INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

THEO JANSSEN

Independence and Hintikka games

PP-2007-13, *received*: April 2007

ILLC Scientific Publications
Series editor: Benedikt Löwe

Prepublication (PP) Series, ISSN: 1389-3033

Institute for Logic, Language and Computation (ILLC)
University of Amsterdam
Plantage Muidergracht 24
NL-1018 TV Amsterdam
The Netherlands
e-mail: illc@science.uva.nl

Independence and Hintikka games

Theo M.V. Janssen

ILLC
University of Amsterdam
theo@science.uva.nl

Abstract

This paper investigates the formalization of independence in logic, in particular between quantifiers. We first illustrate that this form of independence plays a role in many fields. A logic that has the possibility to express such a form of independence is *Independence Friendly logic* (introduced by Hintikka); its semantics is defined by means of strategies in a sequential game. It will be shown that this logic gives results that are in *not* in accordance with intuitions on independence. An alternative semantics is presented where the logic is interpreted by means of a strategic game. The semantics exploits one assumption: the players are assumed to be *rational* (they do not play a strategy if there is a better one available). It is shown that this semantics gives results that are in accordance with intuitions about independence.

1 Introduction

In this paper we aim at a formalization of independence in logic. First some examples will be given of the kind of independence we are interested in: a quantifier that is independent of another quantifiers although it occurs in its scope. Such independency plays a role in many fields.

Mathematics: continuity

Intuitively a function is *continuous* if it can be drawn as one line. The formal definition is the well known epsilon-delta definition:

The function f is continuous if

(1) $\forall x \forall \epsilon \exists \delta \forall x' [|x - x'| < \delta \rightarrow |f(x) - f(x')| < \epsilon]$

In this definition the choice of δ depends on x, and this dependency is needed for a function like x^2 because that goes steeper and steeper, hence the larger x is, the smaller δ has to be.

For some functions this dependency is not needed because they have a steepest part, examples are $x+1$ and $\sin(x)$. Then δ only depends on ϵ. Such functions are called *uniform continuous*, defined by:

(2) $\forall \epsilon\, \exists \delta\, \forall x\, \forall x' \,[\, |x - x'| < \delta \rightarrow [\, |f(x) - f(x')| < \epsilon]$.

This definition is obtained by a reordering of quantifiers. It would be more attractive if we could express explicitly that δ does not depend on x by writing $\exists \delta_{/x}$ because that is the only difference between continuous and uniform continuous. Then the definition would read:

(3) $\forall x\, \forall \epsilon\, \exists \delta_{/x}\, \forall x' \,[\, |x - x'| < \delta \rightarrow [\, |f(x) - f(x')| < \epsilon]$.

Natural language semantics: *de dicto* - *de re*

The sentence *John believes that a stranger blighted his cow* is ambiguous. It may be the case that John has no particular person in mind, but that he believes that whatever the situation is, it will such that some stranger blighted his cow. This is called the *de dicto* reading. Let $Bel(John)$ denote the believe-alternatives of John, and a the actual world. Then this reading is represented by:

(4) $\exists y\, [cow_a(y) \wedge \forall w \in Bel_a(John)\; \exists x\, [Stranger_w(x) \wedge Blight_w(x, y)]]$.

The other reading, the *de re* reading, is that there is a particular person, indeed a stranger, of whom John believes that he blighted his cow. Its representation is obtained by an rearrangement of the quantifiers

(5) $\exists x \exists y\, [Stranger_a(x) \wedge cow_a(y) \wedge \forall w \in Bel_a(John)\; [Blight_w(x, y)]]$.

It would be attractive to describe this by an explicit indication of the independence (by $\exists x_{/w}$), because that is the difference between the two readings:

(6) $\exists y [\, cow_a(y) \wedge \forall w \in Bel_a(John)\, \exists x_{/w} [Stranger_w(x) \wedge Blight_w(x, y)]]$.

Natural language semantics: branching quantifiers

The most well known type of independence in the semantics of natural languages, and at the same time the most discussed one, is due to Hintikka (e.g. in Hintikka & Sandu (1997), Hintikka (1996)) Examples are (7) and (8):

(7) *Some friend of each townsman and some neighbor of each villager hate each other*

(8) *Some book by every author is referred to in some essay by every critic.*

In the intended meaning of (7) the friend of the townsman can be chosen independent of the neighbor of the villager. This meaning is often discussed in the literature, but we will accept that those sentences have such a meaning. Sentences of this type are called 'branching quantifier sentences' because their meaning could be represented using a branching quantifier. As an example this is done for (7): some selfexplaining abbreviations are used:

(9) $\begin{pmatrix} \forall x_1 & \exists x_2 \\ \forall x_3 & \exists x_4 \end{pmatrix} [(T(x_1) \wedge V(x_3)) \rightarrow (F(x_2, x_1) \wedge N(x_4, x_3) \wedge E(x_2, x_4))]$

If we use the notation $\exists x_{4\,/x_1, x_2}$ to indicate that the choice of x_4 has to be made independent of x_1 and x_2, then the meaning of (7) is represented by:

(10) $\forall x_1 \exists x_2 \forall x_3 \exists x_{4/x_1,x_2}[(T(x_1) \wedge V(x_3)) \rightarrow (F(x_2, x_1) \wedge N(x_4, x_3) \wedge H(x_2, x_4))]$

The semantics for (7), as represented in (10), will be discussed in Section 6.

Physics: mechanics The mechanical properties of a particle are described by its position and its pulse. By some experiment one may measure the values of these features. In classical mechanics the position and pulse of an object can be measured independently of each other. In quantum mechanics (which involves subatomair particles) this is not possible because a measurement of the position disturbs the particle with as a consequence that we are not sure about its pulse (and vice versa). If we determine the one rather precisely, the other becomes unspecific; the relation between the two measurements is expressed by Heisenberg's uncertainty relation $\Delta a \times \Delta p \geq \frac{h}{2*\pi}$. Here Δa is the uncertainty in position, Δp in pulse, and h Planck's constant.

Let x and y be the real values of the pulse and position, and v and u the values measured by M_a and M_p respectively and suppose we make statement φ about the measured values. For quantum mechanics, in which the first measurement has influence on the second measurement, this situation is described by:

(11) $\forall x \exists u \forall y \exists v[u = M_a(x) \wedge v = M_p(x) \wedge \varphi(u, v)]$

The independence of the two measurements in classical mechanics cannot be expressed by some rearrangement of the quantifiers. But if we indicate the independence of v by $\exists v_{/x,a}$ it can be expressed as follows (note the same quantifiers prefix as in (10)):

(12) $\forall x \, \exists u \, \forall y \, \exists v_{/x,a}[u = M_a(x) \wedge v = M_p(x) \wedge \varphi(u, v)]$

2 IF logic

In the last decade of the previous century, Hintikka presented his *Independence Friendly Logic*, henceforth IF logic (see e.g. Hintikka (1996) and Hintikka & Sandu (1997)). This logic extends earlier work in Branching Quantification and Game Theoretical Semantics for logic. The syntactical extension consists of a slash operator that can impose quantifications and connectives to be removed from the scope of another quantifications. E.g. in the formula $\forall x \, \exists y_{/x} \, \varphi(x, y)$, the slash operator in $\exists y_{/x}$ indicates that there exists a y that is *independent of* x such that $\varphi(x, y)$ holds.

The interpretation of IF logic proceeds by means of a game between two players. The one player, we call her ∃loise, aims at verifying the formula on a given model, the other, we call him ∀belard, aims at refuting that. ∃loise chooses the values for existentially quantified variables and chooses from disjunctions a disjunct, likewise does ∀belard for the universal quantifiers and conjunctions. So in the game $\forall x \, \exists y_{/x} \, \varphi(x, y)$ ∃loise has to pick a value for y in *ignorance* of the value chosen by ∀belard for x.

Of course, we are not so much interested whether ∃loise by accident wins or looses a game, but whether she has a strategy to win the game against any play of ∀belard. In order to define this notion, we need a some definitions.

Definition 2.1 *Let ψ be a subformula of φ. The set $Fr_\varphi(\psi)$ of variables free in ψ relative to φ consists of*

1. *the variables that are bound by quantifiers that have ψ in their scope*
2. *the free variables of φ*

Definition 2.2 *Let V be a set of assignments and Y a set of variables. A function f defined on V is called* ***Y-independent*** *on V (independent of Y on V) if for all $v, w \in V$ that assign the same values to variables outside Y, holds that $f(v) = f(w)$.*

Definition 2.3 *Let A be the domain of our model and φ some formula. A* ***choice function*** *for the subformula $\varphi_1 \vee_{/Y} \varphi_2$ of φ is a Y-independent function $c_{\varphi_1 \vee_{/Y} \varphi_2} : A^{Fr_\varphi(\varphi_1 \vee_{/Y} \varphi_2)} \rightarrow \{L, R\}$. A choice function for a subformula $\exists x_{/Y}\, \psi$ in game φ is a Y-independent function $c_{\exists x_{/Y}\, \varphi} : A^{Fr_\varphi(\exists x_{/Y}\, \varphi)} \rightarrow A$.*

Definition 2.4 *A* ***strategy*** *S_φ for ∃loise in game φ played on model $\mathcal{A}$ is a collection of choice functions which for each subformula ψ of φ where ∃loise has to play, provides a choice function c_ψ. Likewise for ∀belard.*

A ***winning strategy*** *for ∃loise in game φ played on model $\mathcal{A}$ is a strategy that guarantees ∃loise to win any play of the game, whatever ∀belard plays, if she uses the choice functions to make her moves. That means,*

1. *If for subformula $\varphi_1 \vee_{/Y} \varphi_2$ $c_{\varphi_1 \vee_{/Y} \varphi_2}(v) = L$ the game continues with determining the truth of φ_1, and if was R, of φ_2.*
2. *If for subformula $\exists x_{/Y}\, \varphi, v, \exists$: $c_{\exists x_{/Y}\, \varphi}(v) = a$, then the game proceeds with determining the truth of $\varphi[^a/_x]$.*
3. *For atomic formulas ∃loise wins if that formula is true in $\mathcal{A}$.*

Likewise we define a winning strategy for ∀belard.

Definition 2.5 *A formula φ is called true on a model $\mathcal{A}$ if ∃loise has a winning strategy for the game φ on $\mathcal{A}$, and false if ∀belard has a winning strategy.*

The independence conditions may cause that neither of the players has a winning strategy, so the law of the excluded middle does not hold for IF-logic.

3 Paradoxical Results

Hintikka's formalization of independence has paradoxical effects: i.e. there are several examples where the results are not in accordance with intuitions on independence. We will present some below, others are given in Janssen (2002).

1. Of course $\exists x\ \exists y\ [x=y]$ is true in every model, with strategy $y:=x$ (read as *choose for* y *the value* x *has*). Surprisingly also $\exists x\ \exists y_{/x}\ [x=y]$ is true, although the strategy $y:=x$ is not allowed. The solution consists in playing two constant strategies: $x:=0$ and $y:=0$. One might say that in the course of the game ∃loise has to forget her choice for $\exists x$ (because of $\exists y_{/x}$), but still remembers her strategy for x. It has been argued that the rules for playing IF logic require rather unnatural assumptions about the properties of the players (Sevenster (2005), van Benthem (2005)); this clearly is an example.

2. One cannot say whether x equals 0 without knowledge about x, so it understandable that $\forall x[x=0 \vee_{/x} x\neq 0]$ is not true on a model with at least two elements. Likewise $\exists x[x=0 \vee_{/x} x\neq 0]$ should not be true on such models . But with Hintikka's formalization it is true with strategy $x:=1$ for $\exists x$, and for $\vee_{/x}$ always R. This is again an application of remembering the strategy for $\exists x$, which indirectly gives ∃loise information about the value of x which she is not allowed to know.

3. One cannot select independent of x a value for y such that the two have different values. So $\forall x\, \exists y_{/x}[y\neq x]$ is not true with at least two elements. But $\forall x \exists z\, \exists y_{/x}[y=z]$ is true. The trick is to use the z as information for the value of x: the winning strategy is $\{z:=y, y:=x\}$. In analogy of card games like bridge, the method is called *signalling*. A value one is not allowed to know (x in $\exists y_{/x}$), becomes available in another way.

4. Consider $\forall x \exists u \forall z\, \exists v_{/xu}[u>x \wedge v>y]$, you may recognize it as a mathematical variant of the branching quantifier sentence (7). This formula is true on the natural numbers: e.g. by $u:=x+1$ and $v:=y+2$. If we add a requirement that the sum of u and v be even, then we create a dependency between the two values. Surprisingly $\forall x \exists u \forall z\, \exists v_{/xu}[u>x \wedge v>y \wedge Even(u+v)]$ is true. A winning strategy is that for u the least even integer is chosen with $u>x$, and for v the least even integer with $v>y$. So, whereas v should be independent of u, the strategies for u and v can be coordinated. I do not consider this as making independent choices.

5. We know that $\forall x\, \exists y_{/x}[y\neq x]$ is not true on a model with at least two elements. One expects that if one has to make such a independent choice twice, it still is not possible. Strangely enough $\forall x[\exists y_{/x}[y\neq x] \vee \exists y_{/x}[y\neq x]]$ is true. At the left, the strategy is to play $y:=0$ and at the right to play $y:=1$, and at $\vee$ to play L if $x=1$ and R otherwise. This shows that φ is not for all formulas equivalent with $\varphi \vee \varphi$.

4 Alternative semantics

We have seen that Hintikka's formalization does not formalize intuitions about independence. Therefore we will give an alternative semantics for IF logic: we propose to analyze the game as a special strategic game. The inspiration came from a paper by Sevenster (2005). He considers a spectrum of possible strategies, and discusses several criteria for imposing a restriction on that spectrum. For instance, a player may choose only strategies of a certain type, other players may know this information and use it to eliminate strategies from their spectrum, etc. We have one natural assumption on what to keep: rational strategies. The resulting semantics is more in accordance with intuitions on independence than Hintikka's semantics.

For the ease of discussion, we assume that the formula consists of a predicate, preceded by a series of quantifiers which may contain slashes. We assume that each quantifier has its own indexed variable, and that they appear in order, first the quantifier for x_1, then for x_2, etc. These restrictions are not essential for the approach, which can be extended to arbitrary IF-formulas.

With a variable x_i is associated a player p_i who determines the value for that variable. Players who determine the values for the existential variables aim at truth value **1**, and players for the universal quantifiers aim at truth value **0**. The notion *strategy* is defined as before, and we need further the following definitions.

Definition 4.1 *Two strategies f and g are called* equivalent *if, for any combination of strategies for the other players, f yields the same result as g.*

Definition 4.2 *A strategy s for player p_i who determines $\exists x_{/Y}$ is* better *than strategy t if:*

1. *Against all combinations of strategies of the other players where t yields **1**, also s yields **1**.*
2. *There is at least one combination of strategies for the other player such that s yields **1** whereas t yields **0**.*

*Analogously, we define 'better' for a player who determines $\forall x_{/Y}$, with the roles of **1** and **0** reversed.*

Definition 4.3 *A strategy s for player is* rational *if there is no strategy t such that t is better than s.*

In the examples one will see that a player may have one rational strategy, several rational strategies, or may have none. Our notion of rationality is known in game theory, I learned it from Genesereth, Ginsberg & Rosenschein (1986). There are other notions of rationality, and this one is not the most popular because its properties are mathematically not so nice under the operation of iteratively removing irrational strategies (we will not do that). For an overview of notions of rationality, see Apt (2004).

The central feature of our semantics is that at the start of the game each player determines what its own rational strategies are. A sentence is defined to be 'true' if all combinations of rational strategies yield the outcome **1**.

Two examples are given below; they are played on the domain $\{1, 0\}$.

1. Consider $\exists x_1 \exists x_2 [x_1 = x_2]$. Player p_1 has two strategies: $x_1 := 0$ and $x_1 := 1$. Player p_2 has 4 strategies; $x_2 := 0$, $x_2 := 1$, $x_2 := x_1$, and $x_2 := (\text{if } x_1 = 1 \text{ then } 0 \text{ else } 1)$. None of the strategies for p_1 is better than the other one

(the combination $x_1 := 1, x_2 := 0$ yields **0**, but also $x_1 := 0, x_2 := 1$ yields **0**). So both p_1's strategies are rational. For p_2 the strategy which yields **1** against all other strategies of p_1 is $x_2 := x_1$. So it is a rational strategy (in fact the only one). All combinations of rational strategies (there are two combinations) yield **1**, so the sentence is true.

2. In $\exists x_1 \exists x_{2/x_1}[x_1 = x_2]$, both players p_1 and p_2 have only two strategies: choose always **0** and choose always **1**. For none of the players there is a strategy that improves all his other strategies, so for both players these two strategies are rational. For some combinations of strategies the sentence yields **1**, for others it yields **0**. Therefore the sentence is not true.

The last example illustrates that our semantics is not equivalent with Hintikka's original semantics and that we get a result that is in accordance with intuitions on independence.

A yet unsolved problem for our approach is that situations may arise in which a player has no rational strategy at all. This may arise in infinite models; consider for instance $\exists x_1 \exists x_2[x_2 < x_1]$ played on the natural numbers. A rational choice for p_2 is $x_2 := 0$ another one might be $x_2 := max(0, x_1 - 1)$. However p_1 has no rational strategies: $x_1 := 1$ is better than $x_1 := 0$, but $x_1 := 2$ is better than $x_1 := 1$ etc. One might say that any combination of rational strategies vacuously yields **1** because there are no rational strategies. However, with this version of rationality, $\exists x_1 \exists x_2[x_2 < x_1] \wedge \exists x_3 \forall x_4[x_3 = x_4]$ would be true (incorrectly because of the second conjunct). So the definition of truth has to be adapted somehow for the case that there are no rational strategies.

5 Properties of the alternative semantics

The following theorem says that the truth of a formula cannot essentially depend on signalling.

Theorem 5.1 *For any rational strategy of a player for φ there is an equivalent strategy that depends only on variables that occur in φ.*

Proof. Without loss of generality, we prove the result for the situation that the formula is of the form $\ldots \exists x_{n+1}\, \varphi$ the player is p_{n+1}, and x_1 is a variable that does not occur in φ. So we have to show that any rational strategy $f_{n+1}(x_1, \ldots, x_n)$ of player p_b can be replaced by an equivalent strategy $f^*_{n+1}(x_2, \ldots, x_n)$, so by one which does not have x_1 as argument. We define this function in two steps.

First define g by:

$$g(y_2, \ldots, y_n) = \begin{cases} b, \text{ where } b \in A \text{ such that} \\ \qquad [\![\varphi]\!]^{\{x_1:b, x_2:y_2, \ldots x_n:y_n, x_{n+1}:f(b, y_2 \ldots, y_n)\}} = 1, \\ \qquad \text{if such an element exists,} \\ \text{arbitrary otherwise} \end{cases}$$

Next define $f^*_{n+1}(x_1, x_2, \ldots, x_n) = f_{n+1}(g(x_2, \ldots, x_n), x_2, \ldots, x_n)$.

Suppose that application of strategy f_{n+1} in a situation that $x_1=a_1,\dots,x_n=a_n$ yields **1**. Then there apparently is a value b for x_1, viz. a_1, such that:

$$[\![\varphi]\!]^{\{x_1:b,x_2:a_2,\dots x_n:a_n,x_{n+1}:f(b,a_2\dots,a_n)\}}=1.$$

Maybe for $g(a_2,\dots,a_n)$ some other value is selected than a_1, say b', but also for that value (because of the definition of g):

$$[\![\varphi]\!]^{\{x_1:b',x_2:a_2,\dots x_n:a_n,x_{n+1}:f(b',a_2\dots,a_n)\}}=1.$$

So under the assumption that $x_1=a_1,\dots,x_n=a_n$ and $f_{n+1}=\mathbf{1}$ strategy f^*_{n+1} yields the same value as f_{n+1}. If $f_{n+1}=\mathbf{0}$ it might even be better. The same argumentation applies for values other than $a_1,a_2,\dots a_n$. So f^*_{n+1} is at least as good as f_{n+1}. Since f_{n+1} was rational, f^*_{n+1} cannot be better, so the two must be equivalent must be equivalent.

End of proof

An example: for $\forall x[\exists u[u=x\wedge(u=0\vee u\neq 0)]$ there is a winning strategy $\{u:=x,(\text{if } x=0 \text{ then } L \text{ else } R)\}$, so with the non occurring x as signal for the value of u. Then there must be one for which the x plays no role, and indeed, it can be replaced by $\{u:=x,(\text{if } u=0 \text{ then } L \text{ else } R)\}$

More important is that the theorem excludes certain paradoxical situations. If in $\forall x\exists z\,\exists y_{/x}[x=y]$ the player for $\exists y_{/x}$ has a winning strategy depending z, there must be one without z because in $x=y$ no z occurs. Since for $\exists y_{/x}[x=y]$ there is no winning strategy that only has x as argument, there cannot be one using z. Indeed, if we consider the strategies, there is no rational strategy for $\exists y_{/x}$ that is winning against all strategies of other players.

Another consequence of the theorem is that an inductive definition of satisfaction seems possible. That is remarkable, because we introduced the game with a way of playing that goes from the outermost quantifier to the innermost. We present here only the case that the slashed quantifier is the last quantifier.

Definition 5.2 *Let $\varphi(\bar{x})$ be a formula with possibly $\bar{x}$ as free variables. Then by $\mathcal{A}\models^+\varphi(\bar{x})[v]$ is understood that for any combination of rational strategies the formula φ gets value* **1** *if for $\bar{x}$ we take $v(\bar{x})$ as value.*

Theorem 5.3 $\mathcal{A}\models^+\exists y_{/\bar{x}}\,\varphi(\bar{x},y)[v]\Leftrightarrow$ *there is a function* $f\colon(Fr(\varphi)\backslash\{y\})\to A$ *such that* $\mathcal{A}\models(\,\varphi(\bar{x},f(\bar{x}))\wedge\forall z[\exists u[\varphi(\bar{x},u)]\to\varphi(z,f(z))]\,)\,[v]$

Sketch of proof. ($\Leftarrow$) The first conjunct says that f yields **1** for the choices made earlier for $\bar{x}$ (given by v). The second conjunct says that if other values for x would have been chosen (captured by $\forall z$) and if there was then a choice (captured by $\exists u$) that would yield the formula **1**, then also strategy f would yield 1. This means that that f cannot be improved by any other strategy, i.e. it is a rational strategy.
($\Rightarrow$) The first conjunct is obvious, the second conjunct says that for other values f cannot be improved, which is a property of rational strategies
End of proof.

The truth definition mentioned in this theorem resembles the one put forward by Janssen (2002). That one is suggested by investigating the intuitions about many examples of independence and an attempt to find the common pattern. The present work can be seen as an argument that such a semantics for independence follows natural from considerations concerning game playing.

An important consequence of having a compositional semantics is that the notion of winning strategy for a certain formula is context independent: if there is a winning strategy for a formula, it is a winning strategy in all contexts, and if there is no winning strategy, then in no context there is one. This property excludes some of the paradoxical examples. There is, on models with more than two object, no winning strategy for $\exists y_{/x}[y \neq x]$, hence in no context this formula can be true, hence $\forall x[\exists y_{/x}[y \neq x] \vee \exists y_{/x}[y \neq x]]$ is not true. Another example is that $\exists x[x = 0 \vee_{/x} x \neq 0]$ cannot be true because $x = 0 \vee_{/x} x \neq 0$ is not true.

6 Branching quantifier sentences

We return to one of the linguistic applications mentioned in the beginnings. We repeat the sentence, and its representation in IF logic:

(13) *Some friend of each townsman and some neighbor of each villager hate each other*

(14) $\forall x_1 \exists x_2 \forall x_3 \exists x_{4/x_1,x_2}[(T(x_1) \wedge V(x_3)) \rightarrow (F(x_2, x_1) \wedge N(x_4, x_3) \wedge H(x_2, x_4))]$

Consider now the following situation. Among the friends of the townsmen and two groups are distinguished, viz. male and female ones, and the same among the neighbors of the villagers. Assume now that hating is a relation between all pairs of male friends and male neighbors, and also between female friends and female neighbors, but not between friends and neighbors of different sexes. In this situation the choices of friends for townsmen and neighbors for villagers have to correspond: in both cases male ones, or female ones. So intuitively the choice for $\exists x_{4/x_1,x_2}$ cannot be made independently of the choice for $\exists x_2$. So in this model sentence (13) should not be true.

As we have seen in the discussion of example $\exists x_1 \exists x_{2/x_1}[x_1 = x_2]$, it is in Hintikka's approach allowed to tune the strategies for x_1 and x_2. Analogously, (14) comes out true in Hintikka semantics: coordinate the strategies such such that they yield male friends and neighbors respectively. This shows that the required independence is not captured by his interpretation.

Let us now consider our analysis of (14). Suppose that in the model under discussion a male friend has been chosen. Then a male neighbor must be chosen as well, say Jacob. And if a female friend had been chosen only the choice for a female neighbor would be winning. But the condition of rationality requires that for female friend the original choice Jacob would be winning as well. That is not the case in the given model, so the formula is not true.

Concluding: for this example our semantics gives the desired result, whereas that is not the case for the traditional game theoretical semantics.

7 Conclusion

We have proposed an alternative interpretation for Hintikka's game theoretical semantics for IF logic. It is game theoretically a natural interpretation, and it yields results that are conform intuitions on independence.

Acknowledgements

I thank Merlijn Sevenster for several discussions on IF; without his inspiring paper (Sevenster 2005) this could not have been done.

References

Apt (2004), 'The many faces of rationalizabilty'. unpublished paper.

Genesereth, M. R., Ginsberg, M. L & Rosenschein, J.S (1986), Cooperation without communication, *in* 'Proc. of the Nat. Conf. of Art Intell, Philadelphia: Coherence and Communication: Reasoning about Agent's Knowledge and Behavior', AAAI Press, pp. 220–226. Standford research report from 1984.

Hintikka, J. (1996), *The Principles of Mathematics Revisited*, Cambridge University Press.

Hintikka, J. & Sandu, G. (1997), Game-theoretical semantics, *in* J. van Benthem & A. ter Meulen, eds, 'Handbook of Logic & Language', Elsevier Science, pp. 361–410.

Janssen, T. M. V. (2002), 'Independent choices and the interpretation of IF-logic', *JoLLI* **11**, 367–387.

Sevenster, M. (2005), A player perspective on IF semantic games, to appear.

van Benthem, J. (2005), The epistemic logic of IF games, *in* R.E. Auxier & L. E. Hahn, eds, 'The Philosophy of Jaakko Hintikka', Library of living philosophers, Open Court Publishers.

Institute for Logic, Language and Computation

ILLC Scientific Publications

Coding for Reports and Dissertations: *Series-Year-Number*, with PP = Prepublication Series; X = Technical Notes; MoL = Master of Logic Thesis; DS = Dissertations.
All previous ILLC-publications are available from the ILLC bureau. For prepublications before 1994, contact the bureau.

PP-2006-01 Ulle Endriss *Monotonic Concession Protocols for Multilateral Negotiation*

PP-2006-02 Tomoyuki Yamada *Acts of Commanding and Changing Obligations*

PP-2006-03 Reinhard Blutner *Optimality Theoretic Pragmatics and the Explicature/Implicature Distinction*

PP-2006-04 Evert van Emde Boas *Ποῖον τὸν μῦθον ἔειπες; Rhetorical Questions in Ancient Greek*

PP-2006-05 Steve Jackson, Benedikt Löwe *Canonical Measure Assignments*

PP-2006-06 Johan van Benthem *A New Modal Lindström Theorem*

PP-2006-07 Johan van Benthem *Information as Correlation versus Information as Range*

PP-2006-08 Guram Bezhanishvili, Johan van Benthem *Modal Logics of Space*

PP-2006-09 Steve Jackson, Farid Khafizov *Descriptions and cardinals below* δ^1_5

PP-2006-10 Benedikt Löwe, Brian Semmes *The Extent of Constructive Game Labellings*

PP-2006-11 Johan van Benthem *Dynamic Logic of Belief Revision*

PP-2006-12 Daisuke Ikegami *Projective absoluteness under Sacks forcing*

PP-2006-13 Benedikt Löwe *Set Theory of Infinite Imperfect Information Games*

PP-2006-14 Benedikt Löwe *Revision Forever!*

PP-2006-15 Yann Chevaleyre, Ulle Endriss, Nicolas Maudet *Tractable Negotiation in Tree-structured Domains*

PP-2006-16 Sylvia Estivie, Yann Chevaleyre, Ulle Endriss, Nicolas Maudet *How Equitable is Rational Negotiation?*

PP-2006-17 Yann Chevaleyre, Ulle Endriss, Jérôme Lang *Expressive Power of Weighted Propositional Formulas for Cardinal Preference Modelling*

PP-2006-18 Merlijn Sevenster *The Complexity of Scotland Yard*

PP-2006-19 Rens Bod *Exemplar-Based Linguistics: How to Get Productivity from Examples*

PP-2006-20 Rens Bod *Towards a General Model of Applying Science*

PP-2006-21 Johan van Benthem, Jelle Gerbrandy, Barteld Kooi *Dynamic Update with Probabilities*

PP-2006-22 Vincent Kieftenbeld *Notions of Strong Compactness without the Axiom of Choice*

PP-2006-23 Brian Semmes *Multitape Games*

PP-2006-24 Brian Semmes *A Game for the Borel Functions*

PP-2006-25 Nick Bezhanishvili, Dick de Jongh *Intuitionistic Logic*

PP-2006-26 Tomasz Sadzik *Exploring the iterated update universe*

PP-2006-27 Benedikt Löwe *Space bounds for infinitary computation*

PP-2006-28 Eric Pacuit *Some Comments on History Based Structures*

PP-2006-29 Eric Pacuit *A Note on Some Explicit Modal Logics*

PP-2006-30 Johan van Benthem, Patrick Blackburn *Modal Logic: A Semantic Perspective*

PP-2006-31 Merlijn Sevenster, Tero Tulenheimo *Partially ordered connectives and Sigma-1-1 on finite models*

PP-2006-32 Merlijn Sevenster *Henkin quantifiers: logic, games, and computation*

PP-2006-33 Benedikt Löwe, Eric Pacuit *An abstract approach to reasoning about games with mistaken and changing beliefs*

PP-2006-34 Olivier Roy *Commitment-Based Decision Making for Bounded Agents*

PP-2006-35 Nina Gierasimczuk, Jakub Szymanik *Hintikka's Thesis Revisited*

PP-2006-36 Jakub Szymanik *A note on some neuroimaging study of natural language quantifiers comprehension*

PP-2006-37 Fenrong Liu *Diversity of Agents*

PP-2006-38 Dick de Jongh, Fenrong Liu *Optimality, Belief and Preference*

PP-2006-39 Audrey Yap *Product Update and Looking Backward*

PP-2006-40 Marcin Mostowski, Jakub Szymanik *Semantical bounds for everyday language*

PP-2006-41 Fenrong Liu *Preference Change and Information Processing*

PP-2006-42 Ulle Endriss, Eric Pacuit *Modal Logics of Negotiation and Preference*

PP-2006-43 Yann Chevaleyre, Ulle Endriss, Sylvia Estivie, Nicolas Maudet *Multiagent Resource Allocation in k-additive Domains: Preference Representation and Complexity*

PP-2006-44 Johan van Benthem, Eric Pacuit *The Tree of Knowledge in Action: Towards a Common Perspective*

PP-2006-45 Johan van Benthem, Fenrong Liu *Dynamic logic of preference upgrade*

PP-2006-46 Andreas Blass, Ioanna M. Dimitriou, Benedikt Lwe *Inaccessible Cardinals without the Axiom of Choice*

PP-2006-47 Tero Tulenheimo, Merlijn Sevenster *Approaches to Independence Friendly Modal Logic*

PP-2006-48 Dick de Jongh *The Incompleteness Theorems, their content and their meaning*

PP-2006-49 Johan van Benthem *Computation as Conversation*

PP-2006-50 Yann Chevaleyre, Ulle Endriss, Sylvia Estivie, Nicolas Maudet *Reaching Envy-free States in Distributed Negotiation Settings*

PP-2006-51 Jesus Cerquides, Ulle Endriss, Andrea Giovannucci, Juan A. Rodriguez-Aguilar *Bidding Languages and Winner Determination for Mixed Multi-unit Combinatorial Auctions*

PP-2006-52 Nina Gierasimczuk *The Problem of Learning the Semantics of Quantifiers*

PP-2006-53 Merlijn Sevenster, Tero Tulenheimo *Finite model theory for partially ordered connectives*

PP-2006-54 Martin van Hees, Olivier Roy *Intentions and Plans in Decision and Game Theory*

PP-2006-55 Yann Chevaleyre, Ulle Endriss, Jérôme Lang and Nicolas Maudet *A Short Introduction to Computational Social Choice*

PP-2006-56 Joost J. Joosten *Semantics for sub-intuitionistic logics*

PP-2006-57 Vincent Kieftenbeld, Benedikt Löwe *A classification of ordinal topologies*

PP-2006-58 Raquel Fernández, Ulle Endriss *Abstract Models for Dialogue Protocols*

PP-2006-59 Johan van Benthem *Rationalizations and Promises in Games*

PP-2007-01 Fenrong Liu *Diversity of Agents and their Interaction*

PP-2007-02 Jacob Vosmaer *Essentially Sigma-1 formulae in Sigma L*

PP-2007-03 Johan van Benthem *Logic and Reasoning: Do the Facts Matter?*

PP-2007-04 Johan van Benthem *Situation Calculus Meets Modal Logic*

PP-2007-05 Johan van Benthem *Man Muss Immer Umkehren*

PP-2007-06 Johan van Benthem *Inference in Action*

PP-2007-07 Johan van Benthem *LOGIC GAMES: from tools to models of interaction*

PP-2007-08 Johan van Benthem, Jelle Gerbrandy, Eric Pacuit *Merging Frameworks for Interaction: DEL and ETL*

PP-2007-09 Johan van Benthem, Olivier Roy, Patrick Girard *Everything else being equal: A modal logic approach to ceteris paribus preferences*

PP-2007-10 Johan van Benthem *Actions that Make us Know*

PP-2007-11 Sebastian Bader, Pascal Hitzler, Steffen Hölldobler, Andreas Witzel *A Fully Connectionist Model Generator for Covered*

PP-2007-12 Krzystof Apt, Andreas Witzel *A Generic Approach to Coalition Formation*

PP-2007-13 Theo Janssen *Independence and Hintikka games*

PP-2007-14 Theo Janssen *Compiler correctness and the translation of logics*

PP-2007-15 Sujata Ghosh, Benedikt Löwe, Erik Scorelle *Belief Flow in Assertion Networks*

PP-2007-16 Sara L. Uckelman, Joel Uckelman *Modal and Temporal Logics for Abstract Space-Time Structures*

PP-2007-17 Peter Øhrstrøm, Sara L. Uckelman, Henrik Schärfe *Historical and Conceptual Foundation of Diagrammatical Ontology*

X-2006-01 Stefan Bold, Benedikt Löwe *The general inductive argument for measure analyses with additive ordinal algebras*

X-2006-02 Joost J. Joosten *Credit cards, computationele complexiteit en consistentie uitspraken*

X-2006-03 Nick Bezhanishvili, Lena Kurzen, Rachel Sterken *Student Papers from an Intuitionistic Logic Project*

X-2006-04 Ulle Endriss, Jérôme Lang *Computational Social Choice 2006*
X-2007-01 Johan van Benthem *Intelligent Interaction: Dynamic Trends in Todays Logic*
X-2007-02 Johan van Benthem *An Interview on Logic and Games*
MoL-2005-01 Reut Tsarfaty *'binyanim ba'avir': An investigation of Aspect Semantics in Modern Hebrew*
MoL-2005-02 Boaz Leskes *The Value of Agreement: a new Boosting Algorithm*
MoL-2005-03 Evangelos Tzanis *Algebraizing Hybrid Logic*
MoL-2005-04 Michael Franke *Pseudo-Imperatives*
MoL-2005-05 Floris Roelofsen *Exploring Logical Perspectives on Distributed Information and its Dynamics*
MoL-2005-06 Teresita Mijangos *Penalty Logic and Genomic Encoding*
MoL-2005-07 Samson Tikitu de Jager *Analysing the complexity of games on graphs*
MoL-2005-08 Gustavo Lacerda da Melo *Automating Normal Science: Reusing Exemplars in Quantitative Explanations*
MoL-2005-09 Scott Grimm *The Lattice of Case and Agentivity*
MoL-2006-01 Edgar Andrade *Meaning and Form in Event Calculus*
MoL-2006-02 Dirk Buschbom *Understanding As A Teleonomical Concept: Fundamentals of the Hermeneutical game*
MoL-2006-03 Ioanna Dimitriou *Strong limits and Inaccessibility with non-wellorderable powersets*
MoL-2006-04 Clive Nettey *Link-Based Methods for Web Information Retrieval*
MoL-2006-05 Yanjing Wang *Indexed Semantics and Its Application in Modelling Interactive Unawareness*
MoL-2006-06 Jill Cirasella *Classical and Quantum Algorithms for Finding Cycles*
MoL-2006-07 Martin Müller *Accent and Focus in OT: a Cross-Linguistic Perspective*
MoL-2006-08 Gaëlle Fontaine *Axiomatization of ML and Cheq*
MoL-2006-09 Jacob Vosmaer *Connecting the profinite completion and the canonical extension using duality*
MoL-2007-01 Jonathan Zvesper *A Revised Version: Belief Revision and Epistemic Acts*
MoL-2007-02 Wouter Koolen *Discovering the truth by conducting experiments*
DS-2005-01 Balder David ten Cate *Model theory for extended modal languages*
DS-2005-02 Willem-Jan van Hoeve *Operations Research Techniques in Constraint Programming*
DS-2005-03 Rosja Mastop *What can you do? Imperative mood in Semantic Theory*
DS-2005-04 Anna Pilatova *A User's Guide to Proper names: Their Pragmatics and Semanics*
DS-2005-05 Sieuwert van Otterloo *A Strategic Analysis of Multi-agent Protocols*
DS-2006-01 Troy Lee *Kolmogorov complexity and formula size lower bounds*
DS-2006-02 Nick Bezhanishvili *Lattices of intermediate and cylindric modal logics*
DS-2006-03 Clemens Kupke *Finitary coalgebraic logics*
DS-2006-04 Robert Špalek *Quantum Algorithms, Lower Bounds, and Time-Space Tradeoffs*
DS-2006-05 Aline Honingh *The Origin and Well-Formedness of Tonal Pitch Structures*
DS-2006-06 Merlijn Sevenster *Branches of imperfect information: logic, games, and computation*
DS-2006-07 Marie Nilsenova *Rises and Falls. Studies in the Semantics and Pragmatics of Intonation*
DS-2006-08 Darko Sarenac *Products of Topological Modal Logics*
DS-2007-01 Rudi Cilibrasi *Statistical Inference Through Data Compression*
DS-2007-02 Neta Spiro *What contributes to the perception of musical phrases in western classical music?*
DS-2007-03 Darrin Hindsill *It's a Process and an Event: Perspectives in Event Semantics*