The Philosophical Motivation for Proof-Theoretic Harmony

MSc Thesis *(Afstudeerscriptie)*

written by

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under the supervision of **Dr. Luca Incurvati**, and submitted to the Board of Examiners in partial fulfillment of the requirements for the degree of

MSc in Logic

at the *Universiteit van Amsterdam.*

Date of the public defense:	**Members of the Thesis Committee:**
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Abstract

This thesis presents, discusses, and evaluates the philosophical motivations for proof-theoretic harmony - one of the central concepts of logical inferentialism - and relates them to the corresponding formal notions. It will be argued that the principle of innocence manages the objections against the philosophical motivations for harmony in the most satisfying way. Since the principle of innocence is formulated regarding the deductive system as a whole, this strongly suggests that the formal harmony requirement needs to be a *global* one. The considerations regarding the corresponding formal notions endorse this view, in particular it will be shown how *local* constraints fail to rule out constants such as quantum disjunction and *bullet*, the proof-theoretic variant of the Liar sentence. The further aim of this thesis is to emphasize the role of the structural rules and the *context* of a rule, both made explicit by the sequent calculus, for the inferentialistic behaviour of a logical constant.

Contents

1 The Meaning of the Logical Constants

In the ideal world nations, friends, lovers, and music compositions are all in harmony; should logic, or the logical constants, be added to this list? This thesis is concerned with the just raised question, in particular it outlines and discusses the possible philosophical motivations for proof-theoretic harmony and their formal counterparts. The current chapter serves as a background for the remaining chapters.

1.1 The Background

The demand for harmony has its meaning-theoretic roots in a use-theoretic approach to meaning (Dummett 1973, 1991; Murzi and Steinberger 2015: 1). Contrary to referential, or truth based, approaches, the use-theoretic approach gives the practice and regularities a central stage in the order of explanation of semantic notions. Inferentialism is a specific interpretation of such a use-theory of meaning; regarding the use of an expression it gives a primary role to the *inferences* in which an expression can feature. Given this meaning-theoretic background it is possible to offer a harmony constraint for the language as a *whole* (see Brandom 1994, 2000). However, this thesis is just concerned with the logical fragment of the language. According to this view - *logical inferentialism* - the meaning of the logical constants is determined by the inferences in which a constant can feature.

Quite often the logical inferentialist adopts the position that the meaning of a logical constant λ is given by a specific set of inference rules: its introduction and elimination rules. Consider for example conjunction. According to the just sketched view its meaning is constituted by:

$$\frac{A \qquad B}{A \wedge B}\,\text{I-}\wedge \qquad\qquad \frac{A \wedge B}{A}\,\text{E-}\wedge \qquad\qquad \frac{A \wedge B}{B}\,\text{E-}\wedge$$

Here A and B are arbitrary formulas. It is a delicate issue *which* inferences of a logical constant λ are constitutive for the meaning of λ. For example Gentzen adopted the view that solely the introduction rules are meaning constitutive. What matters for now is that from an inferentialistic perspective the just presented rules for conjunction are self-justifying. This way, the inferentialist tries to avoid the problem, famously raised by Lewis Carroll (1895), to come up with a non-circular justification or explanation of the fundamental rules of logic like *modus ponens*. Since the rules for λ are self-justifying, they are not based upon some independent determined meaning of λ.

In the case of conjunction it seems quite natural to base the just presented inference rules upon the semantics of conjunction, given by, for example, the corresponding truth tables or a truth function. Definitely, this is not the strategy of the authors in the harmony debate; the inference rules for the constants are provided without an appeal to a prior semantic notion. In other words, it is from the current perspective possible to put forward any combination of rules to fix the meaning of a logical constant.

Traditionally, this is the stage where harmony enters the debate. The self-justifying character of the rules leads to the introduction of problematic constants. The next section introduces the most famous example: the connective *tonk*. Besides the association with *tonk*, the harmony requirement is often related to authors who have revisionary aims in logic. Among others, Dummett (1973, 1991) is a prominent example of someone who attacks classical logic in favour of intuitionistic logic by meaning-theoretic considerations and, ultimately, the requirement of harmony. Despite the prominence of the dispute between classical logic and intuitionistic logic, or realism versus anti-realism, this thesis is not primarily concerned with such revisionary issues.

1.1.1 The Problem: Tonk

Arthur Prior introduced in his *The Runabout Inference Ticket* (1960) the just mentioned connective *tonk*. It has the following introduction rule:

$$\frac{A}{A \ tonk \ B}\ \text{I-}tonk$$

In addition, the constant has the following elimination rule:

$$\frac{A \ tonk \ B}{B}\ \text{E-}tonk$$

Together with the transitivity of the deducibility relation these rules lead to the following situation:

$$\cfrac{\cfrac{A}{A \ tonk \ B}\ \text{I-}tonk}{B}\ \text{E-}tonk$$

In other words, once *tonk* and its corresponding rules are added to a language, it is possible to deduce any B once some arbitrary A has been established. Hence, the system to which *tonk* is added leads to triviality, since every B becomes provable. Prior's (implicit) suggestion is that it is a mistake to define the meaning of the connectives solely in terms of their rules; the project of the logical inferentialist fails. Stevenson (1961) made this suggestion explicit. He argued, in line of the traditional semantic approach, that the meaning of a connective has to be defined by a truth function in the meta-language.

Another approach is, contrary to Stevenson, to remain faithful to the idea that the meaning of the logical constants is given by their rules. This is where the harmony requirement comes in. The general idea is, first of all put forward by Belnap (1962), to come up with additional requirements to rule out problematic connectives like *tonk*. The rules for the constants which satisfy the harmony requirement are meaning conferring, whereas the rules for the other constants are disharmonious and thereby do not confer meaning.

1.1.2 The Motivation

Presented this way, the role of a formal harmony requirement is to select the meaningful constants, and in particular to rule out problematic constants like *tonk*. As just a cure for *tonkitis*, the demand for harmony seems quite *ad hoc*. Hence, it is one of the main goals of this thesis to outline and discuss the philosophical *motivations* for a harmony requirement, and to outline how a particular motivation has implications for the corresponding formal notion.

The formal notion of harmony is the most prominent aspect in the current harmony debate; it offers a precise specification of harmony by which it is possible to decide whether a constant or a deductive system is harmonious or not. On the other hand, the philosophical motivation for harmony is often just mentioned, and not widely discussed or defended. This turns the question about the relevance for a harmony requirement into a more urgent one. If it is not clear what the philosophical motivation is, and whether this motivation is correct or not, then one easily doubts the demand for harmony.

In order to start the presentation and discussion of the motivations for harmony and their corresponding formal notions it is necessary to settle some issues. The first section already indicated the main problem: which set of inference rules determines the meaning of the logical constants. Traditionally, the harmony debate is only concerned with the *operational rules*. These are the introduction and elimination rules which are specific for a logical constant. On the other hand the *structural rules* are not specific for a logical constant, but they are part of the deductive system as a whole. Since these rules do influence the kind of inferences which are allowed - the next section will discuss this at length - they seem to offer a threat to the traditional focus of the harmony debate on the operational rules.

Regarding the distinction between operational and structural rules the mode of representation seems, as Steinberger argues, to matter. In particular, one can distinguish between the setting offered by natural deduction and the one offered by the sequent calculus. Therefore, the plan is to start by presenting these proof systems. Once the mode of representation is made clear, a further discussion of the interplay between operational and structural rules is possible.

1.2 The Sequent Calculus

After Hilbert's axiomatic system, Gerhard Gentzen invented both natural deduction and the sequent calculus. These systems mark the beginning of proof theory as it is known nowadays. It will be assumed that the reader is familiar with natural deduction and the corresponding inference rules. Of course the latter depends on the system which is chosen, but when this choice matters the system or the inference rules which are used will be made explicit. Furthermore, it will be assumed that the reader is aware of vacuous discharge and multiple discharge in a natural deduction setting.

Since the sequent calculus is a more unusual setting an introduction is useful. Like natural deduction, the proofs in the sequent calculus are represented as trees, but contrary to the former system the sequent calculus has at each node *sequents* instead of formulas. Sequents often look like $\Gamma \Rightarrow \Delta$. The antecedent of this sequent is Γ, the consequent is Δ, and $\Rightarrow$ is just an arbitrary symbol to distinguish between the antecedent and the consequent. Moreover, the antecedent and the consequent are *multisets* of formulas, and not sets. Traditionally, the intuitive meaning of such a sequent is that the conjunction of all the formulas in Γ implies the disjunction of all the formulas in Δ (Gentzen 1935/1964: 290).[1] Both Γ and Δ can be empty, but usually they contain arbitrary sequences of formulas (idem).

For example, conjunction has the following operational rule to introduce it in the antecedent ("left") of a sequent:

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta}\ \text{L-}\wedge$$

In addition, the following rule introduces conjunction on the right:

$$\frac{\Gamma \Rightarrow A, \Delta \qquad \Gamma \Rightarrow B, \Delta}{\Gamma \Rightarrow A \wedge B, \Delta}\ \text{R-}\wedge$$

In a similar vein, the sequent calculus offers for each logical constant operational rules to introduce the constant on the left and on the right of a sequent. However, and here the sequent calculus becomes relevant for the present purposes, the sequent calculus contains some structural rules. The most prominent one is the *Cut* rule. In general, the rule is as follows:

$$\frac{\Gamma \Rightarrow A, \Delta \qquad \Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ \mathit{Cut}$$

[1] Some authors, in particular Restall (2005), would disagree with such an interpretation. According to him denial is prior to negation, so he offers the by Steinberger called *denial interpretation* of a sequent (2011c: 350). According to this interpretation it is incoherent to assert all the formulas in Γ while denying all the formulas in Δ.

The *Cut* rule captures, compared to the setting of natural deduction, explicit the transitivity of the consequence relation. In addition to the *Cut* rule, Gentzen (1935/1964) came up with three other structural rules for the sequent calculus. Each of these rules has two similar versions, one for the antecedent and one for the consequent of a sequent. The first rule is called *Thinning* or *Weakening*, and allows one to add arbitrary formulas on the right or the left of a sequent:

$$\frac{\Gamma \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta}\text{ L-}Weakening$$

Together with the version for the right hand side, *Weakening* corresponds to vacuous discharge for natural deduction (Steinberger 2009a: 38; Hjortland 2010: 173). Multiple discharge corresponds to the structural rules called *Contraction*:

$$\frac{\Gamma \Rightarrow \Delta, A, A}{\Gamma \Rightarrow \Delta, A}\text{ R-}Contraction$$

This is just the version for the right, the version for the left hand side is similar. The final structural rule has as well two similar versions, this is *Interchange* for the antecedent:

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, B, A \Rightarrow \Delta}\text{ L-}Interchange$$

Besides the operational rules for the logical constants and the just presented structural rules the sequent calculus contains axioms. The standard axiom is *Identity*, which boils down to $A \Rightarrow A$ and it mirrors the reflexivity of the consequence relation. *Identity* is almost always the starting point of a derivation in the sequent calculus.

So far so good. By providing independent structural rules, instead of incorporating them into the discharge policies of, for example, implication, the sequent calculus allows to distinguish more strictly between operational and structural rules. Steinberger uses this observation to adopt the view that the structural rules do not matter for the meaning of the logical constants (2009a: 39-40). They are part of the broader deductive system and not, as natural deduction suggests, part of the operational rules for the logical constants.[2]

Before the issue of the structural rules can be discussed in full detail, the so called "context" of a sequent should be taken into account. Consider, for example, the just presented operational rule for conjunction on the left. It introduced in the conclusion sequent the formula $A \wedge B$, which is

[2]Restrictions on the structural rules or the "context", which is explained below, lead to substructural logics. Two familiar examples are Anderson's and Belnap's Relevance Logic (see, for example, 1962) and Girard's Linear Logic (1987).

thereby defined as the *principal formula* (Dicher 2016a: 729). Furthermore, the *active formulas* of an operational rule are the principal formula and the formulas which are used to introduce the principal formula. The other formulas are the *passive formulas*. In the case of conjunction on the left the active formulas are $A \wedge B$, A, and B, whereas the formulas occurring in Γ and Δ constitute the passive formulas.

The latter are called the context of a rule, or simply the context. Notice, first of all, that the context does matter. In particular, Gentzen showed how one can obtain intuitionistic logic instead of classical logic by the restriction that at most one formula is allowed to stand on the right hand side of a sequent.[3] For the present purpose the urgent question is whether the context of a rule is as well part of the meaning of the considered constant. Some authors (Paoli; Restall) adopt the view that issues regarding the context are about the whole deductive system. Hence, according to such a view the context does not (even partly) determine the meaning of the connectives.

Quite recently, Dicher (2016a, 2016b) has put forward a more nuanced analysis of the interplay between the context, operational, and structural rules. He distinguishes, regarding the context, between minimal structural requirements which are needed to provide a proper meaning for the considered connective, and supplementary structural properties to account for the (potential) interaction with the other connectives of the deductive system (2016a: 754). The former is an integral part of the meaning of a connective, and thereby the connective imposes a constraint upon the way the context of a deduction is defined. The latter kind of properties are not intrinsic for a connective, so this part of the context is not meaning constitutive.

Consider, for example, disjunction. According to Dicher's analysis, disjunction does not need an additional context to give it a proper meaning (idem: 741). In other words, the following rules (called $\times$) capture the intrinsic meaning of disjunction (Dicher 2016b: 596):

$$\frac{A \Rightarrow B}{A \Rightarrow B \times C}\ \text{R-}\times \qquad \frac{A \Rightarrow C}{A \Rightarrow B \times C}\ \text{R-}\times \qquad \frac{B \Rightarrow A \qquad C \Rightarrow A}{B \times C \Rightarrow A}\ \text{L-}\times$$

These rules do not need a context, usually represented by Γ and/or Δ, to capture the intrinsic meaning of the connective. Compare now the just presented rules ($\times$) with the standard (classical) rules for disjunction ($\vee$) in the sequent calculus:

$$\frac{\Gamma, A \Rightarrow \Delta \qquad \Gamma, B \Rightarrow \Delta}{\Gamma, A \vee B \Rightarrow \Delta}\ \text{L-}\vee \qquad \frac{\Gamma \Rightarrow A, B, \Delta}{\Gamma \Rightarrow A \vee B, \Delta}\ \text{R-}\vee$$

[3] Due to Hacking (1979) this observation is called a "seemingly magical fact". For a more recent discussion of this issue, see Milne (2002).

These rules do indeed contain a context, so they allow more formulas on both the left and right and side of the sequences. Thereby it is for example possible to have just one rule to introduce disjunction on the right hand side. Since, according to Dicher's view, $\times$ captured already the intrinsic meaning of disjunction, the occurrence of multiple formulas on the left and right hand side of the sequent has for disjunction just an external, structural function. The role of these formulas is to account for the way disjunction can interact with the other constants in the deductive system.

Of course one can vary the structural context of the rules for a connective. For example, one obtains the rules for quantum disjunction if the left hand side rule for standard disjunction is restricted by not allowing collateral assumptions in the minor premises; Γ needs to be empty (Steinberger 2011b: 278).[4]

1.2.1 Operational and Structural Meaning

Recall the core claim of logical inferentialism: the meaning of the logical constants is given by the inference rules who govern their use. As already encountered, Steinberger, and almost everyone in the harmony literature, adopts the position that solely the operational rules are constitutive for the meaning of the constants. However, the structural rules influence, beyond doubt, the inferential use of a constant (Hjortland 2010: 177).

It is, for example, not possible to derive $A \to (B \to A)$ without vacuous discharge or $Thinning$. Another example is the distributivity of conjunction over disjunction; one needs, even in a rich context, $Weakening$ and $Contraction$ (Dicher 2016b: 597). Hence, the obvious question is why the structural rules are not as well relevant for the meaning of the connectives.

The immediate answer to the latter question is to appeal to Paoli's distinction between the *operational meaning* and the *global meaning* of a logical constant λ (Paoli 2003: 537; Hjortland 2010: 168). The operational meaning is given by the specific rules for λ: the introduction and elimination rules. On the other hand the global meaning is given by the class of sequents containing λ.

One can phrase the debate whether the global meaning is as well relevant for the connectives in the standard meaning-theoretic terms of *atomism* and *holism*[5] (Steinberger 2009a: 219; Hjortland 2010: 159-160). According to atomism operational meaning is all there is, whereas holism

[4] It will turn out that quantum disjunction has a pivotal rule in the current harmony debate. However, it should be emphasized that it is just *called* quantum disjunction. In other words, the discussions in this thesis have nothing to do with quantum logic or quantum theory.

[5] Hjortland usus Gentzianism and Hilbertianism respectively; it boils down, in general, to the same positions.

states that all the rules of a deductive system contribute to the meaning of the logical constants: a constant has an overall meaning which consists of both the operational and the global meaning (Hjortland 2010: 160).

Steinberger rejects holism, according to him it is implausible that a constant has an overall meaning. His reason is that an overall meaning implies that in order to grasp the meaning of a constant all the deductive inferences in which the constant is involved should be taken into account (2009a: 49). This sounds indeed as an exorbitant requirement. Steinberger adopts the view that it is sufficient to grasp the "core inferential use" of a constant, given by the operational rules. Accordingly, he rejects the holistic position.

However, Steinberger concludes as well that there is no decisive argument in favour of logical atomism (2009a: 226). The principle of separability is the underlying idea of logical atomism. According to this principle the inference rules governing the deductive behaviour of a connective should only mention the considered connective, and no other connectives (Steinberger 2011a: 630). Hence, by the inferentialistic assumption the meaning of a logical constant is purely *local*.

Interestingly, Steinberger does not adopt the principle of separability (2011a: 631). According to him molecularism, adopted by both Steinberger and Dummett, is not sufficient to argue in favour of separability. Molecularism solely requires that the logical constants are together separable from the language such that they constitute a semantic cluster. Thereby it does not imply that all the logical constants must be separable from each other.

The problem of the just presented positions seems that they are too extreme. Intuitively, contrary to atomism, the fact that $A \to (B \to A)$ is provable by vacuous discharge in classical logic - or by $Weakening$ in the sequent calculus - and not in relevance logic, seems relevant for the meaning of classical implication. Contrary to holism, as it is presented by Steinberger, the following derivation, which uses again $Weakening$, is intuitively not relevant for the meaning of conjunction:

$$\dfrac{\dfrac{p \wedge q \Rightarrow q}{p \wedge q, r \Rightarrow q}\text{ L-}Weakening}{(p \wedge q) \wedge r \Rightarrow q}\text{ L-}\wedge$$

If the intuition just described is correct, then the obvious but difficult task is to distinguish between the inferences which are relevant for the meaning of a constant and the inferences which are irrelevant. Definitely, the atomist would answer that the operational rules offer such a distinction: these rules constitute the meaning of a connective.

On the other hand, the *reasonable* holist would state that there is no such a strict distinction, but each constant has a scale of more and less relevant inferences. For example, on the scale for conjunction the usual introduction rule is the most relevant inference for its meaning. In the case of implication the most important rule seems to be *modus ponens*. However, the inference of $A \to (B \to A)$ is relevant as well for the meaning of implication in a classical setting, although it has not the same meaning-constitutive status as implication elimination.

The advantage of this proposal is that still some inference rules play an important, or decisive, role for the constitution of the meaning of the connectives. On the other hand it uses the straightforward insight that other rules, in particular the structural ones, do play a role in the inferential behaviour of a connective, without being committed to the position that every inference containing a constant λ is relevant for the meaning of λ. The obvious disadvantage is that the distinction offered by the scale of a constant is a vague one; it does not offer such a strict distinction as atomism.

Given the view of the reasonable holist, this thesis does not aim to rule out atomism. It just aims to sketch a plausible alternative such that there is even more pressure to include the role of the structural rules and the context in the discussion of the harmony debate. Probably it turns out that in the end atomism is the correct view. However, this cannot be simply assumed, and it is not a settled issue. Although it is not yet settled, the next section aims to make some distinctions more precise, in order to use the terminology for the remaining chapters.

1.2.2 Global and Local Constraints

By the observation that structural rules determine partly the inferential behaviour of a connective it seems plausible that they will play a role in the remaining chapters. Given that the notion of harmony should be made precise, it needs to be clear whether it is as well concerned with the structural rules or not.

In order to do this the *local-global* distinction is adopted. A harmony requirement is called *local* if it is just a restriction on a pair of inference rules; the introduction and elimination rules for a particular constant. On the other hand a harmony requirement is completely *global* if both the structural rules and the rules for other constants are used in order to check whether a constant is harmonious or not. Finally, to complicate the distinction, a harmony requirement is *semi-global* if it just contains structural rules, and no rules for other constants are needed. The remaining chapters offer examples of all these three versions. Moreover, they check, if necessary, whether the philosophical motivation for harmony implies a global or a local harmony constraint. Notice that global or semi-global requirements do not intend to provide a criterion whether the structural rules are as such harmonious or not. These requirements just indicate that the structural rules are needed to check whether a constant is harmonious or not.

1.3 Structure of the Thesis

The structure of the next three chapters is quite straightforward. Each chapter presents a philosophical motivation for harmony, discusses, and evaluates it. Subsequently, it is related to the corresponding formal notion. Moreover, the strengths and weaknesses of the formal notions are outlined. The final, fifth, chapter has a concluding character. It aims to bring the observations of this project together and fit them into a corresponding analysis, discussion, and final conclusion. The next chapter, on the principle of innocence, is the first step towards it.

2 The Principle of Innocence

The current chapter discusses the principle of innocence as a philosophical motivation for harmony. First of all the principle is introduced and related to Steinberger's idea of harmony. The second section introduces the formal notion of a conservative extension in order to evaluate Rumfitt's objection against Steinberger's harmony. Furthermore, the astronomy and the truth predicate argument, meant to reject the principle, are discussed as well. Finally, the formal counterpart is presented, and its shortcomings are identified.

2.1 Innocence and General Harmony

The idea behind the principle of innocence is that logic should not affect the non-logical regions of the language:

> "*it should not be possible, solely by engaging in deductive reasoning, to discover hitherto unknown (atomic) truths that we would have been incapable of discovering independently of logic.*" (Steinberger 2011a: 619-620).

The principle should guarantee that the application of logical inference to non-logical sentences does not lead to the assertion of unjustifiable non-logical sentences. Hence, the principle guarantees according to Steinberger the correct applicability of logic to non-logical expressions (idem: 620).

The harmony requirement is, as Steinberger argues, a natural way to secure the innocence of logic. The idea is to guarantee innocence by fixing the meanings of logical constants in the right way (idem: 620). To understand Steinberger's line of reasoning, it is necessary to sketch his meaning-theoretic assumptions. He adopts both Dummett's use-theoretic approach and the corresponding two-sided model of meaning (idem: 617). According to this model the meaning of a statement is given by its I- and E-principles (idem: 618). The I-principles are based upon in a verificationist theory of meaning, and offer the conditions to assert a sentence (Dummett 1973: 221). The E-principles are based upon a pragmatist theory of meaning and indicate the consequences of a statement.

By these meaning-theoretic assumptions, one obtains, according to Steinberger, the right meaning of a logical constant by providing an equilibrium between the introduction and elimination rule. The introduction (I) rules represent the I-principles, whereas the elimination (E) rules capture the E-principles of the two-sided model of meaning. Such an equilibrium between rules contains two aspects. On the one hand the elimination rule should not derive more than is allowed by the introduction rule, on the other hand the elimination rule should be able to exploit all the inferences which are allowed by the introduction rule. This idea of a balance between the introduction and elimination rule of a given constant is defined as *general harmony*.

Since the goal is to obtain a balance, there are two ways to disturb it, so a logical constant can be disharmonious for two reasons. If the elimination rule allows, compared to the introduction rule, too many inferences then the rules are in *E-strong disharmony* (idem: 621). If the elimination rule allows too few inferences, then the constant suffers from *E-weak disharmony*.[6]

Notice that in order to guarantee the innocence of logic it seems solely needed to rule out the E-strong disharmonious constants. By this kind of disharmony, one is able, by logic, to discover more (atomic) truths than one should be able to. Hence, Steinberger's need to rule out E-weak disharmony is, just by the principle of innocence, superfluous. On the other hand, Steinberger argues that logic offers indirect grounds for the assertion of non-logical sentences: by a correct deduction one can assert a non-logical sentence from a set of premises (idem: 619). Once the elimination rules are too restrictive, logic offers too few indirect grounds to assert non-logical sentences. Or, to phrase it in terms of Steinberger's meaning-theoretic assumptions, an elimination rule which suffers from E-weak disharmony fails to capture the pragmatist aspect of the two-sided model of meaning.

One might object, as Steinberger admits (idem: 621), that the assertion can be made by other means than deductive inference. For example, consider the connective $\circ$ with $A, B \vdash A \circ B$ as introduction rule, and as elimination rule just $A \circ B \vdash B$.[7] Clearly, this constant suffers from E-weak disharmony; A is needed to introduce the connective, but it is not attainable by the elimination rule. However, consider the following situation:

$$\dfrac{\dfrac{\dfrac{\begin{matrix}\Pi_1\\ A\end{matrix} \qquad \begin{matrix}\Pi_2\\ B\end{matrix}}{A \circ B}\,\mathrm{I}\circ}{B}\,\mathrm{E}\circ \qquad\qquad \Pi_1}{A, B}$$

By simple repeating the proof of A, which is Π_1, one can still obtain, in a purely deductive way, the grounds to introduce the connective. It should just be allowed by the deductive system to repeat premises in a proof. Steinberger might object that the meaning of $\circ$ is still mistaken, since there is

[6]Strictly speaking there are four ways a constant can be disharmonious; the disharmony can also be formulated in terms of the introduction rules. Steinberger does not give priority to introduction rules or elimination rules so this thesis uses just E-strong and E-weak disharmony, without taking a stance about the importance of elimination rules compared to the introduction rules, unless indicated otherwise (Steinberger 2011a: 621-622).

[7]By the sign $\vdash$ this thesis indicate that the right hand side is deducible from the left hand side of it. It marks the deducibility relation. In this case, B is deducible from $A \circ B$.

not a balance between its I and E principles. From his two-sided model of meaning perspective this sounds correct, but it should be emphasized that it is solely by his meaning-theoretic assumption that the principle of innocence should rule out E-weak disharmony as well.

Furthermore, Steinberger's preference for a *local* harmony requirement strongly influences the way the principle of innocence is related to *general harmony*. The latter is, by the requirement of a balance between each pair of inference rules, a local requirement, whereas the principle of innocence does not imply a local notion. The principle is formulated about logic as a whole, so a *global* notion seems to be sufficient to guarantee it.

To conclude, if one adopts just the principle of innocence, and not Steinberger's meaning-theoretic assumptions, then it seems to be sufficient to come up with a global requirement which just rules out E-strong disharmony. In line of this observation, the next section presents the global requirement of *conservativeness*.

2.2 Conservativeness

The role of conservativeness, or a conservative extension, goes back to Belnap's reply to Prior's *tonk* challenge (Belnap 1962). Belnap argued that *tonk* fails as a connective since it produces a non-conservative extension of the original logical system. The notion of a conservative extension is made precise as follows:

> **Systematic Conservativeness:** Let L and L' be languages with I and I' being their corresponding deductive systems. Moreover, $L' = L \cup \{*\}$, where $*$ is a new logical operator with corresponding deduction rules which are added to the deductive system. The language L' is then a conservative extension of L if for a sequent $\Gamma \Rightarrow B$ in the language L it is the case that $\Gamma \vdash_{I'} B$ only if $\Gamma \vdash_I B$.

The definition is due to Steinberger (2011a: 624), and it is distinguished from other, slightly different definitions, since systematic conservativeness is explicitly formulated for extending the logical system. More informally, conservativeness requires that by adding a logical operator to the language, nothing new about the old part of the language should become provable.

Clearly, conservativeness rules out *tonk*. In a standard deductive system, take for example a standard deductive system for classical or intuitionistic logic such as formalized by van Dalen (2008), it is definitely not possible to prove for an arbitrary A and B that $A \vdash B$. Since this is exactly what *tonk* allows, adding it to a standard system would lead to a non-conservative extension of the existing language.

Belnap emphasizes that conservativeness is context dependent (1962: 133). Whether the addition of a connective to a language leads to a conservative extension depends on the logical system to which it is added. This can lead to the situation that in one case a connective is conservative, and in the other case it produces a non-conservative extension of the system. Such a situation does not seem ideal, since it would depend upon the context whether a constant confers meaning or not.

One obvious way to solve this issue is to come up with a standard base system. This system would serve as the standard background to check whether a particular constant leads to a conservative extension or not. Belnap's logical base system contained just the atomic sentences and structural assumptions, an approach which is recently adopted by Dicher (Belnap 1962: 132; Dicher 2016a: 749).

Such an approach leads to a *semi-global* version of conservativeness which considers the constant in isolation, and not how it is related to the other (operational) rules in the deductive system (Dicher 2016a: 748-749). Belnap's base system consists of the structural rules *Cut*, *Identity*, *Contraction*, and *Weaking*, whereas Dicher just adopts *Cut* and *Identity*. *Contraction* and *Weakening* are strongly related to the *context* of a rule, which is, according to Dicher's analysis as presented in the previous chapter, not always purely part of the meaning of a connective. Hence, these rules are not included in the base system which is used to check whether a constant is conservative or not.[8]

Notice that such a localized and stabilized version of conservativeness should not be identified with systematic conservativeness. For example, adding classical negation to the implicational fragment of classical logic leads to a non-conservative extension (Steinberger 2009a: 57). On the other hand, classical negation is perfectly compatible with the stabilized version.[9] Quantum and standard disjunction offer another example. Both constants are conservative if just the semi-global version is used. On the other hand, as will be explained later on, adding standard disjunction to a system which contains both quantum disjunction and conjunction results in a non-conservative extension.

Steinberger rejects the stabilized version of conservativeness because it does not guarantee the principle of separability (2009a: 57). The example just provided illustrates this point. Adding standard disjunction leads to a non-conservative extension: new logical theorems about the old language become provable. Hence, the meanings of the old vocabulary have been effected and this implies, according to Steinberger, that the meanings of the original constants (in this case conjunction and quantum disjunction)

[8]Unfortunately, and as already mentioned in the first chapter, this thesis will not enter the debate *which* structural rules are harmonious, and which(sub)set of them should be included in the base system.

[9]Here this thesis can just guess, but this might be the reason why systematic conservativeness is more prominent in the harmony debate, since the authors have often strongly revisionary aims.

were not fully determined by their inference rules.

This rejection raises two objections. First of all, as the previous chapter mentioned, Steinberger simply assumes the principle of separability (idem: 58). Moreover, more recently he stated that a decisive argument for separability is lacking (2011a: 631). Given these observations, it seems exorbitant to reject conservativeness solely by separability. Secondly, it might be questioned whether it is indeed the (intrinsic) *meaning* of the constants which is affected by adding standard disjunction to the system. However, this point will be discussed in the concluding chapter.

2.2.1 Rumfitt on Conservativeness

Rumfitt argues, contrary to Steinberger, that the conservativeness requirement is sufficient to guarantee the innocence of logic (Rumfitt 2016: 24). As already mentioned, to guarantee just the innocence of logic it seems sufficient to come up with a global requirement which rules out E-strong disharmony. This is, in a nutshell, Rumfitt's point. Since conservativeness prevents against E-strong disharmony, no additional requirement is needed to secure the innocence of logic.

On the other hand, Steinberger rejects the conservativeness requirement because it fails to rule out E-weak disharmony (2011a: 624). Consider the connective $\circ$, presented in the section 'Innocence and General Harmony'. Adding it to a base system with just atomic sentences and structural rules it clearly produces a conservative extension. However, the constant is E-weak disharmonious so conservativeness alone cannot prevent against this type of disharmony.

In addition, Steinberger opposes conservativeness as a harmony requirement since it is a global requirement, whereas he is looking for a local one (idem: 625). Steinberger states that harmony is a property which is ascribed to a pair of inference rules of a connective, and the global character of conservativeness fails to capture this idea. It seems that he simply assumes that a purely local requirement is the right way to guarantee innocence:

> "*The best way to do this (at a local level) is by requiring that the introduction and elimination rules that govern the meanings of the logical constants be exactly commensurate in strength*" (Steinberger 2011a: 620).

According to this quote the local level is not defended, but simply, by putting it into brackets, assumed as the right level for harmony. Definitely, Steinberger is correct that conservativeness fails to rule out E-weak disharmony and that it is a (semi) global requirement. However, these two criteria are not implied by the principle of innocence, but only by Steinberger's further meaning-theoretic assumptions. Thereby Rumfitt's observation that the principle of innocence as such is guaranteed by the conservativeness requirement is correct. However, for the sake of the argument Steinberger's line of reasoning will be followed, so E-weak disharmony is still included in the discussion.

2.3 Arguments against Innocence

The previous sections simply assumed that the principle of innocence as such is correct. However, Steinberger is aware that the principle of innocence is not beyond criticism, since he admits in a footnote that the innocence of logic raises the question how the usefulness of logic should be explained if it does not deliver new knowledge (2009a: 60). Unfortunately for the purpose of this thesis, Steinberger does not discuss the question.[10] Notice that Steinberger's remark that the principle of innocence implies that logic does not deliver new knowledge is incorrect. The sole problem is that it cannot deliver new knowledge which is (in principle) not attainable by non-logical means, but this does not imply that logic cannot lead to new knowledge at all.

Regarding the status of the principle of innocence it is remarkable that the principle is not widely discussed. For example Murzi and Carrara just take it for granted, according to them it is a "common motivation" for the harmony requirement (2014: 19). Another example of this approach is Griffiths (2014). Contrary to this view, the current section discusses two objections, raised by Rumfitt and Read, against the innocence of logic.

2.3.1 The Astronomy Argument

Rumfitt provides a counterexample which aims to show that logic can deduce conclusions for which it is not possible to provide, even in principle, direct evidence (2016: 23). For further reference the example provided is called *the astronomy argument*, and the argument runs as follows:

> **P1**: By astronomical theory and appropriate observations Rumfitt knows that a body B is either in region R of the Andromeda Galaxy or it is in a black hole.
>
> **P2**: By some further observations Rumfitt knows that B is not in R.
>
> **C**: It can be deduced that B is in a black hole (Rumfitt 2016: 23-24).

[10] The question whether logic is useful was addressed by Cohen and Nagel (1934). According to their *paradox of inference* a deductive inference cannot be both valid and useful (idem: 173). Bar-Hillel and Carnap (1953) tried to solve this by their theory of (empirical) semantic information. In Hintikka's view they did not succeed, and he called this the *scandal of deduction* (1973). Furthermore, the theory of semantic information led to the Bar-Hillel-Carnap paradox. For more recent literature on these issues see for example D'Agostino and Floridi (2009) and Duží (2010).

Since it is not possible to discover, even in principle, the conclusion by direct observations the example shows, according to Rumfitt, that it is indeed possible to discover by deduction hitherto unknown atomic truths (idem: 24). More importantly, there are no other ways to discover these truths than by deductive reasoning. Rumfitt concludes solely from this example that the principle of innocence is too strict and that it does not correspond with the way logic is actually used.

First of all it should be noticed that the conclusion of the astronomy argument is not reached "solely by engaging in deductive reasoning". P1 is partly based upon empirical observations, and P2 is purely based upon observations. It is correct that only by deduction it is possible to derive 'C', but the argument is more empirical than, for example, *tonk*. In the latter case there is no specific observation needed; it is sufficient to pick an arbitrary A in order to derive an arbitrary B. On the other hand, the astronomy argument is based upon a specific empirical observation.

The core problem of the astronomy argument is that it is purely the specific content of P1, in particular the notion of a black hole, on which the strength of the argument is based. Because of some of the characteristics of a black hole, it is by definition impossible to discover directly information about what it might contain. Hence, the fact that knowledge of the conclusion of the argument is, even in principle, not directly attainable is already contained in one of the premisses.

The question is - without delving too much into astronomy theory, which is clearly beyond the scope of this thesis - whether the knowledge that a region R is in a black hole is indeed knowledge about the *world*. The astronomy argument seems, to a certain degree, similar to a case in which the premises are about fiction. If logical reasoning is applied to the latter case the expected conclusion will be on fiction as well. Even in this case deduction leads to a conclusion which cannot be discovered directly, without logic, but it seems not justifiable to state that the deduction enlarges the knowledge about the world.

More in general the question is whether a deduction which contains one or more premises whose constituents are (in principle) not directly discoverable is indeed a counterexample against the principle of innocence. In particular in the case of deductive reasoning the content of the conclusion of an argument is, by definition of logical consequence, so strongly based upon the content of the premises. The main objection against this line of reasoning is some kind of dogmatism about the principle. In order to defend the principle one can adopt the strategy that every supposed counterexample is after all not about the world, hence not a counterexample against the principle. This strategy would turn the principle of innocence into a dogmatic, not falsifiable notion.

In addition, one might wonder how Rumfitt knows that P1 is indeed the case. By P2 it is clear that Rumfitt cannot know that B is in region

R of the Andromeda Galaxy. On the other hand it also quite unclear how Rumfitt might be able to know that B is in black hole. Hence, it is not a surprise that starting from non attainable knowledge leads to a conclusion about non attainable knowledge as well. To conclude, the current section aimed to show that the astronomy argument alone is not sufficient to reject the principle of innocence.

2.3.2 The Truth Predicate Argument

The starting point of Read's criticism is Prawitz's observation that the addition of a harmonious pair of inference rules does not always lead to a conservative extension of the system to which it is added (Read 2016: 411; Prawitz 1994: 374). More specific, Prawitz provided the example of the addition of the Tarskian truth predicate to Peano arithmetic. By Gödel's incompleteness theorem, the result is a non-conservative extension of the original system. The thought behind this example is that logic can indeed create new content, so in line with Rumfitt's astronomy argument Read aims to show that the principle of innocence is too strict.

As Steinberger points out, the strength of the example lies in the rules governing the truth predicate (2011a: 635). The thought is that whatever formal (local) harmony requirement will be used, the rules for the truth predicate will satisfy these requirements. The rules for the *T-schema* are quite simple: $A \vdash Tr(A)$ is the introduction rule and $Tr(A) \vdash A$ is the corresponding elimination rule. However, Steinberger criticizes Read and Prawitz's counterexample because it is, according to him, not the addition of the truth predicate and the corresponding inference rules which results in a non-conservative extension.

The result of just adding the truth predicate and its inference rules is a conservative extension of Peano arithmetic (Ketland 1991: 76 and Halbach and Leigh 2014[11]). By the addition of the entire Tarskian theory the consistency of Peano arithmetic becomes provable which is, by Gödel's second incompleteness theorem, definitely not provable in the original language of just Peano arithmetic. The point is that the counterexample provided by Prawitz and Read does not work, because the non-conservativeness is not due to the introduction and elimination rule for the truth predicate (Steinberger 2011a: 635-636).

Although Read's rejection of innocence is just based upon a mistaken counterexample, his final remark regarding the principle is worth mentioning. According to Read two ideas have dominated the views in philosophy about logic in the twentieth century (2016: 412). The first one is that logical consequence is just formal, and the second one is that logic is empty. Both ideas are according to him connected to each other, since they share

[11]This is the most recent entry in the SEP, this is the reason why it is more recent than Steinberger's own article.

the view that the non-logical terms contain all the content and that logic has no content at all. This is, according to Read, the genuine, and mistaken, basis of the principle of innocence.

These claims are, unfortunately, not further spelled out by Read. Only the statement that logic is empty is accompanied by a quote from Wittgenstein's *Tractatus*. However, without delving too much into these issues, it is clear that these broad statements cannot just be taken for granted. First of all it should be mentioned that, although Wittgenstein was definitely influential, there are other views on logic in twentieth century philosophy. For example Quine's view that logic is the most inner part of the holistic web of belief (1951; 1960), and Putnam's more extreme view that logic is empirical (1969).

Furthermore, the statement that logic is formal is far too general, for example Dutilh Novaes (2011) presents different ways in which logic can be formal.[12] More importantly, one might wonder whether the fact that logic is formal implies indeed that logic has no content. Frege argued for example that logic provides indeed content about the natural numbers (MacFarlane 2002: 29). However, as MacFarlane explains, logic is according to Frege not purely formal in the Kantian sense of it, which means that logic is completely abstracted from the semantic content (idem: 28). The latter means that the semantic content of concepts is for logic completely indifferent such that logic is "unrestrictedly formal" (idem: 29). The point is, as MacFarlane (2000) outlines as well, that there are, besides the Kantian notion, several ways in which logic can be formal. Hence, Read should make the claim that logic is formal more precise in order to claim that the formality of logic implies that it has no content.

Ultimately, Read does not seem to care that much about the motivation for the harmony requirement:

> "*What has to be accepted is that, although their ultimate aim is the same, namely, an account of the meaning of logical constants in purely proof-theoretical terms, different authors have different conceptions of harmony*" (2016: 412).

His rejection of the principle of innocence boils down to the by Steinberger already disproved truth predicate argument. Therefore, Read does not succeed in his aim to reject the principle of innocence.

2.3.3 Evaluation

It might be useful to sum up some of the main elements of the two previous sections. First of all the astronomy argument is too questionable to reject

[12] In addtion MacFarlane's PhD thesis (2000) is the most natural starting point for an overview of what is meant by the claim that logic is formal.

the principle of innocence solely by this counterargument. The truth predicate argument was simply incorrect. In other words, it seems that there is not yet a decisive argument to reject the principle. Secondly, it has been argued that the principle of innocence can be captured by a global harmony requirement which prevents against E-strong disharmony. The notion of a conservative extension satisfies these requirements, but Steinberger rejected it by his additional meaning-theoretic assumptions. Therefore, the next section delves further into this aspect; it explores what Steinberger actually proposes as a formal harmony requirement.

2.4 Steinberger's formal counterpart

Clearly, Steinberger's formalization is a local one. Furthermore, it should prevent against both E-weak and E-strong disharmony. Steinberger indicates more explicit how a formal account of harmony should look like:

> "*Our discussion strongly suggests that ultimately an adequate formulation of harmony will have to be a local constraint that must incorporate an account of stability so as to entail normalizability*" (2011a: 639).

In order to make this quote precise, and Steinberger's formalization as well, some conceptual clarification is needed.

2.4.1 Intrinsic Harmony and Normalization

Before normalization can be introduced some other formal notions need to be defined. Steinberger follows Dummett's formulation, so this will be done here as well. First of all let λ be a logical constant. A *local peak* for λ is a part of a deduction where the introduction rule for λ is followed immediately by the elimination rule for λ (Dummett 1991: 248). If there is such a local peak one tries to apply a *levelling* procedure. To *level* a local peak a deduction should be provided in which the premises for the introduction rule of λ lead to the conclusion of the elimination rule for λ, but without using the rules for λ (idem: 248). More specific, the conclusion of the introduction rule and the major premiss of the elimination rule is called a (λ-) *maximum*, and it is this sentence which is eliminated by the levelling procedure (Steinberger 2011: 626). For example, this is the local peak for conjunction:

$$\dfrac{\dfrac{\begin{matrix}\Pi_1 \\ A\end{matrix} \qquad \begin{matrix}\Pi_2 \\ B\end{matrix}}{A \wedge B}\, I\wedge}{A}\, E\wedge$$

Obviously, the detour via conjunction introduction and elimination is superfluous, since A was already proved. Hence, the local peak for conjunction can be levelled or eliminated (of course B could be as well the conclusion of the local peak). If it is possible to level a local peak for a constant, then the constant satisfies Dummett's requirement of *intrinsic harmony* (1991: 250). It is a local requirement since the levelling procedure for a local peak is done in isolation, so it is not needed to take the rules for other constants into account.

The thought behind the normalization theorem is quite similar to the levelling procedure for local peaks. The goal of normalization is to show that in a proof detours are avoidable such that there is a direct route from the premisses to the conclusion (Steinberger 2011a: 627). More precise, a proof is in *normal form* if no further reduction procedures can be applied to it (idem: 627-628). The other reduction procedure, beside the levelling procedure, is called a *permutative reduction* (idem: 627). By this procedure it is possible to rearrange the order of the application of inference rules in the proof such that a local peak can be created. Once there is such a local peak the usual levelling procedure can be applied to it. Now a proof system is *normalizable* if every proof of it can be turned into normal form (idem: 628). Contrary to intrinsic harmony, normalizability is a global property since it depends upon the combination of inference rules within a system. Therefore, the normalization requirement alone is from Steinberger's perspective not the correct one for harmony.

However, Steinberger argues that normalizability would be the best global requirement for harmony:

> "*It guarantees, therefore, that there can be no semantic spill-over from the logical to the non-logical regions of language. In other words, it guarantees the requirement of innocence*" (idem: 632).

The reason is, as Steinberger briefly explains, that once a conclusion is reached by a proof which contains an operator not occurring in the premisses or in the conclusion, a normalizable system provides another deduction without this operator. The detours are avoidable, so this part of the proof is after all innocent. By the levelling of local peaks the normalization theorem guarantees that the premisses and the conclusion are directly linked to each other. It is thereby not possible to influence by a normalizable system the non-logical sentences of the language. Unfortunately for Steinberger, *intrinsic harmony* does not imply normalization.

2.4.2 E-weak disharmony

The example of quantum disjunction, which is originally due to Dummett (1991), shows that the notion of intrinsic harmony faces some problems. The introduction rule for quantum disjunction is similar to the one for standard disjunction, but the elimination rule is slightly different: no collateral hypotheses are allowed in the minor premises (Steinberger 2011a: 628). Quantum disjunction is in this thesis denoted by $\star$.

First of all quantum disjunction shows that intrinsic harmony is not a guarantee against E-weak disharmony. Quantum disjunction is intrinsically harmonious, but has a too weak elimination rule and leads therefore to E-weak disharmony (idem: 629). This is due to Dummett's observation that once the standard disjunctive operator is added to a system which contains just conjunction and quantum disjunction the latter constant collapses into standard disjunction (idem: 628). Hence, according to Steinberger's view, quantum disjunction elimination fails to fully exploit the corresponding introduction rule.

Secondly, the example shows that the addition of standard disjunction to a system of quantum disjunction and conjunction results in a non-conservative extension of the original system (idem: 629). The reason is that the law of distributivity for quantum disjunction, which is $A \wedge (B \star C) \vdash (A \wedge B) \star (A \wedge C)$, becomes provable in the system once standard disjunction is added to it. Since the law of distributivity contains just constants of the system $\{\wedge, \star\}$, the result is a non-conservative extension.

Thirdly, the new system is not normalizable, whereas the original system is. According to Steinberger, this is again due to the weakness of the elimination rule for quantum disjunction: it fails to accommodate a reduction procedure in order to create a local peak (Steinberger 2011a: 629). The example he uses is the following:

$$\dfrac{\dfrac{A \star B \quad \dfrac{[A]_1}{A \vee B}\,\mathrm{I}\vee \quad \dfrac{[B]_2}{A \vee B}\,\mathrm{I}\vee}{A \vee B}\,\mathrm{E}\star 1,2 \qquad \dfrac{\Gamma_1,[A]_3}{C} \qquad \dfrac{\Gamma_2,[B]_4}{C}}{C}\,\mathrm{E}\vee 3,4$$

In this proof the application of I-$\vee$ and E-$\vee$ is not yet a local peak because of E-$\star$. An application of the permutative reduction procedure in order to create a local peak for disjunction leads to the following situation:

$$\dfrac{A \star B \qquad \dfrac{\dfrac{[A]}{A \vee B}\,\mathrm{I}\vee \quad \dfrac{\Gamma_1,[A]}{C} \quad \dfrac{\Gamma_2,[B]}{C}}{C}\,\mathrm{E}\vee \qquad \dfrac{\dfrac{[B]}{A \vee B}\,\mathrm{I}\vee \quad \dfrac{\Gamma_3,[A]}{C} \quad \dfrac{\Gamma_4,[B]}{C}}{C}\,\mathrm{E}\vee}{C}\,\mathrm{E}\star$$

It is, however, not possible to apply the final step of the proof, E-$\star$, because this is only allowed if $\Gamma_1 - \Gamma_4$ are empty. Therefore, the permutative reduction procedure cannot be applied which means that the system is not normalizable.

The final problem is that the normalizability of the system $\{\star, \wedge\}$ shows that even the normalization requirement does not rule out constants which suffer, intuitively, from E-weak disharmony. In terms of a global requirement an additional principle seems to be needed to rule out this kind of disharmony as well. Steinberger admits the importance of E-weak disharmony, it is according to him the fundamental problem of $\star$, so in his view another local requirement is needed to prevent against E-weak disharmony (2011a: 629).

Recently, Murzi and Steinberger have proposed a procedure to solve the problem of E-weak disharmony (2015: 16). They propose an *expansion* procedure to guarantee that the elimination rules are strong enough. An expansion procedure shows that the conclusion of the introduction rule can be extended by an application of both the elimination rule and the introduction rule of the same constant. The result of these applications should be the conclusion of the introduction rule. An appropriate example is implication:

$$\dfrac{\dfrac{\begin{array}{c}\Pi \\ A \to B\end{array} \qquad [A]^1}{B}\,\mathrm{E}{\to}}{A \to B}\,\mathrm{I}{\to}, 1$$

Originally, Π already proved $A \to B$, and the above expansion shows that $A \to B$ remains provable once the derivation is extended by the full use of both I-$\to$ and E-$\to$. Now, according to this proposal, the inference rules for a constant are harmonious if and only if there are both levelling procedures and expansion procedures for the introduction and elimination rules. This sounds promising, but Murzi and Steinberger do not extend it further and just illustrate it with conjunction. In the light of this section it is worth checking whether the expansion procedure enables to distinguish between $\vee$ and $\star$. This leads immediately to a problem, since Murzi and Steinberger are not explicit whether it is necessary for a correct expansion procedure to apply the E-rule before the I-rule. If this is not necessary, then the following straightforward procedure works for both $\vee$ and $\star$:

$$\dfrac{\begin{array}{c}\Pi \\ A \star B\end{array} \qquad \dfrac{[A]}{A \star B}\,\mathrm{I}\star \qquad \dfrac{[B]}{A \star B}\,\mathrm{I}\star}{A \star B}\,\mathrm{E}\star$$

If the just presented expansion procedure is allowed, then it clearly does not succeed in distinguishing quantum disjunction from standard disjunction. The other option - that it is necessary to have the introduction rule as final step of the expansion procedure - faces another problem. According to the latter interpretation an expansion procedure for disjunction would look like this:

$$\dfrac{\dfrac{\begin{array}{c}\Pi \\ A \vee B\end{array} \qquad \begin{array}{c}\Pi,[A] \\ A\end{array} \qquad \begin{array}{c}\Pi,[B] \\ A\end{array}}{A}\,\mathrm{E}\vee}{A \vee B}\,\mathrm{I}\vee$$

This expansion procedure is not possible for quantum disjunction because collateral hypotheses are used in the minor premisses of the elimination rule. However, the above proof assumes that Π proves one of the two disjuncts, in this case A, such that Π can derive $A \vee B$. This assumption is in general unjustified, and only plausible in specific circumstances. Clearly, a procedure which is questionable, or simply implausible, is not the right requirement for harmony. Hence, the situation for $\vee$ and $\star$ is that either the expansion procedure works for both constants, or the procedure becomes implausible, even for standard disjunction. After all, the expansion procedure fails to distinguish between $\vee$ and $\star$.

To conclude, Steinberger's preference for a local harmony requirement and his claim that normalization is the best global notion to guarantee the innocence of logic leads to his demand for a local stability requirement. Stability should prevent against E-weak disharmony and, in addition with the local notion of intrinsic harmony, imply normalization. A local stability requirement was for Steinberger not on the market, and it just turned out that the proposed expansion procedure cannot rule out intuitively E-weak disharmonious constants. Since it was already concluded that the principle of innocence does not imply a local harmony requirement, it seems wise to check whether it is possible to guarantee innocence by global notions.

2.4.3 Going Global?

The obvious starting point for a purely global notion is normalization. The previous section showed by the example of quantum disjunction that *intrinsic harmony* faces some problems. Furthermore, it highlights two important points for the normalization requirement. Firstly it raises the question whether it is in principle problematic that a requirement is to a certain degree context dependent. Secondly, the fundamental problem does not seem to be context dependency, but to specify the logical operator that causes the non-normalizability of the system. If the latter is possible then the constant identified as the cause of the problems can be regarded as disharmonious. The already familiar example of conjunction, standard disjunction and quantum disjunction illustrates this problem.

Clearly, the system $\{\wedge, \vee\}$ is normalizable, and, as Steinberger states, the system $\{\wedge, \star\}$ is normalizable as well (2011a: 629). Hence, the fact that the system $\{\wedge, \star, \vee\}$ is not normalizable cannot be immediately explained by appealing to a system with less logical constants. It is indeed the interplay between quantum disjunction and standard disjunction that causes the troubles. At first sight the failure of the permutative reduction is due to the elimination rule for quantum disjunction: because of the specific requirement for this rule the final step cannot be applied. However, the question is why it is not the application of standard disjunction elimination which causes the problem. It is because of this application that the quantum elimination rule cannot be applied. In other words, it does not seem possible to decide justifiably which elimination rule is incorrect.

Another option is to bite the bullet and adopt the position that harmony is identified with normalization. Hence, harmony would be a property of the deductive system as a whole. On this account the systems $\{\wedge, \vee\}$ and $\{\wedge, \star\}$ are harmonious, whereas $\{\wedge, \star, \vee\}$ is disharmonious.

The obvious disadvantage of the harmony equals normalization account is that harmony cannot be directly ascribed to logical constants. In order to check whether a constant is harmonious or not it is needed to check whether it is part of a harmonious system. It seems even more problematic that a constant can be both harmonious and disharmonious. Both quantum and standard disjunction are part of normalizable and non-normalizable systems, so they are both harmonious and disharmonious.[13]

Systematic conservativeness, or *global* conservativeness, faces the same problems as just identified for normalization. In particular, it faces the major problem that a constant can be both harmonious and disharmonious. If standard disjunction is added to the system of $\{\wedge, \star\}$ then it leads to a non-conservative extension, hence it would be disharmonious. On the other hand, it is harmonious if it is added to a system which contains, for example, just conjunction and implication. Given this major problem the best option for the conservativeness requirement seems to be to adopt its *semi-global* version. Each constant is tested in isolation by a base system which solely contains atomic sentences and (a subset) of the structural rules.

2.5 Conclusion

The conclusion of the current chapter is threefold. First of all the chapter argued that the principle of innocence does not strictly imply Steinberger's *general harmony*, a (semi)global requirement can secure the innocence of logic as well. Moreover, the principle of innocence as such does not imply the need to rule out E-weak disharmony as well. Secondly, two arguments against the principle of innocence were evaluated and it turned out that both of them do not succeed in their goal to reject the principle. The astronomy argument was too specific, and, more importantly, too questionable to serve as a rejection of the innocence of logic. The truth predicate argument was already discussed and rejected by Steinberger (2011a). Thirdly, the chapter considered several ways to guarantee the innocence of logic by a formal harmony requirement. In line with the first conclusion both local and global requirements were discussed.

It turned out that, both on a local and a global level, it is quite

[13]A way to solve this is to require that a constant is only harmonious if the addition of it to a normalizable system leads to a normalizable system as well. However, this faces the problem that standard and quantum disjunction would be disharmonious. In other words, this requirement is too strong.

hard to rule out E-weak disharmony. Although the principle of innocence does not imply it as such, the formal parts of the remaining chapters will still, for the sake of the argument, check whether a further requirement solves the challenge raised by E-weak disharmony. In addition, the next two chapter offer two other motivations for the harmony requirement.

3 The Inversion Principle

The version of the harmony thesis which is based upon *the inversion principle* is the most traditional one. It relies strongly upon Gentzen's remark, already mentioned in the first chapter, that the meaning of each connective is given by the introduction rule(s) (Rumfitt 2016: 3). The current chapter contains three main sections. The first section presents the inversion principle and the idea that the introduction rules capture the meaning of the connectives. The second section discusses these two ideas. Thirdly, the chapter presents the formal notion of harmony which makes the informal notions of the first section precise.

3.1 Introduction Rules and Inversion

> "*The introduction rules represent, as it were, the "definitions" of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequences of these definitions*" (Gentzen 1934/1964: 295).

This is Gentzen's famous remark on which the idea is based that purely the introduction rules determine the meaning of the logical constants. The introduction rules are according to this view *self-justifying* (Dummett 1991: 251). Therefore, the elimination rules are justified by their reference to the corresponding introduction rule (idem: 252).

In terms of the broader meaning-theoretic assumptions, Gentzen's idea is based upon a *verificationist* theory of meaning (idem: 252). This is due to the idea that for a constant λ the introduction rules of λ specify the direct or "canonical" grounds for the assertion or truth of a sentence with λ as main connective (Rumfitt 2016: 5). Thereby this view, by its focus upon the introduction rules, opposes the two-sided model of meaning which includes both the I-rules and the E-rules. Originally, Prawitz (1974) and Dummett (1991) have further worked out Gentzen's idea.

As always, conjunction is an easy illustration of the role Prawitz and Dummett have in mind for the introduction rules. Suppose that $G1$ and $G2$ are direct grounds for respectively the formulas A and B. Accordingly, the introduction rule for conjunction provides direct grounds for the formula $A \wedge B$ by combining $G1$ and $G2$ (Rumfitt 2016: 5). Contrary to direct grounds, a formula can as well be established by *indirect* grounds (Dummett 1991: 252). For example one can conclude $A \wedge B$ by the formulas C, $C \to (A \wedge B)$, and the usual elimination rule for implication (Rumfitt 2016: 5).

The point of harmony in this context is that the elimination rule of λ must be faithful to the meaning of λ, which is given by the introduction rule. Rumfitt uses Negri and von Plato's inversion principle to make this more precise:

"*Whatever follows from the direct grounds for deriving a formula must follow from that formula*" (Negri and von Plato 2001: 6; Rumfitt 2016: 6).

This sounds quite abstract, but the example of the previous paragraph illustrates it clearly. The direct grounds for $A \wedge B$ were $G1$ and $G2$, and A followed from $G1$ and B from $G2$. Hence, the inversion principle requires that both A and B follow from the conjunction, which leads to the usual elimination rules for conjunction: $A \wedge B \vdash A$ and $A \wedge B \vdash B$.

In other words, a constant is harmonious if the corresponding introduction and elimination rules satisfy the inversion principle. Obviously, conjunction is according to this account a harmonious constant. More generally, the introduction rule of λ is given and thereby justified, and it has to be shown that the elimination rule of λ is in harmony with the introduction rule (Dummett 1991: 253). Later on, this chapter outlines how Read makes this informal notion precise by his General-Elimination Harmony (GE-Harmony). However, the justification of inversion and the corresponding meaning-theoretic primacy of the introduction rules are firstly discussed in the next section.

3.2 The Justification of Inversion

Rumfitt identifies one argument in favour of the inversion principle (2016: 8). Suppose, for the sake of the argument, that C is a formula which follows from any of the direct grounds for the assertion of a formula D. Furthermore, suppose that D is asserted. If the assertion of D is correct, then it seems that one of the direct grounds for D must obtain. By the first supposition C follows from any of these grounds, so the formula C must obtain if D is correctly asserted. Hence, it seems in line with the inversion principle to conclude that C must follow from D.

This argument faces according to Rumfitt two serious problems. First of all it assumes implicitly that logical consequence is about the preservation of correct assertibility, so it rejects the standard view that consequence is concerned with the preservation of truth (idem: 8-9). If logical consequence is about the preservation of truth then the assumption that D is asserted changes into the assumption that D is true (idem: 9). However, from the latter assumption it does not follow that one of the direct grounds for D must obtain. Since D might be true but still unassertible the argument needs to suppose that consequence is about the preservation of assertibility.

The second problem is that in order to conclude that C follows from the correct assertion of D, which is assumed, it must be argued that by correctly asserting D also one of its *direct* grounds obtains (idem: 9). By the supposition that D is correctly asserted it just follows that *a* ground for D obtains, and not necessarily a *direct* ground. In other words, the underlying problem is to find the right notion of a *direct* ground.

3.2.1 Direct and Indirect Grounds

The distinction between direct and indirect grounds is as well relevant for the first problem raised by Rumfitt. Once it is assumed that consequence is about the preservation of assertibility, this notion must provide a new way to distinguish between correct and incorrect ways of reasoning; without appealing to the notion of truth (idem: 11). Prawitz's and Dummett's account of logical consequence is, according to Rumffit, the only one which aims to do this, and this one is strongly based upon the distinction between direct and indirect grounds for assertion.[14]

Logic provides, according to the Dummett/Prawitz account, indirect grounds for atomic assertions. These indirect grounds must be faithful to the direct grounds for the assertion of the atomic statements. It is, for example, possible to assert by logic indirectly B on the basis of the premises $A \to B$ and A. However, the application of implication elimination must be faithful to the direct grounds for the assertion of B. Therefore, logical consequence is about the transformation of any direct grounds for the premises into a direct ground for the conclusion (idem: 12).

Dummett elaborates further on such a notion of consequence:

> "*an argument or proof convinces us because we construe it as showing that, given that the premisses hold according to our ordinary criteria, the conclusion must also hold according to the criteria we already have for its holding*" (1991: 219).

He illustrates this account by Euler's proof concerning the Seven Bridges of Koningsberg problem. The proof shows that someone who is observed to cross every bridge in Konigsberg crosses at least one bridge twice. Such an observation is made "by the criteria we already possessed for crossing a bridge twice" (1991: 219).[15] In other words, the conclusion - that the person crosses at least on bridge twice - holds according to the criteria that are already provided for crossing a bridge twice. Rumfitt criticizes this account of logical consequence, and the corresponding illustration, for two reasons.

Firstly, he argues that Euler's proof does not imply that the criteria for crossing a bridge twice are actually applied (2016: 12). It implies the counterfactual that if there is an observer on each bridge, then at least one

[14]According to Rumfitt Prawitz's and Dummett's account are essentially the same, so they are here discussed together (2016: 11-12).

[15]There is an intimate connection between the just developed line of reasoning and the principle of innocence. Steinberger uses these arguments to construct the principle of innocence, and Dummett's statement that the meaning of logical sentences should not affect the meaning of the other regions of the language is also due to this section.

observer will observe twice the person who aims to cross every bridge. Since it is a counterfactual claim, Dummett's account of the validity of the proof is inconsistent with some *metaphysical* possibilities (idem: 12-13). Rumfitt offers the example of a neurological condition which leads to a catatonic state for every observer before any observation of a person who aims to cross a bridge twice (idem: 13). In such a world, the counterfactual is false, but it is, by Euler's proof, still true that the promenader crossed one bridge twice if he crossed every bridge.[16]

Secondly, Rumfitt questions the exact notion of a *direct* ground (idem: 13). On the one hand the notion needs to be broad enough to capture the idea that only if a direct ground for a formula could have obtained a ground for the assertion of a formula should obtain. On the other hand the introduction rules, by which it is possible to assert complex sentences, constrain the notion of direct grounds. This leads to the requirement that there can be no ground for the assertion of a complex sentence unless this assertion could be justified by the introduction rule for the main connective of the considered sentence.

The latter requirement is what Dummett calls the *fundamental assumption*:

> "*if we have a valid argument for a complex statement, we can construct a valid argument for it which finishes with an application of one of the introduction rules governing its principal operator*" (1991: 254).

Dummett and Rumfitt agree that the argument in favour of the inversion principle is purely based upon the fundamental assumption, so it is worth to check whether the assumption is actually plausible.

3.2.2 The Fundamental Assumption

Since the assumption is in particular concerned with the introduction rules and the possible main connectives of a formula, a case distinction is made to discuss its plausibility.

The first case is easy, since Rumfitt admits that the assumption holds for conjunction (2016: 13). It seems quite plausible that the assertion of $A \wedge B$ is as well a commitment to the assertion of A and the assertion of B. Hence, the introduction rule for conjunction could have been applied as the final step of the argument.

In the case of disjunction, Rumfitt doubts whether the assumption is plausible. Dummett uses the distinction between individual and collective

[16] Ironically, even in the *actual* world Euler's proof is not observable, since the infrastructure of the bridges in Kaliningrad, as Koningsberg is nowadays called, differs significantly from the situation when Euler gave his proof.

possession to rule out some obvious counterexamples (1991: 266). According to Dummett the assumption is only plausible in the case of collective possession of information. In particular for statements about the past this is relevant, so according to the assumption originally one of the members of the community could have asserted the disjunction by an application of the introduction rule.

Despite of this, Rumfitt still presents a counterexample against the fundamental assumption. The example is the following one: "At the moment when Brutus first stabbed Caesar in Pompey's Theatre, there was either an odd or an even number of people in the Agora in Athens" (2016: 14). To assert correctly such a statement about the past it is sufficient that someone at the relevant time *could* have turned appropriate observations into a verification of the asserted statement (1991: 268). The fundamental assumption turns this, as Rumfitt states, into the necessary condition that someone at the relevant time could have made observations to justify the statement (2016: 14).

However, in the case of this disjunctive statement the latter, necessary, condition cannot be fulfilled. Recall that the fundamental assumption requires that either a direct ground for the statement "At the moment when Brutus first stabbed Caesar in Pompey's Theatre an odd number of people were in the Agora" must obtain, or a direct ground for the claim that "at that moment an even number of people were in the Agora". At the relevant time it was impossible to observe one of these two statements (idem: 15). Due to distance between Pompey's Theatre and the Agora, two observers are needed. Moreover, the observer at the Theatre needs to communicate with the observer in the Agora to tell the latter when to count the number of people.

Such a fast communication was, of course, not possible at the relevant time, so Rumfitt concludes that it is not possible to obtain a direct ground for the assertion of the disjunctive statement. Hence, he concludes that the fundamental assumption excludes the assertion of this kind of statements, whereas it seems, according to him, a plausible assertion. The upshot is that the assumption excludes too much assertions, and therefore Rumfitt concludes that the assumption is false for disjunctions (idem: 15).

The question is whether Rumfitt's counterexample is satisfying. One objection is that it seems solely by our current standards a plausible assertion to make. At the relevant time everyone was aware of the fact that it is impossible to check such a statement, and one might even wonder who would consider the number of people in the Agora at the time Brutus first stabbed Caesar. However, discussing this in full detail is not necessary, since the conclusion will be that the fundamental assumption is actually false. This conclusion is due to other constants.

Whereas the fundamental assumption holds for conjunction, it fails for both universal statements and "if, then" statements, of course captured by the universal quantifier and implication. Dummett himself admitted that the assumption is false due to these two constants (1991: 278). It is remarkable that Rumfitt argues at length against the fundamental assumption, and shows that it fails as well for negation, while Dummett has already concluded that the assumption is implausible. Dummett's rejection of the assumption turns an exhaustive evaluation of Rumfitt's counterexamples into a quite superfluous activity. It is, however, still interesting to explore the reason why the assumption fails in particular for these constants, which will be done in the next section.

Given the rejection of the fundamental assumption the only option for the proponents of the inversion principle seems to be to develop another notion of consequence which does not rely upon it. However, the main problem of the inversion principle is not, as the next section aims to argue, the fundamental assumption, but the underlying meaning-theoretic assumption. In order to see this it is time to turn to the well-known counterpart of the introduction rules; the elimination rules.

3.2.3 Elimination Rules

As Steinberger notices, Gentzen's remark is often mentioned, but the position is hardly supported by some well-developed arguments (2011a: 622). Contrary to Gentzen's idea, it has been claimed that for some constants the elimination rule is more important for the meaning of the constant than the introduction rule. In other words, it boils down, at least in Dummett's case, to the question *when*, for example, it is plausible to assert "if A then B". According to the view based upon Gentzen's remark this is most plausible once there is an effective method to transform any proof of A into a proof of B (Dummett 1991: 273).[17] The opposite view holds that "if A then B" is assertible once there are grounds that in any case when A is assertible B is assertible as well.

Regarding implication, Dummett adopts the position that the latter happens far more often than the former (idem: 273). Hence, the elimination rule has meaning-theoretic primacy in the case of implication, a position which is adopted by Rumfitt as well (2000: 790). Dummett's reason is that indicative conditionals are - except the conditionals which express an intention - outside mathematics asserted based upon experience, which favours the elimination rule (1991: 273).

The relevant point for the plausibility of the inversion principle is whether a statement which is asserted on the basis of the meaning-theoretic primacy of the elimination rule could be derived by a deduction with as final step the introduction rule for the considered connective. The problematic case for implication is according to Dummett the one with a disjunctive

[17]Dummett does not presume bivalence, hence implication is not considered truth-functionally by the truth or falsity of the antecedent and the consequent (1991: 272).

consequent (1991: 273). He offers the following example: "If you ask him for a loan, he will either refuse or make an outright gift to you" (idem). Although the statement is based upon experience, it is unknown on which principle the decision to refuse or make an outright gift was made. Hence, it is not possible to make a specific prediction which disjunct will hold in the specific case. Therefore, Dummett sticks to the view that implications, based upon experience, are asserted on the basis of a generalized version of it (idem: 274). This leads immediately to the case of the universal quantifier.

In the case of the universal quantifier Dummett adopts as well the view that its meaning is not purely captured by the introduction rule; the elimination rule is also meaning-constitutive (idem: 275). The reason is that there are, in general, two kinds of ground to assert a universal statement: (i) by a logical derivation which ends in a free variable statement (with the usual restrictions on the variables), and (ii) by an empirical induction. Obviously, the former procedure is adequately captured by the introduction rule, but the latter procedure is, by its non-logical and more pragmatic character, better captured by the elimination rule. The main characteristic of a universal statement, based upon an inductive generalization, is that it allows to assert that for every object in the relevant domain the considered property holds. Definitely, this is better captured by the elimination rule.

The final problem arises once it is tried to construe, in case (ii), a deduction with as final step the introduction rule for the universal quantifier. Since the original assertion was based upon empirical induction, some non-logical principles are as well needed in the new (sub)derivation in order to be able to apply the introduction rule (idem: 275-277).[18] This means that in order to adopt the fundamental assumption in its narrow sense - that the last step in a derivation of the universal quantifier can be an application of the introduction rule - one implies that the universal quantifier is, by the appeal to non-deductive principles, not a purely logical constant (idem: 278). Since this is not acceptable, Dummett rejects the fundamental assumption.

A way to capture the just outlined difference between the importance of the introduction and elimination rules is by Peacocke's notion of an *obvious* rule (1987: 154). This notion is a *primitive* one, so it is quite difficult to make it more precise and develop it in full detail. Although a well-developed account is missing, intuitively, in the case of implication and the universal quantifier the elimination rule is the *obvious* one. On the other hand, disjunction provides an example of a constant for which the introduction rule is obvious.

Hjortland's formal explanation of the latter observation is that it is

[18] Dummett does not discuss in full detail the complicating issues concerning empirical induction. It is beyond the scope of this thesis to add such a discussion here. The option that induction can be regarded as deductive reasoning, by for example the use of probabilities, is also not taken into account. See for example Strawson (1952) as a starting point for a criticism of such an approach to inductive reasoning.

due to the discharge of assumptions in the case of implication introduction and disjunction elimination (2010: 159). These so-called hypothetical rules are thereby less obvious than the other, categorical, rules. This seems a plausible explanation, but it does not contribute to a notion of harmony which is purely based upon either introduction or elimination rules. Since it is for each constant different which rule has the most meaning-theoretic power it is quite hard to put forward a general motivation for prioritizing one of the two for all the constants.

Regarding the question whether the introduction or the elimination rules should be the starting point, it is useful to distinguish between a philosophical and a formal perspective. From a formal perspective it seems a quite natural option to prioritize either the introduction or the elimination rules and to develop an account which leads to harmonious counterparts of these rules. Although the choice between introduction and elimination rules is, given the considerations of the current section, quite arbitrary, the next sections offers such a formal notion of harmony.

From a philosophical perspective it seems that two options are left. First of all it might depend upon the constant which rule has meaning-theoretic primacy. For example Edwards suggests that in the case of the universal quantifier its meaning is captured by the elimination rule, whereas the existential quantifier is captured by its introduction rule (1995: 97).

The second option is that both the introduction and elimination rule have meaning-theoretic importance; they need to be taken into account together. This does not rule out the possibility that for some constants one kind of rule is *more* important than the corresponding I- or E-rule. Such an approach corresponds with the view suggested in the first chapter according to which there is a scale of rules that determine the meaning of a constant. However, the other, less important, rule has still *some* meaning-theoretic power.

Despite of the shortcomings of the current notion of harmony it is still interesting to present a formal notion of harmony which aims to capture the inversion principle. This will be done by Read's account of harmony. Probably it turns out that, although GE-Harmony does not have a satisfying motivation, it offers a formal advantage which can be used to improve the other accounts of harmony.

3.3 General-Elimination Harmony

Read claims that his account of harmony is the one which is indeed based upon Gentzen's famous remark (Read 2000: 129; Read 2010: 562). Philosophically, Read's motivation for his account of harmony is rather weak. The just mentioned claim that his account is the best analysis of Gentzen's remark might turn out to be correct, but the problem is that Read uses it as a motivation for his notion of harmony. Since Gentzen did not provide a further, well motivated, reason for his claim, Read's motivation is, by his appeal to Gentzen, just an argument based upon authority.

The other motivation appeals to the autonomy of the logical constants. Once the meaning of the constants is purely given by the introduction rules these constants are self-justifying and autonomous (Read 2000: 131). The reason for this claim is the familiar one that no more and no less can be inferred from a formula than is provided by the grounds for the introduction of the formula (idem). Since the meaning is fully captured by the introduction rule the constant is self-justifying and autonomous.

The main objection against this way of reasoning is that it seems perfectly possible to substitute "introduction rules" by "elimination rules", and the logical constants are still self-justifying and autonomous. As the previous section argued there is no principal reason to give meaning-theoretic primacy to the introduction rules, so Read's argument seems to fail. Furthermore, it is possible to argue that the logical constants are still self-justifying and autonomous once both the introduction and elimination rules are equally meaning-constitutive. Only an appeal to *non-logical* principles for the justification of the logical constants seems to threaten the autonomy of logic. The self-justifying character of a constant is threatened by a more *global* account of harmony, because in such a case other constants or logical rules need to be taken into account as well. However, this is still no reason to suppose that only the introduction rules guarantee the autonomy of the constants.

From a more formal perspective Read's account offers the advantage that it is sufficient to come up with the introduction rules, since the elimination rules are just consequences of these rules. More importantly, the formal procedure to derive the elimination rule for a logical constant leads to an harmonious pair of inference rules. Dicher offers a quite accurate description of the algorithm to determine harmonious elimination rules (2016b: 589). The major premise of the E-rule is the conclusion of the introduction rule, and in addition the number of subderivations in the E-rule equals the number of introduction rules. Furthermore, each subderivation assumes the premisses of the I-rule, and concludes an arbitrary formula C. Finally, the application of the E-rule discharges the assumptions and concludes that C is the case. For a more formal description, suppose that the introduction rules for a constant λ schematically look like this:

$$\frac{\Pi_1}{A}\,\text{I-}\lambda \qquad\qquad \frac{\Pi_n}{A}\,\text{I-}\lambda$$

According to this schema Π_1 - Π_n denote the grounds for the assertion of a formula A in which λ is the main connective (Read 2000: 130). Given such an introduction rule the harmonious elimination rule for λ is the following:

$$\dfrac{A \qquad \begin{matrix}[\Pi_1]\\ C\end{matrix} \qquad \begin{matrix}[\Pi_n]\\ C\end{matrix}}{C}\;\mathrm{E}\lambda$$

The application of E-λ discharges the assumptions $\{\Pi_1 \text{ - } \Pi_n\}$.[19] This procedure constitutes Read's account of a harmonious pair of inference rules, called General-Elimination (GE) Harmony by Francez and Dyckhoff (2012). As the name strongly suggests, these schema's define a *general* version of the usual elimination rules of the logical constants. For example, the general elimination rules for conjunction are, given the usual introduction rule, the following:

$$\dfrac{A \wedge B \qquad \begin{matrix}[A]\\ \Pi\\ C\end{matrix}}{C}\;\mathrm{GE}\wedge \qquad\qquad \dfrac{A \wedge B \qquad \begin{matrix}[B]\\ \Pi\\ C\end{matrix}}{C}\;\mathrm{GE}\wedge$$

By reflexivity, or the allowance to repeat assumptions, these general elimination rules can be simplified to the usual ones where C is substituted for A and B respectively (Read 2010: 565). This results in the standard elimination rules for conjunction: $A \wedge B \vdash A$ and $A \wedge B \vdash B$. The main characteristic of Read's GE-Harmony is, besides the algorithmic character of the E-rules, that derivations can be inverted (idem: 564). In other words, once the introduction rule for λ is immediately followed by an application of the elimination rule such a detour is superfluous:

$$\dfrac{\dfrac{\Pi_{1-n}}{A}\,\mathrm{I}\lambda \qquad \begin{matrix}[\Pi_1]\\ C\end{matrix} \qquad \begin{matrix}[\Pi_n]\\ C\end{matrix}}{C}\;\mathrm{E}\lambda$$

In Dummett's terminology this *local peak* for λ can be levelled, since Π_{1-n} already proved C. Hence, the formula A, in which λ is the main connective, can be eliminated. The above procedure suggests an intimate connection between GE-Harmony, intrinsic harmony and normalization.

Interestingly, Read's GE-Harmony does not imply normalization, and even some other important properties are not guaranteed by it. The main reason for this is the connective *bullet*: $\bullet$. It has the following introduction rule:

[19] There are some slightly different, merely notational, templates for inducing elimination rules given the introduction rule(s). The main difference is that in the case of introduction rules where the discharge of assumption is involved *higher-order* rules are needed (Schroeder-Heister 1984; Hjortland 2010: 110; Dicher 2016b: 590). However, for the present purpose the just outlined algorithm is sufficient.

$$\dfrac{\begin{matrix}[\bullet]\\ \Delta\\ \bot\end{matrix}}{\bullet}\ \mathrm{I}\bullet$$

Here, 'Δ' indicates that $\bot$ is a derivation from $\bullet$ (Read 2010: 571). The corresponding, harmonious, elimination rule for $\bullet$ is according to the algorithm the following one:

$$\dfrac{\bullet \quad \bullet \quad \begin{matrix}[\bot]\\ \Delta\\ C\end{matrix}}{C}\ \mathrm{E}\bullet$$

Consider now the case when $C = \bot$ (Hjortland 2010: 114). This case leads, together with the supposition that $\bot \Rightarrow \bot$ is by reflexivity trivial, to a special version of the elimination rule:

$$\dfrac{\bullet \quad \bullet}{\bot}\ \mathrm{E'}\bullet$$

By contraction, again a structural rule, this can be simplified to a rule with just one instance of $\bullet$. Hjortland shows, in a far more easy way than Read originally did, how the rules for $\bullet$ lead to a situation in which $\Gamma \vdash A$ for any A and any Γ, even when Γ is empty (2010: 114-115):

$$\dfrac{\dfrac{\dfrac{\dfrac{[\bullet]^1}{\bot}\ \mathrm{E'}\bullet}{\bullet}\ \mathrm{I}\bullet[1]}{\bot}\ \mathrm{E'}\bullet}{A}\ EFSQ$$

In other words, $\bullet$ leads to triviality since it is possible to derive any A. Furthermore, as Read shows, bullet is equivalent to its own negation, and together with the above proof and the standard rules for negation it is possible to show for any A that it is equivalent to $\neg A$ (2010: 572). Since Prawitz (1965: 44) showed that normalization entails that $\bot$ is not derivable, GE-Harmony does not imply normalization, and it produces a non-conservative extension (Read 2010: 572). Hence, Read concludes that harmony should be clearly distinguished from normalization and conservativeness (idem: 574).

This is quite worrying. If GE-Harmony is the correct notion of harmony then it does not prevent against inconsistency or triviality. In the light of *tonk* this is quite ironic. Since *tonk* shows that once there is an A established it becomes possible to derive any B, *bullet* is a more problematic constant than *tonk*.

However, Read's claim that his account is most faithful to Gentzen's remark is correct. It is indeed the case that the introduction rules serve as a definition for the logical constants; the elimination rules are just a consequence of them. Moreover, GE-Harmony captures the inversion principle since it guarantees that the elimination rules infer solely what may be inferred given the grounds for the assertion of the introduction rule (Read 2010: 562). The algorithm accomplishes the former, whereas the reduction procedure guarantees the latter.

The constants who satisfy Read's GE-Harmony are according to him *coherent*, and the meaning of these rules can be read off transparently (idem: 561). So even $\bullet$ is a coherent and harmonious constant, although it is still inconsistent and the introduction rule is self-contradictory (idem: 574). Hence, coherent rules can be inconsistent, and consistent rules can be incoherent (idem: 571). Moreover, it should be mentioned that the inconsistency of the rules for $\bullet$ is in Read's view not problematic or a reason to revise his notion of harmony (2000: 142). The constant is just the proof-theoretic variant of the Liar paradox, and it should not be excluded by a harmony constraint (Read 2010: 574).

Read's motivation to bite, in this case both literally and metaphorically, the bullet is that his notion of harmony is a useful way to analyse the inconsistency of $\bullet$ (2000: 142). The Belnapian strategy to rule out constants like $\bullet$ by harmony is according to Read just one way to deal with Liar-like paradoxes. Read's attitude towards such paradoxes is to represent sentences like "this sentence is false" by $\bullet$ and the corresponding harmonious rules, and to study them subsequently in more detail (2008: 20). In the end, the result of the analysis can still be that the rules for $\bullet$ are mistaken (Read 2000: 142).

Such a view raises a number of questions, in particular for the usefulness of Read's harmony requirement. Clearly, GE-Harmony cannot prevent against inconsistency and a trivial deducibility relation. Read's supposition that Belnap's strategy is just an option, and that his variant to deal with the paradoxes is equally reasonable is questionable as well. Unfortunately, Read's claim is not further illustrated by other examples of such a strategy, which would have been helpful since the general strategy in philosophy is to come up with solutions that *avoid* or rule out paradoxes. Finally, the consequences of Read's argumentation for his distinction between rules that confer and do not confer meaning questions even more the usefulness of GE-Harmony.

The problem is, as Read admits, that inharmonious rules can still confer meaning. This leads to a situation with three kind of rules: (i) rules who are, according to GE-Harmony, harmonious and are thereby guaranteed to confer meaning, (ii) inharmonious rules who confer a coherent meaning, and (iii) inharmonious rules who lack such a coherent meaning. As already outlined, the first class of rules, the harmonious ones, faces the serious problem that it contains inconsistent rules.

Read's favourite example are the rules for the modal operators $\diamond$

and $\Box$ (2008). The traditional rules, to which he refers as the Curry-Prawitz rules, provide the correct meaning for the modal operators, but they do not satisfy the constraint provided by GE-Harmony (idem: 20). However, it remains unclear what this implies for the status of the modal operators. Read admits that inharmonious inference rules are not as such a source of incoherence; these rules can still define the meaning of the given connectives (idem: 12). On the other hand Read proposes harmonious rules for the modal operators, hence these rules are autonomous and coherent (idem: 14). The latter seems to be the sole point of Read's account: a constant is autonomous/harmonious if and only if the introduction rule provides the necessary and sufficient conditions for the assertion of the constant (idem: 12). Despite of this, it remains unclear how Read is able to distinguish between rules that confer meaning and rules that do not confer meaning, since GE-Harmony allows only to distinguish between coherent and incoherent rules.

In other words, it remains quite unclear what harmony is good for in Read's view. Neither does it rule out inconsistent constants like *bullet*, nor does it select the constants that confer meaning. If this is the correct idea of harmony, then one wonders why one should even consider to adopt the principle of harmony. Compared to the discussion of the previous chapter, GE-Harmony is not able to prevent against E-weak disharmony which is Steinberger's reason to reject it (2009a: 133). In particular, Read's procedure to read off the elimination rule for a constant cannot distinguish between the quantum elimination rule and the standard elimination rule for disjunction.

3.4 Conclusion

The verdict of both the philosophical motivation and the corresponding formal account, GE-Harmony, is negative. It turned out that the fundamental assumption, upon which the philosophical motivation relies, was mistaken. The ultimate reason for the failure of the assumption is due to the meaning-theoretic assumption, based upon Gentzen's remark, that the introduction rules determine the meaning of the logical constants. For some constants it seems more natural to give meaning-theoretic primacy to the elimination rule, or to include both rules, and indeed for those constants the fundamental assumption failed.

Read's GE-Harmony succeeded in the goal to capture Gentzen's remark and make it precise. However, it has, beside the lack of a proper philosophical motivation, some shortcomings. Most importantly, GE-Harmony fails to exclude the problematic constant *bullet*, and thereby it leads to triviality and inconsistency. Furthermore, it is not clear how GE-Harmony is able to distinguish between constants with a correct and an incorrect meaning. This questions the general project of GE-Harmony, since it seems that it just captures Gentzen's (problematic) remark, and nothing more. Notice that, since GE-Harmony is closely related to the philosophical motivation, even in the case one disagrees with the way this chapter rejected the latter, an adoption of Gentzen's remark still leads to serious problems by the corresponding formal account. The next chapter checks whether Tennant offers a more promising motivation, and whether his formal notion can solve the problems so far encountered.

4 Tennant's Harmony: Deductive Equilibrium

Tennant's notion of harmony and the corresponding philosophical motivations is the final account which is discussed in this thesis. A complicating aspect in the presentation of Tennant's account is, as Steinberger notices as well, its long history (2009a: 115). It goes back to Tennant's *Natural Logic* (1978), and has undergone some major and minor revisions over the years. Tennant's major argument for harmony is due to his *Anti-Realism and Logic*, and the argument is concerned with the entrenchment of the logical connectives into the language.

In addition to Tennant's Aetiology of Entrenchment, Tennant offers in *The Taming of the True* (1997) another motivation for harmony. Very briefly, the argument is that harmony needs to secure that the logical constants in the empirical sciences are the same constants in mathematics (idem: 23-24). However, the argument is based upon a number of highly questionable, implausible and naive assumptions about in particular the empirical sciences and partly about mathematics. For example Tennant does not take seriously the problems raised by the Duhem-Quine thesis how to respond in a reasonable way to the rejection of a statement or a theory. Moreover, in a recent article (2014) on harmony even Tennant himself does not mention his *Taming of the True* as a motivation for harmony. Because of this it has been decided not to present and discuss this argument, since it would just result in a, probably disappointing, discussion and after all a rejection of the argument. Therefore, the next section focuses on Tennant's story about the entrenchment of the connectives.

4.1 The Aetiology of Entrenchment

Tennant motivates the requirement of harmony by a story about the introduction of the logical constants into the language (1987: 77). The method examines the preconditions to introduce successful new (logical) expressions into the language. His method is one of "logical reconstruction" in which each innovation (i.e. the introduction of a new logical constant) has an "evolutionary endorsed point" (idem: 91). He offers a "speculative reconstruction", which is not about the actual evolution of any natural language. The philosophical goal of Tennant story is just to provide a "possibility proof" of the entrenchment of the connectives. These methodological characteristics will raise some questions, but the next section starts by, for the sake of the argument, just a description of Tennant's story.

4.1.1 The Entrenchment of the Connectives

The starting point of the story is a community whose members speak a language which contains only atomic sentences (idem: 77). In addition, the speakers of the language have inference rules in which just atomic sentences are involved (idem: 77-78). Tennant continues the story by showing how each logical constant is, one at a time, added to the existing language.

The first constant which is added to the language is, as one might expect, conjunction. Suppose that someone, a "linguistic innovator", makes the utterance "*A und B*" (idem: 78). Definitely, just such an utterance does not turn "*und*" into a significant part of the existing language. In order to achieve this one should provide the assertion conditions of the new utterance. Since the language contains at this stage just atomic sentences the assertibility conditions of A *und* B have to be constituted by the assertibility conditions of its constituents (idem: 79). This leads, according to Tennant, to the conclusion that A *und* B is assertible when A is assertible and B is assertible, which implies the familiar introduction rule for conjunction.

In Tennant's view there are, given the situation, "no other possibilities" for the assertibility conditions of A *und* B (idem: 79). In addition, Tennant admits that it might be difficult to explain the usefulness of the emergence of conjunction as the sole logical operator of the language (idem: 82). His motivation to start with conjunction is that it illustrates his harmony principle so well (idem: 80). Subsequently, the corresponding elimination rules for conjunction are justified by the idea that there must be an effective method to turn warrants for asserting the premises into warrants for the assertion of the conclusion (idem: 81). This boils down to the already familiar reduction procedure (see Dummett's *intrinsic harmony* in the chapter on the principle of innocence) which is therefore, at this stage, Tennant's formal notion of harmony.

The second constant which is introduced into the language is negation. Strictly speaking, this is not entirely true since Tennant regards the absurdity constant, which is in his view "a punctuation mark", as primitive and develops subsequently an account for negation (idem: 82-83). It is, in particular in the case of negation, good to be aware that Tennant is driven by revisionary aims.[20] However, these revisionary aims are not prominent in Tennant's story about the entrenchment of negation. He just introduces negation by the notion of absurdity - which leads to the standard introduction rule for negation - and he does not show how the corresponding elimination rule should look like. Tennant appeals, again, to the well-known reduction procedure to derive the correct elimination rule (idem: 90).

Disjunction is introduced and entrenched as the final propositional constant. This allows Tennant to introduce it in terms of the elimination rule. In order to introduce disjunction suppose that the language, which

[20]More specific, he wants to argue in favour of intuitionistic relevant logic, which means that, in addition to classical negation, he wants to drop the *Ex Falso* rule. For a more formal discussion see for example Tennant 1994; a philosophical discussion of the status of $\perp$ and the corresponding implications for *Ex Falso* and negation can be found in Tennant 2004.

already contains implication, negation, and conjunction, offers two hypothetical proofs of C from the assumptions A and B respectively (idem: 91). In other words, the sentence $(A \to C) \wedge (B \to C)$ is asserted. Now suppose that someone wants to turn this sentence into a conditional with C as the consequent. A new expression is introduced to bring the two antecedents of the former sentence together: $(A \vee B) \to C$. Recall that the grounds for the assertion of the latter statement remain the same, a hypothetical proof of C from A respectively B. In addition, suppose that there is a proof of the new expression $A \vee B$ as well (idem: 92). By *modus ponens*, which is already entrenched in the language, it becomes possible to conclude C. In other words, this is, in a slightly different way, the usual elimination rule for disjunction. Subsequently, the usual introduction rules for disjunction follow immediately.

The reason why Tennant offers this story about the introduction of disjunction into the language is that he does not want to be committed to the view that just the introduction rules fix the meaning of the logical constants (idem: 93). According to him even the elimination rule might fix the meaning of a constant, and subsequently the corresponding harmonious introduction rule has to be acknowledged. Obviously, this is important for the kind of formal harmony requirement Tennant is looking for.

4.1.2 The Plausibility of the Entrenchment

The previous section just presented Tennant's story of the entrenchment of the connectives in a descriptive way. However, the story raises a number of questions. First of all Rumfitt criticizes the account of negation which follows from the story. In addition, one might question the way Tennant introduces conjunction. Moreover, the way disjunction is introduced seems even more questionable. Finally, the status of Tennant's story raises the fundamental question whether a plausibility story implies a harmony requirement at all.

According to Rumfitt, Tennant's account does not explain how the connectives have obtained their *actual* meaning (2016: 29). The introduction and entrenchment of negation illustrates the criticism since it does not, according to Rumfitt, correspond to the way negation is actually used (idem: 29-30). Rumfitt argues that nowadays "not" has a classical meaning, so Tennant fails to explain how negation, which was entrenched in the language in a non-classical way, became an operator with a classical meaning. Without discussing Tennant's account of negation in full detail, this objection does not seem to undermine Tennant's story as such.

Tennant simply does not claim that his story explains how the connectives have obtained their actual meaning. He is explicit that his story is just a logical or speculative reconstruction, and not an explanation of the actual meanings of the connectives. Rumfitt's claim that the actual meaning of the connectives is different than the ones in Tennant's story seems correct, but it is not a rejection of Tennant's account of entrenchment. It just shows that the connectives might have been entrenched in another way, but Tennant simply does not claim that his story necessarily provides the correct way the connectives are entrenched into the language.

The second potential problem has to do with the way Tennant introduces conjunction into the language. Tennant claims that the assertibility conditions of A *und* B have to be constituted by the assertibility conditions of its constituents, hence he concludes that this boils down to the standard introduction rule for conjunction. However, it does not follow immediately that the utterance A *und* B is made because the so called "linguistic innovator" knows both the assertibility conditions of A and B. An alternative is that the linguistic innovator knows that either A must be assertible or B must be assertible, but that he is unsure which one is assertible. Hence, he utters A *und* B (or A *oder* B) with different assertibility conditions. In other words, the assertibilty conditions of A *und* B are, contrary to Tennant's suggestion, not straightforward.

Moreover, it is regarding conjunction remarkable that Tennant admits that it is difficult to explain the usefulness of it in a language with only atomic sentences. This seems to be a problem for Tennant's story since he claimed that the introduction of each constant has an "evolutionary endorsed point". The previous section explained that Tennant's reason to start with conjunction was that it explained his harmony principle so well. Subsequently, Tennant appeals to the notion of a reduction procedure to make his account of harmony precise. The problem is that Tennant's final notion of harmony, which is discussed in the next section, is *not* a reduction procedure. Thereby Tennant rejects his own claim that conjunction illustrates his harmony principle so well. After all, there does not seem to be a reason to start the story of entrenchment by conjunction.

The third problem arises by Tennant's introduction of disjunction. Recall that Tennant introduces it via its elimination rule, and that the need to introduce a disjunction operator was based upon the need to turn a sentence with conjunction as the main connective into one with implication as the main connective. However, in the case of disjunction it is quite plausible to imagine that disjunction was introduced into the language by a situation as sketched in one of the previous paragraphs: a person knows that A or B must be the case, but not which one. Hence, the disjunction $A \vee B$ is introduced.

It is remarkable that Tennant admits that the just sketched view offers an (even more) plausible story about the introduction of disjunction (1987: 92-93). Strangely enough Tennant is, partly, aware of the shortcomings of his story about disjunction but he sticks to his own original story. The reason is that, although he admits that the pressure to introduce disjunction might come from the introduction rule, he just wants to take into account the possibility that the elimination rule fixes the meaning of a logical constant. Given the considerations in the chapter on the inversion principle this seems to be a fruitful approach, but it is questionable whether disjunction is the correct constant to illustrate such an approach.

Recall that in the chapter on the inversion principle implication and the universal quantifier were used to illustrate the view that even the elimination rule might fix (partly) the meaning of a constant. Disjunction was considered as a constant for which the introduction rule is *obvious* and meaning constitutive. Of course Tennant can challenge this view by arguing that for disjunction the elimination rule is meaning constitutive. However, the problem is the plausibility of Tennant's story about the entrenchment of disjunction. Intuitively, it seems highly implausible that the need for disjunction was pushed by the demand to turn a conjunction into an implication. Tennant does not support his example by further arguments, thereby this is left to the reader. He claimed that the introduction of each logical constant served an "evolutionary endorsed point", but in this case Tennant does not make explicit why it is useful or needed to turn a conjunction into an implication. The alternative, sketched by the introduction rule for disjunction, seems far more useful and plausible.

Finally, the fundamental question is raised whether just a possible story implies the need for a harmony constraint. Since Tennant admits that he provides a speculative story and that his aim is to give a possibility proof, the argument for harmony seems rather weak. The sole reason for harmony he provides is that a constant which is not in harmony does not have a stable meaning and can therefore not be "entrenched" into the language (Tennant 1987: 94; Rumfitt 2016: 28). Unfortunately, Tennant does not argue in more detail in favour of this claim, and he does not even provide further meaning-theoretic requirements, at least not in this section, by which he is able to support and justify the demand for harmony by the Aetiology of Entrenchment.

In addition, one might question whether Tennant's possibility proof is at all a serious possibility. The current section criticises in particular the way Tennant introduces disjunction, and even the entrenchment of conjunction raised some questions. Moreover, Tennant did not succeed in making his claim explicit that the introduction of each constant served an evolutionary endorsed point. It is unclear why Tennant still sticks to his particular story of the entrenchment of the connectives. The most probable guess is that he is strongly driven by revisionary aims and that he wants to come up with a formal account of harmony which deviates from the work of Prawitz and Dummett. Regarding the former it is illustrative that both in the beginning and in the end of the considered chapter Tennant strongly opposes classical logic and the need to revise it. The aim to oppose Prawitz and Dummett was illustrated by Tennant's, implausible, introduction of disjunction via its elimination rule. To conclude, both by its questionable and its "possible" character Tennant's motivation for harmony is rejected.

Regarding Tennant's formal notion of harmony it is not quite clear, given the Entrenchment argument and the corresponding analysis, how to proceed. However, since the current section rejects Tennant's argument as a proper motivation for harmony, his formal account is in particular presented because it is interesting in its own right.

4.2 Deductive Equilibrium

Tennant's formal notion has undergone some changes during the years, so this thesis follows Steinberger's approach by just presenting the most recent one. Despite the shortcomings of Tennant's motivations for harmony it is still useful to discuss the corresponding formal notion. In particular it is relevant to check whether it can solve the still open problem of E-weak disharmony, illustrated by standard disjunction and quantum disjunction. It will be shown how Tennant tries to accomplish this, and how this leads, both for him and Steinberger, to some serious problems.

4.2.1 The Equilibrium

In order to introduce Tennant's formal approach some terminology is needed. The *strongest* proposition[21] with property P is the proposition A if any other proposition with property P is deducible from A (Tennant 2014: 19). Along the same lines, the *weakest* proposition with property P is the proposition B if any other proposition with property P can deduce B. By these notions Tennant presents the first part of his harmony principle. If λ is an arbitrary (binary) logical constant then λ is harmonious if it satisfies the following two conditions:

(S) $A\lambda B$ is the strongest conclusion possible under the conditions put forward by I-λ.
(W) $A\lambda B$ is the weakest major premise possible under the condition put forward by E-λ (Tennant 2014: 25).

In addition to these two conditions Tennant offers some constraints to show (S) and (W) respectively. To show that (S) is fulfilled one needs to:

(i) exploit all the conditions described by I-λ.
(ii) make full use of E-λ, but one may not make any use of I-λ (Tennant 2014: 25).

By substituting E-λ for I-λ and *vice versa* one obtains the constraints to show that (W) is the case. Disjunction ($\vee$) and quantum disjunction ($\star$) will be used to illustrate and criticize the first part of Tennant's harmony.

[21] By 'proposition' Tennant means the logical equivalence class of sentences. In other words, if A is a sentence then 'proposition' A means the logical equivalence class of which A is a member (2014: 19).

Consider standard disjunction. Suppose that X is an arbitrary formula which satisfies the conditions put forward by I-$\vee$ with A and B as premises. This implies that A entails X and B entails X (Rumfitt 2016: 27). Hence, by E-$\vee$ it follows that $A \vee B$ entails X, so $A \vee B$ satisfies (S). Now suppose, to show (W), that X is an arbitrary formula which can function as the major premise of E-$\vee$. It follows that whenever A entails C and B entails C, then X entails C as well. By I-$\vee$ it follows immediately that both A and B entail $A \vee B$, so X entails $A \vee B$ and thereby the latter formula satisfies (W).

A similar way of reasoning shows how $\star$ satisfies both (S) and (W). Again, suppose that X is an arbitrary formula which satisfies the conditions of I-$\star$ with A and B as premises. This leads to the following inference rules (†) (Steinberger 2009a: 118-119):

$$\dfrac{\begin{array}{c}\Pi\\ A\end{array}}{X}\,\dagger \qquad\qquad \dfrac{\begin{array}{c}\Pi\\ B\end{array}}{X}\,\dagger$$

Where Π is a derivation of A and B respectively. Now it needs to be shown that $A \star B \vdash X$ by making full use of E-$\star$ and not using I-$\star$. This is accomplished by the following derivation (Steinberger 2009a: 119):

$$\dfrac{A \vee B \qquad \dfrac{[A]}{X}\,\dagger \qquad \dfrac{[B]}{X}\,\dagger}{X}\,\text{E-}\star$$

Hence, $\star$ satisfies (S). Moreover, Steinberger (2009a: 119) shows that $\star$ satisfies (W) as well. In other words, the current requirements are satisfied by both standard disjunction and quantum disjunction. To avoid this shortcoming Tennant introduces an additional requirement. The I and E rules for λ are harmonious (with a small 'h') if they satisfy (S) and (W) (Rumfitt 2016: 27). According to the further requirement a pair of rules is Harmonious (with a capital 'H') if the introduction rule is the strongest rule which is in harmony with the elimination rule, and the elimination rule is the strongest rule which is in harmony with the introduction rule (Tennant 2014: 22). Together with (S) and (W) this maximality requirement constitutes Tennant's formal notion of harmony.

Clearly, the additional requirement succeeds in the aim to exclude quantum disjunction and to keep standard disjunction. Both constants satisfy (S) and (W), but the elimination rule of standard disjunction is stronger, so quantum disjunction is ruled out by the additional requirement. However, Tennant's account of harmony still faces a further problem which is pointed out by Steinberger. The problem has to do with the rules for the, so far almost neglected, quantifiers.

4.2.2 The Quantifiers

The standard rules for both the existential and the universal quantifier offer some additional restrictions on the use of the parameters. For example in the case of existential elimination the rule is as follows:

$$\dfrac{\exists x A(x) \qquad \begin{matrix} \Gamma, [A[a/x]] \\ \Delta \\ \psi \end{matrix}}{\psi}\ \text{E-}\exists$$

The usual restriction for this rule is that the parameter ‘a’ may not occur in Γ, $\exists x A(x)$ or ψ. As Steinberger outlines the just presented standard elimination rule satisfies, together with the standard I-rule[22] for $\exists$, the requirements of Tennant’s (S) and (W); hence it is harmonious with a lower ‘h’ (Steinberger 2009b: 658-659). The problem is, as Steinberger subsequently points out, that E-$\exists$ is not the *strongest* rule which is in harmony with I-$\exists$ (idem: 659-660). He introduces the rule E’-$\exists$, which is the same as E-$\exists$ only without any restrictions on the parameters. The new rule E’-$\exists$ is stronger than the standard one, so the pair I-$\exists$ and E’-$\exists$ is Harmonious with a capital ‘H’, contrary to the standard rules for the existential quantifier.

This is certainly problematic since the new, Harmonious, elimination rule has some unwelcome consequences. In particular it allows one to prove that $F(a) \vdash F(b)$ for any individual ‘b’ (Steinberger 2009b: 660). In other words, once it is proved for one atomic sentence that the predicate ‘F’ holds, then the new E-rule for the existential quantifier allows to prove that ‘F’ holds for any atomic sentence. This way the new rule serves as a kind of first-order *tonk* rule which is clearly something that needs to be avoided. Hence, Tennant’s maximality condition fails to select the correct pair of rules for the quantifiers.[23]

However, Tennant (2010) provides an argument against Steinberger’s criticism of the maximality principle in the case of the quantifiers. Tennant’s argument appeals to the sequent calculus setting, and it is in essence that in order to show that the new pair of rules I-$\exists$ and E’-$\exists$ satisfies (S) and (W) one needs to use the structural rule *cut* (idem: 465). Even in the case of the traditional rules for the existential quantifier *cut* is needed to show that (S) and (W) hold (idem: 463). The point is, according to Tennant, that in the latter case the application of *cut* is justified, whereas it lacks such a justification in the former case (idem: 465-466). In Tennant’s view it is a mistake to presuppose the availability of *cut*; it is not a *primitive* structural rule but an admissible rule (idem: 465).

[22] To be complete, this rule is as follows: $A[t/x] \vdash \exists x A(x)$, and it has no significant additional restrictions.

[23] A similar argument is available for the universal quantifier. See Steinberger (2009a: 125-126) for the corresponding proofs.

In order to show the admissibility of *cut* one needs to use a reduction procedure for the rules of the logical constants (idem: 466; Tennant 2012). The reduction procedure was already presented in the chapter on the principle of innocence: it shows that the rules for a constant are - in Dummett's terminology - *intrinsically harmonious.* The point is that a reduction procedure is available for the traditional rules of the existential quantifier, but not for the new rules without any restrictions on the parameter (Tennant 2010: 466-467). Hence, the theorem which establishes the admissibility of *cut* is not provable for the language which contains the new quantifier; therefore the rules of the new quantifier do not satisfy (S) and (W) since *cut* cannot be used to show this.

Steinberger is not convinced by Tennant's response. He argues that the appeal to the admissibility of *cut* is both superfluous and problematic. It is superfluous because the admissibility of *cut* uses a reduction procedure, and therefore it appeals to Dummett's *intrinsic harmony.* Since intrinsic harmony already rules out *E-strong* disharmony the appeal to the admissibility of *cut* to block the problematic, E-strong disharmonious, quantifiers, is superfluous (Steinberger 2011b: 275-277). Intrinsic harmony is sufficient to block E-strong-disharmony; the admissibility of *cut* and (S) and (W) are not needed.

The second objection put forward by Steinberger is that the admissibility of *cut* is a *global* property and not a *local* one (idem: 277). Steinberger provides, again, the example of quantum disjunction: in the systems $\{\vee, \wedge\}$ and $\{\star, \wedge\}$ *cut* is admissible, but not in the system $\{\vee, \star, \wedge\}$ (idem: 278). If *cut* would be admissible in the latter system then $\star$ would collapse into $\vee$, and the law of distributivity would become provable for quantum disjunction. In other words, it depends upon the deductive system and the interplay between the rules of the system whether *cut* is admissible or not. Thereby it is a *global* requirement which Steinberger strongly opposes.

By these two objections Steinberger rejects the admissibility of *cut* as a formal harmony requirement (idem: 279). Instead of Tennant's (S) and (W) requirements Steinberger proposes intrinsic harmony together with the maximality principle as the correct notion of harmony. The former prevents against strong disharmony and the latter against weak disharmony, so together they would constitute a satisfying notion of harmony. However, Steinberger's proposed modification of Tennant's account still faces some problems.

First of all the chapter on the inversion principle showed that even the problematic constant $\bullet$ is intrinsically harmonious, which means that Steinberger's modification cannot exclude $\bullet$ as a disharmonious constant. Secondly, there is the worry - as the next section will argue - that the maximality condition is, contrary to Steinberger's demand, not a *local* constraint.

4.2.3 Intrinsic Harmony and Maximality

The maximality conditioned stated that given a (intrinsic) harmonious pair of rules for a constant λ one selects the *strongest* E-rule which is in harmony with the I-rule, and subsequently one selects the strongest I-rule which is in harmony with the E-rule. The problem is, as Steinberger already stated (2009a: 121-122), what is meant exactly by the *strength* of a rule. In the case of quantum and standard disjunction it seems clear at face value that the elimination rule of standard disjunction is the stronger one, since it offers no restrictions on the use of collateral assumptions in the minor premises. Although this is quite intuitive, for a formal notion a more precise criterion is needed. The first candidate for such a criterion seems to be the derivation that $\star$ collapses into $\vee$ once standard disjunction is added to a language which contains quantum disjunction:

$$\dfrac{A \star B \qquad \dfrac{[A]}{A \vee B}\,\text{I-}\vee \qquad \dfrac{[B]}{A \vee B}\,\text{I-}\vee}{A \vee B}\,\text{E-}\star$$

However, there is a similar derivation which shows that standard disjunction collapses into quantum disjunction. Just replace $\star$ by $\vee$ and $\vee$ by $\star$ in the above derivation and one obtains the same result. Hence, this criterion cannot capture the intuition that standard disjunction is stronger than quantum disjunction.

The second, and probably most natural, candidate is to come up with a formula φ which is provable in the language L which contains standard disjunction, and unprovable in the language L' which contains quantum disjunction. The standard example is the law of distributivity: $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$ whereas $A \wedge (B \star C) \nvdash (A \wedge B) \star (A \wedge C)$. However, to derive this formula one needs the rules for conjunction as well. In other words, if this procedure is adopted to show that E-$\vee$ is stronger than E-$\star$, then the maximality condition is turned into a *global* constraint.

Until Steinberger provides a local alternative to show in a precise way that one rule is stronger than another rule he is committed to the position that the maximality constraint is after all a *global* one. This is at odds with Steinberger's demand of locality. Of course this is, in general, not a reason to reject the maximality principle, but for Steinberger the situation is different. He clearly rules out any *global* harmony requirement, so by the argument of this section he can plainly not, without violating his own requirements, adopt the maximality principle as a harmony requirement.

The second problem has to do with the already discussed constant *bullet*. Recall that *bullet* is by definition intrinsic harmonious, so it can just be ruled out by the maximality condition. Given the E-$\bullet$ rule this is almost impossible, since the E-rule allows one to derive $\Gamma \vdash A$ for any A and any Γ, even when Γ is empty. It seems quite hard to come up with an elimination rule for • which is stronger - and even if this can be done *bullet* would still be a highly problematic constant. Hence, *bullet* satisfies the requirements of intrinsic harmony and maximality and is therefore, according to the current proposal, a harmonious constant. In the article in which Steinberger finishes his discussion of Tennant's account he just mentions *bullet* in a footnote (2011: 277).[24] In this footnote Steinberger states that he is not convinced that Read's *bullet* operator refutes intrinsic harmony. According to him it just shows that the logical inferentialist needs an account of permissible introduction rules, which is not yet available.

Tennant rejects the *Ex Falso* rule, so he would presumably respond to the problems raised by • in a different manner. Since the derivation $\Gamma \vdash A$ involves a crucial application of *Ex Falso* (see the derivation in the chapter on the inversion principle), he can block the problematic consequences of • by arguing that it uses invalid inference rules. However, such a response is not uncontroversial because one needs to adopt Tennant's (unusual) analysis of both $\perp$ and the *Ex Falso* rule. For example Steinberger adopts to a certain extent Tennant's analysis of $\perp$, but he regards *Ex Falso* as a structural rule. Given Steinberger's demand for a local harmony constraint he cannot solve the problematic consequences of • by harmony. This is a more general problem of this type of response; whether or not the analysis of Tennant is correct, even he does not exclude • by the formal harmony notion.

4.3 Conclusion

This chapter presented and discussed Tennant's motivation for harmony and the corresponding formal notion. The Aetiology of Entrenchment argument faced two major problems. First of all Tennant's story about the entrenchment of the connectives was just a possible story, and not necessarily the correct one. This made it quite difficult to adopt it as a motivation for harmony, since a speculative story does not seem to be a convincing justification. Secondly, the story led to a number of objections, which undermine its possible character as well.

Tennant's formal notion offered as main advantage that the introduced maximality condition can distinguish between the elimination rules for standard and quantum disjunction. However, it has been argued that in order to show this in a strictly formal way one needs to turn the maximality condition into a global constraint. Another problem for Tennant's formal notion is that it cannot rule out the problematic constant *bullet*.

[24] In his PhD thesis Steinberger does not mention *bullet* and the corresponding problems at all.

5 Analysis and Conclusion

The final chapter aims to summarize the main observations and conclusions of the previous chapters, and to offer a final analysis of them. The first two sections summarize respectively the philosophical motivations and the corresponding formal requirements. The third section tries to bring these two together. The fourth section is an intermediate one: it offers some shortcomings, open problems, and suggestions for further research. Finally, the chapter provides the main conclusion of this project.

5.1 The Motivations

Three motivations for harmony were presented: the principle of innocence, the inversion principle, and the entrenchment of the connectives. The principle of innocence stated that it should not be possible to discover, solely by logic, (atomic) truths about the world which are not discoverable independently of logic. The inversion principle was based upon the idea that only the introduction rules provide the meaning of the logical constants. Finally, the entrenchment of the connectives was a possibility story offered by Tennant. Of all these justifications for harmony, the principle of innocence turned out to be the most satisfying one.

The idea that it is the introduction rule which is (primarily) meaning-constitutive for all the constants, based upon a remark made by Gentzen, turned out to be mistaken. In particular in the case of implication and the universal quantifier there seemed to be good reasons to give meaning-theoretic primacy to the elimination rule, instead of the introduction rule. Therefore, the inversion principle as a motivation for harmony was rejected. In addition, the previous chapter rejected Tennant's story about the entrenchment of the connectives as a motivation for harmony. Tennant admitted that the story offered just a "possibility proof", which is a rather weak motivation to impose a harmony requirement. Moreover, his story about the entrenchment of the connectives raised a number of questions, which led to the conclusion that after all even its possible character was highly doubtful. Hence, Tennant's motivation for harmony was rejected as well.

The principle of innocence faced two main objections: the astronomy argument and the truth predicate argument. The latter argument was already disproved in a correct way by Steinberger. The former argument, offered by Rumfitt, was too specific and questionable to reject the principle of innocence just on the basis of this argument. It aimed to disprove the main claim of the principle, but it has been shown that this was mainly due to the content of one of the premises which was about the notion of a black hole. Since this was the sole and after all unconvincing argument against the principle of innocence, the innocence of logic is adopted as the most promising motivation for harmony.

Steinberger combined the principle of innocence with his further meaning-theoretic assumptions, namely the two-sided model of meaning and the aim to offer a *local* harmony requirement for the logical constants. Both aspects strongly influenced his quest for a formal harmony requirement to capture the innocence of logic. By his meaning-theoretic assumptions the formal notion of harmony should be a local one which prevented against both E-weak and E-strong disharmony. However, it has been argued that the principle of innocence as such just implies a (semi) global harmony requirement which prevents against E-strong disharmony. The next section provides an overview of the formal requirements such that the principle of innocence (and Steinberger's meaning-theoretic assumptions) can be related to these formal notions in the correct way.

5.2 The Formal Notions

In order to remain faithful to Steinberger's quest, the starting point is to relate the local accounts to the problem of E-weak disharmony. Intrinsic harmony, GE-Harmony, and Deductive Equilibrium (Tennant's harmony with a small 'h') were all *local* notions.[25] All these local notions failed to rule out E-weak disharmony; in particular they could not distinguish between quantum and standard disjunction.

Hence, the *expansion procedure* and *the maximality principle* were put forward to rule out E-weak disharmony by a local constraint. Unfortunately, the expansion procedure was not yet applied to the case of standard and quantum disjunction. It turned out that it failed to distinguish between these two constants in a proper way. The maximality principle succeeded in the aim to rule out quantum disjunction, but it has been shown that it was after a global constraint, and not a local one. In other words, all the local accounts failed to prevent against E-weak disharmony. It might be suggested that this conclusion is a coincidence; a yet unknown local constraint can after all rule out quantum disjunction. However, in the light of the first chapter this does not seem to be a coincidence, but due to some fundamental reasons.

First of all, suppose that a purely local requirement is indeed only concerned with the meaning of the particular constant to which it is applied. Now recall, from the first chapter, how the sequent calculus presents standard disjunction and, accordingly, quantum disjunction:

$$\frac{\Gamma, A \Rightarrow \Delta \qquad \Gamma, B \Rightarrow \Delta}{\Gamma, A \vee B \Rightarrow \Delta}\ \text{L-}\vee$$

[25] One might object that in order to derive the standard rules for the connectives from the rules as obtained by the procedure of GE-Harmony that one needs additional structural rules which might threaten its strictly local character. However, GE-Harmony faces more serious problems, so there is no need to discuss this point in more detail.

This is the standard rule to introduce disjunction in the antecedent, and one obtains quantum disjunction by the restriction that Γ should be empty. This means that the distinction between the two disjuncts is purely due to the different *contexts* of the rules. According to the analysis as presented by Dicher, the context can be intrinsic for the meaning of a constant or it can provide a (potential) way to interact with other constants. In the case of disjunction Dicher argued that the context just played the latter role. Hence, the distinction between standard and quantum disjunction is a structural one which effects the structure of derivations and the way (quantum) disjunction can interact with other constants (Dicher 2016b: 592).

By this observation it is, as Dicher emphasizes as well, no surprise that purely local accounts fail to identify the problems of quantum disjunction.[26] Local accounts are, by definition, just concerned with restrictions on a pair of inference rules, and not with the way constants interact with other constants in the deductive system. To capture the latter by a harmony requirement, one needs a global requirement which contains both structural rules and the rules of the other constants in the deductive system.

In other words, if one wants to distinguish between standard and quantum disjunction, and to prevent E-weak disharmony, it is wise to go global. Beside the just presented maximality principle, this thesis presented two other global requirements: conservativeness and normalization. As the connective $\circ$ showed, the former does not prevent against E-weak disharmony. The normalization requirement was also not able to capture the difference between quantum and standard disjunction. The systems $\{\wedge, \star\}$, $\{\wedge, \vee\}$ are normalizable, whereas the system $\{\wedge, \star, \vee\}$ is not normalizable. However, it is not possible to decide justifiably which elimination rule causes the non-normalizability.

The latter observation seems to emphasize that it is indeed the *interplay* between quantum and standard disjunction which causes the troubles. If there would be something wrong with either quantum or standard disjunction then one might expect that one can show in a precise way which elimination rule is incorrect. Since normalization cannot show this, the most plausible explanation of the non-normalizability is that quantum and standard disjunction are put together in the same deductive system.

On the other hand, both conservativeness and normalization offered the advantage that they both rule out *bullet*, the other problematic constant. *Bullet* was created by Read's GE-Harmony, and therefore it can also not, by definition, be ruled out by intrinsic harmony. Another advantage is that both conservativeness and normalization seem to capture the innocence of logic. Rumfitt argued in favour of conservativeness, whereas

[26]It does surprise Dicher that this observation played almost no role in the harmony debate. He speculates that it is due to the natural deduction setting and in particular Dummett's revisionary aims to reject classical logic in favour of intuitionistic logic (2016b: 592-593).

Steinberger stated that normalization was the best global requirement to guarantee the innocence of logic. Given these considerations, the final task is to relate the principle of innocence, and probably Steinberger's additional meaning-theoretic assumptions, to the just presented formal accounts and their corresponding problems.

5.3 Innocence and the Meaning of the Logical Constants

It is time to combine the observations and put them together into one analysis. However, it is tricky to relate the principle of innocence to just one formal requirement, since further meaning-theoretic assumptions might play a role as well. Hence, a strict case distinction will be made.

The first case (a) is easy; suppose that the inferentialist adopts the principle of innocence and no additional meaning-theoretic assumptions. As already mentioned, a natural candidate to guarantee innocence is conservativeness. Definitely, if the role of harmony is to select the constants that confer meaning, the identification of harmony with conservativeness would lead to the conclusion that the meaning of the constants is a *global* one. Given this unusual conclusion in proof-theoretic semantics, a promising alternative is the stabilized (*semi-global*) version of conservativeness. Moreover, the latter offers the advantage that a constant is simply harmonious or not, whereas the former case has the disadvantage that a constant can be both harmonious and disharmonious. The other, even more local candidate, to guarantee innocence is intrinsic harmony. However, it fails to rule out *bullet*; a constant which is not innocent. Hence, in the first case the stabilized version of conservativeness is the best way to go.

Conservativeness does not rule out improper balanced rules such as $\circ$, so the more ambitious inferentialist might not be satisfied with the proposal as suggested in the previous paragraph. He wants, beside the principle of innocence, to include the two-sided model of meaning in his quest for harmony: a formal account of harmony should prevent against both E-strong and E-weak disharmony. Let this be the second case (b). Since the inferentialist is, at least in the current harmony debate, almost always (implicitly) looking for a local harmony requirement, he faces a serious dilemma in the second case.

By the analysis of the previous section, the inferentialist cannot have both of the following two demands at the same time: (i) a local harmony requirement, and (ii) rule out E-weak disharmony and therefore remain faithful to the two-sided model of meaning. This combination is not possible since it has been argued that one cannot distinguish between standard and quantum disjunction by a purely local constraint. Hence, one of these two needs to go. Giving up (ii) leads to the situation as described by the first case. The consequences of giving up (i) are less straightforward, so this can be explored in more detail. Let this be the *global* version of the second case (b-*global*).

At first sight, to give up (i) and to be forced to adopt the view that the meaning of the logical constants is after all a global one, might seem like a high price to pay. Recall, however, that in the first chapter it has been argued that the distinction between a local and a global meaning is not as strict as it is often suggested. A global meaning, as the reasonable holist would state, is not an immediate commitment to the view that *all* inferences are relevant for the meaning of a connective. One just extends the meaning-constitutive inferences by, for example, the laws of distributivity. Suppose, for the sake of the argument, that the inferentialist is willing to go global to rule out E-weak disharmony; which options are still on the table?

Clearly, (global) conservativeness and normalization are ruled out. Conservativeness is not able to prevent against E-weak disharmony, and normalization cannot identify whether standard or quantum disjunction causes the failure of the reduction procedure. The sole option which is left over is Tennant's maximality requirement. Given the discussion in the corresponding chapter it needs to be accompanied by a requirement to avert disastrous consequences for the quantifiers. Adding a more local requirement to accomplish this does not seem to be a fundamental problem; hence Steinberger's proposal to guarantee harmony by both intrinsic harmony and the maximality requirement seems the best option in the case of b-*global*.

Whereas Steinberger presents it as a local harmony requirement, this thesis disproved the claim; the maximality requirement is a global one. So far so good. Intrinsic harmony and the maximality requirement prevent against E-weak disharmony and seem to guarantee the innocence of logic. However, this option faces two major objections.

The first objection is that intrinsic harmony and the maximality condition cannot rule out *bullet* as a disharmonious constant. Intrinsic harmony cannot, by definition, rule out *bullet* and the maximality condition is not the kind of requirement to be able to exclude *bullet*. Moreover, to keep logic innocent one cannot appeal to the notion of conservativeness, since it clashes with the maximality condition. According to the former, one should not add standard disjunction to a system with conjunction and quantum disjunction since it leads to a non-conservative extension. On the other hand, the maximality condition prefers standard disjunction above quantum disjunction. A possible solution for this problem is that one starts by applying the stabilized version of conservativeness - this rules out *bullet* - and accordingly the maximality condition is used to select the strongest set of rules.

The second, and more fundamental, objection is that the option to go global, in particular by adopting the maximality condition, misses the point. The maximality condition strongly suggests that there is something wrong with quantum disjunction. According to the condition quantum disjunction is the weaker constant, and standard disjunction is the stronger one.

Hence, the former should be rejected and the latter constant is the preferable option. However, another analysis implies that the rules for quantum disjunction do not cause the problems of, for example, non-normalizability. The mistake has to be found in the *interplay* between quantum and standard disjunction. Dicher answers to the, rhetorical, question "What is wrong with quantum disjunction?" as follows:

> "*Not much, it would seem. Plenty can go wrong in its presence, but this has less to do with it, and more with the management of the deducibility context.*" (Dicher 2016b: 595)

In line of this quote Dicher shows, in the sequent calculus, *how* the law of distributivity for quantum disjunction is proved in the system $\{\wedge, \vee, \star\}$.[27] He highlights the fact that it is the applicability of *cut* in the derivation which forces quantum disjunction into a problematic structural context such that it becomes possible to derive the law of distributivity (idem: 593). In other words, the problem is not to be found in the rules for the two disjuncts, but in the way they are forced - although their contexts are incompatible - to interact with each other by the structural rule of *cut*. The lesson is simply that one should not force constants with different contexts into the same deductive system. In the case that such a situation arises, one should not blame the constants but the cross framework application of *cut*.

The suggested solution in the case of b-*global* is after all not convincing. It does not guarantee the innocence of logic, due to *bullet*, and, more importantly, it does not analyse quantum and standard disjunction and their corresponding problems in the right way. Since the principle of innocence does not imply the need to rule out E-weak disharmony it is, after all, the stabilized version of conservativeness which is the most satisfying harmony requirement. It prevents against constants such as *tonk* and *bullet*; these constants are clearly not innocent. Moreover, it is, since it is a stabilized version, not possible that a constant is both harmonious and disharmonious.

5.4 Shortcomings, Suggestions, Open Problems

It is, as it seems, unavoidable that the just presented thesis contains some shortcomings and raises some open problems. This section mentions, among others, some of them.

5.4.1 Introduction Rules

First of all it might be suggested, following Steinberger, that in addition to a harmony requirement an account of permissible introduction rules is needed. Although such a theory is not yet available, this thesis completely

[27] See Dicher 2016b, page 594 for a full exposition of this proof.

neglected the issue how a proper introduction rule should look. For example, one might adopt the constraint that an introduction rule should, in Dummett's terminology, be *pure* (1991: 257). This imposes a rule to let just one logical constant figure in it.[28] Definitely, this raises further questions about the motivation to adopt such a requirement. In any case, further work needs to be done to offer an account of permissible introduction rules in a comprehensive, well motivated, way.

5.4.2 The Two-Sided Model of Meaning

It turned out that the two-sided model of meaning played a crucial role in the debate which kind of formal requirement should be adopted. Only by the two-sided model the need to balance the introduction and elimination rules was given a prominent place. As a consequence of this, it was needed to take E-weak disharmony into account as well. Unfortunately, mainly due to reasons of space and the way this thesis was set up, the two-sided model of meaning is not discussed at all. In particular, it might be questioned whether the inferentialist is indeed forced to adopt it, or whether he can set it aside.

For example: Brandom adopts, from a strongly inferentialistic perspective, the view that the logical constants must satisfy the conservativeness requirement (Brandom 2000: 68-69; Rumfitt 2016: 25). This suggests that Brandom is not concerned with E-weak disharmony, so it would be interesting to check how his inferentialistic assumptions are related or can threaten the two-sided model of meaning. Moreover, given the observation that in order to prevent E-weak disharmony one needs to go global, it would be interesting to explore whether such a global strategy is compatible with the two-sided model of meaning. Finally, the reader might object that inferentialism was as well not discussed or defended. Definitely this is correct, but the inferentialistic assumption does not influence the question which kind of harmony requirement is the correct one. Thereby it serves as a background assumption of this thesis - a defence of it was beyond the scope of this project.

5.4.3 The Threat from Circularity

An even more fundamental objection seems to be the following one. The major goal of this thesis was to discuss the question whether logic, or the logical constants, should be in harmony. Hence, in order to be able to answer this question it was needed to separate the logical fragment from the remaining part of the language. This view was called logical inferentialism.

[28] Notice that a *pure* rule seems a straightforward requirement once a strictly *local* meaning for the connectives is the goal. Otherwise, other constants might be involved in the meaning-constitutive rules.

Now one can reason as follows. In order to be able to distinguish the logical fragment from the non-logical part of the language, it is needed that these two aspects of the language are not intertwined. Otherwise, it would not be possible to separate the logical fragment in a proper way from the non-logical fragment. Since this assumption separates these two parts of the language in a proper way, it seems that the principle of innocence is already contained in one of the main assumptions of this thesis. If this is correct, then it seems that one should not question the principle itself, but the background assumption of logical inferentialism.

One way to approach this issue is from a general meaning-theoretic perspective. The assumption of the logical inferentialist is that the logical fragment of the language can be separated from the remaining language, by Steinberger called *minimal molecularism* (2009a: 34). The holist, about the language as a whole, might challenge this view and argue in favour of a view which is, for example, similar to the one put forward by Quine in his *Two Dogma's of Empiricism* (1951).

An approach which fits probably better in the current debate, and might be more fruitful as well, is to focus specific upon the role of *logic*. The section which discussed the truth predicate argument already mentioned Read's statements that logic is formal and that it has no content. In addition, the section indicates that the statement that logic is formal needs further specification. Moreover, one can think of other roles of logic, for example its supposed *normative* force. It is likely that a further investigation of the precise role of logic in the (linguistic) practice might help to evaluate whether the assumption of logical inferentialism is plausible.

5.4.4 Uniqueness

Definitely, the conservativeness requirement played a crucial role in the previous section: it captured the principle of innocence in the best way. Historically, the role of conservativeness in the harmony debate goes back to the first serious reply to Prior's *tonk* challenge, namely the one provided by Belnap. However, Belnap added a further requirement: *Uniqueness*. Formally, uniqueness states that two constants which share the same rules but are notated differently should be interderivable (Belnap 1962: 133).

Suppose, for example, that *und* has the same introduction and elimination rules as *and* ($\wedge$).[29] Then, in order to satisfy uniqueness, it should be possible to derive A *und* $B \vdash A \wedge B$, and conversely (idem: 134). This condition might be useful; the constant $\circ$, which was not ruled out by conservativeness, can be excluded by it. In order to see this, suppose that $\div$ is a notational variant of $\circ$. However, it is not possible to prove

[29] In addition, suppose that the deductive context is fixed, for example by the base system of the conservativeness requirement.

$A \circ B \vdash A \div B$. By $A \circ B$ it is solely possible to conclude B, whereas in order to introduce $A \div B$ both A and B are needed.

Although it might be a promising criterion, until this section uniqueness was not mentioned at all. Thereby the thesis simply followed the development in the harmony debate: uniqueness is, according to Dicher, conservativeness' "forgotten twin" (2016b: 598). In any case, it is in line of the approach of this thesis to not simply adopt uniqueness as an additional requirement. Belnap's motivation to adopt uniqueness as an additional criterion was based upon his analogy with the way definitions are put forward in mathematics (1962: 133). Dicher mentions that a further defence of uniqueness might be needed, but Belnap already indicated how such a defence would look like.

5.4.5 Structural Rules and the Context

Finally, the so-called structural aspect of the context of a rule raises a number of questions. An interesting question is how issues regarding the communicative context and the structural rules need to be settled. For example, in the case of the (problematic) interplay between quantum disjunction and standard disjunction it has been suggested that the cross framework application of *cut* needs to be banned. On the other hand, in the case of *tonk* the moral is that there is something wrong with the connective, and not with the application of *cut*. This case might be quite straightforward, but one can probably imagine more problematic situations.[30]

More in general, there is the issue how disputes at the structural level should be solved or decided. Sometimes there is the tension to move things up to the structural level, for example by Steinberger's statement that *Ex Falso* should be regarded as a structural rule, but it does not settle the disputes regarding these constants or rules. Dicher's analysis complicates this issue, because according to him the operational rules carry, by their context, some structural information.

If Dicher's analysis is indeed correct, then the latter observation carries some irony. Recall from the first chapter that Steinberger appealed to the sequent calculus to distinguish more strictly between the structural and operational rules; vacuous discharge and multiple discharge are not - contrary to the suggestion made by natural deduction - part of the meaning of implication, but a characteristic feature of the deductive system. However, in the end the sequent calculus blurred the distinction even more. By the presence of the context of an operational rule, every constant carries some structural information.

[30]However, even the case of *cut* and *tonk* is, due to Ripley, not that straightforward. He argues that *cut* is no longer admissible once the rules for *tonk* are added to the deductive system (2015: 31).

It is, unfortunately, not possible to outline and discuss the questions raised in this section in more detail. Hopefully, this thesis contributes to the aim to analyse them extensively; both in a formal and a philosophical way.

5.5 Final Conclusion

The thesis is almost finished: it is time to put the main observations together. The conclusion is definitely that a harmony constraint contains, at least partly, a *global* character. This conclusion is supported by both formal and philosophical considerations. In terms of the latter, it turned out that the most satisfying motivation for harmony, the principle of innocence, does not imply a local harmony constraint.

The formal observations emphasized even more the need to go global. Purely local harmony constraints were not able to rule out *bullet* and to prevent against E-weak disharmony; illustrated by quantum and standard disjunction. The principle of innocence implied that *bullet* needs to be ruled out as a meaningful constant. On the other hand, E-weak disharmony is not a problem for the principle of innocence. Moreover, the analysis of this chapter strongly suggested that the aim to rule out quantum disjunction by the maximality condition misses the point. The problem is not quantum disjunction *per se*, but the way it is put together with standard disjunction in a problematic deductive context that causes the troubles.

Hence, the best option is to adopt the conservativeness constraint as the formal counterpart for harmony. It excludes *tonk*, *bullet*, and guarantees the principle of innocence. In order to stabilize conservativeness it seems wise to adopt the *semi-global* version of conservativeness. This means that a base system is adopted which contains just atomic sentences and (a subset) of the structural rules. Accordingly, it is checked for each constant in isolation whether it leads to a conservative extension or not.

By this harmony constraint the meaning of the logical constants contains some global character, but is not as fully global as in the case of, for example, the maximality condition. In addition to the stabilized version of conservativeness, one might add, as the previous section suggested, the requirement of uniqueness. If a further defence of uniqueness is successful, then the harmony requirement mirrors exactly Belnap's original reply to Prior's *tonk* challenge. Metaphorically, the current situation in the quest for proof-theoretic harmony corresponds to harmony in love: we are finally able to stop the quest because we realise that what we were looking for is already there. It just needs to be explored further.

6 Bibliography

Anderson, A.R. and N.D. Belnap, 1962, 'The Pure Calculus of Entailment', *The Journal of Symbolic Logic* 27: 19-52.

Bar-Hillel, Y. and R. Carnap, 1953, 'An Outline of a Theory of Semantic Information', repr. in: Bar-Hillel (ed.), 1964, *Language and Information*, London: Addison-Wesley.

Belnap, N.D., 1962, 'Tonk, Plonk, Plink', *Analysis* 22: 130-134.

Brandom, R.B., 1994, *Making It Explicit*, Cambridge (Mass.): Harvard University Press.

Brandom, R.B., 2000, *Articulating Reasons: An Introduction to Inferentialism*, Cambridge (Mass.): Harvard University Press.

Carroll, L., 1895, 'What the Tortoise said to Achilles', *Mind* 4: 278-280

Cohen, M.R. and E. Nagel, 1934, *An introduction to logic and scientific method*, London: Routledge and Kegan Paul.

D'Agostino, M. and L. Floridi, 2010, 'The Enduring Scandal of Deduction: Is Propositional Logic Really Uninformative?', *Synthese* 167: 271-315.

Dalen, D. van, 2008, *Logic and Structure*, Berlin/Heidelberg: Springer - Verlag.

Dicher, B., 2016a, 'A Proof-Theoretic Defence of Meaning-Invariant Logical Pluralism', *Mind* 125: 727-757.

Dicher, B., 2016b, 'Weak Disharmony: Some Lessons for Proof-Theoretic Semantics', *The Review of Symbolic Logic* 9: 583-602.

Dummett, M., 1973, 'The Philosophical Basis of Intuitionistic Logic' in: *Philosophy of mathematics*, P. Benacerraf and H. Putnam (eds.) Cambridge: Cambridge University Press.

Dummett, M., 1991, *The Logical Basis of Metaphysics*, London: Duckworth.

Dutilh Novaes, C., 2011, 'The Different Ways in which Logic is (said to be) Formal', *History and Philosophy of Logic* 32: 303-332.

Duží, M., 2010, 'The Paradox of Inference and the Non-Triviality of Analytic Information', *Journal of Philosophical Logic* 39: 473-510.

Edwards, J., 1995, 'The Universal Quantifier and Dummett's Verificationist Theory of Sense', *Analysis* 55: 90-97.

Francez, N., and Dyckhoff R., 2012, 'A Note on Harmony', *Journal of Philosophical Logic* 41: 613-628.

Gentzen, G., 1935/1964, 'Investigations into Logical Deduction', *American Philosophical Quarterly* 1: 288-306.

Griffiths, O., 2014, 'Harmonious Rules for Identity', *The Review of Symbolic Logic* 7: 499-510.

Girard, Y, 1987, 'Linear Logic', *Theoretical Computer Science* 50: 1-101.

Hacking, I., 1979, 'What is Logic?', *The Journal of Philosophy* 76: 285-319.

Halbach, V. and G.E. Leigh, 'Axiomatic Theories of Truth', The Stanford Encyclopedia of Philosophy (Summer 2014 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/sum2014/entries/truth-axiomatic/.

Hintikka, J., 1973, *Logic, language games and information. Kantian themes in the philosophy of logic.* Oxford: Clarendon Press.

Hjortland, O.T., 2010, *The Structure of Logical Consequence: Proof-Theoretic Conceptions*, PhD Thesis University of St. Andrews.

Ketland, J., 1999, 'Deflationism and Tarski's Paradise', *Mind* 108: 69-94.

MacFarlane, J., 2000, *What does it mean to say that logic is formal?*, PhD Thesis University of Pittsburgh.

MacFarlane, J., 2002, 'Frege, Kant, and the Logic in Logicism', *The Philosophical Review* 111: 25-65.

Milne, P., 2002, 'Harmony, Purity, Simplicity, and a "Seemingly Magical Fact"', *The Monist* 85: 498-534.

Murzi, J. and M. Carrara, 2014, 'More Reflections on Consequence', *Logique Et Analyse* 227: 223-258.

Murzi, J. and F. Steinberger, 2015/forthcoming, 'Inferentialism', in: *The Blackwell Companion to the Philosophy of Language*, B. Hale, A. Miller, C. Wright (eds.). Draft version: http://floriansteinberger.weebly.com.

Negri, S. and J. von Plato, 2001, *Structural Proof Theory*, Cambridge: Cambridge University Press.

Paoli, F., 2003, 'Quine and Slater on Paraconsistency and Deviance', *Journal of Philosophical Logic* 32: 531-538.

Peacocke, C., 1987, 'Understanding Logical Constants: A Realist's Account', *Proceedings of the British Academy* 73: 153-200.

Prawitz, D., 1965, *Natural Deduction: A Proof Theoretical Study*, Stockholm: Almqvist and Wiksell.

Prawitz, D., 1974, 'On the Idea of a General Proof Theory', *Synthese* 27: 63-77.

Prawitz, D., 1994, 'Book Review of *The Logical Basis of Metaphysics*', by Michael Dummett, *Mind* 103: 373-376.

Prior, A., 1960, 'The Runabout Inference-Ticket', *Analysis* 21: 38-39.

Putnam, H., 1969, 'Is Logic Empirical?', in: R.S. Cohen and M.W. Wartofsky (eds.), *Boston Studies in the Philosophy of Science* 5: 216-241.

Quine, W.V.O., 1951, 'Two Dogmas of Empiricism', *The Philosophical Review* 60: 20-43.

Quine, W.V.O., 1960, *Word and Object*, Cambridge (Mass.): The MIT Press.

Read, S., 2000, 'Harmony and Autonomy in Classical Logic', *Journal of Philosophical Logic* 29: 123-154.

Read. S., 2008, 'Harmony and Modality', in: C. Degremont, L. Kieff and H. Ruckert (eds.), *Dialogues, logics and other strange things: Essays in honour of Shahid Rahman*, pp. 285-303, London: College Publications.

Read, S., 2010, 'General-Elimination Harmony and the Meaning of the Logical Constants', *Journal of Philosophical Logic* 39: 557-576.

Read, S., 2016, 'Harmonic Inferentialism and the Logic of Identity', *The Review of Symbolic Logic* 9: 408-420.

Restall, G., 2005, 'Multiple Conclusion', in: P. Hajek, L. Valdes-Villanueva, and D. Westerstahl (eds.), *Logic, methodology and philosophy of science: Proceedings of the 12th international congress*, London: King's College Publications.

Ripley, D., 2015, 'Anything Goes', *Topoi* 34: 25-36.

Rumfitt, I., '"Yes" and "No"', *Mind* 109: 781-823.

Rumfitt, I., 2016/forthcoming, 'Against Harmony', in: B. Hale, A. Miller, C. Wright (eds.), *The Blackwell Companion to the Philosophy of Language*. See: http://philpapers.org/archive/RUMAH.pdf.

Schroeder-Heister, P., 1984, 'A Natural Extension of Natural Deduction', *The Journal of Symbolic Logic* 49: 1284-1300.

Steinberger, F., 2009a, *Harmony and logical inferentialism*, PhD Thesis University of Cambridge.

Steinberger, F., 2009b, 'Not so stable', *Analysis*: 655-661.

Steinberger, F., 2011a, 'What Harmony Could and Could Not Be', *Australasian Journal of Philosophy* 89: 617-639.

Steinberger, F., 2011b, 'Harmony in a sequent setting: a reply to Tennant', *Analysis* 72: 273-280.

Steinberger, F., 2011c, 'Why Conclusions Should Remain Single', *Journal of Philosophical Logic* 40: 333-355.

Stevenson, J.T., 1961, ‘Roundabout the runabout inference-ticket’, *Analysis* 21: 124-128.

Strawson, P.F., 1952, *Introduction to Logical Theory*, London: Methuen and CO LTD.

Tennant, N., 1978, *Natural Logic*, Edinburgh: Edinburgh University Press.

Tennant, N., 1987, *Anti-Realism and Logic*, Oxford: Oxford University Press.

Tennant, N., 1994, ‘Intuitionistic Mathematics Does Not Need *Ex Falso Quodlibet*’, *Topoi* 13: 127-133.

Tennant, N., 1997, *The Taming of the True*, Oxford: Oxford University Press.

Tennant, N., 2004, ‘Negation, Absurdity and Contrariety’ in: D. Gabbay and H. Wansing (eds.), *Negation*, Kluwer Academic Press.

Tennant, N., 2010, ‘Harmony in a sequent setting’, *Analysis* 70: 462-468.

Tennant, N., 2012, ‘Cut for core logic’, *Review of Symbolic Logic* 5: 450-479.

Tennant, N., 2014/forthcoming, ‘Inferentialism, logicism and a counterpoint’, in: *Essays for Crispin Wright: Logic, Language, and Mathematics*, ed. A. Miller. Volume 2 of a two-volume Festschrift for Crispin Wright, co-edited with A. Coliva. Oxford: Oxford University Press.