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EFFECTIVE TRUTH

G.K. Dzhaparidze

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Effective Truth

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Abstract

I introduce the notion of effective truth defined in terms of game (dialogue) semantics and very much resembling recursive realizability. This notion seems to me a natural alternative for the classical notion of truth which satisfies the "constructivistic" demands. At the same time it serves as a natural justification of "resource-conscious" effects reflected in substructural logics, the logic corresponding to effective truth is a version of linear logic.

The present paper is not a complete work, it is only the first part of a bigger essay planned by the author. The reader will find here a lot of definitions, examples and some philosophy, but no very serious theorems. In particular, there is nothing said about the complexity of the logic of effective truth, no axiomatization with a completeness theorem is given for it. Preliminary results in this direction do exist, but they are not written yet, and the author prefers to abstain from formulating theorems without proofs; this job is reserved for the second part of the above-mentioned work.

1 Introduction

The classical notion of truth has many times been criticized. This is what I, too, am apparently going to do with it in the next few lines, and that is why I would

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like to start with apologizing if the reader finds some of my arguments not very original.

One of the most vulnerable to controversies point of logic is the existential quantifier,

$$\ldots \exists x \phi(x),$$

read as

$$\ldots \textit{there exists } x \textit{ such that } \phi(x),$$

or, sometimes, as

$$\ldots \textit{there can be found } x \textit{ such that } \phi(x).$$

The two readings are usually perceived as synonyms, and still the difference between them is crucial. "There exists" sounds metaphysically, whereas "can be found" means something what deals with the reality. To leave alone the philosophy on the right of "existence" of the classical notion of existence, it simply has no practical meaning, as it can be demonstrated by the following maybe not very serious example. Consider the sentence

For every disease there is a medicine which cures of this disease.

If there is no way of finding for each disease a cure of it but still we somehow know that this sentence is true in the classical sense, we have no reason to be happier than we would be in the case if it were false. Not in the least![1] In general, truth or falsity of a sentence can concern us only as far as this can spell something, can somehow be reflected in the reality.

Of course "can be found" is a relative notion. Found by whom? If by God or another almighty being, then, and in fact only then, "exists" is really a synonym of "can be found". But the most natural answer on the question *by whom* is *by a Turing machine.*

This treatment of the existential quantifier, when existence means being possible to be found by a machine, is captured in the nonclassical concept of "truth" which I suggest in this work and which will be called *effective truth.* The general ideology of the semantics of effective truth is that sentences are considered as certain *tasks, problems* which are to be solved by a machine, that is, by a subject

[1]A naive opponent could object: The classical truth of this sentence means that we can try all the chemical stuffs, one by one, and sooner or later, one of them will work, so we do have a reason to be happy. Then I would give two answers in the same naive manner: First, what the opponent suggests already *is* a way of finding the medicine, and the second: in fact this way is hardly a good way, because the pure sick man will, most likely, be poisoned and dye before we reach the due medicine.

which has an effective strategy for doing this; effective truth means existence of such a strategy.

The most convenient way of shaping this approach is to build all the semantics in terms of *games*:

> a task (problem) = the task (problem) of winning certain game with established rules.

There are two players in our games: I, asserting a sentence, and my opponent who tries to refute it. I am supposed to follow only effective (recursive) strategies, whereas the opponent can use any strategy, he is meant to represent the blind forces of nature, or the devil himself.

The universal quantifier will always mean the opponent's move and the existential quantifier will mean my move. The above "medical" proposition can now be understood as the game each play of which consists of two moves: the first move is done by the opponent, who names an arbitrary disease d, and the second move is mine, I must name a medicine m; the play is won by me, if m really is a cure of d. Effective truth of this proposition can now really be a reason to lead a quiet life: I have an effective strategy (machine) such that, using it, I can always find a cure of any disease sent to me by the devil.

The connective $\vee$ will be treated in the same manner as the existential quantifier. Say,

$$\forall x(x \in P \vee x \notin P),$$

where P is a set, will be understood as the game each play of which, again, consists of two moves: first the opponent chooses an object n, which leads to the position $n \in P \vee n \notin P$, and then I choose between "left" and "right", getting thus one of the positions $n \in P$ or $n \notin P$; the play is won, if n belongs to P and I have chosen "left", or n does not belong to P and I have chosen "right". It is clear that effective truth of this proposition means nothing but decidability of the predicate $x \in P$.

In general, decidability and other similar concepts are easily definable in terms of effective truth. Moreover: in fact saying that a given sentence is effectively true, we always assert that certain relation similar to decidability (but may be much more sophisticated) holds, as, e.g., the binary relation expressed by the formula $\forall x(x \in P \vee x \in Q)$. Effective truth of this sentence means that there is an effective method of choosing for each individual a one of the two sets P, Q such that a belongs to this set.

In the above examples the operator $\vee$ connects atomic formulas (games). In a more general case, to win the game $\alpha_1 \vee \alpha_2$ means that after my choice of one of the components α_i, I must continue and win the game α_i, whereas the other component should be abandoned for ever. There is however one more natural sort of disjunction, denoted by $\bigtriangledown$. The position $\alpha_1 \bigtriangledown \alpha_2$ does not oblige me to choose one of the α_i and reject the other. I can make a move in one of the components, reserving at the same time the other, and switch any moment from α_1 to α_2 and back; the task is, playing in fact simultaneously in the two components, to win at least in one of them.

Exact definitions will be given in the main text (sections 2 and 3), but now, in order to develop intuition, we continue discussing some more "naive" examples.

I am in prison, my prison cell has two doors locked from the outside, the left door and the right door. My goal is to escape, and for that it is enough to pass through one of the doors. I happen to know that tonight one of the doors has been unlocked. Consider the proposition

The left door is unlocked *or* the right door is unlocked.

In order to escape it is enough for me to be able to "solve" this game (problem) with "or" understood as $\bigtriangledown$: it is not necessary to be able to calculate beforehand exactly which door is unlocked, I can simply try both and one of them will turn out to be unlocked. I write "solve" with quotation marks because in this game there are no moves at all and, under our assumption that one of the doors is really unlocked, it is trivially won.

Let us now slightly change the situation: the doors were not locked but mined, and tonight someone has removed the mine from one of the doors. Yes, we need now just $\vee$, and $\bigtriangledown$ will not do any more.

We treat negation $\neg$ in the following way: the rules of the game $\neg\alpha$ are the same as of α, only with the roles of me and the opponent interchanged.

Example: Let C be a version of chess to win which for me means to win a usual chess play within at most 100 moves, being the white player. Then $\neg C$ will be the game to win which for me means not to loose within 100 moves a chess play being the black player.[2]

Notice that the classical principle $\neg\neg\alpha \equiv \alpha$ does hold with our negation: after interchanging the roles twice, each player comes to his initial role. That means that all classical dualities work. In particular, $\forall$ can be defined in terms of ($\exists$, $\neg$) by $\forall x\alpha(x) \equiv \neg\exists x\neg\alpha(x)$.

As the game of chess has been mentioned, the temptation arises to discuss one more example (which is however not very original). Consider the game

$$C \bigtriangledown \neg C,$$

[2]In this example C is not a proposition but a "pure" game, and the propositional connective $\neg$ is thus an operation on games. But this is normal because propositions are for us nothing but games, and propositional connectives — operations on games.

C being defined as above. This is in fact a play on two chessboards, on the left board I am the white player and on the right one — black. My task is to win in the sense of usual chess (but with the within-100-moves amendment) on one of the chessboards. As switching the components is my priority, I have to move only in the case when the chess rules on the both boards oblige me to move, i.e., as soon as the opponent has to move at least on one of the chessboards, I can wait until he makes this move.

I, being not very good at chess, still can win this game even if my opponent is the world champion Kasparov, if I use the following strategy (solution): After Kasparov makes his first move on the right chessboard (where he is the white player), I repeat the same move on the left chessboard (where I am the white player), then copy Kasparov's answer on this move to the right chessboard and so on. This winning strategy can be used for any game of the type $\alpha \nabla \neg\alpha$, which means that the principle $\alpha \nabla \neg\alpha$ is valid in our sense.

As for the game $C \vee \neg C$, where at the very beginning I have to choose one of the chessboards and then win just on it, I have no chance to defeat Kasparov.[3]

To each sort of disjunction corresponds its dual conjunction, so we have two conjunctions $\wedge$ and $\triangle$, $\alpha \wedge \beta$ formally defined as $\neg(\neg\alpha \vee \neg\beta)$ and $\alpha \triangle \beta$ as $\neg(\neg\alpha \nabla \neg\beta)$.

For example, if I play with Kasparov the game $C \nabla \neg C$, from the point of view of Kasparov this is the game $\neg C \triangle C$. He has to win on both chessboards. Besides, he has to move as soon as the chess rules oblige him to move on at least one of the boards.

Paying tribute to linear logic, we shall call $\vee$ *additive disjunction*, $\wedge$ *additive conjunction*, ∇ *multiplicative disjunction* and $\triangle$ *multiplicative conjunction.*

The borrowing some terminology (though not notation) from linear logic is not by accident. The logic of effective truth, i.e. the set of always effectively true sentences (like $\alpha \nabla \neg\alpha$, but not $\alpha \vee \neg\alpha$), called *effective tautologies*, turns out to be an extension of Girard's [3] linear logic (without exponentials and constants, of course), in fact, a proper extension of linear logic + weakening (so called Affine Logic), and the behaviour of our additive and multiplicative connectives is very much similar to the behaviour of those in linear logic.

For a curious reader I give two principles valid in our semantics but not derivable in Affine Logic, $\phi \rightarrow \psi$ defined as $\neg\phi \nabla \psi$:

- $\big((\alpha_1 \triangle \alpha_2) \nabla (\beta_1 \triangle \beta_2)\big) \rightarrow \big((\alpha_1 \nabla \beta_1) \triangle (\alpha_2 \nabla \beta_2)\big)$;[4]

[3] However, taking into account that there is only a finite amount of all possible plays of C, the game $C \vee \neg C$ has an effective solution (winning strategy for me), even if no contemporary machine is strong enough to carry out this strategy. Still, a bit more carefully chosen example would convince us that the game $\alpha \vee \neg\alpha$ is not always effectively solvable.

[4] This formula is taken from [2].

- $\big((\alpha_1 \vee \alpha_2)\nabla(\beta_1 \vee \beta_2)\big) \rightarrow$
  $\big((\alpha_1\nabla(\beta_1 \vee \beta_2)) \vee (\alpha_2\nabla(\beta_1 \vee \beta_2)) \vee ((\alpha_1 \vee \alpha_2)\nabla\beta_1) \vee ((\alpha_1 \vee \alpha_2)\nabla\beta_2)\big).$

The main intuition behind linear logic was that resources should be consciously book-kept. This leads to rejection of such classically or intuitionisticly valid principles as, e.g., $\alpha \rightarrow \alpha \bigtriangleup \alpha$: If *I have one dollar*, it does not imply that *I have one dollar and I have one dollar*, because the latter conjunction can be understood as *I have two dollars.* However this vague intuition had not been formalized as a strict semantics, and linear logic and other "resource" logics originally have been built proceeding from syntax rather than semantics, say, by taking the classical sequential calculus and throwing away the rules like contraction which are definitely unacceptable even from the indefinite "resource-counting" point of view. But it well may be that together with these rejected rules, some "innocent" principles, too, became underivable. Apparently I claim that though the logic of effective truth is strictly stronger than any known contraction-free substructural logic, it still has all rights to be considered as the logic most appropriate to the "resource" ideology.

To be more correct, I believe that the "resource-consciousness" is not primary, but it is an inseparable satellite and consequence of another approach which could be called "dynamic", and it is just the latter initially captured by the game semantics. I mean the following:

Sentences in the classical logic are "static", they are given one of the two values 0 or 1, once and for ever. That's why a (sub)sentence, occuring more than once in a derivation or in another sentence, is still in all reasonable senses the same in each occurence; the amount of these occurences does not matter, contraction works and the principle $\alpha \rightarrow \alpha \bigtriangleup \alpha$ is valid. As for the game semantics, sentences there are treated as something dynamic; two different occurences of one and the same sentence denote *one and the same game*, i.e. one and the same set of potential plays, but in the process of carrying out this game, the two occurences can be realized as *different plays* of one and the same game. This is just what makes, say, $\alpha \bigtriangleup \alpha$ different from α, and "I have one dollar *and* ($\bigtriangleup$) I have one dollar" becomes equivalent to something like "I have two dollars" (each dollar can be spent its own way) rather than "I have one dollar". This is better to illustrate appealing again to chess.

In the above example with the game $C\nabla\neg C$ my winning strategy consisted in fact in providing that the two occurences of C in $C\nabla\neg C$ were realized as one and the same play. However, this trick fails to work with the game $C\nabla(\neg C \bigtriangleup \neg C)$, which is a game on three chessboards (chessboards NN 1,2 and 3, corresponding to the three occurences of C, in their order from the left to the right). Kasparov can play different openings on the second and the third chessboards (where he is the white player), and I can now only provide that the play on the first chessboard coincides with the play on one of the chessboards N2 or N3, let it be N2. Then Kasparov can win on the chessboards NN 1 and 3 and, though I will have won on the chessboard N2, the whole play will be lost by me, because for winning $C\bigtriangledown(\neg C \bigtriangleup \neg C)$ it was necessary for me to win on the first chessboard or to win on both the second and the third chessboards.

I mentioned above that the logic of effective truth is a proper extension of Affine Logic; on the other hand, it is strictly included in classical logic (the language of the latter is meant to be augmented with $\bigtriangledown$ and $\bigtriangleup$, which are identified with $\vee$ and $\wedge$, respectively). Unfortunately no further no further information on this logic will be given in the present paper (see the abstract).

One more thing which is worth to be mentioned here is that as soon as we remove the restriction on my strategies to be effective and allow any strategies, we get classical logic, where the distinction between the additive and multiplicative versions of disjunction and conjunction simply disappears. Thus, classical logic and our variant of "linear logic" result from two special cases of one general semantical approach.

In the end, some historical remarks. Apparently Lorenzen [6] was the first to introduce a game semantics, in the late 50-s. He suggested that the meaning of a proposition should be specified by establishing the rules of treating it in a "debate" (game) between a proponent who asserts the proposition and an opponent who denies it. The aim was to build a semantics for intuitionistic logic, which was formally achieved, though, to my mind, the undertaking hardly can be estimated as very successful and the semantics very "convincing": together with natural rules of linguistic games, Lorenzen had to introduce a number of artificial supplementary rules in order to make the semantics work in general and work for intuitionism in particular. Especially "unconvincing" was the treatment of atoms. In fact that semantics was to describe logical validity rather than truth (enough to mention that no notion of a model was used), and this circumstance makes the approach vicious: logical validity cannot be primary, it must mean nothing but truth in every particular situation (model), and without having a notion of truth one has no right to speak about validity.

Subsequently Lorenz [5] systematicly investigated many variations of the abovementioned supplementary rules, actually remaining in the framework of the same "vicious" approach.

The notion of effective truth introduced in the present paper, though defined in Lorenzen- and Lorenz-style game-semantical terms, is in fact much more similar to, say, Kleene's [4] recursive realizability, specifically, in what concerns the treatment of additive connectives and quantifiers (which are nothing but additives, again). At the same time, because of the unconstructive treatment of implication, the predicate of recursive realizability is nonarithmetical, whereas the predicate of effective truth of an arithmetical sentence (which is allowed to contain the additional connectives $\bigtriangledown$ and $\bigtriangleup$) has the complexity Σ_3^0. Maybe this low complexity ($\Rightarrow$ high constructivity) is the most remarkable feature that distinguishes effective truth from other alternatives to the classical notion of truth presented so far.

To the comparative criticism of recursive realizability should be added that not all the recursively realizable sentences are true in the classical sense (e.g. some sentences of the form $\neg\forall x(\phi(x) \vee \neg\phi(x))$ are recursively realizable!), whereas effective truth is only a "strong version" of classical truth.

I elaborated this semantics some time before writing the present paper, and gave a talk "The logic of effective truth", with all the main definitions, at the *Logic and Computer Science Conference* in Marseille (June' 1992). At the same conference I met Andreas Blass and studied that he had presented a game semantics quite similar to mine (and just for the same language), discribed in [2], where decidability of two fragments of the corresponding logic has been established:

1. The multiplicative propositional fragment, i.e. which uses only the connectives $\neg$, $\bigtriangledown$ and $\bigtriangleup$.

2. The fragment consisting of additive, i.e. $\bigtriangledown$- and $\bigtriangleup$-free, sequents.

However the question whether the unrestricted logic corresponding to the Blass' semantics (or, at least the propositional fragment of it) is decidable, recursively enumerable or even arithmetical, is open.

Together with relativeness of the two semantics, connected first of all with similar treatments of the connectives, there are also considerable differences:

1. Our games are finite (or, at least, well-founded which means that each play is finite), whereas Blass' games are essentially infinite. E.g., even the proof of nonvalidity of $\alpha \vee \neg\alpha$ in [2] uses a counterexample where α is an infinite (undetermined) game, and the proof hopelessly fails as soon as α is interpreted as a well-founded (to leave alone finiteness) game which is always determined in the sense that one of the players has a winning strategy (though not necessarily effective) in it. This infiniteness makes the things only second-order definable, whereas all the metatheory of effective truth can be build in **PA** (arithmetic).

2. At the same time, Blass' semantics does not demand the strategies to be effective, this demand apparently would be unessential there. Thus, the earlier-mentioned effect, — that classical logic and (a version of) linear logic are two special cases, one corresponding to unrestricted strategies and the other to effective strategies,—does not work there.

3. Finiteness of the plays of our games makes definitions simpler and more natural. E.g., a play is assumed to be lost by the player who has to move in the

last position of the play, so we do not need the parameter which indicates which plays are won by which player in a game.

4. The notion of effective truth is only a refinement of the classical notion of truth, and it is based on the same traditional models for the traditional languages. E.g., the standard model of arithmetic becomes now the unique game-theoretical model where each atomic sentence α is a terminal position, with the opponent's obligation to move (which is though impossible to do), if α is true in the standard model, and my obligation to move otherwise; the set of effectively true arithmetical sentences is a proper subset of the set of those true in the classical sense, which means that effective truth is only "truth in a stronger sense". E.g., if α is the Π^0_3 arithmetic sentence which asserts its own not being effectively true, then $\alpha \vee \neg\alpha$ is true but not effectively true.[5] As for Blass' semantics, it hardly allows to speak about "truth" in the standard model of arithmetic, for atoms are to be interpreted as infinite games and in that case it is not clear which game should be the natural interpretation of, say, $a + b = c$. That approach has no notion which could be called "truth", it only suggests the notion of "a game in which I have a winning strategy", these games having nothing to do with the traditional interpretation of languages.

S.Abramsky and R.Jagadeesan [1] revised Blass' game semantics by slightly changing the game rules, and investigated the multiplicative fragment of the corresponding logic, which does not validate weakening any more and is thus closer to the original Girard's linear logic, being still slightly stronger than the latter. This fragment is shown to be decidable, though the questions on the decidability of the whole logic, as well as of its full propositional fragment — again left open. In my opinion this semantics is less natural than that of Blass, at the same time remaining in the framework of the same infinite games, so the above four clauses of comparative criticism concerns Abramsky and Jagadeesan's approach to the same extent as the approach of Blass.

2 Basic notions and facts on games

Definition 2.1 A *net of games* is a triple $N = \langle W, l, R\rangle$, where:

- W is a nonempty set, the elements of which are called *positions* of N.

[5] Indeed, if this additive disjunction were effectively true, a little analysis of our treatment of $\vee$ shows that then either α or $\neg\alpha$ should be such. A further analysis of the situation convinces us that none of this cases is possible: if α is effectively true, then it is classically false, and if $\neg\alpha$ is effectively true, then α is classically true; this is a contradiction because effective truth always implies classical truth.

- l is a function $W \to \{0,1\}$, called the *labelling function*; for an element w of W, (the value of) $l(w)$ is called the *label* of w. Intuitively, $l(w) = 0$ means that I have to move and $l(w) = 1$ that the opponent has to move.

- R is a binary relation (called the *development relation*) on W such that the converse of R is well-founded, i.e. there is no infinite chain $w_0 R w_1 R w_2 \ldots$ of positions. Intuitively, wRu means that the transfer from the position w to the position u is a legal move. In this case u is called a *development* of w (in N). $R(w)$ usually denotes the set of all developments of w. By a *legal sequence of positions* of N we will mean any sequence of positions of N such that each $n+1$–th term of the sequence (if it exists) is a development of the n–th term. Thus, the converse well-foundedness of R means nothing but that each legal sequence of positions of N is finite.

Definition 2.2 A *game* is a quadruple $\langle W, l, R, s\rangle$, where $N = \langle W, l, R\rangle$ is a net of games and s is an element of W. Usually, if N denotes a net $\langle W, l, R\rangle$ of games and $s \in W$, we use $N(s)$ to denote the game $\langle W, l, R, s\rangle$. "A position of $N(s)$" and "a position of N" are synonyms, and s is said to be the *starting position* of $N(s)$.

Convention 2.3 It is not a "legal move" to speak about a function without specifying its type, as it is done very often in this paper (e.g., Definitions 2.4, 2.7, 2.8). Though sometimes the type of a function really does not matter or can be seen from the context, and it would be awkward still to "invent" in each such case a range and a domain for a function purely out of reasons of a correct style.

In order to avoid possible confusion caused by our irresponsible usage of the notion of a function as such, let us fix a "large enough" universe U; namely, we assume that the set of positions of any game we consider is included in U; for safety we can suppose that all natural numbers (and maybe many other things) are in U. Then

- by a *function*, if not specified otherwise, we will always mean a partial function of the type $U \to U$;

- by a *finite function* we will mean a function (in the above sense) defined only for a finite number of arguments.

Definition 2.4 Let $G = \langle W, l, R, s\rangle$ be a game and f_0 and f_1 functions.

1. *The G-play with my strategy f_0 and the opponent's strategy f_1* is a sequence P of positions of N which we construct in the following way:

   a) The first position of P is s.

   b) Suppose w is the n-th position of P. Then:

- if $l(w) = i$, $f_i(w) = u$ and wRu, then the $n+1$-th position of P is u;
- else w is the *last position* of P.

Notice that P is a legal sequence of positions of G and thus is finite.

2. *A G-play* is the G-play with my strategy f and the opponent's strategy g for some functions f and g.

   Observe that a G-play is nothing but a legal sequence $\langle w_1, \ldots, w_n \rangle$ of positions of G such that $w_1 = s$.

3. *A G-play with my strategy f_0* (resp. with the opponent's strategy f_1) is the G-play with my strategy f_0 and the opponent's strategy f_1 for some function f_1 (resp. f_0).

   In other words, a G-play with my strategy f_0 (resp. with the opponent's strategy f_1) is a G play $\langle w_1(= s), \ldots, w_n \rangle$ such that for any $1 \leq i \leq n$ with $l(w_i) = 0$ (resp. with $l(w_i) = 1$), we have:

   - if $f_0(w_i)$ (resp. $f_1(w_i)$) $= u$ for some $u \in D(w_i)$, then $i < n$ and $w_{i+1} = u$;
   - else $i = n$.

Definition 2.5 The *length* of a game $N(s) = \langle W, l, R, s \rangle$ is the least ordinal number α such that for every w with sRw, $\alpha >$ *the length of* $N(w)$. Thus, if s has no developments, the length of $N(s)$ is 0.

Very roughly, the length of a game G is the maximal possible length of a G-play.

Definition 2.6 Suppose $G = \langle W, l, R, s \rangle$ is a game, P a G-play and w the last position of P. Then:

- if $l(w) = 1$, then we say that P is *won by me* and *lost by the opponent*;
- if $l(w) = 0$, then we say that P is *lost by me* and *won by the opponent*.

Simply the words "won" and "lost", without specifying the player, will always mean "won by me" and "lost by me".

Thus, every play is either won or lost. Intuitively, a play is won if a position (the last position) is reached where the opponent has to move (as the label of that position is 1) but cannot, and in a lost play we have the dual situation: I have to move but cannot.

Definition 2.7 Let G be a game.

- A *solution* to G (*my winning strategy* for G) is a function f such that any G-play with my strategy f is won.
- Dually, an *antisolution* to G (the *opponent's winning strategy* for G) is a function g such that any G-play with the opponent's strategy g is lost.

In order to stress that the relation "...is a solution to ..." applied later to formulas interpreted as games, belongs to the family of relations of the type "...realizes ..." which lead to diverse well-known concepts of realizability (see [7]), we give another, equivalent definition of it:

Definition 2.8 A *solution* to the game $N(s) = \langle W, l, R, s\rangle$ is a function f such that:

- if $l(s) = 1$, then for all $w \in R(s)$, f is a solution to the game $N(w)$;
- if $l(s) = 0$, then $f(s) = w$ for some position w such that $w \in R(s)$ and f is a solution to the game $N(w)$.

Note that this definition is correct because the development relation is converse well-founded.

Fact 2.9 *Definitions 2.7 and 2.8 of the notion of solution are equivalent.*

Proof. Let us temporarily call the notion defined in 2.7 "solution-1" and that defined in 2.8 "solution-2". Fix the game $N(s) = \langle W, l, R, s\rangle$ and a function f. We proceed by induction on the length of $N(s)$.

Case 1: $l(s) = 1$.

Suppose f is not a solution-2 to $N(s)$. Then there is $w \in R(s)$ such that f is not a solution-2 to $N(w)$. The length of $N(w)$ is less than the length of $N(s)$, so we can use the induction hypothesis to $N(w)$ and conclude that f is not a solution-1 to $N(w)$. By definition it means that some $N(w)$-play $\langle w_1(=w), \dots, w_n\rangle$ with my strategy f is lost. Note that then the sequence $\langle s, w_1, \dots, w_n\rangle$ is a lost $N(s)$-play with my strategy f, so f is not a solution-1 to the game $N(s)$.

Suppose now that f is not a solution-1 to $N(s)$, i.e. some $N(s)$-play with my strategy f is lost. This play has the form $\langle s, w, \vec{u}\rangle$, where $w \in R(s)$ and $\vec{u}$ is some (may be empty) legal sequence. Now notice that the sequence $\langle w, \vec{u}\rangle$ is an N(w)-play with my strategy f, which, obviously, is lost, too. Thus f is not a solution-1 to the game $N(w)$ and, by the induction hypothesis, f is not a solution-2 to $N(w)$, either. Since $w \in R(s)$, we immediately have by definition that f is not a solution-2 to $N(s)$.

Case 2: $l(s) = 0$.

Suppose f is a solution-1 to $N(s)$. Then $f(s) = w$ for some $w \in R(s)$ (otherwise any $N(s)$-play with my strategy f would be just $\langle s \rangle$ and thus lost). Consider an arbitrary function g. The $N(w)$-play with my strategy f and the opponent's strategy g is $\langle w, \vec{u} \rangle$ for some (possibly empty) sequence $\vec{u}$ of positions and the $N(s)$-play with my strategy f and the opponent's strategy g is $\langle s, w, \vec{u} \rangle$. The latter is won according to our assumption that f is a solution-1 to $N(s)$, and therefore the play $\langle w, \vec{u} \rangle$ is won, too. Thus, for every function g, the $N(w)$-play with my strategy f and the opponent's strategy g is won, and that means that f is a solution-1 to the game $N(w)$. Then, by the induction hypothesis, f is a solution-2 to $N(w)$. Since $f(s) = w$, we immediately get that f is a solution-2 to $N(s)$.

Suppose now f is a solution-2 to $N(s)$. Then $f(s) = w$ for some $w \in R(s)$ such that f is a solution-2 to $N(w)$. By the induction hypothesis, f is a solution-1 to $N(w)$. Again, for an arbitrary function g, the $N(w)$-play with my strategy f and the opponent's strategy g has the form $\langle w, \vec{u} \rangle$ and the $N(s)$-play with my strategy f and the opponent's strategy g is $\langle s, w, \vec{u} \rangle$. Since f is a solution-1 to $N(w)$, $\langle w, \alpha \rangle$ is won and, therefore, $\langle s, w, \vec{u} \rangle$ is won. We conclude that f is a solution-1 to $N(s)$. ♣

Fact 2.10 *A function f is a solution to a game G if and only if for any finite function (see Convention 2.3) g, the G-play with my strategy f and the opponent's strategy g is won.*

Proof. Taking 2.7 as the basic definition of solution, the "only if" direction is trivial. For the "if" direction, suppose f is not a solution to G, i.e. there is a function h such that the G-play P with my strategy f and the opponent's strategy h is lost. Let then g be the function which coincides with h for the positions that participate in P and is undefined for any other object. Clearly the G-play with my strategy f and the opponent's strategy g is the selfsame P. On the other hand, since P (as well as any legal sequence of positions of G) is finite, the function g is finite. Thus, g is a finite function such that the G-play with my strategy f and the opponent's strategy g is lost. ♣

Definition 2.11

- A game is said to be *solvable*, if it has a solution (there is a solution to it).
- A game is said to be *effectively solvable*, if it has an effective (recursive) solution.
- In general, for a class Γ of functions, a game is said to be Γ *-solvable*, if it has a solution in Γ.

Definition 2.12 Let $N = \langle W, l, R \rangle$ be a net of games. We define a net of games called the *tracing version* of N and denoted by $N^* = \langle W^*, l^*, R^* \rangle$ as follows:

- W^* is the set of all legal sequences of positions of N;
- $l^*(\langle w_1, \dots w_n \rangle) = l(w_n)$;
- $\langle w_1, \dots, w_n \rangle R^* \langle u_1 \dots, u_m \rangle$ iff $\langle w_1, \dots, w_n \rangle = \langle u_1, \dots, u_{m-1} \rangle$.

And the *tracing version* of a game $G = N(s) = \langle W, l, R, s \rangle$ is the game

$$G^* = N^*(\langle s \rangle) = \langle W^*, l^*, R^*, \langle s \rangle \rangle .$$

Theorem 2.13 *Let $G = \langle W, l, R, s \rangle$ be a game.*

1. *G is solvable iff its tracing version is.*
2. *Suppose l is recursive and W, R are recursively enumerable. Then G is effectively solvable iff its tracing version is.*

Proof. We prove here only the clause 2 of the theorem. The proof of the clause 1 is essentially the same but a bit simplier. Let W,l,R be as supposed.

($\Rightarrow$): Suppose f is an effective solution to G. Let g be the function $W^* \to W^*$ defined as follows:

- $g(\langle w_1, \dots, w_n \rangle) = \langle w_1, \dots, w_n, u \rangle$, if $f(w_n) = u$, and is, say, undefined, if $f(w_n)$ is undefined.

Note that g is effective. We claim that g is a solution to the game G^*. Indeed, suppose that

$$\langle \langle s \rangle, \langle s, w_1 \rangle, \langle s, w_1, w_2 \rangle, \dots, \langle s, w_1, w_2, \dots, w_n \rangle \rangle$$

(possibly $n = 0$) is a lost G^*-play with my strategy g. Then, as it is easy to verify, $\langle s, w_1, w_2, \dots, w_n \rangle$ is a lost G-play with my strategy f, which contradicts with our assumption that f is a solution to G.

($\Leftarrow$): Suppose g is an effective solution to G^* and M_g is a machine that calculates g.

- Let *a good sequence* mean a legal sequence $\langle w_1, \dots, w_n \rangle$ of positions of G such that $w_1 = s$ and for any $1 \leq i < n$, if $l(w_i) = 0$, then $g(\langle w_1, \dots, w_i \rangle) = \langle w_1, \dots, w_i, w_{i+1} \rangle$.

Observe that any position in a G^*-play with my strategy g is a good sequence. In view of our assumptions about W,l,R and g, it is also easy to see that the good sequences can be recursively enumerated. So, let us fix a recursive list of good sequences.

Let now f be a partial recursive function the value of which for an element w of W is calculated by the following machine M_f:

- First M_f checks (from the beginning) the list of good sequences till the moment when a good sequence $\langle t_1, \ldots, t_e \rangle$ will be found such that $t_e = w$. Then M_f simulates the machine M_g with $\langle t_1, \ldots, t_n \rangle$ on the input of the latter; if M_g halts and gives the output $\langle t_1, \ldots, t_n, u \rangle$ for some $u \in R(w)$, then M_f gives the output u.

The claim is that f is a solution to G. To show this, suppose, for a contradiction, that there is a lost G-play $\langle w_1, \ldots, w_n \rangle$ with my strategy f. Let us first verify by induction on i that

$$\text{for any } 1 \leq i \leq n, \text{ there is a good sequence with the last term } w_i. \qquad (1)$$

That is trivial for $i = 1$ because $w_1 = s$ and $\langle s \rangle$ is a good sequence. Suppose now $i > 1$. Then, by the induction hypothesis, there is a good sequence $\langle u_1, \ldots, u_m \rangle$ with $u_m = w_{i-1}$. If $l(u_m) = 1$, then obviously $\langle u_1, \ldots, u_m, w_i \rangle$ is a good sequence. Suppose now that $l(u_m) = 0$, i.e. $l(w_{i-1}) = 0$. Then, as $\langle w_1, \ldots, w_n \rangle$ is a G-play with my strategy f, we have $w_i = f(w_{i-1})$. According to the definition of f, this means that for some good sequence $\langle v_1, \ldots, v_k \rangle$ with $w_k = w_{i-1}$, we have $g(\langle v_1, \ldots, v_k \rangle) = \langle v_1, \ldots, v_k, w_i \rangle$. Then, clearly, $\langle v_1, \ldots, v_k, w_i \rangle$ is a good sequence, and (1) is proved.

Thus, by (1), there is a good sequence $\langle t_1, \ldots, t_e \rangle$ with $t_e = w_n$. We may suppose that $\langle t_1, \ldots, t_e \rangle$ is the first good sequence with the last term w_n in the list of good sequences.

Let h be a function such that for any $1 \leq i < e$, if $l(t_i) = 1$, then $G(\langle t_1, \ldots, t_i \rangle) = \langle t_1, \ldots, t_i, t_{i+1} \rangle$. Then

$$\langle \langle t_1 \rangle, \langle t_1, t_2 \rangle, \ldots, \langle t_1, t_2, \ldots, t_e \rangle \rangle$$

is an initial segment of the G^*-play with my strategy g and the opponent's strategy h. Since $l^*(\langle t_1, \ldots, t_e \rangle) = 0$ and g is a solution to G^*, $\langle t_1, \ldots, t_e \rangle$ cannot be the last position of this play, i.e. we must have $g(\langle t_1, \ldots, t_e \rangle) = \langle t_1, \ldots, t_e, r \rangle$ for some $r \in R(t_e) = R(w_n)$. But then, by the definition of f, we have $f(w_n) = r \in R(w_n)$, which contradicts with our assumption that w_n is the last position of a G-play (namely of $\langle w_1, \ldots, w_n \rangle$) with my strategy f: at least, the position r must follow w_n in this play. The theorem is proved. ♣

Lemma 2.14 *Suppose $N(s) = \langle W, l, R, s\rangle$ is a game such that $l(s) = 1$ and for each $u \in R(s)$, $N(u)$ is solvable. Then $N(s)$ is solvable.*

Proof. For each $u \in R(s)$, let us fix a solution g_u to $N(u)$. We define a function f and show that it is a solution to $N^*(\langle s\rangle)$, the tracing version of $N(s)$; By Theorem 2.13, that will mean that there is a solution to $N(s)$. So, for any $u \in R(s)$ and $v_1, \ldots, v_n, v_{n+1}$ such that $v_1 = u$ and $g_u(v_n) = v_{n+1}$, let

$$f(\langle s, v_1, \ldots, v_n\rangle) = \langle s, v_1, \ldots, v_n, v_{n+1}\rangle .$$

Any $N^*(\langle s\rangle)$-play with my strategy f looks like

$$\langle\langle s\rangle, \langle s, v_1\rangle, \ldots, \langle s, v_1, \ldots, v_n\rangle\rangle ,$$

where, unless $n = 0$, we have $v_1 = u \in R(s)$. It is easy to verify that if such a play is lost, then ($n \neq 0$ and) $\langle v_1, \ldots, v_n\rangle$ is a lost $N(u)$-play with my strategy g_u. But this is impossible because g_u is a solution to $N(u)$. ♣

Theorem 2.15 *To any game there is either a solution or an antisolution, i.e. exactly one of the players has a winning strategy.*

Proof. Before we start proving, note that almost all the definitions and facts concerning games enjoy perfect duality: we can interchange everywhere "solution" and "antisolution", "I" and "the opponent", "1" and "0", "won" and "lost".

Fix a game $N(s) = \langle W, l, R, s\rangle$.

First observe that both players cannot have winning strategies for $N(s)$, for otherwise the play corresponding to these two strategies should be simultaneously won and lost, which is impossible.

Let h be the length of $N(s)$. We may suppose that for every $s \neq w \in W$, $sR^{trans}w$ (R^{trans}—the transitive closure of R), which means that the length of $N(w)$ for any such w is less than h.

By induction on $\leq h$ lengths we are going to show that for an arbitrary element w of W, one of the players has a winning strategy for $N(w)$. Before using induction, we consider four cases.

Case 1: $l(w) = 0$ and there is $u \in R(w)$ such that I have a winning strategy g for $N(u)$.

Let then $f(w) = u$ and for any $w \neq v \in W$, $f(v) = g(v)$. Since w can never appear in an $N(u)$-play (because of the converse well-foundedness of R), it is clear that f is a solution to $N(u)$, whence, by 2.8, f is a solution to $N(w)$.

Case 2: $l(w) = 1$ and there is $u \in R(w)$ such that the opponent has a winning strategy g for $N(u)$.

Dual to the previous case: we can define an antisolution f to $N(w)$.

Case 3: $l(w) = 1$ and for any $u \in R(w)$ there is a solution to $N(u)$.
Then, by Lemma 2.14, there is a solution to $N(w)$.
Case 4: $l(w) = 0$ and for any $u \in R(w)$ there is an antisolution to $N(u)$.
Dual to the case 3, with the conclusion that there is an antisolution to $N(w)$.
Now it remains to show that one of the above cases always takes place. Indeed,

Suppose $l(w) = 0$ and the case 4 is "not the case", i.e. there is $u \in R(w)$ such that $N(u)$ has no antisolution. Since the length of $N(u)$ is less than the length of $N(w)$, we can use the induction hypothesis to $N(u)$ and conclude that I have a winning strategy for $N(u)$, i.e. we deal with the case 1.

Suppose now $l(w) = 1$ and the case 3 does not take place, i.e. there is $u \in R(w)$ such that $N(u)$ has no solution. Then, by the induction hypothesis, there is an antisolution to $N(u)$, which means that we deal with the case 2. ♣

3 Sentences as games

The language L which we consider in this section is that of classical first order predicate logic (without identity, functional or individual constants) together with the two additional binary connectives $\bigtriangledown$ and $\bigtriangleup$.

More precisely,

Definition 3.1 The alphabet of the language L consists of:

- *predicate letters*:

$$P_1^0, P_2^0, P_3^0, \ldots, P_1^1, P_2^1, P_3^1, \ldots, P_1^2, P_2^2, P_3^2, \ldots, \ \ldots$$

  the upper index corresponds to the arity of the given predicate letter.

- Individual *variables*:

$$v_1, v_2, v_3, \ldots.$$

- *Propositional connectives*: $\neg$ (negation), $\vee$ (additive disjunction), $\wedge$ (additive conjunction), $\bigtriangledown$ (multiplicative disjunction), $\bigtriangleup$ (multiplicative conjunction).

- *Quantifiers*: $\exists$ (existential quantifier), $\forall$ (universal quantifier).

- Technical signs: , (comma), ((left parenthesis),) (right parentesis).

Definition 3.2

- α is an *atom*, if $\alpha = P_i^0$ or $\alpha = P_i^n(x_1, \ldots, x_n)$, where ($n \geq 1$ and) $x_1, \ldots, x_n$ are variables.

- α is a *negated atom*, if $\alpha = \neg\beta$ for some atom β.

Definition 3.3 *Formulas* of L are the elements of the smallest class Fm such that, saying "α is a formula" for "$\alpha \in Fm$", we have:

1. Atoms and negated atoms are formulas.
2. If α and β are formulas, then $(\alpha) \vee (\beta)$, $(\alpha) \wedge (\beta)$, $(\alpha)\nabla(\beta)$, $(\alpha) \bigtriangleup (\beta)$ are formulas.
3. If α is a formula and x is a variable, then $\exists x(\alpha)$ and $\forall x(\alpha)$ are formulas.

We often omit some parentheses in formulas, if this does not lead to any confusions.

Thus, in the formal language we prefer to restrict the scope of $\neg$ only to atoms. However, we introduce $\neg\alpha$ for complex formulas as an abbreviation defined as follows:

Definition 3.4

- $\neg(\neg\alpha) =_{df} \alpha$
- $\neg(\alpha \vee \beta) =_{df} \neg\alpha \wedge \neg\beta$
- $\neg(\alpha \wedge \beta) =_{df} \neg\alpha \vee \neg\beta$
- $\neg(\alpha\nabla\beta) =_{df} \neg\alpha \bigtriangleup \neg\beta$
- $\neg(\alpha \bigtriangleup \beta) =_{df} \neg\alpha\nabla\neg\beta$
- $\neg(\exists x\alpha) =_{df} \forall x\neg\alpha$
- $\neg(\forall x\alpha) =_{df} \exists x\neg\alpha$.

We define a *free occurence* of a variable x in a formula in the usual way: such an occurence is free, if it is not in the scope of an occurence of $\exists x$ or $\forall x$.

A *closed formula* or *sentence* is a formula without free occurences of variables.

Definition 3.5 Suppose $\mathcal{D}$ is a nonempty set (of "individuals"). A *sentence* (resp. an *atomic sentence*) *with parameters in* $\mathcal{D}$ is a pair $\langle\alpha, f\rangle$, where α is a formula (resp. an atomic formula) and f is a function that assigns to each variable occuring freely in α an element of $\mathcal{D}$.

We can think of sentences with parameters in $\mathcal{D}$ as formulas in which the free variables are "substituted by elements of $\mathcal{D}$", and write $\alpha(a_1, \ldots, a_n)$ for $\langle\alpha(x_1, \ldots, x_n), f\rangle$, if $f(x_1) = a_1, \ldots, f(x_n) = a_n$.

Definition 3.6 A *model* is a triple $M = \langle \mathcal{D}_M, \ell_M, \mathcal{R}_M \rangle$ such that:

- $\mathcal{D}_M$ is a nonempty set, called the *domain of individuals*;
- ℓ_M is a function {atomic sentences with parameters in $\mathcal{D}_M$} $\to \{0, 1\}$, called the *prelabelling function*;
- $\mathcal{R}_M$ is a converse well-founded binary relation on {atomic sentences with parameters in $\mathcal{D}_M$}, called the *predevelopment relation.*

Definition 3.7 A model M is said to be *elementary*, if the relation $\mathcal{R}_M$ is empty.

Definition 3.8 Let M be a model. We define $N_M = \langle W_M, l_M, R_M \rangle$, the *net of L-games induced by M*, as follows:

- W_M is the set of all sentences of L with parameters in $\mathcal{D}_M$.
- 1. $l_M(\alpha) = \ell_M(\alpha)$, if α is an atom;
  2. $l_M(\alpha \vee \beta) = l_M(\exists x \alpha) = 0$;
  3. $l_M(\alpha \wedge \beta) = l_M(\forall x \alpha) = 1$;
  4. $l_M(\neg \alpha) = 1 - l_M(\alpha)$;
  5. $l_M(\alpha \nabla \beta) = max\{l_M(\alpha), l_M(\beta)\}$;
  6. $l_M(\alpha \bigtriangleup \beta) = min\{l_M(\alpha), l_M(\beta)\}$.
- $\phi R_M \psi$ iff one of the following holds:
  1. ϕ, ψ are atoms and $\phi \mathcal{R}_M \psi$;
  2. $\phi = \alpha * \beta$, where $* \in \{\vee, \wedge\}$, and $\psi = \alpha$ or $\psi = \beta$;
  3. $\phi = *x\alpha(x)$, where $* \in \{\exists, \forall\}$, and $\psi = \alpha(a)$ for some $a \in \mathcal{D}_M$;
  4. $\phi = \neg\alpha$, $\alpha R_M \alpha'$ for some α' and $\psi = \neg\alpha'$.
  5. $\phi = \alpha * \beta$, where $* \in \{\nabla, \bigtriangleup\}$, and:
     - $l_M(\alpha) = l_M(\phi)$, $\alpha R_M \alpha'$ for some α' and $\psi = \alpha' * \beta$, or
     - $l_M(\beta) = l_M(\phi)$, $\beta R_M \beta'$ for some β' and $\psi = \alpha * \beta'$.

The standard model of arithmetic defined below will serve as an example of an elementary model.

Definition 3.9 We fix the three predicate letters P_1^2, P_1^3, P_2^3 and call them *arithmetical predicate letters*; those formulas (sentences, atoms) which contain only arithmetical predicate letters, we call *arithmetical formulas* (sentences, atoms), and the fragment of our language L which allows only arithmetical formulas — the *arithmetical language.*

Definition 3.10 The *standard model of arithmetic*, denoted by S, is the following elementary model $\langle \mathcal{D}_S, \ell_S, \emptyset \rangle$:

$\mathcal{D}_S$ is the set $\omega = \{0, 1, 2, \ldots\}$ of natural numbers;

for any nonarithmetical atomic sentence α with parameters in ω, $\ell_S(\alpha) = 0$ (in fact we don't care about the values of ℓ_S for nonarithmetical atoms), and for any natural numbers k, l, m we have:

- $\ell_S(P_1^2(k,l)) = 1 \iff l = k;$
- $\ell_S(P_1^3(k,l,m)) = 1 \iff m = k + l;$
- $\ell_S(P_2^3(k,l,m)) = 1 \iff m = k \times l.$

In the sequel we will usually use the expressions $b = a$, $c = a+b$, $c = a \times b$ instead of $P_1^2(a,b)$, $P_1^3(a,b,c)$, $P_2^3(a,b,c)$, respectively.

Example. The sequence of the following sentences with parameters in ω is a legal sequence of positions of N_S, in fact a won play (for an explanation of the notation N_S see Definition 3.8):

1. $(0 = 1 \vee \forall v_1 \exists v_2 (v_1 = v_2)) \bigtriangleup (\exists v_1 \forall v_2 (v_1 + v_2 = v_2) \bigtriangledown 2 = 3)$
2. $\forall v_1 \exists v_2 (v_1 = v_2) \bigtriangleup (\exists v_1 \forall v_2 (v_1 + v_2 = v_2) \bigtriangledown 2 = 3)$
3. $\forall v_1 \exists v_2 (v_1 = v_2) \bigtriangleup (\forall v_2 (0 + v_2 = v_2) \bigtriangledown 2 = 3)$
4. $\exists v_2 (124 = v_2) \bigtriangleup (\forall v_2 (0 + v_2 = v_2) \bigtriangledown 2 = 3)$
5. $124 = 124 \bigtriangleup (\forall v_2 (0 + v_2 = v_2) \bigtriangledown 2 = 3)$
6. $124 = 124 \bigtriangleup (0 + 18 = 18 \bigtriangledown 2 = 3)$.

Example. The standard model of arithmetic S is an example of an elementary model. We now describe a natural example of a nonelementary model $T = \langle \mathcal{D}_T, \ell_T, \mathcal{R}_T \rangle$:

- $\mathcal{D}_T = \mathcal{D}_S = \omega$.
- ℓ_T coincides with ℓ_S for all atoms except those of the form $P_1^1(n)$, for which we have: $\ell_T(P_1^1(n)) = 1$ if and only if n is the Gödel number of an arithmetical sentence which has the label 1 in the standard model;
- for any atomic sentences α and β with parameters in ω, $\alpha \mathcal{R} \beta$ if and only if, for some $m, n \in \omega$, the following holds: $\alpha = P_1^1(m)$, $\beta = P_1^1(n)$, m and n are the Gödel numbers of some arithmetical sentences γ_m and γ_n, respectively, and γ_n is a development of γ_m in the standard model.

Roughly, we have augmented the arithmetical language with an additional predicate $P_1^1(x)$, which is interpreted as "x is the Gödel number of a true arithmetical sentence".

The sequence of the following five sentences is a legal sequence of positions of N_T; in fact it is a won play, though its first sentence is not true in T, which means that if the opponent had been a bit more "clever", he could have won the play. $\ulcorner\alpha\urcorner$ below is the standard notation for the Gödel number of α:

1. $\forall v_1(v_1 = 5 \vee P_1^1(v_1))$
2. $\ulcorner\forall v_1\exists v_2(v_1 = v_2)\urcorner = 5 \vee P_1^1(\ulcorner\forall v_1\exists v_2(v_1 = v_2)\urcorner)$
3. $P_1^1(\ulcorner\forall v_1\exists v_2(v_1 = v_2)\urcorner)$
4. $P_1^1(\ulcorner\exists v_2(7 = v_2)\urcorner)$
5. $P_1^1(\ulcorner(7 = 7)\urcorner)$.

Identifying $\triangledown$ with $\vee$ and $\triangle$ with $\wedge$, we can think of our language L as the language of classical predicate logic. A model in classical logic is understood as a pair $M = \langle\mathcal{D}_M, \mathcal{G}_M\rangle$, where $\mathcal{D}_M$ is a nonempty set (domain of individuals) and $\mathcal{G}_M$ is a function that assigns to each n-ary predicate letter P of the language an n-ary relation $\mathcal{G}_M^P$ on $\mathcal{D}_M$. There is one-to-one correspondence brtween such functions $\mathcal{G}_M$ and functions ℓ_M of the type {atomic sentences with parameters in $\mathcal{D}_M$} $\to$ $\{0,1\}$, which is established by setting that for any n-ary predicate letter P and any individuals $a_1, \dots, a_n \in \mathcal{D}_M$,

$$G_M^P(a_1, \dots, a_n) \Leftrightarrow \ell_M(P(a_1, \dots, a_n)) = 1.$$

Thus, a classical model is nothing but a model in our sense but without a predevelopment relation. In view of this, we can think of each classical model $M = \langle\mathcal{D}_M, \ell_M\rangle$ as the elementary model $M = \langle\mathcal{D}_M, \ell_M, \emptyset\rangle$. In the sequel we shall use "classical model" and "elementary model" as synonyms.

Definition 3.11 Let M be an elementary (classical) model and ϕ a sentence with parameters in $\mathcal{D}_M$. Then $CV_M(\phi)$, the *classical value* of ϕ in M, is defined by the following induction on the complexity of ϕ:

- $CV_M(\alpha) = \ell_M(\alpha)$, if α is an atom;
- $CV_M(\neg\alpha) = 1 - CV_M(\alpha)$;

- $CV_M(\alpha \vee \beta) = CV_M(\alpha \nabla \beta) = max\{(CV_M(\alpha), CV_M(\beta)\}$;
- $CV_M(\alpha \wedge \beta) = CV_M(\alpha \bigtriangleup \beta) = min\{CV_M(\alpha), CV_M(\beta)\}$;
- $CV_M(\exists x \alpha(x)) = max\{CV_M(\alpha(a)) : \ a \in \mathcal{D}_M\}$;
- $CV_M(\forall x \alpha(x)) = min\{CV_M(\alpha(a)) : \ a \in \mathcal{D}_M\}$.

As we see, from the point of view of classical values, ∇ is the same as $\vee$ and $\bigtriangleup$ the same as $\wedge$.

Theorem 3.12 *Let M be an elementary model and ϕ a sentence with parameters in $\mathcal{D}_M$. Then the classical value of ϕ in M is 1 if and only if the game $N_M(\phi)$ is solvable.*

Proof. ($\Rightarrow$:) Suppose $CV(\phi) = 1$ and show, by induction on the complexity of ϕ, that $N_M(\phi)$ is solvable.

Case 1: ϕ is an atom or a negated atom. Then $R_M(\phi) = \emptyset$ and $l_M(\phi) = CV(\phi) = 1$. Obviously any (even empty) function is a solution to $N_M(\phi)$, because the only possible $N_M(\phi)$-play is $\langle \phi \rangle$.

Case 2: $\phi = \alpha_1 \vee \alpha_2$. Then $max\{CV_M(\alpha_1), CV_M(\alpha_2)\} = 1$. We may suppose that $CV_M(\alpha_1) = 1$. Then, by the induction hypothesis, there is a solution g to $N_M(\alpha_1)$. Let $f(\phi) = \alpha_1$ and for any sentence $\gamma \neq \phi$, $f(\gamma) = g(\gamma)$. We claim that f is a solution to $N_M(\phi)$. Indeed, suppose there is a lost $N_M(\phi)$-play with my strategy f. It will look like $\langle \phi, \gamma_1, \ldots, \gamma_n \rangle$ for some $\gamma_1, \ldots, \gamma_n$, $n \geq 1$. Observe that then $\langle \gamma_1, \ldots, \gamma_n \rangle$ is a lost $N_M(\alpha_1)$-play with my strategy g, which is impossible because, according to our assumption, g is a solution to $N_M(\alpha_1)$.

Case 3: $\phi = \alpha_1 \wedge \alpha_2$. Then $CV_M(\alpha_1) = CV_M(\alpha_2) = 1$ and, by the induction hypothesis, both $N_M(\alpha_1)$ and $N_M(\alpha_2)$ are solvable. Now, since α_1 and α_2 are the only developments of ϕ, it follows by Lemma 2.14 that $N_M(\phi)$ is solvable.

Case 4: $\phi = \alpha_1 \nabla \alpha_2$. Then $max\{CV_M(\alpha_1), CV_M(\alpha_2)\} = 1$. We may suppose that $CV_M(\alpha_1) = 1$. By the induction hypothesis, there is a solution g to $N_M(\alpha_1)$.

Let f be such a function that for any sentences β_1, β_1' and β_2, if $g(\beta_1) = \beta_1'$, then $f(\beta_1 \nabla \beta_2) = \beta_1' \nabla \beta_2$.

Roughly, to play an $N_M(\alpha_1 \nabla \alpha_2)$-play with my strategy f means to play, using the strategy g, only in the left component of the multiplicative disjunction and to do nothing in the right component. Suppose there is a lost $N_M(\phi)$-play

$$\left\langle \beta_1 \nabla \gamma_1, \ldots, \beta_n \nabla \gamma_n \right\rangle$$

(where $\beta_1 \nabla \gamma_1 = \alpha_1 \nabla \alpha_2 = \phi$) with my strategy f. Let $k_1 < \ldots < k_m$ be all the numbers k in the interval $1 < k \leq n$ such that $\beta_{k-1} \neq \beta_k$. Intuitively,

$k \in \{k_1, \ldots, k_m\}$ means that the position $\beta_k \nabla \gamma_k$ has appeared as a result of moving in the left component of the multiplicative disjunction; all the other positions appear as a result of the opponent's move in the right component and they are not interesting for us. It is now easy to see that $\langle \alpha_1, \beta_{k_1}, \ldots, \beta_{k_m} \rangle$ is a lost $N(\alpha_1)$-play with my strategy g, which is impossible because g was my winning strategy for this game. Thus no $N_M(\phi)$-play with my strategy f can be lost, f is a solution to $N_M(\phi)$.

Case 5: $\phi = \alpha_1 \triangle \alpha_2$. Then $CV_M(\alpha_1) = CV_M(\alpha_2) = 1$. By the induction hypothesis, there are solutions g_1 and g_2 to α_1 and α_2, respectively.

Let f be such a function that for any sentences β_1, β_1', β_2 and β_2',

- $f(\beta_1 \triangle \beta_2) = \beta_1' \triangle \beta_2$, if $l_M(\beta_1) = 0$ and $g_1(\beta_1) = \beta_1'$;
- $f(\beta_1 \triangle \beta_2) = \beta_1 \triangle \beta_2'$, if $l_M(\beta_1) = 1$, $l_M(\beta_2) = 0$ and $g_2(\beta_2) = \beta_2'$.

Intuition: To use the strategy f in an $N_M(\alpha_1 \triangle \alpha_2)$-play means to use the strategy g_1 in the first component of the multiplicative conjunction and the strategy g_2 in the second component. Suppose there is a lost $N_M(\phi)$-play

$$\left\langle \beta_1 \nabla \gamma_1, \ldots, \beta_n \nabla \gamma_n \right\rangle$$

(where $\beta_1 \triangle \gamma_1 = \alpha_1 \triangle \alpha_2 = \phi$) with my strategy f. As this play is lost, the label of its last position $\beta_n \triangle \gamma_n$ is 0, i.e. one of the positions β_n, γ_n has the label 0. We may suppose that $l_M(\beta_n) = 0$. Let then $k_1 < \ldots < k_m$ be all the numbers k in the interval $1 < k \leq n$ such that $\beta_{k-1} \neq \beta_k$. Thus, $k \in \{k_1, \ldots, k_m\}$ means that the position $\beta_k \nabla \gamma_k$ has appeared as a result of moving in the left component of the multiplicative conjunction. Now it remines to verify (what can be easily done) that $\langle \alpha_1, \beta_{k_1}, \ldots, \beta_{k_m} \rangle$ is a lost $N_M(\alpha_1)$-play with my strategy g_1, which is impossible because g_1 was my winning strategy for this game. This contradiction proves that f is a solution to $N_M(\phi)$.

Case 6: $\phi = \exists x \alpha(x)$. Similar to the case 2.

Case 7: $\phi = \forall x \alpha(x)$. Similar to the case 3.

($\Leftarrow$:) We have just showed that if $CV_M(\phi) = 1$, then there is a solution to $N_M(\phi)$. In the dual way we can show that if $CV_M(\phi) = 0$ (i.e., if $CV_M(\phi) \neq 1$), then there is an antisolution to $N_M(\phi)$, which rules out solvability of $N_M(\phi)$. ♣

Thus truth (in the classical sense) of a sentence in a classical (elementary) model can be defined in terms of solvability of the corresponding game. It seems quite natural to extend this notion (and use the same word "true") to all models, not restricting it to only elementary models, what is actually done in the first clause of Definition 3.13; this extension is conservative in the sense that as soon as a classical model is considered, the two notions of truth — the old and the new — are equivalent.

Definition 3.13 Let α be a sentence and M a model (not necessarily an elementary one).

- α is said to be *true* in M, if the game $N_M(\alpha)$ is solvable.
- α is said to be *effectively true* in M, if the game $N_M(\alpha)$ is effectively solvable.
- In general, for a class Γ of functions, α is said to be Γ-*true* in M, if the game $N_M(\alpha)$ is Γ-solvable.

Fact 3.14 *A sentence ϕ is true in all models if and only if ϕ is true in all elementary models.*

Proof.

The ($\Rightarrow$) direction is trivial.

($\Leftarrow$): Suppose M is a nonelementary model and ϕ is false (not true) in M. Let $M' = \langle \mathcal{D}_{M'}, \ell_{M'}, \emptyset \rangle$ be the following elementary model:

- $\mathcal{D}_{M'} = \mathcal{D}_M$;
- for any atom α, $\ell_{M'}(\alpha) = 1$ iff $N_M(\alpha)$ is solvable.

We claim that ϕ is not true in M'. To prove this, by Theorem 3.12, it is enough to show that $CV_{M'}(\phi) = 0$. Suppose, for a contradiction, that $CV_{M'}(\phi) = 1$. We will be done if we prove that then

$$N_M(\phi) \textit{ is solvable}, \tag{2}$$

because this contradicts with our assumption that ϕ is not true in M. We prove (2) by induction on the complexity of ϕ.

Case 1: ϕ is an atom. As $CV_{M'}(\phi) = 1$, we have $\ell_{M'}(\phi) = 1$, which, by our choice of M', means that $N_M(\phi)$ is solvable.

Case 2: ϕ is a negated atom $\neg\psi$. As $CV_{M'}(\neg\psi) = 1$, we have $\ell_{M'}(\psi) = 0$, which, by our choice of M', means that $N_M(\psi)$ is not solvable. Then, by Theorem 2.15, there is an antisolution g to $N_M(\psi)$. Let f be such a function that for any atoms α and β, if $g(\alpha) = \beta$, then $f(\neg\alpha) = \neg\beta$. It is not hard to see that f is then a solution to $\neg\psi$.

Cases 3-8: ϕ is a complex formula. We can literally repeat the reasoning from the proof of Theorem 3.12 (cases 2-7), only changing in the latter "CV_M" for "$CV_{M'}$".

(2) is thus proved and we are done. ♣

Having different notions of truth, we can define different notions of tautology:

Definition 3.15 Let α be a sentence.

- α is said to be a *tautology*, if α is true in every model.
- α is said to be an *effective tautology*, if α is effectively true in every model.
- In general, for a class Γ of functions, α is said to be a Γ-*tautology*, if α is Γ-true in every model.

In view of 3.11, 3.12, 3.13 (first clause) and 3.14, the set of tautologies in our sense coincides with the set of classical tautologies, where the two sorts of disjunction, as well as the two sorts of conjunction, are simply identified. This recursively enumerable set is given by, say, the classical sequential calculus, in which the rules for $\vee$ are duplicated also for $\bigtriangledown$ and the rules for $\wedge$ are duplicated also for $\bigtriangleup$. The continuation of the present work will be devoted to studying the set of effective tautologies.

References

[1] S.Abramsky and R.Jagadeesan, *Games and Full Completeness for Multiplicative Linear Logic.* Dept. of Computing, Imperial Colledge of Science, Technology and Medicine, Technical Report DoC 92/24, September, 1992.

[2] A.Blass, *A game semantics for linear logic.* Annals of Pure and Applied Logic, v.56 (1992), pp. 183-220.

[3] J.Y.Girard, *Linear logic.* Theoretical Computer Science, v.50-1 (1987), pp. 1-102.

[4] S.C.Kleene, *Introduction to Metamathematics.* New York, 1952.

[5] K.Lorenz, *Dialogspiele als semantische Grundlage von Logikkalkülen.* Arch. Math. Logik Grundlag., v.11 (1968), pp.32-55, 73-100.

[6] P.Lorenzen, *Ein dialogisches Konstruktivitätskriterium.* In: Infinitistic Methods (PWN, Warsaw, 1959), pp.193-200.

[7] A.S.Troelstra, *Realizability.* ILLC Prepublication Series for Mathematical Logic and Foundations ML-92-09. Amterdam, 1992.

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