Inquisitive Semantics:
Two Possibilities for Disjunction*

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Abstract. We introduce an inquisitive semantics for a language of propositional logic, where the interpretation of disjunction is the source of inquisitiveness. Indicative conditionals and conditional questions are treated on a par both syntactically and semantically. The semantics comes with a new logical-pragmatical notion which judges and compares the compliance of responses to an initiative in inquisitive dialogue.

1 Introduction

In this paper we introduce an *inquisitive semantics* for a language of propositional logic. In inquisitive semantics, the semantic content of a sentence is not identified with its informative content. Sentences are interpreted in such a way that they can embody both data and issues.

The propositional language for which we define the semantics is *syntactically hybrid*. By this we mean that the syntax of the language does not distinguish categories of declarative and interrogative sentences.[1] The language will also turn out to be *semantically hybrid*. By this we mean that some sentences of the language are both informative and inquisitive. Plain contingent disjunctions will count as such.[2]

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[1] You may think that this turns the language into an unnatural one, but that is not so. There are also hybrid natural languages which lack a formal syntactic distinction between interrogatives and declaratives.

[2] In hybrid natural languages, one of the strategies to express polar questions is to use disjunction of a sentence and its negation.

The language will enable us not only to express simple polar questions such as: "Will Bea go to the party?", but also conditional questions like: "If Alf goes to the party, will Bea go as well?", and alternative questions like: "Will Alf go to the party, or Bea?".

The natural use of an inquisitive language lies in dialogues that have the purpose of raising and resolving issues. We will introduce a logical notion that judges whether a sentence φ *is compliant to* a sentence ψ. We look upon φ as a response to an initiative ψ, and require φ to be strictly and obediently related to ψ. Compliance is a very demanding notion of dialogue coherence.[3]

2 Two Possibilities for Disjunction

The main purpose of this paper is to present inquisitive semantics and the logical notions that come with it. Our aim here is not to extensively motivate the semantics from a linguistic perspective. Nevertheless, we start with a short discussion of some phenomena concerning disjunction in English. Consider the simple disjunction in (1), and the interrogative in (2).

(1) Alf or Bea will go to the party.
(2) Will Alf or Bea go to the party?

It is generally acknowledged that the interrogative in (2) has different intonation patterns. On one pattern, the two responses in (3) are the most compliant ones.

(3) a. Yes. Alf or Bea will go to the party.
b. No. Neither of them will go.

Of course, though not answers in this case, these are equally good responses to the indicative sentence in (1), confirming and rejecting what (1) says, respectively.

On another intonation pattern, perhaps the more common one, (2) is not a yes/no-question, but has an alternative interpretation, which has two *different* most compliant responses, one of which is (4).

(4) Bea will go.

There are a couple of things to note. One is that (4), and its alternative, could be an equally compliant response to (1), but not no matter what. The disjunction in (1) *also* has different intonation patterns. And the pattern where (4) is a natural response is not the same as the pattern that most naturally elicits the responses in (3).

That is not to say that something that *amounts to* (4) could not be given as a reaction to (1) when it has the intonation pattern where the responses in (3) are the most expected ones, but the most 'appropriate' way to do it, would then be by means of (5) rather than (4).

[3] In Groenendijk (1999), I defined a similar notion which I called 'licensing'. I switched to the term 'compliance', because it has a negative ring to it, and thus communicates more clearly that being non-compliant can easily be a virtue rather than a vice.

(5) Yes. (In fact,) Bea will go.

First, (1) as such is confirmed, which sort of clears the way to elaborate on this by stating one of the disjuncts of (1).

Conversely, on the alternative interpretation of (2), it is not impossible to respond with something that amounts to (3b), denying both alternatives, but a more compliant way to do it, would be by using (6), rather than (3b).

(6) Well, (actually,) neither of them will go.

If we consider (1) again on the intonation pattern where (4) is ok, (6) (without the 'actually') and (3b) could both be used equally well.

What's the upshot of these (amateur) observations? It is that there is not *that* much of a difference between the indicative disjunction in (1) and the interrogative in (2). Both have a yes/no-interpretation, and both have an alternative interpretation, set apart by intonation. And what that suggests is that the alternative interpretation of the disjunction in (1) makes it into something that is not a plain assertion, that is not just informative, but also *inquisitive*, in much the same way as the alternative question interpretation of (2) is.

So, even though English is not syntactically hybrid, it seems to exhibit hybrid semantic features. Be this as it may, we now turn to our hybrid logical language.

3 Hybrid Propositional Syntax

The syntax of our propositional language is stated in a reasonably standard way.

Definition 1 (Hybrid Propositional Syntax). *Let $\wp$ be a finite set of propositional variables.*[4] *The set of* sentences of $L_\wp$ *is the smallest set such that:*

1. *If $p \in \wp$, then $p \in L_\wp$*
2. *$\bot \in L_\wp$*
3. *If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \to \psi) \in L_\wp$*
4. *If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \wedge \psi) \in L_\wp$*
5. *If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \vee \psi) \in L_\wp$*

What is not completely standard about the way the syntax is stated, is that it appears to intend to be economic in not explicitly introducing negation, whereas at the same time the definition does not stop after introducing implication, but explicitly brings up conjunction and disjunction as well. This suggests that, unlike in classical propositional logic, conjunction, disjunction and implication are not interdefinable in the usual way with the aid of negation.[5]

[4] The assumption that the set of atoms is finite only plays a marginal role in this paper, but is of importance, e.g., for proving functional completeness. See footnote 14.

[5] Like in this syntactic feature, inquisitive logic bears semantical and logical resemblances to intuitionistic logic. Inquisitive logic is a so-called intermediate logic. Except for one more footnote, I will not address the matter. See Mascarenhas (2008).

What is also a bit surprising perhaps, is that it is not immediately obvious from the syntax that questions are there. The last item in the following small list of notation conventions brings them optionally to the syntactic surface.

Definition 2 (Notation Conventions).

1. $\neg\varphi := \varphi \to \bot$ *2.* $\top := \neg\bot$ *3.* $!\varphi := \neg\neg\varphi$ *4.* $?\varphi := (\varphi \vee \neg\varphi)$

We will refer to $!\varphi$ as an *assertion*, and to $?\varphi$ as a *question*. Defining these operations implicates that the law of double negation and the law of the excluded middle do not generally hold under the inquisitive interpretation of the language.

If you have ever taught an introductory class in logic, you know that translating from natural language to the logical language at hand, or the other way around, is not always an easy affair for your students. When you start teaching them inquisitive propositional logic the next semester, the situation wil not improve. But here is a piece of pedagogical advice.

Especially when the native language of your students is not syntactically hybrid — if it is Dutch or English, for example — hammer the following into their heads: Keep in mind that our logical language is syntactically hybrid, and has only a single sentential category. A formula of the form $?p$ is just short for $p \vee \neg p$, and has the same syntactic status, it does not suddenly become an interrogative sentence, there are none.[6]

Hence, tell your students not to criticize the logical language for allowing $\neg?p$ as a well-formed sentence because in their mother tongue negating interrogatives is out of bound: $\neg?p$ simply stands for $\neg(p \vee \neg p)$, and apart from being a contradiction, there is nothing wrong with that sentence.

A multi-categorial language like English also has, what one might call, syntactically hybrid constructions, such as conditional interrogatives, where the antecedent has the same syntactic status as in indicative conditionals, and the consequent is interrogative. The construction as a whole should rather be characterized as an interrogative.[7] In our hybrid logical language, $p \to q$ and $p \to ?q$, where the latter is short for $p \to (q \vee \neg q)$, do not have a distinct syntactic status.

This also means that the logical language allows for a formula like $?p \to ?q$, of which it is dubious whether it has a 'direct' translation in English, in terms of a conditional with an interrogative antecedent. But, of course, both $(p \vee \neg q) \to ?q$ and $(p \vee \neg p) \to (q \vee \neg q)$, which are abbreviated by $?p \to ?q$, do have syntactic correlates in English.

The examples given sofar mainly pertain to prevent your students from making the mistake of literally translating formulas of the form $?\varphi$ into their native

[6] If you are unfortunate enough, and an introductory course in Philosophy of Language — or any other course where your students were inflicted with Speech Acts — went ahead of your logic course in the curriculum, you have to stress on top of this: We are doing logical *semantics*, the question mark in the logical language has nothing to do with illocutionary force.

[7] This is how things are set up syntactically in Velissaratou (2000), where the logical language that deals with conditional questions has two distinctive categories: interrogatives and declaratives. Our present investigations started from her analysis.

tongue by mistakingly treating them as the corresponding interrogatives in all contexts. But in the other direction, translation problems are lurking as well.

Consider our English examples (1) and (2) in the previous Section, a plain disjunction which your students no doubt translate as $p \vee q$, and an alternative interrogative, which they will be tempted to translate as $?(p \vee q)$.

You may forgive them for not noticing the prosodical ambiguity. But even if so, whether $?(p \vee q)$ is the best translation in the hybrid logical language of the English interrogative in (2) under its alternative interpretation, is not obvious. $?(p \vee q)$ is short for $p \vee q \vee \neg(p \vee q)$, which has a direct translation into an indicative English sentence. Given the discussion in Section 2 concerning the similarity in responses to (1) and (2), it is doubtful whether $p \vee q \vee \neg(p \vee q)$, and hence whether $?(p \vee q)$, is the most appropriate translation for (2). Perhaps we might better opt for translating both the indicative in (1) and the interrogative in (2) as $p \vee q$ in the hybrid logical language.

In the logical language we can also form $!(p \vee q)$ and $?!(p \vee q)$, where the former is short for $\neg\neg(p \vee q)$, and the latter can also be written as $!(p \vee q) \vee \neg !(p \vee q)$, or as $!(p \vee q) \vee \neg(p \vee q)$, if we may assume that triple negation reduces to single negation. We have to see whether the inquisitive interpretation is going to corroborate this, but we might have the suspicion that the prosodic ambiguity we observed for both (1) and (2), is reflected in the logical language in the difference between $p \vee q$ on the one hand, and $!(p \vee q)$ and $?!(p \vee q)$ on the other.

4 Inquisitive Semantics

We state the semantics for a language $L_\wp$ relative to a set $W_\wp$ of *suitable possible worlds for* $L_\wp$, where a world $w \in W_\wp$ is a valuation function with the set of propositional variables $\wp$ as its domain and the two values $\{1, 0\}$ as its range.[8]

For a declarative language, a standard way to define the interpretation of the sentences of the language is by the notion $w \models \varphi$, which can be read as: w confirms the information provided by φ. This will not suffice to interpret our hybrid language, where sentences may not only provide information, but may also raise issues.

The simplest way to deal with this is to evaluate sentences relative to pairs of worlds, and define the interpretation of the language in terms of the notion $(w, v) \models \varphi$, which is informally characterized as follows:

Agreement Two worlds w and v *agree on* φ, $(w, v) \models \varphi$ iff

(i) both w and v confirm the information provided by φ; and

(ii) the answer to issues raised by φ is the same in w and v.

It wil be clear from this informal description of agreement, that if we consider $(w, w) \models \varphi$ only the first clause applies, and $(w, w) \models \varphi$ simply boils down to: w confirms the information provided by φ. Some further properties are noted in

[8] We will often suppress the subscript $\wp$ on L and W.

Theorem 2. They follow from the definition of the inquisitive interpretation of our hybrid propositional language, which reads as follows.[9]

Definition 3 (Inquisitive Semantics). *Let $\varphi \in L_\wp$, and $w, v \in W_\wp$.*

1. $(w, v) \models p$ *iff* $w(p) = 1$ *and* $v(p) = 1$
2. $(w, v) \not\models \bot$
3. $(w, v) \models (\varphi \vee \psi)$ *iff* $(w, v) \models \varphi$ *or* $(w, v) \models \psi$
4. $(w, v) \models (\varphi \wedge \psi)$ *iff* $(w, v) \models \varphi$ *and* $(w, v) \models \psi$
5. $(w, v) \models (\varphi \to \psi)$ *iff for all pairs* π *in* $\{w, v\}$: *if* $\pi \models \varphi$, *then* $\pi \models \psi$

The definition has pretty familiar looks,[10] except for the clause for implication, which quantifies over the four pairs (w, v), (v, w), (v, v), and (v, v). It can easily be read from the other clauses that to consider (v, w) next to (w, v) is redundant.[11]

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[9] There are four different situations where we get that $(w, v) \not\models \varphi$: (○○) neither w nor v confirms the information provided by φ; (●○) w confirms the information provided by φ, but v does not; (○●) the other way around; (●○) both w and v confirm the information provided by φ, but the answer to the issues raised by φ is not the same in w and v. Together with the situation (●) where $(w, v) \models \varphi$, there are five possible situations to distinguish. This gives the tools to define the semantics in terms of a five-valued valuation function relativized to two worlds: $V_{w,v}(\varphi) \in \{$●, ●●, ●○, ○●, ○○$\}$. We can use truth tables to explicate the meanings of the logical operations.

There is an interesting connection with Nelken & Francez (2001), who — in response to the claim made in Groenendijk & Stokhof (1997) that an extensional semantics of questions is impossible — also interpret a query language in terms of an 'extensional' five-valued semantics. However, the two sets of values differ in that for our single value ●, they distinguish the two values 'true' and 'resolved', and that our two values ●○, ○● are collapsed into a single value 'unknown'.

As is already obvious from Nelken & Francez (2001) as such, their 'extensional' semantics is seriously flawed, and as is explicitly admitted in Nelken & Shan (2006), who analyze questions in the setting of modal logic, what one needs is a 'minimally intensional' interpretation that needs to take only two worlds into account.

There is also a cryptic remark in Ten Cate and Shan (2007: p. 69) that: "To test a LoI entailment [Logic of Interrogation, see Groenendijk (1999)], it suffices to consider structures with only two possible worlds." It was Balder ten Cate — in his capacity as an anonymous referee for the submitted abstract of the talk on which this paper is based — who suggested the semantics relative to pairs of worlds as it is defined here, as a logically more simple alternative for the update semantics I gave for the language in the abstract. (See footnote 14.)

[10] The clause for disjunction may also look familiar to you if (like Robert van Rooij, thanks) you have read David Lewis' paper: *'Whether' report* (Lewis (1982)). There, Lewis considers to treat *whether A or B* clauses as *wheth A or wheth B* along the following lines:

$$\models_{i,j} \textit{whether } A \textit{ or } B \text{ iff } \models_{i,j} \textit{wheth } A \text{ or } \models_{i,j} \textit{wheth } B$$

$$\models_{i,j} \textit{wheth } A \text{ iff } \models_i A \text{ and } \models_j A$$

The notion $\models_{i,j}$ is conceived of as an application of the technique of double indexing.

[11] This motivates the use of the notation (w, v) to denote pairs, instead of $\langle w, v \rangle$.

We will discuss implication more extensively later, but note that to inspect whether (w, v) agree on $\varphi \to \psi$, we not only check whether if (w, v) *agree* on φ, (w, v) agree on ψ as well. Also in case (w, v) do *not agree* on φ, because w confirms the information provided by φ whereas v does not (or the other way around), we still keep on checking in that case, whether w (or v) also confirms the information provided by ψ.[12]

Although, as I announced in the introduction, we will see later that the semantics gives rise to a new logical notion of complicance that rules the use of the inquisitive language in dialogue, an orthodox notion of entailment in terms of agreement suggests itself as well.

Definition 4 (Entailment).

$$\varphi_1, \ldots, \varphi_n \models \psi \textit{ iff}$$
$$\forall w, v \in W\colon \textit{if } (w, v) \models \varphi_1 \;\&\; \ldots \;\&\; (w, v) \models \varphi_n, \textit{ then } (w, v) \models \psi$$

The notion of entailment is well-behaved and has interesting properties. To note one, from the way in which implication is defined, it immediately follows that under the inquisitive interpretation of the language the deduction theorem holds.

Theorem 1 (Deduction Theorem). $\varphi \models \psi$ *iff* $\models \varphi \to \psi$

Logical equivalence of two formulas is defined as usual as mutual entailment.

Definition 5 (Equivalence). $\varphi \Leftrightarrow \psi$ *iff* $\varphi \models \psi \;\&\; \psi \models \varphi$

We add two more notions that only pertain to information provided by φ.

Definition 6 (Consistency and Assertiveness).

1. φ is consistent *iff* $\exists w \in W\colon (w, w) \models \varphi$
2. φ is assertoric *iff* $\exists w \in W\colon (w, w) \not\models \varphi$

With these logical notions in place, we turn to the discussion of the semantics.

4.1 Informativeness and Inquisitiveness

The first clause in Definition 2 implies that a propositional variable p does not raise an issue. The definition says that to see whether w and v agree upon p, it is sufficient to see whether $w(p) = v(p) = 1$. The situation cannot occur that despite it being the case that $(w, w) \models p$ and $(v, v) \models p$, still p embodies some issue to which the answer in w and v is different, i.e., that $(w, v) \not\models p$. We have that $(w, v) \models p$ iff $(w, w) \models p$ and $(v, v) \models p$.[13]

[12] Compare the discussion in footnote 9 about the four ways in which a pair of worlds may *not* agree on a sentence. The clause for implication reads the way it does, to deal in an appropriate way with these different ways in which the antecedent may not be agreed upon, and to tell us what should be the case for the consequent then.

[13] Note that this is just a blunt stipulation in the vein of: "Eines kann der Fall sein oder nicht der Fall sein und alles übrige gleich blieben." (Ludwig Wittgenstein, *Tractatus 1.2.1)* Under such Logical Atomism it becomes indeed *Unsinnig* to ask *Why?* when atomic sentences are involved. But is it really such nonsense?

This is different for a disjunction like $p \vee q$. There we can have that $(w, w) \models p \vee q$ and $(v, v) \models p \vee q$, whereas $(w, v) \not\models p \vee q$, as can be shown as follows.

Let w be a world where $w(p) = 1$ & $w(q) = 0$, and v a world where $v(p) = 0$ & $v(q) = 1$. Then we have that $(w, w) \models p$ and $(v, v) \models q$, and hence, according to clause 3 of Definition 2, both $(w, w) \models p \vee q$ and $(v, v) \models p \vee q$.

At the same time we have that $(w, v) \not\models p$, because $v(p) = 0$; and $(w, v) \not\models q$, since $w(q) = 0$. According to the definition, this means that $(w, v) \not\models p \vee q$. So, we have shown that, unlike in the case of atomic sentences, there are worlds w and v such that $(w, w) \models p \vee q$ and $(v, v) \models p \vee q$, whereas $(w, v) \not\models p \vee q$.

Two such worlds w and v do not agree on $p \vee q$, because although w and v both confirm the information provided by $p \vee q$, the answer to the issue that $p \vee q$ raises, the issue whether p or q, is different in w and v, in w the answer is p, in v it is q.

We have just shown that an atomic sentence p is not inquisitive, and that a disjunction like $p \vee q$ is inquisitive, according to the following definition:

Definition 7 (Inquisitiveness and Informativeness).

1. φ is inquisitive *iff* $\exists w, v \in W\colon (w, w) \models \varphi$ *&* $(v, v) \models \varphi$ *&* $(w, v) \not\models \varphi$.
2. φ is informative *iff* φ *is consistent and* φ *is assertoric.*

We also define:

(a) φ *is* hybrid *iff* φ *is informative and* φ *is inquisitive.*

(b) φ *is* contingent *iff* φ *is consistent, and* φ *is inquisitive or assertoric.*

What inquisitiveness of φ requires is that there are pairs of worlds that satisfy the information provided by φ (which implies that φ is consistent, and hence contingent), but where the two worlds differ in their answer to an issue raised by φ, which implies that φ indeed does raise an issue, otherwise two such worlds could not be found.

As we have seen, informativeness and inquisitiveness do not exclude each other, $p \vee q$ is both informative and inquisitive, and hence semantically hybrid.

Given the way the interpretation of $\bot$ is defined, it counts as neither informative, nor inquisitive. I.e., $\bot$ is not contingent.The same will hold for $\top$, the difference being that whereas $\top$ is consistent and not assertoric, $\bot$ is assertoric and not consistent.

4.2 Negations and Assertions

Before we turn to negation, we note that on the basis of the informal description of $(w, v) \models \varphi$ in terms of agreement, we may expect the following to hold, which indeed it does, given the way the semantics is defined:

Theorem 2 (Symmetry and Reflexive Closure of Agreement).

1. $\forall w, v \in W\colon (w, v) \models \varphi \Rightarrow (v, w) \models \varphi$
2. $\forall w, v \in W\colon (w, v) \models \varphi \Rightarrow (w, w) \models \varphi$ *&* $(v, v) \models \varphi$

The proof runs by induction on the complexity of φ.[14]

From the definition of inquisitiveness and the last item in Theorem 2, the following follows immediately.

Proposition 1 (Non-Inquisitiveness).

$$\varphi \textit{ is not inquisitive iff } \forall w, v \in W\colon (w,v) \models \varphi \Leftrightarrow (w,w) \models \varphi \;\&\; (v,v) \models \varphi.$$

Let us now consider $\neg\varphi$ which abbreviates $\varphi \to \bot$. Clause 5 of Definition 2 tells us that for w and v to agree on $\varphi \to \bot$, it should hold for the four pairs (w,v), (v,w), (w,w), and (v,v), that if such a pair agrees on φ it agrees on $\bot$. The interpretation of $\bot$ tells us that no pair agrees on $\bot$. Hence, For w and v to agree on $\varphi \to \bot$, it should hold that $(w,v) \not\models \varphi$, $(v,w) \not\models \varphi$, $(w,w) \not\models \varphi$, and $(v,v) \not\models \varphi$. Given Theorem 2 and the way negation is introduced in the language, this boils down to:

Proposition 2 ($\neg$Negation). *$(w,v) \models \neg\varphi$ iff $(w,w) \not\models \varphi$ & $(v,v) \not\models \varphi$.*

Two worlds agree on a negation as soon as neither of the two confirms the information provided by φ. As we saw to be the case for atomic sentences, negations raise no issue. From Propositions 1 and 2 it immediately follows that:

Proposition 3 (Negation). *$\neg\varphi$ is not inquisitive.*

That negations are never inquisitive, is of course behind the fact that disjunction, conjunction and implication are not interdefinable in the usual way with the aid of negation. Disjunction is the indispensable source of inquisitiveness in the language. And if we were to define implication and conjunction in terms of disjunction and negation we do not in general obtain the interpretation now assigned by the semantics to formulas of these forms.

Since $!\varphi$ is defined as double negation, $!\varphi$ is not inquisitive. And we can write:

Proposition 4 (!Assertion). *$(w,v) \models\, !\varphi$ iff $(w,w) \models \varphi$ & $(v,v) \models \varphi$*

This means that for any formula φ, $!\varphi$ delivers its interpretation in classical logic. From Propositions 1 and 4 it follows that every non-inquisitive sentence can be written as an assertion:

Proposition 5 (Assertion). *$!\varphi \Leftrightarrow \varphi$ iff φ is not inquisitive.*

[14] In inquisitive *update semantics*, a *state σ for $L_\wp$* is defined as a reflexive and symmetric relation on a subset of $W_\wp$. In the semantics we recursively define the effect of updating σ with φ, $\sigma[\varphi]$. The relation with the present semantics is given by: $\sigma[\varphi] = \{(w,v) \in \sigma \mid (w,v) \models \varphi\}$. Salvador Mascarenhas proved a Functional Completeteness Theorem, which says that for any two states σ and σ', $\sigma' \subseteq \sigma \subseteq W_\wp^2$: there is a finite sequence of sentences $\varphi_1, \ldots, \varphi_n \in L_\wp$ such that $\sigma[\varphi_1] \ldots [\varphi_n] = \sigma'$. We don't need the full language to achieve this, $\{\neg, \vee\}$ suffices. Since conjunction corresponds to sequencing, if we add $\wedge$, we can move from any state to any of its substates with a single formula of the language. The assumption we made that $\wp$ is finite, is essential for the functional completeness proof. (See Mascarenhas (2008).)

Given this fact, we will often refer to non-inquisitive sentences as *assertions*.

Note that Proposition 5 tells us that the law of triple negation holds. Since negation is not inquisitive, $!\neg\varphi \Leftrightarrow \neg\varphi$. And iteration of ! is superfluous: $!!\varphi \Leftrightarrow !\varphi$.

Also, since $!(p \vee q)$ is not inquisitive, it is not equivalent with the hybrid disjunction $p \vee q$. The assertion $!(p \vee q)$ raises no issue, $!(p \vee q)$ just embodies the truthconditional content of disjunction in classical logic.

Remember the discussion in Section 2, where we observed that the English indicative disjunction (1), like its interrogative sister (2), is prosodically ambiguous between a yes/no-interpretation, and an alternative interpretation. As for the latter, for both (1) and (2), the hybrid disjunction $p \vee q$ suggests itself as a proper translation. As for the yes/no-interpretation of the indicative (1), the assertion $!(p \vee q)$ seems to cover its meaning. And if we take the disjunction $!(p \vee q) \vee \neg(p \vee q)$, i.e., $?!(p \vee q)$ we get a polar question that suits (2) on its yes/no-reading. This brings us to questions.

4.3 Questions

Consider the atomic question $?p$, which abbreviates $p \vee \neg p$. The interpretation of disjunction tells us that $(w, v) \models p \vee \neg p$ iff $(w, v) \models p$ or $(w, v) \models \neg p$, both worlds agree on p or both worlds agree on $\neg p$. This means that $(w, v) \models ?p$ iff $w(p) = v(p) = 1$ or $w(p) = v(p) = 0$. From this it is clear that $?p$ is not informative, and is inquisitive, $p \vee \neg p$ is contingent, it is an inquisitive question.

Given the interpretation of disjunction and the interpretation of negation given in Proposition 4, we can write:

Proposition 6 (?Questions).

$$(w, v) \models ?\varphi \textit{ iff } (w, v) \models \varphi \textit{ or } (w, w) \not\models \varphi \;\&\; (v, v) \not\models \varphi$$

If we consider $(w, w) \models ?\varphi$, we get that $(w, w) \models ?\varphi$ iff $(w, w) \models \varphi$ or $(w, w) \not\models \varphi$, which trivially holds, hence $\forall w \colon (w, w) \models ?\varphi$. Given how assertiveness is defined, the first item in the next fact holds, from which the second item immediately follows.

Proposition 7 (Questions).

1. *$?\varphi$ is not assertoric.*
2. *$?\varphi \Leftrightarrow \varphi$ iff φ is not assertoric.*

Given this fact, we will often refer to non-assertoric sentences as *questions*. (Note that since $\top$ is neither assertoric nor inquisitive, it counts both as an uninformative, non-assertoric assertion, and as a non-contingent, non-inquisitive question.)

The last item in Proposition 7 implies that iteration of ? is superfluous: $??\varphi \Leftrightarrow ?\varphi$. The fact that iteration of both ! and ? are superfluous, makes it easy to state things in general about assertions and questions.

4.4 Conditionals: Divide and Conquer

The two specific cases of conditionals with a question, and conditionals with an assertion as consequent, behave more standardly than the clause for implication in Definition 2 might suggest.

Proposition 8 (Conditional Questions and Conditional Assertions).

1. $(w, v) \models \varphi \to ?\psi$ *iff* $(w, v) \models \varphi \;\Rightarrow\; (w, v) \models ?\psi$
2. $(w, v) \models \varphi \to !\psi$ *iff* $(w, w) \models \varphi \to \psi$ *and* $(v, v) \models \varphi \to \psi$

Clause 5 in Definition 2 requires for $(w, v) \models \varphi \to \psi$ that it holds for each of the pairs π we can form from w and v, i.e., (w, v), (v, w), (w, w), and (v, v) that: if $\pi \models \varphi$, then $\pi \models \psi$. Given symmetry of agreement (Theorem 2), among those four, we can dismiss (v, w).

Furthermore, since $?\psi$ is not assertoric, i.e., $\forall w\colon (w, w) \models ?\psi$, in evaluating $\varphi \to ?\psi$, we can dismiss the two identity pairs as well. This also means that if ψ is not assertoric, then neither is $\varphi \to \psi$.

Conversely, since $!\psi$ is not inquisitive, i.e., $(w, v) \models \psi$ iff $(w, w) \models \psi$ & $(v, v) \models \psi$ (Proposition 1), in evaluating $\varphi \to !\psi$, we only have to consider the two identity pairs. This also means that if ψ is not inquisitive, $\varphi \to \psi$ is not inquisitive either, and behaves like classical material implication:

Proposition 9 (Non-Inquisitive Conditionals). $\varphi \to !\psi \Leftrightarrow !(\varphi \to \psi)$

So, we have seen how the two specific cases of conditionals with non-inquisitive and non-assertoric consequents behave more standardly than the clause for implication in Definition 2 might suggest. But we can actually show that any conditional reduces to a combination of these two simple cases.

The following fact tells us that every sentence φ can be divided in a *theme* $?\varphi$ and a *rheme* $!\varphi$.

Proposition 10 (Division). $\varphi \Leftrightarrow ?\varphi \wedge !\varphi$

From this it immediately follows that every conditional $\varphi \to \psi$ can be written as $\varphi \to (?\psi \wedge !\psi)$. Next, we use the following distribution fact.

Proposition 11 (Distribution 1). $\varphi \to (\psi \wedge \chi) \Leftrightarrow (\varphi \to \psi) \wedge (\varphi \to \chi)$

This allows us to rewrite the conditional $\varphi \to (?\psi \wedge !\psi)$ as the conjunction of conditionals $(\varphi \to ?\psi) \wedge (\varphi \to !\psi)$. Finally, applying the equivalence in Proposition 9 to the second conjunct, we arrive at the following fact.

Proposition 12 (Conditional Division). $\varphi \to \psi \Leftrightarrow (\varphi \to ?\psi) \wedge !(\varphi \to \psi)$

Any conditional can be rewritten as the conjunction of a conditional question and a classical material implication.[15]

[15] Actually, we may take this to mean that the theme of a plain conditional, the question on the background, is the corresponding conditional question (rather than the

4.5 Disjunctive Antecedents

Consider the simplest example $p \to ?q$ of a conditional question. We get that $(w, v) \models p \to ?q$ iff $w(p) = v(p) = 1 \Rightarrow w(q) = v(q)$. I.e., either $w(p) = v(p) = 1 \Rightarrow w(q) = v(q) = 1$ or $v(p) = w(p) = 1 \Rightarrow w(q) = v(q) = 0$. Which means that $p \to ?q$ is equivalent with the disjunction $(p \to q) \vee (p \to \neg q)$. This is a special instance of the following equivalence:[16]

Theorem 3 (Mascarenhas Theorem). $!\varphi \to (\psi \vee \chi) \Leftrightarrow (!\varphi \to \psi) \vee (!\varphi \to \chi)$

This equivalence does not hold generally also in case of inquisitive antecedents. The following pair of examples is a case in point: $(p \vee q) \to ?r \not\Leftrightarrow !(p \vee q) \to ?r$. With the Mascarenhas Equivalence, $!(p \vee q) \to ?r$ corresponds to a disjunction of two assertions: $((p \vee q) \to r) \vee ((p \vee q) \to \neg r)$. We will show that $(p \vee q) \to ?r$ corresponds to a longer disjunction of four assertions.

First we note another distribution fact.

Proposition 13 (Distribution 2). $(\varphi \vee \psi) \to \chi \Leftrightarrow (\varphi \to \chi) \wedge (\psi \to \chi)$

This means that $(p \vee q) \to ?r$ is equivalent to $(p \to ?r) \wedge (q \to ?r)$, which is Mascarenhas-equivalent to $((p \to r) \vee (p \to \neg r)) \wedge ((q \to r) \vee (q \to \neg r))$, to which we apply the (last) distribution fact:

Proposition 14 (Distribution 3).

$$(\varphi \vee \psi) \wedge (\chi \vee \theta) \Leftrightarrow (\varphi \wedge \chi) \vee (\varphi \wedge \theta) \vee (\psi \wedge \chi) \vee (\psi \wedge \theta)$$

This gives us four disjuncts, two of which are $(p \to r) \wedge (q \to r)$ and $(p \to \neg r) \wedge (q \to \neg r)$, which by Proposition 13 reduce to the first two of the following four disjuncts, which together are equivalent with $(p \vee q) \to ?r$:

$((p \vee q) \to r) \vee ((p \vee q) \to \neg r) \vee ((p \to r) \wedge (q \to \neg r)) \vee ((p \to \neg r) \wedge (q \to r))$.

What we have arrived at, is that there are, as we will call them, four *possibilities* for the sentence $(p \vee q) \to ?r$. In this case, since the sentence is an inquisitive question, the four possibilities correspond to four possible answers.[17]

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corresponding questioned conditional). Thus, inquisitive semantics may be taken to give a logical explanation for the idea ventured in the first sentence of Ramsey's famous footnote, known as the Ramsey Test: (Ramsey (1931))

> If two people are arguing "If p will q?" and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense "If p, q" and "If $p, \neg q$" are contradictories.

These two 'contradictories' are the two answers to the conditional question that we just found to be the theme of a conditional. See also Grice's paper on 'Indicative Conditionals' in Grice (1989).

[16] The hard part of this equivalence, from left to right (and with $\neg\varphi$ instead of $!\varphi$), is known as the Kreisel-Putnam Axiom and corresponds to an admissible rule in intuitionistic propositional logic. Salvador Mascarenhas has proved that it is also valid in inquisitive propositional logic. This result is crucial in obtaining a disjunctive normal form. (See footnote 18.)

[17] We discovered this nice feature of the semantics by surprise. Tikitu (Samson) de Jager programmed the semantics. The program spits out the possibilities (see below)

4.6 Possibilities

Given the properties of $(w, v) \models \varphi$, as stated in Theorem 2, the relation between worlds of 'to agree on a sentence φ', corresponds to a set of sets of worlds.

Definition 8 (Possibilities). *Let* $\varphi \in L_\wp$. P is a possibility for φ in $W_\wp$ *iff*

1. $P \subseteq W_\wp$ *&* $\forall w, v \in P\colon (w, v) \models \varphi$*; and*
2. $\forall P' \subseteq W_\wp$*: if* $P \subseteq P'$ *&* $\forall w, v \in P'\colon (w, v) \models \varphi$*, then* $P' = P$.

A possibility for a sentence φ is a largest set P of worlds (a proposition), such that for any two worlds $w, v \in P\colon w$ and v agree on φ.[18]

A sentence φ is inquisitive iff there is more than one possibility for φ; φ is not inquisitive iff there is a single possibility for φ. The set of possibilities for $\perp$ in W is $\{\emptyset\}$. The set of possibilities for $\top$ is $\{W\}$. A sentence φ is informative iff the union of the set of possibilities for φ is not empty and is not equal to W.

If there is more than one possibility for a sentence φ, then each possibility corresponds to a proposition that fully resolves the issue raised by φ. Unions of (some but not all) possibilities for φ correspond to propositions that partially resolve the issue raised by φ.

The possibilities for a sentence may or may not mutually exclude each other. E.g., in the cases of $p \vee \neg p$ and $?p \wedge ?q$ they do, in the cases of $p \vee q$ and $p \rightarrow ?q$ they don't. We call sentences which have no overlapping possibilities classical sentences.[19]

Definition 9 (Classical Sentences).

φ is classical *iff for any two possibilities* $P \neq P'$ *for* φ *in* $W\colon P \cap P' = \emptyset$.

All non-inquisitive sentences, all assertions $!\varphi$ are classical. If $?\varphi$ is a classical question, then the set of its possibilities forms a partition of W.

For classical sentences φ it holds that if P is a possibility for φ, then for *all* $w \in P$, there is no other possibility P' for φ such that $w \in P'$. This is a bit weaker:

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that a formula gives rise to. This is particularly helpful for formulas with more than two propositional variables, which are hard to picture. We ran the program on $(p \vee q) \rightarrow ?r$, expecting to get out the two possibilities for $!(p \vee q) \rightarrow ?r$. Panic struck when the program predicted four possibilities. But after analyzing what came out, the program — and the semantics — turned out to be right. There is a reading of the question: "If Alf or Bea goes to the party, will Chris go as well?", that has the four possible answers that Tikitu's program came up with.

[18] Salvador Mascarenhas has proved that the set of possibilities for a sentence φ can be syntactically characterized as a disjunction of assertions, where each assertion characterizes a possibility. Any sentence can be transformed into its Inquisitive Disjunctive Normal Form, which has this property. (Mascarenhas (2008))

[19] Christopher Brumwell has proved that a language which has $\{\neg, \wedge, ?\}$ as its only basic operators, where $\neg$ and ? are interpreted as indicated in Proposition 2 and 6, is functionally complete (see footnote 14) relative to all equivalences relations on subsets of the set of worlds. So, this language characterizes the classical fragment of the full language.

Definition 10 (Semi-Classical Sentences).

φ is semi-classical *iff for every possibility P for φ in W, there is some world $w \in P$ such that for no possibility $P' \neq P$ for φ in W: $w \in P'$.*

We could say that in case of a semi-classical sentence, the possibilities for that sentence are truly *alternatives*, in the sense that each of them has a unique part not shared by any of the other possibilities. Although not classical, $p \vee q$ and $p \to ?q$ are semi-classical.

An example of a sentence that is not semi-classical, is the disjunction of questions $?p \vee ?q$. It is a *choice-question.* In responding to $?p \vee ?q$ one can choose between answering $?p$ and $?q$. There are four possibilities for $?p \vee ?q$ that correspond to the two answers to $?p$ and the two answers to $?q$. Any world in the possibility that p is either contained in the possibility that q or in the possibility hat $\neg q$, and similarly for the other three possibilities.[20]

5 Inquisitive Logic

If we ask ourselves what the natural purpose of an inquisitive language is, the obvious answer is: to raise and resolve issues; a purpose best suited in dialogue. Then a natural task for a logic that comes with inquisitive semantics is to address moves in a dialogue concerned with cooperatively raising and resolving issues.

Following the lead of the 'normative' status of the logical notion of entailment in judging validity of argumentation, we can take inquisitive logical notions to judge 'correctness', or 'coherence', or 'compliance' of a response to an initiative in a cooperative inquisitive dialogue.

Here we can draw from general insights in dialogue studies.[21] Two fundamental dialogue coherence relations for a response to an initiative are the following:

Two Dialogue Coherence Relations

(i) Answer an issue raised by an initiative *(informative relation)*; or

(ii) Replace the issue by an easier to answer subissue *(inquisitive relation).*

The inquisitive option is second choice, a cooperative responder takes recourse to it only if he lacks the information for even a partial fulfilment of the first option. And note that if the initiative is a question, we may assume that the initiator certainly has no full answer to it, but she just may have a bit of a partial answer to her own question. Hence, it can make sense for the responder to ask a counter question, if only because when *that bit* of the issue were resolved, it may become possible for him to provide a full(er) answer to the initial question.[22]

[20] It has been debated whether disjunctions of questions exist in natural language, for example by Szabolcsi (1997). But, surely, there is nothing wrong with: "Answer 3 of the following 5 questions: ...". This easily translates in our hybrid logical language as a long disjunction,of which each disjunct is a conjunction of 3 of the 5 questions.

[21] See, e.g., Asher & Lascarides (1998), Hulstijn (2000), and Roberts (1996).

[22] For similar reasons, it may also be sensible not to respond with a subissue, but with an objectively speaking unrelated question, which subjectively, for the responder, is related to the issue posed by the initiator.

Of course, the less inquisitive such a counter question is, the better the chances are that this bit of the original bigger issue, turns out not to be an issue for the initiator.

If we go from here, then one can take it to be the case, that the general direction an inquisitive dialogue strives for, is to move from *less informed* to *more informed* situations, and from *more inquisitive* to *less inquisitive* situations.

If we look at entailment from this perspective, we see that more informativeness of φ as compared to ψ is measured by $\varphi \models \psi$, whereas less inquisitiveness of φ as compared to ψ runs in the opposite direction, and is measured by $\psi \models \varphi$.

It is not too difficult to design a logical relation that measures informativeness and inquisitiveness in these opposite direction in one go. We call it homogeneity.

Definition 11 (Homogeneity). *φ is at least as homogeneous as ψ, $\varphi \succeq \psi$ iff*

1. *For all $w \in W$: if $(w,w) \models \varphi$, then $(w,w) \models \psi$, and*
2. *For all $w, v \in W$: if $(w,w) \models \varphi$ & $(v,v) \models \varphi$ & $(w,v) \not\models \varphi$, then $(w,v) \not\models \psi$.*

The second clause holds trivially for assertions, since the antecedent can never be the case. The first clause holds trivially for questions. The most essential features of homogeneity, and its hybrid relation to entailment, are listed below:

Proposition 15 (Homogeneity).

1. *If $\varphi \succeq \psi$, then $!\varphi \models !\psi$*
2. *$!\varphi \succeq !\psi$ iff $!\varphi \models !\psi$*
3. *If $!\varphi \Leftrightarrow !\psi$, then $\varphi \succeq \psi$ iff $?\psi \models ?\varphi$*
4. *$?\varphi \succeq ?\psi$ iff $?\psi \models ?\varphi$*
5. *$!\varphi \succeq ?\psi$*
6. *$\bot \succeq \varphi$*
7. *$\top \succeq ?\varphi$*

Although homogeneity gives the *general* direction an inquisitive dialogue strives for, as is particularly clear from the fact that *any* assertion is at least as homogeneous as *any* question (item 5 in the list), we need some more specific directions that tell us, e.g., *which* assertions are proper responses to *which* questions. The logical notion of relatedness, defined in terms of possibilities, does that.

Definition 12 (Relatedness). φ is related to ψ, $\varphi \propto \psi$ *iff*

every possibility for φ is the union of a subset of the set of possibilities for ψ

Relatedness is defined generally for all kinds of sentences, but if φ is an assertion, for which there is only a single possibility P, relatedness of φ to ψ requires that P is the union of a subset of the set of possibilities for ψ, which, in case ψ is inquisitive, is as close as you can logically expect to get, in characterizing partially resolving the issue raised by an initiative ψ.

By homogeneity we can measure whether the information contained in one sentence more fully resolves an issue, than the information contained in another sentence. Were it not for the borderline case of non-contingent $\bot$, which is more homogeneous than any contingent sentence, and is also related to every sentence, we could equate the most homogeneous related responses to an inquisitive initiative ψ with those sentences φ that completely resolve the issue ψ raises.

In other words, under the general constraint of contingency of a response, relatedness, combined with homogeneity, tells us how well a sentence does in resolving an issue. This concerns the informative dialogue coherence relation.

Concerning the inquisitive dialogue coherence relation, we get a similar story. First of all, if a question $?\varphi$ is related to and at least as homogeneous as a question $?\psi$, it is indeed guaranteed that $?\varphi$ is at least as easy to answer as $?\psi$.[23]

Secondly, by homogeneity we can measure whether one question is a more minimal subissue of some issue, than another question is. Were it not for the borderline case of non-contingent $\top$, which is the most homogeneous 'question', and is also related to every question, we could equate the most homogeneous related questions to some question $\psi?$ with the minimal subquestions of $\psi?$.

In other words, under the general constraint tof contingency of a response, relatedness, combined with homogeneity, tells us how well a question does in replacing an issue by an easier to answer subissue. This concerns the inquisitive dialogue coherence relation.

We put our findings together in the following definition of compliance that deals with both dialogue coherence relations.

Definition 13 (Compliance). φ is a compliant response to ψ *iff*

1. *φ is contingent; and*
2. *φ is related to ψ; and*
3. *φ is at least as homogeneous as ψ.*

This qualitative notion of compliance embodies a comparative quantitative notion as well: among contingent sentences which are related to an initiative, homo-

[23] Relatedness by itself does not in general achieve this. A case in point is that $?p$ is related to $?p \vee ?q$, where at the same time $?p$ is *more* inquisitive, i.e., less homogeneous, than $?p \vee ?q$: $?p \vee ?q \succ ?p$, and hence $?p \models ?p \vee ?q$. The following fact says that when we restrict ourselves to semi-classical sentences, relatedness of φ to ψ as such, already guarantees that φ is not more inquisitive than ψ.

For semi-classical φ and ψ: if $\varphi \propto \psi$ then $\varphi \succeq \psi$

For classical sentences, we can characterize relatedness in terms of entailment.

For classical φ and ψ: $\varphi \propto \psi$ iff $\psi \models ?\varphi$

In the logic of interrogation of Groenendijk (1999), which deals with classical sentences only, a similar fact appears concerning its notion of licensing, which corresponds to the present notion of relatedness. The fact was used in ten Cate & Shan (2007) in axiomatizating the logic. For non-classical φ and ψ this fact does not always hold. For example, we have that $p \to q \propto p \to ?q$, but $p \to ?q \not\models ?(p \to q)$. Instead we have that $?(p \to q) \models p \to ?q$, and $p \to ?q \not\propto ?(p \to q)$.

geneity prefers more informative sentences, and among two equally informative sentences, it prefers less inquisitive sentences.[24] These are the borderline cases:

Proposition 16 (Ultimate Compliance). *Let ψ be a contingent initiative.*

1. φ is a *least compliant response* to ψ iff φ is equivalent to ψ.
2. φ is a *most compliant response* to ψ iff there is a single possibilty P for φ, such that P is a possibility for ψ as well.
3. If ψ is a question, φ is a *most compliant non-informative response* to ψ iff φ is a polar subquestion of ψ.

Note that in case the initiative is a polar question, the most compliant non-informative responses coincide with the least compliant responses. If the responder has no answer to a polar question, there is no significant move to make.

Similarly, in case the initiative is a contingent assertion, the least and most compliant responses coincide: repeating the initiative, at most rephrasing it a bit, is the only compliant move to make.

This is why we characterized compliance informally as strict and obedient relatedness. Compliance as such does not allow for critical responses. Logically speaking, it is just a small step to allow for critical responses: also permit compliance to the *theme* $?\psi$ of an initiative ψ. Emotionally, though, say for a parent with a maturing child, this may be a big step. But that's another story.

6 Conclusion: Inquisitive Pragmatics

It will not have escaped your attention that the way the logical notion of compliance is defined bears resemblances to the Gricean Cooperation Principle and its Maxims of Quality, Relation and Quantity. This may give rise to the expectation that implicatures are around the corner.

Consider the example of a hybrid disjunction $p \vee q$ as an initiative. There are, up to equivalence, only two most compliant responses: p and q. In particular, the more homogeneous sentence $p \wedge q$ is blocked, because it is not related to $p \vee q$. Apparently, according to the initiator, it does not count.

How can that be? We have taken it to be the case that a cooperative dialogue strives for more homogeneous situations. In principle, the initiator should be interested in obtaining the information whether $p \wedge q$ on top of the information that p (or q). By blocking $p \wedge q$ as a response, the initiator *suggests* that: not both p and q. And by responding with just p to $p \vee q$, the responder signals that he goes along with that suggestion. Hence, his answer p *implicates* that $\neg q$. Cooperatively, initiator and responder have agreed upon exclusive disjunction.

Of course, the responder may have reasons for not following the exclusiveness suggestion made by the initiator, just as he may have reasons not to accept the

[24] There is a clearcut syntactic connection between an initiative and compliant responses, and comparison thereof, when we consider inquisitive disjunctive normal forms (see footnote 18): cut off, and/or take the assertive closure of disjuncts, the more the better, as long as one remains.

informative content, which excludes that neither p nor q. In both cases, the responder opts for not being compliant. In such situations, the appropriate way to do this, is not to bluntly reject the information provided, with: "Neither p nor q!"; or to protest against the suggestion being made with: "Both p and q!". Compliant non-compliant responses are rather: "Well, actually, neither p nor q"; and: "Well, in fact, both p and q.", thus explicitly signalling awareness of the non-compliance of one's response. (See also the examples in Section 2.)

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