MERGING OBSERVATION AND ACCESS IN DYNAMIC LOGIC

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Abstract Rational agents base their actions on information from observation, inference, introspection, or other sources. But this information comes in different kinds, and it is usually handled by different logical mechanisms. We discuss how to integrate external 'updating information' and internal 'elucidating information' into one system of dynamic epistemic logic, by distinguishing two basic informational actions: 'bare seeing' versus 'conscious realization'.

1 Rational agency, logic, and the problem of information diversity

Logical dynamics in rational agency Was Gödel's diary kept in German or in English? I can fly to Princeton and check (if Google Earth does not zoom in that close these days), or try to infer the answer in comfort at home from the facts in John Dawson's biography, or failing all that, I might call some East Coast friend who knows the answer. Rational agents have a great variety of ways for getting new information, and they use it for a great variety of purposes. *Observation*, *inference*, and *communication* all seem equally respectable ways for agents to obtain information, and in the program of 'logical dynamics' (van Benthem 1996), they should indeed all be handled on a par. In my paper 'Tell it Like It Is' (*Journal of Peking University*, Humanities and Social Science Edition, No. 1, 2008, pp. 80 – 90), I describe the logical challenges which arise in this way, sketching how current dynamic logics of information update, belief revision, preference change, and strategic interaction all address aspects of this total behaviour of rational agents, which then need to be integrated. [1] In particular, since all these activities are information-driven, our logical systems need to be able to integrate the various kinds of information that are relevant to the above picture. But then, we have a problem of compatibility – and it is of long standing by now:

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[1] The further paper 'Logic, Rational Agency, and Intelligent Interaction', an invited lecture at the 13th International Congress of Logic, Methodology, and Philosophy of Science, Beijing 2007, *Proceedings* to appear with College Publications, London, lays out the program in greater detail.

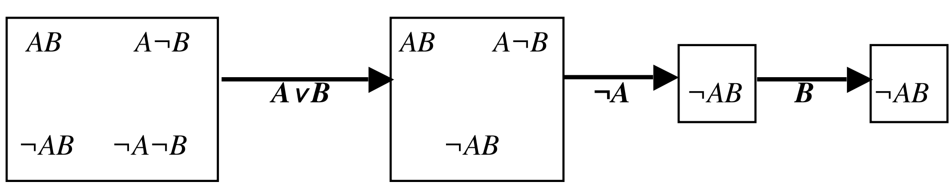
Information diversity in logic Of the three mentioned sources of information that can be used in action, two seem similar, viz. *observation* and *communication*. An observation is an answer to a question we pose to a special omniscient agent, viz. Nature, while conversely, an answer to a question may be seen as the observation of some fact – if we accept what the speaker says. Of course, there are some differences, too, but in principle, current *dynamic-epistemic logics* (cf. van Ditmarsch, van der Hoek & Kooi 2007) handle the information flow in observation and communication rather well. But the next stage of our program runs into a problem. Though entangled with the first two, the third basic process generating information, viz. *inference*, does not seem to have the same status!

The problem of inferential information As pointed out in the survey chapter van Benthem & Martinez 2008 in the forthcoming *Handbook of the Philosophy of Information*, there is no authoritative account of the 'information' that flows in inference, though it is clearly different from the information that flows through observation. Just think of the following folklore example (cf. van Benthem 1996).

Example Inference adds no semantic information.
Consider the propositionally valid inference [2]

from premises $A \vee B$ and $\neg A$ to conclusion B.

In the initial situation, an agent has no knowledge at all about A and B, and hence there are 4 options as to their truth and falsity. The premises then perform successive 'updates' on this information state: the first rules out *1* option, the second *2*, as in the following diagram:



But now we have a problem: the actual inference step at the end does not appear to do anything useful for the agent! For, updating with the statement B would not change the information state any more, and hence there is no change in information state. ♣

[2] This favourite from Greek Antiquity is also the engine of the currently popular *Sudoku* puzzles.

Something must have gone wrong here. Clearly, inferences *are* useful, and our human practice in solving puzzles shows that we intertwine inference and observation all the time. [3] It has even been claimed that all of science is one grand harmonious co-existence of two very similar sources: empirical experimentation and mathematical deduction. How to make sense of this in one coherent notion of logical information? This problem exists in many variants, and it has even been called the 'scandal of deduction'. The mentioned chapter van Benthem & Martinez 2008 discusses many attempted solutions in logic, but it also occurs in the philosophy of science with the question how deduction can be informative in the development of science, given that it excludes no empirical situations. There are also closely related problems in other fields, such as the problem of explaining in which sense *computation* is informative (Abramsky 2008). To make sense of fine-grained inferential dynamics, we need an account of what changes when we make valid inferences, and this must work together with successful 'elimination accounts' of observation-based update.

Information dynamics and syntax There are many approaches here. Some admit that single inference steps are trivial, but somehow, information emerges through accumulation of large numbers of small steps. But one can also analyze individual inferences themselves as information-changing events. An elegant example is the 'logic of proofs' of Artemov 1994, which keeps track of inference steps that make knowledge of a proposition explicit by building a proof term for it. Also in this line is the computational logic approach of d'Agostino & Floridi 2007 to extracting 'deductive information', or the modal logic of explicit proof steps of Jago 2006. In this paper, I will only need the minimal 'folklore' sense in which individual inferences increase syntactic information: they add new formulas to a current set Σ of formulas that we have already seen to be true. Thus,

> The above inference from the premises $A \vee B$ and $\neg A$ to conclusion B
> takes any set of formulas Σ containing $\{A \vee B, \neg A\}$ to the set $\Sigma \cup \{B\}$.

In what follows, I will give this viewpoint a general twist, making inference just a special case of various actions that elucidate the information given to us by different sources.

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[3] Van Benthem 1996, Chapter 1, points this out for the case of popular puzzles like 'Master Mind', which can be described in terms of update only, but in practice, involves crucial use of inferences.

Creating access to worlds Treating the syntactic view on par with the range picture of semantic information, one may view the set Σ of formulas available at a world w as an agent's *access* to that world. In particular, the above inference from $A \vee B$ and $\neg A$ to B, though adding no semantic information, did increase the agent's access to the final world by adding an explicit property B. More generally, external observations create implicit knowledge by eliminating worlds, while actions of inference increase internal access to the remaining worlds, thus upgrading implicit knowledge to explicit knowledge. Why is this access useful? Many cognitive activities involve computation on syntactic code, and hence the agent's access determines its potential for *inference*, but also other relevant activities, such as *model checking* to find truth values of formulas at worlds in the current model.

How can we make sense of these informal ideas? Technically, we will use the *two-level semantic-syntactic format* for information states proposed in van Benthem 2008A. This system works with pairs (w, Σ) consisting of worlds w plus a set of formulas Σ which partially describe w, merging ideas from dynamic-epistemic logic with the 'awareness logics' of Fagin, Halpern & Vardi 1990, which use worlds with sets of formulas, too. [4]

Implicit versus explicit 'seeing' In particular, such a two-level system allows at once for information update at both levels: worlds may be removed, but also, sentences may be added. In this setting, I will explore the idea that the fundamental distinction in the preceding discussion is not between 'observation' and 'inference', but rather between two types of observational event: (a) *'implicit observation'* of new true facts, versus (b) acts of *'explicit realization'* making our current implicit knowledge explicit. A related distinction in colloquial terms is this. The phrase "I see" can mean that I am really observing some empirical fact, or that something suddenly becomes clear to me. We will treat both types of action in the sections to come, and then develop some of their theory, including a complete combined dynamic logic of the two sorts of informational event. At the end, we return to the issue of inferential dynamics, and how this fits with the new proposals made here.

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[4] Related ideas are found with 'belief bases' in belief revision theory (Gärdenfors & Rott 2008).

But first, let us review the observation-oriented basics of dynamic-epistemic logic itself.

2 Dynamic logic of public observation

Here is a very brief summary of epistemic logic of knowledge and related attitudes, including its modern dynamic sense (Baltag, van Ditmarsch & Moss 2008). We will use this system to make sense of what we said informally about observation-based semantic information, and to detect some delicate phenomena that would not be clear informally. Moreover, the system will serve as a methodological model for the desired extensions to internal 'access dynamics' through acts of realization or inference.

Epistemic base language and range models The base language has operators $K_i\varphi$ for 'agent i knows that φ', interpreted over models $\boldsymbol{M} = (W, \{R_i\}_{i\in I}, V)$ where the R_i are epistemic accessibility relations among the worlds for the agents. [5] More precisely:

$$\boldsymbol{M}, s \models K_i\varphi \text{ iff } \boldsymbol{M}, t \models \varphi \text{ for all } t \text{ with } R_i st.$$

Details of this framework can be looked up in any standard text, including the standard complete axiom systems for validity. One famous axiom will be discussed below:

$$K(\varphi\to\psi) \to (K\varphi \to K\psi) \qquad \textit{Modal Distribution}$$

This has been considered the basis of 'logical omniscience', saying that agents know the logical consequences of what they know. We will return to the true role of this axiom in Section 3.4 below, as a 'licenser' for inferential actions. Right now, we only note that we will use equivalence relations in our models, validating the logic of 'multi-*S5*'.

Knowledge as information Please note that we are not using epistemic logic here as an account of the philosopher's notion of *knowledge*. As argued in van Benthem 2006A, the operator K_i should really be read as a more implicit notion of

"to the best of agent i's information",

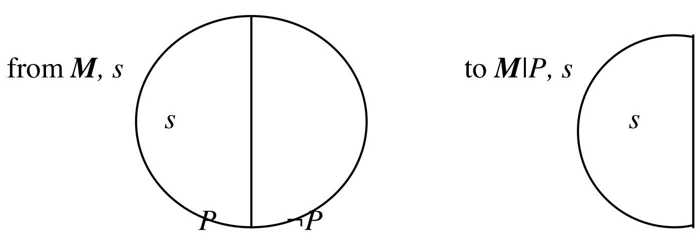
[5] 'Worlds' here can be as light as hands in a card game, or the possible states of a traffic light.

viewing the accessibility relations R_i as defining agents' current range of uncertainty, or their semantic information. Agents' implicit knowledge lies encoded in this semantic range, but it need not be directly accessible to them for explicit manipulation.

The semantic range picture also come with another common sense idea, viz. that receiving new information decreases the agent's current range of uncertainty, while the ideal information state is just the singleton set $\{w\}$ with w the actual world. Our next task is to make these informational events themselves an explicit part of the logic.

Note In what follows, we will restrict attention to *single agents*, suppressing indices. This is because the distinctions we want to make concern single agents in the first place.

Information dynamics: observation and communication For our purposes, it suffices to consider the so-called *logic of public announcements:* events $!P$ of new *hard information* which change irrevocably what I currently know. These can be linguistic communications from some perfectly reliable source, or public inter-subjective observations. Formally, such an event triggers a change in the current epistemic model $(\boldsymbol{M}, s)$ with actual world s. More specifically, $!P$ eliminates all worlds incompatible with P, thereby zooming in on the actual situation. Thus the current model $(\boldsymbol{M}, s)$ changes into its definable sub-model $(\boldsymbol{M}|P, s)$, whose domain is the set $\{t \in \boldsymbol{M} \mid \boldsymbol{M}, t \models P\}$. In a picture, one goes



Typically, truth values of epistemic formulas may change in such an update step: agents who did not know P now do after the event $!P$. This switching leads to subtle phenomena, but one can keep track of them in the following formalism.

Language and dynamic logic of public announcement The following states the basics:

Definition The language of *public announcement logic PAL* extends epistemic logic with action expressions denoting the preceding update steps: [6]

Formulas	P:	$p \mid \neg\phi \mid \phi \vee \psi \mid K_i\phi \mid C_G\phi \mid [A]\phi$
Action expressions	A:	$!P$

The fundamental semantic clause for the dynamic action modality is as follows:

$$\boldsymbol{M}, s \models [!P]\, \phi \quad \text{iff} \quad \text{if } \boldsymbol{M}, s \models P, \text{ then } \boldsymbol{M}|P, s \models \phi$$ ♣

When used with an epistemic logic for several agents, this language can also describe the effects of multi-agent conversation and communication. Indeed, it suffices for solving well-known puzzles like the 'Muddy Children' (Fagin, Halpern, Moses & Vardi 1995), while also throwing new light on old philosophical issues (van Benthem 2004).

As for a matching calculus of 'hard information flow', since these ideas are still less familiar than 'hard core' static epistemic logic, here is what the complete logic looks like:

Theorem PAL is axiomatized completely by the usual complete laws of epistemic logic plus the following *recursion axioms*:

$[!P]q$	$\leftrightarrow$	$P \rightarrow q$	*for atomic facts q*
$[!P]\neg\phi$	$\leftrightarrow$	$P \rightarrow \neg[!P]\phi$	
$[!P]\phi\wedge\psi$	$\leftrightarrow$	$[!P]\phi \wedge [!P]\psi$	
$[!P]K\phi$	$\leftrightarrow$	$P \rightarrow K(P \rightarrow [!P]\phi)$ [7]	

These axioms are the 'recursion equations' of public information flow, performing step-by-step compositional analysis of epistemic effects of incoming hard information. In particular, the final equivalence relates the knowledge agents get after receiving new information to conditional knowledge they already had before. [8]

[6] Through the items *!P* and $[A]\phi$, the two clauses of this definition involve a mutual recursion.

[7] The axiom for common knowledge involves a subtlety: van Benthem, van Eijck & Kooi 2006.

[8] Strictly speaking, this assumes *perfect memory* and other idealized epistemic features of agents.

Computing knowledge gain This system supplies the facts behind our earlier account of observation-based information flow. For instance, we can show as follows that a public announcement of an atomic fact q leads to knowledge of q by the following calculation:

$$[!q]Kq \leftrightarrow (q \rightarrow K(q \rightarrow [!q]q)) \leftrightarrow (q \rightarrow K(q \rightarrow (q \rightarrow q))) \leftrightarrow (q \rightarrow KT) \leftrightarrow T.$$

In the same way, public announcement produces knowledge after public announcement of any 'factual assertion' which contains no epistemic operators. But things may get more complicated with non-factual expressions:

Example Surprises in information flow.
Using the axioms, a similar calculation for the *'Moore sentence'*

$\neg Kq \wedge q$ ("you don't know it, but q"),

shows that it leads to knowledge of q – but thereby, it also leads to knowledge of its own negation, since the formula Kq implies $\neg(\neg Kq \wedge q)$. ♣

By finding such phenomena beyond the common sense view of 'learning the facts', *PAL* serves as a powerful lens for detecting new and surprising features of information flow.

Further developments For technical details of public announcement logic, including its bisimulation-based model theory, cf. van Benthem 2006B. Richer systems of *dynamic-epistemic logic* (*DEL*; cf. Baltag, Moss, Solecki 1998, van Benthem, van Eijck & Kooi 2006, van Ditmarsch, van der Hoek & Kooi 2007) deal with observational information flow in more complex multi-agent settings, such as card games, where not all players have equal observational access to events like your drawing a card from the stack.

3 Dynamic-epistemic logic with informational access

Let us summarize our earlier proposal: in standard epistemic semantics, agents may 'know' many things as encoded in the accessibility structure of some epistemic model. But they need not have *conscious access* to this knowledge, via linguistic formulations describing relevant properties. This access may be obtained by various means, inference, model-checking, and perhaps further procedures. We will not commit to any specific scenario here, but proceed to a minimal modeling of access and actions that change it.

3.1 Epistemic logic with informational access to worlds

As it happens, a slight change in the earlier epistemic models suffices, using a little syntactic fine-structure to make room for further dynamics. As we have already explained in our Introduction, agents can only '*access*' worlds through some description in their language, representing explicit information which they have about that world.

Definition (Static epistemic access language.) The *epistemic access language* has the following inductive syntax for formulas in a propositional language with an added epistemic modality, and a modality of 'explicit information' to be explained below: [9]

$$p \mid \neg\varphi \mid \varphi \vee \psi \mid K\varphi \mid I\varphi$$

In this language, the *factual propositions* are those constructed without using K or I. ♣

Later on we will extend this standard static syntax definition to include dynamic modalities for model changing events. Intuitively, one can think of $K\varphi$ as the agent's having the implicit information that φ, whereas $I\varphi$ says that it has the explicit information that φ. For simplicity, in what follows, we do not provide subscripts for knowledge and information of different agents. Next, we formulate our semantics for this language:

Definition (Models and truth conditions) *Epistemic access models* $\boldsymbol{M} = (W, W^{acc}, \sim, V)$, are epistemic models as defined earlier with an additional set W^{acc} of '*access worlds*', being *pairs* (w, X) of a standard world w and a set of *factual* formulas X representing the agent's access to this world. In these pairs, we require that all formulas in X be true at (w, X), something which will be guaranteed by the truth conditions to be stated later on in this definition. Moreover, there is a constraint: epistemically indistinguishable worlds have the same access set. [10] Next, here is how formulas are interpreted at access worlds:

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[9] For simplicity, we define the single-agent version only – though both observation and inference can be social phenomena: think of experiments performed by groups, or multi-agent argumentation.

[10] Thus, alternatively, we could have given a *function* from worlds to access sets which assigns the same values to all epistemically indistinguishable worlds.

$\boldsymbol{M}, (w, X) \models p$ iff $w \in V(p)$

Booleans are interpreted as usual,

$\boldsymbol{M}, (w, X) \models K\varphi$ iff *for all* $v \sim w$: $\boldsymbol{M}, (v, X) \models \varphi$

$\boldsymbol{M}, w, X \models I\varphi$ iff $\varphi \in X$ & $M, w, X \models \varphi$ ♣

In particular, $I\varphi$ says the agent is explicitly aware of the truth of φ. This is close to the 'awareness logic' of Fagin, Halpern & Vardi 1990, and our further system may be viewed as a 'dynamification' of their system, explaining how awareness sets may change dynamically. Intuitively, we think of the access worlds, rather than the original worlds W, as that part of the total model which encodes the agents' current information state.

The logic of this setting is easy to axiomatize. We get the usual laws of epistemic logic, while the I-operator will have few laws of its own, given the syntactic character of its semantics. But here are some, due to the special way we built our models:

Fact $I\varphi \rightarrow \varphi$ and $I\varphi \rightarrow KI\varphi$ are valid principles of our static logic.

Proof This depends on the following features of our semantics. First, the 'Veridicality' $I\varphi \rightarrow \varphi$ holds because this was put into our truth clause for the I operator. Next, we have 'Implicit Introspection' $I\varphi \rightarrow KI\varphi$ because of the epistemic uniformity in our assignment of access sets. Note that, together, the two principles imply that $I\varphi \rightarrow K\varphi$, as desired. ♣

Discussion: two limitations Our definition has at least two severe limitations.

First, we only put *factual propositions* in access sets. Our reason for this is Veridicality. Unlike factual formulas, complex epistemic formulas can change their truth value when new information comes, witness the above Moore-type sentences of the form $\neg Kq \wedge q$, which became false after their announcement, since q became known. Thus, we get into trouble with Veridicality, as the current access set might become 'obsolete'. This problem seems solvable, but our paper just describes the mechanics of a simplest base system.

Secondly, we have assumed that an agent's access sets are *the same across epistemic ranges*: we only model access to implicit knowledge, not to individual worlds. This epistemic uniformity is much more restricted than our general discussion may have suggested. Again, this restriction may be lifted, and we will discuss it once more below.

3.2 Two dynamic updates that change access models

Van Benthem 2008A distinguishes two sorts of events that can change these models in the earlier dynamic style. Indeed, given the two-fold structure of the states, the earlier action of announcement naturally splits into two versions. One changes the range of worlds, but not the access, which may be described as 'implicit seeing' that something is the case. The other is a conscious form of seeing where the observed proposition makes it into the agent's access set. This distinction may be compared with the two kinds of 'seeing' discussed in Barwise & Perry 1983. The implicit form corresponds to naked infinitive expressions like "Isolde saw a pirate ship land", which does not imply that the princess was aware of the nature of the ship. The other reading entails epistemic awareness of the fact as described, as enforced by linguistic constructions with the verb "see" followed by *that*-clauses, as in "Isolde saw that a pirate ship was landing". [11]

The following notions from van Benthem 2008A capture the spirit of this distinction:

Definition (Implicit and explicit observation) Let $(\boldsymbol{M}, (w, X))$ be an epistemic access model whose actual world (w, X) satisfies the factual formula φ in our language. First, an *implicit public observation* $!\varphi$ transforms this model into a new epistemic access model $(\boldsymbol{M}|\varphi, (w, X))$, where $\boldsymbol{M}|\varphi = (W, \{(w, X) \in W^{acc} \mid \boldsymbol{M}, (w, X) \models \varphi\}, \sim, V)$. Next, an *explicit public observation* $+\varphi$ transforms the current model $(\boldsymbol{M}, (w, X))$ into a new model $(\boldsymbol{M}|^{+}\varphi, (w, X))$, where $\boldsymbol{M}|^{+}\varphi = (W, \{(w, X \cup \{\varphi\}) \in W^{acc} \mid \boldsymbol{M}, (w, X) \models \varphi\}, \sim, V)$. ♣

An implicit public observation restricts the agent's current range to only those worlds in the model satisfying the announced proposition, but there is no reflection of this update in the agent's (remaining) awareness sets. In addition to this, an explicit public observation

[11] For a more dramatic example, that "Oedipus killed his father on the road' does not mean that he was aware that he was killing his father – even though the ensuing classical tragedy shows that the Gods were not willing to give him the benefit of this logical distinction.

adds an immediate reflection in the agent's access to the worlds that remain. As stated earlier, the restriction to factual assertions makes sure that Veridicality is not endangered.

Right now, we would like to rephrase the above set-up to a 'cleaner' distinction:

3.4 Separating concerns: world dynamics and access dynamics

Intuitively, the preceding is not yet the right division of labour, as events of explicit seeing and implicit seeing 'overlap' in their effects on a model. We would rather like to think of two more 'orthogonal' acts: one being bare observation $!\varphi$, and the other an act of mere addition to access sets, more in the spirit of the following intuitive notion:

Definition Acts of 'realization'.
Over epistemic access models, *'realizing that φ'* ($\#\varphi$) means adding a formula φ to all awareness sets in our current range, provided that $K\varphi$ holds in the actual world. ♣

We can think of this as a special case of explicit public observation, where the formula is already true in all worlds that we have left, so no elimination takes place, and what remains mere growth of access. Taking the latter as primitive – with an obvious corresponding model operation $(\boldsymbol{M}|^{\#}\varphi, (w, X))$, we could then also 'factor' explicit seeing into two consecutive actions: implicit public observation, followed by an act of realizing:

Fact Explicit observation is definable using implicit observation and realization. [12]

But realization is more general, since one can 'realize' any formula, factual or not, using perception, inference, and so on. Indeed, an act of realization is justified by any means which leads to implicit knowledge: the use of a model checker on the current epistemic model, the use of a logical system drawing inferences, and so on. We will return to this underlying dynamics in the next section. Here are two special cases of our setting:

Realization and inference We can now model effects of steps of inference by realization. Suppose we know both φ and $\varphi \rightarrow \psi$, for instance, since they were public announced. [13]

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[12] This only works for factual assertions. For non-factual assertions, more care would be needed.

[13] Our earlier calculations in *PAL* showed how $[!\varphi]K\varphi$ is valid for factual assertions φ. Of course, this does not imply that φ belongs to the access set: $[!\varphi]I\varphi$ is *not* a valid law in our semantics.

Thus we have $K\varphi$ and $K(\varphi \rightarrow \psi)$ true, as implicit knowledge of the agent. The Modal Distribution axiom $K(\varphi\rightarrow\psi) \rightarrow (K\varphi \rightarrow K\psi)$ now tells us that $K\psi$ also holds. While this axiom is often misread as saying the agent *may consciously infer that* ψ, it really states the closure property that ψ, too, is implicit knowledge. To make this point a bit more forcefully, the epistemic axiom by itself does not provide the realization that ψ is true, but it does provide a *license* for the agent to realize that this is so! But in our dynamic system, that realization is a further action by itself.

In terms of our system then, the agent can now perform a realization action $\#\psi$ putting the conclusion explicitly into the access set. [14] In particular, our logic will validate the following principle, which shows how semantic update and realization cooperate:

$[!p][!(p \rightarrow q)]\langle +q\rangle Iq$ *('observation enables realization')*

More generally, *any sound inference in our logic* is available for an event of realization.

Our final concern is if reasoning with all this still works as smoothly as in Section 2:

3.5 A complete dynamic logic

First, we need to enrich the earlier syntax to look as follows, where we add dynamic modalities for implicit observation and explicit realization of factual formulas:

$$\varphi ::= p \mid \neg\varphi \mid \varphi\vee\psi \mid K\varphi \mid I\varphi \mid [!\varphi]\psi \mid [\#\varphi]\psi$$

The semantics of these operators is straightforward. For instance,

$$\boldsymbol{M}, (w, X) \models [\#\varphi]\psi \text{ iff } (\boldsymbol{M}|^{\#}\varphi, (w, X)) \models \psi$$

It is easy to write a complete set of recursion axioms for the actions proposed here: [15]

Theorem The dynamic epistemic logic of informational access is axiomatized completely by (a) the static logic of the epistemic access language with K and I,

(b) the *PAL* recursion axioms augmented with one for the I modality:

$$[!\varphi]I\psi \leftrightarrow (\varphi \rightarrow I\psi)$$

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[14] Once again, this is close to the earlier 'awareness logics', except for our new dynamic spin.

[15] Velazquez Quesada 2008 has a similar analysis for his dynamic logic of inference.

(c) the following principles for the dynamic modality of realization:

$[\#\varphi]q \quad \leftrightarrow \quad K\varphi \rightarrow q \qquad$ *for atomic propositions q*

$[\#\varphi]\neg\psi \quad \leftrightarrow \quad K\varphi \rightarrow \neg[\#\varphi]\psi$

$[\#\varphi](\psi \wedge \chi) \quad \leftrightarrow \quad [\#\varphi]\psi \wedge [\#\varphi]\chi$

$[\#\varphi]K\psi \quad \leftrightarrow \quad K\varphi \rightarrow K[\#\varphi]\phi$

$[\#\varphi]I\psi \quad \leftrightarrow \quad K\varphi \rightarrow (I\psi \vee \psi = \varphi)$ [16]

Proof As for soundness, the first axiom says that implicit public observation does not change access, while the second says that realization does not change ground facts. The next two axioms are standard for a modality describing partial function. The recursion axiom for knowledge states the precondition for realization, and then says that no accessibility structure is changed. Finally, the recursion axiom for access expresses that only the formula φ has been added to the access set. As for (relative) completeness, it is easy to see that, working inside out on dynamic modalities, the axioms reduce every formula of the dynamic language into an equivalent one of the static language. ♣

Illustration Here is how realization yields explicit knowledge under the right condition:

$$<+\varphi>I\varphi \leftrightarrow \neg[+\varphi]\neg I\varphi \leftrightarrow \neg(K\varphi \rightarrow \neg[+\varphi]I\varphi) \leftrightarrow (K\varphi \wedge [+\varphi]I\varphi) \leftrightarrow$$
$$(K\varphi \wedge (K\varphi \rightarrow (I\varphi \vee \varphi = \varphi))) \leftrightarrow (K\varphi \wedge (K\varphi \rightarrow True)) \leftrightarrow K\varphi.$$

A similar calculation proves the earlier-mentioned validity $[!p][!(p \rightarrow q)]<+q>Iq$. ♣

Thus, we have shown how an enriched dynamic-epistemic logic with observational access works in the same style as its purely semantic versions, while greatly extending the scope of modeling phenomena concerning different sorts of logical information, including events of observation, inference, and sheer awareness-raising 'realization'.

3.6 Lifting the limitations?

Our system is still subject to the severe limitations mentioned earlier.

Beyond factual propositions We only deal with announcement and realization of factual assertions, and this does not fit well with the *PAL* dynamics of $!\varphi$ in Section 2, which

[16] There is a little harmless abuse of notation here, since '$\psi = \varphi$' is not in our formal language.

announces arbitrary formulas φ. Also, there is nothing in acts of realization per se which would not allow for arbitrary formulas φ to be 'realized', provided the agent knows them in the sense of $K\varphi$. Indeed, our axioms seem valid on this broader reading, too. The issue is just how to deal with Veridicality when the model changes by modifying access sets. There are various option for this, but we must leave them for some future occasion.

Realization and introspection Next, here is one reason for being more liberal with the kinds of formula that can be realized, thereby turning implicit knowledge into explicit knowledge. Beyond the already discussed case of inference, now, *introspection* becomes an activity! [17] For instance, suppose that the agent knows that φ: $K\varphi$. This may become explicit knowledge $I\varphi$ through realization. But also, thanks to the validity of the *K4* axioms in the epistemic structure of the model, the act of realizing $K\varphi$ *itself* is available, and it would lead to the truth of $IK\varphi$. Thus again, the usual K4-axiom would not constitute introspection by itself, but it would *license* an act of introspection.

Local versus global access Giving up the assumption that access is the same throughout the agent's epistemic range, access worlds (w, X), (v, Y) with $w \sim v$ but $X \neq Y$ would become admissible in our models. In this generalized setting, further dynamic processes beyond inference and introspection might be analyzed in terms of our logical system. Earlier on, we mentioned *model-checking* of formulas φ at worlds w as a procedure which tells us more about current worlds. [18] The effects of steps of such a procedure could be modeled as changing local access to specific worlds. Our present logic can accommodate such refinements, and it would be of interest to investigate them further.

4 Coda: inferential dynamics

Acts of realization are licensed whenever implicit knowledge holds, for whatever reason. In particular, this idealization means that each valid principle of our logic is available for an act of realization. Thus, 'logical omniscience' still lurks just below the surface. To

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[17] This desideratum relates to the analysis in Liu 2006 of parametrizable 'agent abilities'.

[18] Ramanujam 1999 investigates explicit knowledge in the form of general *algorithmic knowledge* which means the agent implicitly knows P and has some *algorithm* to determine the truth of P. He then asks the question: if the algorithm is model checking, what model is the agent checking?

improve on this, we need richer settings. So far, an agent's access consisted of formulas she explicitly knows to be 'locally' true in the current world. To model more specific operations which modify access, more detail is needed. In the case of inference, this requires global principles valid in the model: the agent's explicit *inferential apparatus*. Representing this requires putting these rules in the models, which has been proposed by many authors, starting from Dung 1995; cf. the extensive discussion in Allo 2007.

Syntactic inferential dynamics Jago 2006 uses *logic programming* models for inferential information flow. Let states be 'Herbrand sets' of propositional literals $(\neg)p$, while information generating events are applications of Horn-clause rules

$$p_1 \,\&\, \ldots, \,\&\, p_n \rightarrow q$$

from some given set: a logic program, or just some set of rules the agent has available. Rule application to sets containing the premise literals lets these information states grow. Thus, an initial information state might change step-by-step as in the following example:

$$\boxed{p, \neg q, r} \xrightarrow{(p \,\&\, r) \rightarrow s} \boxed{p, \neg q, r, s} \xrightarrow{(t \,\&\, r) \rightarrow q} \boxed{p, \neg q, r, s, \neg t}$$

Many simple puzzles involve reasoning of this kind: the already mentioned *Sudoku* wave is a good example. d'Agostino & Floridi 2007 give a beautiful elaboration of calculi of this sort, which describes how 'surface information' can be obtained fast from given sets of formulas. General *PDL*-style dynamic logics of inference can be set up in the same style, working with arbitrary rules which add items to arbitrary sets of formulas, whose action can be described by means of modal operators of the form $<rule>\phi$. In this way, one can re-interpret the usual theory of logic programming (Doets 1994) as a dynamic logic, which even includes *PDL*- and μ–calculus-style closure operators. [19]

Merging with semantics Combined systems of syntactic inferential dynamics with *PAL*-style semantic update were proposed in van Benthem 2008A, while an elaborate follow-up version is in Velazquez Quesada 2008. The latter performs a dynamification of *PAL*

[19] Van Benthem 1996, Ch. 10, is an extensive study of dynamic aspects of logic programming.

with explicit seeing restricted to factual formulas, while introducing additional explicit inferential actions. This language can formulate combined assertions like this:

$$Ip \rightarrow [+q]\langle (p \& q) \rightarrow r \rangle r$$

This says that, if the agent realizes that p in her current information state, and she then explicitly observes that q, then she can come to realize that r by performing an inference step invoking the rule $(p \& q) \rightarrow r$ available to her as global knowledge in the model.

5 Related work

We already mentioned Fagin, Halpern & Vardi 1990 as essentially the static logic of our system, which may be seen as a dynamification of awareness logics. Ramanujam 1999 is a congenial general account of 'algorithmic knowledge'. Next, Jago 2006 contains much relevant material on inference dynamics and an excursion on learning things through being told. A follow-up to this was Quesada 2008 on combining *PAL* with inferential dynamics. Still within the setting of dynamic-epistemic logic, Hoshi 2008 makes distinctions similar to ours in the context of the system *TPAL*, a protocol-based temporal extension of *PAL* (cf. van Benthem, Gerbrandy, Pacuit 2007) which constrains possible announcement sequences. The resulting distinction between what might be called *epistemic information* (based on truth in the worlds) and *procedural information* (based on agents' knowledge of the protocol) gives syntactic control over which announcements can be made at all, and hence it becomes possible to encode explicit actions of inference within the protocol. [20] [21] Finally, we should mention the dissertation Renne 2008 which develops models consisting of worlds plus associated syntactic 'evidence' similar to ours, in a combination of dynamic-epistemic logic with the 'justification logic' of proof and evidence in the style of the 'New York School' of Artemov, Fitting, and others.

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[20] The recursion axiom for knowledge in this protocol setting is $[!P]K_i\phi \leftrightarrow \langle !P\rangle T \rightarrow K_i(\langle !P\rangle T \rightarrow [!P]\phi)$. The agent will know that ϕ after $!P$ has taken place, iff, when that event is executable at all, he knows beforehand that, conditional on its executability, $!P$ will produce effect ϕ.

[21] Van Benthem 2008B connects this up with information models for intuitionistic and modal logic.

6 Conclusion and further directions

The current system is a simple and workable combination of semantic and syntactic notions of information which fits our intuitive ideas, using the already existing methods of dynamic-epistemic logics. That observation is the main point of this paper.

Alternatives Indeed, it seems possible to recast our system even more closely to the elimination style of *PAL*. Instead of adding syntax sets to worlds, we then follow 'impossible worlds semantics' for para-consistent reasoning (Priest 1997). More precisely, we increase the sets of worlds themselves with explicit valuations on formulas, assigning truth values in those worlds. But initially, we allow any sort of assignment, not necessarily those following the correct semantic evaluation according our truth definition. Now, we can see acts of explicit realization as seeing that some formula must have truth value 1. This will then eliminate all those 'impossible worlds' where its value is different. [22] We think that the two approaches are largely equivalent, but this remains to be seen. This way of rephrasing things is not just a technical exercise, since it might also facilitate comparisons with other frameworks in the foundations of logic today.

Further issues Our system raises many further questions, both smaller and larger in scope. A few of these have already been mentioned in Section 3.6. Here are some more:

(a) Can we formulate 'access' at some *higher abstraction level* than brute syntax? This question relates to the long-standing issue of finding an appropriate level for defining abstract 'propositions': cf. van Benthem & Martinez 2008.

(b) 'Access' also makes sense in *multi-agent settings*. For instance, one may think of conversation or argumentation as producing public access to commitments' made by the participants. What are multi-agent versions of our system dealing with this?

(c) Access makes just as much sense for agents' beliefs as for their knowledge. What about designing doxastic versions of our system, adding 'access' in our style to *dynamic-doxastic logics* of belief revision (van Benthem 2007A, Baltag & Smets 2006), and then

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[22] Similar points have been made in d'Agostino & Floridi 2008, Jago 2007 about modeling inference dynamics by elimination steps like those of *PAL*, but now in 'impossible worlds' models.

bring in 'explicit' realizing counterparts to events of getting soft information? Do we need different versions of the *I* operator in this setting?

(d) Beliefs goes together with forms of *non-monotonic inference*. How can we add inferential dynamics of non-monotonic logics like circumscription or abduction?

(e) Acts of realization may also be governed by *temporal protocols* (Hoshi 2008). How to generalize our system to that setting?

Congratulations So many things to do in modern logic, but also, so many new colleagues to join forces with! It has been a great pleasure attending, and helping to organize, the first Chinese Workshop on Logics for Interaction and Rational Agency (*LORI 2008;* http://www.illc.uva.nl/LORI/cfp.html), in the summer of 2007 in Beijing. It is an even greater pleasure to work with the distinguished colleagues in this new Chinese Logic Journal, which will no doubt become a landmark publication in the years to come.

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