A formalization of Spinoza's Ethics, Part 1: Consequences for interpretation.

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written by

Pablo Sierra Márquez
(born 31/08/1990 in Seville, Spain)

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INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Abstract

In order to formalize the first book of Spinoza's *Ethics*, we first provide a philosophical interpretation of his philosophy from a perspective aiming at connecting his main ideas in a formal language. This interpretation emphasizes on the original content of Spinoza's main ideas such as God, infinity, existence, and the true idea rather than the historical content of those concepts for our porpoise is to set the grounds for a formal interpretation of those notions. This work mainly focuses on the *Ethics*, the *Treatise on the Emendation of the Intellect* and Letter XII as sources to study those concepts. Based on the interpretation we give a formal analysis of the main concepts found in *Ethics, I* focusing mainly on the notion of *dependence* which rules the formal language as the basic relation between the elements of the language, together with its inverse relation *causation*.

Finally we provide a formal language that accounts for the axioms, definitions and the first twenty-three propositions of *Ethics, I*. This language consists on a extension of First Order Logic with a dependence relation and dependency graphs as models for interpreting the language. Then we give a proof that the set of axioms in our language is consistent and a proof for the twenty-three first proposition of *Ethics, I*, with some exceptions.

Contents

1 Introduction

The main goal of this work is to define a formal language that can be used to interpret the first part of Spinoza's main work the *Ethics* from a formal point of view. The motivation behind this goal is twofold firstly to get a deeper insight into Spinoza's philosophy and secondly to use his original ideas as new perspectives in the formal treatment of classical philosophical notions. The first part of this work consists in a study of some relevant concepts in Spinoza's philosophy related to the idea of God from a perspective in which the formal aspects of those concepts are highlighted; this is, we will focus on the way in which the concepts are inter-related, not from a historical point of view[1], but with the goal of narrowing down the inter-relations those concepts have and how to capture them in a formal language. The second part of this work consists of both creating a formal language based on that interpretation and formalizing the first part of the *Ethics*, *De Deo*, using that language. The reasons for this enterprise is mainly focused on one problem that is found in Spinoza's philosophy–the connection between the infinite and the finite. This problem contains other issues in Spinoza's philosophy which are connected to it, like the transition from the first book to the second book, the shift from an eternal perspective into a temporal perspective, and the relation of the human mind to God. By studying the relevant concepts in Spinoza's philosophy and giving them a formal treatment, I wish to accomplish both a better understanding of the problem and a solution to it. A secondary, but no less important, goal is to set the basis of a formal language based on the first book of the *Ethics* that sets a solid ground on which to continue that work in the future and hopefully work our way throughout the entire book.

This first part, titled *De Deo*, sets the ontology of his system, mainly focused in the idea of God–or Nature– as he famously states[2]. The maze, that the first part of the *Ethics* represents, has a very clear end, which Spinoza states clearly at the beginning of the appendix to this first part of his main work:

> I have now explained the nature and properties of God: that he necessarily exists, that he is one alone, that he is and acts solely from the necessity of his own nature, that he is the free cause of all things and how so, that all things are in God and are so dependent on him that they can neither be nor be conceived without him, and lastly, that all things have been predetermined by God, not from his free will or absolute pleasure, but from the absolute nature of God, his infinite power.

Thus this work focuses on the concepts related to God and the ontology defined in the first part. Those relevant properties of the most perfect being can be found in several propositions throughout the first part of the *Ethics*, which will be the objective of our formalization. The second chapter of this work will focus on the theoretical approach to Spinoza's philosophy from a formal point of view, discussing the main concepts and providing a new perspective into his philosophy. In the same way that Descartes arrived to these two ideas, self-understanding and God, as the foundation of truth and correct knowledge, Spinoza focuses his effort in the understanding of the idea of God and the study of our own reflective knowledge. Taking these two ingredients from the Cartesian philosophy, he constructs his philosophy by the method of the true idea[3], i.e., the idea of God and the process of making ideas from this idea, which would

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[1]One of Spinoza's philosophy peculiarities is that the language he uses is defined by using classical concepts from scholastics in a completely different way in which those concepts were defined in the philosophical tradition that preceded him. One of the objectives of the way in which he defines those concepts are precisely to break from the philosophical tradition, but I claim that that wasn't his only objective and that those concepts indeed are defined with a philosophical system in mind. That is precisely the reason why he is suited for giving new perspectives also for modern interpretations of those concepts. So in my interpretation we will not focus on the discussion between the modern and traditional interpretation of these concepts that he introduced, but on the other part.

[2]Ethics, IV, Preface. Spinoza (2002), p. 321.

[3]Principles of Cartesian Philosophy, Appendix Containing Metaphysical Thoughts, Part 1, Chapter 6. Spinoza (2002), p. 187.

secure the veracity of them. This method will be explained in the second chapter when we will focus on understanding, the notion of true idea and the method of forming ideas. The third chapter accounts for the formalization of his philosophy. The fourth chapter of the present work contains the description of the formal language, together with the formalization of the definitions, axioms and the first twenty-three proposition of the first book of the *Ethics*. Due to the nature of this work, which contains both formal and philosophical arguments, people might have different interests when reading this paper; thus the division of the paper is made in a way that there is a separation between those two parts. The current order of the chapters is recommended by the author, but one might start in chapter 4 if their interest is more formal and then trace back the formalization to the interpretation.

The first concern people might have with my decision of attempting a formalization of Spinoza is to ask: "What is it worth for?" My answer to that question is that it is obvious that Spinoza saw in Euclid's *Elements* what can be called a pseudo axiomatic method which he took as the method for the correct use of our understanding. Sadly he didn't have a more advanced axiomatic system to get inspired by at his time. Nowadays the development of mathematical logic has given us very numerous techniques, systems and tools to develop axiomatic systems on which, I am pretty sure, Spinoza would have taken as inspiration. So a formalization of Spinoza seems to be necessary in order to give his philosophy more modern relevance and to continue developing his work, mainly not because it needs it, but because the tools at that time were not enough to give it a sturdier consistency on the axiomatic arguments. For instance, the inclusion of graphs in the formalization might be seen as giving the *Ethics* a visual and more intuitive element, like that given by the visual constructions that made the *Elements* a very popular work, because of the great help they introduced. I hope that the reader bears in mind that this is a preliminary work with the intention to be expanded in the future in an accumulative way. This requires the work presented in this paper to be a solid ground from which to keep on expanding it in the future. The biggest issue with the formalization of Spinoza is not what it is worth, but the loss of expression and complexity that it entails. This is due to the fact that this work does not intend to translate the *Ethics* into a formal language, but to pin-point the fundamental structure that underlies Spinoza's philosophy and to give formal interpretation of it. This claim entails that there is a much more fundamental structure to which the first book can be reduced–at least the 23 first propositions of the first book which are the ones we treat in this work, and at the same time support the rest of the complexity and expression lost in this process. It is a big price to pay, but I think that the benefit we could gain from this process makes it worthwhile.

For a long time I have been trying to interpret Spinoza's philosophy from a formal point of view. Since I first discovered such a unique style of writing and started to be interested in logic I found that the ideas and the logic found in the *Ethics* were susceptible of being boarded from a formal perspective, not just because of the style of the text, or the organization of the ideas in a geometrical way, i.e., in the same way in which the *Elements* from Euclid are written, but because of their level of abstraction, and the power of expression they have could provide a very interesting perspective from a formal point of view. My attempts with several distinct formal systems in trying to capture Spinoza's ideas always arrived at the same dead end–they weren't capable of grasping his ideas. Now this problem might be because the systems were not the right ones, or because the ideas that can be found in Spinoza's philosophy are definitely not suited for a formalization. Nonetheless, after a deep analysis of the most relevant notion in his philosophy from a formal point of view, I came to the conclusion that no existing formal system will ever be suited, but not because of either of the previous reasons. The main reason for this is the same as the one which defines him as such a hated and forgotten philosopher–his ideas were ground-breaking as Spinoza was not a follower of any school of thought or any philosophy of his time, not even Cartesianism. Strictly speaking. If something can define Spinoza is that his ideas were only shared by other philosophies in terminology but in nothing else. He intentionally used the same term for concepts and ideas as different schools of philosophy and that were used since the beginning of it, but his definitions and use of those ideas were intended to both capture his views on some philosophical problems and to destroy the classic use of those notions. This is the

reason why a formal interpretation of Spinoza required a very basic formal language capable of being adapted to his ideas.

Another difficulty that was found in the process of formalizing Spinoza's ideas was the problem of either trying formally to translate his text into a system that will account for all the proofs and the propositions and the deductive process between them, or just trying to brew a formal system out of his definitions, axioms and propositions, and reinterpret the *Ethics* from them. The first option was immediately discarded, although this was considered just a possibility to take into account. Nonetheless there is a system of ideas to be found in his philosophy; that is the objective of this work and that can be captured following some of his notions, definitions and ideas, not only in the *Ethics*, but throughout his entire work. The reason why this is an interesting enterprise is that we have one of the most powerful systems in the history of philosophy, but rather difficult to understand, and we can achieve a double benefit from treating it formally. Firstly, better insights on some of his ideas that will enlighten his whole philosophy and secondly, the formal value of some ideas in his philosophy can also bring new perspectives on some formal problems and basic ideas, which do not have a clear philosophical intuition behind them. Although many authors have discussed and interpret the formal ideas in Spinoza as well as his 'logic', none of them have done this from a formal point of view. Some have done a great job in interpreting Spinoza's philosophy and researching on the importance of some concepts, such as modality, causality, or in which way to interpret the 'geometrical order' and much more, but they have always done it from a purely philosophical point of view. What I propose here is a deep insight into Spinoza's notions from a logical point of view with the objective of making that jump from a purely philosophical interpretation into a formal one. Joining formal logic and philosophy in Spinoza's work creates a great opportunity to give a chance for this great philosopher to be considered within the world of formal sciences and to modernize a philosopher that I still feel has a lot to teach us about relevant things.

The only author that has tried a formal approach to Spinoza is Charles Jarrett[4]. His main work–though not the only one–focusing on a formal interpretation of the *Ethics* is *The logical structure of Spinoza's Ethics, Part 1*. In this book Jarrett formalizes the first part of Spinoza's main book using first order logic together with modal logic. Although his work is original in that enterprise, we propose a different approach to the same goal. In his book he literally translates the first part into a first order language word by word–that is–he has a big set of predicates for almost each concept that appears in the book. He claims in the introduction to his paper that:

> The history of the interpretation of Spinoza, and more specifically, the very great divergence among the interpretations, inclines one to suspect that no interpreter can cast off the biases of his own outlook, in order to give a relatively objective interpretation of Spinoza[5]

and that is precisely the reason for this work–to give an interpretation that is rooted in a formal logic to overcome that very problem. On the other hand, we interpret the first part from our own perspective,and give arguments to explain why my reinterpretation of the elements of the *Ethics* is as honest as possible with his philosophy, taking into account the aim of this work. His work might be seen as being very honest with Spinoza's , since he doesn't leave out anything in the first part. Nonetheless, we try to go deeper into the challenge. In this work I claim that there is an underlying formal structure for Spinoza's philosophical system that can be identified throughout his works and not just in the *Ethics*, and my objective is to give interpretation of his philosophy that supports this claim, together with a formal language that provides a formal translation of that interpretation and use it to formalize the first part of the *Ethics*. The reason for this is that I do not want solely to formalize his work, but to brew a logic out of his philosophy following the *Ethics*. His groundbreaking ideas being the starting point–they shall account for

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[4]Jarrett (1978)
[5]ibid., p. 16.

the proofs on the Ethics since, contrary to Deleuze, I claim that Spinoza's philosophy is in the propositions and not in the scholiums.

The basis of this work relies on the importance of the axiomatic value that Spinoza saw in the *Elements* and the epistemological value gained by following this axiomatic ordering. We proceed to explain this connection between geometry and the method followed in the *Ethics*. We start by recalling the comment that Leibniz made to the *Ethics*:

> Here is a noteworthy observation concerning the infinite. Since there is one infinity greater than another, will there be something more eternal than something else? For instance, a thing can exist before any time imaginable, and yet not from eternity, because its time, in existence, will not be absolutely infinite, but infinite only in relation to us. Therefore there was a time when it did not exist, but that time is infinitely remote from now. This is just as an infinitely small line is in relation to a point.[6]

In this comment, Leibniz starts from the act that there are infinite things greater than other infinite things. We can relate this statement to the fact that in Spinoza we find indeed different sizes of infinities, not because of the physical sense of *size*, but related to how they are related to each other in the dependence ordering. For instance, God's eternity is greater in this sense than the eternity of the infinite immediate mode, since the latter depends on the former, but the interesting comparison is between something being eternal and something having an infinite duration. Leibniz makes such a brilliant comment when comparing these two since, as he claims, they are equated by us; this is by our imagination. The key to understand this quote, and the further analogy, is that, although our minds are able to compare those existences, they are of a different nature and, therefore, completely different. One thing is eternal and therefore cannot be conceived as having neither a beginning nor an end; the other thing has a duration so big that, compared to us, it becomes indeterminate whether it has a beginning or an end, but that does not mean that it hasn't. What is interesting in the analogy that Leibniz does with the relation between an infinitely small line and a point is what he implies when he says "... infinite only in relation to us". As we have explained, Leibniz refers to the fact that the nature of a line and the nature of a point become the same in the case that we take the line to be infinitely small, in the same way that the existence of God and the existence of the universe become similar–by indetermination and the use of imagination. A line, like the existence of an object, can be divided or extended at will by the imagination, but this process might make it too small or too big for our imagination and it can become indeterminate for us. Spinoza uses the same example in Letter 12 as the one used by Leibniz: "So it is nonsense, bordering on madness, to hold that extended Substance is composed of parts or bodies really distinct from one another. It is as if, by simply adding circle to circle and piling one on top of another, one were to attempt to construct a square or a triangle or any other figure of a completely different nature. [...] A parallel case is presented by those who, having convinced themselves that a line is made up of points, have devised many arguments to prove that a line is not infinitely divisible.[7]" Let us pay attention to that analogy because it encloses the key to understand what "difference in nature" means for Spinoza, and why the geometrical method is so relevant for him as an archetype of the use of our intellect. The difference between a point and a line in geometry is a difference in nature–a difference in the way we conceive them. The nature of a point is indivisibility–this is a point which cannot be made of points or anything else, i.e., it doesn't need the concept of any other thing to be conceived. On the other hand, a line is "length without breadth[8]" which means that its nature is to be able to be extended or reduced and in that sense a line can be divided into two lines and so on a and so forth,–reduced or extended *ad infinitum*. In E1P15 we find an example that can be used to illustrate this: "Lastly, if from one point in an infinite quantity two lines, AB and AC, be drawn of fixed and determinate length, and thereafter be produced to

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[6]Leibniz (2013), p. 66.
[7]Spinoza (2002), p. 788.
[8]Euclid (2007), p. 6.

infinity, it is clear that the distance between B and C continues to increase and finally changes from a determinate distance to an indeterminate distance." Following Leibniz's note, we see that this is the same case if we decrease the length of the line– imagine that we take a vertical line from B to C and we move it towards point A; assume that there is a moment in which the distance becomes equal to the point A, but, by definition a line can be divided so could point A, and this is a contradiction. That's what Spinoza meant by "indeterminate"–at some point the line becomes so small that is inconceivable for us, but it will never become a point because that will mean that a line has a part which is indivisible, which is absurd. This will be the same as to say that by dividing a body up to infinity at some point we will erase all quantity from it we will find the quality of extension itself. Spinoza says in the letter of the infinite: "Finally, there are things that can be called infinite, or if you prefer, indefinite, because they cannot be accurately expressed by any number, while yet being conceivable as greater or less. For it does not follow that things which cannot be adequately expressed by any number must necessarily be equal, as is sufficiently evident from the given example and from many others. [9]" The last sentence of this quote makes clear that we cannot even use the notion of equality for those infinities that cannot be captured by our minds, since they have turned indeterminacies for us. This would be like saying that there are only two types of indeterminacy for Spinoza, which is explicitly stated against.

What Spinoza is claiming with the key sentence: "A line cannot be made up of points" is going to be explored now, since it embodies the very root of his philosophy. The first argument that goes against the idea that lines are made up of points is what both Spinoza and Leibniz stated, from the fact that the nature of a line is to be divided, or prolonged, as much as desired, does not follow the fact that at some point we could reach a point. For this would mean that we have changed the nature of a line by a process of our imagination, and that we arrived at the fact that a line is no longer a line, since it cannot be divided further. The other argument against that idea is that, if we accept that lines are made up of a point, then we will have to admit that lines are discontinuous. Let me explain: If a point is what has no parts, i.e., it is a solid unity that cannot be divided, then if we take a line, and we draw a point on the middle of it, then, suddenly the two halves of the line are disconnected, i.e. we can no longer travel from one line to the other since a point represents a *break* in the line. If we admit that that point is indeed constitutive of the line, then the first segment of the line cannot be prolonged further beyond that point, because we have reached a limit–a border–that we cannot pass through since for that to be the case we will have to admit that the line passes through the space that the point encloses making it divisible; otherwise it could not be prolonged, since they are the same operation, changing only in which direction is performed–inwards or outwards. Let me give an example to illustrate this. Take a piece of paper and draw a line; now, draw a point in the middle of the line, and after that try to draw the same line again, and think what happens when you cross the point with your pen. You have two options–first you admit that the point is part of the line and that, when we first draw it, the point was already there, included in the line, and we crossed it in the same way as we did on our second drawing; second, you admit that, the point is a *construction on the line* and not part of it. If you were to accept the first option, like modern mathematics do, you would also have to admit that if a line is made up of points, you actually just draw a row of points connected with each other forming a line[10], but, if that is the case, let me ask you a further question: "What is in between the points?" You might answer "Nothing"–because they are adjacent points and there is no *breaks* in between them but, if that is the case, if the points are adjacent to each other leaving no space between them, then there is a common part to two distinct points in which they are touching. You would finally have to admit that a point has parts, since you distinguished between the part of a point that is common to both, and the one that is not.

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[9]Spinoza (2002), p. 787.

[10]There is an interesting connection here between this example and the famous *formal intuition of time* that Kant talks about through the drawing of a line, in B155, which connects both views on time. In the same way as Spinoza claims that time cannot be taken as being a succession of moments, Kant claims that the continuum of time is apprehended through that process of the drawing of a line. It will be interesting to further develop this connection between the epistemological continuum that a line represents for our minds and how we can understand other concepts through that same idea.

If we turn this discussion to the *Elements*, people might argue against this argument that definition 3 states that: "The ends of a line are points[11]", or that definition 4 states that: "A straight line is a line which lies evenly with the points on itself[12]", even postulate 1, which states that: "To draw a straight line from any point to any point.[13]", but I claim that none of these represent a problem for the Spinozistic argument and now I proceed to explain why. The important thing to keep in mind is that points are *on* lines and not *in* them[14]. This whole argument relies on a change of perspective, from a constructivist point of view to a bounded one. By "constructivist" point of view I mean the view that geometry is a science that generates lines, figures and spaces from an empty space, via process of construction; by "bounded" I mean a perspective which claims that geometry is the process of apprehension of a given, completely filled space through the use of the concept of *point*. Under the latter view, we can understand now those two definitions and the postulate without any problem. If we go back to definition 3, and its use against my argument of lines not being made up of points, we see that we could reinterpret this definition as saying: "We call a line the delimitation of the given space between any two points", and this reinterpretation goes together with my argument, since it establishes the connection between a line at the points that form it, which is not of composition but of *generation*; this is, we can generate a line with any two points, but we do not have to admit that the line is made up of infinite points, but it is made up of the space that the two points delimit. The difference between the verbs "to limit" and "to delimit" is a fundamental distinction here. It is the difference between being the last part of something, and establishing the point that the thing cannot pass or reach. In this last sense the boundary doesn't need to be part of the thing, whereas in the previous sense it is; this is the same difference we have in the mathematical symbols [,] and (,) used for intervals. Points in that sense *delimit* lines, in the sense that they are a limit for lines in the mathematical sense of *limit*. Points are delimitations used not only to conceive, but also to differentiate, lines. This is just a good analogy to understand how Spinoza conceives reality. When we think about an object or a mode we are just delimiting a part of Nature. We will come back to this when we deal with infinity and number in Spinoza. What I want to state with that interpretation is that lines depend on points to be understood. This interpretation of definition 3 actually makes definition 4 more relevant, since it represents a specific way of conceiving the space delimited by the two points, which we call *straight*, but the key to understand all of this is again, to change our perspective of geometry as working on a filled space that we delimit with our constructions and not as working on an empty space by generating figures. If we think for a moment what this interpretation means for the relation between lines and points, since points are not *in* the lines, we could calculate the ratio of approximation of the line to the point by means of another perpendicular line. In the example I invoked before in EIIP15, we find that this interpretation on the relation between lines and points could have been what lead Spinoza and Leibniz to conceive what differential calculus is, since the relation between the approximation of a line to a point, as its limit, encloses a differential relation[15].

From the bounded perspective, geometry becomes the science that apprehends space with the idea of "indivisible unit", since any concept or postulate in the *Elements* depends on the concept of point. Now we have to explain how our imagination and understanding work together in geometry in order to separate one from the other and have a proper understanding of, in this particular case, space. Some at this moment might come up with another argument against my view and say 'Following you idea that lines are not made up of points, how can you prolong a segment into a bigger one, since based on my argument, the segment cannot *cross* any of its end points?', but this is really not an issue for my interpretation since points are drawn *on* lines, and are not part of them, so we can freely make the segment bigger or smaller as we please. Our imagination works freely on geometry, but our understanding tells us what is the correct method of proceeding in geometry. This is the moment in which Euclidean geometry

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[11]Euclid (2007), p. 6.

[12]*ibid.*

[13]*ibid.*, p. 7.

[14]This is obviously not the way in which Euclid wrote his masterpiece in the original language, but the English sense of "inclusion" and "supported by" helps us understand the key difference.

[15]Duffy (2006), p. 299

and Spinoza's philosophy connect with each other–both are based on the same epistemological principle. We must start from a (true) idea that works as the "principle of intellection of everything[16]" and proceed following a method that allows us to understand anything based on the true idea and that secures us that the truth is going to be preserved from the idea into the statements we construct. In the same way I described how the bounded perspective begins with a given space that tried to understand by the use of concepts, Spinoza's philosophy represents the same process; this is we find ourselves in a given world which we have to understand through the use of our intellect and imagination and the concept we use in this case is not that of point, but *God*. Not to get lost, all this argument has been explored because we wanted to explain how finite and infinite are related in Spinoza's philosophy and, after this elucidation on what Spinoza saw in geometry as a paradigm for reasoning, we are set to explain it. The key to understand this relation between finite and infinite is to understand the relation of a point and a line in Euclid's Elements, as we have described. A line cannot be understood unless we considered the notion of point–in the same way, modes cannot be understood unless we have the notion of God. There is an added difficulty here, since we are talking about infinite and finite, points in geometry are finite and lines are infinite, whereas in Spinoza's philosophy it is the other way around–modes are finite and God is infinite, but the important relation is that of dependence between them, and the relation to our intellect, and even more the comparison between their different *existence*. The analogy with Spinoza resides in the comparison between line and points and the existence of modes and God. A point cannot be divided or extended in the same way as the substance's existence. On the other hand a line can be divided and extended at will, through the imagination, in the same way as the existence of modes. This difference in nature– a difference in composition–is transferred into time, space and number for Spinoza. Another argument in support of this view is what we found in the already mentioned letter: "Therefore many who are not used to distinguishing mental constructs from real things have ventured to assert that Duration is composed of moments, thus falling into the clutches of Scylla in their eagerness to avoid Charybdis. For to say that Duration is made up of moments is the same as to say that Number is made up simply by adding noughts together.[17]" Let's try to understand this analogy. To say that duration–which is the indefinite continuation of existence–is made up of moments, or in a similar way to say that extension is made up of objects, or to say that a line is made up of points, or that movement and stillness is made of movements of objects, is to claim that something can be divided into thing of a completely different nature, this will be like saying that something is composed of things from a completely different nature; for the same reason, Spinoza claims, number cannot be made up just by adding noughts. So, following the analogy, the composition by moments of duration is compared with the addition of noughts as the making of number. The reason for this is that noughts–*nullitatum* in the original letter which is the plural of *nullitas* which designates non-existence or emptiness–are of a completely different nature of that of number. Our finite minds perceive things in the same way, i.e., as finite things and perceived through the imagination, the objective of our understanding is to apprehend the nature of things in relation to the concept of God so we can understand them. The goal in hand was to establish the ways of the understanding for the world given, although we do not pay attention to it yet; formally so that we could proceed further by focusing on the finite side of Spinoza's philosophy, and establish a formal epistemology that allow us to understand the world, in the same as the point allows us to understand space in geometry.

For the formalization we will use First Order Logic with some predicates that will capture some of Spinoza's most relevant concepts in such a way that we can relate them in the language we want to develop. We will center the formalization around one main relation between the elements of our language, modes and substances which is the relation of *dependence*. I will discuss that this is the main relation under which we can understand almost any part of Spinoza's philosophy; together with the relation of *causation*. These two are the relations that reign throughout the first book. As models for this language we use *dependence graphs* which we will shown that satisfied the axioms given for this language which, are based on the definitions and axioms of *Ethics I*. These graphs allow us to capture the dependence relation that exist between the different

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[16]See Matheron (2011), p. 48.
[17]Spinoza (2002), p. 789.

elements of the first book. Once we have explain the language and formalized the definitions and axioms from the first book, we will proceed with the soundness proof in order to show that the set of axioms in our language is satisfied by the graphs. After that we will proceed with the formalization and demonstration of the first twenty-three propositions, with some exclusions. Finally, in the conclusion, I will discuss how the language can be expanded without a change in the axioms and definitions, following the spirit of the book. The most relevant part of the conclusion will deal with how to solve the problem of the transition from the infinite element to the finite elements of Spinoza's philosophy in which I will sketch how to deal philosophically and logically with this problem with the models and the language in hand. I will also discuss how the graph could be expanded to model for Prior's temporal logic and the importance of including a temporal treatment for the further development of the formalization.

2 Insights from Spinoza's philosophy

2.1 The method of the True Idea

Spinoza took from Descartes the pillars of his philosophy which can be reduced to three main ideas: The division of reality in thought and extension, the reflexive character of our understanding, and the idea of God. Without paying attention to the differences between the two philosophies, since that will require a whole study itself, we are going to focus on these ideas to highlight the main points of Spinoza's philosophy. The dichotomy of reality, i.e., extension and thought, is not a duality of reality but an ideal one, it is a duality of our understanding of reality. Ideas and objects have no a priori relation except the one they found in our existence as human beings. We are beings composed of both qualities: we are an object that forms ideas. This view implies that the only things that exist for us are ideas and objects, since our own existence is defined in those terms. This distinction comes from the Cartesian distinction between *res extensa* and *res cogitans*–nonetheless Spinoza takes it beyond that distinction. His view on this problem is that they are not two different things, but different qualities of the same being. That union is also found in us (human beings) and Spinoza had the ground-breaking idea at his time that mind and body are not just united, but they are actually two sides of the same coin and they interact with each other. Spinoza defines the mind as

> The first thing that constitutes the essence of the mind is nothing else but the idea of a body actually existing[18]

And in a letter to Schuller he claims

> For the power of any thing is defined solely by its essence (EIIIP7), and the essence of mind consists (EIIP13) solely in it being the idea of an actual existing body. Therefore the mind's power of understanding extends only as far as that which this idea of the body contains within itself, or which follows therefrom.[19].

So we see that they form a union only divided by our own understanding. Since Spinoza understands the *res extensa* and *res cogitans* as attributes i.e. the attribute of extension and thought, he takes them to be *that which the intellect perceives of substance as constituting its essence*[20], so I claim that the division that the concept of *attribute* introduces is an ideal one; this is an epistemological difference and not a real one, i.e. the attributes do not represent an ontological unit, but only an epistemological one. We will come back to this issue later in the paper.

The second pillar is the reflexive power of our understanding. Spinoza dedicated a whole work just for the intellect–the unfinished *Treatise on the Emendation of the Intellect* in which he explains his method for directing the intellect in the correct way towards the understanding of Nature, inspired by the Cartesian method. Spinoza sets up the rules for the method of making ideas from ideas; this is–the reflective method. What he took from Descartes is the strategy of starting with an atomic epistemological principle, which for Spinoza is the 'true idea'. This whole method–his logic and his epistemology–is based on a shift of perspective. Usually we try to conceive things as they are, but Spinoza claimed that the true method for understanding is try to conceive things *as we conceive them*. In the *Treatise* he said it straight:

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[18]Ethics, III, Prop. 3. Spinoza (2002), p. 282. (From now on, to quote the Ethics, we would use the following notation, EIIIP3, in which E stands for the Ethics, the next roman number refers to the part of the Ethics, then we can have either D, for definition, A, for axiom, P, for proposition, and finally the number of the previous element. Thus, EIIIP3, stands for Ethics, III, Prop. 3.)

[19]Letter 64. *ibid.*, p. 918.

[20]EID4. *ibid.*, p. 217.

Hence it is evident that certainty is nothing else than the objective essence itself; that is to say, the way in which we become aware of the formal essence is certainty itself. And from this again it is evident that for the certainty of truth no other sign is needed but to have a true idea. For, as we have shown, in order to know, there is no need for me to know that I know. From this, again, it is clear that no one can know what the highest certainty is unless he has an adequate idea or the objective essence of some thing. For certainty and objective essence are the same. Since truth, then, needs no sign, and to have the objective essences of things, or-which is the same thing- their ideas, is enough to remove all doubt, it follows that the true method does not consist in seeking a sign of truth after acquiring ideas; the true method is the path whereby truth itself, or the objective essences of things, or ideas (all these mean the same) is to be sought in proper order[21].

We should not try to understand anything until we have tried to grasp how can we understand anything at all, because the moment we apprehend how is it possible for the human mind to understand something, we would have already understood that thing. Now, this method consists in making ideas of things, i.e *objective essences*, through their formal essence, in the proper order[22]. Spinoza's philosophy is based on this idea of how to define things based on our different ways of conceiving it. No wonder that all definitions found in the *Ethics* are always stated in relation to a conception of the thing defined. This is what is called the *fourth kind of knowledge* in the *Treatise*:

Finally, a thing is perceived through its essence alone when, from the fact that I know something, I know what it is to know something; or, from the fact that I know the essence of the mind, I know that it is united to the body. By the same kind of knowledge we know that two and three are five, and that if two lines are parallel to a third line, they are parallel to one another, and so on[23].

Or the *third kind of knowledge* in the case of the *Ethics*:

Apart from these two kinds of knowledge there is, as I shall later show, a third kind of knowledge, which I shall refer to as *intuition*. This kind of knowledge proceeds from an adequate idea of the formal essence of certain attributes of God to an adequate knowledge of the essence of things. I shall illustrate all these kinds of knowledge by one single example. Three numbers are given; it is required to find a fourth which is related to the third as the second to the first. Tradesmen have no hesitation in multiplying the second by the third and dividing the product by the first, either because they have not yet forgotten the rule they learned without proof from their teachers, or because they have in fact found this correct in the case of very simple numbers, or else from the force of the proof of Proposition 19 of the Seventh Book of Euclid, to wit, the common property of proportionals. But in the case of very simple numbers, none of this is necessary. For example, in the case of the given numbers 1, 2, 3, everybody can see that the fourth proportional is 6, and all the more clearly because we infer in one single intuition the fourth number from the ratio we see the first number bears to the second[24].

In the same way that we make things with things, i.e., objects are form using other objects,

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[21]Treatise on the Emendation of the Intellect, §35. *ibid.*, p. 10.

[22]This description of the method is a fundamental description for what is to come, since, as I will discuss later, the formal essences of things and the proper ordering is key to understand the formal system.

[23]TEI, §22. *ibid.*, p. 8. (From now on we use TEI to refer to the Treatise on the Emendation of the Intellect.)

[24]EIIP40, Scholium 2. *ibid.*, p. 266.

ideas are produced from ideas[25]. This makes composition the main relation among ideas and among objects. Any object and any idea can be seen as composed of other objects and ideas or as composing another object or another idea. As we have said before, there is a relation between ideas and objects which is that of agreement found in human beings–ideas are used to understand objects and objects are represented by them in our intellect. This is where Spinoza focused his *Treatise on the Emendation of the Intellect* and his definition of a true idea. An idea is the subjective representation of an object, where object just means the aim of the idea; this is–ideas are always the idea of something[26]. A true idea is a special kind of idea; this is–an idea which objectivizes the essence of its *ideatum* and, in order to do that, we first need to apprehend the formal essence of its *ideatum*. It is mandatory to quote the passage here:

> A true idea (for we do have a true idea) is something different from its object (ideatum). A circle is one thing, the idea of a circle another. For the idea of a circle is not something having a circumference and a center, as is a circle, nor is the idea of a body itself a body. And since it is something different from its object, it will also be something intelligible through itself. That is, in respect of its formal essence the idea can be the object of another objective essence, which in turn, regarded in itself, will also be something real and intelligible, and so on indefinitely. For example, Peter is something real. Now the true idea of Peter is the objective essence of Peter and is in itself something real, something entirely different from Peter. [...] Hence it is evident that certainty is nothing else than the objective essence itself; that is to say, the way in which we become aware of the formal essence is certainty itself. And from this again it is evident that for the certainty of truth no other sign is needed but to have a true idea. For, as we have shown, in order to know, there is no need for me to know that I know. From this, again, it is clear that no one can know what the highest certainty is unless he has an adequate idea or the objective essence of some thing. For certainty and objective essence are the same.[27][28]

Here we have one of the most cryptic parts of Spinoza's work–the description of what a true idea is and the way to proceed in its method. I will now explain my interpretation of it. The

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[25]To this end, the first point to consider is that this is not a case of an inquiry extending to infinity. That is, to find the best method of seeking the truth, there is no need of another method for seeking the method of seeking the truth, and there is no need of a third method to seek the second method, and so on to infinity. For in that way we should never arrive at knowledge of the truth, or indeed at any knowledge. The case is analogous to that of material tools, where the same kind of argument could be employed. To work iron, a hammer is needed, and to have a hammer, it must be made. For this purpose there is need of another hammer and other tools, and again to get these there is need of other tools, and so on to infinity. In this way one might try to prove, in vain, that men have no power to work iron. But the fact is that at first, with the tools they were born with, men succeeded, however laboriously and imperfectly, in making some very simple things; and then these were made they made other more complex things with less labor and greater perfection; and thus advancing gradually from the simplest works to the making of tools, and from tools to other works and other tools, they have reached a point where they can make very many complex things with little labor. In just the same way the intellect by its inborn power makes intellectual tools for itself by which it acquires other powers for other intellectual works, and from these works still other tools–or capacity for further investigation–and thus makes steady progress until it reaches the summit of wisdom. (TEI, §30. *ibid.*, p. 9.)

[26]By idea I understand a conception of the Mind which the Mind forms because it is a thinking thing. Explication I say *conception* rather than *perception* because the term perception seems to indicate that the Mind is passive to its object whereas conception seems to express an activity of the Mind. (EIID3. *ibid.* p. 244.)

[27]TEI, §33. Spinoza (2002), p. 10.

[28]Idea vera (habemus enim ideam veram) est diversum quid a suo ideato: Nam aliud est circulus, aliud idea circuli. Idea enim circuli non est aliquid, habens peripheriam et centrum uti circulus, nec idea corporis est ipsum corpus: et cum sit quid diversum a suo ideato, erit etiam per se aliquid intelligibile; hoc est, idea, quoad suam essentiam formalem, potest esse objectum alterius essentiae objectivae, et rursus haec altera essentia objectiva erit etiam in se spectata quid reale et intelligibile, et sic indefinite. Petrus ex. gr. est quid reale; vera autem idea Petri est essentia Petri objectiva et in se quid reale, et omnino diversum ab ipso Petro. [...] Hinc patet, quod certitudo nihil sit praeter ipsam essentiam objectivam; id est, modus, quo sentimus essentiam formalem, est ipsa certitudo. Unde iterum patet, quod ad certitudinem veritatis nullo alio signo sit opus, quam veram habere ideam: Nam, uti ostendimus, non opus est, ut sciam, quod sciam me scire. Ex quibus rursum patet, neminem posse scire, quid sit summa certitudo, nisi qui habet adaequatam ideam aut essentiam objectivam alicujus rei; nimirum, quia idem est certitudo et essentia objectiva. TEI, §33-35. Spinoza (1925).

dichotomy *idea-ideatum* is the epistemic evolution of the *cogito-cogitatum*, in the sense that the latter focus on the epistemic process, and the former in the epistemic content. Ideas represent an activity of the mind, in which we apprehend the essence of the things where we aim our understanding, i.e., in this sense the object of an idea is more of an objective–a target–in that way the essence of the ideatum is captured in the idea as an objective essence. Now a true idea consists in capturing that essence through the formal essence of the ideatum, this is the cornerstone of this whole paper resides in this argument: there are two ways of forming ideas; one, just by capturing directly the essence of something into an idea; and second, capturing the essence of something through its formal essence which results in a true idea and the apprehension of certainty. Let me give an example to illustrate this with a Spinozistic argument. We can form the idea of a circle by saying that the essence of a circle is a geometrical figure in which the center is equidistant from the circumference, or we can form the true idea of a circle by saying that the essence of a circle is the movement that a segment describes around a point. But why does the latter description constitutes the true idea of the circle and the former doesn't? There are two answers to this question. First, the reason is because the former description follows from the second, i.e., that property of the circle follows from the latter description. Second, and the correct Spinozistic answer for that question, the description is correct because the essence of the ideatum is apprehended through its formal essence, which in this case is, a circle is a mode; therefore there must be in something else and conceived through another thing; this is its existence and conception must have an external cause–this is the reason why sometimes it is called the *genetic* idea, and this cause is the movements of a segment around a point[29]. The most important argument on which all this work is based is in the distinction between the objective and the formal in Spinoza, and the quote we just saw describes their relation. I claim that the distinction introduced in the passage from the *Treatise of the emendation of the intellect* is the key to understand his method of the true idea, and since it is obvious that Spinoza wasn't thinking about the 'formal' as we do it nowadays–this is mathematically–it is clear that he was very aware of the importance of mathematical method for the truth. This work precisely sets a modern interpretation of that 'formal' part of Spinoza's philosophy as its objective. That formal part in Spinoza's philosophy is nothing but the ontology found in the first part of the *Ethics*.

Spinoza states that we do possess a true idea. With that statement Spinoza is trying to find a true idea from which we can deduce any other idea in order to preserve the truth from that original idea to another. The method of the true idea does not focus on the mental representation of properties that a thing has–this is what imagination does; the method is run by the intellect, and in that sense we should focus only on the formal properties of things when representing them subjectively. The claim that we do possess a true idea is more important that it might seem because Spinoza is also stating the existence of that idea, which is the core element of his method, within our intellect. And by doing that he is already telling us one of the properties that the ideatum of the true idea possess: existence. Not only that that idea exists in our intellect but that the object of that idea must also exist. It is a general requisite that, in order to form true ideas we need the object of our ideas to exist, or be present to us; otherwise we would be talking about fictional ideas. The existence of the true idea is not only a physical existence–nonetheless since that idea, whatever it is, is an idea of something; the existence of that thing must be included also in the objective representation of it. Now, when we talk about the existence of an idea, we are obviously talking about the existence of an intellect representing something subjectively; therefore the existence of any idea is anchored to the existence of an intellect. Spinoza claims that

> From this we may conclude that method is nothing but reflexive knowledge, or the idea of an idea; and because there is no idea of an idea unless there is first an idea, there will be no method unless there is first an idea. So a good method will be one which shows how the mind is to be directed according to the standard

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[29]For example, to form the concept of a sphere, I invent a cause at will, namely, that a semicircle rotates about its center, and a sphere, as it were, is produced by this rotation. Now this is, of course, a true idea, and although we know that in Nature no sphere has ever been produced in this way, this is nevertheless a true perception and a very convenient way of forming the concept of a sphere. TEI, §72. Spinoza (2002), p. 20.

of a given true idea. Again, since the relation between two ideas is the same as the relation between the formal essences of those ideas, it follows that the reflexive knowledge of the idea of the most perfect Being will be more excellent than the reflexive knowledge of other ideas. That is, the most perfect method will be one which shows how the mind should be directed according to the standard of a given idea of the most perfect Being.[30]

According to the argument just given, why is that the case? There are two reasons why the idea of God is the true idea upon which all the method relies. First the idea of God is the simplest idea, because it requires no other idea except itself to be understood. Second, in relation to the existence of this being, since God is eternal, there is always the possibility of an intellect to form the idea of it. That's the reason why Spinoza says that we do possess a true idea, because we always have the possibility to arrive at this idea, and because that idea does not require any other idea to form it.

There is nonetheless another reason to take the idea of the most perfect being as the fundament of our method which has to do with the relation that any other idea has with it. This is a fundamental relation in Spinoza–what we call *dependence*. Since the method we are following is the method of the intellect, this relation of dependence is taken with regard to ideas and the hierarchy in which one idea depends on another, in the correct ordering of the method. The main objective of Spinoza's ontological argument is, not only to show that God exists, but that anything whatsoever depends on him, as he clearly states in EIP15: *Whatever is, is in God, and nothing can be or be conceived without God.* We see that this dependence reaches even further than just ideas–it is also a dependence of existence. Now I would describe the hierarchy of dependence found in the Spinozistic system. First of all we have the substance, or God, which depends on nothing but itself. Then we have what Spinoza calls *attribute* which are nothing but the qualities of the substance; this is what gives an objective content to the rest of the hierarchy. After this we have the principle of composition of the respective attribute and this represent the infinite immediate mode in the Spinozistic system. This is the principle of formation and differentiation of the modifications, *modes*, of the attribute. Finally we have the modes which are nothing but modifications of one quality of the substance. There is a unique mode which deserves a special mention in this process which is the infinite mediate mode, i.e. the union of all modifications. We will see later why the mention of a time condition is relevant here and what's the notion of time in Spinoza, which is strongly related to modality. Depending on how we interpret the attribute, we can put names to all levels in the hierarchy. If we take the attribute to be *extension*, then we have in order: extension, movement and stillness, bodies, the total face of the universe. If we take the attribute to be *thought*, then we have: thought, reflexive knowledge, ideas, the total face of the universe.[31]

Let me now develop the synthetic argument that supports this view. If we take any idea whatsoever, we will easily see that it depends on another idea, through which it is understood. This is true because there exists a condition for any idea to exists on which they depend, the idea of an intellect in act. It is obvious that there cannot be an idea without a intellect forming it. So we can say that any idea depends on a intellect forming it. But at the same time, this intellect couldn't form any idea without the principle of composition of ideas, i.e. reflexive knowledge, as we saw on the quote. Therefore any intellect depends on reflective knowledge to form ideas. Now the next obvious question is, is there an idea that doesn't depend on any of this? What about the idea of substance? This is, what happens with the idea of the perfect being as we have described before? The only thing which we can think of as being more fundamental than reflexive knowledge, i.e. the very capacity of performing the action of forming ideas, is the quality of thought itself. Now this quality has to be taken as a substance and not as modification, otherwise we will be back to the beginning of the argument. Now take the idea of thought in itself, or the thinking substance, we see that there is nothing more fundamental than that, except

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[30]TEI, §38. *ibid.* p. 11
[31]Giancotti (1991). p. 118.

for a being which has thought as one of its qualities[32]. We have sense that any idea whatsoever depends on this being, but now the question remains: what happens with the idea of this being? The crucial point in this argument is the following one: now that we have reached the substance, if we could form its true idea, i.e. the idea that contains objectively the essence of that being, we will be in possession of the idea of the being which depends on nothing else. Now we find ourselves at a very delicate point–it seems that since the idea of God is just an idea, it will follow the same path that any other idea, but that is not the case, because this idea does not require any intellect for it to form it. The only intellect on which the idea of God depends on is God's intellect, or the infinite intellect. Even further, any idea depends on this intellect, since God is an eternal being, its intellect is always in act; otherwise we would have to admit that God's intellect is subjected to time, which is absurd for Spinoza. Therefore we have found the idea on which any other modification of thought whatsoever–even thought itself–depends and, at the same time, is the only idea that does not depend on anything else, except itself. We find a quote that supports this argument in the *Principles of Cartesian Philosophy*:

> [In God there is only one simple idea.] Finally, before bringing this discussion to a close, we ought to deal with the question as to whether there is in God more than one idea or only one most simple idea. To this I reply that God's idea through which he is called omniscient is unique and completely simple. For in actual fact God is called omniscient for no other reason than that he has the idea of himself, an idea or knowledge that has always existed together with God. For it is nothing but his essence and could have had no other way of being.

> [What is God's knowledge concerning created things.] But God's acquaintance with created things cannot be referred to God's knowledge without some impropriety; for, if God had so willed, created things would have had a quite different essence, and this could have no place in the knowledge that God has of himself. Still, the question will arise as to whether that knowledge of created things, properly or improperly so termed, is manifold or only single. However, in reply, this question differs in no way from those that ask whether God's decrees and volitions are several or not, and whether God's omnipresence, or the concurrence whereby he preserves particular things, is the same in all things. Concerning these matters, we have already said that we can have no distinct knowledge. However, we know with certainty that, just as God's concurrence, if it is referred to God's omnipotence, must be no more than one although manifested in various ways in its effects, so too God's volitions and decrees (for thus we may term his knowledge concerning created things) considered in God are not a plurality, even though they are expressed in various ways through created things, or rather, in created things. Finally, if we look to the whole of Nature by analogy, we can consider it as a single entity, and consequently the idea of God, or his decree concerning Natura naturata, will be only one.[33]

This is the negative epistemological definition of the substance, the substance is the principle of understanding from which everything follows without itself following from anything else. The idea on which any other idea depends, without it depending on any other idea. We can see this process as being inspired by the Cartesian method in the sense that by the use of our empirical knowledge and a process of introspection we find what is the fundament of our understanding, the idea of God. Once we have reached this idea, we can start proceeding with the correct method for the understanding, which in Spinoza is the method of the true idea. The true idea, which refers to God, is the negative epistemological principle of our understanding, it doesn't depend on anything, only in itself. This is the argument, take any quality, we have three epistemological

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[32]I take here the idea that there is no difference between substance and attribute for the sake of the argument, but the reason for it will be explained later in the paper. Here we are just assuming that to one substance corresponds one attribute, since we are only dealing with the argument concerning the attribute of thought.

[33]Principles of Cartesian Philosophy, Appendix Containing Metaphysical Thoughts, Part 2, Chapter 7. Spinoza (2002), p. 199.

units of it , i.e., we can take any quality as a substance, as a mode, or as a property. Modes depend on the property, and the property depends on the substance, and the substance depends on nothing, or only in itself. In an analytical way, following the method of the true idea, from any quality taken in itself, it follows how modifications of that quality are formed, or as Spinoza calls them "affections of the substance". Spinoza arrives to this idea of God by a process that could be called negative epistemology: I can give the definition of something by stating the impossibility of conceiving it through another thing. The process of backwards genealogy of our ideas takes us to an end point–an idea that is not generated by another one. Nonetheless we are in possession of that idea, and its definition is a negative one; that is what the idea of God represents in Spinoza's system–the end point. This has formally a very similar role as the point has in the elements in Euclid, it is the negative geometrical principle, "which has no parts", through which any other thing is understood. Epistemologically they serve as the same principle, although objectively they have nothing to do with each other.

2.2 Existence and necessity

In this chapter we are going to focus on the notion of *existence* and *necessity* found in Spinoza, and the relation they have to God. Let's begin with the notion of *existence* first. The main passage I want to focus on is the second proof given in EIP11:

> For every thing a cause or reason must be assigned either for its existence or for its nonexistence. For example, if a triangle exists, there must be a reason, or cause, for its existence. If it does not exist, there must be a reason or cause which prevents it from existing, or which annuls its existence. Now this reason or cause must either be contained in the nature of the thing or be external to it. For example, the reason why a square circle does not exist is indicated by its very nature, in that it involves a contradiction. On the other hand, the reason for the existence of substance also follows from its nature alone, in that it involves existence (EIP7). But the reason for the existence or nonexistence of a circle or a triangle does not follow from their nature, but from the order of universal corporeal Nature. For it is from this latter that it necessarily follows that either the triangle necessarily exists at this moment or that its present existence is impossible. This is self-evident, and therefrom it follows that a thing necessarily exists if there is no reason or cause which prevents its existence.

From that passage we can first subtract the idea that existence is a result; that is existence needs a cause or a reason to be predicated about something or to not be predicated about something. Existence in Spinoza is always treated as "existent by", itself, or something else. The idea that non-existence is also subjected to the same treatment, i.e., if something does not have a cause to exists, it must have a cause or reason not to exist, and not just the absence of it, is doubtlessly one of Spinoza's most original idea about existence. But we have to pay more attention to what Spinoza is telling us in the previous quote. The first line of the quote is introducing some sort of universal law of excluded middle for existence, anything whatsoever either exists or doesn't, and that for each case it must be a reason for that existence or non-existence. The next division is between those things whose existence or non-existence comes from their own nature or comes from something external. Before going on with the division that we can find in the quote, we already have two kinds of being that we can identify–first we have those beings whose existence comes from its own nature, and those beings whose non-existence comes from their own nature.

We see that Spinoza invokes EIP7 of the first part of the *Ethics* which precisely states: *Existence belongs to the nature of the substance.* The proof of this proposition relies on two main things–first that substances cannot be produced by anything else so they must be produced by themselves and, second, the definition of *causa sui*: *By that which is self-caused I mean that whose essence involves existence; or that whose nature can be conceived only as existing.* Spinoza's principle of sufficient reason is found in his *Principles of Cartesian Philosophy*, Axiom 11 in Part I[34]. The definition of *causa sui* is applied only when Spinoza arrives to the conclusion that a substance cannot be caused by any other thing but itself, and this is exactly the same reason as why we cannot conceive any other being on which the substance depends. That reason is that everything depends, and is caused by, God. This means that the notion of cause is not something already included in the idea of a substance, but that it is also grounded in the notion of dependence, and that even though a substance is defined as:

> By substance I mean that which is in itself and is conceived through itself; that is, that the conception of which does not require the conception of another thing from which it has to be formed

the connection between the notion of "causation" and "dependence" is explained in EIA4:

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[34]Of every thing that exists, it can be asked what is the cause or reason why it exists. Spinoza (2002), p. 133

The knowledge[35] of an effect depends on, and involves, the knowledge of the cause.

The conception of substance depends only in itself, this is, there is nothing on which the conception of the substance depends on. Therefore by EIA2:

That which cannot be conceived through another thing must be conceived through itself.

we arrive to the conclusion that since the substance is conceived through itself, it is self-caused, and whatever is self-caused exists, by EID1:

By that which is self-caused I mean that whose essence involves existence or that whose nature can be conceived only as existing.

Let's reconstruct the argument: anything whatsoever either exists or not, and its existence or non-existence must have a cause. This cause needs to come either from its nature or from an exterior thing. Assume it comes from an exterior thing but, if this were the case, then the conception of that thing involves the conception of the substance, but this cannot be because of the definition of substance, therefore it cannot come from an exterior thing. So it must come from its nature. Once we arrive at this point, this is, when we see that a substance's existence or non-existence must come from its own nature, then we can say that a substance is self-caused, and by definition must exists. The conclusion is achieved also if we assume that a substance cannot exist. Assume that a substance is non-existent; then its non-existence must come either from something else or from itself. But as we just saw it cannot come from something else, and it cannot come from itself by definition–therefore a substance exists necessarily[36].

Now we pass on to talk about things which its non-existence comes from its own nature. These kind of things, that we call *impossible*, are different from the substance in the sense that they do not depend on themselves, but they are self-caused but their nature cannot be conceived as existing, or can only be conceived as non-existing. It seem that we are on the edge of falling into a contradiction here, since it should be the case that if something is self-caused it must exists. But here is where we find the important distinction between depending on itself and being self-caused. The difference in dependence between the substance and a square circle is that there is nothing on which the substance depends to be conceived except itself, but in the case of the square circle, we see that it depends on the ideas of circle and square to be formed. In the case of impossible things, therefore, we have a curios case, since they are not self-dependent, but their (non) existence is self-caused, i.e. it comes from its very nature; therefore, although they are not self-dependent, they are self-caused. But the reason for the latter is that, since they are impossible objects, i.e. it is impossible for them to exist, or to be conceived as existing, this thing belongs to no causal chain or, in other words, since it is an impossible thing it has no relation with any part of Nature whatsoever, not even our conception of it; therefore there is nothing that could cause neither its existence nor its non-existence. But as we saw before there has to be a cause or a reason for the existence or non-existence of a thing, and together with what we just said, it is clear that the cause must come from its own nature. If we pay attention to the reconstruction of the argument we did in the previous paragraph, we can see where the difference resides. The self-causality of the substance is deduced from its self-dependence, but the self-causality of the impossible thing is deduced from its very nature, which is being impossible to exist.

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[35]We are following the translation from Samuel Shirley, but in the original Spinoza uses *cognitio*. Opera Omnia, EIA4.

[36]This use of *necessity* does not introduces a modality about the substance existence, it just refers to the logical necessity of the existence of a substance in his system contrary to what other authors claim. Jarret (2010). We will go back to this discussion later.

The most striking statement of the initial quote of this chapter we find in the last sentence: "This is self-evident, and therefrom it follows that a thing necessarily exists if there is no reason or cause which prevents its existence". Spinoza seems to state here a hardcore negative vision on ontology, but what he means is that thing which do not have a reason or cause which denies their existence, necessarily exists. But let's analyze this more carefully. This can also be taken as a universal rule for existence in which Spinoza states that "for any x, if, there is no y different from x, such that y causes x to not exists, then x must exists". Basically what Spinoza is saying is that necessary existence comes from the fact that there is nothing that could cause something to not-exist. We clearly see that that is the case for the substance, since there is nothing on which it depends, and therefore there is nothing that has the substance as an effect, except itself, at all, therefore it must exists necessarily. We can deduce the existence of somethings from the fact that there is no cause that denies their existence. For instance an square circle has a cause that prevents its existence that comes from its very nature. A triangle could have a cause that prevents it from existing, since from its very nature we cannot deduce their existence or not, so it must be an exterior cause for either. But in the case of a substance, its existence follows necessarily because we cannot conceive a reason or cause that will prevent it from existing. This is the reason why Spinoza takes the existence of substance as an *eternal truth*, since its existence represents the very notion of true idea, since just from its definition, we can deduced that it is an existing thing, even before we have any discussion of the relation between existence and time.

We will continue now with the other possibilities left of the idea of existence that we saw before. The other possibility we are going explore now is those things which have an external cause or reason for their existence or non-existence. If we recall one sentence from the initial quote: "But the reason for the existence or nonexistence of a circle or a triangle does not follow from their nature, but from the order of universal corporeal Nature. For it is from this latter that it necessarily follows that either the triangle necessarily exists at this moment or that its present existence is impossible." This seems like a unnecessarily strong statement, but if we analyze it from the point of view of Spinoza's notion of existence we see that it makes sense. We see that time is included in this statement, "present" and that is the key to understand it but, before continuing to examine that quote, we have to first explain the notions of modality that Spinoza has. We find the definition of his notions of modality in the *Treatise*:

> I call a thing impossible if its nature implies that it would be a contradiction for it to exist; necessary, if its nature implies that it would be a contradiction for it not to exist, and possible, if, by its very nature, neither its existence nor its nonexistence implies a contradiction, the necessity or impossibility of its existence being dependent on causes which are unknown to us while we are assuming its existence.[37]

The most important thing to have in mind here is that these notions about modality, are predicated about the existence of things. When we discuss the formal language we will see that this is a fundamental change with the modern formal views on modality. From this quote, and the previous one, we see that existence is strongly related with time when talking about the existence of particular things. The other important thing to bare in mind is that when we talk about the existence of things we do it from either an eternal point of view, or from an actual, in the sense of temporal, point of view. If we go back to the triangle in the quote from E1P11, and we assume that it exists at this moment, then the triangle must necessarily exists, otherwise assuming its non-existence will be a contraction. On the other hand, if we take a non-existent triangle at this moment, then it is an impossible triangle, because its existence will entail a contradiction. In other words, if we draw a triangle on a piece of paper, we have to say that the thing exists necessarily, since it will be absurd to have the triangle in front of us and deny its existence. Nonetheless this necessary existence is not a necessary existence like that of the substance, but is a necessary actual existence, *at this very moment*. There is a shift in modality

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[37]TEI, §53. Spinoza (2002). p. 14.

from the substance, and the infinite modes, to particular things, or finite modes. For instance a substance cannot be impossible, nonetheless that triangle we just talked about can be see as impossible whenever its existence ends, for instance if I just erase the drawing. Now this division of things from an eternal point of view or from an actual point of view is of vital importance to understand both Spinoza's theory of existence, and consequently his notion of modality. After this we can see that is pretty obvious that everything either exists in the present necessarily or its existence is impossible. And this view is stressed again in the third notion, *possible*, in which we assume the existence of the particular thing together with the lack in knowledge about whether the thing is impossible or necessary. This is, possibility is related to fictional ideas, which are ideas about things which we are neither aware of its existence nor of its non-existence.

Now we are going to study Spinoza's notion of time. In the *Principles of Cartesian Philosophy* we find a pretty straight definition of what time is:

> Now in order that duration may be determined, we compare it with the duration of other things that have a fixed and determinate motion, and this comparison is called time. Therefore time is not an affection of things, but a mere mode of thinking, or, as we have previously called it, a being of reason; for it is a mode of thinking serving to explicate duration.[38]

Another explanation on his view on time we find on the Letter 12:

> The definition of Modes, insofar as it is not itself a definition of Substance, cannot involve existence. Therefore, even when they exist, we can conceive them as not existing. From this it further follows that when we have regard only to the essence of Modes and not to the order of Nature as a whole, we cannot deduce from their present existence that they will or will not exist in the future or that they did or did not exist in the past. Hence it is clear that we conceive the existence of Substance as of an entirely different kind from the existence of Modes. This is the source of the difference between Eternity and Duration. It is to the existence of Modes alone that we can apply the term Duration; the corresponding term for the existence of Substance is Eternity, that is, the infinite enjoyment of existence or-pardon the Latin- of being (essendi).

From what we saw in the previous paragraph, and from the second line of this quote, we see that there is already a difference between the present existence and the non-present existence, past or future. We saw that if the thing is existing at this moment we have to say that its existence is necessary, but that doesn't mean that we can conceive them as not existing in some point in the future or in the past. In other words, its present existence started at some point and will stop at some point since both cases do not depend on its own nature. This is just another example of the change of view in modality depending on which type of element we are talking about.

Here we find the first two notions of time in Spinoza, duration and eternity, both anchored to the notion of existence. While duration is defined as "the indeterminate continuance of existence[39]", eternity is defined in E1D8 as:

> Existence itself insofar as it is conceived as necessarily following solely from the definition of an eternal thing[40].

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[38] Principles of Cartesian Philosophy, Appendix Containing Metaphysical Thoughts, Part 1, Chapter 4. *ibid.*, p. 185.

[39] EIID5. *ibid.*, p. 244.

[40] *ibid.*, p. 217.

Eternity is existence, and existence is eternity if we are talking about the substance's existence. Duration is the unknown period between the composition of a particular thing and its decomposition.[41] Duration cannot be applied to substance, because that will mean that there is a beginning and an end of its existence, even if we conceived duration without a beginning and an end, as Spinoza states in the explication of the definition of eternity. But the main reason because we cannot apply that concept to substance's existence is because its existence is not divisible, this is, its existence is taken as infinite, as unlimited and as one. This is not the case for duration and the existence of modes:

> What I have said makes it quite clear that when we have regard only to the essence of Modes and not to Nature's order, as is most often the case, we can arbitrarily delimit the existence and duration of Modes without thereby impairing to any extent our conception of them; and we can conceive this duration as greater or less, and divisible into parts. But Eternity and Substance, being conceivable only as infinite, cannot be thus treated without annulling our conception of them.[42]

Because of the nature of finite modes we can conceive their existence, i.e. duration, as a bigger or smaller period and, most important, we can divide their duration into parts and study them by the notion of time. It has been explain the difference between the nature of attributes and the nature of the modes, and this difference implies a difference in their existence.

We have talked by now about all the different beings, i.e., things that exists by their own nature, things that do not exist because their own nature, thing that exists by an external cause and thing that do not exist by an external cause. They represent the three modalities in Spinoza; necessary, impossible and possible respectively. This is what I understand when Spinoza talks about the nature of things–this is, there is a formal division of things based on the way we have to conceived their existence and the cause of it. That's why I have always claimed that Spinoza's ontology is an epistemology. We have seen that any distinction of being comes from the notion of conceivability. Therefore those distinction are based on the relation that our mental capacities have with reality, which is also included in his ontology. The epistemological reading of the attributes is the key of his all system, no wonder why in his first letter to Oldenburg he uses attribute as he uses his notion of substance in the *Ethics*: "Here it should be observed that by attribute I mean every thing that is conceived in itself and through itself, so that its conception does not involve the conception of any other thing. For example, extension is conceived through itself and in itself, but not so motion; for the latter is conceived in something else, and its conception involves extension.[43]". The substance is just the entity with the highest notion of quality we can conceive, in the sense that any other thing that can be conceived as sharing that quality or modifying it has to be included in the conception of that entity.

That difference in nature is a difference in the way we perceive those things, this is, the difference of nature that we find on his ontology is an epistemological difference. This distinction is based in the two epistemic capacities we have, understanding and imagination. This two capacities are the subjective representation of two sides of Nature, and from this arises the famous dichotomy *Natura naturans* and *Natura naturata*. To understand this distinction is vital to understand how this two types of nature coexist in Spinoza's ontology and why the human point of view is so important as a means of understanding one of those types through the other, that is the reason why we have this double view on the nature of things, which is translated into their existence through the understanding, as one, infinite, indivisible and in itself; and through the imagination as multiple, finite, divisible and in something else. Under this view, Spinoza defines the three modes of imagination that are number, measure and time. In Letter 12 Spinoza

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[41]It is important to understand that in Spinoza there is no genuine destruction, or obliteration of things, they just are composed and decomposed as Deleuze explains brilliantly in his book about Spinoza, in the chapter called: "Modal existence". Deleuze (2005). pp. 201-217.

[42]Letter 12. Spinoza (2002), p. 788.

[43]Letter 2. *ibid.*, p. 762.

says: "Further, from the fact that we are able to delimit Duration and Quantity as we please, conceiving Quantity in abstraction from Substance and separating the efflux of Duration from things eternal, there arise Time and Measure: Time to delimit Duration and Measure to delimit Quantity in such wise as enables us to imagine them easily, as far as possible. *ibid.*, p. 789." I will come back to this vital distinction again in the paper.

2.3 Absolute infinite being

In this section I am going to focus on the idea of God that Spinoza gives in the *Ethics* focusing on one of its most important features: absolute infinity. Let us start with the definition given in the *Ethics*:

> By God I mean an absolutely infinite being, that is, substance consisting of infinite attributes, each of which expresses eternal and infinite essence. Explication: I say *absolutely infinite*, not *infinite in its kind*. For if a thing is only infinite in its kind, one may deny that it has infinite attributes. But if a thing is absolutely infinite, whatever expresses essence and does not involve any negation belongs to its essence[44].

The first thing to notice here is the sense of infinite that Spinoza uses, and how "absolute infinitely being" is compared to a "substance with infinite attributes". Now a common misinterpretation of this definition is that "infinite attributes" is a statement about the number of attributes possessed by the substance, but that entails several problems in Spinoza's system that we will see later when we talk about his view on numbers. The correct reading is that infinity is a property of the attributes, i.e., the absolute being is a being which possesses unlimited qualities. In the *Ethics* we find his definition of finitude

> A thing is said to be finite in its own kind when it can be limited by another thing of the same nature[45]. For example, a body is said to be finite because we can always conceive of another body greater than it. So, too, a thought is limited by another thought. But body is not limited by thought nor thought by body.

Now this infinite is different from that of the *absolute*. The infinity of the absolute is included in a note to the definition of God in EID6. The note goes as follows:

> "I say *absolutely infinite*, not *infinite in its kind*[46]. For if a thing is only infinite in its kind, one may deny that it has infinite attributes. But if a thing is absolutely infinite, whatever expresses essence and does not involve any negation belongs to its essence.[47]"

We also have the distinction made in the Letter 12, which its called the *Letter on the infinite*:

> The question of the infinite has universally been found to be very difficult, indeed, insoluble, through failure to distinguish between that which must be infinite by its very nature or by virtue of its definition, and that which is unlimited not by virtue of its essence but by virtue of its cause. Then again, there is the failure to distinguish between that which is called infinite because it is unlimited, and that whose parts cannot be equated with or explicated by any number, although we may know its maximum or minimum. Lastly, there is the failure to distinguish between that which we can apprehend only by the intellect and not by the imagination, and that which can also be apprehended by the imagination. I repeat, if men had paid careful attention to these distinctions, they would never have found themselves

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[44]EID6. *ibid.*, p. 217

[45]We will use *Nature* with a capital letter when we refer to Nature as a whole, or as equivalent to God, and *nature* when we talk about the formal properties of something.

[46]His emphasis.

[47]*Spinoza (2002), . 217*

overwhelmed by such a throng of difficulties. They would clearly have understood what kind of infinite cannot be divided into, or possess any, parts, and what kind can be so divided without contradiction. Again, they would also have understood what kind of infinite can be conceived, without illogicality, as greater than another infinite, and what kind cannot be so conceived. This will become clear from what I am about to say. However, I shall first briefly explain these four terms: Substance, Mode, Eternity, Duration.[48]

We are going to analyze Spinoza's notion of infinity following these distinctions, and mainly applied to his ontological hierarchy. Although in the letter Spinoza uses those notions of infinity to explain the confusion that authors have had when dealing with that concept, I will use those distinctions applied to his ontological hierarchy, in order to get a better insight of it, and to show that those are not only used by him to explain what he calls in the letter the aids of the imagination, but that they are also applied to his whole system. But before delving into the distinctions, we see at the end of the quote that what underlines all those distinction is the ontological difference between the substance and the modes and the perspective from which we apprehend their existence. This is, from an eternal point of view, or from temporal one. We talked already about these terms in the previous chapter and their definition, but the relevant feature that Spinoza wants to focus on here is the difference these terms have in divisibility, and most important the relation of dependence in which they stand. Substance and modes, the two main ontological entities in Spinoza, are the basis on the analysis I provide of those distinctions given in the letter.

2.3.1   Distinction 1: Infinite by its very nature and unlimited by cause.

We are going to star with his first distinction regarding the infinite which is a question of the origin of the infinity of a thing. This distinction states that when we call something infinite we do it either because it is in its nature to be so, or because its infinity comes from something external, and is drummed into that thing. Let us begin with the relation between nature and infinity. We can think of what does it mean to be infinite by nature if we pay attention to the definition in the *Ethics* that we saw before. If something is infinite by nature it means that it is the only being of that nature there is, or that all other beings of the same nature are equal to it, since this entails that it cannot be limited, therefore it must be infinite. These are the substance and its qualities, or attributes, since it is clear that nothing can limit a quality in itself. We are not talking here about the substance as absolute yet, only about what is infinite in its kind as Spinoza explains in the explanation to E1D6. What is unlimited by cause? Unlimited by cause means that they are infinite, in the sense we just saw, but at the same time they depend on something, which obviously cannot be of the same nature by definition. This is the difference between substance and infinite modes. Now it is time to compare them, so we will explain what they are first. If to be finite is to be limited, then to be infinite is to be unlimited. Nonetheless we have to pay special attention to this notion of unlimited, since it depends on the nature of things and on the relation of dependence with other things of that same nature. Not surprisingly he chooses modes from the only two attributes of God, extension and thought, as an example for the notion of finitude. Limitation is taken here as composition, as size in extension and comprehension in thought, in the sense that, any physical object can be contained in a bigger object because that is exactly their nature, and that any idea can always be taken as part of the conception of another idea. From that definition it follows an implicit feature of finite things, they are infinitely divisible, or composed. In the same way that we can conceive an object, or an idea forming another object or another idea, we can take any object and idea as being formed by other. Nonetheless there are exceptions to this, i.e., we can conceive an object, or idea, which is not contained in the conception of any other object, or idea. These are what Spinoza calls infinite modes, i.e., an object and an idea which we cannot conceived as contained in the conception of any other object or idea.

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[48]*ibid.*, p. 787

In letter 64 Schuller asks Spinoza:

> Fourthly, I should like to have examples of those things immediately produced by God, and of those things produced by the mediation of some infinite modification. It seems to me that thought and extension are of the first kind, and of the latter kind, intellect in thought and motion in extension, etc.[49]

Spinoza's answer is:

> Lastly, the examples you ask for of the first kind are: in the case of thought, absolutely infinite intellect; in the case of extension, motion and rest. An example of the second kind is the face of the whole universe, which, although varying in infinite ways, yet remains always the same. See Scholium to Lemma 7 preceding EIIP14.[50]

Here we find what has been called the modal theory of modes. There are two types of infinite modes that Spinoza talks about, the infinite immediate mode and the infinite mediate mode. In extension those modes are movement and stillness and the total surface of the universe respectively. In thought we have the intellect absolutely infinite and the total surface of the universe. This last one is a bit controversial because Spinoza didn't mentioned it in the letter, nonetheless I agree with the argument that Spinoza didn't mentioned the second kind of infinite mode in the letter corresponding to thought because it is the same as the one in extension.[51]. Formally they represent the principle through which modes of the respective attribute are formed and the set of all modes, this is, of currently existing modes or of all possible modes, depending on the temporal view we adopt. We find here the ontological hierarchy of extension with proper names: extension, movement and stillness, the total surface of the universe and body (which is the finite mode). Extension is an attribute of Nature (God); movement and stillness is what follows immediately from the attribute of extension; in other words, movement and stillness are the conditions of possibility of any modification of extension whatsoever. Now what is said by Spinoza to follow *mediately* is the total face of the universe and what he means is that this doesn't follow directly from the nature of the substance but it is produced by its direct modification, in the sense that it requires the infinite immediate mode to be formed. In the same way we have the ontological hierarchy of thought: thought, absolute infinite intellect, total face of the universe, and idea.

Here we find the relation between the nature of things and what is produced by that nature. In the case of God, or the substance consisting of infinite attributes, the nature of the substance is the nature of its attributes, i.e., the attributes share the same nature since they aren't but the objectivization of the substance's essence, therefore they must share the same nature since the objective essence is grounded in the formal essence. Even though this argument might seem a bit *ad hoc*, we can find several places in Spinoza's work in which he calls the "extensive substance", or even treats attributes in the same way the treats the substance. The argument that there is no transcendence between the substance and its attributes will be explored later when we justify the decision of not including them in the formal language. We see that things follow immediately or mediately. When we say that something follows from another thing, we mean that the thing's nature is cause by another thing's nature, on which it depends. In this case following immediately means that the nature of the mode is caused *directly* by the substance, i.e., there is no other cause in between them. Following mediately means that the mode's nature

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[49] *ibid.*, p. 917

[50] *ibid.*, p. 919

[51] There is a discussion about why Spinoza left unmentioned what is the infinite mediate mode of thought. We cannot find a direct answer to this issue in any of Spinoza's works, nonetheless I claim that by analogy to the infinite mediate mode of extension we can deduce what it is. We follow the argument from Giancotti (1991), in which she claims that Spinoza didn't mentioned it explicitly, but that both the infinite mediate mode from extension and thought are the same, the totality of modifications.

follows from the substance *indirectly*, i.e. there something between the substance and the mode, that causes the nature of that mode. The important thing here is to see that the infinity that the attribute of extension has and the infinity that movement and stillness and the whole face of the universe have are different. As we saw when we talked about existence, this difference is a difference in nature, in the sense that extension is an attribute and movement and stillness and the face of the whole universe are infinite modes. Here we have the ontological structure of Spinoza's system, the *Natura naturans*, together with the *Natura naturata*, which the explains precisely as how I have been arguing about the structure of his ontology:

> Here, before we proceed to something else, we shall briefly divide the whole of Nature–namely, into Natura naturans and Natura naturata. By Natura naturans we understand a being that we conceive clearly and distinctly through itself, and without needing anything beside itself (like all the attributes which we have so far described), that is, God. The Thomists likewise understand God by it, but their Natura naturans was a being (so they called it) beyond all substances.
>
> The Natura naturata we shall divide into two, a general, and a particular. The general consists of all the modes which depend immediately on God, of which we shall treat in the following chapter; the particular consists of all the particular things which are produced by the general mode. So that the Natura naturata requires some substance in order to be well understood.[52]

Although Spinoza seems to have change his mind about this dichotomy of his system later in the *Ethics*:

> Before I go any further, I wish to explain at this point what we must understand by Natura naturans and Natura naturata. I should perhaps say not explain, but remind the reader, for I consider that it is already clear from what has gone before that by Natura naturans we must understand that which is in itself and is conceived through itself; that is, the attributes of substance that express eternal and infinite essence; or (Cor. 1 Pr. 14 and Cor. 2 Pr. 17), God insofar as he is considered a free cause. By Natura naturata I understand all that follows from the necessity of God's nature, that is, from the necessity of each one of God's attributes; or all the modes of God's attributes insofar as they are considered as things which are in God and can neither be nor be conceived without God[53].

He seems to have included the objective interpretation together with the formal structure, contrary to what he says in the *Short Treatise*. Formally: substance, condition of possibility of modification of the substance (general mode) , union of all possible modifications of the substance. Objectively: extension, movement and stillness, Nature as a whole; thought, reflexive knowledge, Nature as a whole. There is something I must discuss before proceeding further, what is the status of the infinite mediate mode, and how is it related to finite things. The infinite mediate mode represents all the possible modifications of the substance taken as a unit, i.e., what follows from the infinite immediate mode, is not the mode of Nature as a whole, but the set of all possible modes, which can be taken as a mode because of the principle of composition of modes[54]. Now the biggest question in the Spinozistic ontology is, how is this mode and the finite, particular, modes related? In Spinoza's philosophy there is a gap between infinite things and finite things[55], Deleuze has tried to answer this question[56]. In this work we do not focus

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[52]Short Treatise on God, Man, and His Well-Being, Chapter VIII. Spinoza (2002), p. 83.

[53]EIP29, Scholium. *ibid.*, p. 234.

[54]EIID7: By individual things [res singulares] I mean things that are finite and have a determinate existence. If several individual things concur in one act in such a way as to be all together the simultaneous cause of one effect, I consider them all, in that respect, as one individual. *ibid.*, p. 244.

[55]See Gueroult (1968), pp.334-344.

[56]Deleuze (2005), pp. 191-200.

on particular things, since this will require a deeper study and a more complex work, the only thing we need to mention here is the fact that the difference between the dependence of the infinite mediate mode from the infinite immediate mode, and the dependence of any finite mode from the infinite immediate mode. The difference resides in the fact that while formally the infinite immediate mode serves as s composition principle, objectively it serves as a principle of differentiation: "Bodies are distinguished from one another in respect of motion-and rest, quickness and slowness, and not in respect of substance.[57]"

In the first part of the *Ethics* we find three propositions, 21, 22 and 23 that deal with this matter. These propositions focus on those thing which are infinite, but whose infinity comes from something external, and not from their own nature. In E1P21, one of the hardest to understand, we find the first mention to these type of modes we were talking before:

> All things that follow from the absolute nature of any attribute of God must have existed always, and as infinite; that is, through the said attribute they are eternal and infinite.

Spinoza is telling us here that things which are a direct consequence of the nature of the substance, which is the same as the attributes as we said before, have an infinite and eternal existence, but this is a completely different existence compare to that of the substance, since they are not self-caused, but caused by something else. These things which Spinoza is talking about here are the immediate infinite modes or, as he says in the response to Schuller, *immediately produced*, i.e, movement and stillness and the absolute infinite intellect. Let's try to analyze these notions in a less complicated way. He is telling us here that movement and stillness is a direct consequence of a substance that has extension as one of its qualities because it is the way in which any modification of its extension can be explained and formed[58]. In other words, any entity that possesses the quality of extension is necessarily subjected to a modification through movement and stillness. And in this sense movement and stillness is infinite, in the sense of un-limited, and eternal like the substance with the fundamental difference that this does not comes from its own nature, but from the nature of the substance. They both have the same nature except for the fact that the nature of the first depends on itself, and the second depends on the first. The argument that Spinoza gives for that proposition is, as usual, based on a *reductio*: assume that movement and stillness is finite, therefore it must be limited by something of the same nature, and since it follows from extension, this is, it depends on extension, it must be limited by it, but this is absurd since that will entail that extension itself is also limited, and that extension is a mode.

In E1P22 we find the next type of infinite mode:

> Whatever follows from some attribute of God, insofar as the attribute is modified by a modification that exists necessarily and as infinite through that same attribute, must also exist both necessarily and as infinite.

Here we find the infinite mediate mode, or as he says in the response to Schuller *what follows from a mediation of an infinite modification*, i.e., the total face of the universe. Even though Spinoza claims that this proposition is proven in the same way as the previous one, there is a difference in what both propositions are talking about. The mode we were talking about before can be considered as the "first" mode in the sense that is the only mode that is directly connected to the substance, there is nothing between substance and what follows immediately from it. In the case of the total face of the universe, this is a mode that follows mediately from the extensive substance, this is it follows from another mode and not from the substance itself. The total face

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[57]EIIL1. Spinoza (2002), p. 252.

[58]To understand this idea in a more physical way, or in his sense in a rational mechanic way check the second part of the *Ethics* in which you can find his short treatise of physics. EIIP13. *ibid.*, p. 251.

of the universe follows from movement and stillness which is a mode. The distinction in infinity between these two infinite modes will be boarded in the next section, here we are dealing with the difference on their cause. In that sense the total face of the universe can be seen as the universal mode containing all other extensive modes. And the principle under which all those modes are formed is movement and stillness, i.e., the thing on which all those modes depend on, even the mediate one, is movement and stillness. In the last proposition we are considering here, E1P23 Spinoza states:

> Every mode which exists necessarily and as infinite must have necessarily followed either from the absolute nature of some attribute of God or from some attribute modified by a modification which exists necessarily and as infinite.

Here we see how Spinoza sates what I was discussing before, any infinite mode has to either follow from the nature of an attribute or from some modification of an attribute. We can see the infinite immediate mode as the smallest mode possible and the infinite mediate mode as the greatest mode possible. Nonetheless, from a dependence point of view, we see that the infinite immediate mode is the mode that do not depends in any other mode, precisely because any other mode depends on it. In a similar fashion, the infinite mediate mode does not depend on any other mode because there is no such thing, since all modes are considered as forming part of this one.

In a note that Leibniz wrote about Spinoza and his idea on the infinite he says: "Here is a noteworthy observation concerning the infinite. Since there is one infinity greater than another, will there be something more eternal than something else? For instance, a thing can exist before any time imaginable, and yet not from eternity, because its time, in existence, will not be absolutely infinite, but infinite only in relation to us. Therefore there was a time when it did not exist, but that time is infinitely remote from now. This is just as an infinitely small line is in relation to a point."[59] we can see here an example regarding the difference in the existence of the infinite modes we were just talking about. Leibniz is talking here about the difference between the existence of movement and stillness, i.e., eternity, and the existence of the face of the whole universe, which has only infinite duration. We see that he has pinpointed the idea that infinities bigger than other is a matter of nature and not a matter or size. There is a difference in conceiving something as being eternal by it's nature, i.e. by definition, and conceiving something as being finite by nature and conceiving its duration extended beyond comprehension; this is becoming indeterminate for our understanding. We will come back to this example later when we talk about existence, and when we deal with the third distinction on the infinite.

2.3.2 Distinction 2: unlimited and inexplicable by number

In this distinction we are not comparing two different things, but we are comparing two different senses of infinity found in the infinite modes that we just discussed in the previous section. Although this distinction is mainly used in the letter to distinguish extension and duration from quantity and time, as we said before this difference is grounded in the difference between substance and modes. But this second distinction becomes very interesting if we use it, as we did before, to get a better insight on the infinite modes. We are going to compare the difference between being infinite because there are no limits for the thing, and being infinite because of the composition of something. Each of the previous modes we were talking about satisfies this distinction in a different way, we are no longer distinguishing these two infinite modes with each other but comparing the two senses of infinity that are found in them. Let's begin with movement and stillness. In what sense is movement and stillness unlimited? The answer is the same as in the case of extension, motion can only be limited by something of the same nature, but since there is no such thing, i.e. there is no other mode that follows immediately

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[59]Leibniz (2013), p. 66.

from the substance, it cannot be the case that it is limited. Movement and stillness cannot be limited for that reason, but movement or stillness is limited when taken as a property of a mode, and in that sense it can be limited by the movement or stillness of another mode. Extension is infinite by its own nature, that's the reason why it cannot be limited, and therefore cannot be conceived as the sum of all extensive objects, but movement and stillness can be seen as infinite in the double sense we are discussing in this chapter. This infinite mode has a double interpretation, we can think of it as the property in itself, or as a property of an object. The former is what we talked about in the previous chapter, since there is no other mode on which movement and stillness depends, then it cannot be limited by something of the same nature. We can ask the question, how can we conceive a property as being limited? If we consider the property in itself it is obvious that it cannot be limited by anything else, in the same way as extension. Since this is the case we can admit that the movement of a mode is only limited by movement or stillness of another mode.

The other interpretation of the infinity of movement and stillness is the opposite of what we just said, i.e. when we take it as a property of and object and being limited by the motion of another object. As Lemma 3 in the short physics treatise that we find in the second book of the *Ethics*[60], which states:

> A body in motion or at rest must have been determined to motion or rest by another body, which likewise has been determined to motion or rest by another body, and that body by another, and so ad infinitum.

Now the interesting question here is how to interpret the motion of the second infinite mode, i.e., the whole face of the universe, since this is the sum of all extensive modes and we argued before that it is also unlimited, it might look like this determination of movement cannot be applied to it as if we were talking about the determination of movement of two pool balls. It seems like the movement of Nature as a whole must be determinate by the movement of another body, while at the same time not being limited by that body. From definition 7 in the first part of the *Ethics* we understand what determination is:

> That thing is said to be free [liber] which exists solely from the necessity of its own nature, and is determined to action by itself alone. A thing is said to be necessary [necessarius] or rather, constrained [coactus], if it is determined by another thing to exist and to act in a definite and determinate way.

To say that the movement of Nature as an individual object is determine by itself, if we attempt to follow the idea that being finite and being limited are the same, is to say that Nature as a whole is unlimited and not determined by anything, is to say that Nature as a whole is a free thing, which would be the same as saying that it exists solely from the necessity of its own nature, and as a mode this is absurd. The union of all existing objects cannot be limited by any of these objects, but it *can* be determined by then. This is when the notion of composition starts to be relevant in the argument. The existence and the motion of Nature as a whole is determined by all the modes that compose it. If we recall Spinoza's answer to Schuller's letter about the infinite modes, we see that Spinoza mention EII Lemma 7 which states that:

> **Lemma 7** Furthermore, the individual thing so composed retains its own nature, whether as a whole it is moving or at rest, and in whatever direction it moves, provided that each constituent part retains its own motion and continues to communicate this motion to the other parts.

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[60]EIIL3. Spinoza (2002), p. 252.

Proof This is evident from its definition, which you will find preceding Lemma 4[61].

We can conceive Nature as one entity, and in this sense it is unlimited because the conception of any mode is included in the conception of Nature as a whole. In the same way as before, we can also think of Nature as the total union of all existing, or possible, modes, and in that sense its different parts cannot be expressed by any number. We find an example that illustrates this in Letter 12:

> For example, all the inequalities of the space lying between the two (non-concentric) circles ABCD in the diagram exceed any number, as do all the variations of the speed of matter moving through that area. Now this conclusion is not reached because of A the excessive magnitude of the intervening space; for however small a portion of it we take, the inequalities of this small portion will still be beyond any numerical expression. Nor again is this conclusion reached, as happens in other cases, because we do not know the maximum and minimum; in our example we know them both, the maximum being AB and the minimum CD. Our conclusion is reached because number is not applicable to the nature of the space between two non-concentric circles. Therefore if anyone sought to express all those inequalities by a definite number, he would also have to bring it about that a circle should not be a circle.

We can think of that space as finite, being enclosed under boundaries, i.e. knowing its maximum and minimum, or we can think of the composition of all different distances or movements within that space. In this example, because of which Leibniz congratulated Spinoza for his ahead-of-time view on mathematics and that he himself will later use to account for differential calculus[62]. We can see how Spinoza is talking about the two main branches of infinitesimal calculus, differential and integral calculus. When we talk about parts of modes we have two main points of view, first the infinitesimal divisibility of any mode into a great number of parts distinct from each other, and the idea that any mode contains an actual infinity in itself conceived as the concurrence of all its different parts. Both infinite modes represent the maximum and the minimum in Spinoza's ontology. In the case of extension, Spinoza talks about the *simplest bodies*, which are not atoms, but bodies which are only differentiated by their difference in movement, i.e., they are the simplest body we can conceive, which are bodies which have nothing else except movement. In that sense movement and stillness is the minimum, and Nature taken as a whole if the maximum:

> We thus see how a composite individual can be affected in many ways and yet preserve its nature. Now previously we have conceived an individual thing composed solely of bodies distinguished from one another only by motion and- rest and speed of movement; that is, an individual thing composed of the simplest bodies. If we now conceive another individual thing composed of several individual things of different natures, we shall find that this can be affected in many other ways while still preserving its nature. For since each one of its parts is composed of several bodies, each single part can therefore (preceding Lemma), without any change in its nature, move with varying degrees of speed and consequently communicate its own

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[61]EIIL4 **Definition** When a number of bodies of the same or different magnitude form close contact with one another through the pressure of other bodies upon them, or if they are moving at the same or different rates of speed so as to preserve an unvarying relation of movement among themselves, these bodies are said to be united with one another and all together to form one body or individual thing, which is distinguished from other things through this union of bodies. (This is the definition that Spinoza refers to). *ibid.*, p. 253.

[62]In Duffy (2006), the views on the infinitesimal calculus from Spinoza and Leibniz are compared. This author also takes into account the arguments from Deleuze and Gueroult in the matter. What I want to focus on here is in the comparison of the geometrical examples found in that paper. One is the already mentioned geometrical example of the two non-concentric circles found in Letter 12. The other is a Leibnizian example found in Leibniz (1969), p. 545. This example also related to that given by Spinoza in EIP15, Scholium, Spinoza (2002), p. 225.

motion to other parts with varying degrees of speed. Now if we go on to conceive
a third kind of individual things composed of this second kind, we shall find that
it can be affected in many other ways without any change in its form. If we thus
continue to infinity, we shall readily conceive the whole of Nature as one individual
whose parts- that is, all the constituent bodies vary in infinite ways without any
change in the individual as a whole[63].

The distinction that names this chapter is found here to be two sides of the same entity.
Nature taken as a whole in unlimited and composed of a great number of parts, the possibility
of this will be explain in the next chapter. In the same way the movement of this mode is
unlimited, but at the same time determined by the movements of all the objects that compose
it. This composition of different modes with different movements is what cannot be captured by
any number. Try to think if the total face of the universe can be expressed by a number. Now
try to think if number is suited to capture all differences in movement and stillness. Since we can
conceive an infinite mode, this is, we can take Nature as a whole as an individual thing, what
can we say about the movement of this object? The movement of this object is an unlimited
movement. Nonetheless it is composed of all the movements of all the objects that form that
universal, or infinite, mode. The movement of Nature taken as a mode is made by the movements
and the differences in movements of all the modes composing it, and this is infinite in the sense
of indeterminate, in the sense of being inexpressible by numbers.

2.3.3   Distinction 3: intellect and imagination.

We are going to discuss now the last distinction between the infinite based on our way
of conceiving it. He says: *Lastly, there is the failure to distinguish between that which we
can apprehend only by the intellect and not by the imagination, and that which can also be
apprehended by the imagination.* This is the most important distinction of all three because
it establishes a clear boundary between the two epistemological capacities that humans have.
The difference between intellect and imagination is in itself the most important distinction in
Spinoza's whole system. The difference between imagination and understanding, or intellect,
can be found in Descartes' *Meditations on first philosophy*:

> I remark besides that this power of imagination which is in one, inasmuch as it
> differs from the power of understanding, is in no wise a necessary element in my
> nature, or in [my essence, that is to say, in] the essence of my mind; for although
> I did not possess it I should doubt less ever remain the same as I now am, from
> which it appears that we might conclude that it depends on something which differs
> from me. And I easily conceive that if some body exists with which my mind is
> conjoined and united in such a way that it can apply itself to consider it when
> it pleases, it may be that by this means it can imagine corporeal objects; so that
> this mode of thinking differs from pure intellection only inasmuch as mind in its
> intellectual activity in some manner turns on itself, and considers some of the ideas
> which it possesses in itself; while in imagining it turns towards the body, and there
> beholds in it something conformable to the idea which it has either conceived of
> itself or perceived by the senses. I easily understand, I say, that the imagination
> could be thus constituted if it is true that body exists; and because I can discover
> no other convenient mode of explaining it, I conjecture with probability that body
> does exist; but this is only with probability, and although I examine all things with
> care, I nevertheless do not find that from this distinct idea of corporeal nature,
> which I have in my imagination, I can derive any argument from which there will
> necessarily be deduced the existence of body.[64]

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[63] EIIL7, Scholium. Spinoza (2002), p. 254.
[64] Meditation VI. §3 Descartes (1996), p. 26. Even though this definition is found in Descartes and not in

Understanding is when our intellect focuses on our own way of conceiving things, imagination is when our intellect focuses on things in themselves. This distinction entails a double view, things are understood when we apprehend the way in which we can form the idea of that thing; things are imagined when we focus on the properties of that thing using our perception. Understanding uses reflection, ideas of ideas, imagination uses perception, ideas of objects.

Under this view we are going to explain the difference between both senses of infinity. In Letter 12 Spinoza says:

> We conceive quantity in two ways: abstractly or superficially, as we have it in the imagination with the help of the senses, or as Substance, apprehended solely by means of the intellect. So if we have regard to quantity as it exists in the imagination (and this is what we most frequently and readily do), it will be found to be divisible, finite, composed of parts, and manifold. But if we have regard to it as it is in the intellect and we apprehend the thing as it is in itself (and this is very difficult), then it is found to be infinite, indivisible, and one alone.

This argument starts from the fact that we have the capacity to delimit things as we please, as we discussed in the chapter before, remember that Spinoza started in the *Ethics* with the definition of finitude and not of infinity. This capacity is tied to the imagination through which we randomly can divided something, or conceive something as bigger or smaller, this is, we can freely impose limits to things, in the same way as we can conceive something as composed of other things. This is not only restricted to quantity, but also to duration. For instance the Zenodian example that he talks about in the Letter 12 explains the misconception that would follow from the attempt to understand duration taken as time, this is the misconception of trying to apply the imagination to duration:

> To make the matter still clearer, take the following example. If someone conceives Duration in this abstracted way and, confusing it with Time, begins dividing it into parts, he can never understand how an hour, for instance, can pass by. For in order that an hour should pass by, a half-hour must first pass by, and then half of the remainder, and the half of what is left; and if you go on thus subtracting half of the remainder to infinity, you can never reach the end of the hour.

What Spinoza tries to say with this example is that if if we understand extension, for instance, in the same way as we imagine it, then we could never consider possible to travel from one point to another. Time, measure and number are aids of our imagination used to understand reality in the easiest way. This is, a meter is a mental tool that we use in order to be able to communicate under a common metric system. Extension is not composed of meters and and centimeters, extension can be divided into meter and centimeters by our imagination. Because when we use the notion of "meter" we are delimiting extension and individualizing it into one unit of quantity. Now this extension cannot be conceive as being divisible because is not, what is divisible is the unit of quantity of one meter which can be divided into centimeters, but the extension it delimits, regardless of how we delimit it, is not divisible.

This distinction has shown us that while the imagination cannot apprehend certain things, like the substance and the infinite modes, the intellect can actually apprehend the particular modes, which belong to the realm of imagination. Particular modes are apprehended by the intellect through their formal essence, but it is incapable of distinguishing between them. On the other hand, the imagination, focused on the objective essence of modes, and not in their relation to the order of Nature, gives us a whole new different way of perceiving things completely

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Spinoza, I claim that the both use it in the same sense. There is no definition of neither of the concepts in Spinoza, nonetheless this Cartesian definition fits perfectly with Spinoza's notions.

different from the one that understanding provides us. Let us recall a quote from Letter 12 explaining this epistemological difference focusing on the notion of existence:

> The affections of Substance I call Modes. The definition of Modes, insofar as it is not itself a definition of Substance, cannot involve existence. Therefore, even when they exist, we can conceive them as not existing. From this it further follows that when we have regard only to the essence of Modes and not to the order of Nature as a whole, we cannot deduce from their present existence that they will or will not exist in the future or that they did or did not exist in the past. Hence it is clear that we conceive the existence of Substance as of an entirely different kind from the existence of Modes. This is the source of the difference between Eternity and Duration. It is to the existence of Modes alone that we can apply the term Duration; the corresponding term for the existence of Substance is Eternity.

From what I said before we have two ways of conceiving the existence of modes, while we only have one way of conceiving the existence of the substance. The first of these two ways is the apprehension of a mode's existence by the imagination, this is, attending to the mode's properties and its present existence. From this we cannot deduce anything regarding its existence, we can only imagine its existence as continuing for an indeterminate period which we can conceive as greater or less, that is the definition of duration. The other way we have to conceive the existence of modes is through the intellect. In this way the existence of the modes are deduced from the order of Nature, this is, formally their existence is explained as following from the existence of the substance.

3 Formalization of Spinoza's philosophy

In this chapter I am going to explain the main decisions and strategies chosen for the formal language that will be presented in the next chapter. Here you will find arguments, based on my interpretation of Spinoza's philosophy explained in the former chapter, explaining the formal language I will use, and its structure. The main objective of this chapter is to explain what are the elements chosen for the formal language, and how they relate to the elements of Spinoza's philosophy. The main principle that governs all this process is found in the *Treatise*, the distinction between the *formal* properties of things, in contrast to the *objective* properties of things:

> Hence it is evident that certainty is nothing else than the objective essence itself; that is to say, the way in which we become aware of the formal essence is certainty itself. And from this again it is evident that for the certainty of truth no other sign is needed but to have a true idea. For, as we have shown, in order to know, there is no need for me to know that I know. From this, again, it is clear that no one can know what the highest certainty is unless he has an adequate idea or the objective essence of some thing. For certainty and objective essence are the same.
>
> Since truth, then, needs no sign, and to have the objective essences of things, or- which is the same thing- their ideas, is enough to remove all doubt, it follows that the true method does not consist in seeking a sign of truth after acquiring ideas; the true method is the path whereby truth itself, or the objective essences of things, or ideas (all these mean the same) is to be sought in proper order.
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> Again, method must necessarily be discourse about reasoning or intellection. That is, method is not reasoning itself which leads to the understanding of the causes of things, and far less is it the understanding of the causes of things. It is the understanding of what is a true idea, distinguishing it from other kinds of perception and examining its nature, so that we may thereby come to know our power of understanding and may so train the mind that it will understand according to that standard all that needs to be understood, laying down definite rules as aids, and also ensuring that the mind does not waste its energy on useless pursuits.
>
> From this we may conclude that method is nothing but reflexive knowledge, or the idea of an idea; and because there is no idea of an idea unless there is first an idea, there will be no method unless there is first an idea. So a good method will be one which shows how the mind is to be directed according to the standard of a given true idea. Again, since the relation between two ideas is the same as the relation between the formal essences of those ideas, it follows that the reflexive knowledge of the idea of the most perfect Being will be more excellent than the reflexive knowledge of other ideas. That is, the most perfect method will be one which shows how the mind should be directed according to the standard of a given idea of the most perfect Being[65].

This idea of the formal essence, and the relation it has with the objective essence of things, is the key to understand his method of the true idea, the truth of the method resides in: *Hence it is evident that certainty is nothing else than the objective essence itself; that is to say, the way in which we become aware of the formal essence is certainty itself.* Therefore the method consist in becoming aware of the formal essences of things and how those formal essences provide us with the basis to form an idea of the thing, i.e., its objective essence. The formal language is based on this idea of focusing on the formal structures and properties of things which grounds the objectivization of them. The goal is to give an explicit formal language that accounts for the formal properties and relations of the elements of his philosophy upon which the objective

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[65]TEI, §35-38. Spinoza (2002), pp. 10-11.

interpretation is built. Our main objective is to focus on the idea of God and everything that involves, with the objective of establishing the basis for this language, which could be expanded in the future to other parts of the *Ethics*, which would, hopefully, take us to his view on ethics. In Spinoza's philosophy, God and all that involves is the very foundation for his ontology and at the same time for this method of the true idea as we just saw, and we follow the same procedure in this formal language. The foundations for this language is the idea of *God* in Spinoza, and all the philosophical terms related to that idea that can be found in his philosophy. My objective with this work is to brew a formal language based on Spinoza's philosophy and some of his relevant formal concepts. This formal language is more of an interpretation of his philosophy from a formal point of view than a thorough formalization of all the *Ethics* word by word. A formal approach to Spinoza requires a decision to be made, either you try to literally translate the *Ethics* into a formal language[66], or you try to interpret his philosophy from a formal point of view.

My main statement is that in the first part of the *Ethics* we can find a formal structure that underlies this whole part and which serves as a basis for all his philosophy, focused on the idea of God. As we saw in the quote the first part sets the basic structure by inquiring on the idea of God and everything that follows from that idea, i.e., the true idea that is at the very beginning of his method. The goal then is to unravel this basic structure in the first part giving formal interpretation of it. It is a must to mention and discuss here the work of Charles Jarrett, since he attempted the former approach to Spinoza. He translated the first part of the *Ethics* into a formal language, but my strategy is different. His work would also be discussed later in this chapter in comparison with my approach to the problem. I will now proceed to explain how the main concepts in the first part are treated in order to give the reader the basic tools that the language uses to interpret formally his philosophy in the first part, with the objective that when we start dealing each definition, attribute or proposition they can make sense of how they are interpreted formally.

Now when we talk about knowledge we are obviously talking from the perspective of the attribute of thought, and when we talk about causation someone might think that we are talking about physical causation. Nonetheless, as I said before in this work we are focusing of the formal essences of things and not on the objective essences. In the *Treatise*, Spinoza claims that: *Finally, a thing is perceived through its essence alone when, from the fact that I know something, I know what it is to know that thing* , and also later in the same work he says: *Hence it is evident that certainty is nothing else than the objective essence itself; that is to say, the way in which we become aware of the formal essence is certainty itself.* He is claiming that to possess the true idea of something, we need to be aware of the process through which we become aware of the formal essence of something, this is, the true knowledge of something does not come from the objective essence of it, i.e., from our perception of it, but from the deduction of its formal essence from the order of Nature.

3.1 Substance and modes

The first and most important part of Spinoza's philosophy are the concepts of *substance* and *mode*, and any formalization of his philosophy should take them as the core of the whole formal language; that is why our language consist on an extension of First Order Logic, with some predicates and relation since, what we seek is to be able to express quantified statements that range over these two elements. Spinoza himself sometimes gives axioms or propositions that are focused on one of the two elements, e.g., EIP23[67] or EIP8[68], but sometimes he just

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[66]See Jarrett (1978).

[67] "Every mode which exists necessarily and as infinite must have necessarily followed either from the absolute nature of some attribute of God or from some attribute modified by a modification which exists necessarily and as infinite." Spinoza (2002), p.231.

[68] "Every substance is necessarily infinite" *ibid.*, p.219.

makes quantified statements about *things*, e.g., EIA1[69], and in this case he is just quantifying about both elements. Even though some of his propositions or axioms are not explicitly stated as a quantification, we can easily see that almost all of his propositions are susceptible of being interpreted as a quantification, e.g., EIP14:

> There can be, or be conceived, no other substance but God

can be interpreted as:

> There is an element, which is God, and every other substance is equal to it

This example shows a recurrent issue in this book, negative existential quantification, but for the sake of the language they are reinterpreted as a universal quantification. The definition of the two main elements of Spinoza's philosophy are taken in this formal approach in a different way than they are found in the *Ethics*, there is no need for an specific definition of what a substance or a mode is, instead we just work with a two-sorted language in which we consider them to be the only two types of elements of our language. We have a set of constants in our language that are modes and substances, and to quantify over then we use variables, i.e., $\{x, y, z, ...\}$ and we denote that any of these variables is a mode or a substance with the predicates M and S. Instead of the definition of *substance* or *mode*, we just define some axioms that capture the relations under which substances and modes are defined in the *Ethics*, e.g., we don't define "substance" as found *Ethics*, instead we have an axiom that says that "for any element, if that element is a substance, then there is no other element such that the substance depends on". We will explain later this notion of dependence, but for now let's focus on these two elements. The domain of our first order language structure consist on the union of two sets of constants that represent substances and modes respectively, in order to be able to quantify over one type of element, or both. Variables are interpreted to range over these constants. The last thing I want to say concerning substances and modes is that we take them to be opposite terms in our language, therefore we included an axiom that it is not based on a statement from the *Ethics*. This axiom states that: "something is a substance if and only if it is not a mode ". I think it is not much trouble to assume that for Spinoza, by the definitions of both substances and modes, if something is a mode, then it cannot be a substance, and vice versa. Using our notation in the logic, we interpret this axiom as: $\forall x(S(x) \leftrightarrow \neg M(x))$.

3.2 Attributes

Let's pass on now to what is consider the third element of Spinoza's philosophy, the attributes. The most controversial part of this formalization, from the point of view of my interpretation of Spinoza's philosophy, is that I do not include the attributes in my formal interpretation of Spinoza, and I will now proceed with the arguments which explain this decision. The first argument is that in Spinoza's philosophy the attributes seem to be a third kind of entity different from substances and modes, but my argument is that it is not really the case, and that attributes are not necessary for this formal structure I want to develop at least for the first part of the *Ethics*. As I said before we found sometimes that attributes are treated in the same way as the substance. In Spinoza's philosophy it doesn't make sense to treat this third element separated from substances and modes if we want to focus on the formal structure of his philosophy. I claim that the attributes belong to the objective properties of things, the reality of things. Descartes call them the extensive and the thinking "res". This duality is a duality of reality, but not of the formal structures of things, i.e., the only things there are, are objects and ideas, but from our formal perspective there are only qualities and modifications of those

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[69]All things that are, are either in themselves or in something else. *ibid.*, p.217.

qualities. But we establish ourselves in this work in a more fundamental level of his philosophy, in an abstract level in which the objectivity of things, the reality of things have not yet been decided and it is not needed in order to account for their formal structure, that is treated in the second part of the *Ethics*.

This whole argument is based on the most important idea that Spinoza had and which is a destruction of all the Cartesian duality, the principle of parallelism. This principle is found in EIIP7:

The order and connection of ideas is the same as the order and connection of things.

Lets focus on this proposition for a moment, this proposition is one of those that Spinoza does not end with a "Q.E.D.", one of those proposition that he treats as more of an indirect axiom. In its proof he says that "This is evident from EIA4; for the idea of what is caused depends on the knowledge of the cause of which it is the effect". So it doesn't really have a demonstration, it is more a principle or axiom than a proposition, and although is found on the second book, we see that follows from an axiom of the first part. The reason why it is not stated until the second book is because Spinoza is following a strategy of his discourse in which he doesn't need this principle in the first part, since he is only dealing with God and not the human mind, i.e., he is only focusing on the formal principles that surround the idea of God. This principle also has a corollary that is worth quoting here in the original, it says: "Hinc sequitur, quod Dei cogitandi potentia aequalis est ipsius actuali agendi potentiae. Hoc est, quicquid ex infinita Dei natura sequitur formaliter, id omne ex Dei idea eodem ordine eademque connexione sequitur in Deo objective." This means that "Whatever follows formally from the Nature of God, follows objectively in the same way from the idea of God". Now we have to be careful here, because as he explains in the scholium:

> At this point, before proceeding further, we should recall to mind what I have demonstrated above- that whatever can be perceived by infinite intellect as constituting the essence of substance pertains entirely to the one sole substance. Consequently, thinking substance and extended substance are one and the same substance, comprehended now under this attribute, now under that. So, too, a mode of Extension and the idea of that mode are one and the same thing, expressed in two ways. This truth seems to have been glimpsed by some of the Hebrews who hold that God, God's intellect, and the things understood by God are one and the same. For example, a circle existing in Nature and the idea of the existing circle-which is also in God-are one and the same thing, explicated through different attributes. And so, whether we conceive Nature under the attribute of Extension or under the attribute of Thought or under any other attribute, we find one and the same order, or one and the same connection of causes–that is, the same things following one another. When I said that God is the cause, e.g., of the idea of a circle only insofar as he is a thinking thing, and of a circle only insofar as he is an extended thing, my reason was simply this, that the formal being of the idea of a circle can be perceived only through another mode of thinking as its proximate cause, and that mode through another, and so ad infinitum, with the result that as long as things are considered as modes of thought, we must explicate the order of the whole of Nature, or the connection of causes, through the attribute of Thought alone; and insofar as things are considered as modes of Extension, again the order of the whole of Nature must be explicated through the attribute of Extension only. The same applies to other attributes. Therefore God, insofar as he consists of infinite attributes, is in fact the cause of things as they are in themselves. For the present, I cannot give a clearer explanation.

It seems that what the corollary and the scholium say are the same, but they are not, one thing is the relation between what follows formally and what follows objectively form God, and

another thing is the relation between different attributes. The argument behind the doctrine of parallelism is not that the order and connection of ideas is the same as the order and connection of things *because* these two attributes are the reflection of each other. The order and connection of ideas is the same as the order and connection of things are the same *because* they are both grounded in the formal essence of God, this is, everything that follows from its nature, and God's objective essence follows from its formal essence in which there is no distinction between objects or ideas, there are merely *modifications*, i.e., modes. Now this formal essence can be interpreted through the attribute of extension or the attribute of thought, and in that sense they both follow the same formal order, which grounds the objective order; that is the argument behind the doctrine of parallelism and that is the reason why I have not taken the attributes in consideration in my formalization of Spinoza, because they are just an objectivization of the formal elements. And in that sense we can interpret from a extensive or thinking perspective all the formal elements of the system, i.e., substance, immediate infinite mode, mediate infinite mode and finite mode.

The second argument supporting my decision of not including the attributes is based on the method of the true idea implicitly explained in the *Treatise* by Spinoza, this method says that we have to follow the formal essences of things in order to possess the truth about things. The biggest problem that follows from this decision is that there are several propositions in the *Ethics* in which the attributes appear that I have to account for in my interpretation in order to be as honest with the text as possible. In the propositions that I take into account in the formalization, the attributes always appear as a mean of comparing substances, or differentiating them. Nonetheless this treatment of the attributes in never a specific one, i.e., Spinoza never specifies which attribute he is talking about, he is just stating the possibility of substances with different or equal attributes. Now this takes us back to the essence of the substance if we follow the definition of the attribute given in the first part:

> By attribute I mean that which the intellect perceives of substance as constituting its essence.

If we consider the formal essence of substance, it is obvious that they all have the same formal essence, except God, but if we are talking about objective essences, then we can take them as the criteria to differentiate or equate substances. For instance if we take EIP2[70], we see that the attributes here serve the purpose of establishing a difference between substances. But in our formal language we interpret that comparison from a different perspective. In EIP2, which is taken as an theorem in my language, Spinoza states that "Two substances having different attributes have nothing in common", and this is what we use to differentiate two substances by the use of attribute, instead we just say that there is no mode common to both substances. This is something obvious in the language of Spinoza, since an attribute encloses all modifications of a substance under that attribute, therefore two different substances cannot have the same modification if they have different attributes, otherwise they will have the same attribute, which is proven in EIP5 that cannot be the case. Thus the reason why we interpret equality between substances is taken in this language to be that they share a modification, since if there one modification common to two substances, then they have to be the same. For the same reason we interpret equality between modes to be taken as "belonging to the same attribute".

We find another clue in EIP4's demonstration, we see that Spinoza himself says: *Therefore, there can be nothing external to the intellect through which several things can be distinguished from one another except substances or (which is the same thing) (EID4) the attributes and the affections of substances.* Therefore attributes are equated to the substances when we compare them, and therefore are not needed as a formal unit. Let me explain this more concretely. As I said before, all formal units can be interpreted objectively from an attribute, therefore when we say following EIP2 that two substances having different attributes have nothing in common it means that if we take two substances that are different, then they have nothing in

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[70] "Two substances having different attributes have nothing in common" *ibid.*, p. 218.

common, and this *nothing* I interpret as having no modes in common. The reason for this is that, what else can this *nothing* means if not modes? Obviously it doesn't account for substances because that's what we are comparing. In the same way it cannot account for attributes because it is assume in the proposition itself that they have different attributes, whatever they are, and the last possibility are modes. Therefore whenever the attributes appear in the *Ethics*, we see that they can be interpreted as means of comparing or differentiating substances. The attributes in Spinoza are taken as the way to connect the substance and modes, so the definition of attribute is substituted for an axiom that captures this connection. Since all modifications of that substance are modifications of that quality, in Spinoza there is one mode which is so to speak the fundamental modification of any attribute, and on which any other modification depends. This is where the infinite immediate mode plays a important role, since movement and stillness, for instance, is this fundamental extensive modification. With this axiom we want to express the fact that there is a unique mode that follows immediately from the substance that represents this fundamental modification.

3.3   Dependence and causation

I have discussed in a previous chapter why Spinoza's philosophy can be seen as an epistemology in which the most relevant concept is *conceivability*, at least for the part that can be considered as his metaphysics, part one of the *Ethics*, viewed from his method of the true idea. He never defines exactly what "conceivability" is but as we saw it is a fundamental idea in his philosophy. I will not discuss the idea of conceivability in Spinoza further apart from what I have already said about the *Treatise* and the idea of God as the fundament of his entire epistemology, since here I just want to focus on how to interpret it formally and why. What should be clear by now is that everything begins with the idea of the being that depends on nothing but itself, this is, the idea of the being that needs nothing else but itself to be conceived. But to base all our language on the idea of conceivability wouldn't be enough, since we do not want an interpretation based on attributes, since conceivability is obviously connected with the attribute of thought. But there is another reason why we cannot take conceivability as the main concept from which interpret the *Ethics* formally since, we also have to account in our language for those statements in the *Ethics* that are not related with knowledge but with physical relation, such as EIA1:   *All things that are, are either in themselves or in something else*[71]. So we have to account for the conception and containment of things as the main relations that appear in the first part. So if we want to follow the strategy of not using the attributes as part of the language, which I will explain latter, we have to find the way to interpret these relations formally.

Lets begin with conceivability. Conceivability is the fundamental relation between ideas, ideas are formed from other ideas, i.e., to form an idea we need of another idea, or ideas, to form the previous one, as we discussed before. Conceivability is the objective fundamental relation between ideas. Now this relation is translated formally into a relation of dependence in which if an idea is formed by others, then that idea depends on those others, in other words we need those ideas to be able to form that idea, this is, the idea is dependent of those others. Therefore to conceive the essence of an idea we need first to conceive the essence of the ideas forming that other idea. The way in which we form ideas establishes an ordering in them. Conceivability requires of a subject forming particular ideas to be studied, but we abstract the content of those ideas and even the subject itself and we are left with the relation of *dependence*. If conceivability is the relation between ideas, in the sense of composition, ideas depend on its components to be conceived. Now is time to deal with the attribute of extension and the objective relation between them which is *containment*. The objective relation between objects is that of containment, i.e., objects are formed by other objects, this is, any object may be seen as containing other objects which composed that object, in the same way that ideas are composed of other ideas. Therefore we see that composition is the objective relation of modes, conceivability in thought

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[71]Notice that Spinoza here relation this *containment* in relation with existence. In the original: *Omnia quae sunt vel in se vel in alio sunt.* He is talking about things which exists, exists in themselves or in other things.

and containment in extension. Now containment is interpreted formally as causation, and the reason is intrinsically related to existence, and a object comes into existence from a Spinozistic point of view when other objects compose together and form that new object, therefore the objects which composed that new-born object cause it's existence by composing with each other, and then we can see them as contained in the new object. Therefore containment can be seen as the dependence of the container by the contained, in order to be formed. An object's existence is caused by the objects contained within it, i.e., an object's existence depends on the existence of the objects forming it, this is, the existence of objects cause the existence of the object. In the same way an idea's essence is conceived through the conception of the ideas forming it, i.e., an idea's essence depends on the essence of the ideas forming it, this is, the conception of the essence of ideas entails the conception of the essence of the idea formed. Containment and conceivability are the two fundamental objective relations on the first part of the *Ethics*. Fundamental because it is the relation under which the basic elements of Spinoza's philosophy are defined. Objective because implicitly Spinoza is already stating these two relations, which correspond to both attributes of extension and thought, in the first part even though he does not talk specifically about any of these attributes. In fact it is in the second book of the *Ethics*, when he states in EIIA5: *We do not feel or perceive any individual things except bodies and modes of thinking.* Even more relevant is the fact that the first two propositions in the second book are: *Thought is an attribute of God; i.e., God is a thinking thing*, and *Extension is an attribute of God; i.e., God is an extended thing*, respectively.

We have found the key to the map that will guide us through the labyrinth of Spinoza's philosophy. If we follow the general structure of Spinoza's philosophy we do not find any relation between one attribute and the other a priori, but it is in this formal interpretation of the fundamental relation in each of the attributes that gives us the hint. We see that the formal interpretation of the relation in each of the attributes can be reduced to a relation of dependence. The formal composition of things is inter preted as conceivability and containment from an objective point of view, we saw that both relations are based on dependence, that's why we take it to be the most relevant relation in Spinoza. Beyond what we just said, this argument is the reason why Spinoza writes EIA4, lets quote it again: *The knowledge of an effect depends on, and involves, the knowledge of the cause.* This is the most important Spinozistic statement from all since it encloses the secret to understand his philosophy. Usually when Spinoza makes the distinction between formal and objective, like those we saw in the *Treatise*, it is interpreted as related to the attribute of thought and related to the attribute of extension, but what I claim, based on the argument I just gave, is that this distinction must be taken as the difference between *Natura naturans* and *Natura naturata*, since the objective distinction between attributes is a distinction found in the *Natura naturata*. The *Natura naturans* is the formal structure, order and connection, of his ontology following from God, therefore there is no difference in attributes when we remain in this side of his metaphysics, but in the *Natura naturata*, i.e., its product, we find this distinction. But this distinction is based on the a posteriori argument that the formal order that both attributes follow is one and the same. In Spinoza's system logical causation and actual causation are one and the same, if we recall the corollary to EIIP7[72]: *Hinc sequitur quod Dei cogitandi potentia aequalis est ipsius actuali agendi potentiae. Hoc est quicquid ex infinita Dei natura sequitur formaliter, id omne ex Dei idea eodem ordine eademque connexione sequitur in Deo objective.* Spinoza says here that everything that follows formally from the idea of God, also follows objectively with the same order and connection.[73]

The principle of pararellism lets us interpret both inclusion and conceivability as a dependence relation, since dependence is more fundamental than containment or conceivability, we take the dependence relation to be the fundamental relation in our language, together with causation which is its inverse relation. This is precisely what I mean when I say that there is a much more fundamental structure in Spinoza's philosophy, the formal structure, on which the objective structure depends. Although causation and dependence are interpreted as being inverse, the only

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[72] "The order and connection of ideas is the same as the order and connection of things." *ibid.*, p. 247.

[73] Neither Elwes nor Shirley's translations do not quite apprehend the message that Spinoza delivers in the original.

difference is that whenever we talk about the existence of something we would use causation, and when we talk about the essence of something, i.e., the way in which the thing is conceived, we would use dependence. Although it might be argued against this argument that in the end I am still taking about the attributes, I claim that in Spinoza the terms *essence* and *existence* are terms that, even though they are obviously related to the attributes, they are even more fundamental in Spinoza's philosophy than the attributes, one could even say that the attributes are nothing but an objective development of these two formal notions into a real world. The main structure underneath all his philosophy is the excision of all there is in what depends on another thing and what depends on itself alone, and this already gives us some sort of ordering of the elements of his philosophy. The selection of the connective $\lhd$ has a particular objective, to represent this relation of dependence, in which $x \lhd y$ means that "x depends on y". The definitions of the formal essences of things, i.e., the definitions of what I take to be the formal units in his philosophy, i.e., substances and modes, are based on the relation of dependence they have. This relation of dependence is what grounds all the formal language. We will talk more about this when we deal with EIA4.

The second relation introduced in this language, which is vital for an understanding of the formal structure in the *Ethics*, is causation. We find causation relations mentioned from the very first definition of the *Ethics*,, as well as in the axioms. This interpretation of causation is similar to the interpretation of dependence, as we just saw. Far from being interpreted just as physical causation, this causation encloses also logical causation. Another argument to support the idea of interpreting both attributes' causation as logical causation can be found in EIP25: *God is the efficient cause not only of the existence of things but also of their essence.* Although this idea is posterior to the part of the *Ethics* we are focusing on, and it depends on EIP15 and EIA4 to be proven according to Spinoza, the key to understand the relation between these two sense of causation, which in Spinoza can be seen as existence (physical causation) and conceivability (logical causation), can be found in EIA4 that we just discussed. It is no coincidence that in both the previous mentioned EIP25 and the principle of parallelism both depend on this axiom. We interpret causation trough as a binary predicate C, thus Cxy means that: x causes y.

The models we use for our language are *dependency graphs*. These are graphs in which the elements are ordered based on their relation of dependence, which is the relation that governs the entire language, thus the reason for choosing these models. These models serve two vital functions in our language. First, as I discussed before, the idea of truth behind Spinoza's philosophy is a matter of method and not of a sign. The way in which I interpret this fact from his philosophy is by showing that these graphs satisfy all the axioms of the language, in the form of a soundness proof that will be provided later. Second, these models that the graphs represent introduces an element that is missing in the *Ethics* and that is a very important element in the *Elements*, the visual aid. Although we do not include any visual proof in this work, these graphs will allow us to do so in a future. These graphs can be seen as schemata that capture all the elements required for our understanding in order to understand something. But, again, since we are not dealing with any particular thing, this use of the graphs will be developed in a future work, they will work as *schemata of understanding.*

3.4   Existence and Modality

Now I am going to focus on the notion of existence in Spinoza and how I decided to interpret it formally. The obvious way will be to interpret it just by the existential quantifier of first order logic, but as we saw before in the chapter about existence this will be a very narrow interpretation. Spinoza never talks about the notion of existence in itself, he always talks about how the existence of something is derived, and more important where does its existence comes from, i.e., existence is always the result of a derivation in Spinoza, it is an effect. And, as we saw before, not only the existence of things but also their non-existence is subjected to this treatment, but it is obvious that it is impossible for something to exist and not exist at the same time. Now, this derivation

of the existence of things relies on the relation of causation we just talked about. Let's recall the conclusion of that chapter, we have three ways of conceiving existence in relation to causation, first the existence of things that come from themselves, second the non-existence that comes from themselves, and third the existence or non-existence that comes from something external. In Spinoza the notion of existence captures the idea of a state, which can be taken as having only two possibilities, that all things have, and how that state changes. We can reinterpret the three previous ways under this view as: things whose existence never changes (eternity) and things whose existence does change (duration). It is obvious that the notion of existence that Spinoza has is closely related to time, but that is something that we will talk about in the next paragraph.

The first idea that Spinoza has about existence that should be considered is what he says in the second demonstration to EIP11, i.e., *For every thing a cause or reason must be assigned either for its existence or for its nonexistence.* This original view on the principle of sufficient reason is the key to understand the connection between existence and causation. Another relevant principle that joins existence and causation is found in the very first definition of the *Ethics*: *By that which is self-caused I mean that whose essence involves existence or that whose nature can be conceived only as existing.* Although Spinoza seems to say something different, since what he explicitly says is that "the existence of x is contained in the essence of x", this is just the objective of what he wants to proof in EIP7[74]. He only uses this definition twice in the first part of the *Ethics*, first the one I just mentioned, and second in EIP24, in which he uses it to prove that modes' existence does not come from themselves: *The essence of things produced by God does not involve existence.* Now regarding the things whose non-existence comes from themselves, i.e., impossible things, we are not going to discuss that part of the division of existence in our language for two reasons, first it is not something of interest for us in this language, since not much can be said of them, and second because the there is a difference between impossible things and this that do not exist at this moment. Something impossible, is something whose non-existence cannot change and that has no relation with anything else in the world, except for the ideas on which it depends to be formed, that is precisely the reason why Spinoza explains it as something self-caused but not existent, as I explained before, i.e., something that depends on something else to be conceived, but that its non-existence comes from its own essence. Nonetheless to fully understand the notion of existence in Spinoza we would have to include a temporal interpretation of it.

We do not give a direct treatment of *existence* for various reasons. As I just said, Spinoza never talks about existence in itself, except always tied to a particular element of his philosophy, e.g., he talks about the existence of the substance or the existence of a particular mode. Their existence is always derived from their causes, e.g., the substance's existence always comes from itself, and the mode's existence comes from another mode. The main reason for not interpreting existence as a predicate in our system is that existence in Spinoza is closer to what can be seen as a boolean evaluation. For instance, in EID8 he says when talking about eternity: *For such existence is conceived as an eternal truth.* Here we see how existence is tied to two other concepts, time and truth. To take existence *as an eternal truth* means that its existence can never change, in the sense that it can never be the case that it does not exists, and that it always exists in the same way, i.e., unchanged. The first idea I had was to include a temporal treatment of existence in order to account for that side of Spinoza's view on existence, but it was an unnecessary addition to the system since in this work we will only deal with eternal and necessary existence. A formal treatment of time will be required in a future expansion of the system, particularly when we deal with particular things, since their existence is always anchored to a moment in time in which they are *presently existent.* Another thing which requires a temporal treatment is the change this particular thing have in their existence, i.e., generation and destruction. But for the moment we only need to deal with eternal and necessary existence, as I said, so there is no need for a temporal treatment of particular existence yet. Nonetheless we still need a way to formally treat modality in Spinoza, which is also tied to existence. In the first book of the *Ethics*

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[74]Existence belongs to the nature of substance. *ibid.*, p. 219.

we find mentions to existence among the definitions and axioms in EID1, EID7, EID8 and EIA7. The most relevant ones are EID1 and EID8, since EID7[75] cares about where does the existence of the thing comes from, but does not deal with existence directly. In EIA7, Spinoza claims that: *If a thing can be conceived as not existing, its essence does not involve existence*, but as I will explain later, when we deal with the axioms one at a time, this axiom is not included in out language. We find a relation between one of the modalities in Spinoza, eternity, and existence in EID7. But the most important mentioned is found in EID1: *By that which is self-caused I mean that whose essence involves existence; or that whose nature can be conceived only as existing.* Here we see that Spinoza relates being self-caused and existing necessarily, since it is not possible to conceive that self-caused thing as not existing.

Since there is no clear explanation of what *existence* is, but, nonetheless, we find some mentions of *necessary existence* or *eternity*, we will focus on this modalities about existence alone. We have talked before about eternity, and we will explore it further when we talk about EID8, but what I want to focus on in this part is the relation between existence, eternity, necessary existence and self-causation. I interpret eternity to be the denomination of the substance's existence in relation to time, I do not say "temporal" interpretation to avoid the confusion with the existence of the modes. Necessary existence I interpret to be the modality of existence of the substance, this is center in the statement "its existence is conceived as an eternal truth". So there is a connection between necessary existence and eternity that is found in the fact that the substance is self-caused, i.e., nothing can make it change, and its existence is a truth of Spinoza's philosophy. That's why I interpret these two notions in my language to be defined in relation to each other. Eternity is defined in terms of self-dependence and in relation to the graphs. The graphs also play an important role in our interpretation of existence since they will be the base from which develop a temporal treatment of existence in a future. The reason for including the graphs into the definition of these concepts is that the evaluation order lets us express something vital for the treatment of EIP21 and EIP22. This evaluation order allows us to interpret something very relevant for Spinoza such as *following directly from* something. This allows us to differentiate from what is caused by God, and what is *directly* caused by it, since pretty much everything is caused by God, that's the reason why it is important to express this relation correctly. This may seem like an *ad hoc* solution, but I will discuss now that it is not the case. These dependency graphs can be seen as models for the human understanding in which we can inquire about the existence of anything by including it in the graph, and understanding its relation to other elements of the system, like God, the human mind, or the whole of the universe. This process lets us trace anything whatsoever back to the true idea which lets us apprehend it following the method of the true idea, as I described before. Although in this work we will only deal with the fundamental elements of those models, its relation with existence will be explained now. Existence here is not only physical existence, but also the relevance they have for the understanding of anything. In other words, in order to understand anything, we need to first understand on which it depends, and how this relation is. I claim that this is what Spinoza means when he says that *the knowledge of the effect involves the knowledge of the cause*, in the sense that trying to understand the effect forces its cause to be included in the inquire as well. Thus, God is not only always-existent in physical sense, but also always-needed for the understanding or anything. Necessary existence is defined as being caused by something eternal directly in the same way we just explained based on the evaluation order. Basically this interpretation of Spinoza's necessity is grounded in the idea that, if something is necessary it cannot be the case that it doesn't exists, which means that the thing is self-caused, or that it is caused by something eternal. For if it follows directly from something eternal, its existence must be necessary, otherwise if it stops existing that will mean that the thing causing it must have changed as well, and since its eternal it cannot be the case.

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[75]That thing is said to be free [fiber] which exists solely from the necessity of its own nature, and is determined to action by itself alone. A thing is said to be necessary [necessarius] or rather, constrained [coactus], if it is determined by another thing to exist and to act in a definite and determinate way. *ibid.*, p. 217.

4 Formal language

4.1 Alphabet

- First order language extended with two unary predicates, S, M, which define the two different sorts of this logic. We have $S(x)$ and $M(x)$. We define an axiom for the metalanguage to say that those predicated partition the domain of variables: $\forall x((S(x) \lor M(x)) \land \neg \exists x(S(x) \land M(x)))$. Every time we use a quantifier we specify with the predicate, which sort are we using, in this sense $\forall x A$, is translated into $\forall x(S(x) \to A)$ or $\forall x(M(x) \to A)$. When we are quantifying without specifying the type, e.g., $\forall x(A)$, we really mean $\forall x((S(x) \lor M(x)) \land A)$ because in no formula it will appear $(S(x) \lor M(x))$ only as the antecedent of an implication, but since $(S(x) \lor M(x))$ is just a theorem of our language it won't be included in those formulas in order to simplify.

- Quantifiers $\{\exists, \forall\}$.

- Two binary relations $\{\lhd, \simeq\}$.

- Binary causation predicate: $\{C\}$.

4.2 Grammar

Terms:

- Variables are terms.

- Constants are terms.

Atomic formulas:

- If $t_1, t_2, ..., t_n$ are terms and P is a n-ary predicate, then $Pt_1...t_n$ is an atomic formula.

Formulas:

- If A is a formula, and x i a variable, then $\exists x A$ is a formula.

- If A is a formula, and x i a variable, then $\forall x A$ is a formula.

- If A is a formula, then $\neg A$ is a formula.

- If A and B are formulas, then $A \land B$, $A \lor B$ and $A \to B$ are formulas.

- Nothing else is a formula.

4.3 Semantics

We define our language $\mathfrak{L}$ to be FOL plus the set of symbols $\{\lhd, C\}$.

We define the Structure for our language to be , $\mathfrak{M} = \langle \mathbb{S}, \mathbb{M}, \lhd, C, g, \models \rangle$.

Where we have a two sorted domain $\mathfrak{D}$ consisting of the elements of the set $\mathbb{S}$ and $\mathbb{M}$, where $\mathbb{S} = \{\mathbf{a}, \mathbf{b}, ...\}$ is a set of constants that represents substances, and $\mathbb{M} = \{a, b, ...\}$ is a set constants that represent modes. Therefore our domain $\mathfrak{D} = \{\mathbf{a}, \mathbf{b}, ..., a, b, ...\}$.

$\lhd$ is a binary, asymmetric, connected and transitive relation between elements of any of the domain, this is $\lhd \subseteq DxD$.

$\simeq$ is a relation of comparison between modes, s.t., if x, y are modes, i.e. $M(x)$ and $M(y)$, then $x \simeq y$ iff $\forall z(S(z))$ and $x \lhd z$ iff $y \lhd z$.

Now C is is the inverse relation of $\lhd$, this is $C \subseteq DxD$.

g is a constant which represents God.

We define now the assignment function v on our structure $\mathfrak{M}$ with domain the set of variables of our language $\mathfrak{L}$ and range a subset of $\mathfrak{D}$. This function gives an assignment $v(x)$ for each variable x. Now we define for each term t of our language $\mathfrak{L}$, the function $T^{\mathfrak{M}}$ which maps assignments to elements of $\mathfrak{D}$:

- If t is a constant a, then $T^{\mathfrak{M}}(v) = a^{\mathfrak{M}}$ for all v.

- If t is a variable x, then $T^{\mathfrak{M}}(v) = v(x)$ for all v.

- If t is the term $f(t_1...t_n)$, then for all v, define $t^{\mathfrak{M}}(v) = f^{\mathfrak{M}}(t_1^{\mathfrak{M}}(v), ..., t_n^{\mathfrak{M}}(v))$.

Now for the interpretation of the model, where we define $\mathfrak{M} \models \phi[v]$, for all ϕ in our language, where v is an assignment on ϕ, as:

- $\mathfrak{M} \models P(t_1, ..., t_n)[v]$ iff $(t_1^{\mathfrak{M}}(v), ..., t_n^{\mathfrak{M}}(v)) \in T^{\mathfrak{M}}(P)$.

- $\mathfrak{M} \models Rt_1t_2[v]$ iff $(t_1^{\mathfrak{M}}(v), t_2^{\mathfrak{M}}(v)) \in R^{\mathfrak{M}}$.

- $\mathfrak{M} \models x = y[v]$ iff $x^{\mathfrak{M}}(v) = y^{\mathfrak{M}}(v)$.

- $\mathfrak{M} \models \neg\phi[v]$ iff $M \not\models \phi[v]$.

- $\mathfrak{M} \models \phi \rightarrow \psi[v]$ iff $M \not\models \phi[v]$ or $M \models \psi[v]$.

- $\mathfrak{M} \models \exists x\phi[v]$ iff there is v' s.t. $v' = v$ for all $y \neq x$ and $M \models \phi[v]$ iff $M \models \phi[v']$.

- $\mathfrak{M} \models \forall x\phi[v]$ iff for all v' s.t. $v' = v$ for all $y \neq x$ and $M \models \phi[v]$ iff $M \models \phi[v']$.

4.4 Definitions

I will now proceed with explaining each statement of the first parts of the *Ethics* at a time. I will start with the definitions. First of all some definitions of the first part of the *Ethics* are taken as axioms in my language and not as definitions *per se* in the modern sense. The reason for this decision is that in Spinoza the relevance of relations between the elements of his philosophy is where the stress of his definitions rest. Most of them define a relation in which the element defined stands for instance, the definition of a mode is a definition of what "is in another thing and is conceived through another thing". Most of the definitions of the first part of the *Ethics* are defined in that way therefore, from a formal point of view they could be taken as defining relations between the elements of the language or as stating general properties of the language. The only things we define in our language are the notions of *infinite*, *God*, *Eternal* and *Determined* plus the modality. The rest of the definitions are included as axioms of the language because of their importance in the demonstration of the propositions of the *Ethics*, they are not just definitions of elements or parts of the language but they are seen as fundamental principles of the language. Let me give an example, EID1[76] although it is a definition, it is used in EIP7

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[76] "By that which is self-caused I mean that whose essence involves existence or that whose nature can be conceived only as existing."

to prove the existence of substance:

> Substance cannot be produced by anything else (EIP6, Corollary) and is therefore
> self-caused [causa sui]; that is (EID1), its essence necessarily involves existence;
> that is, existence belongs to its nature.

Following that spirit we take some of the definitions to be axioms because we already have a
metalanguage that defines what the elements of the *Ethics* are therefore, we do not take them
as introducing any new element of the language they just express truths about the structure of
our language in which terms and relations are connected. I will proceed now to explain them
one by one.

Lets begin with each definition at a time. I will first state the original formulation from
Spinoza and then how I formalized it:

EID1: *By that which is self-caused I mean that whose essence involves existence, or that
whose nature can be conceived only as existing.*

Axiom 1: $\forall x (Cxx \rightarrow Nx)$

In this definition the first thing we encounter is an equalization between the statement "self-
caused" and "essence involving existence". In the second part of the definition it seems that he
also equates "self-caused" to "nature can only be conceived as existing" which clearly we can
take here as necessary existence since paraphrasing it we could say "it cannot be the case that
it doesn't exists". If we look at the definition carefully we see that the comparison happens
between the first statement i.e., "self-caused" and the other two statements just mentioned. So
we can deduce from here that the two other formulas that appear in the definition are definitely
equivalent. Now the interest is in the relation between "self-caused" and the other two and in
order to understand that relation we will take a look at the propositions in which the definition
is used. Spinoza uses this definition in EIP7: "Substance cannot be produced by anything else
(Cor. EIP6) and is therefore self-caused [causa sui]; that is (EID1), its essence necessarily
involves existence; that is, existence belongs to its nature." He also uses this definition in EIP24:
"This is evident from EID1. For only that whose nature (considered in itself) involves existence
is self-caused and exists solely from the necessity of its own nature."

I claimed that this definition states that if something is self-caused or that its essence involves
existence–which is the same–then that thing also exists necessarily. One argument in favor of this
is that there are necessary things which are not self-caused[77]. But a more compelling argument
based in what I just quoted is that only the use of EID1 in EIP24 makes us think that the
definition is a bi-conditional–as Jarret does in his paper[78]. But if we pay attention carefully we
see that it is not the case. If we look to the proof of EIP24 in the original: "Patet ex definitione
1. Id enim cujus natura (in se scilicet considerata) involvit existentiam, causa est sui et ex sola
suae naturae necessitate existit", we can sense a bit more emphasis that the second sentence
uses the equality between "nature involves existence" and "self-caused" in conjunction with the
fact that its existence is necessary which follows from the fact that its essence involves existence.
These are my arguments to interpret EID1 as a conditional, in the way just presented.

EID2: *A thing is said to be finite in its own kind [in suo genere finita] when it can be
limited by another thing of the same nature. For example, a body is said to be finite because
we can always conceive of another body greater than it. So, too, a thought is limited by another
thought. But body is not limited by thought, nor thought by body.*

Definition 1: "We call x infinite iff for all y, different from x, but of the same type as x, we
have that $\neg(x \lhd y)$."

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[77]EIP22

[78]JArret

First of all–as I discussed before–we are not going to define what being finite means in our language rather we define what is to be infinite. The reason for this is that in the first book the notion of "infinite" is used more than the notion of finite. But the most important reason is that in the present work based on the first book we won't use the notion of "finite" since we are not considering modes as part of our work rather we talk about infinite things. There are two things to mention in this interpretation of EID2. First are the notions "kind" and "nature". These notions as I claimed before refer to attribute and type respectively in the same way as we take type here i.e., mode or substance. I have discussed this before. The second thing to discuss here is that limitation here is taken just as dependence since both relations that are exemplified into the definition which are what I have called the fundamental objective relations in each mode–they are both grounded in dependence. We obtain this definition by negating EID2. The inclusion of kind in the original definition restricts this definition to one single quality or attribute in the sense that whether we talk about finite or infinite we do it from the point of view of one quality. But since we are dealing with the formal structure at the moment we just need to keep this restriction in mind for a future work.

EID3: *By substance I mean that which is in itself and is conceived through itself; that is, that the conception of which does not require the conception of another thing from which it has to be formed.*

Axiom 2: $\forall x(S(x) \rightarrow \forall y((x = y \lor \neg(x \lhd y)))$.

The definition of what a substance is, is taken in this work as an axiom that states that if something is a substance, then the only thing that it depends on, is itself and nothing else, that is the reason why we define it as the unique thing which depends on itself.

EID4: *By attribute I mean that which the intellect perceives of substance as constituting its essence.*

Axiom 3: $\forall x(S(x) \rightarrow \exists y(M(y) \land y \lhd x \land \forall z(M(z) \land z \lhd x \rightarrow z \lhd y)))$

As I explained before attributes are not taken into consideration in my formal interpretation of Spinoza nonetheless, their definition is still relevant for the system. I claimed before that attribute are often equated with the substance. The role that the attributes play in Spinoza' system is as a link between substances and modes. This link is embodied in the infinite immediate mode since, any modification of that attribute depends on the infinite immediate mode; that is reason why the attributes' definition is substituted for this relation of dependence of any mode on the infinite immediate mode of its respective attribute. As I have argued before substances and attributes are the same for Spinoza at the *Natura naturans* level, this definition states that for any substance there is a mode on which any other mode that depends on the substance also depends on it. The notion of immediate infinite mode is defined in the system after EIP21 on which Spinoza deal with it.

EID5: *By mode I mean the affections of substance, that is, that which is in something else and is conceived through something else.*

Axiom 4: $\forall x(M(x) \leftrightarrow \exists y(x \neq y \land x \lhd y)$

This definition is pretty straight forward, something is a mode if and only if there is something, different from it, on which it depends. The inclusion of the inequality is essential, otherwise we could have the case that this definition might be applied to a substance, since they depend on themselves, and it would be a mistake.

EID6: *By God I mean an absolutely infinite being, that is, substance consisting of infinite attributes, each of which expresses eternal and infinite essence.*

Definition 2: $g = S(g) \land \forall y \forall z(S(y) \land z \lhd y \rightarrow z \lhd g)$.

The definition of God that Spinoza gives in the *Ethics* is based on two properties that define God as a substance; first, it has infinite attributes second, each of these attributes express infinite infinite and eternal essences. The first property–that is contains infinite attribute– as I just explained means that God is the substance whose qualities are taken as infinite in the sense just defined above in the explanation of EID2 i.e., as unlimited. In other words there cannot be another entity with the same attributes as God, without those attributes being included in those of God. This is the sense of "absolute" in Spinoza[79]. For the second property it is also relevant to quote the explanation that Spinoza gives to this definition:

> I say *absolutely infinite*, not *infinite in its kind*[80]. For if a thing is only infinite in its kind, one may deny that it has infinite attributes. But if a thing is absolutely infinite, whatever expresses essence and does not involve any negation belongs to its essence.

Here we understand how both properties intertwine together God is not only the being that has unlimited properties but it is also the being whose properties include anything whatsoever as long as that thing is positive expression of any of its qualities[81]. Thus my interpretation of this central definition of the *Ethics* captures both senses it says that: "for any substance, and anything that depends on that substances, also depends on God" in other words, any modification of any possible quality whatsoever is just a modification of one of God's own qualities.

EID7: *That thing is said to be free [liber] which exists solely from the necessity of its own nature, and is determined to action by itself alone. A thing is said to be necessary [necessarius] or rather, constrained [coactus], if it is determined by another thing to exist and to act in a definite and determinate way.*

Definition 3.1: We call x free, and we write Lx, where L is the free predicate, iff Cxx

Definition 3.2: We call x constrained, and we write Dx, where D is the constrained predicate, iff $\exists y(x \neq y \wedge Cyx)$

We see again that in the definition of *free* the sentence "exists solely from the necessity of its own nature" which resembles what we find it in EID1 i.e., things whose existence comes from their own nature alone. That's the reason why I define it as just being another sense that Spinoza gives to what is self-caused and from its use throughout the *Ethics*. We see that he uses this definition to talk about things which have been consider "free" in the tradition of metaphysical thought[82]. We also find in this proposition that the formulas determine to "exists" and "act" go together in the causation relation.

EID8: *By eternity I mean existence itself insofar as it is conceived as necessarily following solely from the definition of an eternal thing.*

Definition 4: We say that x is eternal, and we write ξx, iff for any graph G, we have that $x \in G$ and $x \lhd x$, or if $x \in G$ and there is a $y \in G$ s.t. ξy and $y \lhd y$ and $\mathbf{n}(x) = \mathbf{n}(y) + 1$.

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[79]We will come back to this interpretation of "absolute" when we reach EIP21, and we will see that when Spinoza says "absolute nature of an attribute" we can use the same interpretation we just explained.

[80]His emphasis.

[81]This is a very similar positive definition of God to the definition given by Gödel: "Positive means positive in the moral aesthetic sense (independently of the accidental structure of the world). Only then are axioms true. It might also mean pure attribution as oppose to privation (or containing privation). This interpretation [supports a] simpler proof." Gödel (1995), p. 404.

[82]The main arguments in which this proposition is used, have as the main objective to argue against the theological argument that good has free will and that he acts on this free will. Spinoza uses this definition and the arguments found in the *Ethics* to show that that is not the case. For instance in EIP17, in which the definition is used, he claims that: "God acts solely from the laws of his own nature, constrained by none." The next proposition in which he uses this definition is in EIP32, in which he demonstrates that: "Will cannot be called a free cause, but only a necessary cause."

Eternity is compared to existence, but a special kind of existence, the existence that is conceived through the definition of something eternal. This might seem like a circular, since to know whether or not something is eternal, we have to inquire if its existence comes from something eternal. So the problem here is, to say that God is eternal we need to know whether God is eternal or not, since its existence in conceived through itself. Nonetheless from its use in EIP19 we see that the problem is solved: *God is substance (EID6) which necessarily exists (EIP11); that is (EIP7), a thing to whose nature it pertains to exist, or-and this is the same thing-a thing from whose definition existence follows; and so (EID8) God is eternal.* We see here that to see if something is eternal, we need to see if its existence is *conceived through*, or *depends* in our language, its definition, or through something eternal. This last remark is very important, since this makes this definition not only applicable to God, but also to whatever depends directly on it. In other words, were there any other eternal thing besides God, their existence would have to be conceived through God alone, i.e., their existence would have to follow solely, i.e., directly, from God's. This sense of directly is interpreted in the formalization of this definition by the use of the evaluation order that will be explained later. Using that evaluation we can be sure that there is nothing between God and the eternal thing.

Our last definition although not based on any definition from the *Ethics* is relevant for the work at hand–the definition of *necessity*. As I explained before the use of the evaluation here is as essential as its use in the previous definition since we must define it in a way that we can proof that there can be necessary thing different from God, but also include it in this definition.

Definition 5: We call x necessary, i.e., x necessarily exist, and we write Nx, iff $\exists y$ s.t. Cyx, ξy, and s.t. $\mathbf{n}(x) = \mathbf{n}(y) + 1$ or $\mathbf{n}(x) = \mathbf{n}(y)$.

4.5 Axioms

Now it is time to pass on to the axioms of the first part of the *Ethics*:

EIA1: *All things that are, are either in themselves or in something else.*

Axiom 5: $\forall x(M(x) \vee S(x))$

I interpret this axiom to be a sort of excluded middle, since things which are in themselves are substances and things that are in something else are modes therefore, everything must either be a substance or a mode. Spinoza himself uses this axiom in the same way in the proof to EIP15[83]. And I take those two notions to be opposite, that's the reason why I also include the following axiom:

Axiom 6: $\forall x(S(x) \leftrightarrow \neg M(x))$

EIA2: *That which cannot be conceived through another thing must be conceived through itself.*

Axiom 7: $\forall x(\exists y(x \neq y \wedge x \lhd y) \vee^* x \lhd x))$

The interesting fact about this axiom is that it is never used in the *Ethics* which makes you thing what was the reason he had to include it in his work and–even more–why in the first part of the *Ethics*? Spinoza wasn't precisely famous for including random principles in his philosophy and, since the *Ethics* represents such an intertwined system of statements only increases the interest on this axiom. The direct way to interpret this axiom might be to interpret

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[83]Apart from God no substance can be or be conceived (Pr. 14), that is (Def. 3), something which is in itself and is conceived through itself. Now modes (Def. 5) cannot be or be conceived without substance; therefore, they can be only in the divine nature and can be conceived only through the divine nature. But nothing exists except substance and modes (Ax. 1). Therefore, nothing can be or be conceived without God. Spinoza (2002), p. 225.

that if x does not depend on any y different from it, then x is self-dependent since it seems like that is the only way to proof self-dependence. But this axiom can be more useful than just that interpretation. We take it here to be some sort of law excluded middle for the dependence of substances and modes. This is we also included that if something is self-dependent then it cannot depend on anything different from it which becomes very relevant throughout this entire work–that is the reason why the disjunction included in the axiom must be taken as an exclusive one, since something cannot be a mode and a substance at the same time.

EIA3: *From a given determinate cause there necessarily follows an effect; on the other hand, if there be no determinate cause, it is impossible that an effect should follow.*

Axiom 8: $\forall x(\exists y Cyx \rightarrow \exists z Cxz)$

This axiom has a big relevance in Spinoza's notion of causation since it is what defines the causal chains in his system which formally entails seriality of C. Although this axiom only appears twice in the whole book of the *Ethics*–in EIP27 and EVP33–and he uses it just to show that everything which is determined by God cannot turn undetermined. Now since this axiom is not used in any of the proposition that we are dealing with in this work we state it here but we do not include it in the set of axioms that we will use for our formalization of the first 23 proposition of the first book. Nonetheless this axiom will become relevant in a future development of the system into the study of particular things. But the axiom for us entails two formal properties of the relation of causation that is why I have divided the axiom into two parts since the axiom found in the *Ethics* states two different things. First it establishes the notion of a causal chain in which form a determined element x we have that if x is determined i.e., if there is another element y s.t. y is the cause of x, then x itself must have an effect so on and so forth. This is one of the key principles in Spinoza's ontology since it is the way out of an static ontology into a ontological diversity pushed by a causal motion of the entities of in Nature. The second principle that the axiom entails is that nothing is without a cause and without an effect apart from the fact that nothing in Spinoza's ontology can be undetermined or "undetermining" i.e., without any effect. It also sets the impossibility of things like the void or impossible objects. As he says in the "Tratise":

> Therefore, if there were something in Nature having no interrelation with other things, and if there were also granted its objective essence (which must agree entirely with its formal essence), then this idea likewise would have no interrelation with other ideas; that is, we could make no inference regarding it.

Another important feature of this statement is that is it represents the other side of the principle of sufficient cause instead of just saying in Spinoza that nothing is without a reason he also says that nothing is without an effect.

EIA4: *The knowledge of an effect depends on, and involves, the knowledge of the cause.*

Axiom 9: $\forall x, y(Cyx \leftrightarrow x \lhd y)$

What Spinoza is saying here is that if something is the cause of another thing then the knowledge of the latter depends on the knowledge of the former and, the knowledge of the latter involves the knowledge of the latter. Thus we interpret this axiom from a formal perspective and paraphrase it like: if x causes y, then the essence of y depends on the essence of x and, if the essence of y involves the essence of x then y is the effect of x. Now this sense of "involve" that we find in the quote refers to the fact that the knowledge of any effect contains in itself the information of what is its cause. This last part highlights even more the distinction between objective an formal essences if we pay attention to some object and we take its objective essence as the thing apprehended from our mind we cannot be sure that the information about its cause might be included in that knowledge but if we take its formal essence as apprehended by the mind, then we will know for sure what it is its cause i.e., substance or a different mode, and this

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is something that the formal language captures.

EIA5: *Things which have nothing in common with each other cannot be understood through each other; that is, the conception of the one does not involve the conception of the other.*

Axiom 10: $\forall x, y((y \lhd x) \rightarrow \forall z(z \lhd y \rightarrow z \lhd x))$

This is one of these proposition that are stated in a negative way but we interpret it as an universal quantification. I interpret this axiom by its contrapositive i.e., if one thing depends on another then–by transitivity of $\lhd$–everything that depends on the former depends on the latter. Now the question here is what does Spinoza means with this "having something in common". The obvious argument here is to say that Spinoza means that they do not pertain to the same attribute in the sense that they do not have any property in common therefore, there is no dependence relation between them. But I claim that this axiom goes beyond that. In the ordering of Spinoza's metaphysics that we propose in this work based on the relation of dependence which is a transitive relation–this axiom represents a very important property of this ordering. This axiom states that if one thing depends on another then there must necessarily be something that depends on both things by transitivity. Lets thing about if this makes sense if we interpret the dependence relation from an objective point of view. Take any two ideas–say a and b–such that a depends on b i.e., to conceive b we need to conceive a. This axiom states that if that is the case, then anything which requires b in order to be understood also requires a i.e., that in order to conceive it we have to conceive both a and b. The same argument works if we interpret dependence as containment and we talk about objects.

EIA6: *A true idea must agree with that of which it is the idea [ideatum].*

This axiom is not included in my system since we are only focusing in the first part of the *Ethics* and this axiom becomes relevant in the second book whereas in the first one is only used in EIP5 and EIP30. I already commented on EIP5 and the use of this axiom in relation to the method of the true idea. In EIP30 and the other propositions in which the axiom is used in the second book are closely related to the human mind and epistemology which is something we do not treat in this work. This axiom is basically an statement about how we have to follow the method of the true idea in order to proceed correctly.

EIA7: *If a thing can be conceived as not existing, its essence does not involve existence.*

This axiom is also not included in my work because it is only used once in the proof of EIP11–this is–the proof of the existence of God and this axiom is just one of the ways in which Spinoza proves that proposition that only relies in this axiom and the fact that God is a substance.

Apart from the axioms found in the his book we included more axioms. The reason for them is to capture some other properties that the formal language requires nonetheless, I claim that these axioms are not in any kind of disagreement with the philosophy explained in the *Ethics*. I proceed to discuss them now:

Axiom 11: $\forall x, y(M(x) \wedge M(y) \rightarrow (x \lhd y \rightarrow \neg(y \lhd x)))$

This axiom states a property that is implicit in Spinoza's philosophy, which is, that if a mode depends i.e., is conceived or is contained then the other way around cannot be the case i.e., the second mode cannot depend on the first one.

Axiom 12: $\forall x, y(S(x) \wedge S(y) \rightarrow (x = y \leftrightarrow \exists z(z \lhd x \leftrightarrow z \lhd y)))$

This axiom is taken from EIP2, one of those propositions that Spinoza proof directly from a definition, in this case EID3. I claim that here we have the criteria to differentiate, or equate, substances in the *Ethic*. EIP2 goes as follows: *Two substances having different attributes have nothing in common.* I reinterpret it here as: "two substances are the same iff they share a modification". It should be obvious at this point that a "modification" is always the modification

of an attribute, and that's the criteria to equate them, if they share the same modification, which has to be of any one attribute, then they must be the same.

Axiom 13: $\forall x, y(M(x) \wedge M(y) \to (x \simeq y \leftrightarrow \forall z(S(z) \wedge (x \lhd z \leftrightarrow y \lhd z))))$

These axioms are used in my language to have a way to compare substances and modes. Whenever we have that $x = y$ in our language it just means that both variables represent the same constant but we need to state some axioms depending of the type of constants that x and y are. The first axiom does not presents any problems nonetheless, applying the same criteria of equality to modes might be problematic. It seems like an unnecessary strong statement to claim that all modes that depends on one substance are all the same but what I want to express with that axiom is not that all modes are the same in the sense that they are the same kind of modification of any attribute–this is–the fact that they all share the same property; that is the reason for the use of $\simeq$ instead of equality. Since this work is focus on the first part and we do not enter into the objective interpretation of Spinoza's philosophy just yet, I claim that the only notion of equivalence between modes that we need for the first part is that of sharing the same property if they are modifications of the same substance. This axiom is only used in the demonstration of EIP4 in this work and when we talk about that proposition we would come back to this argument of equating their belonging to one substance. Since we are not focusing on modes until the second book and we find no criteria to equate two individual modes we treat the comparison between modes to be always according to a substance, i.e., to an attribute.

Axiom 14: $\forall x \exists y(x \lhd y)$

We have here the principle of sufficient reason captured by this axioms, which Spinoza doesn't explicitly state in the *Ethics*, but he mentions it indirectly as I quoted before.

Axiom 15: $\forall x, y(M(x) \wedge S(y) \to x \neq y)$

Finally we have an axiom that states the difference that in our language two elements from different types can never be the same element.

The first two proposition of the *Ethics* have a peculiar feature in comparison with the rest of the propositions found in the *Ethics* they have no "Q.E.D." at the end of their demonstration and Spinoza take them to be more of an indirect definition than a proposition since, their demonstration follow directly from just some definitions without use of any axiom. In particular the first two propositions of the first part are taken here not as propositions since they follow from some definitions and they capture basic properties of the ordering of this interpretation of Spinoza's philosophy. We interpret EIP1 to be a lemma of our language because it is more important for other propositions than what it says–which is something pretty obvious in his philosophy.

The last general regard about the formalization is that when we proof the propositions we try to be as honest with the strategy followed in the text–this means–that in some proposition we will follow the demonstration that Spinoza provides in his book but in another we will take another path that makes much more sense in our formal language. In the end we are using a completely different language than Spinoza and sometimes the demonstration of the proposition are just a guide to better understand what the proposition's goal is.

EIP1: *Substance is by nature prior to its affections.*

Lemma 1: $\forall x(M(x) \to \exists y(S(y) \wedge x \lhd y))$

What we want to capture with this lemma, is that any mode has to be a modification of any substance on which it depends. Now I will define the language I will use to formalize the first part. I will discuss the interpretation of each proposition at a time when we start proving them in my language. We have already discussed EIP2 above.

Definitions and Axioms

Definition 1: We call x infinite iff for all y, different from x, but of the same type as x, we have that $\neg(x \lhd y)$.

Definition 2: $g = S(g) \land \forall y \forall z (S(y) \land z \lhd y \rightarrow z \lhd g)$.

Definition 3.1: We call x free, and we write Lx, iff x's existence comes solely from itself, i.e., $\forall y(x = y \lor \neg Cyx)$ and Cxx.

Definition 3.2: We call x constrained, and we write Dx, iff $\exists y(x \neq y \land Cyx \rightarrow Ex)$.

Definition 4: We say that x is eternal, and we write ξx, iff for any graph G, we have that $x \in G$ and $x \lhd x$, or if $x \in G$ and there is a $y \in G$ s.t. ξy and $y \lhd y$ and $\mathbf{n}(x) = \mathbf{n}(y) + 1$.

Definition 5: We call x necessary, i.e., x necessarily exist, and we write Nx, iff $\exists y$ s.t. Cyx, ξy, and s.t. $\mathbf{n}(x) = \mathbf{n}(y) + 1$ or $\mathbf{n}(x) = \mathbf{n}(y)$.

Axiom 1: $\forall x(Cxx \rightarrow Nx)$

Axiom 2: $\forall x(S(x) \rightarrow \forall y((x = y \lor \neg(x \lhd y))$

Axiom 3: $\forall x(S(x) \rightarrow \exists y(M(y) \land y \lhd x \land \forall z(M(z) \land z \lhd x \rightarrow z \lhd y))$

Axiom 4: $\forall x(M(x) \leftrightarrow \exists y(x \neq y \land x \lhd y)$

Axiom 5: $\forall x(M(x) \lor S(x))$

Axiom 6: $\forall x(S(x) \leftrightarrow \neg M(x))$

Axiom 7: $\forall x(\exists y(x \neq y \land x \lhd y) \lor^* x \lhd x))$

Axiom 9: $\forall x, y(Cyx \leftrightarrow x \lhd y)$

Axiom 10: $\forall x, y((y \lhd x) \rightarrow \forall z(z \lhd y \rightarrow z \lhd x))$

Axiom 11: $\forall x, y(M(x) \land M(y) \rightarrow (x \lhd y \rightarrow \neg(y \lhd x)))$

Axiom 12: $\forall x, y(S(x) \land S(y) \rightarrow (x = y \leftrightarrow \exists z(z \lhd x \land z \lhd y)))$

Axiom 13: $\forall x, y, z(S(z) \land M(x) \land M(y) \rightarrow (x \simeq y \leftrightarrow (x \lhd z \leftrightarrow y \lhd z)))$

Axiom 14: $\forall x \exists y(x \lhd y)$

Axiom 15: $\forall x, y(M(x) \land S(y) \rightarrow x \neq y)$

Lemma 1: $\forall x(M(x) \rightarrow \exists y(S(y) \land x \lhd y))$

4.6 Interpretation of $\lhd$

We have $\mathfrak{D}$, and the connected, transitive and asymmetric relation $\lhd$, s.t., $\lhd \subseteq \mathfrak{D}X\mathfrak{D}$, with $(a,b) \in \lhd$. We define a graph to be a non-empty set $\mathfrak{G} = (D,T)$, with $D \subseteq \mathfrak{D}$, and $T \subseteq \lhd$, with $\lhd$ being the transitive closure of T.

Now we define the graph conditions depending on each type of element of $\mathfrak{D}$.

For any graph $G = (D,T)$, and any element $d \in D$, we have that $G \models S(d)$ iff, $\forall x \in D$, s.t. $x \neq d$, we have that $\neg(d \lhd x)$.

For any graph $G = (D,T)$, and any element $d \in D$, we have that $G \models M(d)$, iff, $\exists x \in D$, s.t. $x \neq d$, and $d \lhd x$.

Now we define what is the limit of a graph. For any graph $G = (D,T)$, there is a unique element $d \in D$ s.t. $S(d)$, i.e., $\forall y \in D$, $\neg S(y) \vee y = d$, and $\forall x \in D$, we have that $x \lhd d$.

We define now C to be the inverse relation of $\lhd$.

We define now an evaluation order $\mathfrak{n}$, $\mathfrak{n} : D \mapsto \mathbb{N}$, which is a numbering of the objects in the dependency graph so that the following equation holds: $\mathfrak{n}(a) < \mathfrak{n}(b) \rightarrow (a,b) \notin \lhd$ with $a,b \in D$.

For every graph $G = (D,T)$ and every $x \in D$ where x is the limit of G, we have that $\mathfrak{n}(x) = 0$.

General Axioms of type:

Ax 5: $\forall x(M(x) \vee S(x))$

Take any $G = \{D,T\}$, and any $x \in D$. Since $D \subseteq \mathfrak{D}$, then we have that $x \in \mathfrak{D}$, therefore $x \in \mathbb{S}$ or $\mathbb{M}$.

Therefore we have that $M(x) \vee S(x)$.

Ax.15: $\forall x,y(S(x) \wedge M(y) \rightarrow y \neq x)$.

Take a $G = \{D,T\}$, and $x,y \in D$.

Assume that $S(x)$ and $M(y)$. And we want to see that $x \neq y$.

Assume that $x = y$.

If $x = y$ and $S(x)$, then $S(y)$.

If $S(y)$, then by graphic conditions $\forall x \in D$, s.t. $x \neq y$, we have that $\neg(y \lhd x)$.

But also if $M(y)$, then $\exists x \in D$, s.t. $x \neq y$, and $y \lhd x$. Therefore we have a contradiction.

Therefore $x \neq y$.

Ax. 6: $\forall x(S(x) \leftrightarrow \neg M(x))$

Take a $G = \{D, T\}$, and $x \in D$.

Assume that $S(x)$.

If $S(x)$, then by graphic conditions $\forall y \in D$, s.t. $y \neq x$, we have that $\neg(x \lhd y)$.

Assume that $M(x)$.

If $M(x)$, then $\exists y \in D$, s.t. $y \neq x$, and $x \lhd y$. Therefore we have a contradiction.

Therefore $\neg M(x)$.

Now assume that $\neg M(x)$.

If $\neg M(x)$, then by conditions on the graph we have that $\forall x \in D$, s.t. $x \neq d$, we have that $\neg(d \lhd x)$.

Therefore conditions on graph we have that $S(x)$.

General Axioms of $\lhd$:

Ax.14: $\forall x \exists y (x \lhd y)$.

Take any $G = \{D, T\}$, and any $x \in D$. We want to see that there is a y s.t. $x \lhd y$.

Assume that $M(x)$. If $M(x)$, then by graphic conditions, we have that $\exists y \in D$, s.t. $y \neq x$, and $x \lhd y$.

Therefore there is a y s.t. $x \lhd y$.

Assume that $S(x)$. If $S(x)$, then x is the limit of the graph and we have that for any $y \in D$ $y \lhd x$.

Since $x \in D$, then we have that $x \lhd x$.

Therefore since G and x were arbitrary, we have that for any x, there is a y s.t. $x \lhd y$.

Ax. 7: $\forall x(\exists y(x \neq y \wedge x \lhd y) \vee^* x \lhd x))$

Take any $G = \{D, T\}$, and any $x \in D$.

For any x we have that $M(x) \vee S(x)$, since $x \in \mathfrak{D}$.

Assume $M(x)$.

If $M(x)$, then by conditions on frames we have that $\exists y \in D$, s.t. $x \neq y$, and $x \lhd y$. Therefore the Axiom is satisfied.

Assume $S(x)$.

If $S(x)$, then by conditions on frames we have that $\forall y \in D$, s.t. $y \neq d$, we have that $\neg(x \lhd y)$.

Now by seriality of $\lhd$ we have that $\exists z$ s.t. $x \lhd z$.

Since $\forall y \in D$, s.t. $y \neq d$, we have that $\neg(x \lhd y)$, we have that $z = x$.

Therefore if $z = x$ and $x \lhd z$, then $x \lhd x$. And the Axioms is satisfied.

Ax 10: $\forall x, y((y \lhd x) \rightarrow \forall z(z \lhd y \rightarrow z \lhd x))$

This is true because of the transitivity of $\lhd$.

General Axioms for Causation:

Ax.9: $\forall x, y(Cyx \leftrightarrow x \lhd y)$.

This is true by definition of C.

Ax. 1: $\forall x(Cxx \rightarrow Nx)$.

Take any graph $G = \{D, T\}$, and a $x \in D$.

Assume that Cxx. We want to see that Nx, i.e., we want to see that there is a $y \in G$ s.t. Cyx, ξy and $\mathfrak{n}(x) = \mathfrak{n}(y)$.

Since C is the inverse of $\lhd$, we have that $x \lhd x$.

Since $x \lhd x$, we have that ξx.

Now since $x = x$, then we have that $\mathfrak{n}(x) = \mathfrak{n}(x)$.

Therefore we have that Nx.

Axioms of Substance:

Ax. 2: $\forall x(S(x) \rightarrow \forall y((x = y \vee \neg(x \lhd y)))$.

Take a graph $G = \{D, T\}$ and a $x \in D$.

Assume that $S(x)$.

Now take any $y \in D$. We have that either $S(y)$ or $M(y)$.

Assume $M(y)$.

By conditions on frames, if $S(x)$ then $\forall y \in D$, s.t. $y \neq x$, we have that $\neg(x \lhd y)$.

As we saw when we deal with Ax.14, we have that since $M(y)$ and $S(x)$, then $x \neq y$.

Therefore we have that $\neg(x \lhd y)$.

Assume that $S(y)$.

By the uniqueness of the limit of a graph, we have that for any y, $\neg S(y) \lor y = x$.

Therefore we have that $x = y$.

Therefore if $S(x)$, then for any y, we have that $x = y \lor \neg(x \lhd y)$.

Ax.3: $\forall x(S(x) \to \exists y(M(y) \land y \lhd x \land \forall z(M(z) \land z \lhd x \to z \lhd y)))$

Take any $G = \{D, T\}$, and a $x \in D$.

Assume that $S(x)$.

Now since $T \subseteq \lhd$, and $\lhd \subseteq \mathfrak{D} X \mathfrak{D}$, then there is also an $y \in D$, since the smallest graph contains at least two elements.

Now we have that y cannot be a substance, i.e., $\neg S(y)$, otherwise the graph will contain only one element since $S(x)$ is the limit, and by definition, if $S(y)$ then $y = x$.

Therefore $M(y)$.

By definition of a limit, we have that for any $y \in D$, $y \lhd x$.

Now since there are no more elements in the graph, $\forall z(M(z) \land z \lhd x \to z \lhd y)$ is vacuously true.

Ax.3 Definition 1

We call the smallest graph, and we note $G_s = \{D_s, T_s\}$, the graph which contains only two elements in D, i.e., $D = \{x, y\}$, where $S(x)$ and $M(y)$. By evaluation order we have that $x = 0$ and $y = 1$.

Ax.3 Theorem

Axiom 3 is satisfied by any graph.

Take a graph $G = \{D, T\}$ s.t. $D = D_s \cup \{z\}$.

For the same reason as before z cannot be a substance, therefore $M(z)$.

By the definition of limit we have that $z \lhd x$.

Now by connectivity of $\lhd$, we have that $\forall a, b \in \mathfrak{D} : a \neq b \implies (a, b) \in \lhd \lor (b, a) \in \lhd$.

Therefore we have that either $z \lhd y$ or $y \lhd z$, but since $\lhd$ is asymmetric it cannot be the case that $z \lhd y \land y \lhd z$. Either way we have that the axiom is satisfied.

Since any graph is an extension of G_s, we can proof the validity of the axiom for any graph by

induction on the number of elements in D.

Ax.12: $\forall x, y(S(x) \wedge S(y) \rightarrow (x = y \leftrightarrow \exists z(z \lhd x \leftrightarrow z \lhd y)))$

The direction from left to right is trivially true.

The other direction is also trivially true since there is a unique limit for every G, this is, if $S(x)$ and $S(y)$, then $x = y$.

Axioms for Modes:

Axiom 4: $\forall x(M(x) \leftrightarrow \exists y(x \neq y \wedge x \lhd y))$

Take any graph $G = \{D, T\}$, and an $x \in D$.

Assume that $M(x)$. We want to see that $\exists y(x \neq y \wedge x \lhd y)$.

By conditions on graph, we have that if $M(x)$, then $\exists y \in D$, s.t. $y \neq x$, and $x \lhd y$. Therefore $\exists y(x \neq y \wedge x \lhd y)$.

Now assume there is a $y \in D$, s.t. $x \neq y \wedge x \lhd y$. We want to see that $M(x)$.

By conditions on graph, we have that if $\exists y \in D$, s.t. $y \neq x$, and $x \lhd y$, then $M(x)$. Therefore $M(x)$.

Since x was arbitrary, we have that $\forall x(M(x) \leftrightarrow \exists y(x \neq y \wedge x \lhd y))$.

Axiom 13: $\forall x, y, z(S(z) \wedge M(x) \wedge M(y) \rightarrow (x \simeq y \leftrightarrow \forall z(S(z) \wedge (x \lhd z \leftrightarrow y \lhd z))))$

Trivially true by the definition of $\simeq$.

Axiom 11: $\forall x, y(M(x) \wedge M(y) \rightarrow (x \lhd y \rightarrow \neg(y \lhd x)))$

Trivially true by assymetry of $\lhd$.

Lemma 1: $\forall x(M(x) \rightarrow \exists y(S(y) \wedge x \lhd y))$

Take any graph $G = \{D, T\}$, and an $x \in D$.

Assume that $M(x)$. We want to see that $\exists y(S(y) \wedge x \lhd y)$.

By the definition of a limit, we have that there is a $y \in G$ s.t. $S(y)$.

If $S(y)$, then by conditions on frames we have that $\forall x \in \mathfrak{D}$, s.t. $x \neq y$, we have that $\neg(y \lhd x)$.

By connectivity of $\lhd$, we have that if $\neg(y \lhd x)$, then $x \lhd y$.

Therefore $\exists y(S(y) \land x \lhd y)$.

4.7 Propositions from the *Ethics*

Proposition 1 (E1P3) *When things have nothing in common, one cannot be the cause of the other.*

For any x, y, if there is no z s.t. $z \lhd x$ and $z \lhd y$, then neither Cxy nor Cyx.

Proof Take any x, y. Assume that $\neg \exists z(z \lhd x \land z \lhd y)$. We want to see that $\neg Cxy$ and $\neg Cyx$.

Assume that Cxy. Then by Ax.9 we have that $y \lhd x$.

If we have that $\neg \exists z(z \lhd x \land z \lhd y)$, then we have that $\forall z(z \lhd x \rightarrow \neg(z \lhd y))$.

If $y \lhd x$, then by Ax.10 we have that $\forall z(z \lhd x \rightarrow z \lhd y)$.

Take any z, s.t. $z \lhd x$. If $z \lhd x$, then $z \lhd y$. But we also have that $\neg(z \lhd y)$.

We have a contradiction, therefore $\neg Cxy$.

Assume that Cyx. By the same reasoning we can see that a contradiction will follow.

Therefore $\neg Cyx$.

Proposition 2 (E1P4) *Two or more distinct things are distinguished from one another either by the difference of the attributes of the substances or by the difference of the affections of the substances.*

For any x, y, z, w, if $S(x)$, $S(y)$, $M(z)$, $M(w)$, $z \lhd x$ and $w \lhd y$, then $x \neq y$ iff $z \not\simeq w$.

This is the proposition why Ax.13 is relevant. As I explained before, the sense in which two modes are taken here to be different. In the proof that Spinoza gives, he claims that

> All things that are, are either in themselves or in something else (EIA1); that is (EID3 and EID5), nothing exists external to the intellect except substances and their affections.There can be nothing external to the intellect, therefore. through which several things can be distinguished from one another except substances or (which is the same thing) (EID4) the attributes and the affections of substances.

So we see that his goal with this proposition is to show that the only two way to differentiate things is whether they are substances or affections of substances. I interpret this proposition in the following way: If we want to differentiate two things they must be either substances or modes and, the only way to differentiate substances is through their modes; and the only way to differentiate modes is through the substances they modify. He doesn't explicitly state this as the goal of his proposition but you from the way in which he states it you can tell that what he wants to say is that the only way to distinguish two things are through their substances or through their modes and, there are only substances and modes.

Proof Take any x, y, z, w. Assume $S(x) \land S(y) \land M(z) \land M(w) \land z \lhd x \land w \lhd y$. We want to see that $x \neq y \leftrightarrow z \not\simeq w$.

We prove the left to right direction by contrapositive. Assume that $z \simeq w$, and we want to see that $x = y$.

If $z \simeq w$, then by Ax.13 we have that $z \lhd z' \leftrightarrow w \lhd z'$. Now since we have that $z \lhd x$, we have that $w \lhd x$.

By the same axiom we have that since $w \lhd y$ we have that $x \lhd y$.

Therefore since there is a w s.t. $w \lhd y$ and $w \lhd x$, then by Ax.12 we have that $x = y$.

Now assume that $z \not\simeq w$, and we want to see that $x \neq y$.

If $z \not\simeq w$, then $(z \lhd z' \wedge \neg(w \lhd z')) \vee (w \lhd z' \wedge \neg(z \lhd z'))$.

Assume that $w \lhd x$.

Now since $z \lhd x$, by the previous result, we have that $\neg(w \lhd x)$, and we have a contradiction.

Therefore $\neg(w \lhd x)$.

Assume that $z \lhd y$.

Now since $w \lhd y$, by the previous result, we have that $\neg(z \lhd y)$, and we have a contradiction.

Therefore $\neg(z \lhd y)$.

Therefore since z, w were arbitrary, we have that $\forall z'(\neg(z' \lhd x) \vee \neg(z' \lhd y))$. Therefore by Ax.12 we have that $x \neq y$.

Therefore $x \neq y \leftrightarrow z \not\simeq w$.

Proposition 3 (E1P5) *In the universe there cannot be two or more substances of the same nature or attribute.*

For any x, y, z, w, if $S(x)$, $S(y)$, $M(z)$, $M(w)$ and $z \lhd x$ and $w \lhd y$, then, if $z \simeq w$ then $x = y$.

The importance of the previous proposition and the way in which I interpret the equality between modes becomes relevant in this proposition. Here we see that Spinoza has a goal one of the most relevant principles about his philosophy each attribute is unique i.e., each substance–which are taken in my interpretation as substituting the attributes–is unique. And in the same way attributes are unique i.e., for each attribute there is a unique substance that contains that attribute in its essence. So following the previous proposition I interpret this proposition as: if two substance share any modification then they must be the same. From the point of view of the ontology we find in the first part this argument I claim is honest with Spinoza's philosophy. Think whether it is possible that there were two substances both with the attribute of extension or in the way I interpret Spinoza two different extended substances and, one modification of extension that is common to both of them.

I want to analyze now Deleuze's ideas about real and numerical distinction in his book *Expressionism in Philosophy*. Deleuze claims that one of the main themes of the *Ethics* is that "real distinction is not and cannot be numerical[84]" . To understand this we have to take a look at Spinoza's threefold distinction of things in his *Metaphysical Thoughts* although we are going to focus on the real distinction:

What is called a real distinction is that whereby two substances, whether of different

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[84]Deleuze (2005) , pg. 38

or of the same attribute, are distinguished from one another; for example, thought and extension, or the parts of matter. This distinction is recognized from the fact that each of the two can be conceived, and consequently can exist, without the help of the other.[85]

Bearing in mind that this ideas were taken from Descartes let me reconstruct the argument that leads to the conclusion that if as Deleuze says: "Numerical distinction is never real; then, conversely, real distinction is never numerical[86]". The argument goes as follows: following Descartes take the substance of extension and the substance of thought we can distinguish between them because I can conceive one without the other. Now assume there are two substances with the same quality i.e., the substance of extension, how can we distinguish between them? Obviously not qualitatively but with what Duns Scotus called the *haecceitas*[87] i.e., we can numerically differentiate them. We can call them substance 1 and substance 2. In other words, we could instantiate a *quality* several times in different substances. Therefore the only possible distinction between those two substances is a modal one i.e., numerical distinction. But Spinoza defines modal distinction as follows: *Modal distinction is of two kinds, that between a mode of substance and the substance itself, and that between two modes of one and the same substance.*[88]

Therefore two substances cannot be differentiated by modal distinction by definition. This seems a rather plain argument but lets try to expand it. Modal distinction is the distinction made by the imagination so, it can only be used to distinguish between modes or to distinguish between modes and substance but never to distinguish between substances and the reason for that is that distinction between substances has to be a real distinction. The argument can be summarized like this: we can only distinguish between two substances of the same quality by the use of number, numerical distinction is not a real distinction therefore, we cannot distinguish between them so, they are one and the same substance. As Deleuze says–if we buy this argument– then numerical distinction is never a real one i.e., if we recall Spinoza's definition of number we now understand why he calls it an aid of the imagination whenever we classify things we are distinguishing them from other and from the substance but, this is not a real distinction just a modal one a distinction made by the imagination–there is no real distinction between modes and substance.

Proof This proposition is evident from the previous proposition, since if we assume that $z \simeq w$, then we have that $x = y$.

Proposition 4 (E1P6)   *One substance cannot be produced by another substance.*

For any x, y if $S(x)$ and y is different from x, then $\neg Cyx$.

Proof Take any x, y s.t. $S(x)$ and and $x \neq y$. We want to see that $\neg Cyx$.

If $S(x)$, then by Ax.2 we have that $\forall y((x = y \vee \neg(x \lhd y))$. We have that $x \neq y$, therefore $\neg(x \lhd y)$.

Now if $\neg(x \lhd y)$, then by Ax.9 we have that $\neg Cyx$.

Proposition 5 (E1P7)   *Existence belongs to the nature of substance.*

I interpret this proposition as saying that substances necessarily exists, since "Existence belongs to the nature of substance" can be also interpret as "a substance's nature can only be conceived as existing" if we follow my interpretation of EID1. Another reason for this interpre-

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[85]Spinoza (2002), pg. 195

[86]Expressionism in philosophy: Spinoza, pg. 38

[87]Scotus (1950), Ordinatio II, d. 3, p. 1, qq. 5–6, n. 183 (Scotus, 7:481)

[88]*ibid.*, pg. 195.

tation is found in the proof that Spinoza gives to this proposition in which he summons EID1 to claim that "its essence necessarily involves existence".

For any x if $S(x)$, then x's existence is necessary, i.e., Nx.

Proof Take any x and assume that $S(x)$, and we want to see that Nx, this is, we want to see that $\exists y$ s.t. Cyx, ξy, and s.t. $\mathbf{n}(x) = \mathbf{n}(y) + 1$ or $\mathbf{n}(x) = \mathbf{n}(y)$.

Take any G. Take $z \in G$ s.t. z is the limit of G.

By definition of limit, we have that for all z', if $S(z')$, then $z' = y$.

Since $S(x)$, by Ax. 2 and Ax. 7 we have that $x \lhd x$.

By Ax. 9 if $x \lhd x$, then we have that Cxx.

Therefore since G was arbitrary, and since $x \lhd x$, we have that ξx.

By evaluation order, is obvious that $\mathbf{n}(x) = \mathbf{n}(y)$, since $x = y$

Therefore Nx. Q.E.D.

Proposition 6 (E1P8) *Every substance is necessarily infinite.*

For any x s.t. x is a substance, then x is infinite.

This sense of necessity here is not to be interpreted as a modality but as logical necessity i.e., it cannot be the case that something is a substance and is finite.

Proof Take any x, s.t. $S(x)$, we want to see that x is infinite. This is, we want to see that for all y s.t. $S(y)$ and $y \neq x$, $\neg(x \lhd y)$.

If $S(x)$, then by Ax.2 we have that $\forall y((x = y \lor \neg(x \lhd y))$.

Now since $x \neq y$, then $\neg(x \lhd y)$.

Therefore $\forall y(\neg(x \lhd y))$.

EIP9 *The more reality or being a thing has, the more attributes it has.*

This proposition is not included in the formalization because it does nor have any relevance for the present work since this is more of a philosophical proposition concerning the idea of the most perfect being. In this proposition we find what could be seen as an statement about the number of attributes the substance has since as I have claimed many times already "infinite attributes" is not a statement about the number of attributes. Here Spinoza tell us that since God is the most perfect being it should have *all* attributes whatever they are since from all of them we only know two: extension and thought–although this is only stated in the second part of the *Ethics*.

EIP10 *Each attribute of one substance must be conceived through itself*

This proposition has no interested for the present work since it just compares again the substances and the attributes this time by the way in which they are conceived. The reason for this proposition is found in the schooling in which Spinoza explores the possibility of conceiving a substance with more than one attribute. Nonetheless this does change the fact that each of

them must be conceived through itself and not through each other even thou they might share the same substance.

Proposition 7 (E1P11) *God, or substance consisting of infinite attributes, each of which expresses eternal and infinite essence, necessarily exists.*

We want to see that the existence of God's existence is necessary, i.e., we want to see that Ng.

Proof Take g. We want to see that Ng, this is, we want to see that $\exists y$ s.t. Cyg, ξy, and s.t. $\mathbf{n}(g) = \mathbf{n}(y) + 1$ or $\mathbf{n}(x) = \mathbf{n}(y)$.

By definition of g we have that $S(g) \wedge (\forall y \forall z (S(y) \wedge z \lhd y \to z \lhd g))$.

Since $S(g)$, by Prop. 5 we have that Ng.

Therefore Ng. Q.E.D.

EIP12 *No attribute of substance can be truly conceived from which it would follow that substance can be divided.*

EIP13 *Absolutely infinite substance is indivisible.*

These two proposition are also not included in my formal interpretation of the *Ethics* since they account for a relevant philosophical argument about a property of the substance which is not that important in the formal approach. This property is indivisibility. In these two propositions Spinoza is arguing that substance is not formed by other substances and, that regardless of which attribute we consider as belonging to the substance essence it will never be divisible. He also explores this issue in his letter about the infinite that we have discussed before in a deeper way. I do not include any of these proposition in my work, because it becomes clear after the next proposition that there can only be one substance therefore, no substance can be divided into two distinct parts.

Proposition 8 (E1P14) *There can be, or be conceived, no other substance but God.*

For any x, we have that if $S(x)$, then $x = g$.

Proof Take any x. Assume that $S(x)$. We want to see that $x = g$.

We have that g equals $S(g) \wedge \forall y \forall z (S(y) \wedge z \lhd y \to z \lhd g)$.

Take any G.

By definition of a limit we have that for any G there is a $x \in G$ s.t. $S(x)$ and that for any $z \in G$ s.t. $S(z)$, $z = x$.

Therefore since $S(g)$, we have that $g = x$.

Since x was arbitrary, we have that $\forall x (S(x) \to x = g)$.

Proposition 9 (E1P15) *Whatever is, is in God, and nothing can be or be conceived without God.*

For any x, x depends on God, i.e., $x \lhd g$.

Since I interpret both containment and conceivability as dependence, the way in which this proposition is stated captures both senses under the notion of dependence.

Proof Take any x, we want to see that $x \lhd g$.

By Ax.5 we have that $M(x) \lor S(x)$.

Assume $S(x)$. By Prop. 8 we have that $x = g$.

Take any y s.t. $g \neq y$. Since $y \neq g$, we have that $\neg S(y)$, and by Ax.6 we have that $M(y)$.

Since $S(g)$, then by Ax.2 we have that $\forall y((g = y) \lor \neg(g \lhd y))$. Now since $y \neq g$, then we have that $\neg(g \lhd y)$.

Since $\neg(g \lhd y)$ and $g \neq y$, by Ax.9 we have that, $g \lhd g$, and since $x = g$, we have that $g \lhd g$.

Assume $M(x)$. By Lemma 1 we have that $\exists y(S(y) \land x \lhd y)$. Since $S(y)$, by Prop. 8, we have that $y = g$. And since $x \lhd y$ and $y = g$, we have that $x \lhd g$.

Therefore, since x was arbitrary, we have that $\forall x(x \leqslant g)$.

Proposition 10 (E1P16) *From the necessity of the divine nature there must follow infinite things in infinite ways [modis] (that is, everything that can come within the scope of infinite intellect).*

There is an x s.t. x is a mode and x depends on God, and is infinite.

This proposition is key to what would come next in this work, since this is the proposition that introduces the concept of the "infinite mode". It is important to quote it *in extenso*:

> **Proof** This proposition should be obvious to everyone who will but consider this point, that from the given definition of any one thing the intellect infers a number of properties which necessarily follow in fact from the definition (that is, from the very essence of the thing), and the more reality the definition of the thing expresses (that is, the more reality the essence of the thing defined involves), the greater the number of its properties. Now since divine nature possesses absolutely infinite attributes (Def. 6), of which each one also expresses infinite essence in its own kind, then there must necessarily follow from the necessity of the divine nature an infinity of things in infinite ways (that is, everything that can come within the scope of the infinite intellect) Q.E.D.[89]

The key concept to understand here is *infinita infinitis modis*. This has been usually translated as "infinite things in infinite ways" but as I explain before this sense of *infinite* must be taken as as defined in EID2. Under this view it would be a better to translated as follows: "unlimited things in an unlimited way". My interpretation of this proposition–which is a very important argument towards the conclusion of this paper–is that Spinoza here is anticipating what we will see in EIP21, EIP22, and EIP23 i.e., that from the necessity of the divine Nature which consists

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[89]Haec propositio unicuique manifesta esse debet, si modo ad hoc attendat, quod ex data cujuscunque rei definitione plures proprietates intellectus concludit, quae revera ex eadem (hoc est ipsa rei essentia) necessario sequuntur, et eo plures, quo plus realitatis rei definitio exprimit, hoc est, quo plus realitatis rei definitae essentia involvit. Cum autem natura divina infinita absolute attributa habeat (per defin. 6.), quorum etiam unumquodque infinitam essentiam in suo genere exprimit, ex ejusdem ergo necessitate infinita infinitis modis (hoc est omnia, quae sub intellectum infinitum cadere possunt) necessario sequi debent. Q. e. d.

of infinite qualities (attributes) each of them expressing eternal and infinite essence follows modes in an infinite way (infinite modes). Let us inquire now about what I just said.

As I said the key to understand this is to delve into the meaning that "infinita infinitis modis" has in Spinoza's system which under my interpretation is taken as "unlimited unlimited modes". Each mode is nothing but an affection of a substance–this is–a modification of a quality in the substance's essence. The qualities in a substance's essence are the attributes. Thus an "unlimited mode" is the attribute of a substance being modified uniquely in its kind i.e., it is a modification of an attribute of a substance that is not limited by any other modification of the same attribute. Now an "unlimited unlimited mode" is what Spinoza describes as being "absolutely infinite" in the explication to EID6[90]. The "unlimited unlimited modes" are the attributes of a substance which are uniquely in nature being modified uniquely in its kind. This "uniquely" that accompanies "nature" mean here two things. First each quality of that substance is unique in the sense of EIP5 i.e., there is no other substance with the same quality. Second that all qualities of that substance have the same form i.e., they have the same nature in the sense of being unlimited in their own kind.

Since we have already seen that there cannot be other substances but God and that every substance is infinite there is no need to show in this proposition that the attributes are infinite since they have they have the same nature as the substance which in this case is to be absolutely infinite[91]. As I claim before to make a distinction between *extended substance* and the substance which has the quality of extension within its essence in Spinoza is not necessary to understand the formal structure of his philosophy. This proposition is an unraveling of the definition of God into terms that are familiar with the overall strategy of the first book. This is the proposition which finally connects the essence of God i.e., its attributes with their affections i.e., modes. The reason for this interpretation is that in the demonstration of the proposition Spinoza only refers to EID6. Therefore my aim here is to show that from all the different attributes that compose God's essence–which in this work are just taken as substances and as infinite– infinite modes follow.

Proof We want to see that there is a x, s.t. $M(x)$, $x \lhd g$ and x is infinite, i.e., $\forall y(M(y) \to \neg(x \lhd y))$.

By Ax.3, we have that if $S(g)$, then $\exists y(M(y) \land y \lhd g \land \forall z(M(z) \land z \lhd g \to z \lhd y))$.

Therefore we have that there is a y, s.t. $M(y)$ and $y \lhd g$, and $\forall z(M(z) \land z \lhd g \to z \lhd y)$.

Take any z. Assume that $M(z)$. We have two possibilities, either $z = y$ or $z \neq y$.

By the previous proposition we have that $z \lhd g$. We have that $\forall z(M(z) \land z \lhd g \to z \lhd y)$, therefore since $z \lhd g$, then $z \lhd y$.

If $z = y$, then since $M(z)$, and $z \neq g$ and $z \lhd g$, then by Ax. 7 we have that $\neg(z \lhd z)$.

Assume that $z \neq y$.

By Ax.11 we have that if $z \lhd y$, then $\neg(y \lhd z)$. Therefore we have that $\neg(y \lhd z)$. Since z was arbitrary, we have that $\forall z(M(z) \to \neg(y \lhd z))$.

Therefore there is a x s.t. $M(x)$ and $x \lhd g$ and that $\forall y(M(y) \to \neg(x \lhd y))$.

Definition 8 We call x an infinite mode, and we denote it with $M_i(x)$, iff $M(x) \land \forall z(M(z) \to \neg(x \lhd z))$.

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[90]I say "absolutely infinite" not "infinite in its kind". For if a thing is only infinite in its kind, one may deny that it has infinite attributes. But if a thing is absolutely infinite, whatever expresses essence and does not involve any negation belongs to its essence. Spinoza (2002), p. 217.

[91]We will see in EIP21 that Spinoza actually does not mention there the absolute nature of the unique substance, i.e., God, but instead he mentions the absolute nature of an attribute of God.

Now we can define a new type M_i, which is the set of infinite modes, s.t. $\mathbb{M}_i$.

Proposition 11 (E1P17) *God acts solely from the laws of his own nature, constrained by none.*

g is the only free thing.

We take the second corollary of this proposition to be the relevant clause we want to capture with our language i.e., that God is a free cause. The original proposition states can be seen as a step towards the objective of proving that God is free since in the description of the corollary Spinoza says:

> It follows, secondly, that God alone is a free cause. For God alone exists solely from the necessity of his own nature (EIP11 and EIP14's corollary) and acts solely from the necessity of his own nature (EIP17). So he alone is a free cause (EID7).

We see that this proposition is used in its own corollary since to show that God is the only free cause we have to show first that nothing constrains God and, that God is the only thing which is free i.e., self determined which in this cause is taken as self-caused. So we have to show that God's existence comes from itself alone.

Proof We want to see that g is the only free thing, i.e., we want to see first that g is a free thing, and then that it is the only free thing.

First we want to see that g is free, i.e., we want to see that $\forall x(x = g \vee \neg Cxg)$ and that Cgg.

Take any x, by Ax. 5 we have that $M(x) \vee S(x)$.

Assume $S(x)$.

By Prop. 8 we have that $x = g$. Therefore we have that $(x = g \vee \neg Cxg)$.

Assume that $M(x)$.

Since $S(g)$, then by Ax. 2 we have that $\forall x(g = x \vee \neg(g \lhd x))$.

Since $S(g)$ and $M(x)$, by Ax.15 we have that $g \neq x$.

Now, by Ax.9, if $g \neq x$, then $\neg Cxg$. Therefore $(x = g \vee \neg Cxg)$.

Therefore $(x = g \vee \neg Cxg)$.

Since x was arbitrary, we have that $\forall x(x = g \vee \neg Cxg)$.

Now take any x s.t. $x \neq g$.

Since $S(g)$, and $x \neq g$, by Prop. 4 we have that $\neg Cxg$.

If $\neg Cxg$, then by Ax.9 we have that $\neg(g \lhd x)$.

Now by Ax. 7 if $g \neq x$ and $\neg(g \lhd x)$, then we have that $g \lhd g$.

Now by Ax.9 if $g \lhd g$, then Cgg.

We have that $\forall x(x = g \vee \neg Cxg)$ and Cgg, therefore g is a free thing, i.e., Lg.

Now we want to see that it is the only free thing, i.e., we want to see that $\forall x(\neg Lx \vee x = g)$.

Take any x. By ax. 5 we have that $M(x) \lor S(x)$.

Assume that $M(x)$. If $M(x)$, then by Ax. 4, we have that $\exists y(x \neq y \land x \lhd y)$. Therefore by Ax. 7, we have that $\neg(x \lhd x)$.

If $\neg(x \lhd x)$, then by Ax.9 $\neg Cxx$, and x is not free, i.e., $\neg Lx$.

Assume now that $S(x)$. By Prop. 8 $x = g$.

Therefore since x was arbitrary, we have that $\forall x(\neg Lx \lor x = g)$, i.e., God in the only free thing. Q.E.D.

Proposition 12 (E1P18)   *God is the immanent, not the transitive, cause of all things.*

We want to see that God is the cause of everything, this is, for any x we have that Cgx.

The difference introduced here between *immanent* and *transitive* is of a very important philosophical importance although formally it is not that relevant. What Spinoza claims here is that all thing depends on God and because of that God is their cause. In other words God is not the cause of things because he is the first cause and because causation is transitive. Things are caused within God and not from God as an non-transitive origin. Here we just show that everything is caused by God without using transitivity.

Proof By Prop. 9 we have that $\forall x(x \lhd g)$. Now by Ax.9, we have that if $x \lhd g$, then Cgx.

Therefore $\forall x(Cgx)$. Q.E.D.

Proposition 13 (E1P19)   *God [is eternal], that is, all the attributes of God are eternal.*

We want to see that God is an eternal thing, i.e., we want to see that ξg.

Proof We want to see that ξg, this is, we want to see that for any graph G, $x \in G$ and that $x \lhd x$.

By definition of g we have that $S(g) \land \forall y \forall z(S(y) \land z \lhd y \to z \lhd g)$.

Take any graph $G = \{D, T\}$. By definition of a graph, there is a $y \in D$ s.t. $S(y)$ and y is the limit of the graph.

By definition of limit, we have that for all z, if $S(z)$, then $z = y$.

Therefore we have that $g = y$. Therefore $g \in D$.

Since G was arbitrary, we have that for any G, $g \in G$.

Since $S(g)$, by Ax. 2 we have that $\forall y((x = y \lor \neg(x \lhd y))$.

By Ax. 7 we have that $\forall x(\exists y(x \neq y \land x \lhd y) \lor^* x \lhd x))$.

Therefore we have that $g \lhd g$.

We have seen that for for any graph G, $g \in G$, and that $g \lhd g$, therefore ξg.

EIP20   *God's existence and his essence are one and the same.*

What Spinoza is aiming at in this proposition is the fact that the existence and the essence

of God which are proven to be the same in this proposition are both immutable. This means that not only God but its attributes are immutable. My argument resides in the fact that the proposition itself is not used again in the *Ethics*. Nonetheless, its second Corollary[92] is used in the following propositions which are the most important in this first book. Since both its essence ans existence are eternal truths they can never change for they will become false in that case. Again this is a proposition more relevant for a philosophical discussion about God but not very interesting from a formal point of view because since its existence and therefore, essence as well is eternal i.e., outside time. This already means that it is not subjected to any change whatsoever.

Proposition 14 (E1P21)   *All things that follow from the absolute nature of any attribute of God must have existed always, and as infinite; that is, through the said attribute they are eternal and infinite.*

$$\forall x(x \lhd g \wedge \forall y(M(y) \rightarrow y \lhd x \vee y = x) \rightarrow M_i(x) \wedge \xi x)$$

For any x, if $x \lhd g$, and for any y if y is a mode then $y \lhd x$ or $y = x$, then x is an infinite mode and x is eternal.

In this proposition we find the most complex proposition of the first part not only because it seems like Spinoza claims that there are other things that are eternal apart from the substance but also because the proof seems very confusing. It is clear from the way in which the proposition is stated that these other eternal things are not the attributes because he says "All things that follow from the absolute nature of any attribute of God" and also because that has been proven in EIP19. The way I interpret this proposition is that Spinoza is here introducing the concept of infinite immediate modes i.e., modes that are infinite and that are eternal because the follow directly from the absolute nature of God or its attributes–which are the same. This proposition clearly establishes a connection with EIP16 since here we are introduced with a subtype of infinite mode. The key to understand this proposition resides in the sentence: *following from the absolute nature of any attribute of God*. My interpretation of this sentence is that–as I claimed before–for each attribute there is a fundamental mode or modification that defines any other possible mode–or modification–of that same attribute. This proposition claims that that modification is eternal–in the same way as God is–meaning that the modification in itself is beyond time i.e., its existence cannot be conceived through time neither can it be subjected to any change itself. Nonetheless this eternity is different from the one of the substance in that it follows from something else which is eternal as EID8 states.

Although in the proof to this proposition Spinoza does not talk about any of this directly I argue that we can make sense of my argument if we look carefully enough to the proof of this proposition and the use of it in EVP40. First of all, the proof focuses on the "Idea of God in Thought" which is chosen by Spinoza because the idea of God is the only idea that there is in God's intellect although at the end of the first part of the proof he says "Therefore, if the idea of God in Thought, or anything in some attribute of God (it does not matter what is selected, since the proof is universal)...". We have to keep in mind–in order to understand this proposition–that Spinoza is no considering here the idea of God from amongst any other idea. In the quote we just saw he says "or anything in some attribute" as an alternative to the idea of God but we have to keep in mind that this "anything" must satisfy the condition of "following from the absolute nature of an attribute". In the second part of the proof we read:

> Therefore, outside the bounds of the duration of the idea of God (for this idea is supposed at some time not to have existed, or will at some point cease to exist), Thought will have to exist without the idea of God. But this is contrary to

[92] "It follows, secondly, that God is immutable; that is, all the attributes of God are immutable. For if they were to change in respect of existence, they would also have to change in respect of essence (EIP10); that is-and this is self evident- they would have to become false instead of true, which is absurd" *ibid.*, p. 230.

the hypothesis, for it is supposed that when Thought is granted the idea of God necessarily follows.

As I said before we have to be aware that the existence of an idea is tied to an intellect. In that sense the idea of God follows necessarily from the attribute of Thought since from the definition of God and the fact that thought is one of its attributes it follows that he has necessarily the capacity of forming ideas of things and, since he can conceived himself then the idea of God necessarily follows because of the capacity that this particular attribute provides regardless of the *ideatum* but what is important here is the *capacity* and not the idea in itself since saying that the idea of God is eternal means the capacity of God of forming an idea of itself or "anything in some attribute of God" must be beyond time. If we analyze the proof carefully we see that the eternity or the infinity of the idea of God is *never* mentioned. This means that the selection of the idea is not relevant although it has its reasons as we will see in a moment. This makes my argument stronger since we can select anything whatsoever since the importance is that we assume that idea to be finite and to follow from the attribute of thought i.e., for an intellect to be able to form an idea of it. The reason for the selection of the idea of God in this proposition is found in "Principles of Cartesian philosophy, Chapter 7"[93].

This proof only makes sense if we understand that Spinoza is not talking here about the existence of any particular idea but only of those who follow from the absolute nature of the attribute. We are talking about the connection of those ideas to the substance's attributes which in the case of thought is nothing but the capacity of (reflective) understanding that the intellect has i.e., the absolute nature of the attribute of thought. In that sense it can never be subjected to time it must be eternal. We are not talking about any idea at all we are talking about the infinite intellect of God having the thought of anything i.e., idea of something, and this *process* is eternal in the sense that there is no time limitation to his intellect. The argument found in the *Ethics* that supports my interpretation is literally at the end of the book. In the scholium to EVP40 Spinoza says:

> This is what I had resolved to demonstrate concerning the mind insofar as it is considered without reference to the existence of the body. It is clear from this, and also from EIP21 and other propositions, that our mind, *insofar as it understands*[94], is an eternal mode of thinking which is determined by another eternal mode of thinking, and this again by another, and so on ad infinitum, with the result that they all together constitute the eternal and infinite intellect of God.

Our intellect is the same as God's intellect insofar as we are talking about the capacity of the understanding except that we have a time limitation to the existence of our bodies but not to our understanding. In the corollary to the same proposition he says:

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[93]The quote I refer to is very important on this step of the argument so I will quote it *in extenso*:

[In God there is only one simple idea.] Finally, before bringing this discussion to a close, we ought to deal with the question as to whether there is in God more than one idea or only one most simple idea. To this I reply that God's idea through which he is called omniscient is unique and completely simple. For in actual fact God is called omniscient for no other reason than that he has the idea of himself, an idea or knowledge that has always existed together with God. For it is nothing but his essence and could have had no other way of being.

[What is God's knowledge concerning created things.] But God's acquaintance with created things cannot be referred to God's knowledge without some impropriety; for, if God had so willed, created things would have had a quite different essence, and this could have no place in the knowledge that God has of himself. Still, the question will arise as to whether that knowledge of created things, properly or improperly so termed, is manifold or only single. However, in reply, this question differs in no way from those that ask whether God's decrees and volitions are several or not, and whether God's omnipresence, or the concurrence whereby he preserves particular things, is the same in all things. Concerning these matters, we have already said that we can have no distinct knowledge. However, we know with certainty that, just as God's concurrence, if it is referred to God's omnipotence, must be no more than one although manifested in various ways in its effects, so too God's volitions and decrees (for thus we may term his knowledge concerning created things) considered in God are not a plurality, even though they are expressed in various ways through created things, or rather, in created things. Finally, if we look to the whole of Nature by analogy, we can consider it as a single entity, and consequently the idea of God, or his decree concerning Natura naturata, will be only one. Spinoza (2002), p. 199.

[94]My emphasis.

Hence it follows that the part of the mind that survives, of whatever extent it may be, is more perfect than the rest. For the eternal part of the mind (EVP23 and EVP29) is the intellect, through which alone we are said to be *active*[95] (EIIIP3), whereas that part which we have shown to perish is the imagination (EVP23), through which alone we are said to be passive (EIIIP3 and Gen. Def. of Emotions). Therefore, the former (preceding Pr.), of whatever extent it be, is more perfect than the latter.

The same argument can be used for movement. Our body's movement is not something temporal i.e., the change of location of our body is something temporal but the capacity to move can never be temporal but eternal otherwise there must e a time in which my body could never move nor be still at all,but this is something completely absurd since it will be like saying that a body is not a body. The capacity of a body to move is eternal in the same way as the capacity to think of a mind is. In this proposition our goal is to demonstrate that from God it follows an infinite immediate mode i.e., an infinite mode which is eternal. The main feature of this infinite mode is that it follows directly from the absolute nature of God this is, there is nothing else between God and this very mode.

Proof Take any x. Assume that $x \lhd g \wedge \forall y((M(y) \rightarrow y \lhd x \vee y = x)$, we want to see that $M_i(x) \wedge \xi x$.

We want to see that $M_i(x)$, this is, we want to see that $M(x) \wedge \forall z(M(z) \rightarrow \neg(x \lhd z))$.

We have that $x \lhd g$, therefore by Ax.4 we have that $M(x)$.

Take any z. Assume $M(z)$, we want to see that $\neg(x \lhd z)$.

Since $\forall y((M(y) \rightarrow y \lhd x \vee y = x)$, and $M(z)$, we have that $z \lhd x \vee z = x$.

Assume that $z \lhd x$.

By Ax.11 we have that $\neg(x \lhd z)$.

Assume that $z = x$.

Assume that $x \lhd z$.

Since $z = x$, then we have that $x \lhd x$.

Now since $M(x)$, by Ax.4 and 7 we have that $\neg(x \lhd x)$, and we have a contradiction.

Therefore $\neg(x \lhd z)$.

Therefore we have that $\neg(x \lhd z)$.

Since z was arbitrary, we have that $\forall z(M(z) \rightarrow \neg(x \lhd z))$.

Therefore we have that $M(x) \wedge \forall z(M(z) \rightarrow \neg(x \lhd z))$, this is, we have that $M_i(x)$.

Now we want to see that ξx, this is, we want to see that for any graph G we have that $x \in G$, and that there is a $y \in G$ s.t. $x \lhd y$ and ξy.

We have that $x \lhd g$.

By Prop. 13, we have that ξg.

Now take G_s. By definition of G_s, there is an $x \in D_s$ s.t. $x \lhd g$, and since there are no more

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[95]My emphasis.

elements in D_s rather than g, x, $\forall z(M(z) \to \neg(x \lhd z))$ is vacuously true.

Since any graph is an extension of the smallest graph G_s, take any graph $G = \{D, T\}$ with any number of elements in D.

Since $\mathbf{n}(x) = 1$, take any of those elements y, we have that $\mathbf{n}(y) > 1$, and therefore $y \lhd x$.

By Ax. 4 we have that $M(y)$.

By asymmetry we have that $\neg(x \lhd y)$. Since y was arbitrary, we have that $\forall z(M(z) \to \neg(x \lhd z))$.

We have that there is an $x' \in G$ s.t. $M(x')$ and $x' \lhd g$, and that for any y s.t. $M(y)$ we have that $\neg(x' \lhd y)$. This is the mode which is evaluated as 1 in the evaluation order.

Since G was arbitrary, we have that for any G', $x \in G'$, and since $x \lhd g$ and ξg and $g \lhd g$, we have that ξx.

Definition 9   We call x an infinite immediate mode, and we write $M_{ii}(x)$, iff $M_i(x)$ and ξx.

We have now a new type of infinite modes, i.e., infinite immediate mode which is a subset of the set of infinite modes, $\mathbb{M}_{ii} \subseteq \mathbb{M}_i$.

Lemma 2   If something is eternal, then its existence is necessary, i.e., $\forall x(\xi x \to Nx)$.

Proof

Take any x assume that ξx, and we wan to see that Nx, i.e., we want to see that there is a y s.t. ξy and Cyx, and $\mathbf{n}(x) = \mathbf{n}(y) + 1$, or $\mathbf{n}(x) = \mathbf{n}(y)$.

If ξx, then we have that for any graph G, $x \in G$ and that $x \lhd x$, or $x \in G$ and there is a z s.t. $z \lhd z$ and $\mathbf{n}(x) = \mathbf{n}(z) + 1$.

Assume that that for any graph G, $x \in G$ and that $x \lhd x$.

Then, since $x \lhd x$, we have that there is a $y \in G$ s.t. Cyx and ξx and $\mathbf{n}(x) = \mathbf{n}(y)$, since $y = x$.

Assume now that $x \in G$ and there is a z s.t. $z \lhd z$ and $\mathbf{n}(x) = \mathbf{n}(z) + 1$. Therefore Nx

Since for any G, we have that $z \in G$ and $z \lhd z$, then we have that ξz.

Then we have that there is a y s.t. Cyx and ξy and that $\mathbf{n}(x) = \mathbf{n}(y) + 1$. Therefore Nx

Therefore if ξx, then Nx for any x.

Proposition 15 (E1P22)   *Whatever follows from some attribute of God, insofar as the attribute is modified by a modification that exists necessarily and as infinite through that same attribute, must also exist both necessarily and as infinite.*

$$\forall x, z(x \lhd z \land Mii(z) \land \forall y(M(y) \land y \lhd z \land y \neq z \to y = x) \to M_i(x) \land Nx)$$

Here we find the proposition that refers to infinite mediate modes because while they are infinite modes they do not follow from the absolutely nature of an attribute but from a modification of one of them. There is another very important distinction between these two modes–apart from the one just said. The existence of the infinite mediate mode is not like that of the infinite immediate mode i.e., it is not something eternal rather it exists necessarily. Let me explain this

difference. The existence of the infinite immediate mode is eternal because its existence follows directly from the definition of an eternal thing i.e., God. Nonetheless the existence of the infinite mediate mode is necessary–and not eternal–because it follows from something eternal but not eternal by definition but eternal by cause. In the same way there is a difference in their infinity as I explained when discussing Letter XII. Nonetheless we have to show in this proposition here that it is necessary. From this it follows that there is a difference between eternity and necessary existence mainly there are things which are necessary and not eternal whereas if something is eternal then it must be necessary (Previous Lemma). The main difference in Spinoza can be pinpointed to a difference between the way in which we conceive those existences one from an eternal point of view and the other from a temporal point of view. In our interpretation we follow a similar strategy to differentiate them.

This proposition also represents a step forward in Spinoza's ontology since this is the first element that is already included in the *Natura Naturata*. We will talk later in the conclusion about the difficulty that connecting the infinite and the finite entails in his system but this infinite mediate mode is a solution to that problem. As I discussed before there are only two type of infinite modes the most fundamental mode and the mode which contains all other modes except the previous one[96]. Therefore the path from the first book to the second from the *Natura Naturans* to the *Natura Naturata* connects in this proposition since as I will claim later the way to connect the infinite and the finite is through the limitation within the infinite mediate mode.

This second proposition about the infinite modes hides a very important feature of Spinoza's philosophy as displayed in the *Ethics* that will be explored further in the conclusion. If we look at this proposition carefully we see that it has some relevant implications. First of all we see that from this definition and the previous one it follows that a finite mode could never follow from neither infinite modes since that would make the infinite modes finite as we have seen in the proof to the previous proposition–which is invoked as being the same for this proposition:

> **Proof** Suppose this proposition be denied and conceive, if you can, that something in some attribute of God, following from its absolute nature, is finite and has a determinate existence or duration; for example, the idea of God in Thought. Now Thought, being assumed to be an attribute of God, is necessarily infinite by its own nature (EIP11). However, insofar as it has the idea of God, it is being supposed as finite. Now (EID2) it cannot be conceived as finite unless it is determined through Thought itself. But it cannot be determined through Thought itself insofar as Thought constitutes the idea of God, for it is in that respect that Thought is supposed to be finite. Therefore, it is determined through Thought insofar as Thought does not constitute the idea of God, which Thought must nevertheless necessarily exist (EIP2). Therefore, there must be Thought which does not constitute the idea of God, and so the idea of God does not follow necessarily from its nature insofar as it is absolute Thought. (For it is conceived as constituting and as not constituting the idea of God.) This is contrary to our hypothesis. Therefore, if the idea of God in Thought, or anything in some attribute of God (it does not matter what is selected, since the proof is universal), follows from the necessity of the absolute nature of the attribute, it must necessarily be infinite. That was our first point.

Second it follows something even more relevant if we pay attention to the definition of "infinite" that I gave–based on EID2–we see that this proposition might look like a contradiction. Let me explain why: it seems that what this proposition is saying is that modes which follow from

[96]Someone might argue that the mode which contains all modes might, or should, not be a mode itself. My answer to this is found in EIID7:"By individual things [res singulares] I mean things that are finite and have a determinate existence. If several individual things concur in one act in such a way as to be all together the simultaneous cause of one effect, I consider them all, in that respect, as one individual." Although this is a definition from the second book, I claim that EIP22 is the first step into the realm of the *Natura Naturata*, or the last from the *Natura Naturans*.

infinite and necessary modes are infinite and necessary themselves. But we saw that EID2 states that *A thing is said to be finite in its own kind [in suo genere finita] when it can be limited by another thing of the same nature.* Therefore if we accept my interpretation that "nature" is taken to be type in my language it might seem that a a mode which is necessary and infinite cannot follow from a mode which is necessary and infinite since that would make the first one finite. Nonetheless there is a way to make this proposition true. This proposition focuses on the relation between the infinite immediate mode and the infinite mediate mode. The former is eternal and infinite and–by the previous lemma–it is also necessary. Nonetheless this infinite mode is of a different subtype of infinite mode. They represent the two subtypes of infinite modes and in that sense one can follow from the other without violating the definition of infinite of our system. Thirdly it follows that nothing whatsoever can follow from the infinite mediate mode by what I just said neither a finite nor an infinite mode can follow from it. This means that following the first book–and based on the model interpretation I propose–we have reached the last element of Spinoza's ontology at this moment of the first book only contains three elements: God, infinite immediate mode and infinite mediate mode. So in this proposition we have to show that this very mode is indeed a infinite and necessary mode. There is something that I must discuss here involved in the proof of this proposition related to the necessity of this mode. The definition of this second infinite mode is based on the fact that in Spinoza's system this mode represent the collection of all finite modes. The property that bring all those modes together is that they depend on the infinite immediate one thus the reason for the inclusion of the clause $\forall y(M(y) \wedge y \lhd z \wedge y \neq z \rightarrow y = x)$. Even thou in that clause it says that "for any mode, different from the infinite immediate mode, and that depends on it, that mode is equal to the infinite mediate mode", this claim has to be elucidated. Since we are only dealing with the realm of the infinite in this work we don't include the definition of the mediate mode here since we are only dealing with the fact that there can only with two infinite modes and, the infinite mediate mode is a unique one, which I claim is what Spinoza want to say with this proposition. We will come back to this problem in the conclusion, since this proposition is the beginning of the future development of the system. Nonetheless the infinite mediate mode can be explained in terms of ordering. This m is precisely the last element of this ontology because it represents the union of all finite modes that depend on the infinite immediate mode which is basically any possible modification of a certain attribute. In term of ordering the infinite mediate mode is the collection of all modes which depend on the infinite immediate mode i.e., all modes with a value in the evaluation order bigger than two.

Proof Take any x, z. Assume that $x \lhd z$, $M_{ii}(z)$ and $\forall y(M(y) \wedge y \lhd z \wedge y \neq z \rightarrow y = x)$. We want to see that $M_i(x) \wedge Nx$.

We want to see that $M_i(x)$, this is we want to see that $M(x) \wedge \forall z'(M(z') \rightarrow \neg(x \lhd z'))$.

Since $x \lhd z$ and $z \neq x$, we have that $M(x)$.

Since have that $\forall y(M(y) \wedge y \lhd z \wedge y \neq z \rightarrow y = x)$. Take any z' s.t $M(z')$ and $z' \neq x$.

Now by conectivity of $\lhd$, we have that either $z' \lhd z$ or $z \lhd z'$. But since $M_{ii}(z)$, we have that $M_i(z)$, therefore $z' \lhd z$.

Therefore, since $M(z')$ and $z' \neq x$ and $z' \lhd z$, we have that $z = x$.

Since $M(z')$ by Ax. 4, and Ax. 7 we have that $\neg(x \lhd z')$. Since z' was arbitrary we have that $\forall z'(M(z') \rightarrow \neg(x \lhd z'))$.

Therefore we have that $M_i(x)$.

We want to see now that Nx, this is we want to see that $\exists y$ s.t. Cyx, ξy, and s.t. $\mathbf{n}(x) = \mathbf{n}(y) + 1$ or $\mathbf{n}(x) = \mathbf{n}(y)$.

We have that ξz, and Czx, since $x \lhd z$.

Take any G s.t. $x \in G$. Since ξz, we have that $z \in G$.

We have that $x \lhd z$. And since for any y different from z s.t. $M(y)$, we have that $y = x$, then there is no y' between z and x. Therefore $\mathbf{n}(x) = \mathbf{n}(y) + 1$.

Therefore Nx.

Therefore we have that $M_i(x)$ and Nx. Q.E.D.

Definition 10   We call x a infinite mediate mode, and we denote if M_{im}. This mode is an infinite mode, i.e., $M_i(x)$ and is necessary, i.e., Nx, but not eternal, this is, $\neg \xi x$.

We have now the second subtype of infinite modes $\mathfrak{M}_{im} \subseteq \mathfrak{M}_i$. Thus we can define the sent of infinite modes as $\mathfrak{M}_i = \mathfrak{M}_{im} \cup \mathfrak{M}_{ii}$.

Proposition 16 (E1P23)   *Every mode which exists necessarily and as infinite must have necessarily followed either from the absolute nature of some attribute of God or from some attribute modified by a modification which exists necessarily and as infinite.*

$$\forall x, y (Nx \wedge M_i(x) \to x \lhd g \vee x \lhd M_{ii}(y))$$

In this proposition it seems pretty straight forward that what Spinoza is trying to say here is that for any infinite mode, is either from any of the two subtypes of infinite modes that we have defined, i.e., it follows either from the infinite immediate mode or from the absolute nature of an attribute of God.

Proof Take any x. Assume that $Nx \wedge M_i(x)$.

Since Nx, then we have that that $\exists y$ s.t. Cyx, ξy, and s.t. $\mathbf{n}(x) = \mathbf{n}(y) + 1$ or $\mathbf{n}(x) = \mathbf{n}(y)$.

Now since $M(x)$, then by Ax. 4 and Ax.7, we have that $\neg(x \lhd x)$. So we have that $\neg(\mathbf{n}(x) = \mathbf{n}(y))$, therefore $\mathbf{n}(x) = \mathbf{n}(y) + 1$.

Take any G s.t. $x \in G$.

By definition of M_{ii} and the evaluation order, we have that $\mathbf{n}(y) = 1$.

By definition of g and the evaluation order we have that $\mathbf{n}(y) = 0$.

By evaluation order we have that if $\mathbf{n}(x) > 1$, then $\neg(y \lhd x)$.

By definition connectivity of $\lhd$, we have that if $\neg(y \lhd x)$, then $x \lhd y$.

Therefore we have that $x \lhd g$. Therefore we have that $x \lhd g \vee x \lhd M_{ii}(y)$. Q.E.D.

5 Conclusion

Although in this work we mainly focus on the first book of the *Ethics* the interpretation of his philosophy that we developed in this work towards a formalization has taken into account other works by Spinoza not only the *Ethics*. Firstly, by the analysis of the concept of a true idea, the objective and formal essence found in the *Treatise* we were able to elucidate what sort of notion of truth we had to insert into our formalization in order to capture his idea. This resulted in a soundness proof since for Spinoza truth is originated in the true idea and drummed into the propositions by the use of the correct method, thus the reason for the proof that demonstrates the consistency of the axioms. The distinction between formal and objective essence has also proven to be a vital distinction for a formal treatment of Spinoza since it allowed us to make a division that; firstly separates the attributes from the modes and substances and secondly strengthens the relations that these two basic units of Spinoza's philosophy have–dependence and causation– which also govern all his philosophy. The other cornerstone of this interpretation is the interpretation of the concept of "infinity" found in Letter XII allowed us to understand better and to establish a relation between the infinite modes and the substance. This part of the interpretation played a vital role at the end on the formalization since EIP21 and EIP22 are the propositions that tight all the work together and also open the door to a further developed of the system and a continuation of the formalization. Overall the interpretation of Spinoza's philosophy has proven to be fruitful towards a formal view that unifies the concepts in his philosophy providing them the option to be treated formally.

Regarding the formalization the soundness proof and the fact that we have been able to give a formal version of the relevant prepositions of the first book of the *Ethics* entails an axiomatic improvement of the Spinozistic system not only because it has opened a window for Spinoza's philosophy into mathematical language–something that is implicitly an aim of the system due to the axiomatic value it entails. But also because it provides an alternative to the way in which Spinoza himself proves his propositions since his choice of natural language together with the geometric order could be seen as one of the biggest weaknesses of the system presented in the *Ethics*. In this work we sacrifice expressive power for the sake of axiomatic consistency consistency. This change of natural language for a mathematical one entails that most of the times the way in which we have proven the proposition doesn't follow the same path as Spinoza in the original work but I see this as another achievement of the present work since he himself also provides several proofs for some propositions. Another great addition are the dependency graphs which would allow us to add graphs to the proofs of the proposition–although not included in this work–meaning that we will have a visual support just as the *Elements* did.

Both the interpretation of Spinoza's philosophy and the process of formalization resulted in a deeper insight of the *Ethics* since they support each other. The interpretation gave us new perspectives of some concepts included in the *Ethics* since its objective was to take a point of view of those concepts towards the construction of a formal language that captures the philosophical meaning that Spinoza gave to those concepts while at the same time taking them together with the rest of concepts that are included in the system i.e., the interpretation always had in mind that all the concepts would have to work together as pieces of a language and that is what provided more inner-consistency to the philosophical system. Upon that interpretation we build the formal language that account for those concepts interpreted as I just explained. This proven to be a successful enterprise since we were able to create a formal language based on the definitions and axioms of the *Ethics* together with the proposition and their proofs in a consistent way which what our objective all along. Although this may look like the least interesting set of philosophical concepts to board from a formal perspective they set the basis for a future formalization and the inclusion of more concepts and axioms that will be of a lot of interest for discussion on some of those concepts nowadays. I could have chosen the second or third book of the *Ethics* which contain more interesting and relevant concepts for the present day but the method had to be followed and the foundations needed to be set before going further.

6 Further Research

My decision to stop at EIP23 has to do with the fact that from EIP24[97] onwards, Spinoza starts talking about the relationship that God has with what has been left aside in his ontology–the finite modes. The very EIP24 is a hint about this transition since we get another proposition without a Q.E.D., together with the fact that its proof only relies on EID1. In EIP24, together with EIP25[98] and EIP26[99], Spinoza states the relationship of God with the existence, essence and determinacy of particular things, although he doesn't define what a particular thing is until the second part of the *Ethics*. The important question here is whether or not there is a connection between the infinite mediate mode and the finite mode. Anyone who has taken a peek into Spinoza's maze will notice that there is a discontinuity between the infinite and the finite and this discontinuity is found between EIP22 and EIP28. Let me explain this discontinuity: previously I explained my argument for the connection between God and the infinite modes found in EIP21 and EIP22, but now the important question is What is the connection between God and the finite mode? This relationship is not explained in the first part; nonetheless we have an explanation on the relationship between the individual things in EIP28, which is necessary to quote with its proof for this purpose:

> Every individual thing, i.e., anything whatever which is finite and has a determinate existence, cannot exist or be determined to act unless it be determined to exist and to act by another cause which is also finite and has a determinate existence, and this cause again cannot exist or be determined to act unless it be determined to exist and to act by another cause which is also finite and has a determinate existence, and so ad infinitum.
>
> **Proof** Whatever is determined to exist and to act has been so determined by God (EIP26 and EIP24, Corollary). But that which is finite and has a determinate existence cannot have been produced by the absolute nature of one of God's attributes, for whatever follows from the absolute nature of one of God's attributes is infinite and eternal (EIP21). It must therefore have followed from God or one of his attributes insofar as that is considered as affected by some mode; for nothing exists but substance and its modes (EIA1, EID3 and EID5), and modes (EIP25, Corollary) are nothing but affections of God's attributes. But neither could a finite and determined thing have followed from God or one of his attributes insofar as that is affected by a modification which is eternal and infinite (EIP22). Therefore, it must have followed, or been determined to exist and to act, by God or one of his attributes insofar as it was modified by a modification which is finite and has a determinate existence. That was the first point. Then again this cause or this mode (the reasoning is the same as in the first part of this proof) must also have been determined by another cause, which is also finite and has a determinate existence, and again this last (the reasoning is the same) by another, and so ad infinitum.

We see that Spinoza explicitly says in the proof that this kind of mode, i.e., individual things, cannot be determine to exist neither by the infinite immediate mode nor infinite mediate mode and therefore must follow from a finite thing and so on and so forth. Thus, formally we have a discontinuity between the infinite and the finite because there is no connection between infinite modes and finite modes; in other words, we know the origin of the existence of both infinite modes, but we do not know the origin of the existence of a finite mode, i.e., we have an undetermined backwards regression on an endless chain of finite modes that can never reach God–nonetheless, we can relate it to the infinite immediate mode. So the question arises again What is the connection between these two sides of Spinoza's ontology? We saw in chapter 2 that

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[97]The essence of things produced by God does not involve existence. *ibid.*, p. 232.

[98]God is the efficient cause not only of the existence of things but also of their essence. *ibid.*

[99]A thing which has been determined to act in a particular way has necessarily been so determined by God; and a thing which has not been determined by God cannot determine itself to act. *ibid.*

this question is one of the distinctions mentioned in the letter of the infinite. In that chapter we saw how time plays an essential role in the way in which we conceive the existence of the substance and the mode and how those ways define any other conception in which those elements are involved. I claim that the formal interpretation took us to a closer approach to the problem in hand, and gave us a way to start unlocking this problem in Spinoza's ontology.

We can understand this problem better if we treat it from the objective perspective and not just from the formal perspective. For instance, from the point of view of extension, we have movement and stillness, i.e., infinite immediate mode and the totality of the universe, i.e., infinite mediate mode. Now the important question is Where are finite modes, i.e., objects, placed in his ontology? We have two possible answers for this–in between or outside. The first is "Movement and stillness create individual objects and if we take the union of all of those mode, we get to the totality of the universe"; this is the finite objects are the transition between the condition of possibility of modifications of extension, and the totality of the modifications of extension; the second answer is, "the totality of the universe follows from movement and stillness, since this is nothing but the union of all possible movements and stillness and finite objects are just delimitations within this totality, so they are positioned outside the relation between the infinite modes". I claim that this discussion is an essential discussion to understand the two sides of Spinoza's philosophy–one developed in the first part of the *Ethics*, and the other developed in the second part of the *Ethics*, which correspond to the distinction between *Natura naturans* and *Natura naturata* and also, which I will explain now, is the most important one, to the distinction between the temporal point of view, i.e., between *sub specie aeternitatis* and *sub specie temporis*. The first answer to the question posted seems like the most reasonable one, but it has a major problem–it doesn't follow from the argument found in the *Ethics*. On the other hand, the second answer seems to be the right choice, but this entails a division on Spinoza's philosophy. Nonetheless, this division is not irreconcilable and, beyond that, they are complementary. In the same way, I discussed before that, the intellect and the imagination complement each other, and that the intellect has the capacity also to understand what the imagination performs–these two parts of Spinoza's philosophy work in the same way. This division is between the ontology and the epistemology of Spinoza's philosophy or, in other words, between the formal structure and the objective structure. Far from being a problem, this issue gives sense to the division of the *Ethics* and the duality that the set of relevant dichotomies represent. These problems are the very reason for the necessity of that duality on perspective, which is based on the two features used in the definition of individual things in EIID7:

> By individual things [res singulares] I mean things that are finite and have a determinate existence. If several individual things concur in one act in such a way as to be all together the simultaneous cause of one effect, I consider them all, in that respect, as one individual.

The two things that define an individual thing are *finitude* and *determinate existence*. I am going to explain now that this gap between the infinite and eternal world and the finite and temporal entails a change of view for the epistemological part of Spinoza's philosophy is not an irreconcilable division and that, on the contrary, is a necessary division on which the epistemology is grounded. Many interpreters agreed that the first part of the *Ethics* obtains a new relevance in the second part, and this is my own interpretation of that statement.

The human being is a finite thing which has a determinate existence, but the question is How can we deduce its existence from the notion of God? Apparently, based on what I have said, we cannot–although we know that it is nothing but a modification of God and that it is determined to exist and act by another finite modification of God. This is one of the points in which Spinoza separates from the Cartesian argument, and introduces a shift in the famous *Cogito, ergo sum*. For Spinoza, our existence is an evidence that cannot be doubted, but that doesn't mean that it doesn't have to be understood. Again, the problem we are dealing with here is How is the human existence deduced from the existence of God in Spinoza's philosophy?

In other words, How is the existence of a mode explained in which the two attributes of God are united? The answer is that this is not explained, but understood. As Spinoza claims: "from the fact that I know the essence of the mind, I know that it is united to the body[100]", or in EIIP13: "The object of the idea constituting the human mind is the body, i.e., a definite mode of extension actually existing, and nothing else.[101]" The union of these two attributes in the human being is a fact that comes from its present existence–no wonder Spinoza states as an axiom in the second part of the *Ethics*: "Homo cogitat[102]", contrary to Descartes, for Spinoza, we exist, therefore we think, because of our capacity to form ideas that follows from the fact that we are but a modification of the attribute of thought.

The first book of the *Ethics* represents an analytical argument that centers in the idea of God, in which Spinoza develops an ontology based on that being; the second part of the *Ethics* represents a synthetic argument on which the human existence is understood based on the formal structure given in the previous part. In the first part we take an eternal perspective on things, in which everything is taken as following from God, both infinite things and finite things although they are not defined in this first part nor is their existence proven; on the second part we take a temporal perspective on things as we center on the human perspective, but this perspective can only be understood if we interpret it from the first book. The synthetic argument in the second part consists in a process of understanding the particular mode of our mind and how it is united to our body in an unbreakable union. This process begins with the temporal perspective center in ourselves and how we are open to the objective world from that perspective. If the first part was centered on God, the second is centered on the human mind and it is when the temporal perspective and the notion of "present existence" and the intellect in act becomes important. Spinoza himself gives a hint by mentioning the finite intellect in EIP30: "The finite intellect in act or the infinite intellect in act must comprehend the attributes of God and the affections of God, and nothing else.[103]" The understanding of our existence is grounded in the two elements previously mentioned–determinate existence and finitude. These two features are the key to connect the first and the second part of the *Ethics*, and the key concepts that we will explore to set this transition. Future continuation of this work on the second part of the *Ethics* and a formalization of Spinoza's epistemology are duration, number and quantity.

The formalization process that we have followed in this work has given us some deep insights into some of Spinoza's problems and concepts with which are very hard to deal. As I claimed in the introduction, the formal perspective provides a way to connect the concepts of his philosophy together. I think that this is the biggest achievement of this work–the capacity to connect all elements of the formal language in the models–and to be able to trace back the philosophical sense that Spinoza gives them in his masterpiece. This achievement is embodied in the relationship between God, and the infinite modes, and how it is captured by the dependence graphs. I must admit that since the first time I laid eyes on the *Ethics*, EIP21, 22 and 23 were the most obscure ones, and the hardest to understand. The confusion that the use of the term *infinite*, and the pre-conceived notions of our time does not help with this problem at all, but my exploration and interpretation of the notion of *infinite* in Spinoza seems to be fruitful since it is the key concept that connects the system. Although it might seem that the exclusion of the notion of *attribute* is a huge drawback from this treatment, I claim that in the future expansion of the system they will be included and taken into account.

The models for this language,–he dependence graphs–are fundamental for the language, but they also play a vital role in the expansion of the system and the transition from the infinite to the finite. They are the embodiment of both the strength and the weakness of the formalization; they lack expressivity, but they add axiomatic consistency and soundness to the language and, even more important, they are the best outcome of this work for its future development of the language in which they will be treated as a model for the understanding of particular things.

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[100]TEI, §22. *ibid.*, p. 8.
[101]*ibid.*, p. 251.
[102]EIIA2. *ibid.*, p. 244.
[103]*ibid.*, p. 234.

For this very reason, we will need to include a temporal logic in our language in order to deal with the notions of *present existence* and *duration*, since it is required also in EIP28. Duration is defined in EIIA5 as:

> Duration is the indefinite continuance of existing.
>
> **Explication** I say *indefinite* because it can in no wise be determined through the nature of the existing thing, nor again by the thing's efficient cause which necessarily posits, but does not annul, the existence of the thing.

For this purpose we could build models for temporal logic based on the dependency graphs without altering the benefits obtained from it, but adapting them to the new field of particular things. As I said in the introduction, one of the goals of this work is to provide a solution to the problem of transition between the infinite and the finite– now I proceed with it. The solution to that problem I described above is not intended to be a way to solve that problem in Spinoza, but a way to work with it and to be able to include this problem as a step further in the process of formalization. As I claim there is no transition, there are two different aspects of the world depending on our capacities to access it. Existence of particular things is something we have to take as an *evidence* in Spinoza and not as a cosmological result of this ontology. Our existence is taken as an evidence that cannot be doubted, setting a fundamental difference with Descartes' philosophy–we do not even require a process to reach this certainty. Once we have accepted this claim, we can start to include particular things in our language and study them in relation to the infinite modes and God. We do not have to explain how one realm is originated from the other[104], but how they are understood through each other. Nonetheless, this formalization provides a way to deal with this thanks to the dependency graphs and the interpretation of the infinite modes. Also, when we start dealing with the realm of particular things, the notion of attributes are brought back into the language since now they are relevant for the understanding of finite modes and we will gain back that loss of expression. In the models presented in this work, there is no inclusion of finite modes for the reasons explained before–nonetheless, we do have a way to connect both realms. The infinite immediate mode represents the mode on which any particular mode depends and can even be taken as the simplest finite mode, e.g., *simplest bodies.* These finite models are taken to be elements of the infinite immediate mode, and studied in relation to each other, as they are included and they relate to each other within that mode. Thus, both infinite modes can be interpreted as a lower and upper bound, which follow from a recursive definition of particular things that will be given. In this way we can connect the two sides of Spinoza's philosophy, but, for a better philosophical insight of the problem, I will discuss now the relation of the infinite with the finite–this relation is based on the idea that *perspective bounds.*

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[104]Clarke 1998, Part XI, 81–83

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