Institute for Logic, Language and Computation

ON THE INDEPENDENT AXIOMATIZABILITY OF MODAL AND INTERMEDIATE LOGICS

Alexander Chagrov
Michael Zakharyaschev

ILLC Prepublication Series
for Mathematical Logic and Foundations ML-93-17

University of Amsterdam

The ILLC Prepublication Series

1990 *Logic, Semantics and Philosophy of Language*

LP-90-01 Jaap van der Does — A Generalized Quantifier Logic for Naked Infinitives
LP-90-02 Jeroen Groenendijk, Martin Stokhof — Dynamic Montague Grammar
LP-90-03 Renate Bartsch — Concept Formation and Concept Composition
LP-90-04 Aarne Ranta — Intuitionistic Categorial Grammar
LP-90-05 Patrick Blackburn — Nominal Tense Logic
LP-90-06 Gennaro Chierchia — The Variablity of Impersonal Subjects
LP-90-07 Gennaro Chierchia — Anaphora and Dynamic Logic
LP-90-08 Herman Hendriks — Flexible Montague Grammar
LP-90-09 Paul Dekker — The Scope of Negation in Discourse, towards a Flexible Dynamic Montague grammar
LP-90-10 Theo M.V. Janssen — Models for Discourse Markers
LP-90-11 Johan van Benthem — General Dynamics
LP-90-12 Serge Lapierre — A Functional Partial Semantics for Intensional Logic
LP-90-13 Zhisheng Huang — Logics for Belief Dependence
LP-90-14 Jeroen Groenendijk, Martin Stokhof — Two Theories of Dynamic Semantics
LP-90-15 Maarten de Rijke — The Modal Logic of Inequality
LP-90-16 Zhisheng Huang, Karen Kwast — Awareness, Negation and Logical Omniscience
LP-90-17 Paul Dekker — Existential Disclosure, Implicit Arguments in Dynamic Semantics

Mathematical Logic and Foundations

ML-90-01 Harold Schellinx — Isomorphisms and Non-Isomorphisms of Graph Models
ML-90-02 Jaap van Oosten — A Semantical Proof of De Jongh's Theorem
ML-90-03 Yde Venema — Relational Games
ML-90-04 Maarten de Rijke — Unary Interpretability Logic
ML-90-05 Domenico Zambella — Sequences with Simple Initial Segments
ML-90-06 Jaap van Oosten — Extension of Lifschitz' Realizability to Higher Order Arithmetic, and a Solution to a Problem of F. Richman
ML-90-07 Maarten de Rijke — A Note on the Interpretability Logic of Finitely Axiomatized Theories
ML-90-08 Harold Schellinx — Some Syntactical Observations on Linear Logic
ML-90-09 Dick de Jongh, Duccio Pianigiani — Solution of a Problem of David Guaspari
ML-90-10 Michiel van Lambalgen — Randomness in Set Theory
ML-90-11 Paul C. Gilmore — The Consistency of an Extended NaDSet

Computation and Complexity Theory

CT-90-01 John Tromp, Peter van Emde Boas — Associative Storage Modification Machines
CT-90-02 Sieger van Denneheuvel, Gerard R. Renardel de Lavalette — A Normal Form for PCSJ Expressions
CT-90-03 Ricard Gavaldà, Leen Torenvliet, Osamu Watanabe, José L. Balcázar — Generalized Kolmogorov Complexity in Relativized Separations
CT-90-04 Harry Buhrman, Edith Spaan, Leen Torenvliet — Bounded Reductions
CT-90-05 Sieger van Denneheuvel, Karen Kwast — Efficient Normalization of Database and Constraint Expressions
CT-90-06 Michiel Smid, Peter van Emde Boas — Dynamic Data Structures on Multiple Storage Media, a Tutorial
CT-90-07 Kees Doets — Greatest Fixed Points of Logic Programs
CT-90-08 Fred de Geus, Ernest Rotterdam, Sieger van Denneheuvel, Peter van Emde Boas — Physiological Modelling using RL
CT-90-09 Roel de Vrijer — Unique Normal Forms for Combinatory Logic with Parallel Conditional, a case study in conditional rewriting

Other Prepublications

X-90-01 A.S. Troelstra — Remarks on Intuitionism and the Philosophy of Mathematics, Revised Version
X-90-02 Maarten de Rijke — Some Chapters on Interpretability Logic
X-90-03 L.D. Beklemishev — On the Complexity of Arithmetical Interpretations of Modal Formulae
X-90-04 — Annual Report 1989
X-90-05 Valentin Shehtman — Derived Sets in Euclidean Spaces and Modal Logic
X-90-06 Valentin Goranko, Solomon Passy — Using the Universal Modality: Gains and Questions
X-90-07 V.Yu. Shavrukov — The Lindenbaum Fixed Point Algebra is Undecidable
X-90-08 L.D. Beklemishev — Provability Logics for Natural Turing Progressions of Arithmetical Theories
X-90-09 V.Yu. Shavrukov — On Rosser's Provability Predicate
X-90-10 Sieger van Denneheuvel, Peter van Emde Boas — An Overview of the Rule Language RL/1
X-90-11 Alessandra Carbone — Provable Fixed points in $I\Delta_0+\Omega_1$, revised version
X-90-12 Maarten de Rijke — Bi-Unary Interpretability Logic
X-90-13 K.N. Ignatiev — Dzhaparidze's Polymodal Logic: Arithmetical Completeness, Fixed Point Property, Craig's Property
X-90-14 L.A. Chagrova — Undecidable Problems in Correspondence Theory
X-90-15 A.S. Troelstra — Lectures on Linear Logic

1991 *Logic, Semantics and Philosophy of Language*

LP-91-01 Wiebe van der Hoek, Maarten de Rijke — Generalized Quantifiers and Modal Logic
LP-91-02 Frank Veltman — Defaults in Update Semantics
LP-91-03 Willem Groeneveld — Dynamic Semantics and Circular Propositions
LP-91-04 Makoto Kanazawa — The Lambek Calculus enriched with Additional Connectives
LP-91-05 Zhisheng Huang, Peter van Emde Boas — The Schoenmakers Paradox: Its Solution in a Belief Dependence Framework
LP-91-06 Zhisheng Huang, Peter van Emde Boas — Belief Dependence, Revision and Persistence
LP-91-07 Henk Verkuyl, Jaap van der Does — The Semantics of Plural Noun Phrases
LP-91-08 Víctor Sánchez Valencia — Categorial Grammar and Natural Reasoning
LP-91-09 Arthur Nieuwendijk — Semantics and Comparative Logic
LP-91-10 Johan van Benthem — Logic and the Flow of Information

Mathematical Logic and Foundations

ML-91-01 Yde Venema — Cylindric Modal Logic
ML-91-02 Alessandro Berarducci, Rineke Verbrugge — On the Metamathematics of Weak Theories
ML-91-03 Domenico Zambella — On the Proofs of Arithmetical Completeness for Interpretability Logic
ML-91-04 Raymond Hoofman, Harold Schellinx — Collapsing Graph Models by Preorders
ML-91-05 A.S. Troelstra — History of Constructivism in the Twentieth Century
ML-91-06 Inge Bethke — Finite Type Structures within Combinatory Algebras
ML-91-07 Yde Venema — Modal Derivation Rules
ML-91-08 Inge Bethke — Going Stable in Graph Models
ML-91-09 V.Yu. Shavrukov — A Note on the Diagonalizable Algebras of PA and ZF
ML-91-10 Maarten de Rijke, Yde Venema — Sahlqvist's Theorem for Boolean Algebras with Operators
ML-91-11 Rineke Verbrugge — Feasible Interpretability
ML-91-12 Johan van Benthem — Modal Frame Classes, revisited

Computation and Complexity Theory

CT-91-01 Ming Li, Paul M.B. Vitányi — Kolmogorov Complexity Arguments in Combinatorics
CT-91-02 Ming Li, John Tromp, Paul M.B. Vitányi — How to Share Concurrent Wait-Free Variables
CT-91-03 Ming Li, Paul M.B. Vitányi — Average Case Complexity under the Universal Distribution Equals Worst Case Complexity
CT-91-04 Sieger van Denneheuvel, Karen Kwast — Weak Equivalence
CT-91-05 Sieger van Denneheuvel, Karen Kwast — Weak Equivalence for Constraint Sets
CT-91-06 Edith Spaan — Census Techniques on Relativized Space Classes
CT-91-07 Karen L. Kwast — The Incomplete Database
CT-91-08 Kees Doets — Levationis Laus
CT-91-09 Ming Li, Paul M.B. Vitányi — Combinatorial Properties of Finite Sequences with high Kolmogorov Complexity
CT-91-10 John Tromp, Paul Vitányi — A Randomized Algorithm for Two-Process Wait-Free Test-and-Set
CT-91-11 Lane A. Hemachandra, Edith Spaan — Quasi-Injective Reductions
CT-91-12 Krzysztof R. Apt, Dino Pedreschi — Reasoning about Termination of Prolog Programs

Computational Linguistics

CL-91-01 J.C. Scholtes — Kohonen Feature Maps in Natural Language Processing
CL-91-02 J.C. Scholtes — Neural Nets and their Relevance for Information Retrieval
CL-91-03 Hub Prüst, Remko Scha, Martin van den Berg — A Formal Discourse Grammar tackling Verb Phrase Anaphora

Other Prepublications

X-91-01 Alexander Chagrov, Michael Zakharyaschev — The Disjunction Property of Intermediate Propositional Logics
X-91-02 Alexander Chagrov, Michael Zakharyaschev — On the Undecidability of the Disjunction Property of Intermediate Propositional Logics

Institute for Logic, Language and Computation
Plantage Muidergracht 24
1018TV Amsterdam
Telephone 020-525.6051, Fax: 020-525.5101

ON THE INDEPENDENT AXIOMATIZABILITY OF MODAL AND INTERMEDIATE LOGICS

Alexander Chagrov
Tver State University
Tver, Russsia

Michael Zakharyaschev
Institute of Applied Mathematics
Moscow, Russia

ILLC Prepublications
for Mathematical Logic and Foundations
ISSN 0928-3315
Coordinating editor: Dick de Jongh

Research of second author supported by the
Netherlands Organization for Scientific Research (NWO)

received September 1993

On the Independent Axiomatizability of Modal and Intermediate Logics

Alexander Chagrov and Michael Zakharyaschev
Tver State University, Zhelyabova Str.33, Tver 170013
Institute of Applied Mathematics, Miusskaya Sq.4, Moscow 125047
RUSSIA

§**0**. This paper gives a solution to an old problem connected with the efforts to describe the lattices of all normal modal and intermediate logics. The problem is as follows:

> *Does every normal modal or intermediate logic have an independent set of axioms?*

For intermediate logics it was formulated by A. Tsytkin in Logic Notebook [1986, Problem 148].

A way to the negative solution to this problem is opened by the following observation of Kleyman [1983], which is presented here in a form suitable for our purpose:

Lemma 1 *Suppose a logic L_1 has an independent axiomatization. Then, for every finitely axiomatizable logic $L_2 \subset L_1$, the interval of logics $[L_2, L_1] = \{L : L_2 \subseteq L \subseteq L_1\}$ contains an immediate predecessor of L_1, that is a logic $L \subset L_1$ which has no extension lying properly between L and L_1.*

Proof. If L_1 is finitely axiomatizable then the existence of an immediate predecessor of L_1 in $[L_2, L_1]$ follows from Zorn's Lemma.

Suppose now that L_1 has an infinite independent set of axioms $\{\varphi_i : i \in \omega\}$. Since L_2 is a finitely axiomatizable sublogic of L_1, there is $n < \omega$ such that L_2 is contained in the logic with the axioms $\varphi_0, \ldots, \varphi_n$. Let L_3 be the logic with the axioms $\varphi_0, \ldots, \varphi_n, \varphi_{n+2}, \varphi_{n+3}, \ldots$. Since the set of L_1's axioms is independent, $L_2 \subset L_3 \subset L_1$ and $\varphi_{n+1} \notin L_3$. And now again Zorn's Lemma provides us with an immediate predecessor of L_1 in the interval $[L_3, L_1]$. $\dashv$

Thus, to prove that there is a logic without an independent axiomatization it suffices to produce a finitely axiomatizable logic L_2 and its proper extension L_1 having no immediate predecessor in the interval $[L_2, L_1]$.

A lattice (e.g. the lattice of extensions of a given logic) is called *strongly coatomic* if each its interval $[L_2, L_1]$ with $L_2 \subset L_1$ contains an immediate predecessor of L_1. Blok

[1980] proved that the lattice of normal modal logics is not strongly coatomic (more exactly, he showed that the dual lattice of varieties of modal algebras is not *strongly atomic*). However, it seems unlikely that in the interval $[L_2, L_1]$, constructed by Blok and containing no immediate predecessor of L_1, the logic L_2 is finitely axiomatizable; in any case its semantic definition involves the set of squares of natural numbers which can hardly be described by a finite set of axioms.

We will strengthen appropriately Blok's result to construct logics without independent axiomatizations lying above **K4, S4, Grz** and intuitionistic logic, answering incidentally his question concerning the strong coatomicity of the lattices of intermediate logics and modal logics containing **S4**.

§**1**. We use standard notions and notations in the realm of non-classical logic. Here we mention only those of them that have variants.

We denote by $\Box^+\varphi$, $\Diamond^+\varphi$, $\Box^n\varphi$ and $\Diamond^n\varphi$ the formulas $\varphi \wedge \Box\varphi$, $\varphi \vee \Diamond\varphi$, $\underbrace{\Box\ldots\Box}_{n}\varphi$ and $\underbrace{\Diamond\ldots\Diamond}_{n}\varphi$, respectively; $\varphi(\psi/p)$ means the result of replacement of all occurrences of the variable p in φ with ψ.

All modal logics in this paper, except those in the final §, are assumed to be normal, i.e. containing **K** and closed under modus ponens, substitution and necessitation $\varphi/\Box\varphi$. The smallest normal modal logic to contain a logic L and a set of formulas Γ is denoted by $L \oplus \Gamma$. Intermediate logics are consistent extensions of intuitionistic logic **Int** closed under modus ponens and substitution. $L + \Gamma$ means the closure of the set $L \cup \Gamma$ under the latter two rules.

Let L be a logic and Γ, Δ sets of formulas in the language of L. Γ is said to be an *independent set of axioms for* L *over* Δ if, for every $\Sigma \subseteq \Gamma$, L is the closure of $\Sigma \cup \Delta$ under the postulated inference rules of L iff $\Sigma = \Gamma$. For instance, we can say about independent axiomatization of an intermediate logic over **Int** or that of a modal logic over **K**. If Γ is an independent set of axioms for L over $\Delta = \emptyset$ then Γ is called an (*absolutely*) *independent set of axioms for* L. A logic L is *independently axiomatizable* (*over* Δ) if there is an independent set of axioms for L (over Δ).

It is clear that the following lemma holds.

Lemma 2 *If a logic L is independently axiomatizable over a finitely axiomatizable logic then L is absolutely independently axiomatizable.*

As to our semantic apparatus, we use here *differentiated general frames*. Recall that a general frame $\langle \mathfrak{F}, P \rangle$, where $\mathfrak{F} = \langle W, R \rangle$ is a Kripke frame and P a set of possible values in $\mathfrak{F}$, is *differentiated* if, for every two distinct points $x, y \in W$, there is a set $X \in P$ such that $x \in X$ and $y \notin X$. For more information on general frames consult Goldblatt [1976], from which it follows in particular that every normal modal logic is characterized by a class of rooted differentiated general frames.

All our frames are assumed to be transitive. We will define them by drawing diagrams (directed graphs) in which reflexive and irreflexive points are denoted by $\circ$ and $\bullet$, respectively, and, for distinct points x and y, xRy means that there is a directed path from x

to y. We write $x\overline{R}y$ if xRy or $x = y$. So $\mathfrak{F} = \langle W, R\rangle$ is *rooted* if there is $x \in W$ such that $x\overline{R}y$ for every $y \in W$; in this case x is called a *root* of $\mathfrak{F}$.

§**2**. First we give a solution to the independent axiomatizability problem for modal logics containing **K4**. Though afterwards stronger results will be obtained, we prefer to begin with logics above **K4** because in this case our construction is more transparent.

We require a number of modal formulas:

$$\alpha = p \wedge \neg\Diamond p,\ \alpha' = \alpha(\Diamond p/p),\ \alpha'' = \alpha'(\Diamond p/p) = \alpha(\Diamond^2 p/p),$$

$$\alpha_i = \alpha(\Diamond^i\top/p),\ \alpha_{i+1} = \alpha'(\Diamond^i\top/p),\ \alpha_{i+2} = \alpha''(\Diamond^i\top/p),$$

$$\beta = \Diamond\alpha \wedge \neg\Diamond^+\alpha',\ \beta' = \beta(\Diamond p/p),$$

$$\beta_i = \beta(\Diamond^i\top/p) = \Diamond\alpha_i \wedge \neg\Diamond^+\alpha_{i+1},$$

$$\beta_{i+1} = \beta'(\Diamond^i\top/p) = \Diamond\alpha_{i+1} \wedge \neg\Diamond^+\alpha_{i+2},$$

$$\gamma = \Diamond\beta' \wedge \Diamond\alpha'' \wedge \neg\Diamond\beta,\ \gamma' = \gamma(\Diamond p/p),$$

$$\gamma_{i+1} = \gamma(\Diamond^i\top/p) = \Diamond\beta_{i+1} \wedge \Diamond\alpha_{i+2} \wedge \neg\Diamond\beta_i,$$

$$\gamma_{i+2} = \gamma'(\Diamond^i\top/p) = \Diamond\beta_{i+2} \wedge \Diamond\alpha_{i+3} \wedge \neg\Diamond\beta_{i+1}\quad (i \geq 0).$$

Define L_2 as

$$L_2 = \mathbf{K4} \oplus \{ax1, ax2, ax3, ax4, ax5.\psi :\ \psi \in \{\alpha, \beta, \gamma\}\},$$

where

$$ax1 = \alpha_0 \vee \Diamond^+\alpha_1,\ ax2 = \gamma \to \Diamond\gamma,\ ax3 = \gamma \to \Diamond\gamma',$$

$$ax4 = \Diamond\beta' \wedge \Diamond\alpha'' \to \Diamond\gamma,\ ax5.\psi = \Box^+(q \to \neg\psi) \vee \Box^+(\neg q \to \neg\psi).$$

It is not hard to verify that L_2 is consistent. Indeed, all its axioms are valid in the frame shown in Fig. 1 with empty V.

Our first goal is to characterize the constitution of rooted differentiated frames for L_2. To this end we require the following substitution instances of its axioms:

$$ax2.i = \gamma_i \to \Diamond\gamma_i = ax2(\Diamond^i\top/p),$$

$$ax3.i = \gamma_i \to \Diamond\gamma_{i+1} = ax3(\Diamond^i\top/p),$$

$$ax4.i = \Diamond\beta_i \wedge \Diamond\alpha_{i+1} \to \Diamond\gamma_i = ax4(\Diamond^i\top/p)\quad (i \geq 1),$$

$$ax5.\alpha_i = \Box^+(q \to \neg\alpha_i) \vee \Box^+(\neg q \to \neg\alpha_i) = ax5.\alpha(\Diamond^i\top/p),$$

$$ax5.\beta_i = \Box^+(q \to \neg\beta_i) \vee \Box^+(\neg q \to \neg\beta_i) = ax5.\beta(\Diamond^i\top/p),$$

$$ax5.\gamma_{i+1} = \Box^+(q \to \neg\gamma_{i+1}) \vee \Box^+(\neg q \to \neg\gamma_{i+1}) = ax5.\gamma(\Diamond^i\top/p),\quad (i \geq 0).$$

For each $n \geq 1$, by $\mathfrak{F}(n, V)$ we denote the rooted subframe of the frame in Fig. 1 generated by c_n; $\mathfrak{F}(1, V)$ is that frame itself. Here V is a (possibly empty) set of points which see all a_i's and are seen from all c_i's (as it follows from the diagram, b_i's do not see

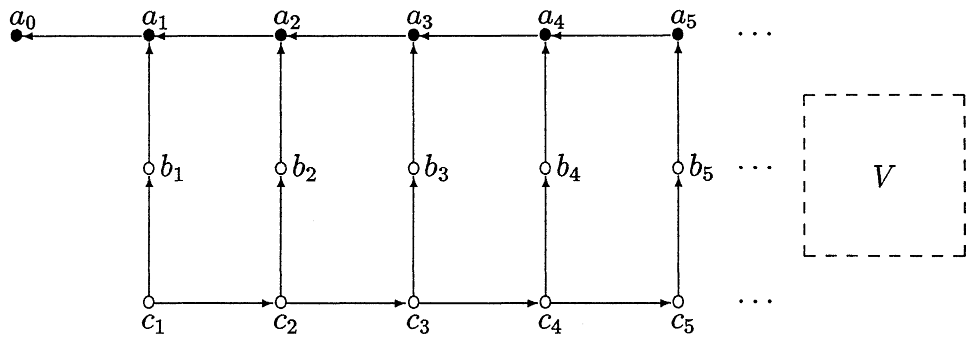

Figure 1:

points in V and are not seen from them); the accessibility relation between points in V is of no concern to us.

Observe that the points a_i, b_{i+1}, c_{i+1}, for $i \geq 0$, are characterized in $\mathfrak{F}(1,V)$ by the formulas α_i, β_{i+1}, γ_{i+1}, respectively, in the sense that under any valuation in $\mathfrak{F}(1,V)$ we have:

$$\{x: x \models \alpha_i\} = \{a_i\},\ \{x: x \models \beta_{i+1}\} = \{b_{i+1}\},\ \{x: x \models \gamma_{i+1}\} = \{c_{i+1}\}.$$

And the points in V are exactly those points in $\mathfrak{F}(1,V)$ at which all $\Diamond\alpha_i$'s are true and all $\Diamond\beta_{i+1}$'s are false, for $i \geq 0$.

Lemma 3 *Suppose $\langle \mathfrak{F}, P\rangle$ is a rooted differentiated frame for L_2. Then $\mathfrak{F}$ is (isomorphic to) a rooted generated subframe of a frame of the form $\mathfrak{F}(1,V)$, for some V, and $\{a_i\}, \{b_{i+1}\}, \{c_{i+1}\}$ are in P, for all $i \geq 0$.*

Proof. Let r be the root of $\mathfrak{F}$. As it was done above, we classify the points in $\mathfrak{F}$ according to which of the formulas α_i, β_i and γ_i are true at them.

Say that a point x in $\mathfrak{F}$ *is of type* a_i (respectively, b_{i+1}, c_{i+1}) if α_i (respectively, β_{i+1}, γ_{i+1}) is true at x; x *is of type* a_ω if $x \models \Diamond\alpha_i$ and $x \not\models \Diamond\beta_j$, for all $i \geq 0$, $j \geq 1$.

Since $\langle \mathfrak{F}, P\rangle \models ax5.\alpha_i$, $\mathfrak{F}$ contains at most one point of type a_i, for each $i \geq 0$. Indeed, suppose there are two distinct points x, y of type a_i. Since $\langle \mathfrak{F}, P\rangle$ is differentiated, there is $X \in P$ such that $x \in X$ and $y \notin X$. Define a valuation $\mathfrak{V}$ in $\mathfrak{F}$ by taking $\mathfrak{V}(q) = X$. Then $r \not\models ax5.\alpha_i$, which is a contradiction. Likewise, for each $i \geq 1$, there are at most one point of type b_i and one point of type c_i.

By the definition of α_i, each point x of type a_i, if any, is irreflexive and must see a point of type a_j, for every $j < i$, and every point accessible from x is of type a_j, for some $j < i$. Therefore, in view of their uniqueness, the points of type a_i, $i \geq 0$, form a descending chain in $\mathfrak{F}$.

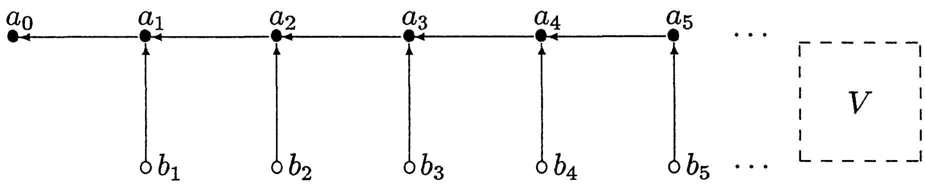

Figure 2:

By $ax3.i$, each point of type c_i for $i \geq 1$, if any, sees a point of type c_j, for every $j > i$, and, by the definition of γ_i, a point of type a_j, for every $j \geq 0$; besides, by $ax2.i$ and the uniqueness of points of type c_i, every such point is reflexive.

If some point x in $\mathfrak{F}$ sees a point of type a_i and neither sees a point of type a_{i+1} nor is of type a_{i+1} itself then, by the definition of β_i, x is of type b_i. Besides, by $ax1$, $ax4.i$ and the properties of points of types c_j and a_j established above, every point accessible from x is of one of the types $a_0, \dots, a_i, b_i$. It follows in particular that x is reflexive. For if x is irreflexive then either it sees only points of types $a_0, \dots, a_i$ and so is of type a_{i+1} itself, contrary to our assumption, or sees a point of type b_i, contrary to the uniqueness of such a point.

It should be clear from the arguments above that each point in $\mathfrak{F}$ is of at most one type. We show now that each point in $\mathfrak{F}$ *is* of some type indeed.

Let x be an arbitrary point in $\mathfrak{F}$. By $ax1$, among the points y such that $x\overline{R}y$ there is at least one point of type a_i, for some $i \geq 0$. If x sees only finitely many points of type a_i, $i \geq 0$, then, as was established above, x is either of type a_i or of type b_i, for some i. If x sees points of type a_i for all $i \geq 0$ then we have the following alternatives. First, x sees no point of type b_j, for $j \geq 1$, which means that x is of type a_ω. Second, x sees a point of type b_j, for some $j \geq 1$, and no point of type b_k, for $0 < k < j$, which means that x of type c_j. We have exhausted all the possibilities, and so each point in $\mathfrak{F}$, in particular r, is of some unique type.

The isomorphism we are after is quite clear now: we map every point of type a_i (respectively, b_{i+1}, c_{i+1}) to a_i (respectively, b_{i+1}, c_{i+1}). The uniqueness of points of types a_i, b_{i+1} and c_{i+1} guarantees that P satisfies the desirable condition. $\dashv$

Now we are in a position to define L_1. Let $\mathcal{C}_1$ be the class of all differentiated frames for L_2 whose underlying Kripke frames have the form shown in Fig. 2. Since $\mathfrak{F}(1, \emptyset) \models L_2$ and the frame in Fig. 2 with empty V is a generated subframe of $\mathfrak{F}(1, \emptyset)$, $\mathcal{C}_1 \neq \emptyset$. We define L_1 as the logic characterized by the class $\mathcal{C}_1$, i.e. put

$$L_1 = \{\varphi : \ \forall \mathfrak{F} \in \mathcal{C}_1 \ \mathfrak{F} \models \varphi\}.$$

Observe that $L_2 \subseteq L_1$; moreover, this inclusion is proper, since $\neg\gamma_1 \in L_1 - L_2$.

Lemma 4 *L_1 has no immediate predecessor in the interval $[L_2, L_1]$.*

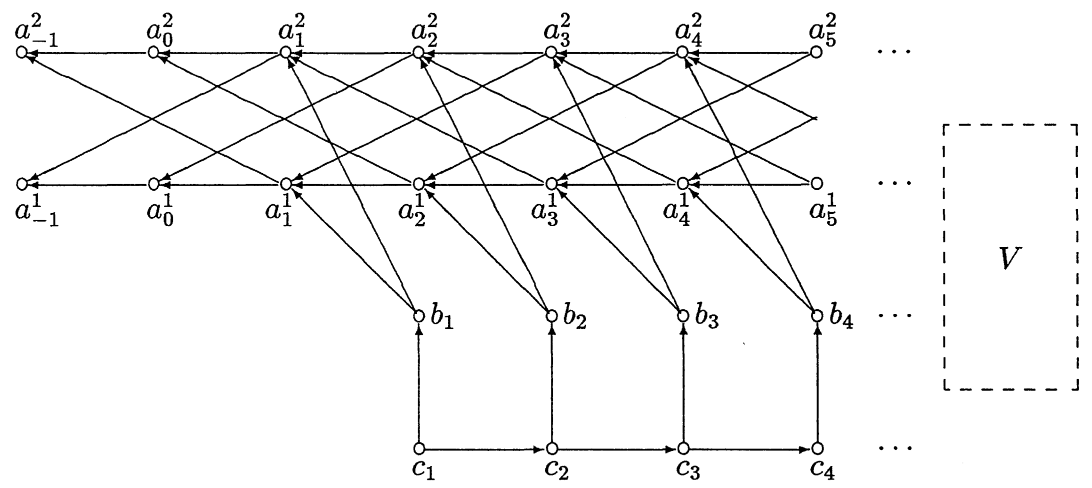

Figure 3:

Proof. Suppose otherwise. Let L be an immediate predecessor of L_1 containing L_2. Since $L \subset L_1$, there exists a rooted differentiated frame $\langle \mathfrak{F}, Q\rangle$ such that $\langle \mathfrak{F}, Q\rangle \models L$ and $\langle \mathfrak{F}, Q\rangle \not\models L_1$. On the other hand, since $L_2 \subseteq L$, we have $\langle \mathfrak{F}, Q\rangle \models L_2$ and so, by Lemma 3, $\langle \mathfrak{F}, Q\rangle$ is of the form $\langle \mathfrak{F}(n, V), P\rangle$, for some $n \geq 1$, V and P. Then $\neg\gamma_n \notin L$; for, as we know, $c_n \models \gamma_n$.

Let $\mathcal{C}'$ be the class of frames containing all the frames in $\mathcal{C}_1$ and also the subframe of $\langle \mathfrak{F}(n, V), P\rangle$ generated by c_{n+1}, and let L' be the logic characterized by $\mathcal{C}'$. By the definition, $L \subseteq L' \subseteq L_1$. Moreover, $\langle \mathfrak{F}(n+1, U), Q\rangle \models \neg\gamma_n$, for every U and Q, from which $\neg\gamma_n \in L'$, and $c_{n+1} \models \gamma_{n+1}$, from which $\neg\gamma_{n+1} \notin L'$, while $\neg\gamma_{n+1} \in L_1$. Therefore, $L \subset L' \subset L_1$, contrary to L being an immediate predecessor of L_1. ⊣

As a consequence of Lemmas 1 and 4 and the fact that L_2 is finitely axiomatizable we obtain our main result:

Theorem 5 *L_1 has no independent axiomatization.*

Remark. It is worth noting that L_1 is recursively axiomatizable. Indeed, using Lemma 3 one can readily prove that

$$L_1 = L_2 \oplus \{\neg\gamma_i : \ i \geq 1\}.$$

§**3.** Now we show how to modify the construction above in order to obtain much stronger logics without independent axiomatizations. First we consider intermediate logics.

The construction in §2 was based upon the frame in Fig. 1 containing the descending chain $a_0, a_1, \ldots$ of irreflexive points. We replace it with "Fine's ladder" consisting of the pairs of reflexive points $a^1_0, a^2_0, a^1_1, a^2_1, \ldots$; see Fig. 3 where the points a^1_{-1} and a^2_{-1} play an auxiliary role (cf. Fine [1974, p.26]).

Since in the case under consideration variable free formulas are not expressive enough — there are only two of them (up to equivalence, of course), namely, $\bot$ and $\top$ — we shall use as a "starting formula" the following one:

$$\delta = (p \to q \vee \neg q) \vee (\neg p \to q \vee \neg q).$$

It is not hard to see that a rooted Kripke frame $\mathfrak{F}$ refutes δ iff it contains a (not necessarily generated) subframe of the form shown in Fig. 4, with a and b having no common successors in $\mathfrak{F}$. Since the frame in Fig. 3 contains only one (modulo interchanging superscripts) subframe of that sort, without loss of generality we may assume that under any valuation refuting δ in the frame we have:

$$a_0^1 \models p,\ a_0^1 \not\models q \vee \neg q,\ a_{-1}^1 \models q,$$

$$a_0^2 \models \neg p,\ a_0^2 \not\models q \vee \neg q,\ a_{-1}^2 \models q.$$

Now, taking the formulas

$$\alpha_{-1}^1 = p \wedge q \to \bot,\ \alpha_{-1}^2 = \neg p \wedge q \to \bot,$$

$$\alpha_0^1 = p \to q \vee \neg q,\ \alpha_0^2 = \neg p \to q \vee \neg q,$$

$$\alpha_{i+1}^1 = \alpha_i^2 \to \alpha_i^1 \vee \alpha_{i-1}^2,\ \alpha_{i+1}^2 = \alpha_i^1 \to \alpha_i^2 \vee \alpha_{i-1}^1,$$

$$\beta_i = \alpha_{i+1}^1 \wedge \alpha_{i+1}^2 \to \alpha_i^1 \vee \alpha_i^2,$$

$$\gamma_{i+1} = \beta_i \to \beta_{i+1} \vee \alpha_{i+2}^1 \vee \alpha_{i+2}^2 \quad (i \geq 0)$$

we obtain, under a valuation refuting δ, a classification of points in the frame in Fig. 3 similar to that in §2:

$$\{x :\ x \not\models \alpha_i^1\} = \{a_i^1\},\ \{x :\ x \not\models \alpha_i^2\} = \{a_i^2\}\ (i \geq -1),$$

$$\{x :\ x \not\models \beta_i\} = \begin{cases} \{b_i\} & \text{if } i \geq 1 \\ \emptyset & \text{if } i = 0 \end{cases},\ \{x :\ x \not\models \gamma_{i+1}\} = \{c_{i+1}\}\ (i \geq 0).$$

Here $x \not\models \varphi \to \psi$ means $x \models \varphi$ and $x \not\models \psi$.

L_2 can be defined by adding to **Int** the following axioms:

$$\beta_0,\ \zeta_2 \to \zeta_1 \vee \delta,\ \zeta_1 \to \eta_1 \vee \xi_2 \vee \xi_2' \vee \delta,$$

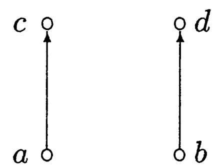

Figure 4:

$$\phi(\xi_0) \vee \xi_1 \vee \xi_1',\ \phi(\xi_1) \vee \xi_1',\ \phi(\xi_2) \vee \xi_1',\ \phi(\eta_1),\ \phi(\zeta_1),$$

where

$$\xi_{-3} = r_1,\ \xi_{-3}' = r_2,\ \xi_{-2} = s_1,\ \xi_{-2}' = s_2,$$
$$\xi_n = \xi_{n-1}' \to \xi_{n-1} \vee \xi_{n-2}',\ \xi_n' = \xi_{n-1} \to \xi_{n-1}' \vee \xi_{n-2}\ (n \geq -1),$$
$$\eta_n = \xi_{n+1} \wedge \xi_{n+1}' \to \xi_n \vee \xi_n'\ (n \geq 0),$$
$$\zeta_n = \eta_{n-1} \to \eta_n \vee \xi_{n+1} \vee \xi_{n+1}'\ (n \geq 1)$$

and $\phi(\varphi \to \psi)$ is an abbreviation for $(t \wedge \varphi \to \psi) \vee (\varphi \to t \vee \psi) \vee \delta$. The meaning and purpose of the axioms above are analogous to those of the axioms in §2; namely, the first axiom is similar to $ax1$, the second one to $ax3$, the third to $ax4$, the forth, fifth and sixth axioms play the same role as $ax5.\alpha$, the seventh is like $ax5.\beta$ and the eighth is like $ax5.\gamma$.

By using these axioms one can prove an analog of Lemma 3 which looks like this: *if a rooted differentiated frame $\langle \mathfrak{F}, P\rangle$ for L_2 refutes δ then $\mathfrak{F}$ is isomorphic to a generated subframe of a frame of the form shown in Fig. 3, with the sets generated by each of the points a_j^i, b_k, c_k, for $i \in \{1,2\}$, $j \geq -1$, $k \geq 1$, belonging to P.* Now, by defining L_1 as the intermediate logic characterized by the class of all differentiated frames validating δ and all differentiated frames for L_2 whose underlying Kripke frames have the form shown in Fig. 3, but with the points c_i's removed, we obtain an analog of Lemma 4 for intermediate logics. Thus we arrive at

Theorem 6 *There is an intermediate logic without an independent axiomatization.*

Lemma 4 (for intermediate logics) provides us with an interval $[L_2, L_1]$ of intermediate logics in which L_1 has no immediate predecessors. This result and the Blok–Esakia Theorem, according to which the lattices of varieties of pseudo–Boolean (alias Heyting) algebras and Grzegorczyk algebras are isomorphic, give a solution to the Blok's [1980] problem:

Theorem 7 (i) *The lattice of varieties of pseudo–Boolean algebras is not strongly atomic.*

(ii) *The lattice of varieties of topological Boolean (and even Grzegorczyk) algebras is not strongly atomic.*

§4. Now we consider the correlation between the independent axiomatizability of intermediate logics and normal modal logics above **S4**. We remind the reader that there is a lattice homomorphism ρ from the lattice of normal extensions of **S4** onto the lattice of extensions of **Int** which is defined as follows: for every normal logic $M \supseteq$ **S4**,

$$\rho M = \{\varphi :\ T\varphi \in M\}$$

where T is the *Gödel translation* prefixing $\Box$ to every subformula of an intuitionistic formula. The logic M is called a *modal companion* of ρM. The set of all modal companions of an intermediate logic $L = \mathbf{Int} + \{\varphi_i :\ i \in I\}$ forms the interval of logics $[\tau L, \sigma L]$, where

$$\tau L = \mathbf{S4} \oplus \{T\varphi_i :\ i \in I\},$$

$$\sigma L = \tau L \oplus \mathbf{Grz} = \tau L \oplus \Box(\Box(p \to \Box p) \to p) \to p,$$

with τ being an isomorphism between the intervals $[\mathbf{Int}, \mathbf{Cl}]$ and $[\mathbf{S4}, \mathbf{S5}]$ and σ the Blok–Esakia isomorphism between the lattices of extensions of **Int** and normal extensions of **Grz** mentioned at the end of §3. For more information on modal companions of intermediate logics and references consult Chagrov and Zakharyaschev [1992].

It follows immediately from these facts and Lemma 4 for intermediate logics that in the intervals $[\tau L_2, \tau L_1]$ and $[\sigma L_2, \sigma L_1]$, where L_1 and L_2 are the intermediate logics constructed in §3, the modal logics τL_1 and σL_1 have no immediate predecessors, respectively. Thus we obtain

Theorem 8 *There are a normal modal logic in the interval* $[\mathbf{S4}, \mathbf{S5}]$ *and a normal logic containing* **Grz** *without independent axiomatizations.*

Remark. It is not hard to modify the proof of Theorems 6 and 8 to construct a normal extension of the Gödel–Löb provability logic **GL** without an independent axiomatization.

Another consequence of the properties of τ and σ mentioned above is

Theorem 9 *For every intermediate logic L, the following conditions are equivalent:*

- *L is independently axiomatizable over* **Int***;*
- *τL is independently axiomatizable over* **S4***;*
- *σL is independently axiomatizable over* **Grz***.*

The maps ρ, τ and σ can be characterized with the help of the apparatus of the modal and intuitionistic canonical formulas, which are denoted here by $\alpha(\mathfrak{F}, \mathfrak{D}, \bot)$ and $\beta(\mathfrak{F}, \mathfrak{D}, \bot)$, respectively; for a brief exposition and further references consult Zakharyaschev [1993]. Namely, a normal logic $M \supseteq \mathbf{S4}$ is a modal companion of an intermediate logic

$$L = \mathbf{Int} + \{\beta(\mathfrak{F}_i, \mathfrak{D}_i, \bot) : \ i \in I\}$$

iff M can be represented in the form

$$M = \mathbf{S4} \oplus \{\alpha(\mathfrak{F}_i, \mathfrak{D}_i, \bot) : \ i \in I\} \oplus \{\alpha(\mathfrak{G}_j, \mathfrak{C}_j, \bot) : \ j \in J\},$$

where each $\mathfrak{G}_j$, for $j \in J$, contains at least one proper cluster; in particular,

$$\tau L = \mathbf{S4} \oplus \{\alpha(\mathfrak{F}_i, \mathfrak{D}_i, \bot) : \ i \in I\},$$

$$\sigma L = \mathbf{S4} \oplus \{\alpha(\mathfrak{F}_i, \mathfrak{D}_i, \bot) : \ i \in I\} \oplus \alpha(\circledcirc, \emptyset).$$

Here $\circledcirc$ is the two point cluster.

Theorem 10 *If an intermediate logic L has an infinite independent axiomatization over* **Int** *then every logic in the interval* $[\tau L, \sigma L]$ *is independently axiomatizable (over* **S4***).*

Proof. Suppose $L = \mathbf{Int} + \{\varphi_i : i \in \omega\}$ with independent axioms φ_i. According to the characterization above, every logic $M \in [\tau L, \sigma L]$ can be represented as

$$M = \mathbf{S4} \oplus \{T\varphi_i : i \in \omega\} \oplus \{\alpha(\mathfrak{F}_i, \mathfrak{D}_i, \bot) : i \in \omega\},$$

where each $\mathfrak{F}_i$, for $i \in \omega$, contains a proper cluster. Therefore,

$$M = \mathbf{S4} \oplus \{T\varphi_i \wedge \alpha(\mathfrak{F}_i, \mathfrak{D}_i, \bot) : i \in \omega\}.$$

The latter axiomatization is independent over **S4**, for otherwise we would have, for some $i \in \omega$

$$T\varphi_i \in M' = \mathbf{S4} \oplus \{T\varphi_j \wedge \alpha(\mathfrak{F}_j, \mathfrak{D}_j, \bot) : j \in \omega, \ j \neq i\},$$

and hence

$$\varphi_i \in \rho M' = \mathbf{Int} + \{\varphi_j : j \in \omega, \ j \neq i\},$$

which is a contradiction. By Lemma 2, M is absolutely independently axiomatizable. ⊣

That L in Theorem 10 is *infinitely* independently axiomatizable over **Int** is essential. For, as is shown by the following theorem, **Int** itself has a modal companion without an independent axiomatization.

Theorem 11 *The interval* $[\tau\mathbf{Int}, \sigma\mathbf{Int}] = [\mathbf{S4}, \mathbf{Grz}]$ *contains a logic without an independent axiomatization.*

Proof (a sketch). We point out how to change the proof of Theorem 6 in order to obtain a logic we need.

As a "starting formula" δ, we take a modal formula which is refuted in a rooted Kripke frame $\mathfrak{F}$ iff $\mathfrak{F}$ contains a subframe shown in Fig. 4, a and b have no common successors in $\mathfrak{F}$ and d (or c) is contained either in a proper cluster or in an infinite strictly ascending chain. Besides, in the frame in Fig. 3 we replace a^2_{-1} with the two point cluster.

Then we construct a finite number of axioms for L_2 in such a way that Lemma 3 holds for every rooted differentiated frame for L_2 refuting δ. And L_1 is defined as the logic characterized by the class of all differentiated (reflexive) frames validating δ and all differentiated frames for L_2 of the form shown in Fig. 3 with a^2_{-1} replaced by the two point cluster and the points c_i, $i \geq 1$, removed. This class contains all the finite partially ordered frames (since all of them validate δ) which means that $\rho L_1 = \mathbf{Int}$. The fact that L_1 has no independent axiomatization is proved in the same way as in §2 and §3. ⊣

That the property of independent axiomatizability is not in general preserved while passing from an intermediate logic to its arbitrary modal companion can hardly be regarded as a great surprise. Many other properties (such as the decidability, finite model property, Kripke completeness, etc.) behave in this respect in the same way. What is rather unexpected is that unlike the other "good" properties of logics (at least those known to us) the independent axiomatizability is not in general preserved under the map ρ.

Theorem 12 *There is an independently axiomatizable normal modal logic $M \supset$* **S4** *such that ρM does not have an independent axiomatization.*

Proof. We are going to construct an independently axiomatizable modal logic M such that $\rho M = L_1$, where L_1 is the intermediate logic without an independent axiomatization constructed in the proof of Theorem 6. By the definition of L_1, each subframe $\mathfrak{G}_i$ of the frame in Fig. 3 generated by b_i, for $i \in \omega$, validates L_1, and so each frame $\mathfrak{F}_i$, which is obtained from $\mathfrak{G}_i$ by replacing b_i with the two point cluster is a frame for τL_1. For $i \in \omega$, we denote by β_i^* the formula

$$T(\alpha_{i+1}^1 \wedge \alpha_{i+1}^2) \to T(\alpha_i^1 \vee \alpha_i^2) \vee (\Box(\Box(r \to \Box r) \to r) \to r),$$

where α_l^k's are taken from the proof of Theorem 6. It is not hard to verify that $\mathfrak{F}_i \not\models \beta_i^*$ and $\mathfrak{F}_j \models \beta_i^*$, for every $j \neq i$. Therefore, the set $\{\beta_i^* : i \in \omega\}$ is independent over τL_1.

Let $\{\varphi_i : i \in \omega\}$ be a set of axioms for L_1 over **Int**. Then, by defining M as

$$\mathbf{S4} \oplus \{T\varphi_i : i \in \omega\} \oplus \{\beta_i^* : i \in \omega\}, \tag{1}$$

we clearly have $\tau L \subset M \subset \sigma L$, with

$$\mathbf{S4} \oplus \{T(\varphi_i) \wedge \beta_i^* : i \in \omega\}$$

being an independent axiomatization of M. ⊣

Remark. It may be of interest that it is impossible to extract an independent set of axioms for M from the axiomatization (1). By using the logic L_1 constructed in the proof of Theorem 6, it is not difficult to construct an intermediate logic with the same property.

§**5.** We conclude the paper with some questions to which we could not find answers.

The first three questions concern the difference between absolutely independent axiomatizability and independent axiomatizability over a finitely axiomatizable logic.

- Is an absolutely independently axiomatizable logic L_1 containing a finitely axiomatizable logic L_2 is independently axiomatizable over L_2?
- Does the conversion of Lemma 1 hold?
- Do Theorems 9 and 10 hold for the case of absolutely independent axiomatizability?

Our forth question is connected with that there are two ways of axiomatizing modal logics, namely, with the rule of necessitation and without it. The results above establish the existence of modal logics having no independent axiomatizations only of the former kind. In the proof of Theorem 5 the rule of necessitation was used together with the formulas $ax3.i$, which can be rewritten as $\Box\neg\gamma_{i+1} \to \neg\gamma_i$, to ensure that $\neg\gamma_i$ is in an extension of L_2 whenever $\neg\gamma_j$ belongs to it, for some $j > i$. Without this rule the set $\{\neg\gamma_i : i \geq 1\}$ is independent over L_2, and it is not hard to show that $L_1 = L_2 + \{\Box^+\neg\gamma_i : i \geq 1\}$. In the proof of Theorem 8 we used the Blok–Esakia isomorphism between the lattices of intermediate logics and *normal* extensions of **Grz**, with the condition of normality being essential here (for details see Chagrov and Zakharyaschev [1992]).

- Do there exist modal logics having no independent axiomatizations without the postulated rule of necessitation?

One can show, using the mystical part V of the frames in Fig. 1 and 3 that all the logics without independent axiomatizations above have rooted frames of infinite width and depth. Besides, the frames in Fig. 1 and 3 are closely related to the frame which was used by Fine [1974] for constructing an incomplete modal logic. So our three final questions are:

- Do there exist Kripke complete (modal or intermediate) logics without an independent axiomatizations?
- Do there exist (modal or intermediate) logics without an independent axiomatizations but with the finite model property?
- Do there exist (modal or intermediate) logics of finite width or finite depth without an independent axiomatizations?

(As to the last question, our conjecture is that such logics do not exist.)

References

W.J. BLOK [1980], *The lattice of varieties of modal algebras is not strongly atomic*, Algebra Universalis, vol.11 (1980), pp.285-294.

A.V. CHAGROV AND M.V. ZAKHARYASCHEV [1992], *Modal companions of intermediate propositional logics*, Studia Logica, vol.51 (1992), pp.49-82.

K. FINE [1974], *An incomplete logic containing* S4, Theoria, vol.40 (1974), pp.23-29.

R.I. GOLDBLATT [1976], *Metamathematics of modal logic*, Reports on Mathematical Logic, vol.6 (1976), pp.41-77; vol.7 (1976), pp.21-52.

YU.G. KLEYMAN [1983], *On certain questions in theory of varieties of groups*, Izvestia of the USSR Academy of Sciences (Mathematical series), vol.47 (1983), pp.37-74. (Russian)

LOGIC NOTEBOOK [1986], Novosibirsk, Institute of Mathematics.

M.V. ZAKHARYASCHEV [1993], *Canonical formulas for modal and superintuitionistic logics: a short outline*, to appear in: M. de Rijke (ed.), Modal logics and its neighbours '92.

The ILLC Prepublication Series

X-91-03 V. Yu. Shavrukov — Subalgebras of Diagonalizable Algebras of Theories containing Arithmetic
X-91-04 K.N. Ignatiev — Partial Conservativity and Modal Logics
X-91-05 Johan van Benthem — Temporal Logic
X-91-06 — Annual Report 1990
X-91-07 A.S. Troelstra — Lectures on Linear Logic, Errata and Supplement
X-91-08 Giorgie Dzhaparidze — Logic of Tolerance
X-91-09 L.D. Beklemishev — On Bimodal Provability Logics for Π_1-axiomatized Extensions of Arithmetical Theories
X-91-10 Michiel van Lambalgen — Independence, Randomness and the Axiom of Choice
X-91-11 Michael Zakharyaschev — Canonical Formulas for K4. Part I: Basic Results
X-91-12 Herman Hendriks — Flexibele Categoriale Syntaxis en Semantiek: de proefschriften van Frans Zwarts en Michael Moortgat
X-91-13 Max I. Kanovich — The Multiplicative Fragment of Linear Logic is NP-Complete
X-91-14 Max I. Kanovich — The Horn Fragment of Linear Logic is NP-Complete
X-91-15 V. Yu. Shavrukov — Subalgebras of Diagonalizable Algebras of Theories containing Arithmetic, revised version
X-91-16 V.G. Kanovei — Undecidable Hypotheses in Edward Nelson's Internal Set Theory
X-91-17 Michiel van Lambalgen — Independence, Randomness and the Axiom of Choice, Revised Version
X-91-18 Giovanna Cepparello — New Semantics for Predicate Modal Logic: an Analysis from a standard point of view
X-91-19 Papers presented at the Provability Interpretability Arithmetic Conference, 24-31 Aug. 1991, Dept. of Phil., Utrecht University

1992 *Logic, Semantics and Philosophy of Langauge*

Annual Report 1991

LP-92-01 Víctor Sánchez Valencia — Lambek Grammar: an Information-based Categorial Grammar
LP-92-02 Patrick Blackburn — Modal Logic and Attribute Value Structures
LP-92-03 Szabolcs Mikulás — The Completeness of the Lambek Calculus with respect to Relational Semantics
LP-92-04 Paul Dekker — An Update Semantics for Dynamic Predicate Logic
LP-92-05 David I. Beaver — The Kinematics of Presupposition
LP-92-06 Patrick Blackburn, Edith Spaan — A Modal Perspective on the Computational Complexity of Attribute Value Grammar
LP-92-07 Jeroen Groenendijk, Martin Stokhof — A Note on Interrogatives and Adverbs of Quantification
LP-92-08 Maarten de Rijke — A System of Dynamic Modal Logic
LP-92-09 Johan van Benthem — Quantifiers in the world of Types
LP-92-10 Maarten de Rijke — Meeting Some Neighbours (a dynamic modal logic meets theories of change and knowledge representation)
LP-92-11 Johan van Benthem — A note on Dynamic Arrow Logic
LP-92-12 Heinrich Wansing — Sequent Caluli for Normal Modal Propositional Logics
LP-92-13 Dag Westerståhl — Iterated Quantifiers
LP-92-14 Jeroen Groenendijk, Martin Stokhof — Interrogatives and Adverbs of Quantification

Mathematical Logic and Foundations

ML-92-01 A.S. Troelstra — Comparing the theory of Representations and Constructive Mathematics
ML-92-02 Dmitrij P. Skvortsov, Valentin B. Shehtman — Maximal Kripke-type Semantics for Modal and Superintuitionistic Predicate Logics
ML-92-03 Zoran Marković — On the Structure of Kripke Models of Heyting Arithmetic
ML-92-04 Dimiter Vakarelov — A Modal Theory of Arrows, Arrow Logics I
ML-92-05 Domenico Zambella — Shavrukov's Theorem on the Subalgebras of Diagonalizable Algebras for Theories containing $I\Delta_0$ + EXP
ML-92-06 D.M. Gabbay, Valentin B. Shehtman — Undecidability of Modal and Intermediate First-Order Logics with Two Individual Variables
ML-92-07 Harold Schellinx — How to Broaden your Horizon
ML-92-08 Raymond Hoofman — Information Systems as Coalgebras
ML-92-09 A.S. Troelstra — Realizability
ML-92-10 V.Yu. Shavrukov — A Smart Child of Peano's

Compution and Complexity Theory

CT-92-01 Erik de Haas, Peter van Emde Boas — Object Oriented Application Flow Graphs and their Semantics
CT-92-02 Karen L. Kwast, Sieger van Denneheuvel — Weak Equivalence: Theory and Applications
CT-92-03 Krzysztof R. Apt, Kees Doets — A new Definition of SLDNF-resolution

Other Prepublications

X-92-01 Heinrich Wansing — The Logic of Information Structures
X-92-02 Konstantin N. Ignatiev — The Closed Fragment of Dzhaparidze's Polymodal Logic and the Logic of Σ_1 conservativity
X-92-03 Willem Groeneveld — Dynamic Semantics and Circular Propositions, revised version
X-92-04 Johan van Benthem — Modeling the Kinematics of Meaning
X-92-05 Erik de Haas, Peter van Emde Boas — Object Oriented Application Flow Graphs and their Semantics, revised version

1993 *Logic, Semantics and Philosophy of Langauge*

LP-93-01 Martijn Spaan — Parallel Quantification
LP-93-02 Makoto Kanazawa — Dynamic Generalized Quantifiers and Monotonicity
LP-93-03 Nikolai Pankrat'ev — Completeness of the Lambek Calculus with respect to Relativized Relational Semantics
LP-93-04 Jacques van Leeuwen — Identity, Quarrelling with an Unproblematic Notion
LP-93-05 Jaap van der Does — Sums and Quantifiers
LP-93-06 Paul Dekker — Updates in Dynamic Semantics
LP-93-07 Wojciech Buszkowski — On the Equivalence of Lambek Categorial Grammars and Basic Categorial Grammars
LP-93-08 Zisheng Huang, Peter van Emde Boas — Information Acquisition from Multi-Agent resources; abstract

Mathematical Logic and Foundations

ML-93-01 Maciej Kandulski — Commutative Lambek Categorial Grammars
ML-93-02 Johan van Benthem, Natasha Alechina — Modal Quantification over Structured Domains
ML-93-03 Mati Pentus — The Conjoinablity Relation in Lambek Calculus and Linear Logic
ML-93-04 Andreja Prijatelj — Bounded Contraction and Many-Valued Semantics
ML-93-05 Raymond Hoofman, Harold Schellinx — Models of the Untyped λ-calculus in Semi Cartesian Closed Categories
ML-93-06 J. Zashev — Categorial Generalization of Algebraic Recursion Theory
ML-93-07 A.V. Chagrov, L.A. Chagrova — Algorithmic Problems Concerning First-Order Definability of Modal Formulas on the Class of All Finite Frames
ML-93-08 Raymond Hoofman, Ieke Moerdijk — Remarks on the Theory of Semi-Functors
ML-93-09 A.S. Troelstra — Natural Deduction for Intuitionistic Linear Logic
ML-93-10 Vincent Danos, Jean-Baptiste Joinet, Harold Schellinx — The Structure of Exponentials: Uncovering the Dynamics of Linear Logic Proofs
ML-93-11 Lex Hendriks — Inventory of Fragments and Exact Models in Intuitionistic Propositional Logic
ML-93-12 V.Yu. Shavrukov — Remarks on Uniformly Finitely Precomplete Positive Equivalences
ML-93-13 V.Yu. Shavrukov — Undecidability in Diagonizable Algebras
ML-93-14 Dick de Jongh, Albert Visser — Embeddings of Heyting Algebras
ML-93-15 G.K. Dzhaparidze — Effective Truth
ML-93-16 Maarten de Rijke — Correspondence Theory for Extended Modal Logics
ML-93-17 Alexander Chagrov, Michael Zakharyaschev — On the Independent Axiomatizability of Modal and Intermediate Logics

Compution and Complexity Theory

CT-93-01 Marianne Kalsbeek — The Vanilla Meta-Interpreter for Definite Logic Programs and Ambivalent Syntax
CT-93-02 Sophie Fischer — A Note on the Complexity of Local Search Problems
CT-93-03 Johan van Benthem, Jan Bergstra — Logic of Transition Systems
CT-93-04 Karen L. Kwast, Sieger van Denneheuvel — The Meaning of Duplicates in the Relational Database Model
CT-93-05 Erik Aarts — Proving Theorems of the Lambek Calculus of Order 2 in Polynomial Time
CT-93-06 Krzysztof R. Apt — Declarative programming in Prolog

Computaional Linguistics

CL-93-01 Noor van Leusen, László Kálmán — The Interpretation of Free Focus
CL-93-02 Theo M.V. Janssen — An Algebraic View On Rosetta

Other Prepublications

X-93-01 Paul Dekker — Existential Disclosure, revised version
X-93-02 Maarten de Rijke — What is Modal Logic?
X-93-03 Michiel Leezenberg — Gorani Influence on Central Kurdish: Substratum or Prestige Borrowing
X-93-04 A.S. Troelstra (editor) — Metamathematical Investigation of Intuitionistic Arithmetic and Analysis, Corrections to the First Edition
X-93-05 A.S. Troelstra (editor) — Metamathematical Investigation of Intuitionistic Arithmetic and Analysis, Second, corrected Edition
X-93-06 Michael Zakharyaschev — Canonical Formulas for K4. Part II: Cofinal Subframe Logics