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The undecidability of second order linear affine logic

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Abstract

Quantifier-free propositional linear affine logic (i.e. linear logic with weakening) is decidable [Kop, Laf2]. Recently, Lafont and Scedrov proved that the multiplicative fragment of *second-order* linear logic is undecidable [LS]. In this paper we show that second order linear affine logic is undecidable as well. At the same time it turns out that even its multiplicative fragment is undecidable. Moreover, we obtain a whide class of undecidabile second order logics which lie between the Lambek calculus (LC) and linear affine logic. The proof is based on an encoding of two-counter Minsky machines in second order linear affine logic. The faithfulness of the encoding is proved by means of the phase semantics.

1 Introduction and summary

Our notation. Linear logic has been introduced by Girard [Gir]. The inference rules of second order linear logic are represented in Table 1. Linear affine logic is linear logic with the weakening rule (see Table 2) [T]. Non-commutative linear logic is linear logic without the permutation rule [Abr91]. Note than non-commutative linear logic has two implications: the right one and the left one ($\multimap$ and $\circ\!-$). We shall abbreviate second order linear logic and linear affine logic as LL2 and LLW2 correspondingly, and the non-commutative versions of these logics as N-LL2 and N-LLW2. We shall use the abbreviations LL, LLW, N-LL, N-LLW for the quantifier-free fragments of the corresponding logics.

There are also intuitionistic versions of all the logics mentioned above. As usual an intuitionistic derivation is a derivation containing only sequents which have no more than one formula in the consequent. The letter **I** stands for intuitionistic logics (e.g. ILL, ILLW and so on).

Connectives and constants of LL are divided into three groups: the multiplicatives ($\otimes$, $\wp$, $\multimap$, 1 and $\bot$), the additives ($\oplus$, &, 0 and $\top$) and the exponentials (! and ?).

In referring to linear logic fragments,
M stands for the multiplicative fragment (i.e. the fragment containing only multiplicatives),
A stands for the additive fragment (i.e. the fragment containing only additives),
E stands for the exponential fragment, (i.e. the fragment containing only exponentials).

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For example, MLL abbreviates the multiplicative fragment of LL, MALLW denotes multiplicative-additive fragment of LLW, and so on.

Also we shall consider the Lambek calculus (LC). In contrast to the traditional notations for connectives of LC: \, /, ·, we shall use the following notations: $\multimap$ and $\circ\!-$ stands for the left and right implications, and $\otimes$ for the tensor product, (see Table 3 for the inference rules). It is clear that LC $\subseteq$ N-ILL $\subseteq$ ILL $\subseteq$ LL $\subseteq$ LLW.

Main results. Lincoln, Scedrov, and Shankar showed the undecidability of IMLL2 and IMALL2 by an embedding of LJ2 [LSS]. Lafont proved the undecidability of MALL2 [Laf1]. Then Lafont and Scedrov proved that MLL2 is undecidable too [LS]. Emms showed an embedding of LJ2 into N-IMLL2. Kanovich demonstrated in [Kan2] the undecidability of N-MLL2, cyclic LL and second order Lambek Calculus (LC2). On the other hand, quantifier-free linear affine logic is decidable [Kop, Laf2]. The decidability problem for second order linear affine logic remained open.

In the current paper we prove the undecidability of LLW2. Also we prove that for any logic L if LC2 $\subseteq L \subseteq$ LLW2, then L is undecidable. In particular, all second-order logics mentioned above are undecidable as well as MLLW2, MALLW2, LLW2, IMLLW2, N-MLLW2, etc. The main ideas of the proof are similar to the ideas of [LS]. Namely, we encode two-counter machines (Minsky machines) in LC2 and LLW2. This encoding is similar to the encodings from [Kan1, Laf1, LS]. In order to obtain the faithfulness of the encoding we use (as in [Laf1, LS]) the phase semantics, but here we need the phase semantics for linear affine logic.

2 Phase semantics

Let us recall some definitions concerning phase semantics [Gir, Laf2]. Phase space is the triple $(M, \bot, K)$, where M is a commutative monoid, $\bot \subseteq M$ and K is a submonoid of the submonoid $J(M) = \{x \in \bot^\bot \mid x \in \{x^2\}^{\bot\bot}\}$. For instance, K may be $\{1\}$.

Let $X, Y \subseteq M$, then, by definition,

$$XY = \{xy \mid x \in X, y \in Y\}, \quad X \multimap Y = \{z \in M \mid \forall x \in X \;\; xz \in Y\}, \quad X^\bot = X \multimap \bot.$$

We say that X is a fact, when there is a set Y such that $X = Y^\bot$. It is easy to see that X is a fact if and only if $X = X^{\bot\bot}$. If X is any subset of M, then $X^{\bot\bot}$ is the smallest fact containing X. By definition,

$$\begin{array}{cc} X \otimes Y = (XY)^{\bot\bot}, & X \wp Y = (X^\bot \otimes Y^\bot)^\bot, \\ X \& Y = X \cap Y, & X \oplus Y = (X^\bot \& Y^\bot)^\bot, \\ 1 = \bot^\bot, \quad \top = M, & 0 = \top^\bot. \end{array}$$

If all atoms p are interpreted by facts $p^\bullet$, then for any formula A we can naturally define a fact $A^\bullet$. Namely, $^\bullet$ commutes with all connectives, and $(\forall \alpha\, A[\alpha])^\bullet$ is defined as

$$\bigcap_{X \text{ is a fact}} A[X]^\bullet,$$

where $A[X]^\bullet$ is an interpretation of $A[\alpha]$, where $\alpha^\bullet = X$. By definition, a formula A is satisfied, if $1 \in A^\bullet$. A sequent $A_1, \ldots, A_n \vdash B_1, \ldots, B_k$ is satisfied, if the formula $(A_1 \otimes \ldots \otimes A_n) \multimap (B_1 \wp \ldots \wp B_k)$ is satisfied. This is equivalent to $(A_1 \otimes \ldots \otimes A_n)^\bullet \subseteq (B_1 \wp \ldots \wp B_k)^\bullet$.

Theorem 1 ([Gir]) *If a sequent Φ is derivable in MALL, then any phase space $(M, \bot)$ satisfies Φ.*

- $(j, p, q-1)$ if $\tau(i) = (-, 2, j, k)$ and $q > 0$ (decrement the second counter),
- (k, p, q) if $\tau(i) = (-, 2, j, k)$ and $q = 0$ (test for zero the second counter).

In other words a Minsky machine has transitions of the following types:

- $(i, p, q) \to (j, p+1, q)$,
- $(i, p, q) \to (j, p-1, q)$ *if* $p > 0$,
- $(i, 0, q) \to (k, 0, q)$,
- $(i, p, q) \to (j, p, q+1)$,
- $(i, p, q) \to (j, p, q-1)$ *if* $q > 0$,
- $(i, p, 0) \to (k, p, 0)$.

The machine stops when $i = 0$. A configuration (i, p, q) is accepted by the machine if, starting from (i, p, q), it eventually stops on $(0, 0, 0)$.

Theorem 3 ([M, Lk]) *There is a Minsky machine for wich the set of accepted configurations is not recursive.*

4 Encoding two-counter Minsky machines

We can encode Minsky machines in the following way. Let us consider two formulas:

$$\varphi[\alpha] = (\alpha \multimap f) \multimap h,$$
$$\psi[\alpha] = (\alpha \multimap g) \multimap e.$$

We construct the following infinite sets of formulas:

$$\varphi_{-1} = a, \quad \varphi_n = \varphi[\varphi_{n-1}], \quad n \in \mathbb{N},$$
$$\psi_{-1} = b, \quad \psi_n = \psi[\psi_{n-1}], \quad n \in \mathbb{N}.$$

Any machine configuration (i, p, q) is encoded by the following formula:

$$c_i \otimes \varphi_p \otimes \psi_q.$$

Here a, b, c_i, e, f, g, h are literals. An increment transition $(i, p, q) \to (j, p+1, q)$ is encoded by the formula

$$\forall \alpha, \beta (c_i \otimes \varphi[\alpha] \otimes \beta \multimap c_j \otimes \varphi[\varphi[\alpha]] \otimes \beta).$$

A decrement transition $(i, p, q) \to (j, p-1, q)$ *if* $p > 0$ is encoded by the formula

$$\forall \alpha, \beta (c_i \otimes \varphi[\varphi[\alpha]] \otimes \beta \multimap c_j \otimes \varphi[\alpha] \otimes \beta).$$

And a test-for-zero transition $(i, 0, q) \to (k, 0, q)$ is encoded by the formula

$$\forall \beta (c_i \otimes \varphi[a] \otimes \beta \multimap c_k \otimes \varphi[a] \otimes \beta).$$

Proof Note that the following sequents are easily derivable: $A^4, C \vdash A^5$; $S_1 \otimes A \otimes S_2, W \vdash A$ and $A, \Sigma, W' \vdash A$. The derivation of the rule $(\tilde{C})$ is the following:

$$\dfrac{\overline{(C^2)^4, C \vdash (C^2)^5} \quad \dfrac{\dfrac{\overline{A^4, C \vdash A^5} \quad \Pi, A^5, C^9, \Gamma \vdash Z}{\Pi, A^4, C, C^9, \Gamma \vdash Z}(CUT)}{\Pi, A^4, C^{10}, \Gamma \vdash Z}(L\otimes)}{\dfrac{\Pi, A^4, C^8, C, \Gamma \vdash Z}{\Pi, A^4, C^9, \Gamma \vdash Z}(L\otimes)}(CUT)$$

And here is the derivations of the rules $(\tilde{W})$ and $(\tilde{W}')$:

$$\dfrac{\overline{S_1 \otimes A \otimes S_2, W \vdash A} \qquad \Pi, A, \Gamma \vdash Z}{\Pi, S_1 \otimes A \otimes S_2, W, \Gamma \vdash Z}(CUT) \qquad \dfrac{\overline{A, \Sigma, W' \vdash A} \qquad \Pi, A, \Gamma \vdash Z}{\Pi, A, \Sigma, W', \Gamma \vdash Z}(CUT)$$

■

Lemma 4.3 *For any transition $(i,p,q) \to (i',p',q')$, if T_k is a formula encoding this transition then the sequent $c_i \otimes \varphi_p \otimes \psi_q, T_k \vdash c_{i'} \otimes \varphi_{p'} \otimes \psi_{q'}$ is derivable in LC2.*

Proof It is easy to see that the following sequents are derivable in LC2:

$$\begin{array}{lcl}
c_i \otimes \varphi_p \otimes \varphi_q,\ \forall\alpha,\beta(c_i \otimes \varphi[\alpha] \otimes \beta \multimap c_j \otimes \varphi[\varphi[\alpha]] \otimes \beta) & \vdash & c_j \otimes \varphi_{p+1} \otimes \varphi_q, \\
c_i \otimes \varphi_p \otimes \varphi_q,\ \forall\alpha,\beta(c_i \otimes \varphi[\varphi[\alpha]] \otimes \beta \multimap c_j \otimes \varphi[\alpha] \otimes \beta) & \vdash & c_j \otimes \varphi_{p-1} \otimes \varphi_q, \\
c_i \otimes \varphi_0 \otimes \varphi_q,\ \forall\beta(c_i \otimes \varphi[a] \otimes \beta \multimap c_k \otimes \varphi[a] \otimes \beta) & \vdash & c_k \otimes \varphi_0 \otimes \varphi_q, \\
c_i \otimes \psi_p \otimes \psi_q,\ \forall\alpha,\beta(c_i \otimes \alpha \otimes \psi[\beta] \multimap c_j \otimes \alpha \otimes \psi[\psi[\beta]]) & \vdash & c_j \otimes \psi_p \otimes \psi_{q+1}, \\
c_i \otimes \psi_p \otimes \psi_q,\ \forall\alpha,\beta(c_i \otimes \alpha \otimes \psi[\psi[\beta]] \multimap c_j \otimes \alpha \otimes \psi[\beta]) & \vdash & c_j \otimes \psi_p \otimes \psi_{q-1}, \\
c_i \otimes \psi_p \otimes \psi_0,\ \forall\alpha(c_i \otimes \alpha \otimes \psi[b] \multimap c_k \otimes \alpha \otimes \psi[b]) & \vdash & c_k \otimes \psi_p \otimes \psi_0.
\end{array}$$

■

We prove the implication (i)→(ii) by induction on the length of the computation. The base of induction holds because of the rule $(\tilde{W}')$:

$$\dfrac{c_0 \otimes \varphi_0 \otimes \psi_0 \vdash c_0 \otimes \varphi_0 \otimes \psi_0}{c_0 \otimes \varphi_0 \otimes \psi_0, (U \otimes T \otimes U \otimes W)^4, C^9, W' \vdash c_0 \otimes \varphi_0 \otimes \psi_0}(\tilde{W}')$$

Now let us verify the induction step. If the computation has the following form:

$$(i,p,q) \to (i',p',q') \to \ldots \to (0,0,0)$$

then one can constact the following derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{Lemma 4.3}}{c_i \otimes \varphi_p \otimes \psi_q, T_k \vdash c_{i'} \otimes \varphi_{p'} \otimes \psi_{q'}} \quad \dfrac{\text{induction hypothesis}}{c_{i'} \otimes \varphi_{p'} \otimes \psi_{q'}, (U \otimes T \otimes U \otimes W)^4, C^9, W' \vdash c_0 \otimes \varphi_0 \otimes \psi_0}}{c_i \otimes \varphi_p \otimes \psi_q, T_k, (U \otimes T \otimes U \otimes W)^4, C^9, W' \vdash c_0 \otimes \varphi_0 \otimes \psi_0}(CUT)}{c_i \otimes \varphi_p \otimes \psi_q, U \otimes T \otimes U, W, (U \otimes T \otimes U \otimes W)^4, C^9, W' \vdash c_0 \otimes \varphi_0 \otimes \psi_0}(\tilde{W})}{c_i \otimes \varphi_p \otimes \psi_q, (U \otimes T \otimes U \otimes W)^5, C^9, W' \vdash c_0 \otimes \varphi_0 \otimes \psi_0}(L\otimes)}{c_i \otimes \varphi_p \otimes \psi_q, (U \otimes T \otimes U \otimes W)^4, C^9, W' \vdash c_0 \otimes \varphi_0 \otimes \psi_0}(\tilde{C})$$

■

Proof of 5.1 and 5.2 Using lemma 2.1, we can calculate $\varphi[\alpha]$, $\varphi[\varphi[\alpha]]$, $(c_i\varphi[\alpha])^\perp$, $(c_j\varphi[\varphi[\alpha]])^\perp$ for any set $\alpha \subseteq M$, where $\alpha = \alpha' \cup M_{l+1}$ and $\alpha' \subseteq M_l$ (see Table 4).

We prove that $\varphi_n^\bullet = \{f_n\} \cup M_2$ by induction on n. For $n = -1$ we have that $\varphi_{-1}^\bullet = a^\bullet = \{f_{-1}\} \cup M_2$ by definition. The induction step holds because of $\varphi[\alpha] = \{f_n\} \cup M_2$, for $\alpha = \{f_{n-1}\} \cup M_2$. The second assertion of lemma 5.1 can be proved similarly.

One can see from Table 4 that $(c_i\varphi[\alpha])^\perp = (c_j\varphi[\varphi[\alpha]])^\perp$ for any $\alpha \neq \{f_{n-1}\} \cup M_2$. Moreover, if the increment transition $(i,p,q) \to (j,p+1,q)$ is accepted, then $(c_i\varphi[\alpha])^\perp \supseteq (c_j\varphi[\varphi[\alpha]])^\perp$ for $\alpha = \{f_{n-1}\} \cup M_2$. Hence, the formula

$$\forall\alpha, \beta(c_i \otimes \varphi[\alpha] \otimes \beta \multimap c_j \otimes \varphi[\varphi[\alpha]] \otimes \beta)$$

is satisfied. By analogy, if the decrement transition $(i,p,q) \to (j,p-1,q)$ is accepted, then $(c_j\varphi[\alpha])^\perp \subseteq (c_i\varphi[\varphi[\alpha]])^\perp$ for $\alpha = \{f_{n-1}\} \cup M_2$. So, the formula

$$\forall\alpha, \beta(c_i \otimes \varphi[\varphi[\alpha]] \otimes \beta \multimap c_j \otimes \varphi[\alpha] \otimes \beta)$$

is satisfied. Finally, if the test-for-zero transition $(i,0,q) \to (k,0,q)$ is accepted, then $(c_k\varphi[a^\bullet])^\perp \subseteq (c_i\varphi[a^\bullet])^\perp$. So the formula

$$\forall\beta(c_i \otimes \varphi[a] \otimes \beta \multimap c_k \otimes \varphi[a] \otimes \beta)$$

is satisfied. Clearly, all formulas encoding transitions for the second counter are satisfied as well. ■

Lemma 5.3 *M satisfies the formula C.*

Proof Let X be a fact. If $1 \in X$, then $X = M$ and $X^4 = X^5$. Otherwise, $XXXX \subseteq M_4 \subseteq \perp$, and $X^4 = (XXXX)^{\perp\perp} = \perp$, because $\perp$ is the least fact. In the same way $X^5 = \perp$. In both cases we have $X^4 = X^5$. Therefore, C is satisfied. ■

Lemma 5.4 *M satisfies the formulas W, W′ and U.*

Proof The formulas W, W' and U are derivable in LLW2. Hence, they are satisfied. ■

Now let us prove (iv)→(i). It follows from (iv) and lemmas 5.2, 5.3 and 5.4 that M satisfies the implication

$$c_i \otimes \varphi_p \otimes \psi_q \multimap c_0 \otimes \varphi_0 \otimes \psi_0.$$

By definition $c_0f_0g_0 \in \perp$. Hence, $(c_0 \otimes \varphi_0 \otimes \psi_0)^\bullet = \{c_0f_0g_0\}^{\perp\perp} = \perp$, because $\perp$ is the least fact. Therefore, we have

$$c_if_pg_q \in (c_i \otimes \varphi_p \otimes \psi_q)^\bullet \subseteq (c_0 \otimes \varphi_0 \otimes \psi_0)^\bullet = \perp.$$

So, (i) holds. ■

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