Interpreting Quantifier Combinations
Hintikka's Thesis Revisited

Nina Gierasimczuk and Jakub Szymanik
Institute for Logic, Language and Computation
University of Amsterdam
Institute of Philosophy, Warsaw University

August 25, 2008

Abstract

We discuss the thesis formulated by Hintikka (1973) that certain natural language sentences require non-linear quantification to express their meaning. Our basic assumption is that a criterion for adequacy of meaning representation is compatibility with sentence truth-conditions. This can be established by observing linguistic behavior of language users. We investigate sentences with combinations of quantifiers similar to Hintikka's examples and propose a novel alternative reading expressible by linear formulae. This interpretation is based on logical and philosophical observations. Moreover, we report on experiments showing that people tend to interpret sentences similar to Hintikka's sentence in a way consistent with our interpretation.

1 Hintikka's Thesis

Jaakko Hintikka (1973) claims that the following sentences essentially require non-linear quantification for expressing their meaning.

(1) Some relative of each villager and some relative of each townsman hate each other.

(2) Some book by every author is referred to in some essay by every critic.

(3) Every writer likes a book of his almost as much as every critic dislikes some book he has reviewed.

Throughout the paper we will refer to sentence (1) as *Hintikka's sentence*. According to Hintikka its interpretation is expressed using Henkin's quantifier as follows:

(4) $$\begin{pmatrix} \forall x \exists y \\ \forall z \exists w \end{pmatrix} ((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))),$$

where unary predicates V and T denote the set of villagers and the set of townsmen, respectively. The binary predicate symbol $R(x,y)$ denotes the relation "x and y are relatives" and $H(x,y)$ the symmetric relation "x and y hate each other".

Branching quantification (also called: partially ordered quantification, Henkin quantification) was proposed by Leon Henkin (1961) (for a survey see (Krynicki and Mostowski, 1995)). Informally speaking, the idea of such constructions is that for different rows the values of the quantified variables are chosen independently. According to Henkin's semantics for branching quantifiers formula (4) is equivalent to the following existential second-order sentence:

$$\exists f \exists g \forall x \forall z \big((V(x) \wedge T(z)) \Rightarrow (R(x,f(x)) \wedge R(z,g(z)) \wedge H(f(x),g(z)))\big).$$

Functions f and g (so-called Skolem functions) choose relatives for every villager and every townsman, respectively. As you can see, the value of f (g) is determined only by the choice of a certain villager (townsman). In other words, to satisfy the formula relatives have to be chosen independently[1]. This second-order formula is equivalent to the following sentence with quantification over sets:

$$\exists A \exists B \forall x \forall z \big((V(x) \wedge T(z)) \Rightarrow (\exists y \in A\ R(x,y) \wedge \exists w \in B\ R(z,w) \\ \wedge \forall y \in A \forall w \in B\ H(y,w))\big).$$

[1]The idea of branching is more visible in the case of simpler quantifier prefixes, like in sentence (8) discussed in Section 3.2 of the paper.

The existential second-order sentence is not equivalent to any first-order sentence (see the Barwise-Kunen Theorem in Barwise, 1979). Not only universal and existential quantifiers can be branched; the procedure of branching works in a very similar way for other quantifiers. Some examples — motivated by linguistics — are discussed in the next section of this paper.

The reading of Hintikka's sentence given by formula (4) is called *the strong reading*. However, it can also be assigned *weak readings*, i.e., linear representations which are expressible in elementary logic. Let us consider the following candidates:

(5) $\forall x \exists y \forall z \exists w \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big)$
$\wedge\, \forall z \exists w \forall x \exists y \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big).$

(6) $\forall x \exists y \forall z \exists w \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big).$

(7) $\forall x \forall z \exists y \exists w \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big).$

In all these formulae the choice of the second relative depends on the one, that has been previously selected. To see the difference between the above readings and the branching reading consider the second-order formula equivalent to the sentence (6):

$$\exists f \exists g \forall x \forall z \big((V(x) \wedge T(z)) \Rightarrow (R(x,f(x)) \wedge R(z,g(x,z)) \wedge H(f(x),g(x,z)))\big).$$

It is enough to compare the choice functions in this formula with those in existential second-order formula corresponding to the sentence (4), to see the difference in the structure of dependencies required in both readings. Of course, dependencies in sentences (5) and (7) are analogous to (6). As a result all the weak readings are implied by the strong reading, (4)(where the both relatives have to be chosen independently), which is of course the reason for the names. Formulae (5)-(7) are also ordered according to the inference relation which occurs between them. Obviously, formula (5) implies formula (6), which implies formula (7). Therefore, formula (5) is the strongest among the weak readings.

By Hintikka's Thesis we mean the following statement:

Hintikka's Thesis. *Sentences like Hintikka's sentence have no adequate linear reading. In particular, Hintikka's sentence should be assigned the strong* *reading and not any of the weak readings.*

Let us stress one point here. Of course, every branching quantifier can be expressed by some single generalized quantifier, so in the sense of definability Hintikka's thesis cannot be right. However, the syntax of branching quantification has a particular simplicity and elegance that gets lost when translated into the language of generalized quantifiers. The procedure of branching does not employ new quantifiers. Instead it enriches the accessible syntactic means of arranging existing quantifiers, at the same time increasing their expressive power. Therefore, the question we are dealing with is: are there sentences with simple determiners such that non-linear combinations of quantifiers corresponding to the determiners are essential to account for the meanings of those sentences? The affirmative answer to this question — suggested by Hintikka — claims existence of sentences with quantified noun phrases, which are always interpreted scope independently.

Because of its many philosophical and linguistic consequences Hintikka's claim has sparked lively controversy (see e.g. Jackendoff, 1972; Gabbay and Moravcsik, 1974; Guenthner and Hoepelman, 1976; Hintikka, 1976; Stenius, 1976; Barwise, 1979; Bellert, 1989; May, 1989; Sher, 1990; Mostowski, 1994; Liu, 1996; Beghelli et al., 1997; Janssen, 2003; Mostowski and Wojtyniak, 2004; Szymanik, 2005; Schlenker, 2006; Gierasimczuk and Szymanik, 2007). In the present article some of the arguments presented in the discussion are analyzed and critically discussed. And in particular, we propose to identify the meaning of Hintikka's sentence with the first-order formula (5):

$$\forall x \exists y \forall z \exists w \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big)$$
$$\wedge \forall z \exists w \forall x \exists y \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big).$$

In the rest of this paper we will refer to this reading as the *conjunctional reading* of Hintikka's sentence.

Our proposal seems to be very intuitive — as we show in the next section — but it is also consistent with human linguistic behaviour. The latter fact is supported by empirical data, which we will present in Section 4.[2]

[2] It is worth noticing that our proposition is a reminiscent of the linguistic representation of reciprocals. For example, according to the seminal paper on "each other" by Heim et al. (1991), Hintikka's sentence has the following structure: `EACH[[QP and QP] ][V the other]`, where "each" quantifies over the two conjuncts,

Our conclusion is that in spite of the fact that Hintikka-like sentences are ambiguous between discussed readings, their dominant meaning is expressible by linear formulae. This of course clearly contradicts Hintikka's thesis.

2 Other Hintikka-like sentences

Before we move on to the central problem let us consider more sentences which fall into the scope of our discussion.

Examples of Hintikka-like sentences were given by Jon Barwise (1979).

(8) Most villagers and most townsmen hate each other.

(9) One third of the villagers and half of the townsmen hate each other.

These sentences seem to be more frequent in our communication and more natural than Hintikka's examples, even though their adequate meaning representation is not less controversial.

Many more examples have been given to justify the existence of nonlinear semantic structures in natural language (see e.g. sentences (10)–(12)).

(10) I told many of the men three of the stories. (Jackendoff, 1972)

(11) A majority of the students read two of those books. (Liu, 1996)

(12) We have been fighting for many years for human rights in China. I recount the story of our failures and successes, and say: "Whenever a representative from each country fought for the release of at least one dissident from each prison, our campaign was a success." (Schlenker, 2006)

3 Theoretical discussion of the thesis

3.1 A remark on possible readings

Let us start with the following remark.

It was observed by Mostowski (1994) that from Hintikka's sentence we can infer that:

which turns the sentence into `[QP1 V the other and QP2 V the other]`, where "the other" picks up the rest of quantifiers anaphorically. This interpretation is very close to our conjunctional proposal.

(13) Each villager has a relative.

This sentence has obviously the following reading: $\forall x(V(x) \Rightarrow \exists y R(x,y))$. It can be false in a model with an empty town, if there is a villager without a relative. However, the strong reading of Hintikka's sentence (see formula (4)) — having the form of an implication with a universally quantified antecedent — is true in every model with an empty town. Hence, the reading of (13) is not logically implied by proposed readings of Hintikka's sentence.

Therefore, the branching meaning of Hintikka's sentence should be corrected to the following formula with restricted quantifiers:

(14) $$\begin{matrix} (\forall x : V(x))(\exists y : R(x,y)) \\ (\forall z : T(z))(\exists w : R(z,w)) \end{matrix} H(y,w),$$

which is equivalent to:

$$\exists A \exists B(\forall x(V(x) \implies \exists y \in A\ R(x,y)) \wedge \forall z(T(z) \implies \exists w \in B\ R(z,w)) \\ \wedge \forall y \in A \forall w \in B\ H(y,w)).$$

Observe that similar reasoning can be used to argue for restricting quantifiers in formulae expressing different possible meanings of all our sentences. However, applying these corrections uniformly would not change the main point of our discussion. We still would have to choose between the same number of possible readings, the only difference being the restricted quantifiers. Therefore, for simplicity we will forego these corrections. From now on we will assume that all predicates in our formulae have non-empty denotation.

3.2 Hintikka-like sentences are symmetrical

It has been observed that we have the strong linguistic intuition that the two following versions of Hintikka's sentence are equivalent (Hintikka, 1973):

(1) Some relative of each villager and some relative of each townsman hate each other.

(15) Some relative of each townsman and some relative of each villager hate each other.

However, if we assume that formula (6), repeated here:

$$\forall x \exists y \forall z \exists w ((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))).$$

is an adequate reading of sentence (1), then we have to analogously assume that an adequate reading of sentence (15) is represented by the formula:

(16) $\forall z \exists w \forall x \exists y ((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))).$

However, (6) and (16) are not logically equivalent, therefore it would be wrong to treat them as correct interpretations of sentences (1) or (15). Therefore, we have to reject readings (6) and (16) from the set of possible alternatives.

Notice that a similar argument works when we consider other Hintikka-like sentences. For instance, it is enough to observe that the following sentences are also equivalent:

(8) Most villagers and most townsmen hate each other.

(17) Most townsmen and most villagers hate each other.

However, the possible linear reading of (8):

(18) $\mathsf{MOST}\ x\ (V(x), \mathsf{MOST}\ y\ (T(y), H(x,y)))$

is not equivalent to an analogous reading of (17). Hence, linear reading (18) cannot be the proper interpretation.

One of the empirical tests we conducted was aimed at checking whether people really perceive such pairs of sentences as equivalent. The results that we describe strongly suggest that this is the case. Therefore, the argument from symmetricity is not only logically valid but also cognitively convincing (see Section 4.4.1 for a description of the experiment and Section 4.5 for our empirical results).

In spite of this observation we cannot conclude the validity of Hintikka's Thesis so easily. First we have to consider the remaining weak candidates, formulae (5) and (7):

(5) $\forall x \exists y \forall z \exists w \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big)$
$\wedge \forall z \exists w \forall x \exists y \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big),$

(7) $\forall x \forall z \exists y \exists w \big((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))\big)$.

Hintikka does not consider them at all, and other authors focus only on formula (7).

Also for different Hintikka-like sentences we still have to differentiate between some possibilities. As an alternative for formula (18) we can consider not only the branching reading (19) (equivalent to (20)):

(19) $\begin{pmatrix} \mathsf{MOST}\ x : V(x) \\ \mathsf{MOST}\ y : T(y) \end{pmatrix} H(x,y)$.

(20) $\exists A \exists B(\mathsf{MOST}\ x\ (V(x), A(x)) \wedge \mathsf{MOST}\ y\ (T(x), B(y) \wedge$
$\forall x \in A\ \forall y \in B\ H(x,y))$.

but also the conjunctional meaning:

(21) $\mathsf{MOST}\ x\ \big(V(x), \mathsf{MOST}\ y\ (T(y), H(x,y))\big)$
$\wedge\ \mathsf{MOST}\ y\ \big(T(y), \mathsf{MOST}\ x\ (V(x), H(y,x))\big)$.

Notice that for proportional sentences, e.g., (8), there is no interpretation corresponding to the weakest reading of Hintikka's sentence, formula (7), as proportional sentences contain only two simple determiners, and not four as the original Hintikka's sentence. This very fact already indicates that the conjunctional form — as a uniform representation of all Hintikka-like sentences — should be preferred over the weakest reading.

To sum up, the symmetricity argument rules out readings with asymmetric scope dependencies. Our space of possibilities contains now: the branching and the conjunctional reading. In the case of Hintikka's sentence we have to deal additionally with the weakest reading, (7). In the next section we give reasons to reject the weakest reading of Hintikka's sentence.

3.3 Inferential arguments

Let us consider the following reasoning:

Some relative of each villager and some relative of each townsman hate each other.
Mark is a villager.

Some relative of Mark and some relative of each townsman hate each other.

In other words, if we assume that Mark is a villager, then we have to agree that Hintikka's sentence implies that some relative of Mark and some relative of each townsman hate each other.

If we interpret Hintikka's sentence as having the weakest meaning (7):

$$\forall x \forall z \exists y \exists w ((V(x) \wedge T(z)) \Rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w))),$$

then we have to agree that the following sentence is true in Figure 1.

(22) Some relative of Mark and some relative of each townsman hate each other.

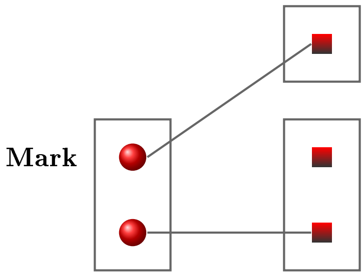

Figure 1: Relatives of Mark are on the left, on the right are two town families.

Mostowski (1994) observed that this is a dubious consequence of assigning the weakest interpretation to Hintikkas's sentence. He claims that sentence (22) intuitively has the following reading:

(23) $\exists x[R(Mark, x) \wedge \forall y(T(y) \Rightarrow \exists z(R(y, z) \wedge H(x, z)))]$.

The above formula (23) is false in the model illustrated by Figure 1. Therefore, it cannot be implied by the weakest reading of Hintikka's sentence which is true in the model. However, it is implied by the strong reading which is also false in the model. Hence, Mostowski concludes that Hintikka's sentence cannot have the weakest reading (7).

If we share Mostowski's intuition, then we can conclude from this argument that the weakest reading, (7), should be eliminated from the set of possible alternatives. Then we are left with two propositions: the branching and the conjunctional interpretation. Both of them have the desired inference properties.

3.3.1 Negation normality

Jon Barwise (1979) in his paper on Hintikka's Thesis refers to the notion of negation normality in a defense of the statement that the proper interpretation of Hintikka's sentence is an elementary formula. He observes that negations of some simple quantifier sentences, i.e., sentences without sentential connectives other than "not" before a verb, can easily be formulated as simple quantifier sentences. In some cases this is impossible. Namely, the only way to negate some simple sentences is by prefixing them with the phrase "it is not the case that" or an equivalent expression of a theoretical character.

Sentences of the first kind are called *negation normal*. For example, sentence:

(24) Everyone owns a car.

can be negated normally as follows:

(25) Someone doesn't own a car.

Therefore, this sentence is negation normal. As an example of statement which is not negation normal consider the following (see Barwise, 1979):

(26) The richer the country, the more powerful its ruler.

It seems that the most efficient way to negate it is as follows:

(27) It is not the case that the richer the country, the more powerful its ruler.

Barwise proposed to treat negation normality as a test for first-order definability. This proposal is based on the following theorem which is a corollary of Craig's Interpolation Lemma[3].

Theorem 1 (Barwise (1979)). *If φ is a sentence definable in Σ^1_1, the existential fragment of second-order logic, and its negation is logically equivalent to a Σ^1_1-sentence, then φ is logically equivalent to some first-order sentence.*

Barwise claims that the results of the negation normality test suggest that people tend to find Hintikka's sentence to be negation normal, and hence definable in elementary logic. According to him people tend to agree that the negation of Hintikka's sentence can be formulated as follows:

[3]The lemma states that: if φ and ψ are sentences such that $\varphi \Rightarrow \psi$ is a logically valid sentence, then there is a sentence θ, called the Craig interpolant of φ and ψ, such that:

- $\varphi \Rightarrow \theta$ and $\theta \Rightarrow \psi$ are logically valid;
- Every relation, function or constant symbol (excluding identity) which occurs in θ occurs in both φ and ψ.

(28) There is a villager and a townsmen that have no relatives that hate each other.

Barwise's claim excludes the branching reading of Hintikka's sentence but is consistent with the conjunctional interpretation. Therefore, in case of Hintikka's sentence we are left with only one possible reading: the conjunctional reading — at least as far as we are convinced by Mostowski's and Barwise's arguments. However, in the case of proportional sentences we still have to choose between the branching and the conjunctional interpretation.

3.4 Complexity arguments

Mostowski and Wojtyniak (2004) claim that native speakers' inclination toward a first-order reading of Hintikka's sentence can be explained by means of Computational Complexity Theory (see e.g. Papadimitriou, 1993). The authors prove that the problem of recognizing the truth-value of the branching reading of Hintikka's sentence in finite models is an NPTIME-complete problem[4]. It can also be shown that proportional branching sentences define an NP-complete class of finite models (see Sevenster, 2006).

Assuming that the class of practically computable problems is identical with the PTIME class (i.e., the tractable version of Church-Turing Thesis; see Edmonds, 1965) — they claim that the human mind is not equipped with mechanisms for recognizing NP-complete problems[5]. In other words, in many situations an algorithm for checking the truth-value of the strong reading of Hintikka's sentence is intractable. According to Mostowski and Wojtyniak (2004) native speakers can only choose between meanings which are practically computable.

The conjunctional reading is PTIME computable and therefore — even taking into account computational restrictions — can reasonably be proposed as a meaning representation.

[4]NP(TIME)-complete problems are computationally the most difficult problems in the NP(TIME) class. In particular, P(TIME)=NP(TIME) iff any NPTIME-complete problem is PTIME computable. P (NP) is the class of problems which can be solved by a (non-deterministic) Turing machine in a number of steps bounded by a polynomial function of the length of a query. See Garey and Johnson (1979) for more details.

[5]This statement can be given independent psychological support (see e.g. Frixione, 2001).

3.5 Theoretical conclusions

We discussed possible obstacles against various interpretations of Hintikka-like sentences. Our conjunctional version for Hintikka-like sentences seems to be very acceptable according to all mentioned properties. It is the only reading satisfying all the following properties.

- It is symmetrical.
- It passes Mostowski's inferential test.
- It is negation normal for Hintikka's sentence.
- Its truth value is practically computable in finite models.

In the case of Hintikka's sentence the conjunctional reading is arguably the only possibility. In general, for proportional sentences it competes only with the branching reading. The next section is devoted to empirical arguments that the conjunctional reading is consistent with the interpretations people most often assign to Hintikka-like sentences.

4 Empirical evidence for the conjunctional reading

Many authors — taking part in the dispute on the proper logical interpretation of Hintikka-like sentences — argued not only from their own linguistic intuitions but also from the universal agreement of native speakers. For instance, Barwise claims that:

> In our experience, there is almost universal agreement rejecting Hintikka's claim for a branching reading (Barwise, 1979).

However, none of the authors have given real empirical data to support their claims. We confronted this abstract discussion with linguistic reality through experiments.

4.1 Experimental hypotheses

Our hypotheses are as follows:

Hypothesis 1. *People treat Hintikka-like sentences as symmetrical sentences.*

This was theoretically justified in the paper of Hintikka (1973) and discussed in Section 3.2. To be more precise we predict that subjects will treat sentences like (29) and (30) as equivalent.

(29) More than 3 villagers and more than 5 townsmen hate each other.

(30) More than 5 townsmen and more than 3 villagers hate each other.

Hypothesis 2. *In an experimental context people assign to Hintikka-like sentences meanings which are best represented by the conjunctional formulae.*

We predict that the default reading for Hintikka's like sentences is best represented by our conjunctional formula. Arguments for that were given throughout the previous section and were summed up in Section 3.5.

Hypothesis 3. *Hintikka-like sentences are understood in the same way in English and Polish.*

We took this opportunity of testing combinations of quantifiers to conduct cross-linguistic comparison. Quantifiers are for the most part logical notions and their presence in language in the form of noun phrases can easily be seen in terms of mathematical operations, like Boolean combinations, over simple determiners. As a result — even though the inventory of determiners varies across different languages — quantifier structures actualize in a very similar way across languages (see e.g. Peters and Westerståhl, 2006). We predict that the comprehension of Hintikka-like sentences is similar in English and Polish — in both languages native speakers tend to choose the conjunctional reading.

4.2 Subjects

Subjects were native speakers of English and Polish who volunteered to take part in the experiment. They were undergraduate students in computer science at Stanford University and in philosophy at Warsaw University. They all had had elementary training in logic so they could understand the instructions and knew the simple logical quantifiers. The last version of the experiment, the one we are reporting on here, was conducted on thirty-two computer science students and ninety philosophy students. However, in the process of devising the experiment we tested fragments of it on many more subjects, getting partial results on which we reported for example at the Logic Colloquium 2006 (see Gierasimczuk and Szymanik, 2007).

The choice of students with some background in logic was made so that our subjects could be trusted to understand instructions which assume some familiarity with concepts of validity and truth. In that manner, we could formulate the task using such phrases as "one sentence implies the other", "inference pattern is valid", and "sentence is a true description of the picture". We did not have to pay the high price of being vague and metaphorical to get enough readability. On the other hand, we do not think that logical training will distort human semantic intuition with respect to the structures we are investigating as they are not part of the standard logical examples.

4.3 Materials

It was suggested by Barwise and Cooper (1981) and empirically verified by Geurts and van der Silk (2005) that the monotonicity of quantifiers influences how difficult they are to comprehend. In particular, sentences containing downward monotone quantifiers are more difficult to reason with than sentences containing only upward monotone quantifiers[6]. Therefore, in the experiment — as we are interested rather in combinations of quantifiers than in simple determiners — we used only monotone increasing quantifiers of the form "More than n" and their combinations in simple grammatical sentences. We used simple determiners, that are relatively easy to process, because we want our subjects to focus on combinations of quantifiers and not on individual ones.

In our tasks the quantifiers referred to shape of geometrical objects (circles and squares). The sentences were Hintikka-like sentences (for examples see Sections 4.4.1 and 4.4.2). All sentences were checked for grammaticality by native speakers.

[6] A quantifier Q_M is upward monotone (increasing) iff the following holds: if $\mathsf{Q}_M(A)$ and moreover $A \subseteq B \subseteq M$ then $Q_M(B)$. The downward monotone (decreasing) quantifiers are defined analogously as being closed on taking subsets.

4.4 Structure of the experiment

The experiment was conducted in two languages and consists of two parts. It was a "pen and paper" study. There were no time limits and it took 20 minutes on average for all students to finish the test. Below we present descriptions of each part of the English version of the test. The Polish test was analogous.

4.4.1 Part I: Are Hintikka-like sentences symmetrical?

The first part of the test was designed to check whether subjects treat Hintikka-like sentences as symmetrical (see Section 3.2 for a discussion).

Let us recall the notion of symmetricity for our sentences. Let $\mathsf{Q}_1, \mathsf{Q}_2$ be quantifiers and ψ a quantifier-free formula. We will say that sentence $\varphi := \mathsf{Q}_1 x\ \mathsf{Q}_2 y\ \psi(x, y)$ is symmetrical if and only if it is equivalent to $\varphi' := \mathsf{Q}_2 y\ \mathsf{Q}_1 x\ \psi(x, y)$. In other words, switching the whole quantifier phrase (determiner + noun phrase) does not change its meaning.

We checked whether subjects treat Hintikka-like sentences with switched quantifier prefixes as equivalent. We presented subjects with sentences pairs φ, φ' and asked whether the first sentence implies the second sentence. There were 20 tasks. Eight of them were valid inference patterns based on the symmetricity. Eight were invalid patterns similar to the symmetric case. In four we have changed nouns following quantifiers, i.e. we had $\varphi := \mathsf{Q}_1 x\ \mathsf{Q}_2 y\ \psi(x, y)$ and $\varphi' := \mathsf{Q}_1 y\ \mathsf{Q}_2 x\ \psi(x, y)$. In the second four we have switched determiners and not whole quantifier phrases, i.e. $\varphi := \mathsf{Q}_1 x\ \mathsf{Q}_2 y\ \psi(x, y)$ and $\varphi' := \mathsf{Q}_2 x\ \mathsf{Q}_1 y\ \psi(x, y)$. Four of the tasks were simple valid and invalid inferences with the quantifiers "more than", "all", and "some".

We constructed our sentences using non existing nouns to eliminate pragmatic influence on subjects' answers. For example, in the English version of the test we quantified over non existing nouns proposed by Soja et al. (1991): mells, stads, blickets, frobs, wozzles, fleems, coodles, doffs, tannins, fitches, and tulvers. In Polish we came up with the following nouns: strzew, memniak, balbasz, protorożec, melark, krętowiec, stular, wachlacz, fisut, bubrak, wypsztyk. Our subjects were informed during testing that they are not supposed to know the meaning of the common nouns occurring in the sentences and that these nonsense words can mean anything. Therefore, subjects were aware that they have to judge an inference according to its logical form and not by semantic content or pragmatic knowledge.

Figure 2 gives examples of each type of task in English.

More than 12 fleems and more than 13 coodles hate each other.
____
More than 13 coodles and more than 12 fleems hate each other.

VALID NOT VALID

More than 20 wozzles and more than 35 fitches hate each other.
____
More than 20 fitches and more than 35 wozzles hate each other.

VALID NOT VALID

More than 105 wozzles and more than 68 coodles hate each other.
____
More than 68 wozzles and more than 105 coodles hate each other.

VALID NOT VALID

Some tulvers are mells.
____
Some mells are tulvers.

VALID NOT VALID

Figure 2: 4 tasks from the first experiment: symmetricity pattern, two invalid patterns and simple inference.

4.4.2 Part II: Branching vs. conjunctional interpretation

The second questionnaire was the main part of the experiment, designed to discover whether people understand Hintikka-like sentences in the conjunctional way. Subjects were presented with nine non-equivalent Hintikka-like sentences. Every sentence was paired with a model. All but two sentences were accompanied by a picture satisfying the conjunctional reading but not the branching reading. The remaining two control tasks consisted of pictures in which the associated sentences were false, regardless of which of the possible interpretations was chosen.

Every illustration was black and white and showed irregularly distributed squares and circles. Some objects of different shape were connected with each other by lines. The number of objects in the pictures varied between 9 and 13 and the number of lines was between 3 and 15. Moreover, the number of objects in pictures where the sentences were false was similar to the number of objects in the rest of the test items. Almost all subjects solved these tasks correctly (90% correct answers). Moreover, subjects, when asked about their strategies, claimed that their decisions were not based on simply counting objects in the pictures. Instead they described a variety of strategies which they tried to use to approach the problem.

The sentences were of the following form, where $1 \leq n, m \leq 3$:

(31) More than n squares and more than m circles are connected by lines.

(32) Więcej niż n kwadraty i więcej niż m koła są połączone liniami.

Notice that the Hintikka-like sentences discussed in Chapter 1 as well as the items in the symmetricity test contain the phrase "each other". However, we decided not to use this phrase in the sentences tested in the main part of the experiments. This was because our previous experiments (Gierasimczuk and Szymanik, 2007) indicated that the occurrence of reciprocal expressions in these sentences made people interpret them as statements about the existence of lines between figures of the same geometrical shape. This surely is not the interpretation we wanted to test. Moreover, interviews with native speakers suggest that in the context of the relation "being connected by lines" omitting "each other" leads to more natural sentences. Additionally, in the Polish version of the sentences there is no possible phrase corresponding to "each other". This is a grammatical difference between Polish and English Hintikka-like sentences.

Figures 3 and 4 show two examples of our tasks. In the first picture the conjunctional reading is true and the branching reading is false. In the second picture the associated sentence is false, regardless of interpretation. The subjects were asked to decide if the sentence is a true description of the picture.

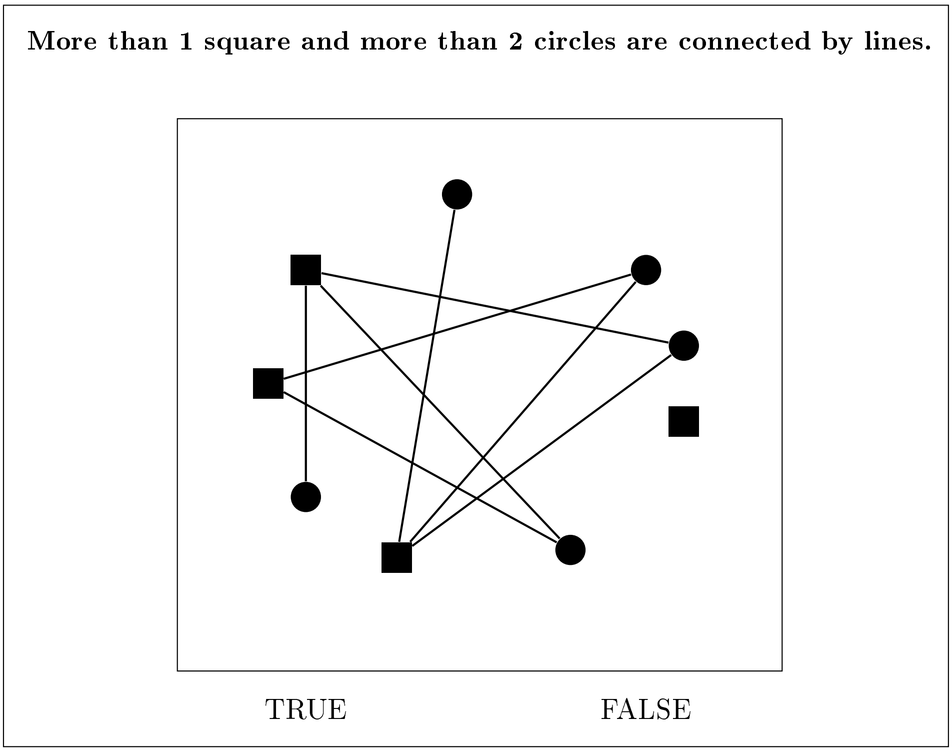

Figure 3: Conjunctional task from the second part of the experiment.

Let us give here a short explanation why we did not show pictures with a branching interpretation — as one might expect. The theoretical arguments given in Section 1 justify the following opposition: either Hintikka-like sentences are mostly interpreted in the conjunctional or mostly in the branching way. We want empirical evidence for conjunctional preferences. In principle we have to compare it with the branching meaning. Notice however, that the branching reading implies the conjunctional reading so it is impossible to achieve consistent results rejecting branching readings and confirming conjunctional reading — at least as long as subjects recognize the inference relations between branching and conjunctional readings, and in our experience most of them do (see Gierasimczuk and Szymanik, 2007). Therefore, we want to prove that people accept the conjunctional reading and not that they reject the branching one. In other words, we are looking for the weakest meaning people are ready to accept. To do this it is sufficient to have tasks with pictures for which the conjunctional reading is true, but the branching

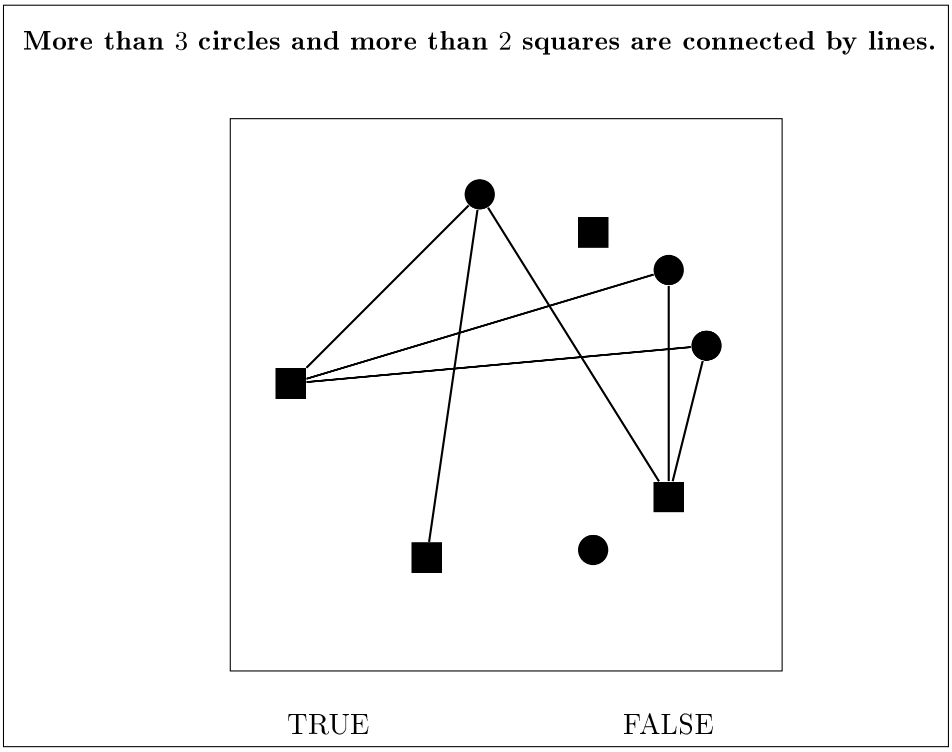

Figure 4: An example of a false task from the second-part of the experiment.

reading is false. As long as subjects accept them we know that they agree with the conjunctional reading and there is no need to confront them with the branching pictures. Of course this does not mean that people in principle reject the branching reading. However, the computational complexity arguments discussed in Section 3.4 suggest that people will reject the branching reading, since its complexity lies beyond human cognitive abilities.

4.5 Results

In the first test (symmetricity problem) we got 90% correct answers in the group consisting of philosophy undergraduates at Warsaw University and 93% correct answers among Stanford University computer science students, where by correct we mean here "correct according to our prediction about symmetricity". With respect to the simple inferences 45 philosophy (50%) and 28 computer science (88%) students answered correctly all questions.

Focusing on the proper symmetricity tasks, 71 subjects among the philosophers (79%, p<0.0001, df=1, $\chi^2 = 30.04$) and 29 computer scientists (91%, p<0.0001, df=1, $\chi^2 = 21.13$) recognized correctly all valid and invalid reasoning with a combination of two quantifiers (see Table 1). This is a statistically significant result for both groups. Therefore, our first hypothesis — that people treat Hintikka-like sentences as symmetrical sentences — was confirmed.

Groups	Polish philosophers	American computer scientists
number of subjects	90	32
all simple inferences correct	45 (50%)	28 (88%)
all symmetricity items correct	71 (79%)	29 (91%)

Table 1: Results of the symmetricity test with respect to subjects who answered all tasks correctly.

In the second test we got the following results. 93% of the answers of the philosophy students and 96% of the answers of the computer science students were conjunctional, i.e., "true" when the picture represented a model for a conjunctional reading of the sentence, and "false" in the two cases where the sentences were false in the pictures no matter how subjects interpreted them. Analysis of the individual subjects' preferences revealed what follows. 85 (94%, p<0.0001, df=1, $\chi^2 = 71.11$) philosophers and 31 (97%, p<0.0001, df=1, $\chi^2 = 28.12$) computer scientists agreed on the conjunctional reading in more than half of the cases. Moreover, 67 (74%, p<0.0001, df=1, $\chi^2 = 21.51$) philosophers and 28 (88%, p<0.0001, df=1, $\chi^2 = 18$) computer scientists chose conjunctional readings in all tasks (see Table 2). All these differences are statistically significant. Therefore, our second hypothesis — that in an empirical context people assign to Hintikka-like sentences meanings which are best represented by the conjunctional formulae — was confirmed.

There was no correlation between mistakes in simple inferences and symmetricity tasks. It seems that the reasoning processes behind these two kinds of tasks are essentially different. We think that simple inferences are more difficult because they are based on comprehension of the semantic content — at least subjects have to recognize monotonicity patterns as predicted

Groups	Polish philosophers	American computer scientists
number of subjects	90	32
most conjunctional answers	85 (94%)	31 (97%)
only conjunctional answers	67 (74%)	28 (88%)

Table 2: Results of the second test with respect to individual preferences.

by Geurts and van der Silk (2005) — as opposed to symmetricity tasks, where the valid reasoning assumes only recognition of a relatively simple syntactic pattern. This conjecture is consistent with the visible variation in performance between the philosophy and computer science students due to differences in background. More extensive mathematical training seems to influence only performance with simple reasoning. Additionally, semantic tasks might be more difficult to solve without pragmatic context than syntactic tasks are. We believe that all these assumptions need to be checked experimentally.

We also found no correlation between non-symmetrical and non-conjunctional answers. Moreover, excluding subjects who answered erroneously the simple inference tasks does not increase the percentage of non-conjunctional profiles.

As to our third hypothesis — that Hintikka-like sentences are understood in the same way by English and Polish native speakers — we did not observe any significant differences in the second test. Therefore, we conclude that with respect to interpretation of quantifier combinations in Hintikka-like sentences there is no difference between English and Polish.

5 Conclusions and perspectives

5.1 Conclusions

We argue that Hintikka-like sentences have readings expressible by linear formulae, despite what Hintikka (1973) and many of his followers have claimed. The reasons for treating such natural language sentences as having Fregean (linear) readings are twofold.

In Section 1 we discussed theoretical arguments. We can sum up them as follows.

(1) For Hintikka's sentence we should focus on four possibilities: a branching reading (4), and three weak readings: (5), (6), (7).

(2) Hintikka's argument from symmetricity given in Section 3.2, together with the results of our first experiment, allows us to reject asymmetric formulae. A similar argument leads to rejecting the linear readings of other Hintikka-like sentences.

(3) The inferential argument from Section 3.3 suggests that the weakest meaning is also not an appropriate reading of Hintikka's sentence. Moreover, for some Hintikka-like sentences an analogous formula does not exist so it cannot be viewed as a universal reading for all of them.

(4) Therefore, there are only two alternatives — we have to choose between the conjunctional (5) and the branching readings (4).

In section 4 we discussed our empirical results. They indicate that people interpret Hintikka-like sentences in accordance with the conjunctional reading, at least in an experimental context.

Additionally, our experimental arguments can be supported by the following observations.

(1) The argument by Barwise from negation normality, discussed in Section 3.3.1, agrees with our empirical results.

(2) Branching readings — being NP-complete — can be too difficult for language users. Conjunctional readings being PTIME computable are much easier in this sense.

Hence, even though we in principle agree that Hintikka-like sentences are ambiguous between all proposed readings, our experiments and theoretical considerations convince us that in most situations the proper reading of Hintikka-like sentences can be given by conjunctional formulae.

5.2 Perspectives

We have tested one of the best known among non-Fregean combinations of quantifiers, the so-called Hintikka-like sentences. As a result we came up with arguments that those sentences can be interpreted in natural language by Fregean combinations of quantifiers. However, there is still some research to be done here. One can find and describe linguistic situations in which Hintikka-like sentences demand branching analysis. For example, the work of Schlenker (2006) goes in this direction. Moreover, it is interesting to ask which determiners allow a branching interpretation at all (see e.g. Beghelli et al., 1997). Finally, we did not discuss the interplay of our proposition with a collective reading of noun phrases (see e.g. Lønning, 1997) and different interpretations of reciprocal expressions (see Dalrymple et al., 1998).

As to the empirical work, we find a continuation toward covering other quantifier combinations exciting and challenging. Some ideas we discussed in the context of Hintikka-like sentences, such as inferential meaning, negation normality, and the computational complexity perspective, seem universal and potentially useful for studying other quantifier combinations.

Acknowledgments

We would like to thank Tadeusz Ciecierski, Paul Dekker, Bogdan Dziobkowski, Bart Geurts, Justyna Grudzińska, Tikitu de Jager, Theo Janssen, Dick de Jongh, Marcin Mostowski, Rick Nouwen, Eric Pacuit, Ingmar Visser, Yoad Winter, Łukasz Wojtyniak. The first author is a recipient of the 2008 Foundation for Polish Science Grant for Young Scientists. The second author was supported by a Marie Curie Early Stage Research fellowship in the project GLoRiClass (MEST-CT-2005-020841).

NINA GIERASIMCZUK
Institute for Logic, Language and Computation
Universiteit van Amsterdam
Plantage Muidergracht 24
1018 TV Amsterdam
The Netherlands e-mail: N.Gierasimczukuva.nl

JAKUB SZYMANIK
Institute for Logic, Language and Computation
University of Amsterdam
Valckenierstraat 65
1018 XE Amsterdam
The Netherlands
e-mail: J.Szymanikuva.nl

References

Barwise, J. (1979). On branching quantifiers in English. *Journal of Philosophical Logic*, 8:47–80.

Barwise, J. and Cooper, R. (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy*, 4:159–219.

Beghelli, F., Ben-Shalom, D., and Szabolcsi, A. (1997). *Variation, Distributivity, and the Illusion of Branching*, volume 65 of *Studies in Linguistic and Philosophy*, pages 29–69. Kluwer Academic Publisher.

Bellert, I. (1989). *Feature System for Quantification Structures in Natural Language*. Foris Publications, Dordrecht.

Dalrymple, M., Kanazawa, M., Kim, Y., Mchombo, S., and Peters, S. (1998). Reciprocal expressions and the concept of reciprocity. *Linguistics and Philosophy*, 21:159–210.

Edmonds, J. (1965). Paths, trees, and flowers. *Canadian Journal of Mathematics*, 17:449–467.

Frixione, M. (2001). Tractable competence. *Minds and Machines*, 11(3):379–397.

Gabbay, D. M. and Moravcsik, J. M. E. (1974). Branching quantifiers, English and Montague grammar. *Theoretical Linguistics*, 1:140–157.

Garey, M. R. and Johnson, D. S. (1979). *Computers and Intractability.* W. H. Freeman and Co., San Francisco.

Geurts, B. and van der Silk, F. (2005). Monotonicity and processing load. *Journal of Semantics*, 22:97–117.

Gierasimczuk, N. and Szymanik, J. (2007). Hintikka's thesis revisited. *The Bulletin of Symbolic Logic*, 17:273–273.

Guenthner, F. and Hoepelman, J. P. (1976). A note on the representation of branching quantifiers. *Theoretical Linguistics*, 3:285–289.

Heim, I., Lasnik, H., and May, R. (1991). Reciprocity and plurality. *Linguistic Inquiry*, 22(1):63–101.

Henkin, L. (1961). Some remarks on infinitely long formulas. In *Infinistic Methods*, pages 167–183. Pergamon Press.

Hintikka, J. (1973). Quantifiers vs. quantification theory. *Dialectica*, 27:329–358.

Hintikka, J. (1976). Partially ordered quantifiers vs. partially ordered ideas. *Dialectica*, 30:89–99.

Jackendoff, R. (1972). *Semantic Interpretation and Generative Grammar.* MIT Press, Cambridge, Massachuset.

Janssen, T. (2003). On the semantics of branching quantifier sentences. In Dekker, P. and van Rooij R., editors, *Proceedings 14th Amsterdam Colloquium*, pages 147–151. University of Amsterdam.

Krynicki, M. and Mostowski, M. (1995). Henkin quantifiers. In Krynicki, M., Mostowski, M., and Szczerba, L., editors, *Quantifiers: Logics, Models and Computation*, pages 193–263. Kluwer Academic Publishers.

Liu, F.-H. (1996). *Branching Quantification and Scope Independence*, pages 155–168. Center for the Study of Language and Information.

Lønning, J. T. (1997). Plurals and collectivity. In van Benthem, J. and ter Meulen, A., editors, *Handbook of Logic and Language*, pages 1009–1053. Elsevier.

May, R. (1989). Interpreting logical form. *Linguistics and Philosophy*, 12(4):387–435.

Mostowski, M. (1994). Kwantyfikatory rozgałęzione a problem formy logicznej. In Omyła, M., editor, *Nauka i język*, pages 201–242. Biblioteka Myśli Semiotycznej.

Mostowski, M. and Wojtyniak, D. (2004). Computational complexity of the semantics of some natural language constructions. *Annals of Pure and Applied Logic*, 127(1-3):219–227.

Papadimitriou, C. H. (1993). *Computational Complexity*. Addison Wesley.

Peters, S. and Westerståhl, D. (2006). *Quantifiers in Language and Logic*. Clarendon Press, Oxford.

Schlenker, P. (2006). Scopal independence: A note on branching and wide scope readings of indefinites and disjunctions. *Journal of Semantics*, 23(3):281–314.

Sevenster, M. (2006). *Branches of imperfect information: logic, games, and computation*. PhD thesis, Universiteit van Amsterdam.

Sher, G. (1990). Ways of branching quantifiers. *Linguistics and Philosophy*, 13:393–442.

Soja, N., Carey, S., and Spelke, E. (1991). Ontological categories guide young children's induction of word meaning: Object terms and substance terms. *Cognition*, 38:179–211.

Stenius, E. (1976). Comments on Jaakko Hintikka's paper "Quantifiers vs. quantification theory". *Dialectica*, 30:67–88.

Szymanik, J. (2005). Problemy z formą logiczną. *Studia Semiotyczne*, 25:187–200.