POLISH LOGIC

some lines from a personal perspective

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Abstract

Without aspiring to historical or systematic completeness, this paper presents an informal survey of some lines in 20th century Polish logic, together with some general historical background, and making special reference to the author's environment in Opole, and the contributions by her teacher Jerzy Słupecki. Further published material can be found in the appended bibliography.

Contents

PREFACE

The flowering of Polish logic is a rich and amazing phenomenon, spanning a century by now. Painting a picture of something this rich and complex is not easy, and many brush strokes by many hands are needed for a grand canvas. This paper by Urszula Wybraniec-Skardowska is one such contribution, following one dimension, prepared for an informal talk at the Stanford logic colloquium in May 2009. It gives a personal view of some main lines in Polish logic and its links to other disciplines, and in particular, adds interesting details on the role of her teacher Jerzy Słupecki and his post-war efforts.

Polish logic has radiated far across our continent, all the way to the Dutch North Sea coast. I even felt its influence in my student days. Reading Rasiowa & Sikorski's masterly book *The Mathematics of Metamathematics* was an eye-opener, and many have followed since. In the 1970s and 1980s, I got to know many mathematical and philosophical logicians in Poland, including Wojciech Buszkowski, Jerzy Perzanowski, Ryszard Wójcicki, and others, and through visits, had my first direct experiences with a vibrant intellectual culture across the Iron Curtain. What surprised me was the intellectual scope and creativity that flourished despite the rigorous of communist rule. I have profited from Polish work on many new themes that set international agendas extending the pre-Second World War tradition: algebraic logic, computational logics of many varieties, such as knowledge representation, program structure, default reasoning, logic and natural language, logic of empirical theories in science, and so on. Many publications testify to these fruitful contacts, and at some stage, my Polish publication ventures with Ossolineum even led to a bank account in a communist country – quite a thrill.

Shared intellectual interests are only one half of a successful human encounter. I have always been touched by the openness, warmth, and generosity of Polish colleagues, and there is also a long history of personal memories, ranging from discussing the first year of Solidarity in a small Krakow apartment to visiting a cemetery with Jan Woleński on the night of a national holiday. At a still more personal level, I recall worrying about roses to bring to Helena Rasiowa's birthday in Warsaw, or home cooking in Professor Wroński's place, after we had taken our young sons up in the Tatry mountains from Zakopane, a delightful pilgrimage where Polish logicians preceded us. Too many memories to relate!

Polish logic continues to amaze us with its vigor and new directions. And contacts with Holland persist, if only through current students, who stand in a century-old tradition: Dutch universities have always had their fair share from Poland. I even note that Urszula's historical maps show her whole country moving westward, surely in the direction of Amsterdam. Happy travels, and welcome…

Johan van Benthem, Holland, July 2009

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Logic Seminar, 29 May 2009, Department of Philosophy, Stanford University[1]

In this talk, we look at some lines in Polish logic of the 20th century. There is no pretense at a complete history of this rich subject: nor do I claim great depth of coverage, for which the reader should seek other sources (I give some references). This text was a light introduction for an informal talk at Stanford, from my personal perspective as a Polish student and logic professional over the last 40 years. My main aim is to tell you about my teacher Jerzy Słupecki and his postwar efforts in Opole, but I place his activities in the earlier tradition of the Warsaw School, as well as some postwar developments in our country.

For a start, to understand Polish logic, in my personal view (which may be controversial to some), you also need to understand a bit of Polish history.

1   Poland in the 20th century[2]

Polish logic and the logic of Polish history are tied together. To me, national independence and freedom of thought and speech are inseparable.

Look at the map of Poland under the Jagiellonian dynasty, in the 17th century, and 20th century Poland after World War I (1918) and World War II (1945):

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[1] I would like to thank Johan van Benthem for his crucial help in preparing this text.

[2] In this brief historical survey, I rely heavily on Roman Marcinek [2005].
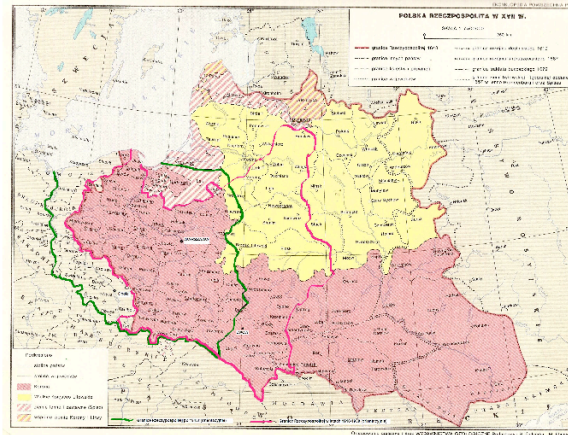
A series of wars in the 17th c. led to territorial loss and economic disaster. Under the Wettin dynasty (18th c.), Poland was a pawn of foreign rulers. The result was the First Polish Partition, among Russia, Austria and Prussia (1772). Attempts at reform (Constitution of May 1791) were blocked by other nations and internal opposition (Targowica, 1792). The Second and Third Partitions (1793, 1795) and the Kościuszko Insurrection (1794) sealed Poland's fate. Hopes of independence revived during the First World War, when Polish volunteers fought on the side of Prussia and Austria (Piłsudski's Legion) and Russia (Puławy Legion). Following revolutions in Russia and Germany, the Poles now claimed independence. On November 11, 1918, Józef Piłsudski became Head of State. Since that date, Poland has been independent. While efforts were under way to rebuild the new state and win international recognition, heavy fighting went on at its borders (Polish-Soviet War, 1919-20; Wielkopolska Uprising, 1918-19; Silesian Uprisings, 1919-21). After a first constitution in 1921, Piłsudski in 1926 acted to restore law and order. The economy rebounded. Polish culture, arts and science could develop freely. Poland seemed safe:

Józef Piłsudski

But history intervened. The invasion by Germany on 1 September 1939 and the Soviet Union on 17 September 1939 begins the Second World War. Poland was again divided, under the Nazi-Soviet pact of 23 August 1939:
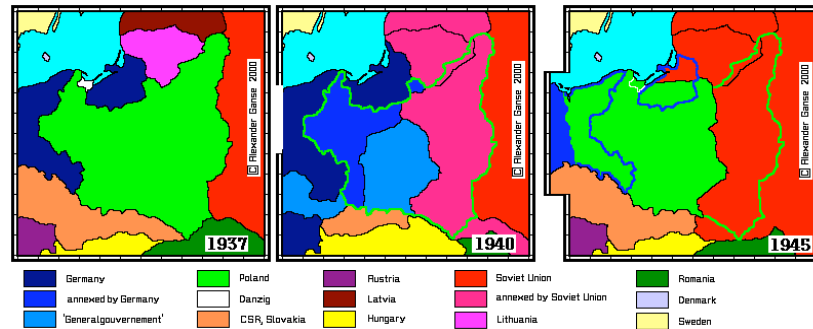
A period of unprecedented persecution and extermination of Polish citizens (especially Polish Jews) began, as well as massive scale pillage. A Polish Government-in-Exile was constituted. Likewise, the Polish Armed Forces were established, fighting hand in hand with the Allies (Narvik, Batle of Britain, Tobruk, Monte Casino). On the home front in Poland itself, the resistance movement gradually evolved into an Underground State. Teaching was conducted in secret. Still, scientific and cultural activity was broken.

Warsaw after a bombing   *The reconstructed Royal Palace*

At Teheran (1943) and Yalta (1945), the "Big Three" put Poland in the post-word Soviet zone. The new rulers used terror (the UB secret police) and the help of Soviet troops, while rigging elections. The political system was modified after Stalin's death (1956), but liberal policies were later abandoned. Student riots in 1968 were suppressed, and in 1970 army and police killed workers on strike in towns on the Baltic coast. In August 1980, strikes at the Gdansk Shipyard and other enterprises led to an independent trade union "Solidarity"

(led by Lech Wałęsa). This relative political freedom was cut short by a period of martial law from December 1981. The last years of communist rule in Poland were marked by a stagnant economy and social apathy.

Round table talks between government and opposition (spring of 1989) led to a partly democratic election that June, which was a sweeping victory for the opposition. Poland was finally on the way towards economic recovery and consolidation of its sovereignty (also through memberships in NATO). Today, the Republic of Poland is a member state of the European Union.

2   How Polish logic started around 1900: the Lvov-Warsaw School

Now that we have seen the historical background before and during the last century, here is our scientific story. We cannot talk about Polish Logic without noting the achievements of The Lvov-Warsaw School and The Warsaw School of Logic (WSL) that grew out of the former. How did these communities arise?

The Lvov-Warsaw School was founded at the end of the 19th century and its activity lasted until the Second World War. Its founder was Kazimierz Twardowski – an associate of Franz Brentano – who, at the age of 29 (in 1895) was appointed professor of philosophy in Lvov, then an Austrian town.

Kazimierz Twardowski

Professor Twardowski was a man with a special charisma as a teacher and tutor, uniquely tied to ordinary friendliness, which built the community of the School. He promoted more than 30 Ph.D. students, who later earned their

professorships. During the inter-war period, his former students held chairs in philosophy departments at all of the Polish universities, with the exception of the Catholic University of Lublin. Among the characteristic features of the School were its serious approach to philosophical studies and teaching of philosophy, dealing with philosophy and propagation of it as an intellectual and moral mission, passion for clarity and precision, as well as exchange of thoughts, cooperation with representatives of other disciplines at home and abroad, and also fruitful collaboration with mathematicians. The School found its own scientific style of philosophizing and met international standards of training, rigor, professionalism and specialization. The Lvov-Warsaw school was the first of its kind in Poland. At the same time, its community managed to establish contacts between its philosophers and world philosophy.

3   The Warsaw Logical School: its main figures and ideas[3]

The cooperation of the Lvov-Warsaw philosophers with mathematicians evolved to a logical school based in Warsaw[4] (WSL) that functioned during the interbellum[5]. Zygmunt Janiszewski (1988-1920), belonging to the first generation of Twardowski's students, a mathematician, also vitally interested in logic and foundations of mathematics, was one the main initiators of the mutual cooperation between mathematicians and philosophers. The WSL

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[3] I rely on extensive excerpts from papers by J. Słupecki [1972] and J. Woleński [1985]a, b.

[4] The Lvov-Warsaw School continued to function as well – but I must restrict my scope here.

[5] In the interbellum, Lvov and Warsaw played major roles in the development of logic. Kraków showed activity as well, but not as significant (there was less acceptance of logic among mathematicians). A pioneer was Jan Śleszyński (1854-1931), professor of logic and mathematics at Jagiellonian University, 1919-1924. Also, Stanislaw Zaręba (1863-1942) contributed greatly to the development of Polish mathematics. Another notable Kraków professor was Witold Wilkosz (1891-1941): a versatile logician, mathematician, philosopher and physicist. But especially, there was Leon Chwistek (1884-1944), who studied mathematics and philosophy, earned his doctor's and professor's degrees (in 1934 he was given the Chair of Mathematical Logic in Lvov, winning a competition with Tarski). In the field of logic, he is best known for his ramified theory of types [1924-1925]; he also contributed many original ideas to other domains, including the philosophy of art.

functioned according to Twardowski's patterns and the *Janiszewski program*, in compliance with which Polish mathematicians, in addition to set theory, topology and their applications, should also thoroughly know mathematical logic and foundations of mathematics. Thus, the WSL had double roots: philosophical and mathematical. The combination of logic and philosophy in the history of the WSL did not limit itself to the fact that it was genetically related to philosophy. There was a close contact between its logicians and philosophers, in particular Tadeusz Kotarbiński, maintained by joint seminars. Stanisław Leśniewski (1886–1939) and Jan Łukasiewicz (1878–1956), the School's two founders, were logicians, philosophers graduated at Lvov. They continued the philosophical thoughts, teaching methods and of scientific research organization of Kazimierz Twardowski, and protagonists of formal techniques in science, similar to those applied in mathematics. And the connection between philosophers and mathematics initiated by Janiszewski and his program for mathematical research manifested itself, among others, in the fact that Warsaw leading mathematicians (Wacław Sierpiński, Stefan Mazurkiewicz, Kazimierz Kuratowski) assignated a considerable role of logic: Jan Łukasiewicz and Stanisław Leśniewski, philosophers by education, were put in charge of chairs in the mathematical environment and community at the Faculty of Mathematics and Natural Sciences. Both began teaching mathematical logic not only in the mathematical environment, but also in the philosophical one. Logic became an attractive subject to study in Warsaw. And even though the WSL was a joint creation of philosophers and mathematicians, logic – as a subject of its research – was not regarded as a part of mathematics or philosophy, but as an autonomous science.
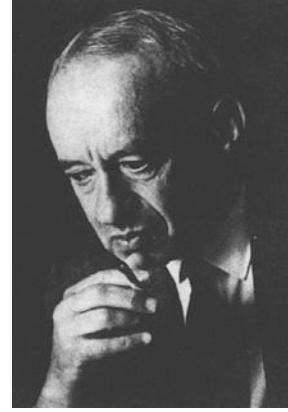
Łukasiewicz, at Warsaw University since 1915, Leśniewski, a professor of the philosophy of mathematics there since 1919, and their student, Alfred Tarski, who obtained his doctor's degree in 1924 with an impressive scientific output, are the three outstanding representatives of the Warsaw School:

Łukasiewicz	*Leśniewski*	*Tarski*

Other well-known members of the School were Adolf Lindenbaum, Stanisław Jaśkowski, Mordechaj Wajsberg, Moses Presburger, Bolesław Sobociński, Andrzej Mostowski, Jerzy Słupecki (my teacher) and Czesław Lejewski. Most were mathematicians, with the exception of Sobociński who graduated in philosophy and Lejewski who studied classical philology[6].

In the WSL, unprecedented results were achieved, sometimes even in Master's theses (Wajsberg, Słupecki). Heinrich Scholz from Münster said [1931]:

"Warsaw became the main centre of logical studies".

Thus, in the lifetime of one generation, 'Polish' logic grew from ground level to the acme of international acclaim. In the well-known book of A. Fraenkel, Y. Bar-Hillel, and A. Levy [1958], p. 200, it is stated that:

"Probably no other country, taking into account the size of its population, has contributed so greatly to the development of mathematical logic and foundations of mathematics as Poland".

and that:

"this curious fact should be explained sociologically."

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[6] Due to limitations of space, in this brief paper, I omit other famous Polish philosophers such as Tadeusz Kotarbiński, Kazimierz Ajdukiewicz, Tadeusz Czeżowski, or Władysław Tatarkiewicz – as their interests were concerned with the Lvov centre rather than the WSL.

The masters – Leśniewski, Łukasiewicz and Tarski – were individualists with different personalities, who shaped their students in many ways. Leśniewski gave the School its theme of synthesis, Łukasiewicz its dynamics, and Tarski the contact with mathematics (see Woleński [1985]a, p. 148). As Jan Woleński writes [1985]a, p.149, there were no divisions in the School into 'old' and 'young', 'beginner' and 'advanced'. The School had a strong emphasis on cooperation, irrespective of social position, views, or character of members. Thus, it combined scientists active in public and academic life (Łukasiewicz was a Minister of Religious Denominations and Public Enlightenment, as well as Deputy President of Warsaw University and twice President of this University) with modest secondary school teachers (Mordechaj Wajsberg, Jerzy Słupecki) – and from the well-off (like Adolf Lindenbaum) to the rather poor, like Alfred Tarski (Woleński [1985]b). Members also differed in socio-political and religious views, or their personalities: Łukasiewicz and Sobociński were conservatives, Lindenbaum and Presburger leaned towards communism. Some were devout Catholics, others followers of Judaism, or atheists. Except the bitter Leśniewski, most were persons of tender heart. Whatever their differences, they united around a shared scientific idea, the charisma of their teachers, awareness of their exceptionality, and their role in the development of logic in the world.

Characteristics of the Warsaw School[7]

The problems the School dealt with mostly belong to mathematical logic. Still, it all started in philosophy. Both Łukasiewicz and Leśniewski earned their doctor's degrees in philosophy at the University of Lvov: the first – in 1902, and the other – in 1912, both under the supervision of Kazimierz Twardowski.

Łukasiewicz started with methodology of empirical sciences. But his [1910] monograph *About the Principle of Contradiction in Aristotle* (published in Polish) adds a short lecture on 'algebraic logic'. The work by Łukasiewicz, besides those of Jan Śleszyński, are the first in Poland on mathematical logic.

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[7] This section largely follows the line of Słupecki [1972].

Łukasiewicz never resumed his research into the methodology of science. Leśniewski's studies in the time before the First World War concerned primarily problems of semantics of colloquial language and antinomies. Some ten years afterwards, he commented [1927], p. 169/170:

> *"I greatly worry about the fact that [these works] were published at all. I would like to disown them […] and acknowledge their bankruptcy […].*

The philosophical education of the WSL founders strongly influenced their output in mathematical logic, and their disciples, inducing great care for intuitive value. Here is what Leśniewski said about early mathematical logic:

> *"[it] can discourage a great number of scientists from dealing with 'logistic', those who are not content with the very delight of putting down signs and transforming formulas and who […] wish to realize the significance of the formulae being transformed […]".*

The representatives of the WSL tied philosophical questions to those of formal logic, solving classical problems of philosophy by its means. For instance, Łukasiewicz was convinced that the three-valued logic he had created indeed cast new light on the problem of determinism. And of course, in the famous work *The Concept of Truth in Languages of Deductive Sciences* (published in Polish in [1933], translated into many languages), Tarski solved one of the fundamental questions of the theory of knowledge in such an undisputable manner that probably no other account could claim. As to self-image, Leśniewski called himself a 'philosopher-apostate', while Łukasiewicz considered himself a philosopher. Still, Łukasiewicz also did purely formal work, while to Leśniewski, logic was always a tool for philosophical questions.

Another feature of the WSL was a drive for full, precise and simplest solutions of problems. This 'perfectionism' (Łukasiewicz) caused the Warsaw logicians to frequently release results with a delay, at the risk of losing priority. They delighted in formally perfecting systems, simplifying axioms several times. A peak achievement was reduction of axioms to only one, as short as possible

in terms of symbols. The most surprising results in this area were achieved by Łukasiewicz and Sobociński, and it is worth recalling that Łukasiewicz and Leśniewski created two types of original and inventive logical symbolism.

Methodology of the sentential calculus and Tarski's influence

I will not describe concrete achievements in detail, since there are many sources. Indeed, in some areas, you probably know as much about the field as I do, thanks to the Tarski biography of Anita and Solomon Feferman [2004]!

A core subject in the WSL was methodology of the propositional calculus, initiated by Łukasiewicz. Tarski's results are the most outstanding. Famous other contributions were by Lindenbaum, Wajsberg on intuitionism, or Jaśkowski's natural deduction system. Tarski's subsequent accomplishments extended to the methodology of all deductive systems. Starting from [1930] a, b, he initiated the abstract study of axiomatic systems, but also the by now standard semantic viewpoint of his paper on the notion of truth [1933]. Tarski had a very busy didactic activity, and pupils who obtained outstanding results already before the War included Andrzej Mostowski and Wanda Szmielew.

A counterpoint: the work of Leśniewski

A highly original strand in the Warsaw School was Leśniewski's creation of the systems of '*prototetics*', '*ontology*' and '*mereology*', trying to improve on the mathematical foundations of Russell and Whitehead. Especially, Leśniewski's ontology has continued to attract interest, for instance from Andrzej Grzegorczyk and Bar-Hillel, and in the early 1970s by Boguslaw Iwanuś. But perhaps his most famous system is the mereology, studied by Tarski and many others, that still finds applications in geometry, biology, and linguistics today.

After Leśniewski's death, Stefan Mazurkiewicz said what many thought:

"His [Leśniewski's] lectures, both written and spoken – were successive floors of one magnificent and uniform building that he was erecting for over 20 years. The same features that give his studies such a high value,

unlimited correctness and solidity, the unbounded will to reach the very foundations of investigated problems, power of dialectics, and acute sense of criticism – were introduced by him in all walks of life".

The Warsaw School: further characteristics, and influence

The notion of a scientific school is a complex one. Members should address a common problem with shared methods of investigation. The WSL satisfied this to a high degree. Also, a true school has results that create a valuable whole. What was the lasting contribution of the WSL?

The 400-page volume *Polish Logic 1920-1939* (Ed. S. McCall [1967]) has translations of 17 articles by Polish logicians. All but two are from the Warsaw School. Also in that period, the *Selected Works* of Łukasiewicz [1961] and [1970] appeared: the first edited by J. Słupecki, the second by L. Borkowski. Half were papers from the interbellum. Also there was a monograph by E.C. Luschei on *The Logical Systems of Leśniewski* [1962]. A wide selection of Tarski's pre-war articles was *Logic, Semantics, Mathematics* [1956] (translated by J. H. Woodger, Oxford 1956; second edition by John Corcoran in 1983).

But a school can also be characterized by personal contacts of its members, an atmosphere of constant discussion and exchange of thoughts. The following excerpt from *Elements of Mathematical Logic* (Łukasiewicz [1929]) illustrates what the cooperation at Warsaw University looked like:

"I owe the most to the scientific atmosphere created at Warsaw University in the field of mathematical logic. It is in discussions with my colleagues, mainly Professor Leśniewski and Assistant Professor Tarski, and often also with students of theirs and mine that I had a chance to comprehend many a notion, absorb new ways of expressing myself, and learning many a new result whatever their authors were like."

4 Polish Logic after the Second World War

1939 was the last year of the WSL. Just before the War, Leśniewski died – and Tarski left for the United States. During the War Lindenbaum, Wajsberg and Presburger were killed. Afterwards, Łukasiewicz, Sobociński and Lejewski worked outside of Poland, and their key members outside of Warsaw.

Polish logic never regained the renown of the WSL, not so much by losses in human resources as in rhythms of scientific activity, and losses to libraries and manuscripts. The new academic life, and lack of freedom in expressing thoughts, did not favor logic either. Lvov found itself outside Poland (it has been in the Ukraine since the War), Warsaw was completely destroyed.

In this context, Alfred Tarski in exile shone all the more. At Berkeley he founded the great Californian School of Logic, a dominant influence after the War. We agree with Jan Woleński, a world expert in the history of logic, that

The California Bay Area

"The most outstanding school of logic in the world after the Second World War was that in California […]. The Californian School of Logic - because of Tarski – was in many ways similar to its Warsaw ancestor."

Present-day Poland

The new Warsaw Centre

After the War, a new center was started by Andrzej Mostowski (1913-1975) who stayed in Warsaw at the time when the situation of logic was difficult.

Andrzej Mostowski

Mostowski kept close contacts with mathematicians like Kazimierz Kuratowski and Wacław Sierpiński. Continuing Tarski's work, his centre studied set theory, model theory, decidability, algebraic and topological methods in logic. Famous names are Helena Rasiowa and Andrzej Grzegorczyk, who has continued the prewar traditions as a logician, mathematician, philosopher and ethicist. A new generation includes logicians like Zofia Adamowicz, Henryk Kotlarski, Michał Krynicki, and Marcin Mostowski (logicians going back to Andrzej Mostowski's school), and Cecylia Rauszer, Ewa Orłowska, Grażyna Mirkowska, Andrzej Skowron, Damian Niwiński (mostly logicians and computer scientists from Helena Rasiowa's circle), who kept up its renown [8].

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[8] One persistent topic has been generalized quantifiers, introduced by Mostowski in 1957.

Other logicians from Warsaw went elsewhere. For instance, Stanisław Jaśkowski (1906-1965), the independent inventor of natural deduction [1934], in 1945 found himself in Toruń (the city of Nicholas Copernicus), continuing his scientific and didactic activity in logic, but also general mathematics.

5   My teacher Jerzy Słupecki, his career and influence[9]

My teacher Jerzy Słupecki (1904-1987) was a typical illustration of how the Warsaw School spread in new ways. He was an ardent propagator of the work of Łukasiewicz, Leśniewski and Tarski. After the War, he found himself in Lublin and then in Wrocław (1948). As his main scientific goal he chose to continue, popularize and extend the studies of the WSL. In particular, it is Słupecki's making Leśniewski's output more systematic and general, that has made it accessible to a much wider audience (see [1953], [1955] and [1958]).

Słupecki's interests during his studies concentrated on mathematical logic. He graduated in 1935 and his M.A., thesis, supervised by Łukasiewicz, won the award of the Department Council. At the start he pursued many-valued sentential logic. A many-valued calculus is 'functionally complete' ('full', in the term of that time) if the truth-tables for its primitive connectives can define every possible truth-value function. Słupecki's first discovery was that 1) Łukasiewicz's three-valued system $Ł$ is not functionally complete, and 2) extending the primitives of $Ł$ with a suitable new functor T yields a system

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[9] See Bryll G, Iwanuś B., Piróg-Rzepecka K. [1983] and Woleński J. and Zygmunt J. [1989].

that is both functionally complete [1936] (English version in McCall S. [1967]) and Post-complete (adding two new axioms to Wajsberg's axioms for $Ł$). In [1939]a, Słupecki formulated a famous criterion of fullness (English translation in 1972). In 1938 he also proved that a broad class of functionally complete logics is finitely axiomatizable, in his Ph.D. thesis *Dowód aksjomatyzowalności pełnych systemów wielowartościowego rachunku zdań* published in [1939]b. It was supervised by Łukasiewicz, and reviewed by Wacław Sierpiński. The thesis was translated into French and English [1971] – and it established his scientific position, not only among the Warsaw logicians.

Significant meaning for logic and philosophy attaches to Słupecki's papers on the intuitive basis of many-valued logics. His papers (in Polish [1964]; in English [1967]; in Russian [1974]), see the issue differently from Łukasiewicz. He believed that it is possible to interpret the system $Ł$ in a deterministic light, and he also considered some dialectic interpretations of Łukasiewicz's logics.

In a further important line of logical research, Słupecki's results on Aristotle's Syllogistic, first presented in a Warsaw seminar of 1938, are well-known, too (Lublin [1946], Wrocław [1948]; English version in 1951). His academic supervisor Jan Łukasiewicz described them as follows [1939]:

> *"the most historical discovery which has been made in the field of syllogistics since Aristotle,"* [and therefore:] *"systematic studies into Aristotle's syllogistics can be considered completed in a sense."*[10]

Logic in Wrocław

Słupecki defended his Habilitation *Studies in Aristotle's Syllogistics* in Krakow (Jagiellonian University, 1947), becoming a full professor in Wrocław (1948). Wrocław was then the home of displaced people from all over Poland, and expatriates from Lvov. A strong group of logicians and philosophers gathered:

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[10] Quotations are from an English translation of Łukasiewicz's views in [1939], in the *Selected Works* of Łukasiewicz [1961] (pp. 226/227) and also in the monograph [1951]. I discuss the significance of Słupecki's research for contemporary logic in my paper [1983].

Maria Lutmanowa-Kokoszyńska, Bogusław Iwanuś, Tadeusz Kubiński, Ryszard Wójcicki, Słupecki's students and then co-workers: Ludwik Borkowski and Witold A. Pogorzelski, Marian Maduch and Urszula Wybraniec-Skardowska. One striking feature were mutual seminars. After the War, Wrocław became perhaps the leading logical center in Poland. Two famous names are Czesław Ryll-Nardzewski and Jerzy Łoś - prominent logicians and mathematicians.

My own environment: the Opole Centre

In 1950, Słupecki started working for the Opole Teacher's Training College, 80 km to the east of Wrocław, where he founded his own center of logic. (It is also why I was asked by Słupecki to move from Wrocław to Opole.) Here, he held the post of the President of the Teacher's Training College for many years. Here, too, he ran all-Polish logical seminars. Opole was also the venue of the All-Polish Conference of Logicians in 1965. Słupecki chaired the Conference of Logicians in 1973 as well, as part of celebrating the Year of Polish Science. Many prominent Polish logicians and mathematicians taught in Opole: Roman Suszko, Stan Surma, and Jan Mycielski, to mention just a few.
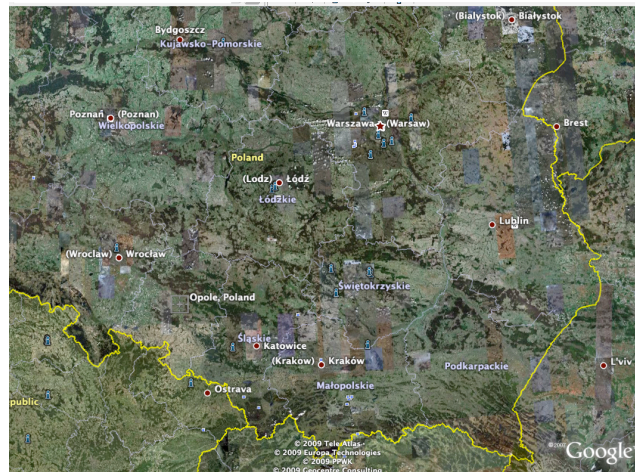
Silesia: Wroclaw, Opole, Katowice

Sentential calculi and methodological issues

The Opole center investigated new three-valued sentential calculi (Krystyna Piróg-Rzepecka [1977]), applying to mathematical theories with conditional definitions (Katarzyna Hałkowska [1979]) for expressions that lack denotation.

This led to the construction of new algebras. My teacher also propagated Tarski's ideas, mainly in formalizing methodology of deductive sciences (see [1962]: the co-author W. A. Pogorzelski) – building with Pogorzelski [1960], and others, theories of deductive systems based on non-classical logics. Enriching Tarski's framework with a 'rejection function' (see Słupecki [1957]), Słupecki inspired my PhD. thesis on a general theory of rejected propositions [1969], on top of Tarski's consequence theory [1930]a, b. This also threw new light on the methodology of the empirical sciences: cf. the dissertation by my colleague Grzegorz Bryll [1969]. Building on this work, Słupecki introduced a new notion of 'Ł-decidability' of deductive systems [1971] (co-authors Bryll & Wybraniec-Skardowska). The Opole logic community still pursues this topic: here I just note that all significant propositional calculi have been tested for Ł-decidability[11].

I cannot mention all people associated with the Opole Centre, but I make an exception for Tadeusz Prucnal, a creative, wise, and kind colleague.

Logic, natural language, and computation

At the instigation of my teacher I prepared my habilitation on the theory of language syntax [1985] (English version with Kluwer [1991]). My theory is indebted to Tarski's concatenation theory [1933], Markov's theory of concrete and abstract words [1954], the Leśniewski-Ajdukiewicz theory of syntactic categories, and the categorial grammar originating with Ajdukiewicz [1935] and Bar-Hillel [1950], [1953], [1964]. In the 1980s, these topics enjoyed great popularity. In 1988, the book *Categorial Grammar*, edited by J. van Benthem, W. Buszkowski and W. Marciszewski appeared (see [1988]). These studies were continued in Poland by W. Buszkowski and his students in Poznań.

Incidentally, it is worth noting that the systems of natural deduction by Jaśkowski and Gentzen had not come to full light until Słupecki and his student and later colleague at Wrocław Ludwik Borkowski published this work.

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[11] A review of results on Ł-decidability is in Bryll [1996] and Wybraniec-Skardowska [2005].

They also expanded the theory to practice by mathematicians, and to colloquial intuitions. The English translation of a Polish book by these authors [1963] was published in Oxford in 1967; it had been translated earlier into Russian in 1965. The method in this book was used in computer-based testing of correctness of proofs of theorems, and by my Opole colleagues and me to teaching logic (see Wybraniec-Skardowska U., Bryniarski E. [1990], Wybraniec-Skardowska U. (ed.) [1991]), using the computer program MIZAR, developed at the University of Białystok by Andrzej Trybulec and his colleagues. This research was encouraged by their supervisor Witold Marciszewski – known, amongst others, as an author of *Dictionary of Logic* [1981] – an elaboration of a set of papers on formal logic contained in *Encyclopedic Dictionary of Semiotics* (Sebeok T. (ed.) [1994]), and encyclopedic publications in Polish.

Słupecki's centre ceased to exist after the death of its founder (1987) and the later elimination of logic from many curricula, including mathematics. Still the three universities in Silesia, Opole, Wrocław, Katowice, organize annual all-Polish conferences on Applied Logic and Foundations of Mathematics. [12]

6   Further trends in Polish logic today

Polish logic and philosophy stil function with vigour today – and again, I can only be brief about all its branches and developments.

Mathematical and philosophical logic

I have already mentioned the strong ongoing tradition of mathematical logic, emanating eventually from the Warsaw School described in the above. As for formal and philosophical logic, a continued influence of the Lvov-Warsaw School is described by Jacek Jadacki [2006] in the book *The Lvov-Warsaw School – The New Generation*, edited by Jacek Paśniczek and himself [2006].

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[12] Janusz Czelakowski should be mentioned here, who moved to Opole in 1992. He is one of the founders of Abstract Algebraic Logic, which studies the interaction of algebra and logic. His *Protoalgebraic Logics* [2001] is still a key monograph. His seminar on algebraic methods in metalogic and the foundations of mathematics gathers young logicians from south-western Poland. He is on of the co-organizers of annual conferences mentioned earlier. One may therefore say that Opole is still a noticeable spot on the logical map of Poland.

Logic and language once more

Polish logic after World War Two owed much to Kazimierz Ajdukiewicz who still taught actively. His early ideas opened a way to study questions and answers in modern logic [1926], the earlier mentioned categorial grammar [1935], and the notion of meaning [1931], [1934]. The specific interest of Polish logicians in questions is due to Tadeusz Kubiński. His monograph [1971], *Wstęp do logicznej teorii pytań* (*An Introduction to the Logical Theory of Questions*), appeared well ahead of *The Logic of Questions and Answers* (Belnap and Steel [1976]); (English version *Outline of the Logical Theory of Questions*, 1980, East Berlin). Kubiński's student Andrzej Wiśniewski developed *'Inferential Erotetic Logic'* in his book [1995] and a series of papers in international journals. Currently Wiśniewski and his students are working on applying Inferential Erotetic Logic in the areas of proof theory and problem-solving. Ajdukiewicz's ideas on meaning have been taken up recently by me [2007] and are also a subject of study in a current research program conducted by Ryszard Wójcicki.[13]

Further themes, and organization

Indeed, Ryszard Wójcicki was for many years head of the Section of Logic in the Polish Academy of Sciences. With Roman Suszko he created, in the late 1960s, an active logical center there. His pupils Janusz Czelakowski, Wiesław Dziobiak, and Jacek Malinowski still pursue logic and algebra in his line. Since the 1960s, he also pioneered logical methods in philosophy of science.

Wojcicki has recently started international conferences "Trends in Logic", with publications under the same title. For many years he was Editor-in-chief of the journal *Studia Logica* founded by Ajdukiewicz in 1955. *Studia Logica* is now a significant international journal (incidentally, one of many founded by Polish logicians); Jerzy Słupecki was on its Editorial Board from the start and its long-time Editor-in-chief, providing also a special column on logic

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[13] One should also mention here Jerzy Pelc who for 40 years conducted all-Poland semiotic seminars in Warsaw, and who authored and edited many papers and books on semiotics.

education. Finally, on the theme of logic events in Poland, Kraków, a former capital of the countru and a strong logical center today (Andrzej Wroński, Jan Wleński, Jerzy Perzanowski, Ewa Żarnecka-Biały, Wojciech Suchoń), is also a capital of annual Conferences on the History of Logic (in November of this year, the 55th of these will be held: a long and illustrious tradition):


Wawel Royal Castle, Kraków

Logic and computer science[14]

My story has mainly followed some lines in mathematical and philosophical logic, with a few excursions into natural language. But I must mention another important development that did not exist at all in the pre-War period. Over the last decades, many Polish logicians found new scope for their talents, namely, in the field of computer science. Names like Rasiowa, Rauszer, Orłowska, Skowron, Niwiński, Apt, Krynicki, Pawlak, Trybulec, Salwicki, Srebrny, Prymuszścinski, Walukiewicz, and many others are internationally famous.

A whole story remains to be told of recent decades of work by Polish logicians at interfaces with computer science: automated deduction, logic programming, foundations of computation in 'algorithmic logic' and the modal mu-calculus, program semantics, data base theory in 'rough sets', generalized quantifiers and complexity theory, algebraic and relational

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[14] For this subsection, I have relied heavily on information from Johan van Benthem (p.c.).

methods in computation, or default logics in AI. [15] But these newer areas require a story for another occasion, and maybe another speaker.
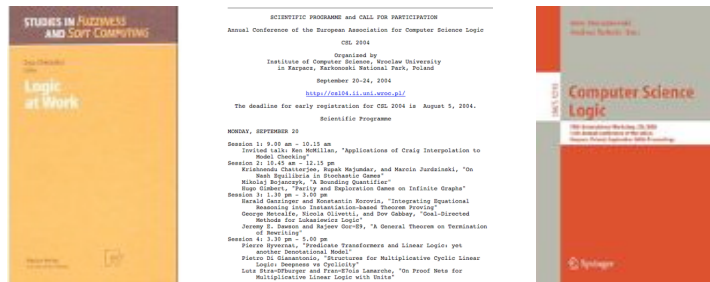
Polish publications on computational logic

The state of logic in Poland today

I end my survey with a more general note. Logic is under some pressure in Poland: many university studies have eliminated the subject. Also, financing scientific research in the field is restricted to a minimum, which even affects the exchange of scientific thoughts. While much of this is an inevitable process of adjusting to new social and political realities – a task for a new younger generation – a further dimension is ever more important. After its enthusiastic influential start, Polish logic now crucially depends on international contacts. After all the national history that I sketched, we are now truly part of one broader world, especially in a current world crisis affecting us all:
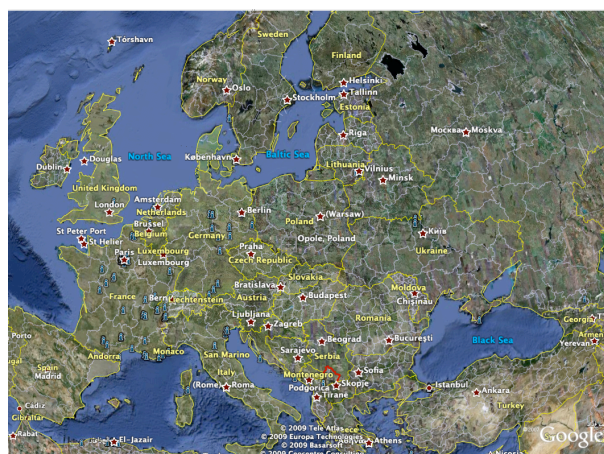
Europe Today

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[15] In the 1980s, a joke said the best conference language for non-monotonic logic was Polish.

In this situation, we Poles recall the famous motto of the Solidarity movement:

"There's no liberty without solidarity".

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