# Diversity of Agents

Fenrong Liu [*]

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## Abstract

Diversity of agents is investigated in the context of standard epistemic logic, dynamic information update, and belief revision. We provide a systematic discussion of different sources of diversities, such as introspection ability, powers of observation, memory capacity, and revision policies. In each case, we show how this diversity can be encoded in a logical system allowing for individual variation among rational agents. We conclude by raising some general issues concerning this view of a logic as a system for encoding a society of diverse agents and their interaction.

## 1 Diversity Inside Logical Systems

Logical systems seem to prescribe one norm for an “idealized agent”. Any discrepancies with actual human behavior are then irrelevant, since the logic is meant to be normative, not descriptive. But logical systems would not be of much appeal if they did not have a plausible link with reality. And this is not just a matter of confronting one ideal norm with one kind of practical behavior. The striking fact is that human and virtual agents are not all the same: actual reasoning takes place in societies of diverse agents.

This diversity shows itself particularly clearly in *epistemic logic*. There have been long debates about the appropriateness of various basic axioms, and they have to do with agents’ different powers. In particular, the modal Distribution Axiom has the following epistemic flavor:

**Example 1.1** Logical omniscience: $K(\varphi \to \psi) \to (K\varphi \to K\psi)$.

Do rational agents always *know the consequences* of what they know? Most philosophers deny this. There have been many attempts at bringing the resulting diversity into the logic as a legitimate feature of agents. Some authors have used “awareness” as a sort of restriction on short-term memory ([FH85]), others have concentrated on the stepwise dynamics of making inferences ([Kon88], [Dun95]). A well-informed up-to-date philosophical summary is found in [Egr04].

The next case for diversity lies in a different power of agents:

**Example 1.2** Introspection axioms: $K\varphi \to KK\varphi$, $\neg K\varphi \to K\neg K\varphi$.

Do agents *know when they know* (or *do not know*)? Many philosophers doubt this, too. This time, there is a well-established way of incorporating different powers into the logic, using different accessibility relations between possible worlds in Kripke models. Accordingly, we get different modal logics: $K$, $T$, $S4$, or $S5$. Each of these modal logics can be thought of as describing one sort of agents. The interesting setting is then one of combinations. E.g., a combined language with two modalities $K_1$, $K_2$ describes a two-person society of introspectively different agents! This gives an interestingly different take on current logic combinations ([GS98], [KZ03]): the various ways of forming combined logics, by “fusions” $S5 + S4$ or “products” $S5 \times S4$, correspond to different assumptions about how the agents *interact*. Effects may be surprising here. E.g., later on, in our discussion of memory-free agents, we see that knowledge of memory-free agents behaves much like “universal modalities”. But in certain modal logic combinations,

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adding a universal modality drives up complexity, showing how the interplay of more clever and more stupid agents may itself be very complex...

Thus, we have seen how *diversity exists inside standard epistemic logic*, and hence likewise in doxastic logic. The purpose of this paper is to bring to light some further sources of diversity in existing logics of information. Eventually, we would want to move from complaints about 'limitations" and "bounds" to a positive understanding of how societies of diverse agents can perform difficult tasks ([GTtARG99]). But our actual contribution is more modest, viz. a discussion of sources of diversity in dynamic logics of information. Section 2 is a brief identification of further parameters of variation for agents beyond those of standard epistemic logic. Section 3 looks at dynamic epistemic logics of information update, showing how limited powers of observation are accounted for, and adding some new systems with bounded memory. Section 4 takes a parallel look at dynamic doxastic logics for belief revision, and shows how different revision policies can be dealt with. Finally, Section 5 is a brief summary, which also identifies some further more ambitious questions.

This paper is based on existing literature plus unpublished work in the author's Master's Thesis ([Liu04]). We will mainly cite technical results, and put them into a hopefully fresh story.

## 2 Sources of Diversity

The diversity of logical agents seems to stem from different sources. In what follows, we shall mainly speak about "limitations", even though this is a loaded term suggesting "failure". The more cheerful reality is of course that agents have various resources, and they use these positively to perform tasks, often highly successfully.

Our epistemic axioms point at several "parameters" of variation of agents:

(a) *inferential/computational power*: making all possible proof steps,

(b) *introspection*: being able to view yourself in "meta-mode".

One further potential parameter relevant to epistemic logic is the "awareness" studied by some authors([FH85]), which suggests some resource like limited attention span, or short-term memory.

Next, consider modern dynamic logics of information, whose motivation sounds closer to actual cognitive practice. These also turn out to incorporate idealizations that suggest further parametrization for diversity. We start with the case of information update.

Consider the basics of public announcements logic ($PAL$): $!\varphi$ in the language is intended to mean "a fact $\varphi$ is truthfully announced". $PAL$ considers the epistemic effects those actions can bring about. In addition to static axioms that "invite diversity", here is one more. The following principle is crucial to the way $PAL$ analyzes epistemic effects of assertions:

$[!\varphi]K_i\psi \leftrightarrow \varphi \rightarrow K_i[!\varphi]\psi$ *Knowledge Prediction Axiom*

But the validity of this axiom presupposes several things, notably *Perfect Observation* and *Perfect Recall* by agents. The event of announcement must be clearly identifiable by all, and moreover, the update induced by the announcement only works well on a unique current information state recording all information received so far. More technically, these points show in a detailed soundness proof for the Knowledge Prediction Axiom in its intended semantics. We will discuss this in Section 3, in the more general framework of "product update" for dynamic epistemic languages ([BMS98]). Thus, we have found two more parameters of diversity in logic. Agents can differ regarding:

(c) *observation*: stipulate agents' powers of observation for current events,

(d) *memory*: stipulate agents' limited memory capacity, e.g., store only the last $k$ events observed, for some fixed $k$.

Can one deal with this inside the logic? As we will see, dynamic epistemic logic with product update can itself be viewed as a calculus of observational powers. And as to memory, [BL04] have shown how to incorporate this into dynamic epistemic logic ($DEL$) for memory-free agents, and we will extend their style of analysis below to arbitrary finite memory bounds.

Yet another source for diversity of agents lies in *belief revision theory* ([AGM85]). This time, agents must revise their beliefs on the basis of incoming information which may contradict what they believed so far. This scenario is different from the preceding one, as has been pointed out from the start in this area ([GR95]). Even for agents without limitations of the earlier sorts, there is now another legitimate source of diversity, viz. their habits that create diversity:

(e) *revision policies*: varying from conservative to radical revision.

Different agents may react differently towards new information: some behave conservatively and try to keep their original beliefs as much as possible, others may be radical, easily accepting new information without much deliberation. However, these policies are not explicitly part of belief revision theory, except for some later manifestations ([Was00]). We will show in this paper, following [Liu04], [BL06], how they can be brought explicitly into dynamic logic as well.

This concludes the list of parameters of diversity that we see in current dynamic-epistemic and dynamic-doxastic logics. It is important to mention that acknowledging this diversity inside logical systems is not a concession to the ugliness of reality. It is rather an attempt to get to grips with the most striking aspect of human cognition: despite our differences and limitations, societies of agents like us manage to cooperate in highly successful ways! Logic should not ignore this, but rather model it and help explain it. Our paper is a modest attempt at systematization toward this goal.

# 3 Dynamic Logics of Information Update

## 3.1 Preliminaries: Product Update

To model knowledge change due to incoming information, the powerful mechanism is dynamic epistemic logic, which has been developed intensively by [Pla89], [Ben96], [BMS98], [Ger99], [DHK06], etc. Since our discussions in this paper will be heavily based on $DEL$, we briefly recall its basic ideas and techniques.

**Definition 3.1** [(epistemic model)] An epistemic model is a tuple $\mathcal{M} = (S, \{\sim_i |i \in G\}, V)$ [1] such that $S$ is a non-empty set of states, $G$ is a group of agents, each $\sim_i$ is a binary epistemic relation on S, and $V$ is a function assigning to each proposition variable $p$ in $\Phi$ a subset $V(p)$ of $S$. ◁

**Definition 3.2** [(event model)] An event model is a tuple $\mathcal{E} = (E, \sim_i, PRE)$ such that $E$ is a non-empty set of events, $\sim_i$ is a binary epistemic relation on $E$, $PRE$ is a function from $E$ to the collection of epistemic propositions. ◁

Note that we have a new function $PRE$ in a event model, the intuition is that it gives the *preconditions* for an action: an event $e$ can be performed at world $s$ only if the world $s$ fulfills the precondition $PRE(e)$.

**Definition 3.3** [(product update)] Let an epistemic model $\mathcal{M}$=$(S, \sim_i, V)$ and an event model $\mathcal{E}$=$(E, \sim_i, PRE)$ be given. The product update model is defined to be the model $\mathcal{M} \otimes \mathcal{E}$=$(S \otimes E, \sim_i', V')$:

- $S \otimes E = \{(s, e) \in S \times E$: $(\mathcal{M}, s) \models PRE(e)\}$
- $(s, e) \sim_i' (t, f)$ iff both $s \sim_i t$ and $e \sim_i f$
- $V'(p) = \{(s, e) \in \mathcal{M} \otimes \mathcal{E}$: $s \in V(p)\}$.

◁

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[1] I will sloppily write it as $\mathcal{M} = (S, \sim_i, V)$ when G is clear from the context.

The *actual world* of the new model is the pair consisting of the actual world in $\mathcal{M}$ and the actual event or action in $\mathcal{E}$. The product rule says that uncertainty among new states can only come from existing uncertainty via indistinguishable actions. The above notions suggest an extension of the epistemic language.

**Definition 3.4** [(dynamic epistemic language)] Let a finite set of proposition variables $\Phi$, a finite set of agents $G$, a finite set of events $E$ be given. The dynamic epistemic language is defined by the rule

$$\varphi := \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid K_i\varphi \mid [\mathcal{E}, e]\varphi$$

where $p \in \Phi$, $i \in G$, and $e \in E$. $\lhd$

Normally, one could also add the usual action operations of composition, choice, and iteration from propositional dynamic logic to the event vocabulary. The language has new dynamic modalities $[\mathcal{E}, e]$ referring to epistemic events, and these are interpreted in the product update model as follows:

$$\mathcal{M}, s \models [\mathcal{E}, e]\varphi \text{ iff } \mathcal{M}\otimes\mathcal{E}, (s, e) \models \varphi.$$

Reduction axioms in $DEL$ play an important role in encoding the epistemic changes. For example, the following axiom concerns agents' knowledge change.

$$[\mathcal{E}, e]K_i\varphi \leftrightarrow PRE(e) \rightarrow \bigwedge_{f\in\mathcal{E}}\{K_i[\mathcal{E}, f]\varphi : e \sim_i f\}.$$

Intuitively, after an event $e$ takes place the agent $i$ knows $\varphi$, is equivalent to saying that if the event $e$ can take place, $i$ knows beforehand that after $e$ (or any other event $f$ which $i$ can not distinguish from $e$) happens $\varphi$ will hold. Such a principle is of importance in that it allows us to relate our knowledge after an action takes place to our knowledge beforehand, which plays a crucial role in communication, and planning in general.

$PAL$ is the simplest case of update logic, in the sense that the event model contains one single event. Moreover, the precondition of the action $!\varphi$ boils down to the fact that $\varphi$ is true, as we will see in the formulas in the next section. In this paper, for easy understanding, we use $PAL$ to motivate our claims, though we also consider things within $DEL$ with a general mechanism of product update.

## 3.2 Public Announcement, Observation and Memory

First, we recall the complete axiom system for public announcement.

**Theorem 3.5** ([*Pla89*][*Ger99*]) *PAL is axiomatized completely by the usual laws of epistemic logic plus the following reduction axioms:*

$(!p)$. $[!\varphi]p \leftrightarrow \varphi \rightarrow p$ *for atomic facts p*

$(!\neg)$. $[!\varphi]\neg\psi \leftrightarrow \varphi \rightarrow \neg[!\varphi]\psi$

$(!\wedge)$. $[!\varphi]\psi \wedge \chi \leftrightarrow [!\varphi]\psi \wedge [!\varphi]\chi$

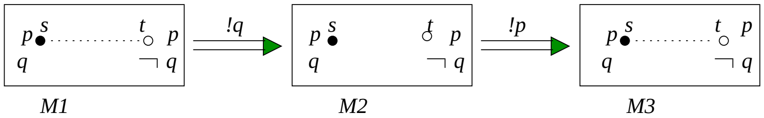
$(!K)$. $[!\varphi]K_i\psi \leftrightarrow \varphi \rightarrow K_i[!\varphi]\psi$

Next, to introduce variety in *observation*, we need to assume a set of possible announcements $!\varphi$, $!\psi$, ...where agent $i$ need not be able to distinguish all of them. This uncertainty can be modelled by an equivalence relation $\sim_i$ between statements which $i$ cannot distinguish. The following principle may then be proved:

**Theorem 3.6** ([*BL04*]) *The following reduction axiom is valid for agents with limited powers of observation:*

$$[!\varphi]K_i\chi \leftrightarrow (\varphi \rightarrow K_i \bigwedge_{!\psi\sim_i!\varphi}[!\psi]\chi)$$

As we have seen from the previous section, *Perfect Recall* assumes that agents can remember all the events happened so far. But in reality there are agents with bounded memory, who can only remember a fixed number of previous events. It is much harder in $PAL$ to model memory difference because the world elimination update procedure shifts agents to ever more informed states. How can they forget? Here is one option (suggested by [BL04]). First, we can reformulate $PAL$ semantics as in [BL06] to never eliminate worlds. The idea is to let announcement $!\varphi$ cut all links between $\varphi$-worlds and $\neg\varphi$-worlds, but keep all worlds in. Now, the resulting "unreachabilities" represent the information agents have so far. One way of describing a memory-restricted agent is then as having forgotten part or all of the "missing links". In the most extreme case, a *memory-free* agent will only acknowledge distinctions made by the last announcement. Apart from that, worlds will be indistinguishable. Agents like this do not satisfy the earlier reduction axiom $(!K)$, see the following example.



**Example 3.7** There are two possible worlds, $s$ and $t$ in $\mathcal{M}_1$, $p$ and $q$ hold at $s$, $p$ and $\neg q$ hold at $t$. After $q$ is announced, we get a new model $\mathcal{M}_2$, in which there is no uncertainty link between $s$ and $t$. Then we have $(\mathcal{M}_2, s) \models p \to K_i(p \to q)$, i.e. $(\mathcal{M}_2, s) \models p \to K_i[!p]q$. After that, $p$ is announced, and we have $\mathcal{M}_3 \not\models K_i q$, since the agent forgot $!q$ already. We look back at $\mathcal{M}_2$: $(\mathcal{M}_2, s) \not\models [!p]K_i q$. The reduction axiom does not hold!

**Fact 3.8** *The correct axiom valid for memory-free agents is*

$$[!\varphi]K_i\psi \leftrightarrow (\varphi \to U[!\varphi]\psi)$$

With the above picture, it is easy to check that the axiom is correct. Here $U\varphi$ is an *universal modality* saying that $\varphi$ holds in all worlds. To restore the harmony of an update logic, one should also extend the update reduction axioms with a clause for the new operator $U$. The following one is valid:

$$[!\varphi]U\psi \leftrightarrow \varphi \to U[!\varphi]\psi$$

Note that it looks like a $(!K)$ clause.

Thus, "logic of public announcement" is actually a family of formal systems, depending on the chosen update rule, which in turn depends on the memory type of the agents.

## 3.3 Adding Memory to Product Update

We now extend the update mechanism to agents with finite memory. As we have seen above, for memory-limited agents, the main point is to try to keep all the possible worlds around, so that the agent can always refer to the possible worlds which have been deleted before. We are still working with the *DEL* models, where information is changed by events. By a *k-memory* agent, we mean an agent that remembers only the last $k$ events before the most recent one. A 0-memory or memory-free agent, then, knows only what she learned from the most recent action; an agent with memory of length 1 knows only what she learned from the most recent action and the one before it, and so on. Now we must define the corresponding updates:

**Definition 3.9** ([Sny04]) Let $\mathcal{M}$ be an epistemic model, $\mathcal{E}_{-k}$ be the $k$-th event model before the most recent one $\mathcal{A}$. The product update for $k$-memory agents is defined as

(1) $\mathcal{M} \times \mathcal{E}_{-k} \times \cdots \times \mathcal{E}_{-1} \times \mathcal{E}$
$=\{(s, a_{-k}, \ldots, a_{-1}, a) : (s, a_{-k}, \ldots, a_{-1}) \in \mathcal{M} \times \mathcal{E}_{-k} \times \cdots \times \mathcal{E}_{-1}$ and $a \in \mathcal{E}\}$

(2) $(s, a_{-k}, \ldots, a_{-1}, a) \sim_i (t, b_{-k}, \ldots, b_{-1}, b)$ iff $(\mathcal{M} \times \mathcal{E}_{-k} \times \cdots \times \mathcal{E}_{-1} \models PRE(a)$ iff $\mathcal{M} \times \mathcal{E}_{-k} \times \cdots \times \mathcal{E}_{-1} \models PRE(b))$ and $a \sim_i b$.

$\lhd$

Compare with the standard product update definition, in the above definition (1) leaves the precondition restriction out. This simply makes it possible to keep all the worlds around. (2) gives restrictions to the states, and defines the uncertainty relation in the new models.

Another alternative is to introduce an auxiliary *copy action* $C!$ which takes place everywhere. It puts those worlds which are to be deleted into a stack, and makes sure agents can always find them when needed.

**Definition 3.10** ([Liu04]) Let $\mathcal{M}$ be an epistemic model, $\mathcal{E}_{-k}$ be the $k$-th action model before the most recent one $\mathcal{E}$. The product update for $k$-memory agents is defined as

(1′) $\mathcal{M} \times \mathcal{E}_{-k} \times \cdots \times \mathcal{E}_{-1} \times \mathcal{E}$
$=\{(\text{s,a}_{-k}, \ldots, a_{-1}, a) : (s, a_{-k}, \ldots, a_{-1}) \in \mathcal{M} \times \mathcal{E}_{-k} \times \cdots \times \mathcal{E}_{-1}$ and $a \in \mathcal{E}$ and $(s, a_{-k}, \ldots, a_{-1}) \models PRE(a)\}$

(2′) For $a_{-k}, \ldots, a_{-1}, a, b_{-k}, \ldots, b_{-1}, b \neq C!$,
$(s, a_{-k}, \ldots, a_{-1}, a) \sim_i (t, b_{-k}, \ldots, b_{-1}, b)$ iff $a_{-k} \sim_i b_{-k}, \ldots, a_{-1} \sim_i b_{-1}$ and $a \sim_i b$

$\lhd$

These two definitions can take care of our goal, namely, to keep those worlds around in the model. The technical difference lies in their different intuitions. In Def 3.9, it is believed that all the possible worlds make sense for bounded memory agents, so one *should not* remove the worlds because of the precondition restriction. Differently, Def 3.10 makes the worlds stay around with the help of the auxiliary copy action, just as memory bounded agents often do in real life.

### 3.4 Discussion

We have identified two new parameters of variation for dynamic updating agents; powers of observation, and powers of memory. $DEL$ as it stands provides a way of modelling the former, while we have shown how it can also be modified to accommodate update for agents with bounded memory.

Of course, this is only the beginning of an array of further questions. In particular, we would like to have a more structured account of memory, as in computer science where we update data or knowledge bases. Update mechanisms are more refined there, referring to memory structure with actions such as information replacement([Liu04]). This is one instance of a more "constructive" syntactic approach to update, complementary to our abstract one in terms of model manipulation. Whether our current semantic method or a syntactic one works better for finding agents' parameters of diversity seems a question worth investigating.

Other questions that come up have to do with the interaction between agents. Our logical systems can describe the behavior of various agents via reduction axioms, that is, schemas with infinitely many concrete instances. But they cannot yet state in one single formula "that an agent is of a certain type". And as they stand, they are even less equipped to describe the interplay of different agents in a compact illuminating way. Thus, we have only established a first toehold for diversity within the bastion of dynamic epistemic logic.

## 4 Diversity in Dynamic Logics of Belief Change

Belief revision describes what happens when an agent is confronted with new information which conflicts her earlier belief. Different policies toward revising beliefs fall within the $AGM$ postulates. We first look at a concrete example of how belief revision can be implemented technically.

### 4.1 Belief Revision as Relation Change

**Example 4.1** ([Ben06]) ($\Uparrow$) Radical revision
$\Uparrow P$ is an instruction for replacing the current ordering relation $\leq$ between worlds by the following: all $P$-worlds become better than all $\neg P$-worlds, and within those two zones, the old ordering remains.

Another possibility would be that just the best $P$-world comes to the top, leaving the further order unchanged. A more general description of such policies can be given as ways of *changing a current preference relation* ([BL06], [Rot06]). Viewed in this way, the dynamic logic for radical revision can be axiomatized completely in $DEL$ style:

**Theorem 4.2** ([*Ben06*]) *The dynamic logic for radical revision* ($\Uparrow$) *is axiomatized completely by a complete axiom system KD45 on the static models, plus the following reduction axioms*

$(\Uparrow p)$. $[\Uparrow \varphi]p \leftrightarrow p$

$(\Uparrow \neg)$. $[\Uparrow \varphi]\neg\psi \leftrightarrow \neg[\Uparrow \varphi]\psi$

$(\Uparrow \wedge)$. $[\Uparrow \varphi](\psi \wedge \chi) \leftrightarrow [\Uparrow \varphi]\psi \wedge [\Uparrow \varphi]\chi$

$(\Uparrow B)$. $[\Uparrow \varphi]B_i\psi \leftrightarrow (E\varphi \wedge B_i([\Uparrow \varphi]\psi|\varphi)) \vee B_i[\Uparrow \varphi]\psi$[2]

The last axiom here shows the doxastic effects of the chosen policy. But it still does so somewhat implicitly. In what follows, we will explore the same issues, but now with a notion of *plausibility* for worlds, which allows us to high-light policies more directly.

## 4.2 Plausibility Change

We first review briefly some previous work in this line. Following [Spo88], a $\kappa$-ranking function was introduced into the dynamic epistemic logic in [Auc03]. A $\kappa$-ranking is a function $\kappa$ from a given set $S$ of possible worlds into the class of numbers up to some maximum $Max$. The numbers can be thought of as denoting degree of surprise. 0 denotes ‘unsurprising’, 1 denotes ‘somewhat surprising’, etc. In other words, $\kappa$ represents a plausibility grading of the possible worlds. This makes it possible to express the degree of beliefs.

Next, we also add plausibilities $\kappa^*$ to the event model $\mathcal{E}$, representing the agents’ view on which event is taking place. With plausibilities assigned to states and events, belief changes via product update. Here is the key formula:

$$\kappa_i'(s,e) = Cut_{Max}(\kappa_i(s) + \kappa_i^*(e) - \kappa_i^s(\varphi)),$$

where $\varphi = PRE(e)$, $\kappa_i^s(\varphi) = min\{\kappa_i(t) : t \in V(\varphi)$ and $t \sim_i s\}$

$$Cut_{Max}(x) = \begin{cases} x & \text{if } 0 \leq x \leq Max \\ Max & \text{if } x > Max. \end{cases}$$

The crucial reduction axiom for belief in [Auc03] is the following:

$$\begin{gathered}[\sigma_j,\psi]B_i^m\varphi \leftrightarrow (\psi_j \rightarrow \bigwedge\{B_i^{l-1}\neg\psi_k \wedge \neg B_i^l\neg\psi_k \rightarrow B_i^{m+l-\kappa_i^*(\sigma_k)} \\ [\sigma_k,\psi]\varphi : \sigma_k \sim_i \sigma_j, \text{ and } l \in \{0,\ldots,Max\}\}) \text{ where } m < Max\end{gathered}$$

Here $\sigma_j$ and $\sigma_k$ are actions, $\psi$ are preconditions. $B_i^m\varphi$ is intended to mean that an agent believes $\varphi$ up to degree $m$, i.e. $\varphi$ is true in all epistemically accessible worlds of $\kappa$-value $\leq m$.

In the following section, instead, we take a more perspicuous approach, using epistemic-doxastic language with propositional constants to describe the plausibility change.

## 4.3 Revision Policies

What follows is taken from the unpublished Master’s thesis [Liu04].

**Definition 4.3** The *epistemic-doxastic language* is defined as

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[2] Some explanations about notations here: $E$ is the existial modality, the dual of the universal modality $U$. The symbol | is to denote an conditionalization and it is intended to mean ‘given that’.

$$\varphi := \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid K_i\varphi \mid q_i^{\alpha}$$

where $p \in \Phi$, a set of propositions, $i \in G$, a set of agents, and $\alpha$ is a $\kappa$-value in $\mathbb{N}$, $q_i^{\alpha}$ are a special type of propositional constants. $\lhd$

The interpretation is as usual, but with the following *truth condition* for the additional propositional constants:

$$(\mathcal{M}, s) \models q_i^{\alpha} \text{ iff } \kappa_i(s) \leq \alpha$$

The update mechanism can now be defined by merely specifying the new $\kappa$-value in a product model $\mathcal{M} \times \mathcal{E}$. To keep our discussion simple, we use just this:

**Definition 4.4** [(Bare addition rule)] The new plausibilities in product models are defined by the following rule:

$$\kappa_i'(s, e) = \kappa_i(s) + \kappa_i^*(e)$$

$\lhd$

**Theorem 4.5** ([*Liu04*]) *The complete dynamic logic of plausibility belief revision consists of the key reduction axioms in Theorem.1 plus the following new one.*

$$[\varphi!]q_i^{\alpha} \leftrightarrow q_i^{\alpha - \kappa_i(\varphi!)}$$

**Proof.** This Axiom captures the plausibility change. By the Bare addition rule, the plausibility of the world in the original model $\kappa_i(w) = \kappa_i(w, e) - \kappa_i(e) = \alpha - \kappa_i(\varphi!)$. ■

In fact, more generally, the plausibility rule can be any function of the plausibility of the previous event and that of the previous state. This is of course the locus for different policies! Moreover, if the update rule is functionally expressible, we can still get a complete logic, though clearly the simple substraction will not work anymore. To illustrate how it works, we now present a proposal which attempts to incorporate more elements into the update rule to characterize the diverse aspects of agents.

**Definition 4.6** ([Liu04]) Let the weight that an agent $i$ gives to the state $s$ be $\lambda$ and the weight to the event $e$ be $\mu$. The plausibility of the new state $(s, e)$ is calculated by the following rule

$$\kappa_i'(s, e) = \frac{1}{\lambda + \mu}(\lambda\kappa_i(s) + \mu\kappa_i^*(e)) \quad (\natural).$$

$\lhd$

The variations of parameter $\lambda$ and $\mu$ describe a range of various agents. For instance, when $\mu$=0, we get *highly conservative agents*, the ($\natural$) rule turns into $\kappa_i'(s, e) = \kappa_i(s)$. It means the agent does not consider the effect of the action. Similarly, $\lambda$=0, the agents are *highly radical*, and $\kappa_i'(s, e) = \kappa_i^*(e)$. When $\lambda = \mu$, we call them *Middle of the road agents* who believe plausibility of states and actions should play an equally important role in determining the plausibility of the new state. In this manner, we have distinguished five types of agents. For an even more general view of agents' behavior towards incoming information, see the continuing work in [Liu06].

In particular, this view of policies challenges one key assumption of $AGM$: the Success Postulate, which accords top status to the new information. Highly conservative agents would not take new information, right from the beginning, the belief revision cannot go on successfully. One can also get complete dynamic logics for various policies, but we will not pursue these here.

### 4.4 Comparisons and Discussion

Our treatment of belief revision provides a simple format of plausibility change, where different policies show in a perspicuous manner in the reduction axiom for the “value constants”. Moreover, our treatment also goes beyond the standard $AGM$ paradigm, in that we allow agents to doubt the current information. Here are a few further issues that come up in this setting, some conceptual, some technical.

First, doubting the current information might also make sense for $PAL$ and $DEL$ scenarios without belief revision. It is easy to achieve this by adding further events to an event model, providing, say, a public announcement $!\varphi$ with a counterpart $!\neg\varphi$ with some plausibility value reflecting the strength of the “dissenting voice”. Likewise, policies with weights for various factors in update make much sense in recently proposed dynamic logics of probabilistic update ([Auc05], [BGK06]).

Incidentally, this $DEL$ approach via modified event models for different policies may also suggest that we can *relocate* policies from “modified update rules” to “modified event models” with a standard update rule. We must leave this comparison to another occasion.

Next, there is an issue about relation-changing views of belief revision as in Section 4.1 versus our plausibility changes. One obvious difference is that plausibility change stays within the realm of *connected* world-ordering relations, whereas relational redefinition need not. On the other hand, plausibility change can describe scenarios such as “add one plausibility point to every $\varphi$-world”, which have no immediate counterpart in terms of relation changes. For comparison, we refer to [Liu06]. Of course, all general questions from Section 3.4 about representing agents and their *types* return here in even greater force.

Finally, a new observation concerning two parameters, revision policy and memory: Technically speaking, the update behavior of highly radical agents is no different from that of memory-free agents, as they simply take the new information without considering what happened before (of course for different reasons). In other words, the event that takes place completely characterizes the “next” epistemic state of the agent. That seems to be related also to notions such as “only knowing” or “minimal knowledge” in [Lev90] and [HJT90]. This observation seems to suggest a way to unify some of our parameters discussed so far.

## 5 Conclusion: A Unified Account of Diversity?

We have investigated many different sources of diversity, some visible in static logics, some in dynamic ones. Besides the old parameters from epistemic logic, namely computation and introspection ability, we have added several new aspects, i.e. observation power, memory capacity and revision policy. Our discussion has been mostly in the framework of dynamic epistemic logic and we have shown how it is possible to allow for a characterization of diversity within the logic. To summarize, look at the following diagram consisting of the main components of dynamic epistemic logic:

$$\left\{\begin{array}{ll} Static\quad language & \text{Epistemic model } \mathcal{M}; \\ Dynamic\quad language & \text{Event model } \mathcal{E}; \\ Product\quad update & \text{Model change } \mathcal{E}\times\mathcal{M}. \end{array}\right.$$

In the previous sections we have shown that the diversity of agents can be explicitly modelled in terms of these logical components. The following table is an outline of our discussions.

| Component | Residence | Diversity |
|---|---|---|
| $\mathcal{M}$ | relations between worlds | introspection |
| $\mathcal{E}$ | relations between actions | observation |
| $\mathcal{M}\times\mathcal{E}$ | update mechanism | memory, revision policy |

As we can see from the table, by introducing parameters of variation to each component, we are able to describe diversity of agents inside the logic system. Note that computation ability is not included in the table, we think its dimension is slightly different.

Still, there remains the issue whether one can have a *general* view of the natural "parameters" that determine differences in behavior of logical agents. Our analysis does not provide such a general account, but at least, it shows more richness and uniformity than earlier ones. Especially, we have seen one possibility to unify the parameters of revision policy and memory capacity at the end of the previous section. If more uniformity is needed, a good challenge would be to unify our observation- and memory-based analysis with diversity in deductive and computational powers. Our current speculation would be that many "idealizations" in standard logic have something to do with passing to a *countable limit*. Closing knowledge under consequence, computes the $k$-step consequences for all successive $k$. Introspection involves computing the transitive closure of the base accessibility relation. And memory goes from ever larger $k$-memory agents to unbounded stacks. At some abstract level, this may all be computing some fixed-point for a closure operator in some superstructure over an existing logic.

Even so, we hope that our account of diversity provides a fresh look at "logical systems". We now see them as vehicles for agents having powers of observation, memory, inference, computation, attention, and so on. Indeed, they begin to look remarkably like us!

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