The Evolution of Combinatorial Phonology

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Abstract

A fundamental, universal property of human language is that its phonology is combinatorial. That is, one can identify a set of basic, distinct units (phonemes, syllables) that can be productively combined in many different ways. In this paper, we review a number of theories and models that have been developed to explain the evolutionary transition from holistic to combinatorial signal systems, but find that in all problematic linguistic assumptions are made, or crucial components of evolutionary explanations are omitted. We present a novel model to investigate the hypothesis that combinatorial phonology results from optimising signal systems for perceptual distinctiveness. Our model differs from previous models in two important respects. First, signals are modelled as trajectories through acoustic space. Hence, both holistic and combinatorial signals have a temporal structure. Second, we use the methodology from evolutionary game theory. Crucially, we show a path of ever increasing fitness from holistic to combinatorial signals, where every innovation represents an advantage even if no-one else in a population has yet obtained it.

1 Introduction

1.1 Natural language phonology is combinatorial

One of the universal properties of human language is that its phonology is *combinatorial*. In all human languages, utterances can be split into units that can be recombined into new valid utterances. Although there is some controversy about what exactly the units of (productive) combination are, there is general agreement that in natural languages – including even sign languages (Deuchar, 1996) – meaningless atomic units (phonemes or syllables) are combined into larger wholes; these meaningful combinations (words, or morphemes) are then further combined into meaningful sentences. These two levels of combination constitute the *duality of* patterning (Hockett, 1960).

In the traditional view, the atomic units are *phonemes* (minimal speech sounds that can make a distinction in meaning), or the distinctive features of these phonemes (Chomsky and Halle, 1968). Signal repertoires that are built-up from combinations of phonemes are said to be "phonemically coded" (Lindblom et al., 1984). For instance, the words "we", "me", "why" and "my", as pronounced in standard British English, can be analysed as built-up from the units "w", "m", "e" and "y", which can all be used in many different combinations. One popular alternative view is that the atoms are *syllables*, or the possible onsets, codas and nuclei of syllables (e.g. Levelt and Wheeldon, 1994). A second alternative theory uses *exemplars*, which can comprise several syllables or even words, as its basic units (e.g. Pierrehumbert, 2001). In this paper we will avoid the debate about the exact level of combination – and the conventional term "phonemic coding" – and instead focus on the uncontroversial abstract property of "combinatorial phonology"¹.

Note that, whichever the real level of combination is, there is no logical necessity to assume that all recurring sound patterns observed in speech, are in fact units of productive combination in the speaker's brain. For instance, if one accepts that syllables or exemplars are the units of combination used by the speaker, phonemes are still a useful level of description to characterise differences in meaning. We distinguish between:

- 1. productively combinatorial phonology, where the cognitive mechanisms for producing, recognising and remembering signals make use of a limited sets of units that are combined in many different combinations. Productive combinatoriality is a property of the internal representations of language in the speaker (I-language).
- superficially combinatorial phonology, where parts of signals overlap with parts of other signals. Superficial combinatoriality is a property of the observable language (E-language). Importantly, the overlapping parts of different signals need not necessarily also be the units of combination of the underlying linguistic representations.

This paper is concerned with mathematical and computational theories of the evolution of combinatoriality of human languages at both these levels. It has often been observed that natural language phonology is *discrete*, in that it allows only a small number of basic sounds and not all feasible sounds in between. In this paper we argue that it is important to distinguish between discreteness per se, and superficial and productive combinatoriality. In section 2, we will review existing models of Liljencrants and Lindblom (1972), Lindblom et al. (1984),

¹In the animal behaviour literature the term "phonological syntax" (coined by Peter Marler, see Ujhelyi, 1996) is often used, and Michael Studdert-Kennedy also uses the term "particulate principle" (coined by W. Abler, see Studdert-Kennedy, 1998). Jackendoff (2002, p.238) uses the term "combinatorial, phonological system" on which our terminology is based.

de Boer (2001) and Oudeyer (2001, 2002, 2005), and argue that they are relevant for the origins of discreteness, but have little to say about the origins of superficial and combinatorial phonology. Nowak and Krakauer (1999) and Nowak et al. (1999) do address the origins of productive combinatoriality, but these models have a number of shortcomings that make them unconvincing as an explanation for its evolution.

In our own model, that we will introduce in section 3, we address the questions of why natural language phonology is both discrete and superficially combinatorial. We assume, but do not show in this paper, that superficial combinatoriality is an important intermediate stage in the evolution of productive combinatoriality.

1.2 The origins of combinatorial phonology

Although discrete, combinatorial phonology has often been described as a uniquely human trait (e.g. Hockett, 1960; Jackendoff, 2002), it is increasingly realised that many examples of bird and cetacean songs (e.g. Doupe and Kuhl, 1999; Payne and McVay, 1971) and, importantly, non-human primate calls are combinatorial as well (Ujhelyi, 1996). For instance, the "long calls" of tamarin monkeys are built up from many repetitions of the same element (e.g. Masataka, 1987), and those of gibbons (e.g. Mitani and Marler, 1989) and chimpanzees (e.g. Arcadi, 1996) of elaborate combinations of a repertoire of notes.

Such comparative data should be taken seriously, but it is unwarranted to view combinatorial long calls in other primates as an immediate precursor of human combinatorial phonology, because there are some important qualitative differences:

- Although a number of building blocks might be used repeatedly to construct a call, it does not appear to be the case that rearranging the building blocks results in a call with a different meaning.
- It is unclear to what extent the building blocks of primate "long calls" are flexible and whether they are learnt.
- In human language, combinatorial phonology functions as one half of the "duality of patterning": together with recursive, compositional semantics it yields the unlimited productivity of natural language, but it is unclear if the single combinatorial system of primates can be seen as its precursor.

Nevertheless, combinatorial phonology must have evolved from holistic systems by natural selection. There are at least two views on what the advantages of combinatorial coding over holistic coding are:

1. It makes it possible to transmit a larger number of messages over a noisy channel (the "noise robustness argument", an argument from information theory, e.g. Nowak and

Krakauer, 1999). Note that this argument requires that the basic elements are distinct from each other, and that signals are strings of these basic elements. The argument does not address, however, how signals are stored and created;

2. It makes it possible to create an infinitely extensible set of signals with a limited number of building blocks. Such productivity provides a solution for memory limitations, because signals can be encoded more efficiently, and for generalisation, because new signals can be created by combining existing building blocks (the "productivity argument", a point often made in the generative linguistics tradition, e.g. Jackendoff, 2002). Note that this argument deals purely with the cognitive aspects, and views the acoustic result more as a side-effect.

These views are a good starting point for investigating the question of *why* initially holistic systems (which seem to be the default for smaller repertoires of calls) would evolve toward combinatorial systems. However, just showing an advantage does not constitute an evolutionary explanation (Parker and Maynard Smith, 1990). At the very least, evolutionary explanations of an observed phenotype, involve a characterisation of (i) the set of possible phenotypes, (ii) the fitness function over those phenotypes, and (iii) a sequence of intermediate steps from an hypothesised initial state to observed phenotype. For each next step, one needs to establish that (iv) it has selective advantage over the previous, and thus can invade in a population without it. In section 2 we will criticise some existing models because they lack some of these required components.

In language evolution, fitness will not be a function of the focal individual's traits alone, but also of those of its conversation partners. That is, the selective advantage of a linguistic trait will depend on the frequency of that trait and other traits in a population (it is "frequency dependent"). Therefore, evolutionary game-theory (Maynard Smith, 1982) is the appropriate framework for formalising evolutionary explanations for language (Nowak and Krakauer, 1999; Komarova and Nowak, 2003; Smith, 2004; van Rooij, 2004; Jäger, 2005). In this framework, the crucial concept is that of an evolutionary stable strategy (henceforth, ESS): a strategy that cannot be invaded by any other strategy (Maynard Smith and Price, 1973). Thus formulated, the challenge is to show that (i) repertoires of signals with a combinatorial phonology are ESSs, and that (ii) plausible precursor repertoires, without combinatorial phonology, are not evolutionarily stable.

There are also theories of the origins of combinatorial phonology that do not assume a role for natural selection. For instance, Lindblom et al. (1984), de Boer (2001) and Oudeyer (2001, 2002) see "self-organisation" as the mechanism responsible for the emergence of combinatorial phonology. These authors use the term self-organisation in a very broad sense, where it can refer to almost any process of pattern formation other than classical, Darwinian evolution. Liljencrants and Lindblom (1972) use an optimisation heuristic, but do not make explicit which process underlies the optimisation. In the next section we will argue that self-organisation and natural selection need not be put in opposition, but can be seen as detailing *proximate* and *ultimate* causes respectively (Tinbergen, 1963; Hauser, 1996), where natural selection modifies the parameters of a self-organising process (Waddington, 1939; Boerlijst and Hogeweg, 1991).

2 Existing Approaches

2.1 Maximising discriminability

Liljencrants and Lindblom (1972) argued that one can understand the structure of the sound systems in natural language as determined by physical factors, such as perceptual discriminability and articulatory ease, and not as the result of arbitrary settings of abstract parameters (as in the theories from the generative phonology tradition, e.g. Chomsky and Halle, 1968). In their paper they focused on the discriminability of vowel repertoires, and proposed the following metric to measure their quality:

$$E = \frac{1}{2} \sum_{i,j \neq i \in R} \frac{1}{d_{ij}^2} = \sum_{i=2}^{|R|} \sum_{j=1}^{i-1} \frac{1}{d_{ij}^2}$$
(1)

where R is a repertoire with |R| distinct sounds, and d_{ij} is the perceptual distance between sound i and sound j. The perceptual distance between vowels is determined by the position of peaks (resonances) in the vowel's frequency spectrum. The frequency of the first and the second peak can be used as coordinates in a two-dimensional space. The weighted Euclidean distance between two such points turns out to be a good measure of perceptual distance between vowels. E is a measure for the quality of the system, where lower values correspond to a better distinguishable repertoire. The E stands for "energy", in analogy with the potential energy that is minimised in various models in physics.

Lindblom & Liljencrants performed computer simulations using a simple hill-climbing heuristic, where at each step a random change to the repertoire is considered, and adopted only if it has a lower energy than the current state. Their results compared favourably to observed data on vowel system distributions. These results were important because they showed that sound systems in natural languages are not arbitrary. Rather, they can be understood as the result of more fundamental principles. However, a number of questions remain. First of all, what in the real world exactly is the optimisation criterion meant to be modelling? It is important to realise that the optimisation criterion in equation (1) is neither maximising the distances between vowels nor minimising the probability of confusion. Minimising $E = \frac{1}{2} \sum_{i,j \neq i \in \mathbb{R}} 1/d_{ij}^2$ by changing the configuration of a set of vowels in a restricted acoustic space, is not necessarily the same a maximising the average distance $\overline{d} = \frac{1}{N} \sum_{i,j \neq i \in \mathbb{R}} d_{ij}$ (or squared distance), nor is it the same as minimising the average confusion probability $\overline{C} = \frac{1}{N} \sum_{i,j \neq i \in \mathbb{R}} P(j \ perceived | i \ uttered)$. At intermediate distances, these three criteria behave very similarly. The crucial difference is at distance 0, where Lindblom & Liljencrants's E goes to infinity, and at large distances, when both the E and \overline{C} measure, but not \overline{d} , approach 0. In section 3.2 we will argue that Liljencrants and Lindblom's E behaves unrealistically, and that minimising the average confusion probability (or equivalently, maximising the *distinctiveness* $D = 1 - \overline{C}$) is a better criterion.

Second, we should ask which mechanism in the real world is responsible for the optimisation. Lindblom himself has referred to both natural selection and self-organisation. The frequency dependence of language evolution makes that natural selection at the level of the individual cannot be equated with optimisation at the level of the population (see Zuidema and de Boer, 2003). Before we can invoke natural selection, we need to do at least a game-theoretic analysis to show that every new configuration of signals in the acoustic space can *invade* in a population where it is extremely rare. We call this the "invasibility constraint". Models of this type will be discussed in the next section. For self-organisation, the mechanism for optimisation has been worked out more precisely. De Boer (2000; 2001) has studied a simulation model of a population of individuals that each strive to imitate the vowels of others, and be imitated successfully by others. He showed that in a process of self-organisation, similar configurations of vowels emerge as in the Lindblom & Liljencrants model, and as found empirically in the languages of the world.

Finally, the important question remains of how to extend these models to more complex signals. The models of Lindblom, Liljencrants and de Boer only deal with vowels and, hence, only with the *discrete* aspect of human phonology. They have little to say about the evolution of superficial and productive combination. Lindblom et al. (1984), and similarly de Boer (1999, chapter 7), have studied models where signals are trajectories, going from a point in a consonant space, to a point in a vowel space. But in these models the issue is still really the emergence of categories, because the sequencing of sounds is taken as given. In this paper, in contrast, we will focus on the emergence of superficial combination.

2.2 Natural selection for combinatorial phonology

Nowak and Krakauer (1999) apply notions from information theory and evolutionary game theory to the evolution of language. They derive an expression for the "fitness of a language". Imagine a population of individuals, a set of possible signals and a set of possible meanings to communicate about. Speakers choose a meaning to express (the intention), and choose a signal for it with a certain probability. Hearers receive the signal, possibly distorted due to a certain degree of noise. Hearers subsequently decode the (distorted) signal and arrive at an interpretation.

Nowak & Krakauer observe that when communication is noisy and when a unique signal is used for every meaning, the fitness is limited by an "error limit": only a limited number of sounds can be used — and thus a limited number of meanings be expressed — because by using more sounds the successful recognition of the current signals would be impeded. They further show that in such noisy conditions, fitness is higher when (meaningless) sounds are combined into longer words. These results are essentially instantiations of Shannon's more general results on "noisy coding" (Shannon, 1948), as is explored in a later paper by the same group (Plotkin and Nowak, 2000).

More interesting is the question how natural selection could favour a linguistic innovation in a population where that innovation is still very rare. Nowak and Krakauer (1999) do not address that specific problem mathematically. They do, however, perform a mathematical, game-theoretic analysis of the evolution of "compositionality", and point out that this analysis can be adapted easily to the case of combinatorial phonology, as is worked out in Zuidema (2005). In such an analysis, all mixed strategies are considered where both holistic and combinatorial signals are used.

Nowak & Krakauer assume that the confusion between holistic signals is larger than the confusion between combinatorial signals, and that there is no confusion between the two types of strategies. From these assumptions it follows that a more combinatorial language can always invade a population with a less combinatorial language. This is the case, because for languages L and L' (with proportions of combinatorial signals x and x', respectively) it turns out that:

$$F(L', L') > F(L, L') > F(L, L)$$
 if $x' > x$. (2)

where F(a, b) is the expected communicative success (fitness) of users of language *a* communicating with users of language *b*.

If L' is very infrequent, then all speakers of language L (the "residents") will have a fitness of approximately F(L, L) and the rare speakers of language L' (the "mutants") will have fitness of approximately F(L, L'), because for both residents and mutants the vast majority of interactions will be with speakers of language L. Once the frequency of mutants starts to rise, the residents will gain in fitness, that is, move toward a fitness F(L, L'). However, the mutants will gain even more by interacting more and more with other mutants, that is, move toward F(L', L'). Hence, these calculations show that strategies that use more combinatoriality can invade strategies that use less. This means that the evolutionary dynamics of languages under natural selection should lead to compositionality and combinatorial phonology.

Although this model is a useful formalisation of the problem and gives some important insights, as an explanation for the evolution of combinatorial phonology (and compositionality) it is unconvincing. The problem is that the model only considers the advantages of combinatorial strategies, and ignores two obvious disadvantages: (1) by having a "mixed strategy" individuals have essentially two languages in parallel, which one should expect to be costly because of memory and learning demands and additional confusion. Nowak & Krakauer simply assume that the second system is in place, and that the hearer interprets all signals correctly, even if x is close to 0, and the number of learning experiences is therefore extremely small; (2) combinatorial signals that consist of two or more sounds take longer to utter and are thus more costly^2 . A fairer comparison would be between holistic signals of a certain duration (where continuation of the same sound decreases the effect of noise) and combinatorial signals of the same duration (where the digital coding decreases the effect of noise). This is the approach we take in the model of this paper, but like Nowak & Krakauer, we will look at invasibility in addition to optimisation.

2.3 Crystallisation in the perception–imitation cycle

A completely different approach to combinatorial phonology is based on "categorical perception". Categorical perception (Harnad, 1987) is the phenomenon that categorisation influences the perception of stimuli in such a way that differences between categories are perceived as larger and differences within categories as smaller than they really are (according to an "objective", cross-linguistic similarity metric). For instance, infants of six months old are already unable to perceive distinctions between sounds that are not phonemes in their native language, something they *were* able to do at birth (Kuhl et al., 1992). Hence, when presented syllables as stimuli ranging from /ba/ to /pa/ in fixed increments, British subjects will hear the first stimuli as ba's, and the last as pa's, without perceiving an intermediate signal (Cooper et al., 1952). Apparently, the frequency and position of acoustic stimuli gives rise to particular phoneme prototypes, and the prototypes in turn "warp" the perception.

Oudeyer (2001, 2002) observes that signals survive from generation to generation because they are perceived and imitated. Because of categorical perception, the imitation will not always be exactly the same as the perceived signal. However, the signals that are produced shape the categories of the next generation. Thus there is feedback between emitted signals, formation of categories and perception. This shapes the repertoire of signals in a cycle from articulation to perception to articulation (the perception–imitation cycle; see also Westermann, 2001 and Westermann and Miranda, 2004, for models of sensory-motor integration and their relevance for imitation and categorical perception).

Oudeyer (2001) presents a model to study this phenomenon. In this model, signals are modelled as points in an acoustic space. The model consists of two coupled neural maps, one for perception and one for articulation. The perceptual map is of a type known to be able to model categorical perception: its categorisation behaviour changes in response to the input data. In addition, the associations between perceptual stimuli and articulatory commands are learnt. Through this coupling between perceptual and articulatory maps, a positive feedback

 $^{^{2}}$ It is, of course, slightly awkward to criticise a model that Nowak and Krakauer (1999) never actually worked out. The point here is that if one takes all the assumptions that they do spell out in the paper, and work out the model as they suggest, the result is unsatisfactory. A better model of the evolution of combinatorial phonology must start with different assumptions.

loop emerges where slight non-uniformities in the input data lead to clusters in the perceptual map, as well as weak clusters in the articulatory map, and hence to slightly stronger nonuniformities in the distribution of acoustic signals. Oudeyer calls the collapse of signals into a small number of clusters "crystallisation".

Oudeyer (2002) generalises these results to a model with (quasi-) continuous trajectories, where a production module triggers a sequence of targets in the articulatory map, which yield a continuous trajectory. Also in this version of the model, well-defined clusters form in the perceptual and articulatory maps. The signals can thus be analysed as consisting of sequences of phonemes.

Oudeyer's model is fascinating, because it gives a completely non-adaptive mechanism for the emergence of combinatorial phonology. However, the question whether recombination increases the functionality of the language, and thus the fitness of the individual that uses it, remains unanswered. If it is not functional, one would expect selection to work against it. In particular, in Oudeyer's first model (2001), where signals are instantaneous, a large repertoire of signals collapses into a small number of clusters. A functional pressure to maintain the number of distinct signals would thus have to either reverse the crystallisation, or combine signals from different clusters.

In his second model (Oudeyer, 2002), signals are continuous trajectories and potentially a much larger distinct repertoire can emerge. However, the functionality of the repertoire is not monitored, and plays no role in the dynamics. It might or might not be sufficient. The number of "phonemes" (the discrete aspect) that forms is a consequence of the parameters and initial configuration, and in a sense accidental. The reuse (the superficial combination aspect) in the model is built-in in the production-procedure. The assumption that signals already consist of sequences of articulatory targets is justified with considerations from articulatory phonetics: constraints from articulatory motor control, it is argued, impose combinatorial structure on any large repertoire of distinct sounds. Even if one fully accepts this argument, the need for a large and distinctive repertoire is a functional pressure. In Oudeyer's model, however, there is no interaction between the number of phonemes that is created, and the degree of reuse (the number of phonemes per signal) that emerges. This issue, which seems the core issue in understanding the origins of combinatorial phonology, is not modelled by Oudeyer. In our model, in contrast, we ensure that the functionality increases rather than decreases.

2.4 Other models and linguistic theories

All other computational models of the evolution of combinatorial phonology that we are aware of, also assume the sequencing of phonetic atoms into longer strings as given. They concentrate rather on the structure of the emerged systems (Lindblom et al., 1984; de Boer, 2001; Redford et al., 2001; Ke et al., 2003) or on how conventions on specific combinatorial signal systems can

become established in a population through cultural transmission (Steels and Oudeyer, 2000). Theories on the evolution of speech developed by linguists and biologists focus on possible preadaptations for speech. MacNeilage and Davis (2000) propose oscillatory movements of the jaw such as used in breathing and chewing as precursors for syllable structure. Fitch (2000) sees sexual selection as a mechanism to explain the shape of the human vocal tract. Studdert-Kennedy (2002) explains the origin of recombination and duality of patterning as the result of vocal imitation.

These models are interesting, and, importantly, bridge the gap with empirical evidence on how combinatorial phonology is implemented in the languages of the world. However, they are of less relevance here, because they do not address the origins of the fundamental, qualitative properties of discrete and combinatorial coding. That is, they leave open the question as to under what circumstances a system of holistically coded signals with finite duration would change into a combinatorial system of signals.

3 Model Design

We will now present the design of a new model that shares features with all existing approaches. Like the Liljencrants and Lindblom (1972) model, it makes use of the concept of "acoustic space", a measure for perceptual distinctiveness and a hill-climbing heuristic. Like the Nowak and Krakauer (1999) model, the measure for distinctiveness is based on confusion probabilities, and our study includes a game-theoretic invasibility analysis. Finally, like Oudeyer (2002), we model signals not just as points, but as trajectories through acoustic space.

In the model, we do not assume combinatorial structure, but rather study the gradual emergence of superficially combinatorial phonology from initially holistic signals. We do take into account the temporal structure of both holistic and phonemically coded signals. We view signals as continuous movements ("gestures", "trajectories") through an abstract acoustic space. We assume that signals can be confused, and that the probability of confusion is higher if signals are more similar, i.e. closer to each other in the acoustic space according to some distance metric. We further assume a functional pressure that maximises distinctiveness.

3.1 The acoustic space

The model of this paper will deal with repertoires of signals, their configuration and the similarities between signals. This requires conceptualising signals as points or movements through a space. An appropriate definition of acoustic space will, as much as possible, reflect the articulatory constraints as well as perceptual similarities, such that signals that cannot be produced fall outside the space, and that points in the space that are close sound similar and are more easily confused. For human perception of vowels, a simple but very useful acoustic space can be constructed by looking at the peaks in the frequency spectrum. These peaks, called formants, correspond to the resonance frequencies in the oral cavity, and are also perceptually very salient. Artificially produced vowels with the correct peaks but otherwise quite different frequency spectra, are recognisable by humans. From experiments where subjects are asked to approximate vowels sounds by manipulating just two formants frequencies, it appeared that a good representation of vowels can be given in just two dimensions, with the first formant as the first dimension, and the *effective second formant* as the second dimension (Carlson et al., 1970). A related approach for defining an acoustic space works with *cepstral coefficients* (Bogert et al., 1963). These define a sequence of features of the signal of decreasing importance. Speech signals can be characterised by a relatively low number of cepstral coefficients (10–15).

Although formants work well for humans, it appears that pitch is a more salient variable in articulations of non-human primates (although they are able to perceive formants as well). Of course, it is difficult to tell what the appropriate acoustic space is for modelling articulation and perception of early hominids that feature in scenarios of the evolution of language (e.g. Lieberman, 1984; Jackendoff, 2002). However, the considerations that will be presented below remain the same, independent of the exact nature of the underlying perceptual dimensions.

3.2 Confusion probabilities

Once we have constructed an acoustic space that captures the notion of perceptual similarity, we can ask how the distance in that space relates to the probability of confusion. Answering that question requires us to make assumptions on the causes of confusion and the nature of categorisation. We can get a general idea, by first looking at the simple example of a 1-dimensional acoustic space with just 2 prototype signals A and B (modelled as points in that space), and a distance d between them:



Now assume that a received signal X, lying somewhere on the continuum, will be perceived as A or B depending on which is closest (*nearest neighbour classification*). Finally, assume a degree of noise on the emitted signals, such that when a signal, say A, is uttered, the received signal X is any signal drawn from a Gaussian distribution around A:



Now we can calculate the probability that an emitted signal A is perceived as B:

$$P(B \text{ perceived}|A \text{ uttered}) = \int_{x=\frac{1}{2}d}^{\infty} \mathcal{N}(\mu=0,\sigma=\delta) \, dx$$
$$= \int_{x=\frac{1}{2}d}^{\infty} \frac{1}{\sqrt{2\pi\delta}} e^{\frac{-x^2}{2\delta^2}} \, dx, \tag{3}$$

where δ is the standard deviation of the Gaussian³. This integral, which describes the surface under the Gaussian curve right of the point $\frac{1}{2}d$, has a number of important features, as illustrated with the solid curves in figure 1.



Figure 1: The probability of confusion as a function of distance for several values of δ . The curves give the theoretical prediction based on the calculations in section 3.2; the points are data from a computational simulation of the confusion probabilities between two trajectories in a 2d acoustic space (discussed in section 3.4).

First of all, at d = 0, the confusion probability is not 100%, as the naive first intuition might be, but 50%. That is, even if two signals are identical, the hearer still has 50% chance of decoding it correctly. Second, with increasing d, the confusion probability first rapidly decreases and then slowly approaches 0. These are crucial properties: even though the confusion probability as a function of distance can have many different shapes depending on the exact type of noise and the exact type of categorisation, the function will always have these general characteristics at d = 0 and in the limit of $d \to \infty$. In contrast, the previously discussed E measure, and summed distance measure, do not have both these properties. For the purposes of this paper, they are therefore not appropriate criteria for optimisation.

If the acoustic space has more than 1 dimension, and if there are more than 2 signals, calculations like in equation (3), quickly get extremely complex, and confusion probabilities are no longer uniquely dependent on distance. We can, however, assume that the confusion probabilities are generally proportional to a function of distance with a shape as in figure 1.

³Equation (3) is the *error function* $\operatorname{erf}(x)$ (modulo a correction factor, see equation 4), and of course the same equation as used for calculating the *p* value in standard statistical tests.

Hence, let f(d) be a function of distance d of that shape, parameterised by the noise level δ :

$$f(d) = \int_{x=\frac{1}{2}d}^{\infty} \frac{1}{\sqrt{2\pi\delta}} e^{\frac{-x^2}{2\delta^2}} dx, = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{d}{2\delta\sqrt{2}}\right).$$
(4)

We assume that confusion probabilities are proportional to their "f-score": $P(B \text{ perceived}|A \text{ uttered}) \propto f(d(A, B))$. But we also know that the probabilities of confusing a signal with any of the other signals in a repertoire (including the signal itself) must add up to 1: $\sum_{X \in R} P(X \text{ perceived}|A \text{ uttered}) = 1$. Hence, we can estimate the probability of confusing signal A with signal B as:

$$P(B \text{ perceived}|A \text{ uttered}) = \frac{f(d(A, B))}{\sum_{X \in R} f(d(A, X))}.$$
(5)

From this, it is now straightforward to define a measure for the *distinctiveness* D of a repertoire. Let D be the estimated probability that a random signal t from R is correctly identified:

$$D(R) = \frac{1}{T} \sum_{t=1}^{T} \frac{f(d(R_t, R_t))}{\sum_{t'=1}^{T} f(d(R_t, R_{t'}))}.$$
(6)

3.3 Trajectory representation

We have a qualitative understanding of how to define the acoustic space with points representing signals, and of how to estimate the confusion probabilities and distinctiveness as functions of the distances between those points. We can now try to extend the model to deal with signals that have a temporal dimension. It would be desirable if the same apparatus can still be used. We therefore define temporal signals as *trajectories*: movements through the acoustic space. In our approach, a trajectory is a connected sequence of points (each of which corresponds to the frequency spectrum of a small interval in the waveform).

To illustrate the feasibility of deriving trajectory representations from acoustic data, we show in figure 2(a) a number of trajectories through vowel space that are based on actual recordings. The graph shows the trajectories from a number of recorded vowels, which correspond to more-or-less stationary trajectories in the space, and from recordings of a number of diphthongs, which correspond to movements from one vowel's region to another. Figure 2(b) and (c) show trajectories through the space defined by cepstral coefficients 1, 3 and 5. In this space we can, to a certain extent, represent consonants as well. Overlaid in both graphs are the resulting trajectories of two recordings, illustrating that the construction is repeatable, albeit with considerable variation.

In the model of this paper we will not worry about the problems of constructing acoustic spaces and drawing trajectories through them. Instead, we will take as our starting point a set of trajectories through an abstract acoustic space. The model is based on piece-wise linear trajectories in bounded 2-D or 3-D continuous spaces of size 1×1 or $1 \times 1 \times 1$. Trajectories



Figure 2: Trajectory representations derived from recorded acoustic data from Dutch native speakers. Each point on a trajectory is given by the cepstral coefficients of the frequency spectrum of a short time interval of the signal. In (a) the first two coefficients are used; in (b) and (c) coefficients 1, 3 and 5 are used.

are sequences of a fixed number of points (parameter P). Each point has a maximum distance (parameter S) to the immediately preceding and following points in the sequence. The following and preceding points to a point can lay anywhere within a circle of radius S with that point at the centre. Trajectories always stay within the bounds of the defined acoustic space.

Signals in the real world are continuous trajectories, but in the model we need to discretize the trajectories. To ensure that we do not impose the combinatorial structure we are interested in, we discretize at a much finer scale than the combinatorial patterns that will emerge. Hence, the points on a trajectory are not meant to model atomic units in a complex utterance. They implement a discretisation of a continuous trajectory that can represent both a holistic or combinatorial signal.

3.4 Measuring distances and optimising distinctiveness between trajectories

How do we extend the distance and distinctiveness measures for points to trajectories? Perhaps the simplest strategy would be to look at a repertoire of trajectories one time-slice at a time, and simply optimise –as before– the distinctiveness between the points (Lindblom et al., 1984). With such an approach, however, the temporal dimension could just as well have been left out, and the approach has little to say about the emergence of superficial combination. Combinatorial phonology emerges –if at all– as a trivial consequence of (i) the formation of categories (phonemes), and (ii) sequencing imposed by the trajectory representation.

Much more interesting is when we measure the distance between complete trajectories and optimise their distinctiveness. In such an approach, there is a role for combinatorial phonology: the confusion probability between two largely overlapping trajectories might be very low, as long as they are sufficiently distinct along one stretch of their length. We define the distance between two trajectories t_i and t_j , as the *average* distance between the corresponding points on the trajectories:

$$d(t_i, t_j) = \frac{1}{P} \sum_{p=1}^{P} d(t_i^p, t_j^p),$$
(7)

where t_i^p is the p-th point on the i-th trajectory in a repertoire, and d(a, b) gives the distance between two points a and b.

This distance measure then provides the input to the same type of distance-to-confusion function that we derived for points (equation 6). For trajectories, it is far from trivial to derive the exact shape of that function analytically, even if the noise and categorisation mechanisms were completely known. However, we have performed computational experiments that demonstrate that our approximation is very accurate. In these simulations, noise was simulated using the DISTURBANCE function as will be defined in section 3.5, and nearest neighbour classification. The dots in figure 1 show results relating distance between two trajectories with the probability that they are confused. The results give an excellent fit with the approximation of equation (6), and thus indicate that the distance-to-confusion function for points is also applicable to trajectories.

One may argue that the distance metric in equation (7) is too simplistic, and does not do justice to the fact that slight differences in timing of two signals will not affect their perceptual similarity much. A possibility for a more realistic measure of distance, that does take into account such timing effects, is "dynamic time warping" which has been used with reasonable success in computer speech recognition (e.g. Sakoe and Chiba, 1978). We have run simulations with this technique, but did not find qualitative differences with the simpler distance measure.

3.5 The hill-climbing heuristic

Now that we have defined a distance metric, it is straightforward to use a hill-climbing heuristic such as Liljencrants and Lindblom (1972) and apply it to much more complex signals. Hillclimbing is an iterative procedure, where at each step a random change to the repertoire is considered, and if it improves the distinctiveness it is applied. Then another random change is considered and the same procedure applies over and over again. In pseudo-code, the procedure looks as follows:

```
% R is a repertoire of signals
% S is the segment length parameter
% \rho is the hill-climbing rate parameter
% \delta is the acoustic noise parameter
for i = 1 to I
R' = \text{CONSTRAIN}(R+\text{DISTURBANCE}(\rho), S);
if HILLCLIMBING-CRITERION(R, R', \delta) then R = R';
end for
```

Here, DISTURBANCE applies random noise (from a Gaussian with $\mu = 0$ and $\sigma = \rho$), to all of the coordinates of a (uniformly) random point on a random trajectory. CONSTRAIN is a function that enforces that all points on the trajectories fall within the boundaries of the acoustic space, and that all segments have maximum length S. Hence, after a random point t_x is moved to a new random position, the CONSTRAIN function first moves it back, if necessary, within the boundaries of the acoustic space; it then moves the two points on both sides of the moved point, t_{x+1} and t_{x-1} , closer, if necessary, such that the distance to t_x is no more than S. The direction from t_x to t_{x+1} or t_{x-1} remains the same. The same procedure is applied iteratively to the neighbours of t_{x+1} and t_{x-1} until the ends of the trajectory are reached. The HILLCLIMBING-CRITERION (R, R', δ) in the basic model, which we call the "optimisation condition" (OP), is defined as follows:

$$OP: \quad D_{\delta}(R') > D_{\delta}(R), \tag{8}$$

where D is the distinctiveness function given in equation (6). Note that this criterion is frequency-independent; in section 4.5 we will consider frequency-dependent criteria.

Hill-climbing is just an optimisation *heuristic*; there is no guarantee that it will find the optimal configuration for the given criterion. Especially when the repertoire considered is relatively complex, the system is likely to move toward a local optimum. Although better optimisation heuristics exist, this problem is in general unavoidable for systems with so many variables. Hence, also in Nature, the optimisation of sound systems has not escaped the problem of local optima; the real optimum is therefore not necessarily interesting for describing the patterns in human speech. Instead, we will concentrate on general properties of the local optima we find, and on the gradual route towards them.

4 Results

We have implemented versions of the basic model as outlined above in C++ and MatLab. We have ran many simulations with a large number of parameter combinations and a number of variations of the basic model. In the following we will first briefly give an overview of the general behaviour of the model in these simulations by means of a representative example, and then give a detailed analysis of why we observe the kind of results that we do. In section 4.5 and 4.7 we will study extensions of the basic model where we test whether innovations can invade in populations where they are rare, and where we evaluate some of the other simplifications made in the basic model.

4.1 An overview of the results

We will describe the behaviour of the model under many different parameter settings by using the representative example depicted in figure 3. Figure 3(a) shows an the equilibrium configu-



(a) 9 instantaneous signals optimised for distinctiveness. D=0.97 ($\delta = 0.1, \rho = 0.1, I = 1000$).

(b) 9 trajectories at same locations as the signals in (a) with small perturbations. D=0.94 ($\delta = 0.1, S = 0.1$).



(c) 9 trajectories optimised for distinctiveness. D=0.98 ($\delta = 0.1, \rho = 0.1, S = 0.1, I = 6000$).

(d) 9 instantaneous sounds at same location as endpoints in (c). D=0.72 (δ = 0.1).

Figure 3: In a combinatorial phonology, distinctiveness of signals at each particular time-slice is sacrificed for better distinctiveness of the whole trajectory. Instantaneous signal (or equivalently, stationary trajectories) will be organised in patterns like (a) and not like (d) when optimised for distinctiveness. For non-stationary trajectories, the same pattern, as in (b), is not stable, but will –after optimisation– in stead be organised like (c). Each individual time-slice, as illustrated with the end-points in (d) is suboptimal, but the whole temporal repertoire is at a local optimum.

ration of 9 point-like signals in an abstract acoustic space, optimised for distinctiveness at an intermediate noise-level ($\delta = 0.1$). This particular configuration is stable: no further improvements of the distinctiveness of the repertoire can be obtained by making small changes to the location of any of the signals. The distinctiveness is D = 0.97; that is, with the given noise level, our estimate of the probability of successful recognition of a signal is 97%.

Figure 3(b) shows 9 trajectories, consisting of 10 points and hence 9 segments each. Each of these trajectories was created by taking 10 copies of one of the points in figure (a) and connecting them. A small amount of noise was added to each point, and the CONSTRAIN function, as described above, was applied to each trajectory, enforcing a maximum distance (S = 0.1) between all neighbouring points on the same trajectory. Due to this perturbation, the distinctiveness of this repertoire of trajectories is somewhat lower, D = 0.94, than of the repertoire in (a). (The definitions of distance and distinctiveness are such that a repertoire of stationary trajectories has the same D as a repertoire of points at the same locations; hence, points and stationary trajectories, if all the same length, are equivalent in the basic model).

What will happen if we now optimise, through hill-climbing, the repertoire of trajectories for distinctiveness? One possibility is that the applied perturbations are nullified, such that the system moves back to the configuration of (a). That is not what happens, however. Rather, the system moves to a configuration as in figure 3(c). This graph shows a number of important features. First, all trajectories start and end near to where other trajectories start and end. The repertoire therefore can be said to exhibit a *superficially combinatorial phonology*: if we label the corners A, B, C and D, we can describe the repertoire as: $\{A, AB, B, CA, BC, C, CD, DB, D\}$. That is, we need only 4 category labels (phonemes) to describe a repertoire of 9 signals. In contrast, the repertoire in (b) is most easily described by postulating 9 categories, one for each trajectory⁴.

Second, some trajectories are bunched up in as small a region as possible, but other trajectories are stretched out over the full length of the space. Third, the configuration of the repertoire appears somewhat idiosyncratic and is in a local optimum⁵. Fourth, at each time-slice the configuration of the corresponding points is in fact suboptimal. For instance, in figure 3(d) just the endpoints of the trajectories in (c) are shown. 8 out of 9 of these points are closer to their nearest neighbour than any of the points in (a). Before we extend these results to simulations with many more trajectories of various lengths, and to acoustic spaces with more dimensions, we will first look at a number of simple cases that explain why the optimised repertoires have these features.

⁴We implicitly assume a model of categorisation here that favours robust and coherent categories.

⁵The stability of this configuration has not been rigorously established, but no qualitative changes have been observed in many thousands of additional iterations of the hill-climbing algorithm.

4.2 The optimal configuration depends on the noise level

To evaluate the role of the noise parameter δ , it is instructive to first look at a simple, 1dimensional example with signals as points. Consider a situation with 3 signals, 2 of which are fixed at the edges of a 1-dimensional acoustic space. The third signal is at distance x from the leftmost signal, and at distance 1 - x from the rightmost signal:



Now what is the optimal distance x for maximising the distinctiveness? As it turns out, the optimal x depends on the noise level δ . Recall that distinctiveness D is defined as the average probability of correct recognition (equation 6). In this case, we have three terms describing the recognition probabilities of each of the three signals. These are:

$$P(t_1 \text{ perceived} \mid t_1 \text{ uttered}) = \frac{f(0)}{f(0) + f(x) + f(1)}$$

$$(9)$$

$$P(t_2 \text{ perceived} \mid t_2 \text{ uttered}) = \frac{f(0)}{f(x) + f(0) + f(1-x)}$$
(10)

$$P(t_3 \text{ perceived} \mid t_3 \text{ uttered}) = \frac{f(0)}{f(1) + f(1-x) + f(0)}$$
(11)

The values of these three functions, for two different choices of the parameter δ are plotted in figure 4(a) and (b). If we add up these three curves, we find, for different values of δ , the curves in figure 4(c). Clearly, for low levels of noise the optimal value of x is x = 0.5. For higher noise levels, however, this optimum disappears, and the optimal configuration has x = 0or x = 1. That is, if there is too much noise, it is better to have several signals overlap.



Figure 4: Distinctiveness as dependent on distance and noise, 1d example. Panel a) shows confusion probabilities in a low-noise environment, panel b) shows them in a high-noise environment. Panel c) compares the probabilities of correct recognition for the low-noise and the high noise conditions. Note that the high-noise condition has a minimum at maximal distance, whereas the low-noise condition has a maximum.

Figure 5(a) shows a 2-dimensional system of 9 points optimised for distinctiveness with a

high noise level ($\delta = 1$). The optimal configuration under these conditions is to have each signal in one of the four corners: 3 corners with 2 signals, and one corner with 3 signals. With this configuration, the distance between the two or three signals that share a corner is d = 0, and their mutual confusability is high. But at least the distance to the other signals is high (d = 1, or $d = \sqrt{2}$).



Figure 5: The noise level determines how many signals can be kept distinct

Maximising distinctiveness is, because of the high noise level, equivalent to maximising summed distance. Consider one of the signals in the top-right corner, and consider moving it to the left, that is, away from the two signals already in that corner. The gain in distance from the top-right corner (Δdtr), will be exactly cancelled out by the loss in distance from the top-left corner (Δdtl). The gain in distance from the bottom-right corner (Δdbr), however, will not compensate for the loss in distance from the bottom-left corner (Δdbl). To see why, consider moving the signal a distance ϵ to the left. The (squared) gain in distance to the top-right is given by:

$$\Delta dbr^2 = [\epsilon^2 + 1] - [1] = \epsilon^2, \tag{12}$$

The (squared) loss in the distance to the top-left by:

$$\Delta dbl^2 = [1+1] - [(1-\epsilon)^2 + 1] = [1+1] - [1-2\epsilon + \epsilon^2 + 1] = 2\epsilon - \epsilon^2, \tag{13}$$

Of course, the summed distance will increase only if (12) is larger than (13), which is never the case if $0 \le \epsilon \le 1$.

In contrast, in figure 5(b) a system of 9 points is shown that has been optimised for distinctiveness at a relatively low noise level ($\delta = 0.1$). Here maximising distinctiveness is not equivalent to maximising summed distance, because of the relatively low noise level. To see why the noise level determines whether it is equivalent, consider a small change to the configuration, for instance moving the central point a bit to the left. Such a change will decrease the distance to some points, and increase the distance to some other points. Now, note that the distance-to-confusion function is approximately linear for relatively small distances (see figure 1). Therefore, maximising distinctiveness corresponds approximately to maximising average distances only if distances are small *relative to the noise level*, or equivalently, if the noise-level is high *relative to the distances*.

4.3 Distinctiveness is a non-linear function of distances

Figure 6 shows another 2-dimensional, 9 signal system. It has, after running the hill-climbing algorithm, converged to a local optimum (a). Why is this configuration stable? Consider moving the signal α at the left-most end of the interval, along that same interval. For each alternative x-coordinate of that signal, we can calculate the estimated probability of confusion with other signals. The *f*-values for all the other signals are plotted in figure (b). For instance, the *f*-value of the central-right signal (its contribution to the confusion about α) goes from very low if α is at the left-most end of the interval to very high (0.3) if α is at the right-most end of the interval.



(a) 9 points at a local optimum; we will consider moving one signal α along the horizontal interval.

(b) The f-scores of all other signals when α is moved along the interval in (a).

(c) The sum of all the f-scores of all other signals when α is moved along the interval in (a).

Figure 6: Figure (a) shows a local optimum of a 9-signal repertoire optimised for distinctiveness. What would happen if we move the signal at the right end of the interval in (a) horizontally to left? The probability of correct recognition of that signal, α , is inversely proportional to the sum of the f-scores of all other signals (see equation 5). Figures (b) and (c) show why this probability is in a local optimum with α at its current location. Parameters: $\delta = 0.3$.

The probability of correct recognition of α , and hence its contribution to the total distinctiveness, is inversely proportional to the sum of all *f*-values. In figure 6(c) we therefore give a plot of the sum of all these values (with the contribution of each signal indicated in different colours). That sum is in a local minimum at the actual location of α , which suggests that –at least initially– distinctiveness will not improve by shifting α to the left. The plot does not tell the whole story, though, because the probability of correct recognition of the other signals will also change due to the new position of α . Nevertheless, it does illustrate that the distinctiveness of a repertoire is a non-linear combination of the distances between the signals. Due to this non-linearity, the resulting stable configurations are sometimes counter-intuitive.

4.4 Why trajectories stretch out

Finally, in figure 7 we explore the question of why many trajectories in our simulations stretch out. In figure (a) we show 5 signals (in the bottom-left corner there are 2 signals on top of each other). The signals are points in the acoustic space, which we will here interpret as *stationary* trajectories of some arbitrary length. The graph shows the configuration that maximises the summed distance between the signals. The figure also gives the distance matrix, that gives for every pair of signals the distance between them. Naturally, the values are $\sqrt{2} \approx 1.4$ (across the diagonals), 1 (horizontally or vertically) and 0 (for the pair in the bottom-left corner). The average distance is $\overline{d} = 10.2/10 = 1.02$.

Figure 7(b) shows an alternative configuration, with the fifth signal in the centre. The distance matrix shows that the distance of the fifth signal to the bottom-left corner has increased, but at the expense of the distances to the three other corners. As a result, the average distance has actually gone down to $\overline{d} = 0.96$. The reason is that this configuration doesn't make optimal use of the longest available distances over the diagonal. Importantly, however, at low noise-levels, the distinctiveness of this configuration is in fact higher than of the configuration in (a). The reason is that with relatively little noise and long distances, the distinctiveness–distance function flattens out. Hence, there is more to be gained from avoiding confusion between the fifth and the bottom-left signal, then there is from maintaining the excessive "safety margin" with the other signals. In other words, the configuration in (b) sacrifices some average distance, to gain a more even distribution of distances and, hence, a lower average confusion probability.

In a restricted space, increasing the distance with one sound, will usually decrease the distance with another sound. That is, there is a crucial trade-off between maximising one distance at the expense of another. Although maximising distinctiveness D will generally lead to larger distances d, due to the non-linear dependence of D on d, that trade-off can work out differently when maximising D than when maximising \overline{d} .

Figure 7(c) shows yet another configuration, now with the fifth trajectory stretched out over the whole diagonal. As is clear from the given distance matrix, this configuration yields larger distances than in (b). To go from (b) to (c) there is no trade-off. The distances from the central, fifth signal to the top-left and bottom-left corners can be increased without decreasing the distances to the other two signals. The reason is that the distance between a stationary trajectory t and a stretched out trajectory t' is equal to the distance between t and the centroid of t' when t (like the top-right and bottom-left signals) is on a line through all the points of t', but larger when it's not (like the top-left and bottom-right signals). The distinctiveness in (c) is always larger than in (b). In figure 8 and 9 we show results from running the basic model under various parameter settings, including with repertoires with many trajectories and with 3-dimensional acoustic spaces. These results show that the observations made in the simple systems above, generalise to a wide range of conditions.

4.5 Locally optimal repertoires are evolutionary stable strategies

So-far, we have seen that repertoires of signals with a temporal structure will, when optimised for distinctiveness, not be organised in as many little clumps as needed, but instead stretch out. Rather than staying away as far as possible from other trajectories along its whole length, each trajectory will be close to some trajectories for some of its length, and close to other trajectories elsewhere. In qualitative terms, these systems show superficially combinatorial phonology. The model represents progress from existing work, because it deals with the categorical and combinatorial aspects as well as with the trade-off between them. It shows a possible sequence of fit intermediates, and, hence, a route up-hill on the fitness landscape.

We have not, however, dealt with the invasibility constraint from section 2.1. Will an innovation be able to invade and become established in a population where it is very infrequent? In other words: are systems that show combinatorial phonology evolutionary stable? To investigate these questions, we adapt the definition of distinctiveness to tell us something about pairs of languages. This way we can ask the question: how well will a repertoire R' do when communicating with a repertoire R? Pairwise distinctiveness \mathcal{D} is defined as follows:

$$\mathcal{D}(R,R') = \sum_{t=1}^{T} \frac{f(d(R_t, R'_t))}{\sum_{t'=1}^{T} f(d(R_t, R'_{t'}))}.$$
(14)

The quantity $\mathcal{D}(R, R')$ can be interpreted as the estimated probability of a signal uttered by a speaker with repertoire R, to be correctly interpreted by a hearer with repertoire R'.

When we now consider the invasion of a *mutant* repertoire R' into a population with *resident* repertoire R, four measures are of interest: $\mathcal{D}(R, R)$, $\mathcal{D}(R, R')$, $\mathcal{D}(R', R)$ and $\mathcal{D}(R', R')$. That is, how well does each of the repertoires fare when communicating with itself or with the other repertoire, in the role of speaker or of hearer? Specifically, for the invasion of R', it is necessary that $\mathcal{D}(R', R) > \mathcal{D}(R, R)$ or $\mathcal{D}(R, R') > \mathcal{D}(R, R)$, or some weighted combination of these requirements (depending on the relative importance of speaking and hearing). That is, a successful mutant must do better against the resident language, than the resident language does against itself. Can such situations arise?

Interestingly, this situation turns out to be very common. Consider the following 1d example:



The configuration on the right (B) is better on all accounts. Obviously, there will be less



Figure 7: Why do trajectories stretch out? Three configurations and their distance matrices. Abbreviations: bl: bottom-left, tr: top-right, tl: top-left, br: bottom-right, c centre.



Figure 8: A repertoire in a 2d acoustic space optimised for distinctiveness. Common parameters: $T = 9, P = 20, 2d, S = 0.2, \rho = 0.2, \delta = 0.25$. Figures (a-d) show the configurations at various stages of the hillclimbing process. Figures (e-m) show each of the individual trajectories in figure (d).



Figure 9: Repertoire in a 2d and 3d acoustic space optimised for distinctiveness.

confusion between its signals because they are further apart (when x = 0.1 and $\delta = 0.1$, $D(A) = \mathcal{D}(A, A) = 0.70$ vs. $\mathcal{D}(B, B) = 0.84$). But configuration B will even do better when communicating with A, both as a hearer ($\mathcal{D}(A, B) = 0.78$) and as a speaker ($\mathcal{D}(B, A) = 0.76$).

The HILLCLIMBING-CRITERION (R, R') is redefined as follows in each of the conditions "hearer benefits" (HB), "speaker benefits" (SB) or "equal benefits" (EB):

HB:
$$\mathcal{D}(R, R') \ge \mathcal{D}(R, R)$$
 (15)

SB:
$$\mathcal{D}(R', R) \ge \mathcal{D}(R, R)$$
 (16)

EB:
$$\frac{1}{2} \left(\mathcal{D} \left(R', R \right) + \mathcal{D} \left(R, R' \right) \right) \ge \mathcal{D}(R, R)$$
 (17)

It turns out that all the stable configurations we found in simulations with the optimisation criterion (OP, eq. 8), are also stable under criteria HB, SB and EB. Thus, locally optimal repertoires are evolutionary stable strategies.

4.6 Not all ESSs are locally optimal

ESSs are strategies that cannot be invaded by any other strategy. In evolutionary game theory, ESSs are therefore considered likely outcomes of an evolutionary process. However, if there are many ESSs in a given system, the initial conditions will determine which ESS will emerge ("equilibrium selection"). In our simulations with the HB, SB and EB conditions, we also observe ESSs that do not correspond to the locally optimal configurations that we found with the OP condition.

Figures 10(a-d) show the configuration of the repertoire at different numbers of iterations of the hill-climbing algorithm under the HB condition. Figure 10(i) gives the pairwise distinctiveness measures for each combination of these 4 configurations. At the diagonal of this matrix are the distinctiveness scores of each configuration. As is clear from this matrix, each next configuration can invade a population with previous repertoire. In bold-face we see the approximate evolutionary trajectory (the actual steps in the simulation are much smaller). Figure (d) is an ESS. However, figures (e-f) show that this configuration is not stable when the OP criterion is used. Figure 10(j) gives the pairwise distinctiveness measures for each combination of these 5 configurations. The diagonal elements in this matrix are the highest values in their row and column, which shows that none of these configurations could have invaded a population using (d) under the HB (or SB, or EB) condition. Nevertheless, once adopted, communication is more successful with every next configuration (as the diagonal elements show). The locally optimal configuration in (f), however, is an ESS under all four conditions.

We find suboptimal ESSs in simulations with the SB and EB conditions as well. Figure 11 shows stable configurations that emerge each of the four conditions. Interestingly, these suboptimal ESSs disappear when a different distance-to-confusion function is used. In figure 12 we used:

$$f(d) = \frac{1}{1 + e^{(\frac{1}{\delta}d^2)}},\tag{18}$$

instead of equation (4). In this, and other simulations (2d and 3d) with that same function, all ESSs observed show the same type of superficially combinatorial phonology that we found in the OP condition.

It is difficult to evaluate the relevance of suboptimal ESSs. It appears that their existence depends on the details of the distance-to-confusion function. Moreover, their stability depends on the assumption of deterministic evolutionary dynamics implicit in equations (15-17). Finally, whether or not they will emerge in evolution depends on the initial configuration.

4.7 Individual-based model

As a final test of the appropriateness of the basic model, we studied an individual-based simulation of a *population* of agents that each try to imitate each other in noisy conditions. This simulation is similar to the model described above, but now each agent in the population has its own repertoire, and it tries to optimise its own success in imitating and being imitated by other agents of the population. Hence, a random change, as before, is applied to a random trajectory of a random agent in the population. If this change improves the imitation success in interaction with a number of randomly chosen other individuals in the population, it is kept. Otherwise, it is discarded.

This version of the model is similar to the imitation games of de Boer (2000). That paper only modelled point-like signals (vowels) and did not investigate combinatorial phonology. The game implemented here is a slight simplification of the original imitation game. First, all agents in the population are initialised with a random set of a fixed number of trajectories. Then



Figure 10: Locally optimal repertoires are ESSs, but not all ESSs are locally optimal. (a-d) show configurations in an evolutionary simulation with the hearer benefit condition (HB, D(R, R') > D(R, R)) at various time steps; (d) is an ESS in the HB condition; (e-h) show results from a simulation in the optimisation condition (OP, D(R', R') > D(R, R)) that used (d) as its initial condition. (h) is an ESS in all conditions (OP, HB, SB, EB) considered. (i) shows a matrix that gives the pairwise distinctiveness scores for every combination of configurations in (a-d); (j) the matrix that gives the pairwise distinctiveness scores for every combination of configurations in (d-h). The approximate evolutionary trajectory is indicated with bold-face in these matrices. Parameters are: T=9, P=10, D=2, N=0.05, S=0.1.



Figure 11: Not all ESSs are locally optimal. Results from 4 simulations, each with the initial condition as in figure 10a. Different payoff functions lead to different ESSs, although for all payoff functions considered, locally optimal configurations as in (a) are stable. Parameters are: T = 9, P = 10, D = 2, $\rho = 0.2$, $\delta = 0.2$, S = 0.1.



Figure 12: A 16 trajectories signal system in a 3d acoustic space, after 1000 and 11000 iterations. At each time step, a small random change is considered, and only adopted if it represents an improvement according to the pairwise distinctiveness criterion (equation 14). Here, the confusion-probabilities are proportional to $\frac{1}{1+e^{(\frac{1}{\delta}d^2)}}$, where $\delta = 0.1$ and d is the average segment-by-segment Euclidean distance. Parameters are: $P = 15, S = 0.1, \rho = 0.05, \delta = 0.1$. The final distinctiveness is $D(R) = \mathcal{D}(R, R) = 0.94$.

for each game, a speaker is randomly selected from the population. This speaker selects a trajectory, and makes a random modification to it. Then it plays a number of imitation games (50 in all simulations reported here) with all other agents in the population. In these games, the *initiator* utters the modified trajectory with additional noise. The *imitator* finds the closest trajectory in its repertoire and utters it with noise. Games are successful if the imitator's signal is closest to the modified trajectory in the initiator's repertoire. If it turns out that the modified trajectory is kept, otherwise the original one is restored.

For vowel systems, it has been shown that optimising a single repertoire leads to similar systems as a population-optimisation system (compare de Boer, 2000; Liljencrants and Lindblom, 1972). It can be shown that for trajectories the same is true, under the condition that noisy distortions of trajectories do not distort the shape of these trajectories too much. This is illustrated in figure 13.



Figure 13: Comparison of population-based models with the optimisation model. Frame (a) shows the (five) trajectories of four agents (from a population of ten), when noise preserves the shape of trajectories. Notice the similarities with the optimised trajectories in frame (c). If noise does not preserve shape of trajectories, the trajectories tend not to stretch out, as shown in the frame (b). Although it is rather hard to see, there are four clusters in the corners, and one in the middle.

In this figure the left frame shows the system of five trajectories that resulted from playing imitation games in a population, using shape-preserving noise. Frame (c), for reference, shows a system of five trajectories that resulted from optimising distinctiveness as in the basic model. It can be observed that in both cases, the corners are populated by four trajectories, which are bunched up. The fifth trajectory, in contrast, follows the diagonal. As before, an analysis in terms of phonemes suggests itself: the four corners are basic phonemes, while the fifth trajectory uses one as the corners as a starting phoneme and the opposite corner as the ending phoneme. Both models result in similar systems of trajectories.

The middle frame, on the other hand, shows that when noise does not preserve shape of

trajectories, a system results in which all trajectories are bunched up and an analysis in terms of phonemes is therefore not possible. As noise in real signals is band limited, it follows that shape will always be preserved to some extent. For computational reasons, we have not performed simulations in the population condition with more than 5 trajectories. However, although less clean and not fully conclusive, the results from the individual-based model seem to be consistent with the observations with the basic model.

4.8 Measuring combinatorial phonology

So far, we have relied on an intuitive notion of what it means for a repertoire of trajectories to show combinatorial phonology. As the paper investigates the emergence of categorical and combinatorial coding on a qualitative level, this has not been an obstacle. However, for a more quantitative understanding of the emergence of combinatorial phonology, a numerical measure of the degree of recombination would be useful. When the basic building blocks of the repertoire are known, it is straightforward to give a measure of the degree of combination. An example would be the number of times a given building block is being reused: $\varphi = \frac{N}{k}$, where φ is the measure of recombination (phonemicity), N is the number of words in the repertoire and k the number of building blocks.

A difficult problem, however, is finding the basic building blocks, given a repertoire of signals. The traditional linguistic procedure for indentifying phonemes in an unknown language, relies on the notion of "minimal pair": a pair of words that have different meanings and differ only in one sound. An example would be the English pair "keel" and "cool". Hence, an hypothesis on a candidate phoneme can be validated by providing a minimal pair that differs only in that phoneme.

This analysis might seem straightforward, but when put to use in an automatic phoneme discovery procedure, a number of pitfalls emerge. First of all, it already makes use of an existing set of basic building blocks (usually the phonetic categories as defined by the International Phonetic Association). Secondly, it needs to decide what is meaningful variation and what is variation caused by automatic influence from neighbouring sounds. In the example, the k-like sounds at the beginning of the words are different, too, but it is generally accepted that this is due to the influence of the vowels that follow. They are said to be allophones. However, this can only be deduced from the physics of articulation, not from the signals themselves.

A final problem in the analysis is how to decide that two building blocks are equal. In the case of the two different k-sounds, it can be argued that they are instances of the same building block, because there are no minimal pairs and their articulation is very similar. However, the case is not so clear-cut for other examples. A well-known example is the case of English "h" and "ng". The former occurs only at the beginning of syllables, and the latter occurs only at the end. There are therefore no minimal pairs, and they could be considered allophones, but

because of their articulatory differences, this is never done. There are cases in other languages that are even more problematic, and it is not uncommon to find disagreement amongst linguists about the exact set of building blocks that are used in a language.

In the case of artificially generated sets of trajectories, these problems are even more apparent. One needs to make assumptions about what kind of building blocks are possible, what constitutes natural behaviour for a trajectory between building blocks, and how to define similarity between building blocks. In the analyses that we have presented, we have implicitly assumed that building blocks are points, that trajectories tend to follow straight lines from point to point and that points that are close together are the same building block. We do not yet know how to automate the process of extracting building blocks and therefore cannot calculate the phonemicity of a repertoire automatically.

5 Discussion and Conclusion

We have shown that optimising trajectories for acoustic distinctiveness results in superficial combinatorial phonology. When optimising a repertoire of temporally extended trajectories in an abstract acoustic space, the trajectories tend either to occupy the corners of the available space or to stretch out from corner to corner. It appears as if trajectories become *far apart where possible* and *close together where necessary*. A repertoire with this structure can be analysed as reusing certain points as building blocks of its trajectories and thus to have combinatorial structure. As there is nothing in the trajectory that explicitly codes this structure and as agents that would use these repertoires of trajectories need not be aware of their structure, their combinatorial nature is purely *superficial*.

Most of the results presented here were obtained through direct optimisation of repertoires of trajectories. However, we have also shown that superficially combinatorial repertoires of trajectories can emerge in a population of agents that try to imitate each other as well as possible. Apparently agents that strive for maximum success in imitation in noisy conditions, using information from simple interactions (imitation games) alone, converge towards repertoires that are similar to repertoires that are optimised directly.

Finally, we have shown that repertoires of trajectories that are optimised for acoustic distinctiveness (and thus combinatorial) are evolutionary stable. Agents that have a repertoire of trajectories that is more optimised for acoustic distinctiveness can invade a population of agents that have less optimal (but otherwise similar) repertoires, at least if the only fitness criterion is the robustness to acoustic noise of their repertoires. Conversely, a population of agents with an optimal repertoire cannot be invaded by agents with less optimal repertoires. We have also shown that there is a path of ever increasing fitness towards the optimal (and combinatorial) repertoire.

Our model differs from other attempts to explain combinatorial speech in several ways.

First of all, both holistic and (superficially) combinatorial signals have temporal structure. All signals in the model are of the same duration. Secondly, our model does not use articulatory targets. The resulting structure is purely emergent and therefore called "superficial". In fact, no distinction is made between holistic and combinatorial signals in the model; the difference only becomes apparent when analysing the structure of the repertoires.

We argue that agents can make use of this structure to evolve towards productive use of recombination. When the structure becomes available in the population, it becomes advantageous for agents to make use of it. They can use it to store the repertoire of trajectories more compactly, to perceive and produce trajectories in a more robust way and eventually to more easily create new trajectories. In this way, agents that use combinatorial structure productively can invade a population of agents that do not. This is only possible when there already exists a repertoire that is superficially combinatorial. Only then is there a path of continuously increasing fitness towards productive combinatorial coding, and eventually, to phonemic speech. We have shown that optimisation for acoustic distinctiveness can result in such a repertoire.

Natural language phonology is categorical and combinatorial. What we have shown in this paper is that these properties have functional significance: they aid the reliable recognition of signals by the hearer. We have also shown that there is a path that leads from a signal system without these properties, to one with those properties. Crucially, we have shown that each step on this path represents an improvement, both when it first appears in a population and when it is already common.

It turns out that a categorical and combinatorial system of speech sounds is advantageous even for a population of speakers and listeners that is not aware of this structure. Thus combinatorial structure can emerge in a culturally transmitted system of calls before there are cognitive (genetic) adaptations for using this structure. This makes it unnecessary for combinatorial phonology to have emerged through purely genetic evolution. Rather, natural selection shapes the parameters of the self-organising process and cultural self-organisation determines what genetic adaptations will be beneficial. Hence, *self-organisation is the substrate of evolution* (Waddington, 1939; Boerlijst and Hogeweg, 1991; Kirby and Hurford, 1997; Smith, 2004).

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