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GOING STABLE IN GRAPH MODELS

Inge Bethke

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Going Stable in Graph Models

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Abstract

In this paper, a procedure is given for depleting graph models in a way such that the representable functions are exactly the stable ones.

1 Introduction

There is a class of models for the untyped λ-calculus whose members can be described in a very elegant and easy way, the class of so-called *graph models.*

All that is needed for the construction of such a graph model, is an infinite set A together with an embedding $(.,.) : A^{<\infty} \times A \rightarrow A$ of the Cartesian product of the collection of finite subsets of A and A into A. One then obtains a total application operation $\bullet$ on $\mathcal{P}(A)$ by defining for $X, Y \in \mathcal{P}(A)$

$$X \bullet Y = \{a : \exists Z \subseteq Y \ (Z, a) \in X\}.$$

The most prominent graph models are the P_ω-models due to Plotkin [1972] and Scott [1969], and the D_A-models due Engeler [1981]:
A P_ω-model is based on an embedding $(.,.) : \omega^{<\infty} \times \omega \rightarrow \omega$ which is obtained from an injective coding $p : \omega \times \omega \rightarrow \omega$ and a bijection $e : \omega^{<\infty} \rightarrow \omega$ by defining

$$(Z, n) = p(e(Z), n).$$

D_A-models are obtained from underlying sets $G(A)$ which have the property that $G(A)^{<\omega} \times G(A) \subseteq G(A)$: simply take any nonempty set A, and let $G(A)$ be the least set containing A and all ordered pairs (Z, b) consisting of a finite set $Z \subseteq G(A)$ and an element $b \in G(A)$, assuming that elements of A are distinguishable from ordered pairs.

Any graph model is a model for the untyped λ-calculus, since $(\mathcal{P}(A), \subseteq)$ is a reflexive cpo through the Scott-continuous mappings $F : \mathcal{P}(A) \rightarrow [\mathcal{P}(A) \rightarrow \mathcal{P}(A)]$ and $G : [\mathcal{P}(A) \rightarrow \mathcal{P}(A)] \rightarrow \mathcal{P}(A)$ given by

$$F(X)(Y) = X \bullet Y \text{ and } G(f) = \{(Z, a) : a \in f(Z)\}.$$

It follows that the functions representable in $\mathcal{P}(A)$ are exactly the Scott-continuous ones. For more details about graph models, the reader is referred to Barendregt [1984] and Schellinx [1991].

Recently, in connection with the semantics of classical linear logic, attention has turned to so-called *coherence spaces.* Coherence spaces are just a very special class of Scott domains which, however, come along with so-called *stable* morphisms, i.e. morphisms that are Scott-continuous and preserve in addition meets of pairs bounded above or, in category-theoretic terms, pullbacks.

As graph models are so easily conceived, the question then arises whether, given an infinite set A and an embedding $(.,.) : A^{<\omega} \times A \rightarrow A$, one can deplete $\mathcal{P}(A)$ in a way such that the representable functions are exactly the stable ones. In this paper we shall show that this is indeed the case.

The paper is organized as follows: In section 2, we collect some well-known notions and facts concerning coherence spaces. In section 3, we render precise the notion of a stable graph model and show that, although the universe may be heavily depleted, one still ends up with a model for the untyped λ-calculus. Section 4 is addressing the question of existence of stable graph models for arbitrary embeddings. Here, the key notion is the notion of a *meager* set. We describe a construction procedure for stable graph models from meager sets.

2 Coherence Spaces

To fix our terminology and notation, we shall collect in this section a few well-known notions and facts concerning coherence spaces and briefly review the theory of λ-structures obtained from them. Our exposition is based in part on Girard [1986] and Girard, Taylor and Lafont [1989].

Definition 2.1 A *coherence space* is a set (of sets) $\mathcal{A}$ which satisfies:

i) *Down-closure*: if $X \in \mathcal{A}$ and $X' \subseteq X$, then $X' \in \mathcal{A}$,

ii) *Binary completeness*: if $\mathcal{A}' \subseteq \mathcal{A}$ and if for all $X, X' \in \mathcal{A}' \,(X \cup X' \in \mathcal{A})$, then $\bigcup \mathcal{A}' \in \mathcal{A}$. □

In particular, we have the *undefined object*, $\emptyset \in \mathcal{A}$. One may therefore consider $\mathcal{A}$ as a cpo (partially ordered by inclusion), and as such it is *algebraic*, i.e. any set is the directed union of its finite subsets. So coherence spaces are a very special sort of cpos. However, they are better regarded as undirected graphs.

Elements of the set $\bigcup \mathcal{A}$ are called *atoms*. This set will also be denoted by $|\mathcal{A}|$. The *compatibility relation* between atoms is defined by

$$a \frown a' \text{ iff } \{a, a'\} \in \mathcal{A}.$$

This constitutes a reflexive symmetric relation on $|\mathcal{A}|$, so $(|\mathcal{A}|, \frown)$ is a graph, called the *web* of $\mathcal{A}$.

The construction of the web of a coherence space is a bijection between coherence spaces and (reflexive symmetric) graphs. From the web one can recover the coherence space by

$$X \in \mathcal{A} \longleftrightarrow X \subseteq |\mathcal{A}| \ \wedge \ \forall a, a' \in X \, a \supsetsim a'.$$

So a coherence space $\mathcal{A}$ is the set of all coherent subsets of $|\mathcal{A}|$.

Whereas in Scott-style domain theory the functions between domains are exactly those which preserve directed joins, this is no longer the case here.

Definition 2.2 Given two coherence space $\mathcal{A}$ and $\mathcal{B}$, a function f from $\mathcal{A}$ to $\mathcal{B}$ is *stable* if

i) if $X \subseteq X' \in \mathcal{A}$, then $f(X) \subseteq f(X')$ (monotonicity)

ii) if $\mathcal{A}'$ is a directed subset of $\mathcal{A}$, then $f(\bigcup \mathcal{A}') = \bigcup f(\mathcal{A}')$ (directed union)

iii) if $X, X', X \cup X' \in \mathcal{A}$, then $f(X \cap X') = f(X) \cap f(X')$ (stability). □

Whereas the first two conditions are entirely familiar from the topological setting, the third - the stability property itself - does not have any obvious topological significance. However, if the ordered sets $\mathcal{A}$ and $\mathcal{B}$ are considered as categories, then i) states that f is a functor, ii) that it preserves directed joins and iii) that it also preserves pullbacks.

As such, the collection of stable functions from $\mathcal{A}$ to $\mathcal{B}$ is not presented as a coherence space. However, it can be considered as belonging to this very special class of spaces. Here the crucial observation is that for a given stable function, a fixed argument and a finite portion of its value, there is a finite least part of that argument which suffices to give that value portion. Or loosely speaking, if one has some information on the output, one knows which part of the input was used to get it.

Lemma 2.3 (Normalisation Lemma) If f is a stable function from $\mathcal{A}$ to $\mathcal{B}$, $X \in \mathcal{A}$ and $b \in f(X)$, then there is a finite $Z \subseteq X$ such that

$$b \in f(Z) \ \wedge \ \forall Y \subseteq X (b \in f(Y) \longrightarrow Z \subseteq Y).$$

PROOF. See e.g. Girard [1986]. □

Since a stable function f from $\mathcal{A}$ to $\mathcal{B}$ is determined by its values on least finite sets, f has a unique graph representation, called *trace.* This gives a bijection $\mathcal{T}$ between the set of stable functions and the coherence space of traces with an obvious inverse $\mathcal{F}$ which maps traces onto stable functions.

Theorem 2.4 (Representation Theorem) Let $\mathcal{A}$ and $\mathcal{B}$ be coherence spaces and $\mathcal{A}_{fin}$ be the set of finite sets in $\mathcal{A}$.

i) Define a compatibility relation on $\mathcal{A}_{fin} \times |\mathcal{B}|$ by
$< Z, b > \asymp < Z', b' >$ iff

1. $Z \cup Z' \in \mathcal{A} \longrightarrow b \asymp b'$,
2. $Z \cup Z' \in \mathcal{A} \wedge b = b' \longrightarrow Z = Z'$.

Moreover, let $[\mathcal{A} \rightarrow_s \mathcal{B}]$ be the set defined by

$$X \in [\mathcal{A} \rightarrow_s \mathcal{B}] \longleftrightarrow X \subseteq \mathcal{A}_{fin} \times |\mathcal{B}| \ \wedge \ \forall x, x' \in X \ x \asymp x'.$$

Then $[\mathcal{A} \rightarrow_s \mathcal{B}]$ is a coherence space.

ii) Let f be a stable function from $\mathcal{A}$ to $\mathcal{B}$. Define the *trace* of f, $\mathcal{T}(f)$, by

$$\mathcal{T}(f) = \{< Z, b > \in \mathcal{A}_{fin} \times |\mathcal{B}| \ : \ b \in f(Z) \wedge \forall Z' \subseteq Z (b \in f(Z') \longrightarrow Z' = Z)\}.$$

Then $\mathcal{T}(f) \in [\mathcal{A} \rightarrow_s \mathcal{B}]$.

iii) Let $X \in [\mathcal{A} \rightarrow_s \mathcal{B}]$. For $Y \in \mathcal{A}$, define $\mathcal{F}(X)(Y)$ by

$$\mathcal{F}(X)(Y) = \{b \in |\mathcal{B}| \ : \ \exists Z \subseteq Y \ \ < Z, b > \in X\}.$$

Then $\mathcal{F}(X)$ is a stable function from $\mathcal{A}$ to $\mathcal{B}$.

iv) $\mathcal{T}$ and $\mathcal{F}$ are mutually inverse constructions, i.e. for all stable functions f from $\mathcal{A}$ to $\mathcal{B}$ and all $X \in [\mathcal{A} \rightarrow_s \mathcal{B}]$ one has $f = \mathcal{F}(\mathcal{T}(f))$ and $X = \mathcal{T}(\mathcal{F}(X))$.

PROOF. See e.g. Girard [1986]. □

Coherence spaces can be used to give a semantics to the untyped λ-calculus. Here one can proceed in the same way as in the case of reflexive complete lattices or reflexive complete partial orders, that is

Definition 2.5 Let $\mathcal{A}$ be a coherence space.

i) $\mathcal{A}$ is *reflexive* if $[\mathcal{A} \rightarrow_s \mathcal{A}]$ is a retract of $\mathcal{A}$, i.e. there are stable functions

$$F : \mathcal{A} \rightarrow [\mathcal{A} \rightarrow_s \mathcal{A}]$$
$$G : [\mathcal{A} \rightarrow_s \mathcal{A}] \rightarrow \mathcal{A}$$

such that $F \circ G = id_{[\mathcal{A} \rightarrow_s \mathcal{A}]}$.

ii) Let $\mathcal{A}$ be reflexive via the maps F and G.

1. For $X, Y \in \mathcal{A}$, define

$$X * Y = \{a \in |\mathcal{A}| \ : \ \exists Z \subseteq Y \ \ < Z, a > \in F(X)\}.$$

2. Let ρ be a valuation in $\mathcal{A}$. Define the interpretation $[\![\ ]\!]_\rho : \Lambda \to \mathcal{A}$ by induction as follows

$$[\![x]\!]_\rho = \rho(x)$$

$$[\![MN]\!]_\rho = [\![M]\!]_\rho * [\![N]\!]_\rho$$

$$[\![\lambda x.M]\!]_\rho = G(\mathcal{T}(\lambda X \in \mathcal{A}.[\![M]\!]_{\rho(x:=X)})). \ \square$$

Checking that $[\![\ ]\!]_\rho$ is well-defined is a boring but straightforward exercise. For this and the following theorem we refer the reader to e.g. Girard [1986].

Theorem 2.6 Let $\mathcal{A}$ be a reflexive coherence space via F, G and let $\mathcal{M} = (\mathcal{A}, *, [\![\ ]\!])$. Then

i) $\mathcal{M}$ is a λ-model.

ii) $\mathcal{M}$ is extensional iff $G \circ F = id_{\mathcal{A}}$. $\square$

In the next section we shall show that stable graph models arise as reflexive coherence spaces.

3 Stable Graph Models

Let us first make precise what we mean by a stable graph model.

Definition 3.1 Let A be an infinite set and $(.,.) : A^{<\omega} \times A \to A$ be an embedding of the Cartesian product of the collection of finite subsets of A and A into A. For $X, Y \in \mathcal{P}(A)$, define

$$X \bullet Y = \{a \in A : \exists Z \subseteq Y \ (Z, a) \in X\}.$$

Then $\mathcal{A} \subseteq \mathcal{P}(A)$ is a *stable graph model* if

(i) $\mathcal{A}$ is a coherence space,

(ii) $\mathcal{A}$ is closed under application, i.e.

$$\forall X, Y \in \mathcal{A} \ \ X \bullet Y \in \mathcal{A},$$

(iii) for all $X, X', Y, Y' \in \mathcal{A}$, if $X \cup X', Y \cup Y' \in \mathcal{A}$, then

$$(X \cap X') \bullet (Y \cap Y') = X \bullet Y \cap X' \bullet Y',$$

(iv) every stable function from $\mathcal{A}$ to $\mathcal{A}$ is representable in $\mathcal{A}$, i.e. if f is a stable function from $\mathcal{A}$ to $\mathcal{A}$, then

$$\exists X \in \mathcal{A} \forall Y \in \mathcal{A} \ \ X \bullet Y = f(Y). \ \square$$

Note that this definition allows for stable graph models $\mathcal{A}$ the atom set $|\mathcal{A}|$ of which may be properly included in the carrier set A: clearly $|\mathcal{A}| \subseteq A$, the reverse inclusion, however, does not follow from the above definition. What does follow from the definition is that, contrary to ordinary graph models, $\mathcal{A} \neq \mathcal{P}(A)$ (see below).

Any stable graph model may in a natural way be regarded as a reflexive coherence space such that both the canonical application operations, $*$ and $\bullet$, coincide. It follows that any stable graph model is in fact a λ-model.

Lemma 3.2 Let $\mathcal{A}$ be a stable graph model.

(i) For $(Z,a),(Z',a) \in |\mathcal{A}|$, if $(Z,a) \frown\!\!\!\!\smile (Z',a')$, then

1. $Z \cup Z' \in \mathcal{A} \longrightarrow a \frown\!\!\!\!\smile a'$,
2. $Z \cup Z' \in \mathcal{A} \ \wedge \ a = a' \longrightarrow Z = Z'$.

(ii) For $X \in \mathcal{A}$, $\{< Z,a >: (Z,a) \in X\} \cap (\mathcal{A}_{fin} \times |\mathcal{A}|) \in [\mathcal{A} \rightarrow_s \mathcal{A}]$.

(iii) For $X \in [\mathcal{A} \rightarrow_s \mathcal{A}]$, $\{(Z,a) :< Z,a > \in X\} \in \mathcal{A}$.

(iv) $\{(Z,a) :< Z,a > \in \mathcal{A}_{fin} \times |\mathcal{A}|\} \subseteq |\mathcal{A}|$

(v) $\mathcal{A} \neq \mathcal{P}(A)$

PROOF. (i) Let $(Z,a),(Z',a')$ be compatible and suppose that $Z \cup Z' \in \mathcal{A}$.
1. Then $\{(Z,a),(Z',a')\} \in \mathcal{A}$ and $\{(Z,a),(Z',a')\} \bullet (Z \cup Z') = \{a,a'\} \in \mathcal{A}$. Hence $a \frown\!\!\!\!\smile a'$.
2. Suppose that $a = a'$. From 3.1.(iii) it follows that

$$\{a\} = \{(Z,a)\} \bullet (Z \cup Z') \cap \{(Z',a)\} \bullet (Z \cup Z') = (\{(Z,a)\} \cap \{(Z',a)\}) \bullet (Z \cup Z').$$

Thus $\{(Z,a)\} \cap \{(Z',a)\} \neq \emptyset$; hence $(Z,a) = (Z',a)$; therefore $Z = Z'$, since $(.,.)$ is injective.
(ii) Clearly, $\{< Z,a >: (Z,a) \in X\} \cap (\mathcal{A}_{fin} \times |\mathcal{A}|) \subseteq \mathcal{A}_{fin} \times |\mathcal{A}|$, and from (i) it follows that this set is coherent.
(iii) Let $X \in [\mathcal{A} \rightarrow_s \mathcal{A}]$. Then $X = \mathcal{T}(f)$, for some stable f. Pick $X_f \in \mathcal{A}$ representing f. We shall show that $\{(Z,a) :< Z,a > \in X\} \subseteq X_f$.
Let $(Z,a) \in \{(Z,a) :< Z,a > \in X\}$. Then, by 2.4(ii)

$$(\dagger) \quad a \in f(Z)$$

and

$$(\ddagger) \quad \forall Z' \subseteq Z\, (a \in f(Z') \longrightarrow Z' = Z).$$

From $(\dagger)$ it follows that $a \in X_f \bullet Z$. Thus $(Z',a) \in X_f$, for some $Z' \subseteq Z$. But then $a \in X_f \bullet Z' = f(Z')$. So $Z = Z'$ by $(\ddagger)$.
(iv) Let $< Z,a > \in \mathcal{A}_{fin} \times |\mathcal{A}|$. Then $\{< Z,a >\} \in [\mathcal{A} \rightarrow_s \mathcal{A}]$. Thus $\{(Z,a)\} \in \mathcal{A}$, by (iii).

(v) Suppose the $\mathcal{A} = \mathcal{P}(A)$. Pick $a \in A$ and put $Z = \{(\emptyset, a), (\{a\}, a)\}$. Then $Z \in \mathcal{A}$. Hence $\{<\emptyset, a>, <\{a\}, a>\} \in [\mathcal{A} \to_s \mathcal{A}]$ by (ii). But $\{<\emptyset, a>, <\{a\}, a>\}$ is incoherent by 2.4(i)2. □

Theorem 3.3 Let $\mathcal{A}$ be a stable graph model and ρ be a valuation in $\mathcal{A}$. Define the interpretation $[\![\ ]\!]_\rho : \Lambda \to \mathcal{A}$ by induction as follows.

$$[\![x]\!]_\rho = \rho(x)$$

$$[\![MN]\!]_\rho = [\![M]\!]_\rho \bullet [\![N]\!]_\rho$$

$$[\![\lambda x.M]\!]_\rho = \{(Z, a) : < Z, a > \in \mathcal{T}(\lambda X \in \mathcal{A}.[\![M]\!]_{\rho(x:=X)})\}.$$

Then $\mathcal{M} = (\mathcal{A}, \bullet, [\![\ ]\!])$ is a λ-model and the functions representable in $\mathcal{M}$ are exactly the stable ones. Moreover,

$$\mathcal{M} \text{ is extensional iff } |\mathcal{A}| = \{(Z, a) : < Z, a > \in \mathcal{A}_{fin} \times |\mathcal{A}|\}.$$

PROOF. For $X \in \mathcal{A}$ and $Y \in [\mathcal{A} \to_s \mathcal{A}]$, define

$$F(X) = \{< Z, a > : (Z, a) \in X\} \cap (\mathcal{A}_{fin} \times |\mathcal{A}|)$$

$$G(Y) = \{(Z, a) : < Z, a > \in Y\}.$$

From the preceding lemma it follows that $F : \mathcal{A} \to [\mathcal{A} \to_s \mathcal{A}]$ and $G : [\mathcal{A} \to_s \mathcal{A}] \to \mathcal{A}$. Clearly, F and G are both stable and $F \circ G = id_{[\mathcal{A} \to_s \mathcal{A}]}$. Hence $\mathcal{A}$ is reflexive via F and G. Moreover, observe that for $X, Y \in \mathcal{A}$ one has that

$$X \bullet Y = \{a \in |\mathcal{A}| : \exists Z \subseteq Y ((Z, a) \in X \wedge < Z, a > \in \mathcal{A}_{fin} \times |\mathcal{A}|)\},$$

since $\mathcal{A}$ is downwards closed and closed under $\bullet$. Thus

$$X \bullet Y = \{a \in |\mathcal{A}| : \exists Z \subseteq Y \ < Z, a > \in F(X)\} = X * Y.$$

It therefore follows from theorem 2.6 that $\mathcal{M}$ is a λ-model.
Clearly, $\bullet$ is monotone and continuous in its second argument, and meets, by 3.1.(iii), the stability condition. This means that every representable function is stable. The converse is given by 3.1(iv). So the functions representable in $\mathcal{M}$ are exactly the stable ones.
Finally, observe that

$\mathcal{M}$ is extensional $\longrightarrow G \circ F = id_{\mathcal{A}}$, by theorem 2.6(ii)
$\longrightarrow |\mathcal{A}| \subseteq \{(Z, a) : < Z, a > \in \mathcal{A}_{fin} \times |\mathcal{A}|\}$
$\longrightarrow |\mathcal{A}| = \{(Z, a) : < Z, a > \in \mathcal{A}_{fin} \times |\mathcal{A}|\}$, by 3.2(iv)
$\longrightarrow G \circ F = id_{\mathcal{A}}$
$\longrightarrow \mathcal{M}$ is extensional, by theorem 2.6(ii). □

Corollary 3.4 Let $\mathcal{A}$ be an extensional stable graph model. Then $|\mathcal{A}| \neq A$.

PROOF. Let $\mathcal{A}$ be extensional and suppose that $|\mathcal{A}| = A$. Pick $a \in A$ and put $Z = \{(\emptyset, a), (\{a\}, a)\}$. Then $(Z, a) \in A$, hence $(Z, a) \in |\mathcal{A}|$, and therefore, by theorem 3.3, $Z \in \mathcal{A}_{fin}$. Thus $Z, \{(\emptyset, a)\} \in \mathcal{A}$, $Z \bullet X = \{(\emptyset, a)\} \bullet X$, for all $X \in \mathcal{A}$, but $Z \neq \{(\emptyset, a)\}$. So $\mathcal{A}$ is not extensional. Contradiction. □

In the next section we shall address the question whether, given an embedding $(.,.) : A^{<\omega} \times A \to A$, there always exists a stable graph model $\mathcal{A} \subseteq \mathcal{P}(A)$.

4 Existence of Stable Graph Models

In this section we shall propose a procedure to produce nontrivial stable graph models for arbitrary nonsurjective and rather 'well-behaved' surjective embeddings.

Let us start with fixing an embedding $(.,.) : A^{<\omega} \times A \to A$.

Definition 4.1

(i) Let $A' \subseteq A$. A' is *meager*[1] if

$$\forall (Z, a) \in A'(\, Z = \emptyset \;\wedge\; a \in A'\,).$$

(ii) Let $A' \subseteq A$ be meager. Define $A_n \subseteq A$ recursively by

$$A_0 = A'$$

$$\Gamma_0(X) \longleftrightarrow X \subseteq A_0$$

$$A_{n+1} = A_n \cup \{(Z, a) \in A : \Gamma_n(Z) \;\wedge\; \Gamma_n(\{a\})\}$$

$$\Gamma_{n+1}(X) \longleftrightarrow X \subseteq A_{n+1} \;\wedge$$

1. $\forall Y(\,\Gamma_n(Y) \longrightarrow \Gamma_n(X \bullet Y)\,)$
2. $\forall (Z, a), (Z', a) \in X(\,\Gamma_n(Z \cup Z') \longrightarrow Z = Z'\,)$

(iii) Let $A' \subseteq A$ be meager. Define $\mathcal{A}_{A'} \subseteq \mathcal{P}(A)$ by

$$X \in \mathcal{A}_{A'} \longleftrightarrow \forall Z \in \mathcal{P}(X) \cap A^{<\omega}\, \exists n\, \Gamma_n(Z) \quad □$$

We shall prove that $\mathcal{A}_{A'}$ is indeed a stable graph model. First observe that

Lemma 4.2 For $n \in \omega$,

(i) $\forall X \forall X'(\,\Gamma_n(X) \;\wedge\; X' \subseteq X \longrightarrow \Gamma_n(X')\,)$

(ii) $\forall X(\,\forall a, a' \in X\; \Gamma_n(\{a, a'\}) \longrightarrow \Gamma_n(X)\,)$

[1] It should be noted that the notion of a meager set is not taken from set theory, but follows Barendregt and Longo [1980].

PROOF. By induction on n.
(i) The base case is trivial. For the induction step let $X' \subseteq X$ be such that $\Gamma_{n+1}(X)$. To prove $\Gamma_{n+1}(X')$, let Y be such that $\Gamma_n(Y)$. Then $\Gamma_n(X \bullet Y)$, and as $X' \bullet Y \subseteq X \bullet Y$, it follows from the induction hypothesis that $\Gamma_n(X' \bullet Y)$. The second condition is trivial.
(ii) The base case is trivial. For the induction step let X be such that $\Gamma_{n+1}(\{a, a'\})$, for all $a, a' \in X$. To prove $\Gamma_{n+1}(X)$, let Y be such that $\Gamma_n(Y)$ and let $b, b' \in X \bullet Y$. Then $\Gamma_{n+1}(\{(Z,b),(Z',b')\})$, for some $(Z,b),(Z',b') \in X$ such that $Z, Z' \subseteq Y$. Thus $\Gamma_n(\{(Z,b),(Z',b')\} \bullet Y)$, i.e. $\Gamma_n(\{b,b'\})$. It follows from the induction hypothesis that $\Gamma_n(X \bullet Y)$. The second condition is immediate.
□

Lemma 4.3 For $n \in \omega$,

(i) $\forall Z \forall a((Z,a) \in A_{n+1} \longrightarrow \Gamma_n(Z) \ \wedge \ \Gamma_n(\{a\}))$

(ii) $\forall m \forall X(\Gamma_n(X) \longrightarrow \Gamma_{n+m}(X))$

(iii) $\forall m \forall X \forall X'(\Gamma_n(X) \ \wedge \ \Gamma_n(X') \ \wedge \ \Gamma_{n+m}(X \cup X') \longrightarrow \Gamma_n(X \cup X'))$

PROOF. By simultaneous induction on n.
For $n = 0$, the truth of (iii) is immediate.
$(\text{i})_0$ This is trivial for $(Z,a) \in A_1 \backslash A_0$. And if $(Z,a) \in A_0$, then $Z = \emptyset \subseteq A_0$ and $a \in A_0$, since A_0 is meager.
For $(\text{ii})_0$ we have to show that $\Gamma_m(X)$ holds for all m and $X \subseteq A_0$. Clearly, $\Gamma_0(X)$ if $X \subseteq A_0$. To prove $\Gamma_{m+1}(X)$, let Y be such that $\Gamma_m(Y)$. Then, as $X \subseteq A_0$ and A_0 is meager, it follows that $X \bullet Y \subseteq A_0$. Hence by the induction hypothesis $\Gamma_m(X \bullet Y)$. The second condition follows from the fact that $Z = \emptyset$, for all $(Z,a) \in X$.
For the induction step assume that $(\text{i})_n$-$(\text{iii})_n$ hold.
$(\text{i})_{n+1}$ Let $(Z,a) \in A_{n+2}$. If $(Z,a) \in A_{n+2} \backslash A_{n+1}$, then $\Gamma_{n+1}(Z)$ and $\Gamma_{n+1}(\{a\})$. If $(Z,a) \in A_{n+1}$, then $\Gamma_n(Z)$ and $\Gamma_n(\{a\})$, by $(\text{i})_n$. Therefore $\Gamma_{n+1}(Z)$ and $\Gamma_{n+1}(\{a\})$, by $(\text{ii})_n$.
We prove $(\text{ii})_{n+1}$ by induction on m. For $m = 0$, this is immediate. For the induction step let X be such that $\Gamma_{n+1}(X)$. Then $X \subseteq A_{n+1} \subseteq A_{n+1+m+1}$. Now let Y be such that $\Gamma_{n+1+m}(Y)$. Put $Y' = \bigcup\{Z : \exists a((Z,a) \in X \ \wedge \ Z \subseteq Y)\}$. Then $X \bullet Y = X \bullet Y'$.

Claim. $\forall a, a' \in Y' \ \Gamma_n(\{a,a'\})$: Let $a, a' \in Y'$. Then $a \in Z, a' \in Z'$, for some $(Z,b),(Z',b') \in X$. Since $X \subseteq A_{n+1}$, it follows from $(\text{i})_n$ that $\Gamma_n(Z)$ and $\Gamma_n(Z')$. Moreover, $Z \cup Z' \subseteq Y$, and hence by 4.2(i) $\Gamma_{n+1+m}(Z \cup Z')$. Whence $\Gamma_n(Z \cup Z')$ by $(\text{iii})_n$; therefore $\Gamma_n(\{a,a'\})$, again by 4.2(i).
It now follows from the claim and 4.2(ii) that $\Gamma_n(Y')$. Whence $\Gamma_n(X \bullet Y')$, and so $\Gamma_n(X \bullet Y)$. Therefore $\Gamma_{n+m+1}(X \bullet Y)$ by $(\text{ii})_n$.
For the second condition let $(Z,a),(Z',a) \in X$ and assume that $\Gamma_{n+1+m}(Z \cup Z')$. Then $\Gamma_n(Z \cup Z')$ by $(\text{i})_n$ and $(\text{iii})_n$. Thus $Z = Z'$.
$(\text{iii})_{n+1}$ Let X, X' be such that $\Gamma_{n+1}(X)$, $\Gamma_{n+1}(X')$ and $\Gamma_{n+1+m}(X \cup X')$. To

prove $\Gamma_{n+1}(X \cup X')$, let Y be such that $\Gamma_n(Y)$. Then $\Gamma_n(X \bullet Y)$, $\Gamma_n(X' \bullet Y)$. Put $Y' = \bigcup\{Z : \exists a(\,(Z,a) \in X \cup X' \ \wedge \ Z \subseteq Y\,)\}$. Then $Y' \subseteq Y$ and therefore $\Gamma_n(Y')$ by 4.2(i). Thus $\Gamma_{n+m}(Y')$ by (ii)$_n$. Whence $\Gamma_{n+m}((X \cup X') \bullet Y')$. Now observe that

$$(X \cup X') \bullet Y' = (X \cup X') \bullet Y = X \bullet Y \cup X' \bullet Y.$$

Hence $\Gamma_n(X \bullet Y \cup X' \bullet Y)$ by the induction hypothesis, i.e. $\Gamma_n((X \cup X') \bullet Y)$. For the second condition let $(Z,a),(Z',a') \in X \cup X'$ be such that $\Gamma_n(Z \cup Z')$. Then $\Gamma_{n+m}(Z \cup Z')$ by (ii)$_n$. Therefore $Z = Z'$. □

We are now in the position to prove

Theorem 4.4 Let $A' \subseteq A$ be meager. Then $\mathcal{A}_{A'}$ is a stable graph model. Moreover, $\mathcal{A}_{A'}$ is extensional provided $(.,.)$ is surjective.

PROOF. In order to prove that $\mathcal{A}_{A'}$ is a stable graph model, we shall check the conditions 3.1(i)-(iv), i.e.
$\mathcal{A}_{A'}$ is a coherence space: Clearly, $\mathcal{A}_{A'}$ is downwards closed. For binary completeness let $\mathcal{A}' \subseteq \mathcal{A}_{A'}$ be such that $X \cup X' \in \mathcal{A}_{A'}$, for all $X, X' \in \mathcal{A}'$. To prove that $\bigcup \mathcal{A}' \in \mathcal{A}_{A'}$, let $Z \subseteq \bigcup \mathcal{A}'$ be finite. Now let $a, a' \in Z$. Then $a \in X$, $a' \in X'$, for some $X, X' \in \mathcal{A}'$. Thus $\{a, a'\} \subseteq X \cup X' \in \mathcal{A}_{A'}$. Hence $\Gamma_n(\{a, a'\})$, for some n. Combining 4.3(ii) and the finiteness of Z yields the existence of some m such that for all $a, a' \in Z$ $\Gamma_m(\{a, a'\})$. Therefore $\Gamma_m(Z)$ by 4.2(i).
$\mathcal{A}_{A'}$ is closed under $\bullet$: Let $X, Y \in \mathcal{A}_{A'}$ and $Z \subseteq X \bullet Y$. Pick finite $Z' \subseteq X$ and $Z'' \subseteq Y$ such that $Z = Z' \bullet Z''$. Then $\Gamma_n(Z')$, $\Gamma_m(Z'')$, for some n, m. Let $k = \max\{n, m\}$. Then $\Gamma_{k+1}(Z')$, $\Gamma_k(Z'')$ by 4.3(ii). Therefore $\Gamma_k(Z' \bullet Z'')$, i.e. $\Gamma_k(Z)$.
For all $X, X', Y, Y' \in \mathcal{A}$, if $X \cup X', Y \cup Y' \in \mathcal{A}$, then $(X \cap X') \bullet (Y \cap Y') = X \bullet Y \cap X' \bullet Y'$: Since $\bullet$ is monotone in both its arguments, it follows that

$$(X \cap X') \bullet (Y \cap Y') \subseteq X \bullet Y \cap X' \bullet Y',$$

for all $X, X', Y, Y' \in \mathcal{A}_{A'}$. For the reverse assume that $X \cup X', Y \cup Y' \in \mathcal{A}_{A'}$ and let $a \in X \bullet Y \cap X' \bullet Y'$. Then there are $(Z,a) \in X, (Z',a) \in X'$ such that $Z \subseteq Y$, $Z' \subseteq Y'$. Let n, m be such that $\Gamma_n(\{(Z,a),(Z',a)\})$, $\Gamma_m(Z \cup Z')$ and put $k = \max\{n, m\}$. Then $\Gamma_{k+1}(\{(Z,a),(Z',a)\})$ and $\Gamma_k(Z \cup Z')$ by 4.3(ii). Therefore $Z = Z'$. Hence $(Z,a) \in X \cap X', Z \subseteq Y \cap Y'$; therefore $a \in (X \cap X') \bullet (Y \cap Y')$.
Every stable function from $\mathcal{A}_{A'}$ to $\mathcal{A}_{A'}$ is representable in $\mathcal{A}_{A'}$: First observe that by 4.3(ii), if $< Z, a > \in (\mathcal{A}_{A'})_{fin} \times |\mathcal{A}_{A'}|$, then there is an n such that $(Z,a) \in A_n$. Now let f be a stable function from $\mathcal{A}_{A'}$ to $\mathcal{A}_{A'}$ and put $X = \{(Z,a) :$
$< Z, a > \in \mathcal{T}(f)\}$. Let $Z \subseteq X$ be finite. Then $Z \subseteq A_{n+1}$, for some n. To prove $\Gamma_{n+1}(Z)$, let Y be such that $\Gamma_n(Y)$ and $a, a' \in Z \bullet Y$. Then there are $Z', Z'' \subseteq Y$ such that $(Z',a),(Z'',a') \in Z$. By 4.2(i) $Z' \cup Z'' \in \mathcal{A}_{A'}$, and

$< Z', a >, < Z'', a' > \in \mathcal{T}(f)$. Hence $\{a, a'\} \in \mathcal{A}_{A'}$, since $\mathcal{T}(f)$ is coherent. So $\Gamma_{n+m}(\{a, a'\})$, for some m. But as $Z \subseteq A_{n+1}$, it follows from 4.3(i) that $\Gamma_n(\{a\})$, $\Gamma_n(\{a'\})$. Therefore $\Gamma_n(\{a, a'\})$. Whence $\Gamma_n(Z \bullet Y)$ by 4.2(ii). The second condition follows almost directly from the fact that $\mathcal{T}(f)$ is coherent. So $X \in \mathcal{A}_{A'}$. Clearly, X represents f.

Finally suppose that $(.,.)$ is surjective. Then $|\mathcal{A}_{A'}| \subseteq \{(Z, a) :< Z, a > \in \mathcal{A}_{A'_{fin}} \times |\mathcal{A}_{A'}|\}$, and, since $\mathcal{A}_{A'}$ is a stable graph model, the reverse inclusion follows from 3.2(iv). Hence $\mathcal{A}_{A'}$ is extensional by theorem 3.3. □

Every embedding comes along with a meager set, namely $\emptyset$. However, $\mathcal{A}_\emptyset = \{\emptyset\}$, the trivial stable graph model. In order to obtain nontrivial stable graph models, one has to look for nonempty meager sets.

Corollary 4.5 If $(.,.)$ is nonsurjective, then $(.,.)$ has a nontrivial stable graph model.

PROOF. Let $A' = \{a : \neg\exists Z, a' \ (Z, a') = a\}$. Then $A' \neq \emptyset$. And, since $\Gamma_0(A')$ holds, we can conclude that $A' \in \mathcal{A}_{A'}$. Therefore $\mathcal{A}_{A'} \neq \{\emptyset\}$. □

We shall end this section with an example of a nontrivial extensional stable graph model.

Example 4.6 This example lives in P_ω with the following standard coding: For $n, m \in \omega$, define

$$p(n, m) = \frac{1}{2}(n + m)(n + m + 1).$$

For $Z \in \omega^{<\infty}$, define

$$e(Z) = n \longleftrightarrow Z = \{k_0, ..., k_{m-1}\} \text{ with } k_0 < k_1 ... < k_{m-1} \ \wedge \ n = \Sigma_{i<m} 2^{k_i}.$$

Now define $(.,.) : \omega^{<\infty} \times \omega \to \omega$ by

$$(Z, n) = p(e(Z), n).$$

Then $(.,.)$ is a bijection with $(\emptyset, 0) = 0$. Hence $\{0\}$ is meager. □

References

[] H.P. Barendregt, **The Lambda Calculus. Its Syntax and Semantics**, 2^{nd} edition, North-Holland, Amsterdam, 1984.

[] H.P. Barendregt, G. Longo, *Equality of λ-terms in the model T^ω*, in: Hindley, J.R. and Seldin, J.P. (eds.), **To H.B. Curry: Essays on Combinatory Logic, Lambda-Calculus and Formalism**, Academic Press, New York, 1980, pp. 303-339.

[] J-Y. Girard, *The system F of variable types, fifteen years later*, **Theoretical Computer Science**, vol. 45 (1986), pp. 159-192.

[] J-Y. Girard, P. Taylor, Y. Lafont, **Proofs and Types**, Cambridge University Press, Cambridge, 1989.

[] E. Engeler, *Algebras and combinators*, **Algebra Universalis**, Vol. 13 (1981), pp. 389-392.

[] G. Plotkin, **A set-theoretical definition of application**, Memo MIP-R-95, School of AI, Edinburgh, 1972, 32 pp.

[] H. Schellinx, *Isomorphisms and Nonisomorphisms of Graph Models*, **JSL**, vol. 56 (1991), pp. 227-249.

[] D. S. Scott, **Models for the λ-calculus**, Manuscript, 1969, 53 pp.

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