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IST is more than an algorithm to prove ZFC theorems

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Abstract

There is a sentence in the language of IST, Nelson's internal set theory, which is not equivalent in IST to a sentence in the $\in$-language. Thus the Reduction algorithm, that converts bounded IST formulas with standard parameters to provably (in IST) equivalent $\in$-formulas, cannot be extended to all formulas of the IST language.

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Introduction.

Internal set theory IST was invented by Nelson [1977] as an attempt to develop nonstandard mathematics from a unified axiomatical standpoint. This theory has demonstrated its ability to ground various branches of nonstandard analysis, see e.g. van den Berg [1987], F. and M.Diener [1988], F.Diener and Reeb [1989], Reeken [1992].

It is regarded as one of the advantages of IST that there exists a simple algorithm, introduced also by Nelson, to transform sentences in the language of IST to provably equivalent (in the sense of provability in IST) sentences formulated in the ZFC language. This algorithm, together with Nelson's theorem that IST is a conservative extension of ZFC, is used sometimes (see e.g. Nelson [1988]) to give back to the statement that IST is nothing more than a new way to investigate the standard ZFC universe.

This is true, indeed, so far as *bounded* IST formulas are considered. (The mentioned algorithm works for these formulas only.)

It is the aim of this paper to demonstrate that there is a certain, explicitly given sentence in the IST language which is not provably equivalent in IST to a sentence in the $\in$-language. Thus the IST truth cannot be completely reduced to the ZFC truth.

A sentence of this kind has to be undecidable in IST; actually the sentence we consider belongs to a type of undecidable sentences discovered and studied in Kanovei [1991]. It is as follows:

$$(*)\quad \forall F\,[\forall^{\mathrm{st}} n\,(F(n) \text{ is standard}) \;\longrightarrow\; \exists^{\mathrm{st}} G\,\forall^{\mathrm{st}} n\,(F(n) = G(n))\,].$$

(n is assumed to range over integers, F and G over functions defined on integers and taking arbitrary values.)

Theorem 1. *Let* Φ *be an arbitrary* $\in$*-sentence. Then the equivalence* $\Phi \longleftrightarrow (*)$ *is not a theorem of* IST *unless* IST *is inconsistent .*

(Take notice that ZFC and IST are equiconsistent.) The idea of the proof is to use a pair of (transitive) models, V and V', of a sufficiently large fragment of ZFC, elementary equivalent with respect to Φ, and then define their extensions, ${}^*V$ and ${}^*V'$ respectively, models of the corresponding finite fragment of IST, such that $(*)$ is false in ${}^*V$ but true in ${}^*V'$. Then, since ${}^*V$ and ${}^*V'$ are elementary extensions of V and V' respectively with respect to $\in$-sentences, Φ is either simultaneously true or simultaneously false in both ${}^*V$ and ${}^*V'$. This proves the theorem.

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Preliminaries

Theory IST was introduced by Nelson [1977]. The IST language contains, together with equality, the membership predicate $\in$ and the standardness predicate $\mathtt{st}$. Formulas of this language are called $\mathtt{st}$-$\in$-*formulas* while formulas of the ZFC language are called $\in$-*formulas*, and also *internal* formulas. Two abbreviations are very useful: $\exists^{\rm st} x \dots$ and $\forall^{\rm st} x \dots$ (there exists standard x ..., for all standard x ...).

IST contains all axioms of ZFC (Separation and Replacement are formulated in the $\in$-language) together with the following three additional principles or (schemes of) axioms.

Idealization I: $\forall^{\rm stfin} A \; \exists x \; \forall a \in A \; \Phi(x,a) \;\longleftrightarrow\; \exists x \; \forall^{\rm st} a \; \Phi(x,a)$
for any internal formula $\Phi(x,a)$.

Standardization S: $\forall^{\rm st} X \; \exists^{\rm st} Y \; \forall^{\rm st} x \; [\, x \in Y \;\longleftrightarrow\; x \in X \;\&\; \Phi(x) \,]$
for any st-$\in$-formula Φ.

Transfer T: $\exists x \; \Phi(x) \;\longrightarrow\; \exists^{\rm st} x \; \Phi(x)$
for any internal formula $\Phi(x)$ with standard parameters.

The formula Φ can, of course, contain *arbitrary* parameters in I and S.

Thus $\mathsf{IST} = \mathsf{ZFC} + \mathsf{I} + \mathsf{S} + \mathsf{T}$.

Definition. Let V be a transitive set. ${}^*V = \langle {}^*V;\, {}^*{\in},\, {}^*{=},\, {}^*\mathtt{st} \rangle$ is an IST-*like* extension of V if and only if, first, axioms I, S, T hold in ${}^*V$, and second, there exists an 1-1 embedding $^* :\; V$ onto a subset of ${}^*V$ satisfying

- $x \in y \;\longleftrightarrow\; {}^*x \;{}^*{\in}\; {}^*y$ and $x = y \;\longleftrightarrow\; {}^*x \;{}^*{=}\; {}^*y$ for all $x, y \in V$, and

- ${}^*\mathrm{st}\,X \longleftrightarrow \exists\, x \in V\,({}^*x \mathrel{{}^*{=}} X)$ for all $X \in {}^*V$.

It is not assumed, in general, that ${}^*{=}$ coincides with the true equality on ${}^*V$, but ${}^*{=}$ has to be an equivalence relation and satisfy the logic axioms for equality with respect to ${}^*{\in}$ and ${}^*\mathrm{st}$.

Proof of Theorem 1.

Assume on the contrary that Φ is an $\in$-sentence such that the equivalence $\Phi \longleftrightarrow (*)$ is a theorem of IST, therefore of a theory

$$\mathsf{IST}' = \mathsf{ZFC}' + \mathsf{I} + \mathsf{S} + \mathsf{T},$$

where ZFC' is a finite fragment of ZFC. Having this fixed, we start to argue in ZFC. The final aim is to obtain a contradiction.

Ground ZFC' models.

It is a consequence of the ZFC Reflection principle that there exist cardinals ϑ of both countable and uncountable cofinality such that $\mathbf{V}_\vartheta$ is an elementary submodel of the universe of all sets with respect to Φ and all formulas of ZFC'.

Let ϑ be the least among the countably cofinal while ϑ' among the uncountably cofinal cardinals of this kind. We use the sets $V = \mathbf{V}_\vartheta$ and $V' = \mathbf{V}_{\vartheta'}$ as the ground ZFC' models. Take notice that Φ is either true in both V and V' or false in both V and V'.

The next step is to define IST-extensions (therefore models of IST'), ${}^*V$ and ${}^*V'$, of V and V' respectively, such that $(*)$ is true in ${}^*V'$ but false in ${}^*V$. The extensions are constructed as ultrapowers via a kind of *adequate* ultrafilters of Nelson [1977]. (Original Nelson's construction includes infinite number of successive ultrapowers; we show here that this can be managed an one-step construction.)

The "falsity" extension

Thus we define ${}^*V$ as an ultrapower of V using the *index set*

$$I = \mathcal{P}^{\mathrm{fin}}(V) = \{i \in V : i \text{ is finite}\},$$

and an arbitrary *ultrafilter* U over I containing all sets of the form $I_a = \{i \in I : a \in i\}$, $a \in V$.

We introduce a convenient tool, the quantifier "there exist U-many" by

$$\mathbf{U}\, i\, \varphi(i) \quad \text{if and only if} \quad \{i \in I : \varphi(i)\} \in U.$$

The following is the list of properties of $\mathbf{U}$ implied by the definition of an ultrafilter and (this regards (U5)) the choice of the ultrafilter U.

(U1) $\varphi \longleftrightarrow \mathbf{U}\, i\, \varphi$ whenever i is not free in φ;

(U2) if $\forall i\, [\varphi(i) \longrightarrow \psi(i)]$ then $\mathbf{U}\, i\, \varphi(i) \longrightarrow \mathbf{U}\, i\, \psi(i)$;

(U3) $\mathbf{U}\, i\, \varphi(i)\ \&\ \mathbf{U}\, i\, \psi(i) \longleftrightarrow \mathbf{U}\, i\, [\varphi(i)\ \&\ \psi(i)]$;

(U4) $\mathbf{U}\, i\, \neg\varphi(i) \longleftrightarrow \neg\, \mathbf{U}\, i\, \varphi(i)$;

(U5) if $a \in V$ then $\mathbf{U}\, i\, (a \in i)$.

To introduce the extension, we put

$$^*V_r = \{f : f \text{ is a function, } f : I^r \longrightarrow V\}, \qquad \text{for all } r \in \omega.$$

In particular, $^*V_0 = \{^*z : z \in V\}$, where $^*z = \{\langle 0, z\rangle\}$, since $I^0 = \{\emptyset\}$.

The set $^*V = \bigcup_{r\in\omega} {^*V_r}$ is what we call *the falsity extension.*

To continue notation, we let, for $F \in {^*V}$, $r(F)$ denote the unique r satisfying $F \in {^*V_r}$. If $F \in {^*V}$, $q \geq r = r(F)$, $\mathbf{i} = \langle i_1, ..., i_r, ..., i_q\rangle \in I^q$, then we put $F[\mathbf{i}] = F(i_1, ..., i_r)$. Note that $F[\mathbf{i}] = F(\mathbf{i})$ whenever $r = q$. We define finally $^*z[\mathbf{i}] = z$ for all $^*z \in {^*V_0}$ and $\mathbf{i} \in I^r$, $r \geq 0$.

Let $F, G \in {^*V}$ and $r = \max\{r(F), r(G)\}$. We set

$$F \mathrel{^*\!\!\in} G \quad \text{if and only if} \quad \mathbf{U}\, i_r\, \mathbf{U}\, i_{r-1} ... \mathbf{U}\, i_1\, (F[\mathbf{i}] \in G[\mathbf{i}]);$$

$$F \mathrel{^*\!\!=} G \quad \text{if and only if} \quad \mathbf{U}\, i_r\, \mathbf{U}\, i_{r-1} ... \mathbf{U}\, i_1\, (F[\mathbf{i}] = G[\mathbf{i}]);$$

of course $\mathbf{i}$ denotes the sequence $i_1, ..., i_r$.

The definition of standardness in $^*V$ is given by:

$$^*\mathrm{st}\, F \quad \text{if and only if} \quad \text{there exists } x \in V \text{ such that } F \mathrel{^*\!\!=} {^*x}.$$

So up to the relation $\mathrel{^*\!\!=}$ the level $^*V_0$ is just the standard part of $^*V$.

Let, finally, Ψ be a formula with parameters in $^*V$. We define $r(\Psi) = \max\{r(F) : F$ occurs in $\Psi\}$. If in addition $r \geq r(\Psi)$ and $\mathbf{i} \in I^r$, then let $\Psi[\mathbf{i}]$ denote the result of replacing each F that occurs in Ψ by $F[\mathbf{i}]$. Clearly $\Psi[\mathbf{i}]$ is a formula with parameters in V.

Proposition 2. $\langle {}^*V;\ {}^*\!=,\ {}^*\!\in,\ {}^*\mathrm{st}\rangle$ *is a model of* IST' *and an* IST*-like extension of* V. *Finally,* $(*)$ *fails in* $^*V$.

Proof. The following principal statement plays the key role.

Lemma 3. [Łoś Theorem] *Let* Ψ *be an internal formula with parameters in* $^*V$ *and suppose that* $r \geq r(\Psi)$. *Then*

$$\Psi \textit{ is true in } {}^*V \quad\longleftrightarrow\quad \mathbf{U}\, i_r \dots \mathbf{U}\, i_1\ (\Psi[i_1, \dots, i_r] \textit{ is true in } V).$$

Proof of the lemma. The proof goes by induction on the logical complexity of Ψ. We abandon easy parts of the proof, based on properties (U2), (U3), (U4) of the quantifier $\mathbf{U}$, and consider the induction step $\exists$. Thus the lemma is to be proved for a formula $\exists\, x\, \Psi(x)$ in the assumption that the result holds for $\Psi(F)$ whenever $F \in {}^*V$. We denote $r = r(\Psi)$.

The direction $\longrightarrow$. Suppose that $\exists\, x\, \Psi(x)$ holds in $^*V$. Then $\Psi(F)$ holds in $^*V$ for some $F \in {}^*V$. Let $p = \max\{r, r(F)\}$. To convert the reasoning into a more convenient form, we let $\mathbf{i}$ and $\mathbf{j}$ denote sequences

$$\langle i_1, \dots, i_r\rangle\ \ (\in I^r) \qquad \text{and} \qquad \langle i_1, \dots, i_r, \dots, i_p\rangle\ \ (\in I^p)$$

respectively. Further let $\mathbf{U}\,\mathbf{i}$ and $\mathbf{U}\,\mathbf{j}$ denote sequences of quantifiers

$$\mathbf{U}\, i_r \dots \mathbf{U}\, i_1 \qquad \text{and} \qquad \mathbf{U}\, i_p \dots \mathbf{U}\, i_r \dots \mathbf{U}\, i_1.$$

Thus $\mathbf{U}\,\mathbf{j}\ \Psi(F)\,[\mathbf{j}]$ holds by the induction hypothesis. We note that, for all $\mathbf{j}$, $\Psi(F)\,[\mathbf{j}] \longrightarrow \exists\, x\, \Psi(x)\,[\mathbf{j}]$. Hence $\mathbf{U}\,\mathbf{j}\ \exists\, x\, \Psi(x)\,[\mathbf{j}]$ is true by (U2). Moreover the formula $\exists\, x\, \Psi(x)\,[\mathbf{j}]$ coincides (graphically) with $\exists\, x\, \Psi(x)\,[\mathbf{i}]$ because $r(\exists\, x\, \Psi(x)) = r \leq p$. Hence, deleting the superfluous quantifiers by (U1), we obtain $\mathbf{U}\,\mathbf{i}\, \exists\, x\ \Psi(x)\,[\mathbf{i}]$.

The direction $\longleftarrow$. Let $\Psi(x)$ be $\Psi(x, G, H, \dots,)$, where $G, H, \dots \in {}^*V$. Suppose that $\mathbf{U}\,\mathbf{i}\, \exists\, x\ \Psi(x)\,[\mathbf{i}]$ holds, that is,

$$\mathbf{U}\,\mathbf{i}\ [\,\exists\, x\ \Psi(x, G[\mathbf{i}], H[\mathbf{i}], \dots) \ \text{ is true in } \ V\,].$$

For each $\mathbf{i} \in I^r$, if there exists some $x \in V$ such that $\Psi(x, G[\mathbf{i}], H[\mathbf{i}], ...)$ is true in V, then we let $F(\mathbf{i})$ be one of x of such kind; otherwise let $F(\mathbf{i}) = \emptyset$. By definition, $F \in {}^*\!V_r$ and

$$\forall\, \mathbf{i} \in I^r\, [\, \exists\, x\ \Psi(x)\, [\mathbf{i}] \ \longrightarrow\ \Psi(F)\, [\mathbf{i}]\,],$$

therefore $\mathbf{U}\,\mathbf{i}\ \exists\, x\ \Psi(x)\, [\mathbf{i}] \ \longrightarrow\ \mathbf{U}\,\mathbf{i}\ \Psi(F)\, [\mathbf{i}]$ by (U2). Recall that the left-hand side of the last implication has been supposed to be true. So the right-hand side is also true. Then $\Psi(F)$ holds in ${}^*\!V$ by the induction hypothesis, and we are done.

The just proved lemma easily implies logical equality axioms for ${}^*\!\!=$, and Transfer, therefore all ZFC', in ${}^*\!V$. Standardization is evident because every set V of the form $V = \mathbf{V}_\vartheta$ has the property that if $Y \subseteq X \in V$ then $Y \in V$. We prove Idealization.

Thus let $\varphi(x, a)$ be an internal formula with parameters in ${}^*\!V$. We denote $r = r(\varphi)$ and prove the following:

$$\forall^{\mathrm{stfin}} A\ \exists\, x\ \forall\, a \in A\ \varphi(x, a) \ \longrightarrow\ \exists\, x\ \forall^{\mathrm{st}} a\ \varphi(x, a)$$

in ${}^*\!V$. (The implication $\longleftarrow$ does not need a special consideration because it follows from Standardization that elements of finite standard sets are s-tandard, see Nelson [1977].) Lemma 3 converts the left-hand side to the form:

$$\forall^{\mathrm{fin}} A\ \subseteq\ V\ \mathbf{U}\, i_r \ldots \mathbf{U}\, i_1\ \exists\, x\ \forall\, a \in A\ (\varphi(x, a)\, [i_1, ..., i_r]).$$

Recall that I consists of all finite subsets of V, so we may replace the variable A by i, having in mind that $i \in I$. Further define $\tilde{A} : I^{r+1} \longrightarrow V$ by $\tilde{A}(i_1, ..., i_r, i) = i$. Then $\tilde{A} \in {}^*\!V$. The left-hand side takes the form

$$\forall\, i\ \mathbf{U}\, i_r \ldots \mathbf{U}\, i_1\ (\exists\, x\ \forall\, a \in \tilde{A}\ \varphi(x, a))\, [i_1, ..., i_r, i].$$

Changing $\forall\, i$ by $\mathbf{U}\, i$, we obtain $\exists\, x\ \forall\, a \in \tilde{A}\ \varphi(x, a)$ in ${}^*\!V$ again by the lemma. So, to verify the right-hand side of Idealization, it suffices to prove ${}^*a \in \tilde{A}$ in ${}^*\!V$ for all $a \in V$. This is equal to

$$\mathbf{U}\, i\ \mathbf{U}\, i_r \ldots \mathbf{U}\, i_1\ (a \in \tilde{A}\, [i_1, ..., i_r, i]),$$

by the lemma, and then to $\mathbf{U}\, i\ \mathbf{U}\, i_r \ldots \mathbf{U}\, i_1\ (a \in i)$ by the definition of $\tilde{A}$. So apply (U5) and complete the proof of Idealization in ${}^*\!V$.

Thus ${}^*V$ is an IST' model. One can easily verify the required properties of the embedding $*$. To complete the proof of Proposition 2 it remains to show that $(*)$ does not hold in ${}^*V$.

Let $\langle \kappa_n : n \in \omega \rangle$ be a sequence of ordinals cofinal in ϑ. (We recall that ϑ has countable cofinality.) Let $F \in {}^*V_0$ be defined by

$$F(i) = \{\langle n, \kappa_n \rangle : \langle n, \kappa_n \rangle \in i\} \quad \text{for all } i \in I.$$

It is true in ${}^*V$ by Lemma 3 that F is a function defined on a subset of integers, and, for every $n \in \omega$, it is also true in ${}^*V$ that $F({}^*n)$ is defined and equal to ${}^*\kappa_n$, hence standard. Thus the left-hand side of $(*)$ is satisfied by F.

The right-hand side cannot be satisfied since it would imply that there exists $g \in V$ such that $g(n) = \kappa_n$ for all n, which is impossible.

Corollary 4. *Φ is false in ${}^*V$, therefore in V.*

The "truth" extension

Let ${}^*V'$ be defined the same way as ${}^*V$ above, but starting from V'.

Proposition 5. *$\langle {}^*V'; {}^*{=}, {}^*{\in}, {}^*\mathrm{st} \rangle$ is a model of IST' and an IST-like extension of V. Finally, $(*)$ holds in ${}^*V'$.*

Proof. We check the last statement. Thus let $F \in {}^*V'$ be such that the following is true in ${}^*V'$:

> F is a function, every standard $n \in {}^*\omega$ belongs to the domain of F, and $F(n)$ is standard for every standard $n \in {}^*\omega$.

By the definition of standardness, there exists a function $f : \omega \longrightarrow V'$ such that $F({}^*n) \mathrel{{}^*{=}} {}^*(f(n))$ for all $n \in \omega$. By the choice of ϑ' (uncountable cofinality) there exists $\kappa < \vartheta'$ such that $f(n) \in \mathbf{V}_\kappa$ for all $n \in \omega$. This easily implies that actually $f \in V'$, and therefore $F({}^*n) \mathrel{{}^*{=}} ({}^*f)({}^*n)$ for all $n \in \omega$, the right-hand side of $(*)$.

Corollary 6. *Φ is true in ${}^*V'$, therefore in V'.*

Thus finally Φ is true in V' and false in V, a contradiction with the choice of V, V' as models elementary equivalent with respect to Φ.

Question

Does there exist, in ZFC, a transitive set V, a model of a previously fixed finite fragment of ZFC, which has IST-like extensions of both types, those in which $(*)$ holds and those where $(*)$ fails ? The answer is affirmative provided there is a cardinal ϑ such that $\mathbf{V}_\vartheta$ is a model of the full ZFC (then we may take $V = \mathbf{V}_\vartheta$, where ϑ is the least among such cardinals), but we are unable to get it without extra assumptions. If this is actually impossible, then, perhaps, $(*)$ still corresponds to something in ZFC, not in the direct form mentioned in Theorem 1, of course.

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