Two Qubits for C.G. Jung's Theory of Personality

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Abstract

We propose a formalization of C.G. Jung's theory of personality using a four-dimensional Hilbert-space for the representation of two qubits. The first qubit relates to Jung's four psychological functions **T**hinking, **F**eeling, **S**ensing and i**N**tuition, which are represented by two groups of projection operators {**T**, **F**} and {**S**, **N**}. The operators in each group are commuting but operators of different groups are not. The second qubit represents Jung's two perspectives of extraversion and introversion. It is shown that this system gives a natural explanation of the 16 psychological types that are defined in the Jungian tradition. Further, the system accounts for the restriction posed by Jung concerning the possible combination of psychological functions and perspectives. Interestingly, the unitary transformation called X-gate in the quantum computation community realizes the cognitive operation connected to Jung's idea of the shadow. The empirical consequences of the present model are discussed and it is shown why the present praxis of personality diagnostics based on classical statistics is insufficient.

1 Introduction

Pioneer geneticist and evolutionary biologist J.B.S. Haldane can be seen as one of the first researchers who realized that quantum mechanics is linked to living systems and thought; cf. Sultan Tarlaci (e.g. 2003). In his important paper Haldane (1934) points out that many characteristics of mind are comparable to those of atomic particles: both arise from dynamical systems, both exhibit a continuity and wholeness, both are at once localized yet spatially diffused.

About ten years earlier, C.G. Jung published his theory about psychological types (Jung, 1921) – after almost 20 years of practical experience and work as a specialist in psychiatric medicine. In this book Jung gave a careful analysis of the universals and differences of Human personalities. Jung thought that people were born with an inborn predisposition to type, perhaps at the quantum level (Meier, 1992) and that the positive combination of both nature and nurture would see that predisposition expressed healthily. In Jung's theory there are no pure types. There is a set of psychological opposites, equally valuable but realized with different preferences for different personalities. Type preferences themselves are the bridge between the conscious and the unconscious. Jung's holistic picture of the Self is difficult to reconcile with classical ideas of physical symbol systems. Instead, we will argue that a simple quantum mechanic model is sufficient to express the bulk of Jung's theory.

Though it is not implausible to assume that Jung (anticipating Haldane) felt that the mind and the Self are “resonance phenomena” that are associated with the wave-like aspect of atomic particles, he didn't make any attempt to express his theory by using the language of quantum mechanics. To develop logically stringent theories was not Jung's strongest talent, and this is perhaps one of the main reasons why Jung never was acknowledged as one of the big forerunners in unifying psychology, eastern thinking and quantum physics. Regrettably, Jung's cooperation with Nobel prizer Wolfgang Pauli didn't help to lift Jung's informal theory of personality onto a more stringent level. Instead, their common reflections were directed far beyond psychology and physics, entering into the realm where the two areas meet in the philosophy of nature.

In the present article we propose a simple formalization of the crucial traits of C.G. Jung's theory of personality by using the formulation of quantum theory as currently used in the context of quantum computation (e.g. Vedral, 2006). Recently, it has been argued by a number of researchers, that the basic framework of quantum theory can find useful applications in the cognitive domain (e.g. Aerts, Czachor, & D'Hooghe, 2005; Atmanspacher, Römer, & Walach, 2002; Busemeyer, Wang, & Townsend, 2006; Franco, 2007; Khrennikov, 2003). The present application to personality diagnostics is new.

In section 2 we will present the basic traits of C.G. Jung's theory and also mention two related approaches: the Myers-Briggs type indicator and the framework of socionics. Section 3 introduces basic concepts of quantum theory. Our formal model will be introduced and discussed in section 4, and section 5 concludes the paper with a general discussion.

Though there is some polemics between and within the different personality schools that follow C.G. Jung, we feel free to ignore these aspects and to concentrate on approaching a formal model of personality that is using C.G. Jung's ideas in its originally fresh, independent and anti-dogmatic way. Hence, our focus will be on the fruitfulness of certain formal ideas, not on a hair-splitting and pedantic justification of various aspects of the offspring of C.G. Jung's theory of personality.

2 C.G Jung's theory of personality in a nutshell

Taking the stance of 'normal psychology', Jung basically assumes that all people have broadly the same psychological equipment of apperception and responsiveness. Where people differ is in the way that each of them typically makes use of the equipment. Accordingly, we are confronted with two main questions for the psychologist:

- What are the essential components of the equipment?
- How do people differ in using these components to form their habitual mode of adaptation to reality?

Jung's answer to the first question claims that all people are equipped with four psychological functions, called **T**hinking, **F**eeling, **S**ensing and i**N**tuition, which are realized in one of two different attitudes: **I**ntroversion and **E**xtraversion. Normally, people use all four psychological functions. However, they have different preferences for what functions they use predominantly. Jung claims that it is exactly these differences that constitute the different types of personality.

Jung's typology of personality is pretty popular now and the introvert-extravert dimension is the most popular part of the theory. We find this dimension in several theories, notably Hans Eysenck's, although often hidden under alternative names such as "sociability" and "surgency". Introverts are people who prefer their internal world of thoughts, feelings, fantasies and dreams, while extroverts prefer the external world of things, events, people and activities. The words have become confused with ideas like shyness and sociability, partially because introverts tend to be shy and extroverts tend to be sociable. But Jung intended for them to refer more to whether you more often faced toward the persona and outer reality, or toward the collective unconscious and its archetypes.

Whether we are introverts or extroverts, we need to deal with the world, inner and outer. And each of us has their preferred ways of dealing with it, ways we are comfortable with and good at. Jung's four basic ways, or functions, are explained now in a bit more detail:

> Ich unterscheide vier Funktionen, nämlich *Empfindung*, *Denken*, *Gefühl* und *Intuition*. Der Empfindungsvorgang stellt im wesentlichen fest, dass etwas ist, das Denken, was es bedeutet, das Gefühl, was es wert ist, und Intuition ist Vermuten und Ahnen über das Woher and das Wohin. (Jung, 1936, p. 270)[1]

[1] In English translation: "I consider four functions, namely *sensation*, *thinking*, *feeling*, and *intuition*. *Sensation* tells us that something exists; *thinking* tells you what it is; *feeling* tells you whether it is agreeable or not; and *intuition* tells you whence it comes and where it is going".

Thinking means evaluating information or ideas rationally, logically. Jung called this a *rational* function, meaning that it involves decision making or *judging*, rather than the simple intake of information. **F**eeling, like thinking, is a matter of evaluating information, this time by weighing one's overall, emotional response. **S**ensing means what it says: getting information by means of the senses. A sensing person is good at looking and listening and generally getting to know the world. Jung called this an *irrational* function, meaning that it involved *perception* rather than judgment of information. **I**ntuiting is a kind of perception that works outside of the usual conscious processes. It is irrational or perceptual, like sensing, but comes from the complex integration of large amounts of information, rather than simple seeing or hearing. Jung said it was like seeing around corners.

We all have these psychological functions. We just have them in different proportions. Each of us has a *superior* function, which we prefer and which is best developed in us, a *secondary* function, which we are aware of and use in support of our superior function, a *tertiary* function, which is only slightly less developed but not terribly conscious, and an *inferior* function, which is poorly developed and so unconscious that we might deny its existence in ourselves.

Across the different types of personality, there are several restrictions which determine what functions can be realized under what attitude at what position in the rank ordering of the functions. For understanding these restrictions it is important to see that the functions are organized as equally valuable psychological opposites. **T**hinking and **F**eeling constitute a pair of opposites [rational opposites], and also the pair **S**ensation/i**N**tuition [irrational opposites].

> Der Denktypus zum Beispiel muss notwendigerweise immer das Gefühl möglichst verdrängen und ausschließen, weil nichts so sehr das Denken stört wie das Gefühl, und umgekehrt muss der Fühltyp das Denken tunlichst vermeiden, denn nichts ist dem Gefühl schädlicher als das Denken. (Jung, 1923)[2]

Hence, the first restriction is that if the superior function is rational/irrational, then the secondary function must be irrational/rational. Otherwise they cannot support the superior function. Plausibly, the alternation of rational and irrational function is continued along the ranking hierarchy.

There is a second restriction which constraints the attitudes that can be connected with the different psychological functions:

> Neben diesen eben erwähnten Qualitäten der unterentwickelten Funktionen kommt letzteren auch die Eigentümlichkeit zu, dass sie bei bewusster introvertierter Einstellung extrovertiert sind und umgekehrt, dass sie also zugleich die bewusste Einstellung kompensieren. Man darf daher erwarten, bei einem introvertierten Intellektuellen zum Beispiel extravertierte Gefühle zu entdecken. (Jung, 1923)[3]

Hence, the restriction is that opponent (or *dual*) functions have different attitudes. Possibly the rational of this restriction is related to the evolution of typologies. As Jung mentions in chapter X of his book "Psychologische Typen" (Jung, 1921), the personality of an individual conflicts if the Self has to realize two opponent functions in the same attitude. There is an evolutionary pressure to avoid such constellations. This pressure results in a kind of dynamics that is called type dynamics.[4]

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[2] In English translation: "The thinking type, for instance, has to necessarily repress and eliminate the feeling, because it's the feeling that interferes the most with thinking. On the contrary the feeling type has to urgently avoid thinking, because nothing is more harmful to the feeling than that."

[3] In English translation: "Apart from these last-mentioned qualities of the inferior functions, the latter also possess the feature that they are extraverted assuming a conscious introverted attitude (and vice versa), which means that they compensate their conscious attitude at the same time. Consequently one can expect, for example, to discover extraverted feelings in an introverted intellectual."

[4] See, for instance, http://en.wikipedia.org/wiki/Myers-Briggs_Type_Indicator#cite_ref-Myers_1-4/index.html.

Obviously, these restrictions do not constraint the free choice of the superior function, and, consequently, Jung proposed eight basic psychological types, four with the extraverted attitudes and four with the introverted attitude: E/I **F**eeling type (corresponding to the regions 1 & 8 in Figure 1), E/I i**N**tuition type (region 2 & 3), E/I **T**hinking type (region 4 & 5), and E/I **S**ensing type (region 6 & 7). These eight basic types discussed by Jung can be further refined into 16 psychological types depending on what is considered as the secondary function. As Figure 1 makes pretty clear, there are only two options in each case for fixing the secondary function, and this is exactly the content of the first restriction mentioned above.

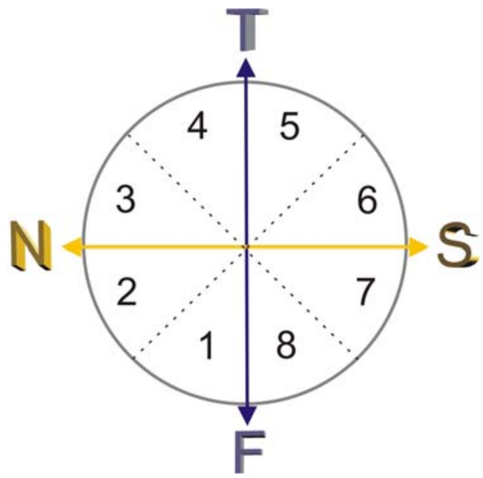

Figure 1: Two pairs of opposite psychological functions: **T**hinking and **F**eeling [rational opposites], **S**ensation/i**N**tuition [irrational opposites]. Jung takes this two-dimensional representation in order to demonstrate the dependencies between the psychological functions. For example, **T**hinking and **F**eeling are opposites and conflict with each other (assuming one fixed attitude). However, their effect can be modified by using the irrational functions (**S**ensation and i**N**tuition, respectively).

Notably, there are two different systems that make use of 16 types. The first system is the *Myers-Briggs type indicator* (MBTI). The second system is the scheme of *socionics*.

MBTI was developed by Katharine Cook Briggs and her daughter, Isabel Briggs Myers, based on a roughly simplified picture of C.G Jung's ideas. The MBTI classifies a person's personality along four dichotomous categories. In each case the emphasis is on an either-or preference (somewhat akin to your preference for being either right or left handed). In the MBTI the first element indicates the preferred attitude (**E**xtraverted/**I**ntroverted), the second element indicates the preferred irrational function – whether you tend to take in new information as it is (**S**ensing) or connect it with ideas of what could be (i**N**tuition), the third indicates the preferred rational function – whether you value emotions and values over logic and reason (**F**eeling) or the other way around (**T**hinking), and the fourth element indicates whether the rational function is more important than the irrational one, i.e. whether you prefer planned order and quick decisions (**J**udging) or spontaneity and contemplation (**P**erceiving).[5]

Socionics was developed in the 1970s and 80s mainly by the Lithuanian researcher Aušra Augustinavičiūtė. The name socionics is derived from the word "society" since Augustinavičiūtė believed that each personality type has a distinct purpose in society, which can be described and explained by socionics. The system of socionics is in several respects

[5] Myers believed that the first extraverted function decides about **P**erceiving or Judging, in contrast to C.G. Jung who referred it to the first function (without considering the qualification as extraverted or introverted). Consequently, the MBTI is using the reverse order of the first two functions in case of introverted people. Socionics is criticising this point.

similar to the MBTI; however, whereas the latter is dominantly used in the USA and West Europe the former is mainly used in Russia and East Europe.

Despite of several similarities there are also important differences. For instance, the MBTI is based on questionnaires with so-called forced-choice questions. Forced-choice means that the individual has to choose only one of two possible answers to each question. Obviously, such tests are self-referential. That means they are based on judgments of persons about themselves. Socionics rejects the use of such questionnaires and is based on interviews and direct observation of certain aspects of human behavior instead. However, if personality tests are well constructed and their questions are answered properly, we will expect results that often make sense. For that reason, we don't reject test questions principally but we have to take into account its self-referential character. Another difference relates to the fact that socionics tries to understand Jung's intuitive system and to provide a deeper explanation for it, mainly in terms of informational metabolism (Kepinski, 1972). Further, socionics is not so much a theory of personalities *per se* but much more a theory of type relations providing an analysis of the relationships that arise as a consequence of the interaction of people with different personalities.

The 16 psychological types correspond to the 8 sectors in Figure 1 if we take into account that the two dominant (conscious) psychological functions can be either in the extraverted attitude or in the introverted attitude. Table 1 gives the complete classification in a system based on the first two dominant functions and in the closely related type indicator developed by Myers-Briggs (MBTI).

Extravert			Introvert		
1	1EF 2EN	ENFJ	1IF 2IN	INFP	1
2	1EN 2EF	ENFP	1IN 2IF	INFJ	2
3	1EN 2ET	ENTP	1IN 2IT	INTJ	3
4	1ET 2EN	ENTJ	1IT 2IN	INTP	4
5	1ET 2ES	ESTJ	1IT 2IS	ISTP	5
6	1ES 2ET	ESTP	1IS 2IT	ISTJ	6
7	1ES 2EF	ESFP	1IS 2IF	ISFJ	7
8	1EF 2ES	ESFJ	1IF 2IS	ISFP	8

Table 1: 16 psychological types. Following the Jungian tradition, the first two psychological functions are given with the corresponding attitude (extraverted/ introverted). Further, the closest pendant in the MBTI is specified.

Here are some examples illustrating the types of forced-choice questions used in the empirical type test à la Myers-Briggs:

(1) **E**xtraverted/**I**ntroverted opposition
   a. At parties, do you stay late with increasing energy or leave early with decreased energy? (E/I)

b. In doing ordinary things are you are more likely to do it the usual way, or do it your own way? (E/I)
c. When the phone rings, do you hasten to get to it first, or do you hope someone else will answer? (E/I)

(2) **F**eeling/**T**hinking opposition
a. In making decisions do you feel more comfortable with feelings or standards? (F/T)
b. In approaching others is your inclination to be personal or objective? (F/T)
c. In order to follow other people do you need trust, or do you need reason? (F/T)

(3) **S**ensing/i**N**tuition opposition
a. Which seems the greater error: to be too passionate or to be too objective? (S/N)
b. Are you more attracted to sensible people or imaginative people? (S/N)
c. Facts speak for themselves or illustrate principles? (S/N)

Taking C.G. Jung's theory seriously, the expression of a person's psychological type is more than the sum of the four individual preferences expressed by the MBTI. This is because of the way in which the preferences interact through type dynamics and type development. Although the interpretation of the MBTI acknowledges the role of type dynamics and type development, these concepts do not enter the test procedure. As an example assume that for a person X the test results into a balance between **E**xtraversion and **I**ntroversion (i.e. 50% E, 50% I). Assume further, that we also find a balance between **T**hinking and **F**eeling and a low percentage of the irrational functions. Obviously, the type of *extraverted thinkers* and the type of *introverted thinkers* are both in agreement with this test results. Unfortunately, there is no way to discriminate the two types by simply testing the percentage of E, I, T and F. The reason is that due to type dynamic in the case of extraverted thinkers the ET function is superposed with the IF function. And in the case of introverted thinkers the IT function is superposed with the EF function. In both cases we get 50% E, 50% I, 50% F and 50% T. Hence, though there is a big difference between the personality types that agree with the test results, there is no way to find out the correct Jungian type by using the MBTI. What we need are particular test questions that directly test for the functions in a certain attitude.[6]

For good reasons C.G. Jung was relatively vague concerning the details of type dynamics. However, he seems to claim that the auxiliary function has the same attitude as the first function (otherwise the auxiliary function could not support the first function)[7], and he insists in claiming that the unconscious inferior function always has the opposite attitude of the superior function.[8]

[6] Hence, C.G. Jung had good reasons to describe the four psychological functions not in isolation but always related to a certain attitude, extraverted or introverted. The MBTI ignores the wisdom of this decision. Socionics follows the Jungian tradition in a more respectful way. For example, socionics discusses the psychological functions always in the context of the extraverted or introverted attitude. Unfortunately, socionics doesn’t like to develop corresponding tests that take this point into account.
[7] In his concise description of C.G. Jung’s theory of personality Anthony Stevens (1994) presents a picture (Figure 2) that exactly makes his point and demonstrates that the first two psychological functions have the opposite attitude as the last two functions.
[8] The MBTI seems to deviate from this position in assuming that the auxiliary function has the opposite attitude as the superior function, hence ESTJ would conform to the following four functions: 1 ET 2IS 3N 4IF and INFP, to take the shadow example, is 1IF 2EN 3S 4ET. In our opinion, it is extremely difficult to empirically justify these details. That doesn’t exclude the possibility that a good theory leads to a decision at this point making speculations superfluous. For references about MBTI see http://en.wikipedia.org/wiki/Myers-Briggs_Type_Indicator#cite_ref-Myers_1-4/index.html; for references about socionics, see http://en.wikipedia.org/wiki/Socionics and http://www.socionics.us/intro.shtml#1/index.html.

3 Basic concepts of quantum theory

Modern quantum theory is a conceptual framework relating *states* and *observables* in a dynamic way. States are describing aspects of the physical world, observables relate to meaningful questions we can ask about the world. States are modelled within a vector space. This idea tells us that states can enter into certain systematic operations. For instance, we can add two existing states resulting in a sum vector. The addition of states relates to the phenomenon of superposition. Superposition is mainly known from the physics of waves. However, it also applies in the cognitive domain where we find superpositions of percepts, such as in the domain of colours, faces, and music (see, for example Gärdenfors, 2000). Further, we can multiply a vector with a scalar. That means that we lengthen or shorten the relevant vector. However, things are more complicated if we take the wave-like character of physical objects into account. Then we also need an operation to change the phase of the corresponding wave. This is described mathematically by a scalar multiplication with a complex number of unit length. Such complex scalars are usually written as $e^{i\Delta}$ where angle Δ describes the corresponding phase shift.

Another aspect of the idea to model states by vectors is the desire to determine the similarity of two states. The relevant operation is the so-called scalar product. The scalar product of two vectors *u* and *v* relates to the product of the lengths of the two vectors multiplied with the *cosine* of the angle between the two vectors. If this angle is $\pi/2$ we will say the two vectors are orthogonal to each other – meaning that they are maximally dissimilar.

Formally, states are described as elements of a Hilbert space. A Hilbert space $\mathcal{H}$ is a complete complex vector space upon which an inner product (= scalar product) is defined. The scalar product of two vectors *u*, *v* in $\mathcal{H}$ is written in the form $\langle u|v\rangle$. I assume some familiarity with the notion of a vector space and an inner product. For details, the reader is referred to introducing textbooks in quantum information science, e.g. Vedral (2006).

In the following we will make use of finite Hilbert spaces only, i.e. Hilbert spaces which are spanned by a finite system **S** of linearly independent vectors, which can be assumed to be pairwise orthogonal, i.e. the scalar product of two different vectors in **S** is zero.

In quantum theory, *observables* are modelled by "normal" linear operators of the Hilbert space. Intuitively, such observables ask about the value of a certain real-valued variable, e.g. what is the momentum/energy/place… of a particle? The value also can be discrete, e.g. 1 standing for *yes*, –1 standing for no, and 0 standing for *don't know*. The expected, averaged answer to the question asked by an observable **a** in a certain state *u* is formally expressed by the scalar product between *u* and the state $\mathbf{a}|u\rangle$ that results by applying the operator **a** to the state $|u\rangle$. Hence, for the expected answer we can write

(4) $\langle \mathbf{a}\rangle_u = \langle u|\mathbf{a}|u\rangle$

The expected answer may differ from the real answer given after performing the relevant experiment. However, if repeating the experiment, the expected answer reflects the average of the real answers given in exact replications of the experiment.

Observables are often uncertain when measured in a certain state. The root mean square deviation (=standard deviation) is the standard mathematical measure for calculating the uncertainty of an observable in a given state. It is defined as follows:

(5) $\Delta_u(\mathbf{a}) = \mathrm{sqrt}(\langle \mathbf{a}^2\rangle_u - \langle \mathbf{a}\rangle_u{}^2)$.

For each physical observable there are some states where the answer is absolutely certain, i.e. the standard deviation is zero. These states are called eigenstates of the operator. In such states, the application of the operator to the states results in a scalar multiplication of the state.

In can be shown that under very general conditions each operator has exactly n orthogonal eigenstates where n is the dimension of the underlying Hilbert space (spectrum theorem).

If you have more than one observable, then an important question is if there are states where a simultaneous measurement of these observables can lead to a definite, absolutely certain result. The famous answer given by Heisenberg is that states with definite values for both observables, say **a** and **b**, exist if and only if the ordering of the two observables doesn't matter, i.e. **ab–ba** = 0. For example, states with definite position **X** and simultaneously definite momentum **P** do not exist in quantum mechanics. In this case, a non-zero canonical commutation relation applies: **XP–PX** = ih/2π, where h is the Planck constant.

The size of this ordering effect determines a lower boundary for the product of the uncertainties of **a** and **b**. This is the content of the famous Heisenberg uncertainty principle:[9]

(6) $\Delta_u(\mathbf{a})\, \Delta_u(\mathbf{b}) \geq \frac{1}{2} \langle \mathbf{ab}-\mathbf{ba}\rangle_u$

In the case of position and momentum, the predicted lower boundary is h/4π.

The simplest non-trivial physical system is a two state system, also called a *qubit*. In such a system each proper observable has exactly two (orthogonal) eigenvectors, i.e. two states where the question asked by the corresponding observable has a certain outcome. Of course, a qubit can realize an infinite set of states but two states only relate to eigenstates of an observable.

Formally, an arbitrary state of a qubit can be written as

(7) $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with $\alpha^2 + \beta^2 = 1$

Hereby the two states $|0\rangle$ and $|1\rangle$ are two orthogonal unit vectors of our two-dimensional Hilbert space. In such a physical system exactly three independent observables are possible. A common choice for these operators is the Pauli operators σ_x, σ_y, and σ_z, which are defined in terms of operators $|u\rangle\langle v|$ with unit vectors *u* and *v*:[10]

(8) a. $\sigma_x = |1\rangle\langle 0| + |0\rangle\langle 1|$
b. $\sigma_y = i|1\rangle\langle 0| - i|0\rangle\langle 1|$
c. $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$

Examples for realizing qubits are the spin of electrons (here the three operators give the spin in directions x, y and z) or the polarization of photons (here σ_z is measuring the polarization in↑-direction, σ_y the polarization in ↗-direction, and σ_z is measuring left and right circularly polarized light). The eigenvectors of σ_z are $|0\rangle$ and $|1\rangle$, with eigenvalues 1 (*yes*) and –1 (*no*). These contrast with the eigenvectors of σ_x, namely $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$, which are simple superpositions of the base states. And the eigenvectors of σ_y are superpositions of the base states including a π/2 phase shift: $\frac{1}{\sqrt{2}}(|0\rangle+i|1\rangle$ and $\frac{1}{\sqrt{2}}(|0\rangle-i|1\rangle)$.

Making use of a particular parameterization of the states $|\psi\rangle$ every state of a qubit can be realized as the point on a three-dimensional sphere, the so-called Bloch sphere.

(9) $|\psi\rangle = \cos(\theta/2)\, e^{-i\Delta/2}\, |0\rangle + \sin(\theta/2)\, e^{+i\Delta/2}\, |1\rangle$

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[9] See appendix A for a simple proof.

[10] The definition of this elementary operator is $|u\rangle\langle v|\,(|w\rangle) =_{def} |u\rangle \cdot \langle v|w\rangle$ for each state $|w\rangle$ of the Hilbert space.

The parameters θ and Δ are nothing else than spherical polar coordinates, $0 \leq \Delta < 2\pi$ and $0 \leq \theta < \pi$.

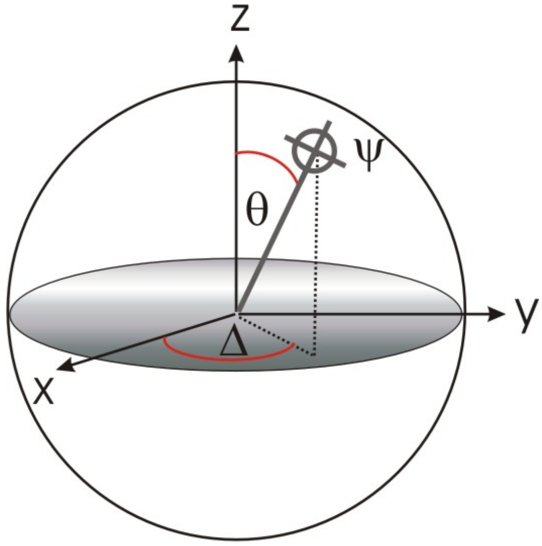

Figure 2: Bloch sphere. Using equation (9) an arbitrary (normalized) state of the two-dimensional Hilbert space can be parameterized by the two spherical polar coordinates θ and Δ. Hereby, Δ corresponds to a phase shift of the two superposing states $|0\rangle$ and $|1\rangle$)

For a simple illustration, consider a photon in a qubit state $|\psi\rangle$, and take $|0\rangle$ as indicating horizontal polarization and $|1\rangle$ as indicating vertical polarization. Then the probability that the object is horizontally polarized (i.e. it collapses into the state $|0\rangle$) is

(10) $\langle \mathbf{0} \rangle_\psi = |\langle 0|\psi\rangle|^2 = \cos^2(\theta/2) = ½(1+\cos(\theta))$

And the probability that it is vertically polarized (i.e. it collapses into the state $|1\rangle$) is

(11) $\langle \mathbf{1} \rangle_\psi = |\langle 1|\psi\rangle|^2 = \sin^2(\theta/2) = ½(1-\cos(\theta))$

Further, we also can calculate the probability that the object is polarized into a direction given by the eigenvectors of σ_x, namely $|\nearrow\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ or $|\nwarrow\rangle = \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$

(12) a. $\langle \nearrow \rangle_\psi = ½\, |\langle 0+1|\psi\rangle|^2 = ½\, (1+\sin(\theta)\cdot\sin(\Delta/2))$
b. $\langle \nwarrow \rangle_\psi = ½\, |\langle 0-1|\psi\rangle|^2 = ½\, (1+\cos(\theta)\cdot\sin(\Delta/2))$

In this case the calculated probability also depends on the phase shift Δ.

Finally, it is straightforward to calculate the corresponding expectations for the Pauli operators in state $|\psi\rangle$:

(13) a. $\langle \sigma_x \rangle_\psi \quad = \quad \sin(\theta)\cdot\cos(\Delta)$
b. $\langle \sigma_y \rangle_\psi \quad = \quad \sin(\theta)\cdot\sin(\Delta)$
c. $\langle \sigma_z \rangle_\psi \quad = \quad \cos(\theta)$

In case of states with no phase shift between the two components $|0\rangle$ and $|1\rangle$ – i.e. $\Delta = 0$ – we see that the operator's σ_y expectations are always zero and the expectations for σ_x and σ_z are sin(θ) and cos(θ), respectively. It is interesting to calculate the standard deviation $\Delta_\psi(\sigma_x)$ and $\Delta_\psi(\sigma_z)$ in this case:

(14) a. $\Delta_\psi(\sigma_x) = |\cos(\theta)|$
b. $\Delta_\psi(\sigma_z) = |\sin(\theta)|$

Figure 3 shows the expectation values for the complementary observables σ_x and σ_z including an indication of the corresponding standard derivations.

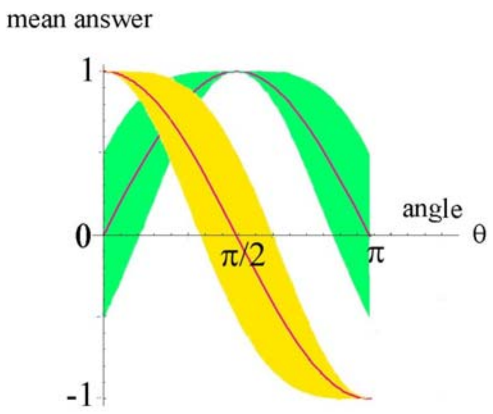

Figure 3: Expectation values for the complementary observables σ_x (in green) and σ_z (in yellow) in case of a cubit states with zero phase shift Δ. The graph also indicates the corresponding standard deviations.

The picture clearly demonstrates the complementary character of the observables σ_x and σ_z: there is no state where a simultaneous measurement of both observables is possible without any uncertainty. Instead, if one observable can be measured without deviation, the other one can be measured with maximal uncertainty only.[11]

In quantum theory, complementary observables are in a sharp contrast to opponent observables. A pair of opponent observables makes a precise simultaneous measurement possible but the values of the two observables are opponent in these cases (e.g. +1 for one observable and −1 for the other, and vice versa). In the present case of Pauli operators we simply can derive the opponent counterparts of the Pauli observables by multiplying them with −1 (corresponding to a phase shift of $\Delta = \pi$). Figure 4 illustrates the opponent observables σ_z and $-\sigma_z$. Evidently, at $\theta = 0$ and $\theta = \pi$ the measurement is sharp and the results are +1, −1 and −1, +1, respectively.

[11] The standard uncertainty principle (5) repeated here in the case of the observables σ_x and σ_z

$$\Delta_\psi(\sigma_x)\,\Delta_\psi(\sigma_z) \geq \tfrac{1}{2}\langle \sigma_x\sigma_z - \sigma_z\sigma_x\rangle_\psi$$

is not strong enough to make this prediction since in case of Δ=0 the lower boundary on the right hand site becomes zero. However, there is an additional term for the lower boundary that usually is dropped but required in the present case in order to give a non-zero lower boundary. The details can be found in appendix A.

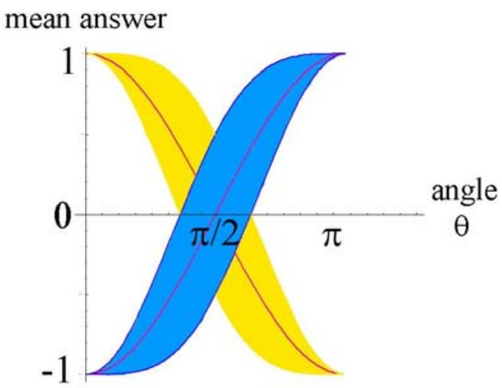

Figure 4: Expectation values for the opponent observables σ_z (yellow) and $-\sigma_z$ (blue) in case of a cubit state with zero phase shift Δ including an indication of the corresponding standard derivations.

As with classical bits it is possible to put qubits together in order to build and store more information. In quantum theory complex systems are constructed by using tensor products ⊗. This operation applies both to vectors of the Hilbert space $|u\rangle \otimes |v\rangle$ and to linear operators **a** ⊗ **b**. If the context excludes misunderstandings, it is convenient to miss out the ⊗. Hence we will write $|011\rangle$ instead of $|0\rangle\otimes|1\rangle\otimes|1\rangle$ and **011** instead of **0**⊗**1**⊗**1**.

In quantum theory, the existence of entangled states of several qubits is of greatest importance. In such entangled states a single qubit doesn't have a definite state. However, the system of the qubits (as a whole) is in a definite state. This can be tested by fixing the first qubit (by a local measurement). Then the result of measuring the second qubit is always definite, i.e. 100% predictable. This leads to so-called nonlocal effects as described in the EPR experiment (Einstein, Podolsky, & Rosen, 1935), at least in the standard physical case where the underlying elementary objects have a spatial distribution such as single electron spin qubits or single photon polarization qubits.

Recently, several researchers have mentioned the cognitive relevance of quantum theory (e.g. Aerts et al., 2005; Atmanspacher et al., 2002; Busemeyer et al., 2006; Franco, 2007; Khrennikov, 2003). The main arguments for this claim are the order-dependence of questions and interference effects found in simple decision tasks (for a recent overview see Blutner, forthcoming). The effect of entanglement is much less spectacular in the cognitive domain since the underlying 'objects' don't have any spatial organization. As we will see, entanglement is closely related to the well-known binding problem of cognitive science (Smolensky, 1990) in this case.

Closing this section we will give a simple illustration of quantum entanglement in a two qubit state. Suppose we have prepared the following entangled pure state describing two objects 1 and 2, which is called the Bell-state:

(15) $\psi_B = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$.

We will demonstrate now that classical probabilities lead to wrong predictions concerning the measurement of (stochastic) *correlations* between certain observables.

Let's assume four binary random variables A_1, A_2, B_1, B_2 with range $\{-1, +1\}$as described in classical probability theory. The intention is that A_1 and A_2 represent the stochastic outcome of two different kinds of measurement concerning the first object, and B_1 and B_2 represent the stochastic outcome of two kinds of measurement concerning the second object. Defining correlations between pairs of these random variables in the usual way,

(16) $C(A_i, B_j) = E_\mu(A_i B_j)$

we are able to derive the following inequality, which is a version of Bell's inequality (cf. Vedral, 2006)[12].

(17) $E_\mu(A_1 B_1) + E_\mu(A_2 B_1) + E_\mu(A_1 B_2) - E_\mu(A_2 B_2) \leq 2$

The inequality expresses an upper boundary which restricts the correlations between the random variables A_i and B_j. The same inequality also should apply in the quantum case if quantum theory can be rewritten as a classical theory with hidden variables (an idea famously associated with Albert Einstein). However, it is possible to demonstrate that quantum mechanics makes predictions that violate the "Bell inequality" in the setup considered in the EPR thought experiment. A simple example is as follows:

(18) A_1: $\sigma_z \otimes I$; A_2: $\sigma_x \otimes I$, B_1: $-\frac{1}{\sqrt{2}} I \otimes (\sigma_z + \sigma_x)$, B_2: $\frac{1}{\sqrt{2}} I \otimes (\sigma_z - \sigma_x)$

Using the corresponding Pauli operators then the following correlations can be calculated in the Bell-state ψ_B:

(19)
a. $E(A_1 B_1) = -\frac{1}{\sqrt{2}} \langle \sigma_z \otimes (\sigma_z + \sigma_x) \rangle_{\psi_B} = \frac{1}{\sqrt{2}}$
b. $E(A_2 B_1) = -\frac{1}{\sqrt{2}} \langle \sigma_x \otimes (\sigma_z + \sigma_x) \rangle_{\psi_B} = \frac{1}{\sqrt{2}}$
c. $E(A_1 B_2) = \frac{1}{\sqrt{2}} \langle \sigma_z \otimes (\sigma_z - \sigma_x) \rangle_{\psi_B} = \frac{1}{\sqrt{2}}$
d. $E(A_2 B_2) = \frac{1}{\sqrt{2}} \langle \sigma_x \otimes (\sigma_z - \sigma_x) \rangle_{\psi_B} = -\frac{1}{\sqrt{2}}$

Calculating the left hand side of inequality (17) yields $2\sqrt{2} > 2$. Hence, we have found a clear violation of Bell's inequality (17) and this demonstrates that quantum theory cannot be replaced by a classical theory with hidden parameters that are stochastically modelled. The importance of this inequality will be demonstrated when the question is raised if cognitive phenomena can be described in terms of classical probabilities or if a more general theory in terms of quantum probabilities is required. The essential point is that when the inequality is violated, it indicates that the underlying state is entangled; that is, the aspects of the state exposed by the different experiments are interdependent.

4 Two qubits for C.G. Jung's theory of personality

The main argument for applying the formal apparatus of quantum theory to the domain of cognition has to do with the flexibility, instability, and context-dependency of meaningful cognitive entities that manifest themselves as fleeting contents of conscious experience. For example, in the domain of language, words are floating freely in a polyvalent state representing a variety of different uses. As the properties of small particles are not absolute and determined until observing them, the properties of word tokens are not determined until conscious apprehension. Similarly, impressions, ideas and opinions are conceptual entities with analogous properties and likewise invite an analysis in terms of quantum theory. Recently, Aerts et al. (2008) have applied a quantum analysis to a cognitive setting where individuals' opinions were probed. Three different questions for the opinion poll were considered:

[12] You can also consult the following web site: http://www.quantiki.org/wiki/index.php/Bell's_theorem

(20) Q1 : Are you in favor of the use of nuclear energy?
Q2 : Do you think it would be a good idea to legalize soft-drugs?
Q3 : Do you think capitalism is better than social-democracy?

Interestingly, in such situations most people don't have a predetermined opinion. Instead, the opinion is formed to a large extend during the process of questioning in a context-dependent way. That means, opinions formed by earlier questions can influence the actual opinion construction. Aerts et al. (2008) assume that in such situations Bell's inequalities can be violated, and, consequently, the values of the corresponding probabilities cannot be fitted into a classical (Kolmogorovian) probability model. Although Aerts et al. (2008), don't present explicit empirical data to prove the point, their argumentation is still convincing. In a related study, Conte et al. (2008) come to a similar conclusion.

Personality test can similarly be seen as a cognitive setting where individual opinions are probed. Forced choice questions such as presented in the examples (1)-(3) are a suitable material for checking the statistical framework and looking for quantum effects. In this section we will show that the phenomenon of entanglement fits very naturally into the Jungean framework, and consequently we expect violations of Bell's inequalities.

In quantum information science, the qubit proves as the simplest possibility to represent forced choice questions. In the last section we have shown how qubits can be modeled using a two-dimensional Hilbert space. Definitely, in this treatment every question/observable can be represented as linear combinations of the three Pauli operators. Ignoring phase shifts for the moment, i.e. assuming $<\sigma_z> = 0$, we are left with two independent operators, σ_z and σ_x. As explained before, the operator σ_z gives a definite *yes* (*no*) answer in case the system is in the base state *0* (*1*). On the other hand, the operator σ_x gives a definite *yes* (*no*) answer in case the system is in the superposed state *0+1* (*0–1*). Opponent questions can be formulated by negation (multiplication with –1). Hence $\{\sigma_z, -\sigma_z\}$ and $\{\sigma_x, -\sigma_x\}$ form two independent systems of opponent questions.

In contrast, a classical bit-state could be described by a system consisting of two possible worlds, {*0*, *1*}. In such a system only one independent system of opponent question is possible asking whether the state *0* (or *1*) is realized. Of course, using the Cartesian product space, n-bit states can be realized. Though correlations between the corresponding questions ('random variables') can be expressed in a classical system with a Kolmogorovian probability measure, the idea of entanglement cannot be expressed in this way.

The following is a straightforward way to express Jung's four psychological functions within a single qubit system.

(21) $T = \sigma_z$, $F = -\sigma_z$ (rational functions)
$S = \sigma_x$, $F = -\sigma_x$ (irrational functions)

This is a stipulation expressing that the rational functions are opposites from each other. The same applies to the irrational functions. The main motivation for this stipulation is that it gives a simple explanation of the eight basis types that result from the different proportions of the expectation values of the psychological functions (cf. formula (13)). This is illustrated in Figure 5.

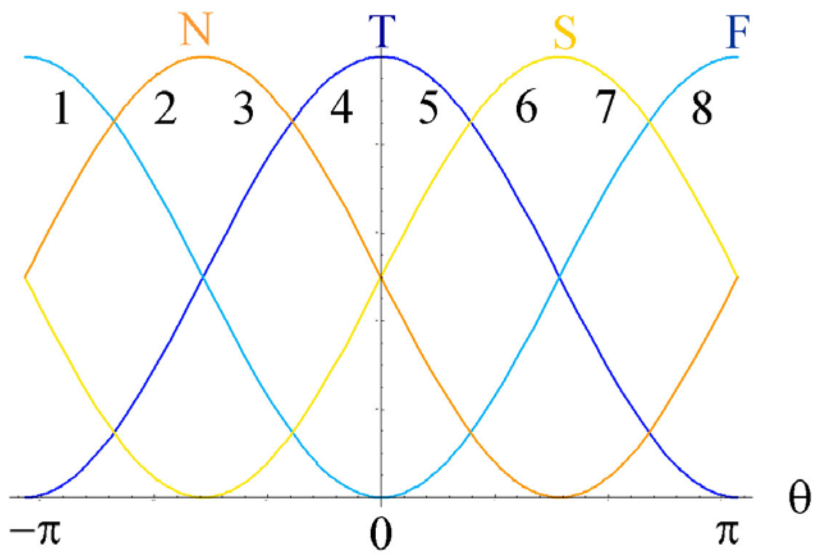

Figure 5: The 8 types as resulting from different proportions of the expectation values for N, T, S, and F.

Obviously, exactly eight configurations of psychological functions can be realized corresponding to the eight segments of Figure 1.

1. F>N>S>T	5. T>S>N>F
2. N>F>T>S	6. S>T>F>N
3. N>T>F>S	7. S>F>T>N
4. T>N>S>F	8. F>S>N>T

Further, this schema satisfies the first restriction we have formulated in connection with C.G Jung's theory: if the superior function is rational/irrational then the secondary function must be irrational/rational, and this alternation is continued along the ranking hierarchy.

In the following we will assume that the different attitudes of a Self (extraverted vs. introverted) can be expressed by a second qubit. This gives the possibility not only to model pure extroverted or pure introverted Selfs but also superpositions of extroverted and introverted types. We assume that a corresponding operator σ_z is available with eigenvectors that represent the two opponent attitude states extraversion and introversion. Hence, we can write down two observables for extraversion and introversion:

(22) $E = \sigma_z, I = -\sigma_z$ (**E**xtraversion, **I**ntroversion)

Though not explicitly discussed in the literature, it also makes sense to introduce an observable that registers states between extraversion and introversion. We will call it M and assume that it gives a definite *yes* answer for an equal superposition of pure introverted and pure extraverted states, i.e.

(23) $M = \sigma_x$ (inter**M**ediate)

For constructing the full Hilbert space we will make use of the tensor product. Hence, if $|\alpha\rangle$ is expressing a certain state of attitude (extraverted, introverted or a superposition of both) and $|\psi\rangle$ a certain psychological state reflecting a certain ranking of the four psychological functions, then $|\alpha\rangle\otimes|\psi\rangle$ is expressing a psychological state $|\psi\rangle$ in the attitude $|\alpha\rangle$.

In discussing the type dynamics crucially involved in C.G. Jung's theory (section 2) we have claimed that each person is realizing more than one psychological function, and we have stressed the point that opponent psychological functions are realized with contrasting attitudes. In the present formal theory, this idea is expressed by the notion of entanglement. Hence, we will claim that the attitudes are entangled with the psychological functions. Formally, we can write such entangled states $|\Psi\rangle$ in the following way,

(24) $|\Psi\rangle = |\alpha\rangle \otimes |\psi\rangle - |\alpha\rangle^{\perp} \otimes |\psi\rangle^{\perp}$

where $\perp$ is an operation that gives the orthogonal state of a certain qubit state. The Bell-state discussed earlier and repeated here

(11) $\psi_B = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

is an instance of the entangled states we have in mind. Using the Bloch sphere and ignoring phase factors we can write $|\psi\rangle$ and $|\psi\rangle^{\perp}$ in the following way:

(25) $|\psi\rangle = \cos(\theta/2)\,|0\rangle + \sin(\theta/2)\,|1\rangle, \qquad |\psi\rangle^{\perp} = \sin(\theta/2)\,|0\rangle - \cos(\theta/2)\,|1\rangle$

Here $|0\rangle$ and $|1\rangle$ are the eigenvectors of the system of rational functions {T, F}; the state $|0\rangle$ conforms to the state with a maximal expectation value of T and the state $|1\rangle$ indicates the state with maximal expectation value of F (= –T).

Not unsurprisingly, we can reinterpret Figure 1 now as a two-dimensional cut of the Bloch sphere showing the possible states $|\psi\rangle$ with a zero phase shift Δ. Obviously, for $\theta = 0$ we get a maximal expectation value of T, for $\theta = \pi/2$ we get a maximal expectation of S, and so on.

In the same way, we can parameterize $|\alpha\rangle$:

(26) $|\alpha\rangle = \cos(\alpha/2)\,|0\rangle + \sin(\alpha/2)\,|1\rangle, \qquad |\alpha\rangle^{\perp} = \sin(\alpha/2)\,|0\rangle - \cos(\alpha/2)\,|1\rangle$

In this case $|0\rangle$ and $|1\rangle$ stand for extraversion and introversion. In section 3 we have seen that quantum entanglement is a phenomenon in which the states of two or more objects are linked together so that one object can no longer be adequately described without fully mentioning its counterpart. This interconnection leads to correlations between observable physical properties of systems. In the present case where we assume that the psychological functions are entangled with the attitudes we expect, for example, that extraverted thinkers have a dominant **F**eeling-function in the introverted attitude. Similarly for introverted **S**ensation types we expect a dominant i**N**tuition-function in the extraverted attitude. As a consequence of this entanglement, we can calculate the following correlations when the operators {E, M} for the attitudes and {T, S} for the psychological functions are involved:

(27)
a. $C_\Psi(E, T) = \frac{1}{\sqrt{2}}\langle E \otimes T\rangle_\Psi = \cos(\alpha+\theta)$
b. $C_\Psi(E, S) = \frac{1}{\sqrt{2}}\langle E \otimes S\rangle_\Psi = \sin(\alpha+\theta)$
c. $C_\Psi(M, T) = \frac{1}{\sqrt{2}}\langle M \otimes T\rangle_\Psi = \sin(\alpha+\theta)$
d. $C_\Psi(M, S) = \frac{1}{\sqrt{2}}\langle M \otimes S\rangle_\Psi = -\cos(\alpha+\theta)$

For instance, we expect a high correlation $C_\Psi(E, T)$ if the personality is characterized by both a high percentage of extraverted thinking and a high percentage of introverted feeling. If the person is characterized by an entangled state $\Psi(\alpha, \theta)$, then the correlation $C_\Psi(E, T)$ is maximum for $\alpha = \theta = 0$, for instance. Bell's inequality (17) can be applied to the present situation and it says the following:

(28) $C_\Psi(E, T) + C_\Psi(M, T) + C_\Psi(E, S) - C_\Psi(M, S) \leq 2$

Inserting the quantum results given in equation (27), the inequality becomes

(29) $2\sin(\alpha+\theta) + 2\cos(\alpha+\theta) \leq 2$

It is not difficult to see that this inequality is violated in case $0 < \alpha+\theta < \pi/2$. The violation is maximum if $\alpha+\theta = \pi/4$, for example $\alpha = 0$ (pure extraversion) and $\theta = \pi/4$ (state between thinking and sensing, i.e. at the border between region 5 and 6 of Figure 1). Calculating the left hand site of inequality (29) gives $2\sqrt{2} > 2$ (you see the close correspondence to the EPR experiment discussed in connection with the Bell state in equation (19)). This means that for extraverted thinkers who are strongly supported by the sensing function, we find a maximum violation of Bell's inequality (28). It is obvious that for other types of personalities other observables must be used in order to measure true violations of the inequality.

Empirical evidence supporting violations of Bell's inequalities proves that quantum mechanics cannot be replaced by a classical theory with hidden variables. Experimentally it is notoriously difficult to prove violations of one of Bell's inequalities. However, in the physical domain there is now convincing evidence for violations (e.g. Aspect, Dalibard, & Roger, 1982). Though recent attempts to prove Bell's inequality violation in the mental domain were not completely successful (cf. Conte et al., 2008), the situation is not hopeless. Possibly it will be more successful in the present case of personality diagnostics. In this connection it is important to bring up that the experimental situation in cognitive science is possibly different from the situation in particle physics. Particles act in agreement with certain probabilistic laws but they cannot directly tell us expectation values. For human subjects, however, we can assume that they can give us information directly related to these probabilities. Hence, we expect that human subjects cannot only give yes/no answers; they also can tell us which answer has a higher or a lower probability. Further, they have intuitions about the certainty of a given answer. Theoretically spoken, they have some insight into the underlying quantum probability. And this possibly can simplify the task of empirically finding Bell violations in the domain of personality types.

Next, we have to discuss the second constraint stating that contrasting attitudes have opponent psychological function. This constraint is an immediate consequence of the (type-dynamic) assumption about the entanglement of attitudes and psychological functions as formally described by (24). As a special instance of (24) we take $\alpha = 0$ corresponding to pure states of extraversion/introversion:

(30) $|\Psi\rangle = |0\rangle\otimes|\psi\rangle - |1\rangle\otimes|\psi\rangle^\perp$

We can calculate now the expectation values for the psychological functions under the two conditions (i) E=1 and (ii) I=1. We consider region 5 only (cf. Figure 1), i.e. we take $0 < \theta < \pi/4$. In this region we have $1 > \cos(\theta) > \sin(\theta) > 0$ (corresponding to extraverted thinkers with sensing as auxiliary function):

(31)
$\langle T / E = 1\rangle_\psi = \cos(\theta)$ $\quad\langle F / I = 1\rangle_\psi = \cos(\theta)$
$\langle S / E = 1\rangle_\psi = \sin(\theta)$ $\quad\langle N / I = 1\rangle_\psi = \sin(\theta)$
$\langle N / E = 1\rangle_\psi = -\sin(\theta)$ $\quad\langle S / I = 1\rangle_\psi = -\sin(\theta)$

$$\langle F / E = 1\rangle_{\psi} = -\cos(\theta) \qquad \langle T / I = 1\rangle_{\psi} = -\cos(\theta)$$

Using these results, we get the ranking 1T 2S 3N 4F for the extraverted attitude, and for the inverted attitude we get the ranking 1F 2N 3S 4T. Since the corresponding functions are opponents ({T, F} at rank 1, {S, N} at rank 2 etc.), the second restriction of C.G. Jung's theory is satisfied. Obviously, this is a consequence of the entanglement between psychological functions and attitudes.

It is possible to rank the eight attitude-specific psychological functions within one and the same ordinal scale if we make certain stipulations about the relative strength ρ of the two parts of the entangled state. So far we have assumed $\rho = 1$, i.e. both parts are equally strong. According to the idea of a dynamic process of type elaboration, the process starts without entanglement ($\rho = 0$) and during the process the parameter ρ is slowly increased up to $\rho = 1$ (reflecting the case of an "ideal" personality which is integrating its own shadow). In dependence on the value of ρ, we have to distinguish two different cases: (a) $\cos(\theta) \cdot \rho < \sin(\theta)$, where the ranking (32a) applies, and (b) $\cos(\theta) \cdot \rho > \sin(\theta)$, where the ranking (32b) applies:

(32) a. 1ET 2ES 3IF 4IN (for $\rho < \tan(\theta)$)
b. 1ET 2IF 3ES 4IN (for $\rho > \tan(\theta)$)

The first configuration (32a) almost agrees with the ordering suggested by C.G. Jung for the first four functions. Only the third and fourth functions are switched. This configuration applies if the type dynamics has not yet fully developed (and/or the type is close to the boundary between ET and ES). In contrast, the exceptional ranking (32b) applies for fully developed personalities which have integrated their own shadow to a high degree. Of course, it is an empirical question if this possibility can be realized in the type dynamic reality.

Finally, I will make some remarks about cognitive operations. In quantum mind theory (Aerts et al., 2008; Aerts et al., 2005; Busemeyer et al., 2006; Khrennikov, 2003), cognitive operations are modelled by unitary transformations (i.e. transformations which do not change the scalar product of two involved states). Interestingly, the unitary transformation called X-gate in the quantum computation community realizes the cognitive operation connected to Jung's idea of the shadow. This transformation maps vectors into its orthogonal counterparts. In the field of socionics many other operations are discussed that can be used to define various relations between different types of personalities. However, a careful discussion of the corresponding unitary operations in the quantum theoretic framework goes beyond this introductory article and must be left for another occasion.

5 Conclusions

The main claim of this article was to demonstrate that quantum theory, as a mathematical construction, provides a natural framework for giving a sound foundation of C.G. Jung's theory of personality. This claim has nothing to do with any speculations about the molecular, biochemical basis of the macro-psychological construction of the Self and its constituents. In this regard we fully agree with Aerts et al. (2008) who state this general methodological point as follows:

> "For clarity, we emphasize that it is the abstracted formalism which is 'borrowed' from quantum theory, not in any way its microphysical ontology of particles and fields. Our approach thus concerns the formal structure of models that are able to describe cognitive entities and processes with contextuality, not the substrate that implements them in the brain." (Aerts et al., 2008, p.1)

The basic tenet is simply that notions from quantum physics fit better with the conceptual, algebraic and numerical requirements of the cognitive domain than the traditional modeling of concepts in terms of Boolean algebras and the classical probabilities based upon it. Using the quantum framework, we were able to demonstrate that the four psychological functions are in strict correspondence to the Pauli operators σ_x, $-\sigma_x$, σ_z, $-\sigma_z$ of a single qubit state. It is straightforward then to describe the eight basis types of personalities (resulting from the different proportions of the four psychological functions) as different proportions of the expectation values of the relevant Pauli operators (ignoring phase shifts in the underlying qubit states). Further, it was shown that the quantum theoretic notion of entanglement is very useful to express the Jungian idea of type dynamics and his observation that opponent psychological functions are realized in one and the same person with contrasting attitudes.

Although we think that our basic assumptions fit naturally with the Jungian framework, there is enough room for speculations concerning the details that have still to be filled. This concerns not only the preference ordering of the eight attitude-specific psychological functions depending on the type dynamics and the critical parameter ρ. It also concerns the assignment of a qualification as conscious or unconscious function. Further, the consequences of assuming non-zero phase factors have not yet been discussed. It is superfluous to say that we were not able to tell anything about an empirical verification of C.G. Jung's theory, or related theories such as proposed by Myers-Briggs or the representatives of socionics.

The present framework provides a mathematical instrument to formulate new ways of empirical testing. Most importantly, the present framework is much more general than the standard statistical framework (as used in MBTI, for instance). The future will show whether we need this generalization, and the answer will be definitely positive if a real violation of one of Bell's inequalities can be demonstrated. Further, the quantum theoretical instruments of unitary operations can be extremely useful to help socionics in classifying and investigating the relations between different types.

Some critics have claimed that the expression *quantum* should be reserved for applications only with a real, non-zero phase. However, also with the present simplifying assumption of ignoring phase shifts, real quantum effects are possible, such as a true uncertainty principle, the existence of entanglement, unitary operations on entangled states, and violations of Bell's inequalities. In quantum mind theory, non-zero phase shifts are usually connected with interference phenomena (e.g. Khrennikov, 2006). Hence, the present treatment can be principally extended to look for interference effects in the domain of personality psychology, including the phenomenon of opinion building in self-reflection.

Acknowledgments

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Appendix A: Proving the Uncertainty Principle

Fick (1968) gives the following general derivation of Heisenberg's uncertainty principle (p. 191ff):

$\boldsymbol{a} = \mathbf{a} - \langle \mathbf{a} \rangle_\psi \mathbf{1}$
$\boldsymbol{b} = \mathbf{b} - \langle \mathbf{b} \rangle_\psi \mathbf{1}$
$\Delta^2(\mathbf{a}) = ||\boldsymbol{a}\psi||^2$
$\Delta^2(\mathbf{b}) = ||\boldsymbol{b}\psi||^2$

We need Schwarz' inequality: $||\varphi|| \cdot ||\chi|| \geq |\langle \varphi|\chi\rangle|$.

$$\Delta^2(\mathbf{a})\, \Delta^2(\mathbf{b}) = ||\boldsymbol{a}\psi||^2\ ||\boldsymbol{b}\psi||^2 \geq |\langle \boldsymbol{a}\psi|\boldsymbol{b}\psi\rangle|^2$$
$$= \langle \boldsymbol{a}\psi|\boldsymbol{b}\psi\rangle\langle \boldsymbol{b}\psi|\boldsymbol{a}\psi\rangle = \langle \psi|\boldsymbol{ab}\psi\rangle\ \langle \psi|\boldsymbol{ba}\psi\rangle$$
$$= \langle \boldsymbol{ab}\rangle_\psi\ \langle \boldsymbol{ba}\rangle_\psi$$

Decomposing the operators $\boldsymbol{ab}$ and $\boldsymbol{ba}$ in the two Hermitean operators ½ ($\boldsymbol{ab} + \boldsymbol{ba}$ and 1/2i ($\boldsymbol{ab} - \boldsymbol{ba}$) we get

$\boldsymbol{ab} = (\boldsymbol{ab} + \boldsymbol{ba})/2 + i\,(\boldsymbol{ab} - \boldsymbol{ba})/2i$
$\boldsymbol{ba} = (\boldsymbol{ab} + \boldsymbol{ba})/2 - i\,(\boldsymbol{ab} - \boldsymbol{ba})/2i$

From that the unequality reads

$$\Delta^2(\mathbf{a})\, \Delta^2(\mathbf{b}) \geq (\langle \boldsymbol{ab} + \boldsymbol{ba}\rangle/2)^2 + (\langle \boldsymbol{ab} - \boldsymbol{ba}\rangle/2i)^2$$

Inserting the definitions for **a** and **b** the following inequality results:

(*) $\quad \Delta^2(\mathbf{a})\, \Delta^2(\mathbf{b}) \geq (\langle \mathbf{ab} + \mathbf{ba}\rangle/2 - \langle \mathbf{a}\rangle\langle \mathbf{b}\rangle)^2 + (\langle \mathbf{ab}\rangle - \langle \mathbf{ba}\rangle/2i)^2$

Erasing the first (non-negative) part, the usual formulation will result:

(**) $\quad \Delta(\mathbf{a})\, \Delta(\mathbf{b}) \geq |½\, i < [\mathbf{a},\mathbf{b}] >|$.

In case of the Pauli operators σ_x and σ_z and state $|\psi\rangle = \cos(\theta/2)\,|0\rangle + \sin(\theta/2)\,|1\rangle$ the inequality (**) makes a trivial statement only:

$\Delta\psi(\sigma_x)\, \Delta\psi(\sigma_z) \geq 0$ (since $½\, |\langle[\sigma_x, \sigma_z]\rangle_\psi| = |\langle\sigma_y\rangle_\psi| = 0$).

However, if we respect the original inequality (*) then we get the following stronger result:
$\Delta\psi(\sigma_x)\, \Delta\psi(\sigma_z) \geq |\sin(\theta)\cos(\theta)|$ (cf. Figure 3).