Institute for Logic, Language and Computation

MODELS OF THE UNTYPED λ-CALCULUS IN SEMI CARTESIAN CLOSED CATEGORIES

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Models of the untyped λ-calculus in semi cartesian closed categories

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Abstract

We show sound- and completeness of two classes of category-theoretical models for the untyped $\lambda\beta$-calculus, classes that are distinguished from (but closely related to) the traditional class of categorical models consisting of reflexive objects in cartesian closed categories.

1 Introduction

In this note we introduce two classes of category-theoretical models for the untyped lambda calculus and show their sound- and completeness with respect to the $\lambda\beta$-calculus (i.e. untyped lambda calculus without η-rule).

Recall that the traditional class of categorical models for the untyped lambda-calculus is that of *reflexive objects in cartesian closed categories*, where an object is called reflexive iff it has its own function-space as retract. Sound- and completeness of the $\lambda\beta$-calculus for this class are well-known (see e.g. Koymans(1984)).

Here we will relax the requirements imposed by this class, and work in categories having only a *semi cartesian closed structure.* Intuitively, semi cartesian closed categories can be seen as non-extensional versions of cartesian closed categories, i.e. the η-rule need not hold and pairing need not be surjective. Generalizing the notion of reflexive object to this new context (section 2) is somewhat subtle: in semi cartesian closed categories reflexive objects need not have their own (semi-)exponent as retract, although an object with that property always gives rise to a reflexive object (but in general the reverse does not hold).

The obvious interpretation of the $\lambda\beta$-calculus on reflexive objects in semi cartesian closed categories is sound, as will be shown in section 3, using an easy

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adaptation of the argument in Barendregt(1984). Completeness is immediate, as this class contains the traditional one as a subclass.

In Hoofman and Schellinx(1991) we considered the class of *iso-objects in weak cartesian closed categories*, where a weak cartesian closed category is a semi cartesian closed one, but *with* surjective pairing, and an iso-object is an object isomorphic to its own exponent. As we argued there, this class provides us with a *fully uniform* definition of models for the untyped lambda calculus: just like models of the *extensional* $\lambda\beta\eta$-calculus are iso-objects in cartesian closed categories (see e.g. Lambek and Scott(1987); they "live in an *extensional* environment"), we take models of the *non-extensional* $\lambda\beta$-calculus to be iso-objects living in a non-extensional environment, viz., in a weak cartesian closed category.

The soundness of the interpretation of the $\lambda\beta$-calculus on objects in this class follows from the fact that they have their own semi-exponent as retract, and thus give rise to *reflexive* objects. For completeness we have to do just a little bit more work: we show that each reflexive object yields an iso-object in a weak cartesian closed category on which the interpretation of the $\lambda\beta$-calculus is identical to that on the original one. In fact, we will show completeness with respect to the class of so-called *mighty* objects in weak cartesian closed categories, objects that in fact are *identical* to their own (semi-)exponent (section 4). Thus we illustrate that ultimately this paper should be seen as an exercise in *"the shifting of one's point of view"*, the opportunity for which is provided by the existence of several non-isomorphic semi(weak) cartesian closed structures on a given category.

The results of this note extend Martini(1992), which shows soundness of $\lambda\beta$ for the class of objects in weak cartesian closed categories that have their own exponent as retract.

2 Reflexive, iso- and mighty objects

We recall the notion of semi cartesian closed category (see Hayashi(1985) and Hoofman(1993)) and introduce the concept of *reflexive object* in categories that are semi cartesian closed.

A *semi-terminal object* in a category C is an object $\mathbf{1}$ in C together with arrows $t_A : A \to \mathbf{1}$ (for all objects A in C) such that $t_B \circ f = t_A$, for all $f : A \to B$ (note that $\mathbf{1}$ is a terminal object iff $t_{\mathbf{1}} = id$).

A *semi-product* of objects A, B in C is an object $A \times B$ together with arrows $\pi : A \times B \to A$, $\pi' : A \times B \to B$, $< f, g >: C \to A \times B$ (for all $f : C \to A, g : C \to B$) such that

1. $\pi \circ < f, g > = f$
2. $\pi' \circ < f, g > = g$

3. $< f, g > \circ h = < f \circ h, g \circ h >$.

(so $A \times B$ is a product of A, B iff $< \pi, \pi' > = id$.)

Let C be a category with a semi-terminal object **1** and semi-products $A \times B$. A *semi-exponent* of objects A, B in C is an object $A \Rightarrow B$ in C together with arrows $\mathbf{ev} : (A \Rightarrow B) \times A \to B$ and $\mathbf{cur}(f) : C \to (A \Rightarrow B)$ (for all $f : C \times A \to B$) such that

1. $\mathbf{ev} \circ < \mathbf{cur}(f) \circ g, h > = f \circ < g, h >$

2. $\mathbf{cur}(f \circ < g \circ \pi, \pi' >) = \mathbf{cur}(f) \circ g$.

(A semi-exponent is an exponent iff it also satisfies $\mathbf{cur}(\mathbf{ev}) = id$.)

A *semi cartesian closed category (semi-CCC)* C then is a category with a semi-terminal object, semi-products and semi-exponents. A *weak cartesian closed category (wCCC)* is a semi-CCC in which the semi-terminal object and the semi-products are in fact a terminal object and products. A *cartesian closed category (CCC)* is a wCCC in which the semi-exponents are exponents.

2.1. EXAMPLE. Let Pow denote the category with as objects powersets $\mathcal{P}(A)$ and as arrows continuous (i.e. directed lub[1] preserving) functions. The category Pow has finite products: $\mathcal{P}(\emptyset)$ is terminal and $\mathcal{P}(A) \times \mathcal{P}(B) := \mathcal{P}(A \uplus B)$ (where $A \uplus B$ stands for the disjoint union of A and B). Furthermore, semi-exponents can be defined on Pow by $(\mathcal{P}(A) \Rightarrow \mathcal{P}(B)) := \mathcal{P}(A^{<\omega} \times B)$ (with $A^{<\omega}$ the set of all *finite* subsets of A), $\mathbf{ev}(F, x) = \{b | \exists (X, b) \in F.X \subseteq x\}$, $\mathbf{cur}(f)(z) = \{(X, b) | b \in f(z \uplus X)\}$. One easily shows that this defines a weak cartesian closed structure (see Hoofman and Schellinx(1991)). But Pow is *not* cartesian closed. This is due to the fact that it is a full subcategory of the categorie **DCPO** of directed complete partial orders: if Pow were cartesian closed, then any exponent $(\mathcal{P}(A) \Rightarrow \mathcal{P}(B))$ would be isomorphic to the complete lattice $[\mathcal{P}(A) \to \mathcal{P}(B)]$ of continuous mappings from $\mathcal{P}(A)$ to $\mathcal{P}(B)$ (see e.g. Jung(1989), lemma 1.21); but the lattice of continuous mappings between the powersets of two one-element sets has precisely *three* elements, so it can not be (isomorphic to) an object in Pow.

As the example shows, the notion of wCCC (semi-CCC) is strictly weaker than that of CCC. Moreover, semi-cartesian closed structures on a category C are not unique up to isomorphism as in the case of cartesian closed structures: there can be many non-isomorphic semi-exponents of objects $A, B \in$ C.

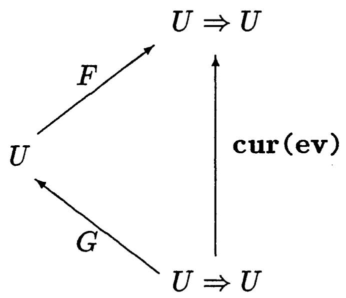
[1]least upper bound

2.2. EXAMPLE. Let s be some (non-empty) set. Put

$$\mathcal{P}(A) \Rightarrow_{\mathsf{s}} \mathcal{P}(B) := \mathcal{P}((A^{<\omega} \times B) \cup \mathsf{s}).$$

Define $\mathbf{cur_s}(f) : \mathcal{P}(A) \to (\mathcal{P}(B) \Rightarrow_{\mathsf{s}} \mathcal{P}(C))$, for $f : \mathcal{P}(A) \times \mathcal{P}(B) \to \mathcal{P}(C)$ by: $\mathbf{cur_s}(f)(z) := \{(X, b) \mid b \in f(z \uplus X)\} \cup \mathsf{s}$. One easily verifies that $\mathbf{cur_s}(f)$ and $\Rightarrow_{\mathsf{s}}$, together with the finite products and $\mathbf{ev}$ from the previous example, forms an alternative weak cartesian closed structure on Pow.

2.3. DEFINITION. A *reflexive object* in a semi-CCC C is given by an object U together with two arrows $F : U \to (U \Rightarrow U)$ and $G : (U \Rightarrow U) \to U$ such that $F \circ G = \mathbf{cur}(\mathbf{ev})$. I.e. U is a reflexive object iff the following diagram commutes:



□

Recall that for an object A in C we say that B is a *retract* of A iff there are $F : A \to B$ and $G : B \to A$ such that $F \circ G = id$. It follows that U is reflexive in a CCC iff $U \Rightarrow U$ is a retract of U. In a semi-CCC a retract $U \Rightarrow U$ of an object U always gives rise to a reflexive object:

2.4. PROPOSITION. *If $U \Rightarrow U$ is a retract of U in C through F, G, then $(U, \mathbf{cur}(\mathbf{ev}) \circ F, G)$ is a reflexive object.* □

However, given a fixed semi cartesian closed structure on a category C, not all reflexive objects U in C can be obtained in this way from a retract , as will be clear from the next

2.5. EXAMPLE. Let $\mathbf{D}_A$ denote Engeler's graph model for the untyped lambda-calculus, with atomset A (see e.g. Schellinx(1991)). So we have $\mathbf{D}_A = \mathcal{P}(U_A)$, where U_A is the smallest set containing A, that satisfies $X \in U_A^{<\omega}$ & $u \in U_A \Rightarrow (X, u) \in U_A$. The powerset $\mathbf{D}_A$ is a reflexive object in the category Pow with respect to the semi-exponent $\Rightarrow_{\mathsf{s}}$ from example 2.2. This is witnessed by the functions $F : \mathbf{D}_A \to (\mathbf{D}_A \Rightarrow_{\mathsf{s}} \mathbf{D}_A) : x \mapsto ((x - A) \cup \mathsf{s})$ and $G : (\mathbf{D}_A \Rightarrow_{\mathsf{s}} \mathbf{D}_A) \to \mathbf{D}_A : x \mapsto \overline{x} - \mathsf{s}$ (where $\overline{x}$ denotes $\{(X, b) \mid \exists (Y, b) \in x.Y \subseteq X\}$) which satisfy $F \circ G = \mathbf{cur}_{\mathsf{s}}(\mathbf{ev})$. Moreover, it is clear that if A is countable and s is uncountable, then $\mathbf{D}_A \Rightarrow_{\mathsf{s}} \mathbf{D}_A$ can not be a retract of $\mathbf{D}_A$ (as $\mathbf{D}_A \Rightarrow_{\mathsf{s}} \mathbf{D}_A$ is of higher cardinality than $\mathbf{D}_A$).

2.6. DEFINITION. Let C be semi cartesian closed. An object U in C such that $U \cong (U \Rightarrow U)$ is said to be an *iso-object*. If in fact U is the *same* as $U \Rightarrow U$, then we will call the object U *mighty*. □

Clearly a mighty object is always an iso-object, while an iso-object always has its own exponent as retract, and hence gives rise to a reflexive object.

3 Interpreting $\lambda\beta$ on reflexive objects in semi cartesian closed categories

Let (U, F, G) be a reflexive object in a semi-CCC. We are going to interpret the untyped lambda-calculus on the reflexive object U, or, more precisely, *in* $\mathrm{Hom}(\mathbf{1} \to U)$ (i.e. the collection of all maps $f : \mathbf{1} \to U$, also denoted by $|U|$, and sometimes referred to as the collection of *points* of U).

Using $\mathbf{ev}$ and F, we define for $g, h : A \to U$:

$$g \cdot_A h = \mathbf{ev}_{U,U} \circ < F \circ g, h > .$$

Writing $\cdot$ for $\cdot_{\mathbf{1}}$ in particular, we get an applicative structure $(|U|, \cdot)$.

We recall some definitions from Koymans(1984):

3.1. DEFINITION.

1. For any $n \geq 0$ the n-fold semi-product U^n is defined by:
   - $U^0 = \mathbf{1}$;
   - $U^{n+1} = U^n \times U$.

2. Let $\Delta = x_1, \ldots, x_n$ be a sequence of distinct variables. The canonical projections, $p^{\Delta}_{x_i} : U^n \to U$, are given by

$$p^{\Delta}_{x_i} = \begin{cases} p^{\Delta \backslash x_n}_{x_i} \circ \pi & \text{if } i \neq n \\ \pi' & \text{otherwise.} \end{cases}$$

   (Here $\Delta \backslash x_n$ stands for the sequence Δ minus its last element.)

3. For any object A and $f_1, \ldots, f_n : A \to U$, we define $\langle f_1, \ldots, f_n \rangle_A : A \to U^n$ by

$$\begin{aligned} \langle\rangle_A &= \mathbf{t}_A \\ \langle f_1, \ldots, f_{n+1} \rangle_A &= < \langle f_1, \ldots, f_n \rangle_A, f_{n+1} > . \end{aligned}$$

4. Let $\Gamma = y_1, \ldots, y_m$ with $\{\Gamma\} \subseteq \{\Delta\}$ (where $\{\Gamma\}$ denotes the set having as elements the variables in the sequence Γ). We define a mapping $\Pi^\Delta_\Gamma : U^n \to U^m$ by $\Pi^\Delta_\Gamma = \langle p^\Delta_{y_1}, \ldots, p^\Delta_{y_m} \rangle_{U^n}$. □

One can interpret the untyped λ-calculus on the reflexive object U in a semi-CCC just as in a CCC. We follow Barendregt(1984), chapter 5, section 5:

3.2. DEFINITION. Given a λ-term M with its free variables among $\{\Delta\}$, inductively define the interpretation $[\![M]\!]_\Delta : U^n \to U$ by:

1. $[\![x_i]\!]_\Delta = p^\Delta_{x_i}$;
2. $[\![PQ]\!]_\Delta = [\![P]\!]_\Delta \cdot_{U^n} [\![Q]\!]_\Delta$;
3. $[\![\lambda x.P]\!]_\Delta = G \circ \mathbf{cur}([\![P]\!]_{\Delta,x})$.

We take the usual precautions regarding names of variables, e.g. in the last case $x \notin \{\Delta\}$. □

In order to prove adequacy of this interpretation, also in the case of a semi-CCC, it suffices to check that the arguments in Barendregt(1984) do not use properties that are not generally valid in a semi-CCC, or, when they do, can be replaced by appropriate *'semi'*-arguments. Also, we have to take into account the definition of reflexive object in a semi-CCC. Let us start with a list of properties regarding the notions introduced above:

3.3. LEMMA. *The following equalities hold in any semi-CCC:*

1. $\langle f_1, \ldots, f_n \rangle_A \circ h = \langle f_1 \circ h, \ldots, f_n \circ h \rangle_B$;
2. $p^\Delta_{x_i} \circ \langle f_1, \ldots, f_n \rangle_A = f_i$;
3. $\Pi^\Delta_{x_i} \circ \langle f_1, \ldots, f_n \rangle_A = \langle f_i \rangle_A$;
4. $p^\Gamma_y \circ \Pi^\Delta_\Gamma = p^\Delta_y$;
5. $\Pi^\Gamma_\Theta \circ \Pi^\Delta_\Gamma = \Pi^\Delta_\Theta$;

6. $\Pi^{\Delta,x}_{\Gamma,x} = < \Pi^{\Delta}_{\Gamma} \circ \pi, \pi' >$

7. $\Pi^{\Delta,x}_{\Gamma} = \Pi^{\Delta}_{\Gamma} \circ \pi$.

PROOF: Straightforward. $\square$

The arguments in Barendregt(1984) use but *one* more equality which is missing here: in general in a semi-CCC it will not be true that $\Pi^{\Delta}_{\Delta} = id_{U^n} \quad (*)$. However the use of this property turns out to be inessential, as one can slightltly modify the arguments and avoid it.

3.4. LEMMA. *The following hold in any semi-CCC:*

1. *Let* $\{\Delta\} \supseteq \{\Gamma\} \supseteq \mathrm{FV}(M)$. *Then* $[\![M]\!]_{\Delta} = [\![M]\!]_{\Gamma} \circ \Pi^{\Delta}_{\Gamma}$.
2. *Let* $\{\Delta\} = \{\vec{x}\} \supseteq \mathrm{FV}(M)$, $\vec{N}$ *fit in* $\vec{x}$ *and* $\Gamma \supseteq \mathrm{FV}(\vec{N})$. *Then*

$$[\![M[\vec{x} := \vec{N}]]\!]_{\Gamma} = [\![M]\!]_{\Delta} \circ \langle [\![\vec{N}]\!]_{\Gamma} \rangle .$$

3. *Let* $\Delta \supseteq \mathrm{FV}(\lambda x.M), \Gamma \supseteq \mathrm{FV}((\lambda x.M)N)$ *and* $\{\Gamma\} \supseteq \{\Delta\}$. *Then*

$$[\![M[x := N]]\!]_{\Gamma} = [\![M]\!]_{\Delta,x} \circ < \Pi^{\Gamma}_{\Delta}, [\![N]\!]_{\Gamma} > .$$

PROOF: Both 1 and 2 are shown by induction on the structure of M, while 3 is a corollary to 2. Barendregt(1984) uses a non-semi property (namely $< \pi, \pi' >= id$) in the argument for 2 in case $M \equiv \lambda y.P$. However, one can do without:

$$\begin{array}{rcl}
[\![(\lambda y.P)[\vec{x} := \vec{N}]]\!]_{\Gamma} & = & [\![\lambda y.P[\vec{x}, y := \vec{N}, y]]\!]_{\Gamma} \\
 & = & G \circ \mathbf{cur}([\![P[\vec{x}, y := \vec{N}, y]]\!]_{\Gamma,y}) \\
\text{(ind.hyp.)} & = & G \circ \mathbf{cur}([\![P]\!]_{\Delta,y} \circ \langle [\![\vec{N}]\!]_{\Gamma,y}, [\![y]\!]_{\Gamma,y} \rangle) \\
 & = & G \circ \mathbf{cur}([\![P]\!]_{\Delta,y} \circ < \langle [\![\vec{N}]\!]_{\Gamma,y} \rangle, \pi' >) \\
\text{(by 1)} & = & G \circ \mathbf{cur}([\![P]\!]_{\Delta,y} \circ < \langle [\![\vec{N}]\!]_{\Gamma} \circ \Pi^{\Gamma,y}_{\Gamma} \rangle, \pi' >) \\
\text{(by 3.3, 7)} & = & G \circ \mathbf{cur}([\![P]\!]_{\Delta,y} \circ < \langle [\![\vec{N}]\!]_{\Gamma} \circ \Pi^{\Gamma}_{\Gamma} \circ \pi \rangle, \pi' >) \\
\text{(by 1 and 3.3, 1)} & = & G \circ \mathbf{cur}([\![P]\!]_{\Delta,y} \circ < \langle [\![\vec{N}]\!]_{\Gamma} \rangle \circ \pi, \pi' >) \\
 & = & G \circ \mathbf{cur}([\![P]\!]_{\Delta,y}) \circ \langle [\![\vec{N}]\!]_{\Gamma} \rangle \\
 & = & [\![\lambda y.P]\!]_{\Delta} \circ \langle [\![\vec{N}]\!]_{\Gamma} \rangle . \qquad \square
\end{array}$$

The adequacy of reflexive objects in a semi-CCC for the interpretation of the λ-calculus then follows:

3.5. PROPOSITION. *Let M, N be two lambda-terms, and $\{\Delta\} \supseteq \mathrm{FV}(MN)$. Then*

$$\lambda \vdash M = N \quad \Rightarrow \quad [\![M]\!]_\Delta = [\![N]\!]_\Delta .$$

PROOF: By induction on the length of proof of $M = N$. The crucial case is that of the β-axiom, and it is here that the reflexivity of U plays a role. The argument in Barendregt(1984) for the cartesian closed cases uses the property $(*)$ (i.e. $\Pi^\Delta_\Delta = id$). The following shows that its use is not necessary:

$$\begin{array}{rcl} [\![(\lambda x.P)Q]\!]_\Delta & = & (G \circ \mathbf{cur}([\![P]\!]_{\Delta,x})) \cdot [\![Q]\!]_\Delta \\ & = & \mathbf{ev} \circ < F \circ G \circ \mathbf{cur}([\![P]\!]_{\Delta,x}), [\![Q]\!]_\Delta > \\ \text{(by refl.)} & = & \mathbf{ev} \circ < \mathbf{cur}(\mathbf{ev}) \circ \mathbf{cur}([\![P]\!]_{\Delta,x}), [\![Q]\!]_\Delta > \\ & = & \mathbf{ev} \circ < \mathbf{cur}([\![P]\!]_{\Delta,x}), [\![Q]\!]_\Delta > . \end{array}$$

But we have by 3.4,1 and 3.3,6 that

$$\begin{array}{rcl} [\![P]\!]_{\Delta,x} & = & [\![P]\!]_{\Delta,x} \circ \Pi^{\Delta,x}_{\Delta,x} \\ & = & [\![P]\!]_{\Delta,x} \circ < \Pi^\Delta_\Delta \circ \pi, \pi' > . \end{array}$$

So we get

$$\begin{array}{rcl} [\![(\lambda x.P)Q]\!]_\Delta & = & \mathbf{ev} \circ < \mathbf{cur}([\![P]\!]_{\Delta,x} \circ < \Pi^\Delta_\Delta \circ \pi, \pi' >), [\![Q]\!]_\Delta > \\ & = & \mathbf{ev} \circ < \mathbf{cur}([\![P]\!]_{\Delta,x}) \circ \Pi^\Delta_\Delta, [\![Q]\!]_\Delta > \\ & = & [\![P]\!]_{\Delta,x} \circ < \Pi^\Delta_\Delta, [\![Q]\!]_\Delta > \\ & = & [\![P[x := Q]]\!]_\Delta, \quad \text{by } 3.4, 3 \qquad \Box . \end{array}$$

We then obtain the following:

3.6. THEOREM. *Any reflexive object U in a semi cartesian closed category determines a λ-algebra $(|U|, \cdot, [\![\ ]\!])$.* $\Box$

3.7. REMARK. Let us define U' to be the collection of all $x \in |U|$ such that $x = x \circ t_1$. Clearly U' is closed under application: if $x, y \in U'$, then $x \cdot y = (x \circ t_1) \cdot (y \circ t_1) = (x \cdot y) \circ t_1$.

Then observe that by 3.4,1 each closed lambda-term M in fact is interpreted as an element $[\![M]\!]_{\langle\rangle}$ of U'. If $t_1 = id$ (i.e. if the semi-terminal object is *terminal*), then $U' = |U|$. In general, however, $(U', \cdot)$ will be a proper substructure of $(|U|, \cdot)$.

3.8. REMARK. We define the interpretation of the $\lambda\beta$-calculus on objects in semi cartesian closed categories that have their own semi-exponent as retract as the interpretation on the corresponding reflexive object. The reader will observe that (due to the fact that $\mathbf{ev}\circ < F\circ g, h >= \mathbf{ev}\circ < \mathbf{cur}(\mathbf{ev})\circ F\circ g, h >$) this is equivalent to the obvious direct definition.

It is quite possible that a given object in a category C is reflexive with respect to *distinct* semi cartesian closed structures on C, as we already saw in example 2.5, where we gave non-standard readings of Engeler's graphmodel $\mathbf{D}_A$ as a reflexive object with respect to semi cartesian closed structures on Pow. The reader might verify however, that the *interpretation* on the object in all these cases is the same as in the 'standard' semi cartesian closed structure on Pow, that of example 2.1. Consequently in all cases the λ-theory associated will be the same.

It is however clear that the interpretation of the untyped $\lambda\beta$-calculus as given above depends essentially on the semi cartesian closed structure at hand, as well as on the functions witnessing reflexivity. One therefore would expect that in general *changing* the structure and/or the witnessing functions will change the corresponding λ-theory. Indeed this is so. For an example we again take a look at a non-standard reading of Engeler's $\mathbf{D}_A$ as a reflexive object in Pow. As semi cartesian closed structure we take that of example 2.2, with the atom-set A as the non-empty set s. As witnessing function for reflexivity we take $F : x \mapsto \overline{x} \cup A$, and G the identity mapping. The model thus obtained corresponds to what is called $\mathbf{D}_A^+$ in Longo(1983). By the results of this same paper the theory of $\mathbf{D}_A^+$ differs from that of $\mathbf{D}_A$. Detailed verification is left to the zealous reader.

4 Blankets and completeness

4.1. DEFINITION. Let C be semi cartesian closed. A *blanket* on C is a family $E = (E_{A,B} | A, B \in \mathsf{C})$ of objects of C together with arrows $r_{A,B} : E_{A,B} \to (A \Rightarrow B)$ and $s_{A,B} : (A \Rightarrow B) \to E_{A,B}$ (where $\Rightarrow$ denotes the semi-exponent in C), such that $r_{A,B} \circ s_{A,B} = \mathbf{cur}(\mathbf{ev})$. □

Intuitively, $E_{A,B}$ can be thought of as providing an alternative for the semi-exponent $A \Rightarrow B$ (see lemma 4.3).

4.2. EXAMPLE. Let C be semi cartesian closed with reflexive object U. We can define a blanket on C by

$$E_{A,B} = \begin{cases} U & \text{if } A = B = U \\ A \Rightarrow B & \text{otherwise} \end{cases}$$

(take $r = F, s = G$ in the first, $r = s = \mathbf{cur}(\mathbf{ev})$ in the second case).

As we saw above, in general semi cartesian closed structures on a category are not unique. In fact, the following shows that a blanket on a semi cartesian closed category gives rise to an *alternative* semi cartesian closed structure on the same category.

4.3. LEMMA. *Let E be a blanket on a category C having a semi cartesian closed structure containing $(\Rightarrow, \mathbf{ev}, \mathbf{cur})$. One obtains an alternative semi cartesian closed structure on C, by keeping the same finite semi-products, and defining $(\Rightarrow', \mathbf{ev}', \mathbf{cur}')$ by*

$$\begin{array}{rcl} (A \Rightarrow' B) & := & E_{A,B} \\ \mathbf{ev}'_{A,B} & := & \mathbf{ev}_{A,B} \circ < r_{A,B} \circ \pi, \pi' > \\ \mathbf{cur}' & := & s_{A,B} \circ \mathbf{cur}. \end{array}$$

PROOF: One easily checks that the defining equations hold for $(\Rightarrow', \mathbf{ev}', \mathbf{cur}')$. For example,

$$\begin{array}{rcl} \mathbf{ev}' \circ < \mathbf{cur}'(f) \circ \pi, \pi' > & = & \mathbf{ev} \circ < r \circ \pi, \pi' > \circ < s \circ \mathbf{cur}(f) \circ \pi, \pi' > \\ & = & \mathbf{ev} \circ < r \circ s \circ \mathbf{cur}(f) \circ \pi, \pi' > \\ & = & \mathbf{ev} \circ < \mathbf{cur}(\mathbf{ev}) \circ \mathbf{cur}(f) \circ \pi, \pi' > \\ & = & \mathbf{ev} \circ < \mathbf{cur}(f) \circ \pi, \pi' > \\ & = & f. \qquad \square \end{array}$$

4.4. REMARK. Note that the alternative structure defined does not change the nature of the (semi-)products and the (semi-)terminal object in C: if originally we have *real* products and/or a *real* terminal object, then the same will be the case in the alternative structure. So if C happened to be *weak* cartesian closed, then a blanket defines an alternative *weak* cartesian closed structure on C; a non-trivial blanket on a CCC will result in a *weak* cartesian closed structure.

4.5. PROPOSITION. *Reflexive objects in semi cartesian closed categories give rise to mighty objects in semi cartesian closed categories. Moreover, the reflexive object and the generated mighty object are identical as λ-algebras.*

PROOF: Let U be a reflexive object in a semi cartesian closed category C. Define a blanket on C as in example 4.2, and apply lemma 4.3 to find a semi cartesian closed structure on C in which $U = (U \Rightarrow' U)$.

Let $(|U|,\cdot,[\![\,]\!])$ be the original λ-algebra, and $(|U|,\star,[\![\,]\!]')$ the λ-algebra given by the generated object. First we note that $f\star_A g = f\cdot_A g$, for all objects A, and all $f,g: A\to U$:

$$\begin{aligned} f\star_A g &= \mathbf{ev}'_{U,U}\circ < id_U\circ f, g > \\ &= \mathbf{ev}'_{U,U}\circ < f, g > \\ &= \mathbf{ev}_{U,U}\circ < F\circ\pi, \pi' > \circ < f, g > \\ &= \mathbf{ev}_{U,U}\circ < F\circ f, g > \\ &= f\cdot_A g. \end{aligned}$$

In particular we find that $\cdot_{\mathbf{1}} = \star_{\mathbf{1}}$, i.e. $\cdot = \star$.

To finish the proof, we show by induction on the complexity of M, that for all lambda-terms M and all $\Delta\supseteq \mathrm{FV}(M)$, we have $[\![M]\!]_\Delta = [\![M]\!]'_\Delta$.

For the basis of the induction, we observe that for all Δ and all x we obviously have $[\![x]\!]_\Delta = p_x^\Delta = [\![x]\!]'_\Delta$.

We proceed as follows:

$$\begin{aligned} [\![MN]\!]'_\Delta &= [\![M]\!]'_\Delta \star [\![N]\!]'_\Delta \\ \text{(ind.hyp.)} &= [\![M]\!]_\Delta \star [\![N_\Delta]\!] \\ &= [\![M]\!]_\Delta \cdot [\![N_\Delta]\!] \\ &= [\![MN]\!]_\Delta; \end{aligned}$$

$$\begin{aligned} [\![\lambda x.M]\!]'_\Delta &= id_U\circ \mathbf{cur}'([\![M]\!]'_{\Delta,x}) \\ \text{(ind.hyp.)} &= \mathbf{cur}'([\![M]\!]_{\Delta,x}) \\ &= G\circ \mathbf{cur}([\![M]\!]_{\Delta,x}) \\ &= [\![\lambda x.M]\!]_\Delta. \qquad \square \end{aligned}$$

In fact we can show something slightly stronger:

4.6. THEOREM. *Reflexive objects in semi cartesian closed categories give rise to mighty objects in* weak *cartesian closed categories. Moreover, the reflexive object and the generated mighty object are identical as λ-algebras.*

PROOF: Let C be semi cartesian closed with reflexive object (U,F,G). Write F' for $\mathbf{cur}(\mathbf{ev})\circ F$, and G' for $G\circ\mathbf{cur}(\mathbf{ev})$. The Karoubi envelope $\mathcal{K}(\mathsf{C})$ of C (see e.g. Hoofman and Schellinx(1991)) is cartesian closed, with reflexive object (id_U, F', G'). If $\mathbf{1}$ is the semi-terminal object in C, then $t_{\mathbf{1}}$ is the terminal object in the Karoubi envelope. The λ-algebra determined by the reflexive object in $\mathcal{K}(\mathsf{C})$ has as its domain of interpretation $\mathrm{Hom}(t_{\mathbf{1}}, id_U)$. This consists of all those $f\in|U|$ such that $f\circ t_{\mathbf{1}} = f$.

Further we observe that

$$\mathbf{ev}\circ < \mathbf{cur}(\mathbf{ev}) \circ F \circ g, h > = \mathbf{ev} < F \circ g, h >,$$

and

$$\begin{aligned} G \circ \mathbf{cur}(\mathbf{ev}) \circ \mathbf{cur}(f) &= G \circ \mathbf{cur}(\mathbf{ev}\circ < \mathbf{cur}(f) \circ \pi, \pi' >) \\ &= G \circ \mathbf{cur}(f \circ < \pi, \pi' >) \\ &= G \circ \mathbf{cur}(f) \end{aligned}$$

Therefore, using remark 3.7, the λ-algebra determined by the reflexive object in $\mathcal{K}(\mathsf{C})$ is just the λ-algebra determined by U in C. By the previous proposition and remark 4.4, there is a weak cartesian closed structure on $\mathcal{K}(\mathsf{C})$ where $id_U = (id_U \Rightarrow id_U)$ and which again gives the same λ-algebra. □

It was shown by Koymans (see Barendregt(1984), chapter 5, section 5) that *every* λ-algebra can be obtained from a reflexive object U in a category C with cartesian closed structure. The above results enable us to rephrase this as follows:

4.7. THEOREM. *A mighty object U in a weak cartesian closed category determines a λ-algebra. Conversely each λ-algebra can be obtained from such an object.*

PROOF: *Soundness* is clear from proposition 2.4, theorem 3.6 and remark 3.8: take the λ-algebra determined by the reflexive object $(U, \mathbf{cur}(\mathbf{ev}), id)$. Conversely, for *completeness*, let some λ-algebra be given. By Koymans's result it can be obtained from a reflexive object U in a category C with cartesian closed structure. By theorem 4.6, U is mighty in a weak cartesian closed structure on C. Moreover, it remains unchanged as a λ-algebra. □

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