Analyzing stories as games with changing and mistaken beliefs

Benedikt Löwe[1,2,3], Eric Pacuit[4], Sanchit Saraf[1,2,5]⋆

[1] Institute for Logic, Language and Computation, Universiteit van Amsterdam, Plantage Muidergracht 24, 1018 TV Amsterdam, The Netherlands; bloewe@science.uva.nl;

[2] Department Mathematik, Universität Hamburg, Bundesstrasse 55, 20146 Hamburg, Germany;

[3] Mathematisches Institut, Rheinische Friedrich-Wilhelms-Universität Bonn, Beringstraße 1, 53115 Bonn, Germany;

[4] Department of Computer Science, Stanford University, 353 Serra Mall, Stanford CA 94305, United States of America; epacuit@stanford.edu;

[5] Department of Mathematics and Statistics, Indian Institute of Technology, Kanpur 208016, India; sanchit@iitk.ac.in

Abstract. The first two authors proposed a simple algorithm for analyzing stories in terms of belief states. In this paper, we use this algorithm to analyse actual stories from a commercial TV crime series, and identify a small number of building blocks sufficient to construct the doxastic game structure of these stories. (June 13, 2008)

1 Introduction

In [7], the authors propose a simple algorithm for analysing stories in terms of belief states based on notions of doxastic logic. The algorithm applies to stories in which all agents have perfect information about the events in the past, but may be mistaken about their (iterated) beliefs about preferences of the players and may change their beliefs and preferences during the course of events.

Whereas in [7, § 4], the algorithm was used to fully analyze a fictitious story about love and deceit, in this paper, we focus on actual stories commercially produced for television broadcasting in this paper. In a *descriptive-empirical* approach we investigate their common structural properties based on a formalization in our system. The doxastic tree structures associated to the stories allow natural definitions of formal properties and complexity that can be further used to classify story types. The results of this paper show that from a large number of possible formal structures, commercial crime stories only use a very small number of doxastically simple basic building blocks (§ 2.4).

In §2 of this paper, we shall introduce our system, modified from [7, § 3] to incorporate event nodes (at which no agent is playing). We also discuss the basic

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building blocks of belief structures that we will later encounter in the analyzed stories. In §3, we discuss the process of taking an actual story and transforming it into a game of mistaken and changing beliefs, focusing in particular about the restrictions that we imposed upon ourselves by the choice of our formal framework. We discuss in detail the distinction between the formal structure of a story and its presentation (in narratology, these components are normally called "story" and "discourse")[1] and its consequences for our formalization endeavour. Finally, in §4, we then present the formalization of ten stories from the first four episodes of the TV series *CSI: Crime Scene Investigation*™ in which we can see that the eight doxastic building blocks from § 2.4 are enough to formalize all stories. In § 5, we summarize the findings of the paper and discuss related work.

2 Definitions and fundamental structures

2.1 Definitions

We give a short version of the definitions from [7, § 3]. As opposed to the discussion there, we shall explicitly use *event nodes*, i.e., nodes in which none of the agents makes a decision, but instead an event happens. Structurally, these nodes do not differ from the standard *action nodes*, but beliefs about events are theoretically on a lower level (of theory of mind) than beliefs about beliefs.

Let I be the finite set of players whom we denote with boldface capital letters. We reserve the symbol $\mathbf{E} \in I$ for the event nodes. If $\vec{\mathbf{P}} = \langle \mathbf{P}_0, ..., \mathbf{P}_n \rangle$ is a finite sequence of players symbols, we write $\vec{\mathbf{P}}\mathbf{P}$ for the extension of the sequence by another player symbol $\mathbf{P}$, i.e.,

$$\vec{\mathbf{P}}\mathbf{P} := \langle \mathbf{P}_0, ..., \mathbf{P}_n, \mathbf{P} \rangle.$$

A tree T is a finite set of nodes together with an edge relation (in which any two nodes are connected by exactly one path). Let $\mathsf{tn}(T)$ denote the set of terminal nodes of T, and for $t \in T$, let $\mathbf{succ}_T(t)$ denote the set of immediate T-successors of T. The **depth** of the tree T is the number of elements of a longest path in T, and we denote it by $\mathrm{dp}(T)$.

We fix I and T and a **moving function** $\mu : T \backslash \mathbf{tn}(T) \to I$, where $\mu(t) = \mathbf{P}$ indicates that it is $\mathbf{P}$'s move at node t. If $\mu(t) = \mathbf{E}$ we call t an **event node**, otherwise we call it an **action node**. We call total orders $\succeq$ on $\mathbf{tn}(T)$ **preferences** and denote its set by $\mathcal{P}$. A map $\succeq : I \to \mathcal{P}$ is called a **description**. We call a functions

$$S : T \times I^{\leq \mathrm{dp}(T)} \to \mathcal{P}^I$$

states, interpreting the description $S(t, \varnothing)$ as the **true state of affairs** at position t. If $S(t, \vec{\mathbf{P}})$ is one of the descriptions defined by the state S, we interpret $S(t, \mathbf{P}\vec{\mathbf{P}})$ as player $\mathbf{P}$'s belief about $S(t, \vec{\mathbf{P}})$.

[1] Alternatively, the term pairs "фабула"/"сюжет" or "histoire"/"récit" are used. From now on, we shall used the term "*discourse*" to refer to the presentation of the narrative.

2.2 The analysis

Given a tuple $\langle I, T, \mu, S \rangle$, we can now fully analyze the game and predict its outcome (assuming that the players follow the backward induction solution). In order to do this analysis, we shall construct labellings $\ell_{S_{\vec{\mathbf{P}}}} : T \to \mathsf{tn}(T)$ where $\ell_{S_{\vec{\mathbf{P}}}}$ is interpreted as the subjective belief relative to $\vec{\mathbf{P}}$ of the outcome of the game if it has reached the node t. For instance, $\ell_{S_{\mathbf{A}}}(t) = t^* \in \mathsf{tn}(T)$, then player $\mathbf{A}$ believes that if the game reaches t, the eventual outcome is t^*.

The labelling algorithm. If t is a terminal node, we just let $\ell_U := t$ for all states U. In order to calculate the label of a node t controlled by player $\mathbf{P}$, we need the $\mathbf{P}$-subjective labels of all of its successors. More precisely: If $t \in T$, $\mu(t) = \mathbf{P}$ and we fix a state U, then we can define ℓ_U as follows: find the U-true preference of player $\mathbf{P}$, i.e., $\succeq = U(t, \varnothing)(\mathbf{P})$. Then consider the labels $\ell_{U_{\mathbf{P}}}(t')$ for all $t' \in \mathsf{succ}(t)$ and pick the $\succeq$-maximal of these, say, t^*. Then $\ell_U(t) := t^*$. Concisely, $\ell_U(t)$ is the $U(t, \varnothing)(\mu(t))$-maximal element of the set $\{\ell_{U_{\mu(t)}}(t')\,;\, t' \in \mathsf{succ}(t)\}$.

Computing the true run of the game. After we have defined all subjective labellings, the true run can be read off recursively. Since our labels are the terminal nodes, for each t with $\mu(t) = \mathbf{P}$ and S, there is a unique $t' \in \mathsf{succ}(t)$ such that $\ell_{S_{\mathbf{P}}}(t') = \ell_S(t)$. Starting from the root, take at each step the unique successor determined by $\ell_S(t)$ until you reach a terminal node.

2.3 Partial states, notation, and isomorphism

Note that in actual stories (as opposed to stories invented for the purpose of formalization, such as the story in [7, § 2]), we cannot expect to have full states. Instead, we'll have some information about players' preferences and beliefs that is enough to run the algorithm described in § 2.2. If $\mathcal{P}^{\mathrm{p}}$ is the set of partial preferences (i.e., linear orders of subsets of $\mathsf{tn}(T)$) and $\mathrm{PF}(X, Y)$ is the set of partial functions from X to Y, then we call partial functions from $T \times I^{\mathrm{dp}}(T)$ to $\mathrm{PF}(I, \mathcal{P}^{\mathrm{p}})$ **partial states**.

In the following, we shall use the letters v_i for non-terminal nodes of T and t_i for terminal nodes. If we write $S(v_i, \vec{\mathbf{P}})(\mathbf{P}) = (t_{i_0}, t_{i_1}, ..., t_{i_n})$, we mean that in the ordering $\succeq := S(v_i, \vec{\mathbf{P}})(\mathbf{P})$, we have $t_{i_0} \succeq t_{i_1} \succeq ... \succeq t_{i_n}$. If in such a sequence, we include a non-terminal node v_i, e.g.,

$$S(v_i, \vec{\mathbf{P}})(\mathbf{P}) = (t_j, v_k),$$

we mean that t_j is preferred over *all* nodes following v_k. Similarly,

$$S(v_i, \vec{\mathbf{P}})(\mathbf{P}) = (v_j, v_k)$$

means that every outcome following v_j is preferred over every outcome following v_k. In particular for the event nodes, we normally phrase preferences in these terms. When we are drawing our game trees, we represent non-terminal nodes by

$\boxed{v_i}\boxed{\mathbf{P}}$ indicating $\mu(v_i) = \mathbf{P}$. In our discussions, we will assume introspection of all agents, i.e., agents are aware of their own preferences and iterations thereof, even though there is evidence that introspection is not necessarily a feature of human mental processes and awareness [10]. This simplifies notation considerably, and there are no indications that failure of introspection is relevant in any of the stories we analyzed.

The notion of partial states give an obvious definition of **isomorphism** of two formalized versions of stories: if $\langle I, T, \mu, S\rangle$ and $\langle I^*, T^*, \mu^*, S^*\rangle$ describes two stories (where S and S^* are partial states), then they are isomorphic if there are bijections $\pi_0 : I \to I'$ and $\pi_1 : T \to T'$ such that

1. π_1 is an isomorphism of trees,
2. $\pi_0(\mathbf{E}) = \mathbf{E}$,
3. $\mu^*(\pi_1(x)) = \pi_0(\mu(x))$, and
4. $S^*(\pi_1(x), \pi_0(\vec{\mathbf{P}}))(\pi_0(\mathbf{P})) = (\pi_1(t), \pi_1(t'))$ if and only if $S(x, \vec{\mathbf{P}})(\mathbf{P}) = (t, t')$ (where $\pi_0(\vec{\mathbf{P}})$ is the obvious extension of π_0 to finite sequences of elements of I).

In § 3.4, we'll discuss whether this notion of isomorphism can encapsulate the informal notion of identity of stories.

2.4 Building blocks of stories

While working with the actual stories, we identified a number of fundamental building blocks that recur in the investigated stories and that can describe all of the stories under discussion. For our reconstruction of the stories, we need eight building blocks (not including a special case in one of the stories, discussed in detail in § 4).

The trivial building blocks are just events or actions that happen with no reasoning at all (described in Figure 1); these could be called doxastic blocks of level -1. We denote them by Ev if it is an event, and by $\mathsf{Act}(\mathbf{P})$ if it is an action by player $\mathbf{P}$. Typical examples are random events or actions where agents just follow their whim without deliberation. Note that being represented by a building block of level -1 does not mean that the *discourse* of the story shows no deliberation: in fact, even in our investigated stories we find examples of agents discussing whether they should follow their beliefs (i.e., perform a higher level action) or not, and finally decide to perform the action without taking their beliefs into account. These would still be formalized as blocks of level -1.

The next level of basic building blocks are those that have reasoning based on beliefs, but not require any theory of mind at all, i.e., building blocks of level 0. The two fundamental building blocks here are *expected event* ($\mathsf{ExEv}(\mathbf{P})$) and *unexpected event* ($\mathsf{UnEv}(\mathbf{P})$), as described in Figure 2.

Moving beyond zeroth order theory of mind, we now proceed to building blocks that require beliefs about beliefs. There are two such building blocks used in our stories, *Unexpected Action* ($\mathsf{UnAc}(\mathbf{P}, \mathbf{Q})$) and *Collaboration gone wrong* ($\mathsf{CoGW}(\mathbf{P}, \mathbf{Q})$) whose structure we give in Figures 3 and 4.

$$\begin{tikzcd}[row sep=small] & x_0 \\ \boxed{v_0 \mid \mathbf{X}} \arrow[ur] \arrow[r] & x_1 \end{tikzcd}$$

$$S(v_0, \varnothing)(\mathbf{X}) = (x_1, x_0)$$

Fig. 1. The basic building blocks Ev (in the case that $\mathbf{X} = \mathbf{E}$) and $\mathsf{Act}(\mathbf{X})$ of *Event* and *Action*.

$$\begin{tikzcd}[row sep=small] & t_0 & t_1 \\ \boxed{v_0 \mid \mathbf{P}} \arrow[ur] \arrow[r] & \boxed{v_1 \mid \mathbf{E}} \arrow[ur] \arrow[r] & x \end{tikzcd}$$

$\mathsf{ExEv}(\mathbf{P})$: $S(v_0, \varnothing)(\mathbf{P}) = (t_1, t_0)$; $S(v_0, \mathbf{P})(\mathbf{E}) = (t_1, x)$; $S(v_1, \varnothing)(\mathbf{E}) = (t_1, x)$
$\mathsf{UnEv}(\mathbf{P})$: $S(v_0, \varnothing)(\mathbf{P}) = (t_1, t_0)$; $S(v_0, \mathbf{P})(\mathbf{E}) = (t_1, x)$; $S(v_1, \varnothing)(\mathbf{E}) = (x, t_1)$

Fig. 2. The basic building blocks $\mathsf{ExEv}(\mathbf{P})$ and $\mathsf{UnEv}(\mathbf{P})$ of *Expected Event* and *Unexpected Event*.

$$\begin{tikzcd}[row sep=small] & t_0 & t_1 \\ \boxed{v_0 \mid \mathbf{P}} \arrow[ur] \arrow[r] & \boxed{v_1 \mid \mathbf{Q}} \arrow[ur] \arrow[r] & x \end{tikzcd}$$

$$S(v_0, \varnothing)(\mathbf{P}) = (t_1, t_0);\ S(v_0, \mathbf{P})(\mathbf{Q}) = (t_1, x);\ S(v_1, \varnothing)(\mathbf{Q}) = (t_1, x)$$

Fig. 3. The basic building block $\mathsf{UnAc}(\mathbf{P}, \mathbf{Q})$ of *Unexpected Action*.

There is an obvious analogue of $\mathsf{ExEv}(\mathbf{P})$ at this level that would be called *Expected Action*, but as it did not occur in the stories we analyzed, we do not give its formal structure. Similarly, there is a natural dual for successful collaborations. Note that $\mathsf{UnEv}(\mathbf{P})$ is structurally the special case of $\mathsf{UnAc}(\mathbf{P}, \mathbf{E})$, but we decided to keep them separate in order to stress the difference in the theory of mind needed to deal with the building blocks.

$$\begin{tikzcd}[row sep=small] & t_0 & t_1 & t_2 \\ \boxed{v_0 \mid \mathbf{P}} \arrow[ur] \arrow[r] & \boxed{v_1 \mid \mathbf{Q}} \arrow[ur] \arrow[r] & \boxed{v_2 \mid \mathbf{E}} \arrow[ur] \arrow[r] & x \end{tikzcd}$$

$$S(v_0, \varnothing)(\mathbf{P}) = (t_2, t_0);\ S(v_0, \mathbf{P})(\mathbf{Q}) = (t_2, t_1);\ S(v_1, \varnothing)(\mathbf{Q}) = (t_2, t_1)$$
$$S(v_0, \mathbf{P})(\mathbf{E}) = (t_2, x);\ S(v_0, \mathbf{PQ})(\mathbf{E}) = (t_2, x);\ S(v_1, \mathbf{Q})(\mathbf{E}) = (t_2, x)$$
$$S(v_2, \varnothing)(\mathbf{E}) = (x, t_2)$$

Fig. 4. The basic building block $\mathsf{CoGW}(\mathbf{P}, \mathbf{Q})$ of *Collaboration gone wrong*.

Finally, we move to the building blocks that use second order beliefs. In our stories, there are only three such building blocks. One of them (in the story *The corrupt judge*) is slightly more complicated due to a component of incomplete information in the story. This will be discussed in more detail in § 4. The other two building blocks are *Betrayal* ($\mathsf{Betr}(\mathbf{P},\mathbf{Q})$) and *Unsuccessful Collaboration with a Third* ($\mathsf{UnCT}(\mathbf{P},\mathbf{Q},\mathbf{R})$) (given in Figures 5 and 6). Again, as with $\mathsf{UnAc}(\mathbf{P},\mathbf{Q})$ and $\mathsf{UnEv}(\mathbf{P}) = \mathsf{UnAc}(\mathbf{P},\mathbf{E})$, we see that *Collaboration gone wrong* is the special case of *Unsuccessful Collaboration with a Third* where the “third” is an event.

$$\begin{tikzcd}
 & t_0 & & t_1 & & t_2 \\
\boxed{v_0 \mid \mathbf{P}} \arrow[ur] \arrow[rr] & & \boxed{v_1 \mid \mathbf{Q}} \arrow[ur] \arrow[rr] & & \boxed{v_2 \mid \mathbf{P}} \arrow[ur] \arrow[r] & x
\end{tikzcd}$$

$$S(v_0,\varnothing)(\mathbf{P}) = (x, t_0);\ S(v_0,\mathbf{P})(\mathbf{Q}) = (t_2, t_1);\ S(v_1,\varnothing)(\mathbf{Q}) = (t_2, t_1)$$
$$S(v_0,\mathbf{PQ})(\mathbf{P}) = (t_2, x);\ S(v_1,\mathbf{Q})(\mathbf{P}) = (t_2, x);\ S(v_2,\varnothing)(\mathbf{P}) = (x, t_2)$$

Fig. 5. The basic building block $\mathsf{Betr}(\mathbf{P},\mathbf{Q})$ of *Betrayal*.

$$\begin{tikzcd}
 & t_0 & & t_1 & & t_2 \\
\boxed{v_0 \mid \mathbf{P}} \arrow[ur] \arrow[rr] & & \boxed{v_1 \mid \mathbf{Q}} \arrow[ur] \arrow[rr] & & \boxed{v_2 \mid \mathbf{R}} \arrow[ur] \arrow[r] & x
\end{tikzcd}$$

$$S(v_0,\varnothing)(\mathbf{P}) = (t_2, t_0);\ S(v_0,\mathbf{P})(\mathbf{Q}) = (t_2, t_1);\ S(v_1,\varnothing)(\mathbf{Q}) = (t_2, t_1)$$
$$S(v_0,\mathbf{P})(\mathbf{R}) = (t_2, x);\ S(v_0,\mathbf{PQ})(\mathbf{R}) = (t_2, x);\ S(v_1,\mathbf{Q})(\mathbf{R}) = (t_2, x)$$
$$S(v_2,\varnothing)(\mathbf{R}) = (x, t_2)$$

Fig. 6. The basic building block $\mathsf{UnCT}(\mathbf{P},\mathbf{Q},\mathbf{R})$ of *Unsuccessful Collaboration with a Third*.

These building blocks can be stacked. We used the symbol x in our building blocks to indicate that this could either be a terminal node (at the end of the story) or a non-terminal node which would now become the top node of the next stack. If the last node of a building block is controlled by an agent, then the doxastic structure of the building blocks overlaps, as the first node of the second block becomes the last node of the first block. In Figure 7, we can see the concatenation of five *Unexpected Actions* and one *Expected Event*.

3 Methodological issues

When we formalize a story that is independently given to us in natural language (possible with pictorial elements; in our case the crime investigation cases from episodes of *CSI: Crime Scene Investigation*™), we are given both *story* and *discourse* (in narratological terms, see Footnote 1). Separating one from the other is a difficult task and the formalizing subject has to make a number of modelling decisions. These issues are intensified by our choice to formalize the stories in our very parsimonious system described in §2 that takes beliefs about preferences and their iterations as the only driving force of actions and is working with an underlying model of perfect information games.

$$\begin{tikzcd}
 & t_0 & t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \\
\boxed{v_0 \mid \mathbf{B}} \arrow[r] \arrow[ur] & \boxed{v_1 \mid \mathbf{W}} \arrow[r] \arrow[ur] & \boxed{v_2 \mid \mathbf{B}} \arrow[r] \arrow[ur] & \boxed{v_3 \mid \mathbf{W}} \arrow[r] \arrow[ur] & \boxed{v_4 \mid \mathbf{C}} \arrow[r] \arrow[ur] & \boxed{v_5 \mid \mathbf{H}} \arrow[r] \arrow[ur] & \boxed{v_6 \mid \mathbf{E}} \arrow[r] \arrow[ur] & t_7
\end{tikzcd}$$

$$\begin{gathered}
S(v_0,\mathbf{B})(\mathbf{W}) = (t_1, v_2);\ S(v_0,\varnothing)(\mathbf{B}) = (t_1, t_0);\ S(v_1,\varnothing)(\mathbf{W}) = (t_2, t_1) \\
S(v_1,\mathbf{W})(\mathbf{B}) = (t_2, v_3);\ S(v_1,\varnothing)(\mathbf{W}) = (t_2, t_1);\ S(v_2,\varnothing)(\mathbf{B}) = (t_3, t_2) \\
S(v_2,\mathbf{B})(\mathbf{W}) = (t_3, v_4);\ S(v_2,\varnothing)(\mathbf{B}) = (t_3, t_2);\ S(v_3,\varnothing)(\mathbf{W}) = (t_4, t_3) \\
S(v_3,\mathbf{W})(\mathbf{C}) = (t_4, v_5);\ S(v_3,\varnothing)(\mathbf{W}) = (t_4, t_3);\ S(v_4,\varnothing)(\mathbf{C}) = (t_5, t_4) \\
S(v_4,\mathbf{C})(\mathbf{H}) = (t_5, v_6);\ S(v_4,\varnothing)(\mathbf{C}) = (t_5, t_4);\ S(v_5,\varnothing)(\mathbf{H}) = (t_7, t_5) \\
S(v_5,\mathbf{H})(\mathbf{E}) = (t_7, t_6);\ S(v_4,\varnothing)(\mathbf{E}) = (t_7, t_6)
\end{gathered}$$

Fig. 7. The concatenation of $\mathsf{UnAc}(\mathbf{B},\mathbf{W})$, $\mathsf{UnAc}(\mathbf{W},\mathbf{B})$, $\mathsf{UnAc}(\mathbf{B},\mathbf{W})$, $\mathsf{UnAc}(\mathbf{W},\mathbf{C})$, $\mathsf{UnAc}(\mathbf{C},\mathbf{H})$, and $\mathsf{ExEv}(\mathbf{H})$. Incidentally, this is the formalization of the story of *The death of Holly Gribbs*, one of the stories discussed in § 4.

In general, we have both too much information (as there are parts of the *discourse* that we want to ignore in the formalization in order to identify the doxastically relevant sequence of actions and events) and too little (as the writers of the story may have left parts of the reasoning processes of the agents to our imagination).

In this section, we discuss a number of methodological difficulties that arise from this issue.

3.1 The sequence of events

The narrative of a TV crime story rarely proceeds chronologically. Often, it starts when the corpse is found, and then proceeds to tell the story of the detectives unearthing the sequence of events that led to the murder. Sometimes, we see scenes of the past in flashbacks, sometimes, they are being reported by agents.

We consider all this part of the *discourse* of the story and will build our structures of actions and events in chronological order. Note that one consequence of this is that our models do not take into account the beliefs of the audience, and thus cannot formally model surprise of the audience coming from the order of narration. However, since this type of surprise is often connected to the fact that the sequence of events resembles an imperfect information game (something that is discussed in more detail in § 3.2), some expansion of the formal model would be necessary in order to adequate describe this.

3.2 Imperfect or incomplete information

A lot of the suspense and enjoyment in crime stories comes from the fact that the audience (and the detectives) do not know who committed the crime. As a consequence, the most natural way to model crime stories would be by imperfect information games or incomplete information games. Our formal model described in § 2 is purely based on a perfect information game model.

Obviously, this leads to a situation where a number of interesting aspects about beliefs prevalent in the stories (namely, beliefs about who is the murderer) cannot be incorporated into our formal models. We see this as a major task for the future to develop a version of our formal model that incorporates some aspects of imperfect or incomplete information but retains its fundamental simplicity (as discussed in § 5.2).

In many cases, imperfect or incomplete information can be mimicked in our system by event nodes. Let us give a simple examples:

> **Example.** *Detective Miller thinks that Jeff is Anne's murderer while, in fact, it is Peter. Miller believes that Jeff will show up during the night in Anne's apartment to destroy evidence and thus hides behind a shower curtain to surprise Jeff. However, Peter shows up to destroy the evidence, and is arrested.*

The natural formalization would be an imperfect or incomplete information games, but the structure given in Figure 8 can be used to formalize the story with **M** representing Miller, **J** Jeff, and **P** Peter. The event node v_1 should be read as "Peter turns out to be Anne's murderer". Nodes t_1 and t_3 are "Jeff (Peter) is the murderer, returns to the apartment and is caught", respectively; nodes t_2 and t_4 are "Jeff (Peter) is the murderer and does not return to the apartment".

We let $S(v_0, \mathbf{M})(\mathbf{E}) = (v_3, v_2)$ (i.e., Miller believes that Jeff will turn out to be the murderer), $S(v_3, \mathbf{M})(\mathbf{J}) = t_3$, $S(v_1, \varnothing)(\mathbf{E}) = (v_2, v_3)$ (i.e., Peter is the actual murderer), and $S(v_2, \varnothing)(\mathbf{P}) = (t_1, t_2)$ (i.e., Peter in fact plans to return to the apartment).

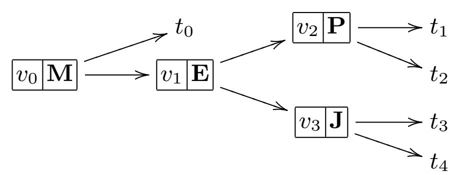

Fig. 8. Mimicking imperfect information by an event node v_1 representing "Peter turns out to be the murderer".

While this structure adequately describes the motivation of Miller and his surprise about catching someone who was not the suspect, it is unable to motivate why Peter chooses to go to the apartment.

However, we found that for the chosen stories from the series *CSI: Crime Scene Investigation*™, the impact on the adequacy of our formalizations was relatively minor. One of the reasons is that "strictly go by the evidence" is one of the often repeated creeds of the CSI members, prohibiting the actors from letting beliefs influence their actions.[2]

[2] A consequence of this is also that the investigators play only a minor rôle in our formalizations, often occurring in event nodes, and rarely making any decisions.

3.3 Not enough information

As mentioned in § 2.3, we will often not have enough information to give the full state, but only enough of the state that allows us to formally reconstruct the sequence of events and actions. In general, this is not a problem, but sometimes, the narrative is ambiguous on what happened or why it happened, and we are not even able to reconstruct the formal structure without any doubts.

We can give an example from the stories investigated in § 4: In the story *Pledging gone wrong*, we see in a brief flashback scene that Kyle murders James. There is a cut, and after that we see that Matt enters, and Kyle and Matt discuss what to do. The whole scene lasts but a few seconds, and the narrative does not give any clue whether Kyle was expecting Matt to enter or not. There are various different ways to formalize this brief sequence of events as described in Figure 9. In option (a), we consider Kyle's action almost as a joint action: he is murdering James under the (correct and never discussed) assumption that Matt will help him to cover this up. In option (b), we allow Matt to consider not helping Kyle, and then have to model Kyle as correctly assuming that Matt will help him, i.e., $S(v_1, \mathbf{K})(\mathbf{M}) = (x, t_1)$ and $S(v_1, \varnothing)(\mathbf{M}) = (x, t_1)$. In option (c), we now model the entering of Matt after the murder as an event and have to decide whether Kyle expected that this happens or not. One could take the casual tone of Kyle when Matt enters as an indication of lack of surprise, and therefore choose $S(v_1, \mathbf{K})(\mathbf{E}) = (v_2, t_1)$.

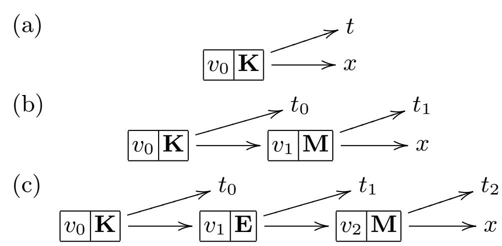

Fig. 9. Three different formalizations of the interaction between Kyle and Matt in the story *Pledging gone wrong*.

Which of the three options is correct? We believe that there is no good answer that does not take into account the story as a whole. In this particular case (see § 4), we decided to go with option (b), as Matt's decision is explicitly relevant in the last scenes of the story when Matt decides to tell the truth. We therefore decided that having a decision node for Matt represents the character of the story most appropriately. It is unlikely that modellings decision like this can always be uncontroversial. The problem of judging what is the natural formalization from the narrative is exemplified once more in § 3.4.

3.4 Relevant information

In § 3.3 we have seen that the narrative sometimes does not allow us to uncontroversially choose the formalization. The dual problem to this is that the *discourse* is often much richer than the structure necessitates. In particular, there is information that may not be relevant, but could be included in the story.

In the following let us give an example for this. Consider the following three examples:

Example 1. *John and Sue are a happily married couple when John's old friend, Peter, suddenly shows up after no contact for seven years, inviting himself for dinner. Peter asks John for a large amount of money without giving any reasons. Sue had always disliked Peter, and after Peter had left, Sue urged her husband not to give him any money. After a long discussion, Peter sighs and agrees to Sue's request. The couple goes to bed, but after Sue is sound asleep, John sneaks into the living room, gives Peter a call and promises to pay. After two weeks, Sue finds out that a large amount of money is missing from their joint bank account.*

Example 2. *... The couple goes to bed, but after Sue is sound asleep, John sneaks into the living room, gives Peter a call and promises to pay. Peter is honestly surprised, as he had not expected this after the rather icy atmosphere at the dinner table. After two weeks, ...*

Example 3. *... John sneaks into the living room, and gives Peter a call, intending to give him the money. However, John did not know how deep in trouble Peter was. After Peter noticed the icy atmosphere at the dinner table, he had taken the elevator to the rooftop of John's apartment building. There, he takes John's call, says "Good bye, John, you were always a good friend", and jumps, before John can tell him that he'll give him the money. John shouts "I'll give you the money" into the phone, but it is too late. When he turns around, Sue is standing behind him.*

$$\begin{tikzcd} & t_0 \\ \boxed{v_0 \,|\, \mathbf{J}} \arrow[ur] \arrow[r] & t_1 \end{tikzcd}$$

Fig. 10. The tree diagram for all three example stories about John, Sue and Peter.

The tree structure of all of these stories is the same, viz. the one depicted in Figure 10. Only the partial states differ slightly. In Example 1, we have $S(v_0, \mathbf{S})(\mathbf{J}) = (t_0, t_1)$ and $S(v_0, \varnothing)(\mathbf{J}) = (t_1, t_0)$ which explains Sue's surprise. In Examples 2 and 3, we have in addition $S(v_0, \mathbf{P})(\mathbf{J}) = (t_0, t_1)$ representing Peter's belief in both stories that John will not give him the money.

Structurally, Examples 2 and 3 are isomorphic in the sense of § 2.3 and slightly different from Example 1. However, we are sure that most readers will agree that Examples 1 and 2 are closer to each other than to Example 3. This difference does not lie in the event and action structure of the stories, but in the *discourse*. In Example 3, Peter's disbelief in John giving him the money intensifies the emotional difference between the terminal nodes t_0 and t_1, and thus creates a different feeling. As the modeller, we would have to make the decision of whether we include $S(v_0, \mathbf{P})(\mathbf{J}) = (t_0, t_1)$ in the formalization of Example 2: Is the brief mention of Peter's surprise worth being included in the formal model?

Difficulties like this are very much related to the research on analogies. Starting with Rattermann and Gentner's "Karla the hawk" stories [12], cognitive scientists have investigated the skill of finding structural analogies. The discussed issue is related to the question of what to do with "nonalignable objects" in analogical reasoning [15].[3]

4 The ten stories formalized

In this section, we shall give the formal structure of ten stories that form the first four episodes of season one of the drama series *CSI: Crime Scene Investigation*™.[4] These four episodes contain ten stories which we list as follows:

1. *The death of Holly Gribbs* (episodes 1 and 2; agents Jim Brass, **B**, Warrick Brown, **W**, Jerrod Cooper, **C**, Holly Gribbs, **H**).
2. *Paul Milander* (episodes 1 and 8; agent Paul Milander, **M**)
3. *The killed house guest* (episode 1; agents Jimmy, **J**, Husband, **H**)
4. *Trick roll* (episode 1; agents victim, **V**, Kristy Hopkins, **K**)
5. *The corrupt judge* (episodes 1, 2 and 4; agents Warrick Brown, **W**, Judge Cohen, **J**)
6. *Winning a fortune* (episode 2; agents Jamie Smith, **J**, Ted Sallanger, **T**)
7. *Faked kidnapping* (episode 3; agents Chip Rundle, **C**, Laura Garris, **L**, the CSI unit, **U**)
8. *Hit and run* (episode 3; agents Charles Moore, **C**, James Moore, **J**)
9. *The severed leg* (episode 4; agents Catherine Willows, **C**, Winston Barger, **W**)
10. *Pledging gone wrong* (episode 4; agents James Johnson, **J**, Jill Wentworth, **W**, Kyle Travis, **K**, Matt Daniels, **M**)

The full formalizations are given in the appendix. Here, we shall reconstruct all ten stories in terms of the basic building blocks given in § 2.4. We write ";" for the concatenation of basic building blocks as defined above.

Two of our stories do not even contain first-order beliefs. These are the stories of *Paul Milander*, formalized as $\mathsf{ExEv}(\mathbf{M}); \mathsf{Ac}(\mathbf{M})$ and *Hit and run*, formalized as $\mathsf{UnEv}(\mathbf{J}); \mathsf{UnEv}(\mathbf{C}); \mathsf{UnEv}(\mathbf{C})$. The story *Paul Milander* (about a forensic expert who uses his knowledge about the procedures of the CSI unit in order to make

[3] Note that our problem of finding the right protocol for formalizing a story given in natural language is very much related to the attempts to formalize detection of analogies (cf. [6, p. 791–792] for an overview of existing models for this task). Also related is *Formal Concept Analysis* (FCA; cf. [3]), a technique for extracting an ontology from a collection of objects. None of these methods work with data given in ordinary language such as our stories.

[4] Cf. [1]. Episode 1, entitled "Pilot", was written by Anthony E. Zuiker and directed by Danny Cannon; Episode 2, entitled "Cool Change" was written by Anthony E. Zuiker and directed by Michael W. Watkins; Episode 3, entitled "Crate 'n Burial", was written by Ann Donahue and directed by Danny Cannon; Episode 4, entitled "Pledging Mr. Johnson", was written by Josh Berman and Anthony E. Zuiker and directed by Richard J. Lewis. For the formalization of the story *Paul Milander*, we had to consider part of Episode 8, entitled "Anonymous", written by Eli Talbert and Anthony E. Zuiker, and directed by Danny Cannon.

a fool out of the police) is an example for a story that provides suspense and surprise for the audience even though there are no mistaken beliefs relevant for the decisions of agents.[5]

Half of our stories involves basic building blocks of at most level 1. These five are listed and formalized in Figure 11. The remaining three stories have blocks of level 2. These are *Faked kidnapping*, formalized as $\mathsf{Betr}(\mathbf{C},\mathbf{L})$; $\mathsf{UnEv}(\mathbf{C})$; $\mathsf{UnEv}(\mathbf{L})$; $\mathsf{Act}(\mathbf{U})$; Ev and *Pledging gone wrong*, formalized as $\mathsf{UnCT}(\mathbf{J},\mathbf{W})$; $\mathsf{CoGW}(\mathbf{K},\mathbf{M})$; $\mathbf{UnEv}(\mathbf{K})$; $\mathsf{Act}(\mathbf{M})$.

The severed leg: $\mathsf{UnAc}(\mathbf{C},\mathbf{W})$.

Winning a fortune: $\mathsf{UnAc}(\mathbf{J},\mathbf{T})$; $\mathsf{UnEv}(\mathbf{T})$; $\mathsf{UnAc}(\mathbf{T},\mathbf{J})$; $\mathsf{UnEv}(\mathbf{J})$.

Trick roll: $\mathsf{UnAc}(\mathbf{V},\mathbf{K})$; $\mathsf{UnEv}(\mathbf{K})$.

The death of Holly Gribbs: $\mathsf{UnAc}(\mathbf{B},\mathbf{W})$; $\mathsf{UnAc}(\mathbf{W},\mathbf{B})$; $\mathsf{UnAc}(\mathbf{B},\mathbf{W})$; $\mathsf{UnAc}(\mathbf{W},\mathbf{C})$; $\mathsf{UnAc}(\mathbf{C},\mathbf{H})$; $\mathsf{ExEv}(\mathbf{H})$.

The killed house guest: $\mathsf{UnAc}(\mathbf{J},\mathbf{H})$; $\mathsf{UnEv}(\mathbf{H})$; $\mathsf{UnEv}(\mathbf{H})$; $\mathsf{UnEv}(\mathbf{H})$.

Fig. 11. Formalizations in terms of basic building blocks of all stories which have blocks of level 1 but no blocks of level 2.

The only story that is not so easy to describe in terms of our building blocks is *The corrupt judge*. It ends with a scene in which the judge threatens Warrick who seems to have the chance to agree to the corrupt judge's demand or not (in which latter case he'd suffer the consequences). So far, this could have been described as a standard building block of *Threat*.[6] However, Warrick chooses to pretend that he agrees and to frame the judge. This is an option that the judge has not considered, and therefore, we formalized it as an option that the judge considers very low on Warrick's preference order. We give the full formalization in Figure 12; using the final three non-terminal nodes as a building block $\mathsf{special}$, we can write this in building blocks as $\mathsf{UnEv}(\mathbf{W})$; $\mathsf{UnAc}(\mathbf{W},\mathbf{J})$; $\mathsf{special}$.

5 General conclusion and related work

In § 4, we have seen that ten crime stories commercially produced for TV entertainment show a lot of recurring structures. A total number of nine basic building blocks (eight from § 2.4 and the special one in § 4) is able to describe the event and action structure of all of the ten stories; most of the building blocks involve only zeroth- and first-order beliefs, and there are only three instances of genuine

[5] Note that the mistaken belief of the CSI agents that Milander is innocent at the end of Episode 1 is of course relevant for the narrative, but not modelled in our system as we are not modelling beliefs about facts, only beliefs about preferences.

[6] Not defined in § 2.4, but easily described by the tree of $\mathsf{Betr}(\mathbf{P},\mathbf{Q})$ and the partial state in which $\mathbf{P}$ believes that $\mathbf{Q}$ believes that he will choose t_2; $\mathbf{P}$ prefers t_1 and believes that $\mathbf{Q}$ prefers t_1 over t_2.

$$\begin{tikzcd}[column sep=small]
 & t_0 & t_1 & t_2 & t_3 & t_4 & t_5 \\
\boxed{v_0 \mid \mathbf{W}} \arrow[r] \arrow[ur] & \boxed{v_1 \mid \mathbf{E}} \arrow[r] \arrow[ur] & \boxed{v_2 \mid \mathbf{W}} \arrow[r] \arrow[ur] & \boxed{v_3 \mid \mathbf{J}} \arrow[r] \arrow[ur] & \boxed{v_4 \mid \mathbf{W}} \arrow[r] \arrow[ur] \arrow[dr] & \boxed{v_5 \mid \mathbf{J}} \arrow[r] \arrow[ur] & t_6 \\
 & & & & & t_7 &
\end{tikzcd}$$

$$S(v_0, \mathbf{W})(\mathbf{E}) = (t_1, v_2);\ S(v_0, \varnothing)(\mathbf{W}) = (t_1, t_0);\ S(v_1, \varnothing)(\mathbf{E}) = (v_2, t_1)$$
$$S(v_2, \mathbf{W})(\mathbf{J}) = (t_3, v_4);\ S(v_2, \varnothing)(\mathbf{W}) = (t_3, v_4);\ S(v_3, \varnothing)(\mathbf{J}) = (t_4, t_3)$$
$$S(v_5, \mathbf{JW})(\mathbf{J}) = (t_5, t_6);\ S(v_5, \mathbf{J})(\mathbf{W}) = (t_4, t_5, t_7);\ S(v_5, \varnothing)(\mathbf{W}) = (t_7, t_4, v_5)$$

Fig. 12. The formalization of *The corrupt judge*

second-order beliefs. Not surprisingly, we see that second-order beliefs typically show up in those parts of the crime stories that do not directly related to solving the crime, but to interpersonal interaction between the agents. While mistaken belief is a relatively common phenomenon, changing preferences and beliefs did not occur in any of the formalized stories.

5.1 The magical number seven and restrictions on orders of theory of mind

There are obvious parallels to the findings in our paper to various other fields. George Miller's "Magical Number Seven, Plus or Minus Two" should come to mind; in [8], he surveys a range of results from the psychology of perception indicating that there is a very concrete upper bound to our power to quantify differences in perception. His examples include length of lines, pitch of tone, hues of colours, size of geometrical objects and many more. Similar limitations have been observed in the understanding of nested relative clauses [9].

Of course, the link between the "Magical Number Seven" and our findings is the experimental research in orders of theory of mind. Both in experimental game theory (as a reaction to the fact that human beings do not seem to follow the mathematical predictions of game theory) and in psychology and cognitive science, researchers have investigated the limits of the capacity of human cognition to reason about iterated beliefs.

In game theory, this led to Herbert Simon's notion of "Bounded Rationality". Stahl and Wilson have investigated levels of belief in games [14] and identified "most participants' behavior ... as being observationally equivalent with one specific type" from their list of five types: 'level–0', 'level–1', 'level–2', 'naïve Nash', and 'worldly'. There is evidence from evolutionary game theory [13] that even in a population with players of arbitrary depth of theories of mind, the simple types will never be driven out of the population (this argument is the foundation of the decision of Stahl and Wilson to restrict their attention to the above mentioned five types as there is little advantage to move beyond level–2 [14, p. 220]).

In psychology, the study of the development and use of second-order beliefs started with Perner and Wimmer [11] and was continued in experiments by Hedden and Zhang [4], Keysar, Lin, and Barr [5], Verbrugge and Mol [17], and Flobbe, Verbrugge, Hendriks, and Krämer [2], to name but a few. The experimental evidence suggests that many adults only apply first-order theory of mind (even this is not always done without errors, cf. [5, Experiment 1]) and few progress to second-order theory of mind and beyond. Our results are perfectly in line with this, showing that stories in commercially produced crime stories rarely use second-order beliefs.

5.2 Future work

In § 3, we have discussed the methodological problems encountered while formalizing the stories. Some of the problems were created by our particular choice of the formal model. A formal model including some aspects of imperfect or incomplete information would have been able to deal much more easily with the issues discussed in § 3.2. This leads to the natural proposal to enhance our formal system by including these aspects; however, this will have to be done with caution in order to retain the simplicity of the system: there are many formal models that can powerfully deal with various aspects of communication and reasoning, but we do not want to jeopardize perspicuity and ease of use of our formal system.

One possible direction is the incorporation of ideas from Dynamic Epistemic Logic. This is briefly discussed in [7, § 5.3], and the work of van Ditmarsch and Labuschagne [16], written in direct connection with the discussions from cognitive science about reasoning about second-order beliefs points into the right direction.

Once a system has been developed that can capture many relevant aspects of stories, larger numbers of stories, also from different genres could be translated into this formal system in order to form a corpus for investigating various important and wide-ranging empirical questions.

1. Is there a bound on the number of building blocks that occur in natural stories?
2. Is there a correlation between type, genre or source and formal properties of its story?
3. Is there a correlation between the perception and emotional effect of a story and its formal structure?

Note that these empirical questions are methodologically very difficult: cognitive aspects of stories such as enjoyment, tension, quality are a property of the story as a whole, i.e., of *story* and *discourse*. For instance, if we want to test whether a story with relevant second-order beliefs is seen as more enjoyable than a story without, we need to make sure that superficial features of the *discourse* of the two stories presented to the test subjects did not affect their emotional states.

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Appendix: Formalizations of the ten stories.

1. *The death of Holly Gribbs* (episodes 1 and 2; agents Jim Brass, **B**, Warrick Brown, **W**, Jerrod Cooper, **C**, Holly Gribbs, **H**).
2. *Paul Milander* (episodes 1 and 8; agent Paul Milander, **M**)
3. *The killed house guest* (episode 1; agents Jimmy, **J**, Husband, **H**)
4. *Trick roll* (episode 1; agents victim, **V**, Kristy Hopkins, **K**)
5. *The corrupt judge* (episodes 1, 2 and 4; agents Warrick Brown, **W**, Judge Cohen, **J**)
6. *Winning a fortune* (episode 2; agents Jamie Smith, **J**, Ted Sallanger, **T**)
7. *Faked kidnapping* (episode 3; agents Chip Rundle, **C**, Laura Garris, **L**, the CSI unit, **U**)
8. *Hit and run* (episode 3; agents Charles Moore, **C**, James Moore, **J**)
9. *The severed leg* (episode 4; agents Catherine Willows, **C**, Winston Barger, **W**)
10. *Pledging gone wrong* (episode 4; agents James Johnson, **J**, Jill Wentworth, **W**, Kyle Travis, **K**, Matt Daniels, **M**)

$$\begin{tikzcd}[column sep=small]
 & t_0 & & t_1 & & t_2 & & t_3 & & t_4 & & t_5 & & t_6 & \\
\boxed{v_0 \mid \mathbf{B}} \arrow[ur] \arrow[rr] & & \boxed{v_1 \mid \mathbf{W}} \arrow[ur] \arrow[rr] & & \boxed{v_2 \mid \mathbf{B}} \arrow[ur] \arrow[rr] & & \boxed{v_3 \mid \mathbf{W}} \arrow[ur] \arrow[rr] & & \boxed{v_4 \mid \mathbf{C}} \arrow[ur] \arrow[rr] & & \boxed{v_5 \mid \mathbf{H}} \arrow[ur] \arrow[rr] & & \boxed{v_6 \mid \mathbf{E}} \arrow[ur] \arrow[rr] & & t_7
\end{tikzcd}$$

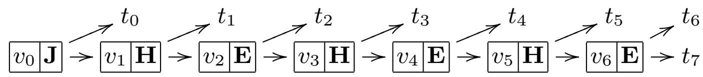
$$S(v_0, \mathbf{B})(\mathbf{W}) = (t_1, v_2);\ S(v_0, \varnothing)(\mathbf{B}) = (t_1, t_0);\ S(v_1, \varnothing)(\mathbf{W}) = (t_2, t_1)$$
$$S(v_1, \mathbf{W})(\mathbf{B}) = (t_2, v_3);\ S(v_1, \varnothing)(\mathbf{W}) = (t_2, t_1);\ S(v_2, \varnothing)(\mathbf{B}) = (t_3, t_2)$$
$$S(v_2, \mathbf{B})(\mathbf{W}) = (t_3, v_4);\ S(v_2, \varnothing)(\mathbf{B}) = (t_3, t_2);\ S(v_3, \varnothing)(\mathbf{W}) = (t_4, t_3)$$
$$S(v_3, \mathbf{W})(\mathbf{C}) = (t_4, v_5);\ S(v_3, \varnothing)(\mathbf{W}) = (t_4, t_3);\ S(v_4, \varnothing)(\mathbf{C}) = (t_5, t_4)$$
$$S(v_4, \mathbf{C})(\mathbf{H}) = (t_5, v_6);\ S(v_4, \varnothing)(\mathbf{C}) = (t_5, t_4);\ S(v_5, \varnothing)(\mathbf{H}) = (t_7, t_5)$$
$$S(v_5, \mathbf{H})(\mathbf{E}) = (t_7, t_6);\ S(v_4, \varnothing)(\mathbf{E}) = (t_7, t_6)$$

Fig. 13. The formalization of *The death of Holly Gribbs*

$$\begin{tikzcd}
 & t_0 & & t_1 & & t_2 \\
\boxed{v_0 \mid \mathbf{M}} \arrow[ur] \arrow[rr] & & \boxed{v_1 \mid \mathbf{E}} \arrow[ur] \arrow[rr] & & \boxed{v_2 \mid \mathbf{M}} \arrow[ur] \arrow[r] & t_3
\end{tikzcd}$$

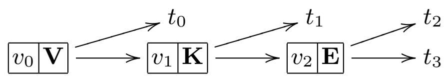
$$S(v_0, \mathbf{M})(\mathbf{E}) = (v_2, t_1);\ S(v_1, \varnothing)(\mathbf{E}) = (v_2, t_1);\ S(v_0, \varnothing)(\mathbf{M}) = (t_2, t_3, t_0, t_1)$$
$$S(v_2, \varnothing)(\mathbf{M}) = (t_2, t_3)$$

Fig. 14. The formalization of *Paul Milander*



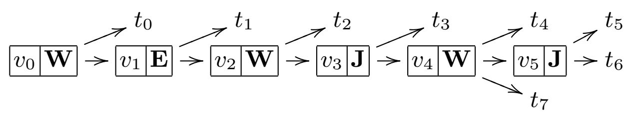
$$S(v_0, \mathbf{J})(\mathbf{H}) = (t_1, v_2);\ S(v_1, \varnothing)(\mathbf{H}) = (t_2, t_1);\ S(v_0, \varnothing)(\mathbf{J}) = (t_1, t_0)$$
$$S(v_1, \mathbf{H})(\mathbf{E}) = (t_2, v_3);\ S(v_2, \varnothing)(\mathbf{E}) = (v_3, t_2);\ S(v_3, \varnothing)(\mathbf{H}) = (t_4, t_3)$$
$$S(v_3, \mathbf{H})(\mathbf{E}) = (t_4, v_5);\ S(v_4, \varnothing)(\mathbf{E}) = (v_5, t_4);\ S(v_5, \varnothing)(\mathbf{H}) = (t_6, t_5)$$
$$S(v_5, \mathbf{H})(\mathbf{E}) = (t_6, t_7);\ S(v_6, \varnothing)(\mathbf{E}) = (t_7, t_6)$$

Fig. 15. The formalization of *The killed house guest*



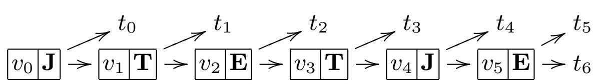
$$S(v_0, \mathbf{J})(\mathbf{K}) = (t_1, v_2);\ S(v_1, \varnothing)(\mathbf{K}) = (t_2, t_1);\ S(v_1, \mathbf{K})(\mathbf{E}) = (t_2, t_3)$$
$$S(v_2, \varnothing)(\mathbf{E}) = (t_3, t_2)$$

Fig. 16. The formalization of *Trick roll*



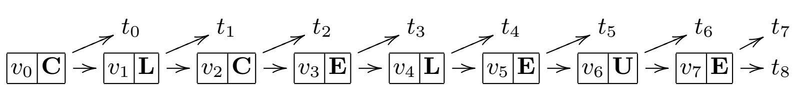
$$S(v_0, \mathbf{W})(\mathbf{E}) = (t_1, v_2);\ S(v_0, \varnothing)(\mathbf{W}) = (t_1, t_0);\ S(v_1, \varnothing)(\mathbf{E}) = (v_2, t_1)$$
$$S(v_2, \mathbf{W})(\mathbf{J}) = (t_3, v_4);\ S(v_2, \varnothing)(\mathbf{W}) = (t_3, v_4);\ S(v_3, \varnothing)(\mathbf{J}) = (t_4, t_3)$$
$$S(v_5, \mathbf{JW})(\mathbf{J}) = (t_5, t_6);\ S(v_5, \mathbf{J})(\mathbf{W}) = (t_4, t_5, t_7);\ S(v_5, \varnothing)(\mathbf{W}) = (t_7, t_4, v_5)$$

Fig. 17. The formalization of *The corrupt judge*



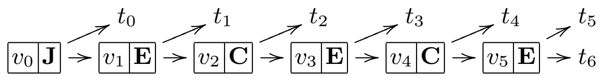
$$S(v_0, \mathbf{J})(\mathbf{T}) = (t_1, v_2);\ S(v_1, \varnothing)(\mathbf{T}) = (v_2, t_1);\ S(v_0, \varnothing)(\mathbf{J}) = (t_1, t_0)$$
$$S(v_2, \mathbf{T})(\mathbf{E}) = (t_2, v_3);\ S(v_1, \varnothing)(\mathbf{T}) = (t_2, t_1);\ S(v_2, \varnothing)(\mathbf{E}) = (v_3, t_2)$$
$$S(v_3, \mathbf{T})(\mathbf{J}) = (t_4, v_5);\ S(v_3, \varnothing)(\mathbf{T}) = (t_4, t_3);\ S(v_4, \varnothing)(\mathbf{J}) = (t_5, t_4)$$
$$S(v_4, \mathbf{J})(\mathbf{E}) = (t_5, t_4);\ S(v_4, \varnothing)(\mathbf{J}) = (t_5, t_4);\ S(v_5, \varnothing)(\mathbf{E}) = (t_4, t_5)$$

Fig. 18. The formalization of *Winning a fortune*



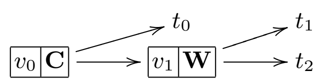
$$S(v_0, \varnothing)(\mathbf{C}) = (t_3, t_2, t_0, t_1);\ S(v_2, \varnothing)(\mathbf{C}) = (t_3, t_2);\ S(v_2, \mathbf{C})(\mathbf{E}) = (t_3, v_4)$$
$$S(v_0, \mathbf{C})(\mathbf{L}) = (t_2, t_1, v_3);\ S(v_0, \mathbf{CL})(\mathbf{C}) = (t_2, v_3);\ S(v_1, \mathbf{L})(\mathbf{C}) = (t_2, v_3)$$
$$S(v_1, \varnothing)(\mathbf{L}) = (t_2, t_1, v_3);\ S(v_3, \varnothing)(\mathbf{E}) = (v_4, t_3)$$
$$S(v_4, \mathbf{L})(\mathbf{E}) = (t_5, v_6);\ S(v_4, \varnothing)(\mathbf{L}) = (t_5, t_4);\ S(v_5, \varnothing)(\mathbf{E}) = (v_6, t_5)$$
$$S(v_6, \varnothing)(\mathbf{U}) = (v_7, t_6);\ S(v_7, \varnothing)(\mathbf{E}) = (t_7, t_8)$$

Fig. 19. The formalization of *Faked kidnappping*



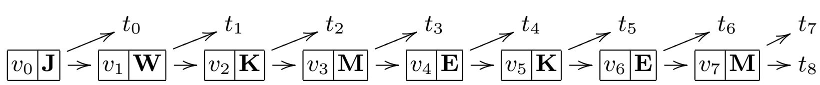
$$S(v_0, \mathbf{J})(\mathbf{E}) = (t_1, v_2);\ S(v_0, \varnothing)(\mathbf{J}) = (t_1, t_0);\ S(v_1, \varnothing)(\mathbf{E}) = (v_2, t_1)$$
$$S(v_2, \mathbf{C})(\mathbf{E}) = (t_3, v_4);\ S(v_2, \varnothing)(\mathbf{C}) = (t_3, t_2);\ S(v_3, \varnothing)(\mathbf{E}) = (v_4, t_3)$$
$$S(v_4, \mathbf{C})(\mathbf{E}) = (t_5, t_6);\ S(v_4, \varnothing)(\mathbf{C}) = (t_5, t_4);\ S(v_5, \varnothing)(\mathbf{E}) = (t_6, t_5)$$

Fig. 20. The formalization of *Hit and run*



$$S(v_0, \mathbf{C})(\mathbf{W}) = (t_1, t_2);\ S(v_0, \varnothing)(\mathbf{C}) = (t_1, t_0);\ S(v_1, \varnothing)(\mathbf{W}) = (t_2, t_1)$$

Fig. 21. The formalization of *The severed leg*



$$S(v_0, \mathbf{J})(\mathbf{W}) = (t_2, t_1);\ S(v_0, \mathbf{JW})(\mathbf{K}) = (t_2, v_3);\ S(v_0, \mathbf{J})(\mathbf{K}) = (t_2, v_3)$$
$$S(v_0, \varnothing)(\mathbf{J}) = (t_2, t_0, t_1);\ S(v_1, \varnothing)(\mathbf{W}) = (t_2, t_1)$$
$$S(v_2, \mathbf{K})(\mathbf{M}) = (t_4, t_3);\ S(v_2, \mathbf{K})(\mathbf{E}) = (t_4, v_5);\ S(v_2, \mathbf{KM})(\mathbf{E}) = (t_4, v_5)$$
$$S(v_2, \varnothing)(\mathbf{K}) = (t_4, t_2, t_3);\ S(v_3, \varnothing)(\mathbf{M}) = (t_4, t_3);\ S(v_4, \varnothing)(\mathbf{E}) = (v_5, t_4)$$
$$S(v_5, \mathbf{K})(\mathbf{E}) = (t_6, v_7);\ S(v_5, \varnothing)(\mathbf{L}) = (t_6, t_5);\ S(v_6, \varnothing)(\mathbf{E}) = (v_7, t_6)$$
$$S(v_7, \varnothing)(\mathbf{M}) = (t_7, t_8)$$

Fig. 22. The formalization of *Pledging gone wrong*