MODAL LOGIC MEETS SITUATION CALCULUS

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Second Draft

1 Where are the differences?

In 1965, my calvinist high school class was invited for a social evening at the home of a catholic family in Den Haag. After all those textbooks on our doctrinal differences, and all our gossip about the much warmer (though of course less studious and less reliable) temperament of the other faith, every step in that house was an adventure. From the corner of my eyes, I quickly noticed that Catholics had two arms and two legs, just like us – in fact, the girl in charge was remarkably pretty. Moreover, the crucifixes which I expected to hang in every closet and over every door, did not materialize, and there were no tell-tale stains on the walls indicating that they had just been removed for our sake. Indeed, the whole evening was remarkable for its complete lack of remarkable events. Young Dutch Catholics were just like us, in their taste in clothes or pop music, their views from religion to politics, and even the adolescent jokes that they found funny.

My initial plan for this paper was another neighbourly visit which seemed long overdue between two Stanford colleagues of some 15 years by now. I wanted to make a crisp comparison between Situation Calculus, the world of John McCarthy and his followers, and Modal Logic, my own habitat for the past decades (see Blackburn, van Benthem & Wolter, eds., 2006, for the current state of the art). Indeed, John himself sometimes grumbles that modal logicians are not paying enough attention to his work, or to his knock-down arguments against their approach. Why don't the crushed admit defeat, and give up? But once in the neighbours' home, my high-school experience repeated itself. When opening a book like Reiter's *Knowledge in Action* from 2001, almost from the first page, I am struck by the great similarities: even the very title seems a borrowing from 'Language in Action' and 'Logic in Action', fashionable book titles in my world since the early 1990s. And so, I find it hard to say which differences I am supposed to high-light and compare. The models of situation theory turn out to be branching trees of possible event sequences, the standard paradigm used by all modern process theories and accounts of rational action (van Benthem & Pacuit 2006), major methods like 'Regression' are standard compositional computation techniques for preconditions of events (Herzig et al. 2006), knowledge is treated in good old philosophical logic style (Fagin et al. 1995; give or take a few notations), and the specific logic programming underpinning analyses of specific scenarios is again a general resource that we all use. To avoid a possible misunderstanding here, I am *not* saying that Reiter's book contains nothing new. To the contrary, I was deeply impressed by its content, and I stand by what I said on its cover! The point I am making here is rather that its conceptual framework of 'situation calculus' largely seems to be the air that we all breathe.

I could elaborate on these sweeping statements here – and in a zero draft of this paper, still lying in my waste basket, I did just that. Instead, I will discuss a few specific interfaces between McCarthy's ideas and modal logic as practiced these days, and end with a few general remarks on what can be expected from 'framework comparison'.

2 From circumscriptive consequence to dynamic logics of belief

2.1 Classical consequence and circumscription

Classical logical consequence from a set of premises ***P*** to a conclusion *C* says that all models of ***P*** are models for *C*. Those models for ***P*** may be seen as the current range of options, encoding what we know. A logical conclusion does not add to that knowledge, but it helps elucidate it. The by now famous insight from McCarthy 1980 was that reasoning in many practical settings of problem solving and planning seems to go beyond this, getting us more out of stated premises by means of special models that are most 'congenial' to the premises. A *circumscriptive* consequence from ***P*** to *C* says that

C is true in all *minimal* models for ***P***

Here, minimality is taken with respect to some relevant comparison order for models: inclusion of object domains, inclusion of denotations for certain specified predicates, and so on. The general idea here is the focus on minimal models over any sort of reflexive transitive order of 'relative plausibility' (Shoham 1988), much as in the Lewis semantics for conditional logic since around 1970 – an analogy which has been often noted (cf. van Benthem 1989). This much is familiar and well-established.

By now, many further notions of non-classical consequence have been brought to light, and their properties studied. Indeed, some logicians would claim that logic is the study of different consequence relations, somewhat in the manner of Bernard Bolzano (van Benthem 2003A). But this diversity of reasoning styles for different purposes also raises quite a few new problems which seem unresolved. What is the nature of this diversity: do we really 'infer' in many different ways, and why? Can we chain these styles of inference into longer arguments, and can we combine different ones in useful ways? And given that these styles are meant to reflect cognitive practice, or 'common sense', what about their computational complexity? Over a first-order language, circumscription has a much more complex notion of validity than classical consequence, so what is the total package of benefits that we are buying at this increased price?

I have no definitive answers to all these questions. But I would like to suggest that a shift in perspective may be helpful – from a steaming jungle of non-classical 'consequence relations' to the current world of modal logics for belief update, belief revision, and other *informational attitudes* and *informational processes* .

2.2 Logics of knowledge and belief

Let's reconsider the puzzles and planning problems that gave rise to non-monotonic logic in the first place. We are given some initial information, and solving the puzzle means finding out what the true situation is. Maybe we also get additional information on the way. Perhaps the central phenomenon in such a scenario is *not inference at all*, but rather our receiving that information, and our subsequent responses. This is the arena of current dynamic logics of information update and belief revision.

Imagine that we are playing the new board game "Kings and Cardinals" (with the board itself an object of public observation) having 'monasteries' and 'advisors' placed here and there. I am looking at the cards in my hand (this is a private observation), and also at the map of medieval Europe on the board. Right now, I know certain things about the outcome of the game for me, while I may believe even more than what I strictly know, based on my expectations about the cards that the other players hold, or their general temperaments: timid, bluffing, and the like. Now, new information comes in, say: you select a new country on the map and place some counters there. This observation changes my current information state. I know more now, and the observation may even speed along further beliefs of mine, such as that you are trying to build a trade route from Burgundy to Bohemia. Of course, these current beliefs may be refuted by further moves of yours, unlike the hard knowledge that I obtained about what's on the board. Solving puzzles, and playing games, seems all about such processes.

Now, I claim that it is illuminating to think about classical versus circumscriptive 'inference' in the same way. Indeed, the very motivation for nonmonotonic reasoning seems epistemic, having to do with managing our knowledge and beliefs – but this key feature *is left implicit*. To me, classical consequence is about the *knowledge update* that takes place when new information comes in. And in tandem with this, I would say that

Circumscriptive inference is about belief formation which takes place on the basis of incoming new information. [1] Clearly, knowledge update and belief revision are intertwined, and they provide mutual support. I think it is this diversity of responses to information which truly explains the modern galaxies of 'notions of consequence'. So, let's look at some modal logics underpinning these phenomena when we shift the focus to processing information.

2.3 Logical dynamics 1: update, belief, and nonmonotonic consequence

Basic epistemic and doxastic logic To make our main points on nonmonotonic reasoning, just assume some standard language with operators $K_i\phi$ for knowledge (agent *i* knows that ϕ') and $B_i\phi$ for knowledge (agent *i* believes that ϕ'). These modal operators are interpreted in semantic models $\boldsymbol{M} = (W, \sim_i, \leq_i, V)$, where the $\sim_i$ are epistemic accessibility relations giving an agent's current range of uncertainty, while the $\leq_i$ are relative plausibility orderings, dependent on given worlds *w*, of the epistemically accessible worlds as seen from *w*. On a straightforward view, knowledge at a world *w* then means truth at all worlds accessible from *w* via $\sim_i$,, while belief at *w* means truth at all most plausible worlds in the $\leq_i$-ordering at *w*. Complete logics for these operators are well-known (cf. again Fagin et al. 1995), and we will omit their formulations here. Instead, we formulate some less-known dynamic variants.

Dynamic-epistemic logic of hard information Perhaps the most typical event producing information is a *public announcement !P* of some true proposition *P* (i.e., true at the actual world *s* in ***M***). E.g., announcing a fact *q* will make you know that *q* – even though there are more subtle phenomena in general, which are irrelevant to us here. But the announcement need not be made in language. Placing a counter on the board in "Kings and Cardinals" leads to public observation with the same effects. There is a widespread intuitive idea of new information as elimination of current possibilities, which arises here as an action of *model change*. The event *!P* takes the current model $(\boldsymbol{M}, s)$ to a new model $(\boldsymbol{M}|P, s)$, viz. the model ***M*** restricted to its unique submodel consisting of just the *P*-worlds. We can see this update mechanism as our response to *hard information*, which can be accepted once and for all, and which affects out knowledge and beliefs in an irreversible sense.

Next, to reason more precisely about this informational process and its effects, we introduce a matching dynamic operator as follows:

$$\boldsymbol{M}, s \models [!P]\phi \quad \text{iff} \quad \boldsymbol{M}|P, s \models \phi.$$

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[1] Incidentally, here, the term 'belief' should not be taken in any deep religious sense: I just mean the temporary views that we entertain as most plausible given our information so far.

The principles which analyze the effects of public announcements on what agents *know* (van Benthem 2006, van Ditmarsch, van der Hoek & Kooi 2007) yield a logical system *PAL* which is axiomatized completely by the usual laws of epistemic logic, known since Hintikka's work in the early 1960s, plus the following *reduction axioms*:

$$[!P]q \leftrightarrow P \rightarrow q \qquad \text{for atomic facts } q$$
$$[!P]\neg\phi \leftrightarrow P \rightarrow \neg[!P]\phi$$
$$[!P]\phi\wedge\psi \leftrightarrow [!P]\phi \wedge [!P]\psi$$
$$[!P]K_i\phi \leftrightarrow P \rightarrow K_i\,(P \rightarrow [!P]\phi$$

These principles analyze in a systematic compositional manner what happens to agents' knowledge when hard information *!P* comes in. In particular, the last axiom is crucial for that purpose – in that it reduces knowledge which results after an announcement to *conditional knowledge* which agents had before the announcement was made. This phenomenon is called 'pre-encoding' in the static language.

Dynamic consequence and update In this dynamic perspective, the classical notion of consequence from premises ***P*** to a conclusion *Q* can be reformulated as follows. Updating the current model with successive public announcements $!P_1, ..., !P_n$ leads to a new model where *Q* is known to all agents, or even more strongly, a model where *Q* has become common knowledge among them. We can write this succinctly and clearly as the following formula which should be valid:

$$[!P_1][\ ...\ ][!P_n]\ C_GQ$$

This dynamic notion of consequence has structural rules which differ from classical consequence. In particular, and unsurprisingly, it is non-monotonic: new information in the premises can change conclusions after update. [2]

Dynamic-doxastic logic of hard information Next, let us turn to the effect of hard information on agents' *beliefs*. As it stands, the above language of belief lacks the power of conditionalizing needed for the required pre-encoding. But one can introduce an operator of *conditional belief* in our models, with

$\boldsymbol{M}, s \models B(_i\phi|\psi)$ iff $\boldsymbol{M}, t \models \phi$ for all worlds *t* which
are minimal for $\lambda xy. \leq_{i,s} xy$ *in the set* $\{u \mid \boldsymbol{M}, u \models \psi\}$.

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[2] Cf. the 'update-to-test' notion of dynamic consequence and its structural rules in van Benthem 1996. A complete representation theorem for public announcement actions is found in van Benthem 2003B.

And then a complete axiomatization becomes possible for belief change under incoming information (van Benthem 2007), on top of any complete static logic of knowledge and belief for the model class chosen, plus the preceding reduction axioms for atomic facts and Boolean operations, and one axiom for conditional beliefs:

$$[!P]\, B_i(\phi | \psi) \quad \leftrightarrow \quad P \rightarrow B_i(\, [!P]\phi \mid P \wedge [!P]\psi) \qquad (\#)$$

As a special case, conditionalizing on *True*, this describes formation of absolute beliefs:

$$[!P]\, B_i\phi \quad \leftrightarrow \quad P \rightarrow B_i([!P]\phi \mid P) \qquad (\$)$$

Again, there is a notion of dynamic consequence here, as above.

First rendering of circumscription Circumscriptive inference seems to lead to beliefs rather than knowledge, since its conclusions may be refuted, and hence retracted on the basis of further evidence. Thus, in the present setting, the dynamic formula $[!\boldsymbol{P}]B_i\phi$ seems an obvious analogue of circumscriptive inferences from premises $\boldsymbol{P}$ to conclusions ϕ. The case with more than one premises is treated similarly. The stated reduction axioms (#), ($) then show the intimate connection of circumscription with *conditional beliefs*, and hence with ordinary conditional logic. [3]

This shift in perspective has clear advantages. Dynamic epistemic and doxastic logic can be studied by ordinary techniques, it allows for unrestricted iteration, and all sorts of combination of operators. Moreover, there is a great variety of possible ways of changing models on the basis of incoming information, which may throw new light on the variety of styles of consequence, from circumscription to abduction and beyond.

But even so, we are still missing a dynamic feature that seems relevant here:

2.4 Logical dynamics 2: belief revision and changing plausibility orders

Soft information and plausibility change In specific applications of circumscription, one fixes a comparison relation between models. And in many nonmonotonic logics, this choice is even left implicit in context, without a trace in the formal language. But since our way of comparing models as to relative plausibility determines agents' beliefs, and hence their 'conclusions', it seems important to have explicit control over how we

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[3] This has been noted many times, as it explains the strong similarities between the structural rules governing circumscriptive inference and the validities of conditional logic over partial (not just 'connected') world comparison orderings. But now, we can also *explain* these analogies as a matter of dynamic logics of the maintenance of explicit knowledge and belief.

choose, and *change*, that ordering. This brings us to the dynamic process of *belief revision,* where triggers for changing belief need not be 'hard information' of the public announcement type, which rules out certain worlds for good. They can rather be 'soft information' affecting just our plausibility ordering of the worlds!

Revision policies, and dynamic logics once more A triggering event which leads to our coming to believe that *P* need only *rearrange worlds* in such a way that the most plausible ones are now *P*: it works by 'promotion' rather than elimination of worlds. Thus, on the earlier models $\boldsymbol{M} = (W, \sim_i, \leq_i, V)$, we change the relations $\leq_i$, rather than the domain of worlds *W* or the epistemic accessibilities $\sim_i$. For greater concreteness, here is a well-known soft trigger from the literature on belief revision, sometimes called 'radical revision'. A *lexicographic upgrade* $\Uparrow P$ is an instruction for changing the current ordering relation $\leq$ between worlds as follows: all *P*-worlds in the current model become better than all $\neg P$-worlds, while, within those two zones, the old plausibility ordering remains. We have a corresponding dynamic modality

$$\boldsymbol{M}, s \models [\Uparrow P]\phi \quad \text{iff} \quad \boldsymbol{M}\Uparrow P, s \models \phi$$

with $\boldsymbol{M}\Uparrow P$ the model $\boldsymbol{M}$ with its order $\leq$ changed as stated above. Again, this language will describe how agents' beliefs change under soft information – and its complete set of principles has just been found (van Benthem 2007). The dynamic logic of lexicographic upgrade is axiomatized completely by any complete axiom system for conditional belief on the static models, plus the following reduction axioms:

$$[\Uparrow P]\, q \quad \leftrightarrow \quad q, \qquad \text{for all atomic proposition letters } q$$

$$[\Uparrow P]\, \neg\phi \quad \leftrightarrow \quad \neg[\Uparrow P]\phi$$

$$[\Uparrow P]\, (\phi \wedge \psi) \quad \leftrightarrow \quad [\Uparrow P]\phi \wedge [\Uparrow P]\psi$$

plus the following crucial compositional principle telling us which conditional beliefs an agent will have after some piece of soft information has come in:

$$[\Uparrow P]\, B(\phi|\psi) \quad \leftrightarrow \quad (E(P \wedge [\Uparrow P]\psi) \wedge B([\Uparrow P]\phi \mid (P \wedge [\Uparrow P]\psi)) \vee (\neg E(P \wedge [\Uparrow P]\psi) \wedge B([\Uparrow P]\phi \mid [\Uparrow P]\psi)$$

This may look forbidding, but it is no more difficult than the principles of the situation calculus in their lusher modern formulations. And there is a reward for grasping these formulas. We can now see explicitly how incoming triggers will affect the plausibility ordering $\leq$ among our worlds, and hence our current beliefs at any given stage, and hence the 'nonmonotonic inferences' available to us on the basis of the ambient order $\leq$.

In addition, there are many further possible ways of taking soft information. E.g., a more conservative form of belief revision puts, not *all P*-worlds on top qua plausibility, but just *the best P-worlds*. This 'co-opts the leaders of the underclass, not all of them'. Complete dynamic logics for these and other policies exist, too (van Benthem 2007).

Circumscriptive inference and belief revision once more So which is the true analogue of a circumscriptive inference $\boldsymbol{P} \Rightarrow_{circ} \phi$ in this dynamic setting? We cannot tell, because we now live in a richer universe of informational events that may determine how we solve our problem, make our plan, or play our game. In particular, there is at least this distinction to be made. Is our non-monotonic inference the result of an incoming hard fact? Then $\boldsymbol{P} \Rightarrow_{circ} \phi$ will be like the above dynamic doxastic formula $[!\boldsymbol{P}]B\phi$. Or, is it the result of an incoming soft fact? Then $\boldsymbol{P} \Rightarrow_{circ} \phi$ will be more like the above dynamic doxastic formula $[\Uparrow\boldsymbol{P}]B\phi$, or some other response to the incoming soft information that $\boldsymbol{P}$. These two versions, 'hard' and 'soft' will have different logical properties, witness their respective reduction axioms as stated above. Thus we find a variety of notions of 'circumscription', depending on how we process a trigger.

For many reasoning purposes, the difference will not matter. But there may be long-term effects on further informational scenarios. The former reading would say that the current $\neg P$-worlds are eliminated from consideration altogether, while the latter would leave them in (though 'demoted'). The circumscriptive reasoner, or theorist, must make up her mind on this, and only the details of some problem-solving scenario can tell which one is more appropriate to the purpose at hand. This is just one of many fine distinctions afforded by a dynamic logic framework which remain implicit in most calculi of nonstandard inference. And options even multiply when we consider different policies for incorporating the soft information into our plausibility ordering.

A conclusion, and a recommendation The intuitions behind circumscriptive inference seem epistemic and doxastic, involving knowledge and belief. They also seem dynamic, involving responses to incoming information about some situation of relevance to us. We have proposed a dynamic epistemic perspective then, where circumscription and other styles of nonmonotonic reasoning are at heart about cognitive attitudes and responses to information. We brought out two different dynamic processes involved in drawing nonmonotonic conclusions: information update, and plausibility change.

Viewed in this way, the study of circumscription and similar nonmonotonic reasoning styles and the current study of dynamic modal logics of knowledge and belief are after the same things. More ambitiously, they may be able to help each other!

3 Logics of time and change

Our next encounter between modal logic and situation calculus concerns the 'theatre' where informational processes play over time. Dynamics leads to temporal logic.

3.1 The common playground: branching time

All of the preceding could be read as a study of information about some unchanging static situation which we do not know yet in its entirety, and at best, events informing us that given situation. But of course, real planning problems are about a world which is also changing as we speak. The preferred universe for that in approaches to rational action is that of a *branching tree of events*, supporting various systems of temporal logic with additional epistemic and doxastic operators. Van Benthem & Pacuit 2006 point out how pervasive this structure is across AI, computer science, linguistics, and philosophy, while the temporal logics and technical results about them are also in close harmony, even when developed independently. Not surprisingly, it is also the grand stage for the Situation Calculus in many of its manifestations which reason about action and change (Shoham 1988, Reiter 2001, Sandewall 1994, Shanahan 1995). We may have different computational cultures, but we all live on the same mathematical planet.

3.2 Events and general dynamic epistemic logic

The dynamic logics of information change discussed in Section 2 have more general event-based versions where update is more sophisticated than mere elimination of worlds, or rearranging plausibility among given worlds (Baltag, Moss & Solecki 1998, van Benthem 2006). General observation of events involves two ingredients:

(a) a current information model ***M*** as above, plus
(b) an epistemic *event model* ***E*** of all relevant possible events,

structured by accessibility relations as before representing what agents know and believe about these events, their 'epistemic access'. E.g., in a card game, my playing specific cards on the table is a publicly observable event, but in drawing a card from the stack, I can see which specific card I am drawing, but you may only know it is one of a certain set. Moreover, you and I may also have different beliefs. Perhaps my smile means that I am drawing the Ace of Hearts, but you cannot know this.

In this setting, *information flows after observation* because events *e* cannot just occur in every world: they come with *preconditions PRE(e)* on their successful execution. E.g., a truthful public announcement *P!* can only happen in *P*-worlds, I can only draw the Ace of Hearts if it is actually on the stack (well, usually: you know...), and so on.

Product update The matching general dynamic epistemic update takes ***M***, ***E*** to a

> *Product* model ***M*** *x* ***E*** with domain $\{(s, e) \mid \boldsymbol{M}, s \models PRE(e)\}$.

Here *(s, e)* records new events occurring at *s*, provided the preconditions of *e* holds. The basic epistemic stipulation is this. Uncertainty among new states can only come from existing uncertainty via indistinguishable events: $(s, e) \sim_i (t, f)$ iff both $s \sim_i t$ and $e \sim_i f$. This amounts to an assumption of agents having 'Perfect Recall' (van Benthem 2006) similar to those discussed in Fagin et al. 1995, Reiter 2001, or in game theory (van Benthem 2001). Finally, the valuation for proposition letters at *(s, e)* may be just copied from that at *s*, or it can change when the event truly changes the world.

The framework can also be extended to deal with beliefs and belief change, providing a generalization for all we have said in Section 2 above (cf. Baltag & Smets 2006).

Dynamic epistemic logics Again there are complete logics governing this (cf. van Benthem, van Eijck & Kooi 2006). Their compositional reduction axioms analyze epistemic, doxastic, and physical effects of events, with dynamic operators now of the forms $[\boldsymbol{E}, e]K_i\phi$, $[\boldsymbol{E}, e]B_i\phi$, etc. Again, the heart of the matter is the ability of our logical languages to *pre-encode* what would become true after certain events. The result of this are again *reduction axioms* performing a systematic compositional analysis of the effects of observed events. Here is the central reduction axiom for the case of knowledge (note the role of the precondition for the event *e*):

$$[\boldsymbol{E}, e]\, K_i\, \phi \;\leftrightarrow\; PRE_e \rightarrow \mathbf{\&}\, \{K_i\, (PRE_f \rightarrow [\boldsymbol{E}, f]\phi)) \mid f \sim_i e \text{ in } \boldsymbol{E}\, \}$$

Dynamic epistemic logic in this style may be viewed as a well-chosen fragment of the richer *epistemic temporal logics* of Fagin et al. 1995, Parikh & Ramanujam 2003. These also formulate longer-term environment of events, into the future and the past.

Finally, dynamic epistemic logics can be extended to deal with actual world change through physical events (cf. van Benthem, van Eijck & Kooi 2006), which are now specified using both 'pre–' and 'post-conditions'. The methodology via compositional reduction axioms and the resulting complete axiomatizations remain the same.

3.3 Regression principles are reduction axioms

One striking illustration of convergence between paradigms is the extensive use of the so-called *Regression Method* in Reiter 2001 for 'pre-computing' the effects of a plan at the initial stage of some process. In an original version of this paper in the summer of 2006, this was to have been one of my key examples of harmony between Situation Calculus and Modal Logic. But a few months ago, I found that Herzig et al. 2006, though still unpublished, makes essentially the same point in some formal detail. Thus, let me just make the point in general terms, because it does seem telling all the same.

The modal mechanics of regression We have a plan consisting of certain actions, perhaps in the form of some structured program, and we want to know about its effects. Now, the crucial observations are these. First, when describing relevant events *e*, we often know explicitly when they can occur, and the resulting *possibility predicates Poss(e)* in the Situation Calculus are just like the above *preconditions*. They define the states where the event can take place. In modal terms, this uses formulas:

(1) $<e>T \leftrightarrow \phi(e),$ where ϕ is a formula referring to the current state only.

In Reiter's language, all these predicates can involve individual terms, and they may come from some pretty lush higher-order languages, far beyond the usual austere modal formalisms – but we will discuss *that* difference in Section 4 below.

Next, we often specify what effects given events will have in terms of conditions on the current state. My lighting the match will result in the cigarette burning (a post-condition) if and only if certain preconditions are met. The resulting *successor state axioms* in the Situation Calculus describe when an event produces a state satisfying a certain atomic predicate in terms of some local condition on the current state. Again, the pattern is familiar from modal logic. Successor state axioms are like the *generalized preconditions* $<\pi>\varphi$ of dynamic logics of programs, where the existential modality says that the proposition φ will hold after some successful execution of the program π:. For the case of specific event, the reduction principle takes the following form

(2) $<e>\psi \leftrightarrow \Psi(e),$ where Ψ is a formula referring to the current state only. [4]

On top of these basic events, we have complex plans , but their structure can be reduced via the usual axioms of dynamic logic, such as the following law for composition:

(3) $<\pi_1;\pi_2>\varphi \leftrightarrow <\pi_1><\pi_2>\varphi$

Finally, and this is a feature not found in propositional dynamic logics, there may be general reduction axioms for existential quantifiers ranging over domains of objects. The following 'Barcan Axiom' from modal predicate logic will hold, e.g., provided that the event does not change the domain by actions of destruction or creation:

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[4] For the situation calculus, where events are usually taken to be *deterministic*, the difference with the universal modal variant $[\pi]\varphi$ also found in propositional dynamic logics is slight.

(4) $<e> \exists x\, \varphi \leftrightarrow \exists x <e> \varphi$.

Now, the Regression Theorem, arguably the central result in Reiter 2001, in both its factual and epistemic variants, says essentially that

> One can *pre-compute in the current state* when some given complex event or plan has given effects following its execution.

The simple proof rests on the following observation, when stated in our modal terms. Given preconditions for events and their atomic effects, all further constructions describing complex effects *can be pushed recursively through the event modality*. This is even an effective method for analyzing given plans, since all steps are recursive.

The modal counterpart But this, of course, was also exactly the point of our reduction axioms and the role of 'pre-encoding' in a suitably expressive static language. Hence the two methods in Situation Calculus and Dynamic Epistemic Logic seem to exploit essentially the same natural features of events and how we understand them in terms of information flow and physical changes. Of course, there are some details in matching here, and differences in notation to be polished away, but what comes out of this contact is the following. Dynamic epistemic logics use Situation Calculus-like machinery, but conversely, they also provide additional insights. In particular, they provide *low-complexity settings* where effects of the regression technology can be studied in vitro, so to speak. We know a lot about the complexity of dynamic-epistemic logics over their static base logics (cf. Lutz 2006, Herzig et al. 2006), and accordingly, we can learn a lot about the design of well-chosen fragments of the full Situation Calculus.

3.4 Knowledge and epistemic temporal logic

The case study of Regression is just one example of a match between frameworks. As we have also observed above, Situation Calculus works over the same branching temporal universe of events as most process theories. And not surprisingly then, its basic mathematical axioms for describing this general 'Theatre' are very much like the *induction axioms* and other basic principles found in most branching temporal logics.

Likewise, once epistemic structure is added to the temporal universe, the discussions of knowledge and informative events in Reiter 2001 are much like those found in Fagin et al. 1995, or the dynamic-epistemic literature. Indeed, the very treatment of knowledge is that of standard epistemic logic, in a rich language which also describes accessibility relations directly. [5] In particular – but we omit details here – Reiter's discussion of

[5] This is the 'Tandem Approach' beloved by modern modal logicians: cf. the chapter "Modal Logic:

reduction axioms for information after events makes essentially the following interesting point, which is also central to the dynamic-epistemic literature. The validity of reduction axioms for knowledge after events through commutation laws as in Section 3 above is not just a convenient technical decision. It makes two major cognitive presuppositions about agents (van Benthem & Pacuit 2006). One is that they satisfy

> *Perfect Recall* Future uncertainties can only arise out of past uncertainties plus uncertainty introduced by partial observation.

The other major presupposition is that the models should satisfy

> *No Miracles* Current uncertainty between worlds persists, unless it is resolved by some newly observed event.

4 Further Comparisons between Situation Calculus and Modal Logic

So far, we have found a striking number of similarities between Situation Calculus and Modal Logic. But this is not to say that we are claiming some mathematical equivalence between the two areas! To the contrary, there are also appreciable differences which one should cherish, since cross-overs to mutual benefit may then be possible.

In particular, one important aspect of current modal and dynamic logics has been left out so far. Possible worlds models tend to make key semantic notions into unanalyzed primitives, and accordingly modal languages are small, often propositional. By contrast, the Situation Calculus use structured objects all around: events have agents and objects, agents know people and their telephone numbers, etc. In line with this, formal languages are usually based on at least predicate logic with atomic predicates, variables, terms crossing between objects and predicates, and other ways of describing individual objects and their properties and interrelations. While this language issue is a big deal in some ways, it seems slight in others. But it does reflect a difference in methodology.

From modal logic to situation calculus In line with much established practice in computational logic, modal logic seeks a *Balance* between expressive power and computational complexity of model checking, satisfiability, and other core tasks (cf. Blackburn, van Benthem & Wolter 2006). The weaker the formalism, the better its computational behaviour, which may be decidable in low complexity classes. Now this poverty would not be much use, except perhaps for its moral virtues, if it were not for the following fact. Weak modal languages often high-light key structures in reasoning in a perspicuous way, allowing us to see essentials which can then be taken to richer

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A Semantic Perspective" in the Handbook Blackburn, van Benthem, & Wolter 2006.

languages afterwards. This was the point of our discussion of regression. In the same vein, a modal logician will say that much of the complexity of the Situation Calculus comes from mixing two types of information in nonmonotonic or temporal reasoning. One are *core principles* of the logical reasoning itself, the other are effects of describing *particular classes of models* ordered by concrete comparison relations – as happens in circumscription. That the resulting mixture of logic and 'mathematics' is complex goes without speaking: but is seems of interest to 'deconstruct' it into core plus extras. Modal Logic then provides fine-structure to the Situation Calculus.

From situation calculus to modal logic Conversely, as a modal logician, I see many virtues and challenges in the language design of the Situation Calculus. First, placing methods in such a richer setting serves as a good reminder that, e.g., our compositional methodology for analyzing information and belief is not restricted to propositional languages – and that the real issue to understand is not total complexity, but *relative complexity* of a dynamic logic over its static base logic.

Object structure and predicate logic Next, even though dynamic epistemic logics and epistemic temporal logics extol the importance of events, they are remarkably silent on the structure of these supposedly crucial entities. And that, while most theories of events in formal semantics endow them with a lot of basic structure, such as their having agents, objects, patients, locations, modes, etc! And agents knowing objects and persons, in addition to mere propositions, were still crucial to epistemic logic as originally developed in philosophy by Hintikka and subsequent authors. The agenda shift in modal logic taking this richer *object structure* inside worlds to the margin of research has occurred largely without strong reasons or public debate. And one can question it. As McCarthy has pointed out a number of times, even though one can get away with propositional encodings for all of this in given finite models, one loses the *genericity* of the modal first-order formulations, which works across a whole family of models at the same time. Finally, predicates, variables , and object terms are essential to sophisticated techniques such as logic programming, which definitely enhance our understanding of nonmonotonic reasoning, and queries about facts associated with it.

Suffice it to say this. Each item in the preceding list represents a challenge to people like me, viz. the design of richer and more generic versions of dynamic epistemic logic!

5 Conclusions: Comparing Frameworks Once More

We have shown how Modal Logic and Situation Calculus share key concerns and can interact with mutual benefit. Indeed, this seems a foregone conclusion, as most major paradigms describing common sense and rational action today, not just these two, are based on similar ideas, and work with similar techniques (cf. again van Benthem & Pacuit 2006). Partly, this is the result of pre-established harmony: great rational minds think alike. Partly also, it seems an effect of the inevitable osmosis in a field. Ideas tend to travel, and get borrowed, even though the receiving 'culture' may not be aware that it is borrowing. Personally, I am more struck by the analogies than by the differences, and I feel that some sort of Convergence Thesis would be good for all of us.

Three components of paradigms But research paradigms are pretty tricky things to compare and merge. In my view, they have at least three components. First, there is a mathematical *core theory* which can be surprisingly light (this is my sense of the axioms of the Situation Calculus, but also, e.g., of dynamic epistemic logic) and easy to equate with core theories elsewhere. But next, there is also a surrounding *modeling practice*, which makes the paradigm attractive in use. This consists in something extra: a historically growing fund of successful analyses of phenomena, that serve as crucial exemplars for new researchers. In this sense, the paradigm of 'Situation Calculus' is also the sum total of its well-known formalizations of common sense reasoning, including its successful modeling practice with 'abnormality' or 'clipping' predicates. Clearly, some frameworks come with more of such a fund by now than others, and these accompanying styles of formalization may be harder to compare, some being more suited to some phenomena than others. Finally, there is a third component to a paradigm, viz. its *research agenda*: which determines what is urgent or not. E.g., whether you are bothered by modal logic-style considerations of computational complexity is a bit like sensitivity to sin: some people have it, some do not. [6]

I am not sure what follows from this second set of observations, since this makes paradigms in our fields much like cultures, with all their inertia. Situation Calculus and Modal Logic do seem different as 'life-styles' in that sense. But then, there is also the undeniable fact that cultures meet and merge – so I hope that this paper can facilitate one such encounter. And we know from evolutionary game theory that, if you repeat even small encounters long enough over time, society as a whole will be affected!

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[6] People who do not often think the above 'Balance' is just a silly game to get papers published.

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