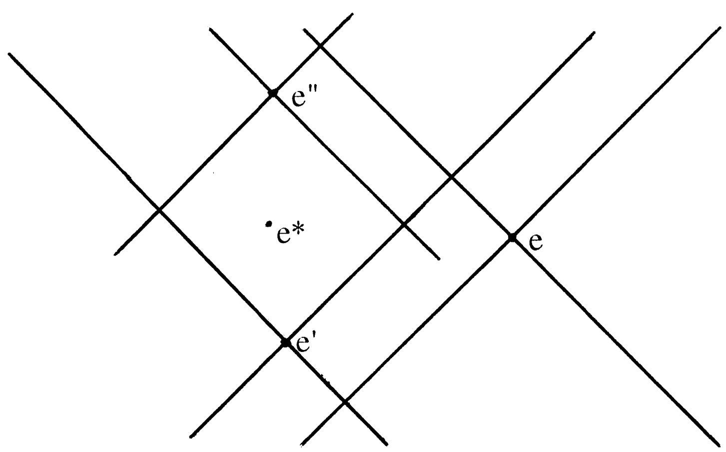

Fig. 1.

future of present-event e, we want it to be in the causal future of some other present-event wrt e. Further, since e' is not in the causal past of present-event e, we want it to be in the causal past of some other present-event wrt e. But, we actually want something stronger. Namely, since e" is in the future of e and since e' is in the past of e, we want to ensure that there will be a present-event wrt e, say e*, which is "causally between" e' and e", that is, a present event wrt e which is both in the causal future of e' and in the causal past of e". Postulate 4 states this condition for the general case.

Postulate 4.[12]
$\forall e, e', e''(e' < e'' \wedge e \mathrel{R} e' \wedge \neg(e \mathrel{R} e'') \rightarrow \exists e^*(e' \leq e^* \wedge e^* < e'' \wedge e \text{ PRES } e^*))$.

Fact 5.[13]
(i) $\forall e, e_1(\neg(e \mathrel{R} e_1) \wedge e \text{ ELS } e_1 \rightarrow \exists e'(e' < e_1 \wedge e \text{ PRES } e'))$.
(ii) $\forall e, e_1(\neg(e \mathrel{R} e_1) \rightarrow \exists e'(e' < e_1 \wedge e \text{ PRES } e'))$.

Fact 5(ii) claims that if e_1 is not realized wrt e, then a present-event wrt e exists in the causal past of e_1.

(iii) $\forall e_1,e_2,e_3(e_1 \mathrel{R} e_2 \wedge e_3 \leq e_2 \rightarrow e_1 \mathrel{R} e_3)$.

Fact 5(iii) says that if e_2 is realized wrt e_1, then everything that is in the causal past of e_2 is realized wrt e_1. This implies that causal pasts of the present-events wrt any event e are realized wrt e.

(iii') $\forall e_1,e_2,e_3(\neg(e_1 \mathrel{R} e_2) \wedge e_2 \leq e_3 \rightarrow \neg(e_1 \mathrel{R} e_3))$.

Fact 5(iii') says that if e_2 is not realized wrt e_1, then everything that is in the causal future of e_2 is not realized wrt e_1.

(iv) $\forall e \forall e'((e \text{ ELS } e' \wedge \neg(e \mathrel{R} e')) \rightarrow \exists e''(e \text{ ELS } e'' \wedge e \mathrel{R} e''))$.
(v) $\forall e \exists e'(e \text{ ELS } e' \wedge e \mathrel{R} e')$

Fact 5(v) rejects one of the possible answers to the question concerning the realization, wrt an event e, of the events belonging to the elswhere of e. Fact 5(v) says that it is not the case that all events from the elsewhere of e are unrealized wrt e.

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[12] There is some redundancy in Postulate 4. Namely, when the events e, e' and e" are such that $e' \leq e$ and $e < e''$, Postulate 4 can be proved using Fact 4(i). But, for the sake of elegance I do not exclude this case from the postulate.

[13] The proofs of Fact 5 as well as of all subsequent facts are given in the Appendix.

Fact 6.

(i) $\forall e, e_1(e \text{ R } e_1 \wedge e \text{ ELS } e_1 \rightarrow \exists e'(e_1 \leq e' \wedge e \text{ PRES } e'))$.

(ii) $\forall e, e_1(e \text{ R } e_1 \wedge \rightarrow \exists e'(e_1 \leq e' \wedge e \text{ PRES } e'))$.

Fact 6(ii) claims that if e_1 is realized wrt e, then either e_1 is a present-event wrt e or a present-event wrt e exists in the causal future of e_1.

(iii) $\forall e, e'(e \text{ PRES } e' \leftrightarrow \forall e^*(e^* \neq e' \wedge e \text{ PRES } e^* \rightarrow e' \text{ELS } e^*))$.

Transitivity of the relation R has been accepted in section 4. I take it here as a postulate on R.

Postulate 5. $\forall e_1 e_2 e_3(e_1 \text{ R } e_2 \wedge e_2 \text{ R } e_3 \rightarrow e_1 \text{ R } e_3)$

Before we look at some consequences of the acceptance of Postulate 5, I define R_e to be the set of all events that are realized wrt event e.

Definition 4. $R_e = \{e' | \, eRe'\}$.

Fact 7.

(i) $\forall e \exists e'(e \text{ ELS } e' \wedge \neg(e \text{ R } e'))$.

Fact 7(i) rejects another possible answer to the question concerning the realization, wrt an event e, of the events belonging to the elswhere of e. Fact 7(i) says that it is not the case that all events from the elsewhere of e are realized wrt e. This fact, together with Fact 5(v), implies that wrt every event e, some elswhere-events are realized and some elswhere-events are not realized.

(ii) $\forall e,e'(eRe' \leftrightarrow R_{e'} \subseteq R_e)$.

(iii) $\forall e,e'(R_e = R_{e'} \leftrightarrow eRe' \wedge e'Re)$.

(iv) $\forall e,e'(eRe' \wedge \neg(e \text{ PRES } e') \rightarrow \neg(e'Re)))$.

(v) $\forall e,e'((eRe' \wedge \neg(e \text{ PRES } e')) \rightarrow \exists e^*(e^* < e \wedge e \text{ PRES } e^*))$.

(vi) $\forall e,e'(R_{e'} = R_e \leftrightarrow e \text{ PRES } e' \wedge e' \text{ PRES } e)$.

(vii) $\forall e,e'(eRe' \wedge e'Re \leftrightarrow e \text{ PRES } e' \wedge e' \text{ PRES } e)$

Finally, I state my last postulate. It makes the final step towards the formalization of C. D. Broad's conception of the present in my interpretation. Broad wrote in his *Scientific Thought:* "Nothing has happened to the present by becoming past except that fresh slices of existence have been added to the total history of the world." I understand this to imply that our relation PRES should be an equivalence relation. In that way, Broad's "slices of existence" would correspond to PRES-classes of equivalence. (Broad's notion of temporal becoming will be used latter in defining the A-series.)

Postulate 6. $\forall e,e'(e \text{ PRES } e' \rightarrow e'Re)$.

Postulate 6 says that if event e' is the present-event wrt event e, then e is realized wrt e'. Our postulates now imply that the relation PRES is symmetric (Fact 8(ii)) as well as transitive (Fact 8(iv)). This, together with Fact 4(i), makes the relation PRES an equivalence relation.

Fact 8.
(i) $\forall e,e'(e \text{ PRES } e' \leftrightarrow eRe' \wedge e'Re)$.
(ii) $\forall e,e'(e \text{ PRES } e' \rightarrow e' \text{ PRES } e)$.
(iii) $\forall e,e'(e \text{ PRES } e' \rightarrow \forall e''(e \text{ PRES } e'' \rightarrow e' \text{ PRES } e''))$.
(iv) $\forall e,e',e''(e \text{ PRES } e' \wedge e' \text{ PRES } e'' \rightarrow e \text{ PRES } e'')$.
(v) $\forall e,e'(e \text{ PRES } e' \leftrightarrow R_e = R_{e'})$.
(vi) $\forall e,e'(eRe' \vee e'Re)$.

Since the relation PRES is an equivalence relation, the set of events is partitioned by PRES into equivalence classes.

Definition 5. $[e] = \{e' | e \text{ PRES } e'\}$.

The partition will be important later in defining the notions of "A-series" and "ontological becoming". The set of equivalence classes is linearly ordered by the relation $\propto$ defined in the following way.

Definition 6. $[e_1] \propto [e_2]$ iff $\neg(e_1Re_2)$.

Fact 9.
(i) $\propto$ is irreflexive.
(ii) $\propto$ is transitive.
(iii) For every two equivalence classes $[e_1]$ and $[e_2]$, either $[e_1] \propto [e_2]$ or $[e_2] \propto [e_1]$ or $[e_1] = [e_2]$ holds.
(iv) $[e_1] \propto [e_2]$ iff $R_{e_1} \subset R_{e_2}$.

It is now easy to see that the relation $\angle$ on the set of R_e-sets (see Def. 4) defined by the condition "$R_{e_1} \angle R_{e_2}$ iff $R_{e_1} \subset R_{e_2}$" would be linear. This fact will be important later for the understanding of the notion of "A-series" that I will propose.

In the previous section I have shown that the relation R is not definable in term of the STR-causal relation. Besides, if we take Postulate 1 to be a full Robb-style axiomatization of Minkowski space-time geometry, then it follows that in every model for Postulate 1 we can find at least one relation R satisfying the rest of the postulates. Choose the relations "simultaneous with" and "earlier than" defined for an arbitrary frame of reference, and take e_1 R e_2 to hold if and only if e_2 is simultaneous with or ealier than e_1 relative to the choosen frame of reference.

Conclusion

In an extended space-time frame the set of events is separated wrt every event e into the set of realized and the set of unrealized events: wrt e, e itself, all events from its causal past and some events from its elsewhere are realized, while all events from its causal future and some events from its elsewhere are not realized. The "upper bound" of realized events wrt e are the events that are in the present of e. The set of realized events wrt any event e', such that e' is in the present of e, is the same as the set of realized events wrt the event e. (For an example see Fig 5.)

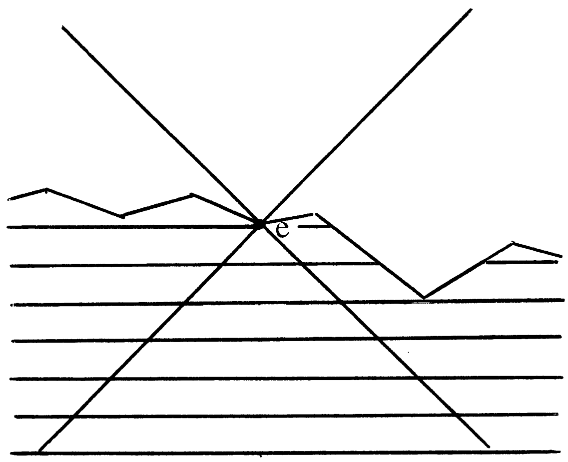

Fig. 5.

6. THE ARGUMENT FROM STR REVISITED

Introduction

I shall now return to the argument from the STR against the ontological separation of space-time and discuss whether it can still be successfully applied to the notion of separation based on the new primitive relation R among events, introduced in the previous section.

Analysis of the argument revisited

Let me first define precisely the notions of past, present and future wrt an event e.

Definition 6.

$\mathbf{Past}_e = \{e' | \; e \; R \; e' \; \& \; \neg(e \; PRES \; e')\}$.

$\mathbf{Present}_e = \{e' | \; (e \; PRES \; e')\}$

$\mathbf{Future}_e = \{e' | \; \neg(e \; R \; e')\}$

Consider now the statements (i) - (iii) from section 2. In our framework they become the following statements:

(i) $e' \in \mathbf{Present}_e$.

(ii) $e' \in \mathbf{Past}_e$.

(iii) $e' \in \mathbf{Future}_e$.

Each of these statements hold independently of frames of reference, since the notions of present, past and future that they employ are not frame relative. For example, e PRES e' and eRe' mean "taking e to be here and now, e' is in the present" and "taking e to be here and now, e' is realized" respectively. Hence, such statements will have the same truth value for any observer whose here and now is the event e. Furtheremore, since the relation PRES is an equivalence relation, all observers that are in each other's presents will agree on the separation of space-time in the past, the present and the future. On the other hand, the following statements (the statements (i') - (iii') from secion 2 are true only relative to a frame of reference:

(i') e is simultaneous with e';

(ii') e' is earlier than e;

(iii') e is ealier than e'.

Statement (i) does not imply statement (i') nor is implied by it. The same holds for the other two pairs of the corresponding statements - for (ii) and (ii') as well as for (iii) and (iii'). Since this is very easy to see, I shall show it only for the pair (i),(i').

Consider event e_1 in Fig. 1 and observers O_1 and O_2 as described in the argument from the STR. Let e_3 be an event in the causal past of e_2 but simultaneous with e_1 for observer O_2 (see Fig. 6). Assume that statement (i') implies statement (i). Then, since e_1 is simultaneous with e_2 for observer O_1, it would follow that e_1 PRES e_2. Further, since e_3 is simultaneous with e_1 for observer O_2, it would follow that e_1 PRES e_3. e_1

PRES e_2 and e_1 PRES e_3 imply, by Fact 4(v), that e_2 ELS e_3. This contradicts the assumption that e_3 is in the causal past of e_2. Hence, (i') does not imply (i).

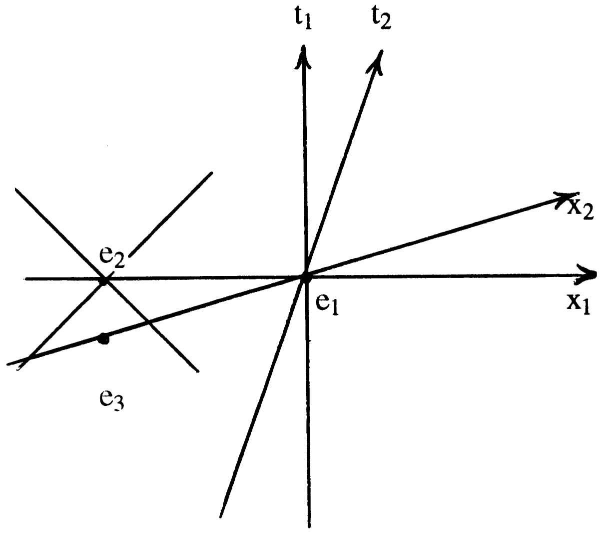

Fig. 6.

Consider again the situation in Fig. 6, with the additional assumption that e_1 PRES e_2 holds. Assume that statement (i) implies statement (i'). Then, since e_2 is not simultaneous with e_1 for O_2, it would follow that e_1 PRES e_2 does not hold. This contradicts the assumption that e_1 PRES e_2 holds. Hence, (i) does not imply (i').

However, Postulates 2 and 3 force relation R to respect the causality relation between events. Besides, the following are the STR-facts: the relation of simultaneity relative to an observer is reflexive; for any two events e and e', if event e' can causally influence event e, then e' is earlier than e relative to all observers. These STR-facts, together with Postulates 2 and 3, force the equivalences rejected above (that is, assumptions 1) - 3) in section 2 to hold under the condition that e and e' are causally connectible. Namely, for any events e and e' such that either $e \leq e'$ or $e' \leq e$, it holds that:

(i) is equivalent to (i');

(ii) is equivalent to (ii');

(iii) is equivalent to (iii').

Conclusion

The answer to the argument from the STR is now clear: the three basic assumptions of the argument are not valid when the notions of past, present and future, that are introduced in this paper, are involved. The notions of past, present and future are based on the ontological distinction between realized and unrealized events. On the other side, frame relative temporal notions such as the notion of "simultaneity", do not have any immediate implications for such a distinction. They reflect the partial "ontological uninformativity" of the STR-causal relation, the relation that I identified as the primitive temporal relation. Only for the events causally connectible to an event e it is certain whether they are realized or not realized with respect the event e. Ontologicaly uninformative in the same way, the frame relative temporal relations can differ from frame to frame without causing any "ontological trouble".

7. ONTOLOGICAL BECOMING

Introduction

In this section I shall sketch my approach to the question of temporal becoming. I shall introduce the notions of "BB-series", "B-series" and "A-series". As I have already mentioned, these notions, based on the ideas of C.D. Broad and D. Zeilicovici, will lead to a variant of the third theory of time. Besides, I shall give a few hints for combining indeterminism and the STR.

BB-series and B-series

I define *the Basic B-series (BB-series)* as the set of point events together with the STR-causality relation ≤. The generating relation of the BB-series is not connected, that is, it is not the case that for any two events e and e' either e ≤ e' or e'≤ e holds. This is the reason why we cannot consider the series generated by the relation ≤ to be a B-series.

I define a *B-series* to be the set of point events together with the frame dependent metrical relation of "being earlier than". This means that the notion of "B-series" is relativized to a frame of reference. In other words, each observer, having its own metrical relation of "earlier than", has its own B-series.

As far as the relation between BB- and B-series is concerned, the earlier mentioned result of Robb implies that B-series are definable in terms of the BB-series.

In section 3 I have accepted the STR-causal relation as the only primitive temporal relation. This now implies that the BB-series is a temporal series. Since a BB-series is definable in the terms of the BB-series, the BB-series are also temoral.

In section 4 I have shown that the relation R is not definable in terms of the STR-causality relation. In other words, in the BB-series, the set of realized events and the set of not realized events with respect to any event cannot be distinguished. The generating relation of the BB-series fails to distinguish between the two sets of events. This job will be done by the A-series.

A-series

I shall use the extended space-time, which is the BB-series together with the primitive relation R as described in section 5, to define the *A-series.*

Let me first discuss the nature of the R relation. Since R is not definable in terms of the STR-causality relation, and since the last relation is accepted as the only primitive temporal relation, relation R is not temporal. Since R yields the distinction between realized and unrealized events wrt an event, I understand it to be an ontological relation. I take each set R_e, that is, each part of the BB-series which contains only events realized wrt e, to be an A-series. Because of the Fact 8(v), for every two events e' and e", belonging to an PRES-equivalence class [e], it will hold that $R_{e'} = R_{e''}$. Hence, there will be as many A-series as equivalence classes generated by the relation PRES. I shall write "A[e]-series" for the A-series whose upper bound of events is the equivalence class [e]. Different A-series emerge by a new "slice of events" from BB-series being realized. Different A-series, having emerged in that way, form a sequence of consecutive A-series.

Explanations of A-determinations I take over from Zeilicovici. A-determinations of an event depend on A-series. They refer to the status of an event in a particular A-series. For example, the sentence "e' is present" holds in an A[e]-series if event e' belongs to the upper limit of that A[e]-series. This view is immune to McTaggart's argument which basically says the following: every event possesses all mutually incompatible A-properties, since all events are first future, then become present and finally become past.[14] In this framework no event has incompatible A-determinations. In every A-series every event has exactly one A-determination, so that no contradiction can arise within a single A-series. Events do have different A-status in different A-series. However, that causes no contradictions since different A-series are involved.

The third theory of time

Being defined in terms of the temporal relation $\leq$ and the ontological relation R, the notion of "A-series" is a temporal as well as an ontological notion. Each A-series gets its temporality from the BB-series, while the BB-series, as well as each B-series, gets ontological dynamism from A-series. To put it in Zeilicovici's terms, the BB-series and B-series express the static theme of time while the song of change is relegated to the voices of A-series.

If by "time" we mean an observer-relative ordered set of moments, where moments are defined as equivalence classes of events under the relation of simultaneity in the observer's B-series, then a consequence of this version of the third theory of time is

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[14] McTaggart (1927), ch. 33.

that the notion of "temporal becoming" changes a bit. Replacement of an old A-series by a new A-series leads to a change of A-determinations of events, but not to a "creation" of a new moment for an observer. The notion of "creation of a new moment" does not make sense any more. For example, it can be the case that, for an observer O, whose here-now is event e, event e' is simultaneous with e, while it holds that wrt e, e' is not realized. Events come into existence, they become, but not the moments of an observer's time. This separation of time from ontology is, according to the view presented in this paper, the most important implication of the STR for the issue of temporal becoming. Because of that, I find it more appropriate to call the replacement of A-series "ontological becoming".
So, the "song of change" does not concern the instants of an observer-time, but the "observer-independent" events.

Oaklander's criticism

Zeilicovici's view was criticised by N. Oaklander.[15] Without going into that discussion, I shall first try to reformulate Oaklander's arguments in such a way that they apply to the view presented in this paper, and then show that these reformulated arguments are not valid. If I understand the arguments correctly, then they would basically in this context run as follows.

(i) If the BB-series is composed of events "waiting" to be realized, then the creation of each new A[e]-series is itself an event, that is, a sort of meta-event, that must undergo realization. But, the realization of that meta-event is itself an event, that is, a sort of meta-meta-event, that must undergo realization. Etc.
(ii) The single BB-series, being the actual history of the world, has to be composed entirely of realized events. That is, for every event in the BB-series all other events in the BB-series are realized. Hence, if we accept Broad's idea that "the sum total of existence is always increasing", we cannot speak of a single BB-series. It turns out that the BB-series simply is the A-series. Thus we cannot say that the BB-series is a time series independently of temporal becoming.
(iii) The single BB-series, being the actual history of the world, has to be composed entirely of realized events. If we insist that there is only one BB-series, than Broad's idea that "the sum total of existence is always increasing" must be abandoned.

I shall first dicsuss argument (i). The creation of an A[e]-series is done by the realization of the events belonging to the class [e]. Thus, to judge the argument, we actually have to answer the question "Is the realization of an event an event?". Realization of an event

[15] Oaklander (1994).

does not change the event in the sense that it changes the "ordinary" properties of the event. Such change would certainly be a sort of second order event, although I cannot find much sense in such a notion of change. But, realization of an event does not give a different event. We talk about one and the same event being realized or being unrealized. Realization of an event does change the event's A-determination, but that kind of change is certainly not an event.

As far as the arguments (ii) and (iii) are concerned, they are both based on the assumption that the BB-series is composed entirely of realized events. The assumption is supposed to follow from the fact that the BB-series represents the actual and not the possible events. However, "actual" does not mean "realized". If an event is realized, than it is certainly actual, but if an event is actual, then it is not necesserily realized. The events that belong to the actual future of the world are still the members, although unrealized, of the actual BB-series.

STR and ideterminism

The problem of combining the STR with indetereminism has led N. Belnap to introduce *branching space-time*, a kind of fusion of space-time with *Prior-Thomason branching time*.[16] Having introduced the notion of ontological separation of space-time, I can suggest another option - a structure that might be called *indeterministic space-time structure*. This structure consists of possible histories, where each possible history is an extended space-time frame, together with a three-place relation between two histories and an event. This relation holds, for example, between histories H and H_1 at e if e belongs to both H and H_1, and A[e]-series in history H is the same as A[e]-series in history H_1. If H and H_1 are in the relation at e, then with respect to e there are at least two possible futures.

8. CONCLUSION

As we have seen, to combine temporal becoming and the STR in a version of the third theory of time, a new primitive relation on the events of space-time had to be introduced. I have shown that we do have the notion of realization of some "elswhere"-events and I have formalized this notion by the new binary relation of "being realized with respect to". One possible criticism of such an approach is that this new primitive relation does not have a place in our physical theory. To answer this criticism I call on Einstein himself for help. Einstein, according to the writting of R. Carnap in his

[16] Belnap (1992).

Autobiography, "once ... said that the experience of the Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics".[17] Although Einstein's theory forces difficult revisions of some common temporal notions, such as the notion of "simultaneity", "being earlier than" and "being later than", it does not force us to give up the notion of "temporal becoming". The STR is a physical theory about time, but it is not everything that can be said concerning time. Still, our philosophical claims must be reconciled with the results of physical science.

APPENDIX

Fact 5.

(i) $\forall e, e_1(\neg(eRe_1) \wedge eELSe_1 \rightarrow \exists e'(e' < e_1 \wedge ePRESe'))$.

Proof: Suppose $\neg(eRe_1)$ and $eELSe_1$. Fact 1(iii') implies that there is e_2 such that $e_2 < e$ and $e_2 < e_1$. $e_2 < e$ implies, by Postulate 2, that eRe_2. Postulate 4 implies that there is e' such that $e_2 \leq e'$, $e' < e_1$ and $ePRESe'$.♥

(ii) $\forall e, e_1(\neg(eRe_1) \rightarrow \exists e'(e' < e_1 \wedge ePRES\, e'))$.

Proof: by Fact 4(i) and Fact 5(i).♥

(iii) $\forall e_1,e_2,e_3(e_1Re_2 \wedge e_3 \leq e_2 \rightarrow e_1Re_3)$.

Proof: Suppose e_1Re_2, $e_3 \leq e_2$ and $\neg(e_1Re_3)$. By Fact 5(ii), $\neg(e_1Re_3)$ implies that there is e' such that $e' < e_3$ and e_1PRESe'. By Postulate 1(ii), $e' < e_3$ and $e_3 \leq e_2$ imply $e' < e_2$. Since e_1PRESe', it follows that $\neg(e_1Re_2)$. Contradiction, since e_1Re_2.♥

(iii') $\forall e_1,e_2,e_3(\neg(e_1Re_2) \wedge e_2 \leq e_3 \rightarrow \neg(e_1Re_3))$.

Proof: Suppose $\neg(e_1Re_2)$, $e_2 \leq e_3$ and e_1Re_3. By Fact 5(iii), e_1Re_3 and $e_2 \leq e_3$ imply $e_1R\, e_2$. Contradiction, since $\neg(e_1Re_2)$.♥

(iv) $\forall e \forall e'((eELSe' \wedge \neg(eRe')) \rightarrow \exists e''(eELSe'' \wedge eRe''))$.

Proof: Suppose $eELSe' \wedge \neg(eRe')$. By Fact 5(i), it follows that there is e" such that $e'' < e'$ and $ePRESe''$. By Definition 3, $ePRESe''$ implies eRe''. It is not the case that $e = e''$, since $e'' < e'$ and $eELSe'$. By Fact 4(iv), it follows that $eELSe''$.♥

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[17] Cited in Schlesinger (1994).

(v) $\forall e \exists e'(eELSe' \wedge eRe')$

Proof: Let e be an arbitrary event and assume that there is no e' such that eELSe' and eRe'. By Postulate 1(vi), it holds that there is e* such that eELSe* holds. By the assumption, it follows that eRe* does not hold. Now, by Fact 5(iv), it follows that there is e" such that eELSe" and eRe" hold. Applying the assumption again, we reach the contradiction: there is e" such that both eRe" and ¬(eRe") hold. Hence, there exist e' such that eELSe' and eRe'.♥

Fact 6.

(i) $\forall e,e_1(eRe_1 \wedge eELSe_1 \rightarrow \exists e'(e_1 \leq e' \wedge ePRESe'))$.

Proof: Suppose eRe_1 and $eELSe_1$. Fact 1(iii) implies that there is e_2 such that $e < e_2$ and $e_1 < e_2$. $e < e_2$ implies, by Postulate 3, that $\neg(eRe_2)$. Postulate 4 implies that there is e' such that $e_1 \leq e'$, $e' < e_2$ and ePRESe'.♥

(ii) $\forall e,e_1(eRe_1 \wedge \rightarrow \exists e'(e_1 \leq e' \wedge e\ PRESe'))$.

Proof: By Fact 4(i) and Fact 6(i).♥

(iii) $\forall e,e'(ePRESe' \leftrightarrow \forall e^*(e^* \neq e' \wedge ePRESe^* \rightarrow e'ELSe^*))$.

Proof: Assume $\forall e^*(e^* \neq e' \wedge ePRESe^* \rightarrow e'ELS\ e^*)$. If ¬(eRe'), then, by Fact 5(ii), there is e^ such that e^< e' and ePRESe^ hold. Then, the assumption implies e'ELSe^. Contradiction. Hence, eRe'. If eRe' and ¬(ePRESe'), then, by Fact 6(ii), there is e^ such that e'< e^ and ePRESe^. Then, the assumption implies e'ELSe^. Contradiction. Hence, ePRESe'. The other direction follows from Fact 4(v).♥

Fact 7.

(i) $\forall e \exists e'(eELSe' \wedge \neg(eRe'))$.

Proof: Let e be an arbitrary event. Postulate 1(vi) implies that there is event e" such that eELSe" holds. If ¬(eR e"), then we are done. So, suppose eRe". By Fact 6(i), eELSe" and eRe" imply that there is e* such that e"≤e* and ePRES e*. It cannot hold that e=e*, since e"≤e* and eELSe". By fact 4(iv), it follows that eELSe*. By the symmetry of ELS, eELSe* implies e*ELSe. Postulate 1(vii) implies that there is e' such that e*<e' and eELSe' hold. By Definition 3, e*<e' and ePRESe* imply ¬(eRe'). Hence, there is e' such that eELSe' and ¬(eRe') hold.♥

(ii) $\forall e,e'(eRe' \leftrightarrow R_{e'} \subseteq R_e)$.

Proof: Suppose eRe' and $e^* \in R_{e'}$. $e^* \in R_{e'}$ means that e'Re*. eRe' and e'Re* imply, by Postulate 5, that eRe*. Hence, $e^* \in R_e$ and $R_{e'} \subseteq R_e$. For the other direction, suppose $R_{e'} \subseteq R_e$. By Fact 2(i), e'Re' and hence $e' \in R_{e'}$, which by assumption implies $e' \in R_e$. Hence, eRe'.♥

(iii) $\forall e,e'(R_{e'} = R_e \leftrightarrow eRe' \wedge e'Re)$.

Proof: $\Rightarrow$ Suppose $R_{e'} = R_e$, that is, $\forall e^*(eRe^* \leftrightarrow e'Re^*)$. By Fact 2(i), eRe and e'Re'. Hence eRe' and e'Re. $\Leftarrow$ Suppose $eRe' \wedge e'Re$. By Fact 7(ii), eRe' implies $R_{e'} \subseteq R_e$, while e'Re implies $R_e \subseteq R_{e'}$. Hence, $R_{e'} = R_e$.♥

(iv) $\forall e,e'(eRe' \wedge \neg(ePRESe') \rightarrow \neg(e'Re)))$.

Proof: Suppose eRe' and ¬(ePRESe'). There are two cases to be considered. (1) If e'<e, than Postulate 3 implies ¬(e'Re). (2) If eELSe', then Fact 6(i) implies that there is e* such that e'<e* and ePRESe*. By Definition 3, it follows that eRe*. Now, assume that e'Re. By Postulate 5, e'Re and eRe* imply e'Re*. By Postulate 3, e'<e* implies ¬(e'Re*). Contradiction. Hence, ¬(e'Re).♥

(v) $\forall e,e'((eRe' \wedge \neg(ePRESe')) \rightarrow \exists e^*(e^* < e \wedge e'PRESe^*))$.

Proof: By Fact 7(iv) and Fact 5(ii).♥

(vi) $\forall e,e'(R_{e'} = R_e \leftrightarrow ePRESe' \wedge e'PRESe)$.

Proof: $\Rightarrow$ Let e and e' be arbitrary and assume $R_e = R_{e'}$, that is, $\forall e^*(eRe^* \leftrightarrow e'Re^*)$. I shall first prove that ePRESe'. According to Fact 1(ii), there are three cases to be considered.

Case I: e≤e'.

If e<e', then, by Postulate 3, ¬(eRe'). By Fact 2(i), e'Re'. This contradicts the assumption. If e=e', then, Fact 4(i) implies e PRES e'.

Case II: e'≤e.

Analogous to Case I.

Case III: eELSe'.

(a) Let eRe'. Suppose ¬(ePRESe'). Then, by Fact 6(i), there is e* such that e'<e* and ePRESe*. By Definition 3, ePRESe* implies eRe*. By Postulate 3, e'<e* implies ¬(e'Re*). This contradicts the assumption that $R_e = R_{e'}$. Hence, ePRESe'.

(b) Let ¬(eRe'). This contradicts the assumption, since e'Re' holds (Fact 2(i)).

Hence, ePRESe'. Showing that e'PRESe is analogous.

$\Leftarrow$ Assume ePRESe' and e'PRESe. Definition 3 implies eRe' and e'Re. This together with Fact 7(ii) imply $R_{e'} \subseteq R_e$ and $R_e \subseteq R_{e'}$. Hence, $R_{e'} = R_e$.♥

(vii) $\forall e,e'(eRe' \wedge e'Re \leftrightarrow ePRESe' \wedge e'PRESe)$

Proof: $\Rightarrow$ By Facts 7(iii) and 7(vi). $\Leftarrow$ By Definition 3.

Fact 8.

(i) $\forall e,e'(ePRESe' \leftrightarrow eRe' \wedge e'Re)$.

Proof: By Postulate 6, Defintion 3 and Fact 7(vii).

(ii) $\forall e,e'(ePRESe' \rightarrow e'PRESe)$.

Proof: Suppose ePRESe'. Fact8(i) implies e'Re and eRe'. Now, suppose $\neg$(e'PRESe). Then, by Fact 6(ii), there is e* such that e < e* and e' PRES e*. By Definition 3, e'PRESe* implies e'Re*. eRe' and e'R e* imply, by Postulate 5, that eRe*. Contradiction, since e < e* implies, by Postulate 3, that $\neg$(eRe*). Hence, e'PRESe.♥

(iii) $\forall e,e'(ePRESe' \rightarrow \forall e''(ePRESe'' \rightarrow e'PRESe''))$.

Proof: Suppose ePRESe' and ePRESe". ePRESe" implies, by Fact 4(iv), that either (I) e=e" or (II) eELSe".

(I) Suppose e=e". By Fact 8(ii), the assumption that ePRESe' implies that e'PRESe. Hence, e'PRESe".

(II) Suppose eELSe". Then, according to Fact 1(ii), there are four following cases to be considered: e"= e', e'< e", e"< e' and e'ELSe".

(a) Let e"= e'. It follows, by Fact 4(i), that e'PRESe".

(b) e'< e" cannot hold. By Definition 3, the assumption ePRESe" implies eRe", while the assumption that ePRESe', together with e'< e", implies $\neg$(eRe").

(c) e"< e' cannot hold. The assumption that ePRESe", together with e"< e' implies $\neg$(eRe'), while the assumption that ePRESe' implies eRe'.

(d) Let e'ELSe". First I prove that e'Re". Suppose $\neg$(e'Re"). By Postulate 5, $\neg$(e'Re") and eRe" imply $\neg$(e'Re). By Fact 8(ii), the assumption that ePRESe' implies that e'PRESe. This implies, by Definition 3, that e'Re. Contradiction. Hence, e'Re" holds. Now I prove that e'PRESe". Suppose $\neg$(e'PRESe"). Then, by Fact 6(i), there is e* such that e"< e* and e'PRESe*. e'PRESe* implies e'Re*. Since ePRESe', eRe' holds. eRe' and e'Re*, by Postulate 5, imply eRe*. But, the assumption that ePRESe" and e"< e* imply $\neg$(eRe*). Contradiction. Hence, e'PRESe".♥

(iv) $\forall e,e',e''(ePRESe' \wedge e'PRESe'' \rightarrow ePRESe'')$.

Proof: Assume ePRESe' and e'PRESe". By Fact 8(ii), the first assumption implies e'PRESe. By Fact 8(iii), e'PRESe and e'PRESe" imply ePRESe".♥

(v) $\forall e,e'(ePRESe' \leftrightarrow R_e = R_{e'})$.

Proof: $\Rightarrow$ By Facts 8(ii) and 7(vi). $\Leftarrow$ By Fact 7(vi).♥

(vi) $\forall e,e'(eRe' \vee e'Re)$.

Proof: Assume $\neg$(eRe'). Then, by Postulate 2 and Fact 1(ii), either (I) e<e' or (II) e ELS e'. (I) If e<e', then, by Postulate 2, e'Re. (II) Suppose eELSe' and $\neg$(e'Re). The summetry of ELS implies e'ELSe. Then, by Fact 5(i), there is e* such that e*< e and e'PRESe*. By Fact 8(ii), e'PRESe* implies e*PRESe'. e*PRESe' implies e*Re'. By Postulate 2, e*< e implies eRe*. By Postulate 5, e*Re' and eRe* imply eRe'. This contradicts the assumption that ¬(eRe'). So, eELSe' implies e'Re.♥

Fact 9.

(i) $\propto$ is irreflexive.

Proof: by Fact 4(i).♥

(ii) $\propto$ is transitive.

Proof: by Postulate 5 and Fact 8(vi).♥

(iii) For every two equivalence classes $[e_1]$ and $[e_2]$, either $[e_1] \propto [e_2]$ or $[e_2] \propto [e_1]$ or $[e_1] = [e_2]$ holds.

Proof: Assume that $[e_1] \propto [e_2]$ and $[e_1] = [e_2]$ do not hold. By Definition 6, the first assumption implies that e_1Re_2 holds. The second assumption implies that $e_1 \neq e_2$ holds. e_1Re_2, $e_1 \neq e_2$, Postulate 3 and Fact 1(ii) imply that either e_1ELSe_2, or $e_2 < e_1$ holds. If $e_2 < e_1$ is the case, then Postulate 3 implies $\neg(e_2Re_1)$. This, by Definition 6 implies $[e_2] \propto [e_1]$. Assume now that e_1ELSe_2 holds. Assume further that e_1PRESe_2 hold. This implies that that $[e_1] = [e_2]$ hold. Contradiction. Hence, $\neg(e_1PRESe_2)$. By Fact 6(i), it follows that there is e* such that e_1 PRES e* and $e_2 <$ e* hold. e_1PRESe* implies that e_1Re* holds. Suppose that e_2Re_1 holds. Then, Postulate 5 implies e_2Re*. Contradiction. Hence, $\neg(e_2Re_1)$. It follows, by Definition 6, that $[e_2] \propto [e_1]$ holds.♥

(iv) $[e_1] \propto [e_2]$ iff $R_{e_1} \subset R_{e_2}$.

Proof: By Postulate 5 and Fact 2(i).

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