The Logical Dynamics of Gossip: an analysis in Dynamic Epistemic Logic

MSc Thesis *(Afstudeerscriptie)*

written by

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under the supervision of **Prof. dr. Sonja Smets** and **Dr. Fernando Velázquez Quesada**, and submitted to the Board of Examiners in partial fulfillment of the requirements for the degree of

MSc in Logic

at the *Universiteit van Amsterdam.*

Date of the public defense:
March 2nd, 2017

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Abstract

This thesis gives a formal logical analysis of a social phenomenon; gossip. Gossip can be used to serve many goals. The Sherlock Holmes' of our society -always looking for an explanation- might use gossip to find an explanation for a surprising observation. Also for the agents that are not so eager to find the truth, for those who are more concerned about their social position instead, gossip might be helpful. Gossip can be used as a tool to exclude or include a specific person from a group. Further, gossip can help to increase or decrease the social bonding within a group. An agent might start to gossip in order to reach any of these goals. Each of them gives the gossip conversation a different structure. This work formally describes and models the epistemic and social sides of gossip and proceeds by giving formal protocols that correspond to a number of different conversation structures. In the formal part we use dynamic epistemic logic, which is able to model the changes in the knowledge and beliefs of agents. In order to deal with the changes in the network relations, we add a social network structure to each epistemic state as well as actions that change those network structures.

Acknowledgements

First and foremost I would like to thank my supervisors Sonja Smets and Fernando Velázquez Quesada. I would like to thank both of them for their invaluable supervision, guidance and enthusiasm. I learned a lot from working with them. Next, I would like to thank the other members of the committee, Benedikt Löwe, Dora Achourioti, Maria Aloni and Frank Veltman, for reading through my thesis and for the valuable remarks made during my defense. I am grateful to my peers of the Master of Logic, during the entire masters they helped me by studying together and proofreading my work. Furthermore, I would like to thank my dearest friends, Evi, Mickey, Marije, Laura, Hanna, Michelle and Marleen, who have been there over the past years to support me. Special thanks to my parents, Bart and Wilma for their spiritual and financial support. And last but surely not least I want to thank Luc, for never ever doubting me and for standing by me throughout the highs and lows of this project.

Contents

Chapter 1

Introduction

> At the turn of the century it was claimed that there might be "an essential identity between the gossip[er] and the genius". [(Logan, 1908, p.106)] The argument went that both types have great intellectual curiosity, excellent recall, and the ability to make novel and original connections between events.[...] (Rosnow and Fine, 1976, p.83).

This thesis is about gossip, a social phenomenon we all are acquainted with. Gossip might seem to be "unintelligent talk" about "irrelevant" ideas (as in the quote above), but its influence is not to be underestimated. From the social point of view, "[g]ossip fulfills a variety of essential social network functions" (DiFonzo and Bordia, 2007a, p.19). The social impact of gossip can be huge; it might lead to social exclusion or, since gossip can also be positive, social inclusion. It is the potential negative social influence that gives gossip its unethical and negative flavour. Apart from the ethical discussion around gossip in philosophy and the discussions around the social influences of gossip in sociology (e.g. Eder and Enke 1991), psychology (e.g.Rosnow and Fine 1976; DiFonzo et al. 2013) and the behavioural sciences (e.g. Houmanfar and Johnson 2004), gossip can also be discussed from an epistemic point of view. Gossip aims at creating "a Knowledge Base of social information between the group (as a projected whole) and its members" (Bertolotti and Magnani, 2014, p.4037). As gossip changes the beliefs and knowledge of gossipers, it has an important epistemic function. Further, gossip is said to have an abductive nature and can be seen as a tool to find a (non-trivial) explanation for a surprising event (Bertolotti and Magnani, 2014). This thesis aims to merge the sociological and formal epistemological view on gossip. Although the paper of Bertolotti and Magnani (2014) is slightly more formal than the average philosophical or sociological ones, gossip is still underexposed from a formal perspective.

In this thesis we use logic as a tool to clarify the structure that gossip can take, by focussing on the epistemic and social dimensions and by making these formally explicit we aim to enhance our understanding of the gossip phenomenon and its effects. First, flow diagrams are used to show the typical order in which communication and reasoning inferences take place during a gossip scenario. Each communication and reasoning act might change the epistemic state of the participants of the gossip scenario. We will use the dynamic epistemic logic proposed by Baltag and Smets (2008) to express the dynamics of the epistemic states of agents during gossip. Based on these diagrams, the logic enables us to define formal protocols for gossip. Those protocols are descriptive and meant to describe the order in which different kinds of epistemic actions typically occur in a gossip scenario. In order to include the social acts and to model the dynamics of the network structure among the participants of gossip, we will extend the logic. Semantically we will add a network structure among the agents (based upon the work of Baltag et al. (2016); Christoff (2016)) to each epistemic state; syntactically we will add atomic propositions that are able to express agents to be connected. Further, we define the social acts of deleting and adding a network relation.

The structure of this thesis is designed as follows: In the next chapter we discuss different definitions of gossip that can be found in the literature. Based on those definitions, this thesis gives a definition to work with in the later chapters. While discussing the meaning and characteristics of gossip, we will list the different goals gossip can fulfil. Some of those goals are epistemic and due to the abductive nature of gossip. Others are social, such as including or excluding an agent from a (social) group. Gossip can be seen as a sequence of communication and reasoning acts. The structure of a gossip scenario is discussed in the second chapter and depends on the goal the gossiper aims to fulfil. We will discuss the possible structures of gossip and illustrate them by flow diagrams. Then, the formal part of the thesis begins. The third chapter focusses on gossip from an epistemic perspective. Dynamic epistemic logic is able to describe and model the changes in the knowledge and beliefs of the participants. The fourth chapter explores the dynamics of the social network structure. It extends the logic of chapter three in a way such that it becomes able to deal with the social side of gossip as well. The third and fourth chapter use the flow diagrams, presented in chapter 2, to describe formal protocols for gossip scenarios. In chapter 3 this is done for gossip with an epistemic goal and in chapter 4 for gossip with a social goal.

Chapter 2

The definition of gossip

This thesis is about gossip. The reader probably has lots of empirical experiences with this social phenomenon outside the academic context. On the one hand, this triggers the interest of the reader and makes academic work about gossip more enjoyable to read. On the other hand, it brings a great difficulty: not only has the reader some informal empirical knowledge, beliefs and intuitions about the topic (which is in general not a bad thing), but also, based on his or her past experiences, the reader has given the word 'gossip' his or her own (implicit) meaning. This results in individual differences in the understanding of 'gossip', depending on cultural and linguistic differences. Those differences might cause the unaware reader to misread and misunderstand academic work about daily topics such as gossip.

Even the reader who is more familiar with gossip in an academic context may give 'gossip' a meaning which differs from the one intended by the writer. This happens because 'gossip' appears in many different disciplines, each with its own (implicit) definition of 'gossip'. Therefore, in order to make clear what this thesis is about, it is of great importance to clarify the definition that is adopted in this work.

This section discusses the intuitive and scientific meaning of 'gossip'. Furthermore, it gives a definition of 'gossip' based upon previous academic work. The given definition describes what this thesis is about. Apart from the next section about the definition of gossip, this thesis is not about the meaning of the word 'gossip' but instead about the social phenomenon defined as (and referred to by) 'gossip'.

The aim of this thesis is to merge the social and epistemic perspective on gossip. Both approaches can be found in the literature. The social perspective is for example emphasised in the definition of DiFonzo and Bordia (2007a):

Definition 1. *Gossip (DiFonzo and Bordia, 2007a, p.19)*
Gossip is evaluative social talk about individuals, usually not present, that arises in the context of social network formation, change and maintenance- that is, in the context of building group solidarity. Gossip fulfills a variety of essential social network functions including entertainment, maintaining group cohesiveness, and establishing, changing, and maintaining group norms, group power structure, and group membership.

The epistemic importance of gossip is emphasised by Bertolotti and Magnani

(2014):

> Gossip is such a fundamental source of knowledge concerning our fellow human beings that it begs indeed for a serious epistemological analysis focusing on the *kind* of knowledge conveyed by gossip, how it actually manages to *share* any knowledge, and its further *knowledge forming* capabilities.
> (Bertolotti and Magnani, 2014, p.4038-4039).

Both citations focus only on one of those two sides of gossip. The next sections will discuss both sides of gossip separately and continue with the combination of both perspectives. The definition given by this thesis, presented at the end of this section, emphasises both the epistemic and the social perspective on gossip.

2.1 The epistemic value of gossip

It is not without any reason that gossip is called a "fundamental source of knowledge" (Bertolotti and Magnani, 2014, p.4038). Gossip depends on the knowledge and beliefs of agents, the ability to share information, to gain knowledge and to adjust beliefs. This section discusses gossip from an empirical perspective.

2.1.1 The content of gossip

The content of gossip emphasises its epistemic value. Gossip is, according to many authors, (e.g. DiFonzo and Bordia 2007a; Rosnow and Fine 1976; Bertolotti and Magnani 2014) evaluative talk about an absent person. This is in line with the view of Houmanfar and Johnson (2004), who state that "the [verbal] stimulus in the gossip-type interaction is primarily information regarding a third party (e.g., I have heard that the new boss has a tendency to be hot tempered)" (Houmanfar and Johnson, 2004, p.119). It is important to note that not every piece of information about a third person necessarily provokes gossip. If the information is either trivial or known to be common knowledge, sharing this information would not make any sense.

The epistemic value of gossip can also be found in the work of Rosnow and Fine (1976). They define gossip as "non-essential (often trivial) news about someone" (Rosnow and Fine, 1976, p.87). This definition tells us that gossip is non-essential news. Although called "non-essential", gossip is defined by Rosnow and Fine (1976) as "news". The information should have some epistemic value in order to provoke gossip. It has to be surprising for at least some agents in the gossip network. Bertolotti and Magnani (2014) state that it is typically a surprising observation about a third party that initiates the gossip. In order to find an explanation for the surprising observation the agent will use abductive reasoning. This abductive process gives rise to the belief of a best, non-trivial explanation, that provokes the observer to gossip.

2.1.2 Communication and reasoning

It is not just the verbal stimulus that gives gossip its epistemic value. Due to reasoning and communication, without which there would not be gossip, the knowledge and beliefs of the agents change constantly during gossip. One way to change the doxastic and epistemic states of an agent in gossip is by inference, based on deduction, induction and last but not least abduction. Like communication, those internal epistemic processes give rise to new beliefs. In a later section, this thesis elaborates on the possible kinds of epistemic and doxastic changes that typically occur in gossip. The most interesting reasoning process that occurs in gossip is abduction. The current work agrees with Bertolotti and Magnani (2014) that gossip has an abductive nature; it typically starts with a surprising observation after which the agents together try to find a best explanation. It seems indeed to be the case that "a significant role of gossip is to function as a *group-based abductive appraisal of social matter*" (Bertolotti and Magnani, 2014, p.4038). In a later section, this thesis will elaborate on the interesting role abduction plays in gossip.

2.1.3 The epistemic goals

One can use gossip in order to achieve an epistemic goal. These epistemic goals have to do with the initial surprising observation that provoked abductive reasoning and the search for the best explanation. By gossiping one can use the knowledge of the group to achieve one of the following two epistemic goals:

1. getting to know an explanation of a surprising observation;
2. getting to know whether a certain proposition, typically one of the possible explanations for the surprising observation, is true.

Apart from those two epistemic goals, gossip can also fulfil some social goals. The next section discusses the social perspective of gossip and presents the social goals of gossip.

2.2 The social perspective of gossip

2.2.1 The content of gossip

The previous sections discussed gossip from an epistemic point of view, but gossip is not just an epistemic tool; it is also a social phenomenon. Recall that the content of gossip is not just information about any event but always about a person. It is the social perspective of knowledge that gives gossip its unethical reputation. Gossip is evaluative talk, either positive or negative, about an absent person.

2.2.2 The context of gossip and its group functions

Also the context of gossip shows that gossip is a social phenomenon. Gossip is said by DiFonzo and Bordia (2007a) to appear in contexts where social networks are built, (re)structured or maintained. For the group as a whole, gossip has the function to establish, change or maintain either the group memberships, the group power structure or the norms of the group. Further, gossip supplies social information and might entertain (DiFonzo and Bordia, 2007a). This thesis neglects the entertaining function of gossip as well as the function to change the power structure or the group norms. However, the proposed models will be able to model the epistemic and social dynamics, also when the goal of gossip is to entertain. Gossip thus might cause the network structure to change. Therefore, one of the goals of this thesis is to model the network changes. Another goal is to model the (dynamic of the) knowledge and beliefs of agents about the network structure.

2.2.3 The social goals of gossip

Apart from the epistemic goals listed above, gossip can be used to achieve several social goals. An agent will typically only start to gossip if he or she wishes to achieve one of the social or epistemic goals. However, note that gossip may have side-effects. For instance, one can start to gossip about a common friend for epistemic reasons; as a side effect (if the gossip is morally considered to be negative), this might lead to social exclusion of the common friend. Further, even if agents have a goal to start gossiping it might not be sure that gossip will make them achieve their goal (if they try to find out information about someone, they might not get that information if it is not there). So there is some uncertainty involved at the start, they have an aim or goal they would like to see fulfilled.

An agent may start to gossip with one of the following social goals, furthermore, all these goals may occur as a side-effect of gossip in general:

1. social inclusion of someone;
2. social exclusion of someone;
3. strengthen bondings within a group;
4. weaken bondings within a group.

This thesis represents strengthening and weakening of the social bonds by means of changes in the agents' knowledge about the network structure. This is where the epistemic and social side of gossip come together. The next section is about the interplay of those two perspectives and explains in more detail what is meant by the strength of a social bonding as well as by strengthening and weakening those bondings between agents.

2.3 The interplay between the epistemic and the social perspective

The content, the context and even the group functions all show us an interplay between the epistemic and social perspective on gossip. Gossip is all about social information. The content is social information (about an absent person) and the group functions are all about social information (about the network structure and about members of the group). For example, sharing information about complex social environments, by means of gossip, helps to achieve one of the group goals of gossip: building group solidarity (DiFonzo and Bordia, 2007a). Further, as Foster (2004) mentions, because of the complexity of the groups, gossip enables agents to obtain information about the members of the groups they are part of that is impossible to obtain directly. Let us again have a look at two of the social goals:

1. strengthen bondings within a group;

2. weaken bondings within a group.

When a group gossips together about an outsider, either positively or negatively, this will strengthen the bonding of the group. The process of gossiping together emphasises the group membership of the participants. Both, the participants as well as random observers of the gossip processes will come to believe, or know about the social bondings between the gossipers. By (negative) gossip about an insider, gossip can also weaken the bondings within a group.

In the formal language, we represent this bonding by a network relation. Such a connection is to be interpreted as something in between a friendship connection and a connection that gives agents the possibility to communicate to each other. If two people are connected, they are able to communicate. The strength of the bond depends on the frequency of communication as well as on how hard it is to communicate with the other agent. A strong connection can be seen as friendship. The question now is how to model the strength of a bonding and how to model strengthening and weakening those bondings. This work proposes to define the strength of the bondings between two people by means of notions such as knowledge and beliefs. The number of people that know/believe a social network relation to exist correlates positively with the strength of the bonding between the agents.

It is intuitive that, in general, the stronger one's bond with someone, the larger the number of people that know about one being socially connected to this other person. Of course there are some exceptions where the bond is strong but unknown by other agents (think about a secret affair). This work proposes that in general, the stronger the connection between two agents, the more often and the easier they will communicate and the more often they will be (seen) together. Hence, the stronger the bonding between two agents, the larger the number of people that will know about this connection. The weaker the bond, the less people will know about it. In case there is no connection between two agents, the stronger this disconnection is, the more people will know about this disconnection.

My claim is that the reverse also holds. Assume that one wants to get some information about someone (person a). Suppose that one knows or believes that person a is socially connected to another agent(person b). Since one is socially connected to person b, one will ask person b to contact a. This will strengthen the social bond between person a and person b. Furthermore, the agents around person a and person b will approach them as if they have a strong connection, which will (most of the time) cause the connection between the two agents to get stronger. The reversed will weaken the bonding. When dealing with the goal of weakening or strengthening the social bondings between agents in the formal setting, this thesis will elaborate on this way to adjust the strength of a social bonding.

2.4 What this thesis is not about

The previous sections discussed the meaning of 'gossip'. This section discusses what gossip is not.

It is important to note that this thesis is not about the so-called 'gossip problem', presented by Baker and Shostak (1972) and Berman (1973). Based upon this problem van Ditmarsch et al. (2017); Apt et al. (2015) and others propose formal protocols for what they call 'gossip'. However, those papers assume that the agents have the (common) goal to get to know as many secrets in as little time as possible. While interesting, those protocols are not about what one generally understands gossip to be; but instead they are about some form of secret sharing. In any case, what is called 'gossip' by these works is not what this thesis is about.

Further, gossip is often confused by rumour or urban legends. DiFonzo and Bordia (2007a,b) therefore explicitly distinguish not only between gossip and rumour as Rosnow and Fine (1976) do, but they add urban legends to the comparison. Those three, as they argue, are often mixed up in daily and academic use. DiFonzo and Bordia (2007a) formulated definitions for those three phenomena, based on what they call 'three main features': 1. the context in which gossip, rumour or urban legend appear, 2. the content of it and 3. the group function it has.

This thesis already discussed the content, context and group functions of gossip. This section briefly discusses the differences with respect to rumour and urban legends. which are defined by DiFonzo and Bordia (2007a) as follows:

Definition 2. *Rumour by DiFonzo and Bordia (2007a, p.13)*
We define rumo[u]r as unverified and instrumentally relevant information statements in circulation that arise in context of ambiguity, danger, or potential threat and that function to help people make sense and manage risk.

Definition 3. *Urban legends by DiFonzo and Bordia (2007a, p.13)*
Urban legends are stories of unusual, humorous, or horrible events that contain themes related to the modern world; are told as something that did or may have happened, variations of which are found in numerous places and times; and contain moral implications.

Urban legends and rumour appear according to DiFonzo and Bordia (2007a), respectively in the context of storytelling and ambiguous or threatening events or situations. They both differ from gossip in content, since both are, in contrast to gossip, typically instrumentally relevant information statements that are unverified (DiFonzo and Bordia, 2007a). Rumour statements are often about events, whereas gossip typically has a person as subject (Rosnow and Fine, 1976; DiFonzo and Bordia, 2007a). Urban legends are even more distinct, they are mostly entertaining narratives. Those differences are fundamental for the definitions of those three social phenomena given by DiFonzo and Bordia (2007a). This thesis will focus merely on gossip, because of the epistemic and network dynamics it brings about.

Bertolotti and Magnani (2014) give two further distinctions between gossip and rumour that are rather important for a clear understanding of what gossip is, based on the work of Coady (2012). First:

> Gossip may well be first-hand. By contrast no first-hand account of an event can be a rumo[u]r, though it may later become one (Coady, 2012, p.87).
> In (Bertolotti and Magnani, 2014, p.4041).

And further:

> As far as rumo[u]r is concerned, unless one deems herself expert in the topic of a rumo[u]r, or has other good reasons for doing so, she will not take a stand to reject a rumo[u]r that she judges false– she will merely not pass it on. Conversely, in gossip everyone is an *expert* about one's acquaintances, and a self-proclaimed moral expert in human affairs (how often do we proclaim ourselves unable to proffer a personal moral say-so on a given situation?): if one is not an accidental bystander, even her silence and passivity will be a form of engagement and collaboration, as gossip makes no (moral nor epistemological) room for indifference.
> (Bertolotti and Magnani, 2014, p.4042-4043).

This last difference between gossip and rumour will be especially useful when describing the protocols for gossip. It states that, in the case of gossip, if the gossip conflicts with one's previous knowledge or beliefs he or she has to speak up.

2.5 How this thesis defines gossip

Gossip is here defined as evaluative social talk about individuals. It can be seen as an interplay between the gossipers' beliefs and knowledge about a specific individual. Furthermore, most of the social network functions are about changes in the network structure of the gossipers. Gossip has not to be confused with rumour or urban legends, which differ from gossip in context, content and group goal (DiFonzo and Bordia, 2007b,a). During a gossip scenario there are at least two features of importance: the change of (social) information in a group and the (potential) changes of social network relations. This work uses the following definition of gossip:

Definition 4. *Gossip*
Gossip is a social epistemic phenomenon. It is provoked by a surprising observation about an individual and has an abductive nature. Gossip can help to get to know the (truth value of an) explanation of this surprising event. The content of gossip is evaluative social news about an individual, usually not present. Because of the social goals gossip can fulfil (i.e. exclusion, inclusion and strengthening or weakening the bonds of a group), it arises in a social context. Those goals can be achieved only by sharing knowledge and beliefs. Furthermore, because of the complexity of the groups, gossip enables agents to obtain information about the members of the groups they are part of that is impossible to obtain directly. Gossip is a fundamental source of (social) knowledge that makes no (moral nor epistemological) room for indifference.

This definition shows two kinds of dynamics in Gossip. First, it shows the influence of gossip on the beliefs and knowledge of agents. Secondly, it states that gossip might change the network structure of a group. In the next chapters of this thesis, we formalize those two types of dynamics.

Chapter 3

The structure of gossip

This chapter discusses the structure of gossip. It discusses the epistemic and doxastic changes that typically occur during acts of gossip. In order to describe and model the epistemic changes during a gossip scenario, we have to deal with two kinds of processes. First, we focus on the external processes, such as observations and communication. Secondly, we analyse the internal epistemic processes, i.e. human reasoning. This section, therefore, is divided into two parts. The first part describes the intra-personal epistemic changes that occur during gossip. The second part discusses the interpersonal changes and the changes that occur in the social network structure during gossip.

3.1 Intra-personal epistemic changes; deduction, induction and abduction

According to Peirce (1958, CP 5.145, CP 2.96 and CP 2.774) there are three kinds of reasoning (or arguments): deduction, induction and abduction. The last one, according to Peirce (1958, CP 5.145, CP 2.96 and CP 2.774) is usually called 'adopting a hypothesis '(Peirce, 1958, CP 5.145, CP 2.96 and CP 2.774). Peirce refers to this last kind of reasoning as presumption (CP 2.774). According to Ma and Pietarinen (2016) Peirce also calls this type of reasoning "hypothetical, retroductive, adductive or presumptive [reasoning]"(Ma and Pietarinen, 2016, p.74).

This thesis assumes the agents to be logically omniscient. Hence they will instantaneously have all knowledge that real agents will obtain by deductive reasoning. The current work will not deal with inductive reasoning. In contrast, this thesis does deal with abduction. The reason that this work models abduction but not induction is the abductive nature of gossip as noted by Bertolotti and Magnani (2014). Later on, this work gives some examples that stress the abductive nature of gossip and show that the example of abduction as given by Bertolotti and Magnani (2014) is actually a combination of abductive and deductive reasoning.

The next subsection discusses the theoretical background of abduction, the role of abduction in gossip scenarios and the way in which we incorporate abductive reasoning in the proposed logic.

3.1.1 Abduction

Peirce famously described the inferential structure of abduction (Peirce, 1958):

Definition 5. *The inferential structure of abduction according to Peirce (1958, CP 5.189)*

1. *The surprising fact, C, is observed;*
2. *But if A were true, C would be a matter of course,*
3. *Hence, there is reason to suspect that A is true.*

This is seen as the standard or classical structure of abductive reasoning. Nevertheless, abductive reasoning itself is more complex and largely discussed in literature. Abduction is an interesting form of reasoning because it is "[o]riginary in respect to being the only kind of argument which starts a new idea."(Peirce, 1958, CP2.774). Furthermore, "[a]bduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea; for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis " (Peirce, 1958, CP 5.171).

This is exactly what makes abduction an interesting reasoning process, often used in daily life, science and, last but not least, gossip. An agent might want to verify new beliefs, potentially about other agents, by scientific means or by gossip.

While there is general agreement about the abductive syllogism, questions were raised about the process of generating explanations (also called conjectures), the way in which the best explanation is chosen, the kind of consequence relation that is used in abductive reasoning, the properties of formulas that define an abductive problem as well as the properties of formulas that define an abductive solution. Last but not least there is literature about algorithms to find abductive solutions.

Abductive reasoning can be seen as a four step reasoning process. First an agent has to recognize an abductive problem. Secondly the agent identifies a set of possible solutions (also called explanations). Then the agent will select one best solution from this set. The agent concludes his abduction by incorporating this explanation with his current information. (Velázquez-Quesada et al., 2013; Nepomuceno-Fernández et al., 2013). It is the selection of the best explanation that has received the most attention in the literature.

One proposal, which is specifically interesting for this thesis because of its dynamic epistemic perspective, is presented by Velázquez-Quesada et al. (2013) and Nepomuceno-Fernández et al. (2013). We will discuss the concepts of abductive problem and abduction solutions later on. As soon as the logical language is defined, we will also discuss how a best explanation can be selected from the set of possible solutions.

Velázquez-Quesada et al. (2013) write:

> The present work proposes an approach to abductive reasoning from an epistemic and dynamic perspective. Instead of understanding abductive reasoning as a process that *modifies a theory* whenever *there is a formula* that *is not entailed by the theory under some*

> *particular consequence relation*, as the traditional definition of an abductive problem does, we propose an approach that understands abductive reasoning as a process that *changes an agent's information* whenever, *due to some epistemic action, the agent has come to know or believe a fact* that she *could not have predicted otherwise.* (Velázquez-Quesada et al., 2013, p.506-507).

Nepomuceno-Fernández et al. (2013) and Velázquez-Quesada et al. (2013) approach abduction from an epistemic and dynamic perspective. One aim of this thesis is to model the epistemic changes during gossip. For that reason, their perspective on abduction is very valuable for this thesis. Hence, this work discusses and adopts their definition of an abductive problem, their definition for an abductive solution and their abduction modality. This thesis also follows Velázquez-Quesada et al. (2013) in the use of not one, but two epistemic attitudes, one to express knowledge, and one to express belief.

> The reason is that an agent typically tries to explain facts that she knows as the result of some observation, but the chosen solution, being a hypothesis that might be dropped in the light of further observations, should not attain the full certainty status. Moreover, the use of two epistemic notions gives us more flexibility to deal with a wider variety of abductive problems and abductive solutions, and makes our analysis closer, we think, to Peirce's original formulation. (Velázquez-Quesada et al., 2013, p.507).

The goal of this thesis is not to model abduction on its own, but to model gossip, which involves abductive reasoning. One of the characteristics of gossip is that the gossiper generally does believe, but does not know whether the proposition he shares is true. One is not absolutely sure about his or her colleague having an affair or being fired, although one has some evidence and therefore the belief that her colleague does have an affair. As the previous section stated, one of the goals of gossip is to gain information, i.e. to verify whether one's belief is true. This thus is in line with the abductive part of the verification of the possible conjecture.

3.2 External and inter-personal epistemic changes: observation and communication

Apart from internal reasoning, also external influence such as observations and communication can cause changes to the doxastic and epistemic states of the agents. The logic thus has to be able to deal with (surprising) observations, since gossip typically starts with one. Furthermore, agents can share knowledge and beliefs by announcements, this provokes the knowledge and beliefs to change. Gossip is not an individualistic phenomenon; according to Bertolotti and Magnani (2014), the actual actors are not individuals but groups as embodying epistemic synergies. Hence, gossip is not an individual phenomenon but an interplay of abductive reasoning between agents. The current work agrees with Bertolotti and Magnani (2014) about the abductive nature of gossip. This section discusses the abductive structure of gossip as described by Bertolotti and

Magnani (2014). We make their proposal more formal and use the proposed structure of gossip in the later part of this thesis.

The intuition behind the work of Bertolotti and Magnani (2014) is as follows: Each agent has his own Individual Knowledge Base, denoted by KB_n^I, where the superscript I stands for 'individual' and where n is a natural number that corresponds to the agent. Further, we describe the Group Knowledge Base (denoted by KB^G). The knowledge of each agent who is part of a certain epistemic group G is added to this Group Knowledge Base KB^G. The Group Knowledge Base thus contains all knowledge that each agent which is part of the group can possibly obtain (in an ideal scenario) by communication with all group members. This notion of group knowledge is in epistemic logic typically called distributed knowledge. The intuition of Bertolotti and Magnani (2014) is shown in the following schema, m is the notation used to represent an anomaly or misbehaviour, but may in principle represent any proposition:

> 1. $m \in KB_1^I$: Anomaly m is witnessed by [i]ndividual 1, and stored in her Knoweldge Base.
> 2. If $m \in KB_1^I$ [then] $m \in KB^G$: If m is stored in [i]ndividual 1's [Individual Knowledge Base KB_1^I], and if no more up-to-date instance is present in KB^G, then m will be transferred to KB^G.
> 3. If $m \in KB^G$ [then] possibly $m \in KB_2^I, ...KB_n^I$: Once received by KB^G, anomaly m can be possibly be transferred to all other connecting Individual Knowledge Bases.
>
> (Bertolotti and Magnani, 2014, p.4046).

The notions of an Individual Knowledge Base and a Group Knowledge Base are the basis of their proposal. They are used to describe the function of gossip as a "group-based abductive appraisal of social mater"(Bertolotti and Magnani, 2014, p.4038). Gossip is an abductive process. It is started by the initial observer of the surprising observation. The goal of the group is to discover an explanation of a surprising event. First the observer will use abduction to provide her best explanation for the surprising event. She gets to believe a specific explanation because of her information base. Note that the concept 'information base' of an agent is not the same as her Individual Knowledge Base. The information base of an agent instead consists of the Individual Knowledge Base plus a certain disposition, which is composed of several components such as the mood of the agent, her feelings and cognitive endowments. The agent then stores the surprising event in the Group Knowledge Bases. This enables others to receive the knowledge about the surprising event. Note that agents may belong to several different groups and hence can have several Group Knowledge Bases. However, the other agents will not just without any effort obtain the knowledge about the surprising event. Typically, the first observer shares only her best explanation for the event and not yet the knowledge of the surprising observation. Next, the hearer aims to assess the abduction that leads to this gossip and tests the conclusion to his own background. This process is called the counter-abduction. Note that it is important to receive the knowledge about the surprising event in order to assess the abduction that led to the best explanation of the initial observer. This counter-abduction might lead to a discrepancy of the best explanation with the Individual Knowledge Base of the hearer. If this discrepancy cannot be solved by either of the two, there will be a new best explanation which again might rise as a topic of gossip (Bertolotti and Magnani, 2014).

The current work proposes to model gossip based on dynamic epistemic network models which look quite different compared to the models used by Bertolotti and Magnani (2014). The dynamic epistemic network models proposed in this thesis, are in contrast to the models of Bertolotti and Magnani (2014), formal logical models. Even though more expressive, the dynamic epistemic network models are in line with the proposal of Bertolotti and Magnani (2014). This thesis will model the network structure of a group of agents together with their beliefs, safe belief and knowledge. By announcements the agent can share their knowledge with the other agents in the model. Agents can change their knowledge and beliefs based on those announcements. In this way, the knowledge of an individual agent becomes potential knowledge for the others. Surprising events will be defined in terms of belief. Abduction will be, based on the proposal of Nepomuceno-Fernández et al. (2013) and Velázquez-Quesada et al. (2013), defined based upon belief upgrades. Counter-abduction can be seen as sequences of actions that aim to assess the initial abductive reasoning of the observer.

The precise structure of gossip, i.e. the order of the announcements and abductive inferences, depends on the goal of the gossiper. The next section discusses the structure of gossip. We will call those structures intuitive protocols. Later on, those intuitive protocols will be used to define formal protocols.

Further, the current thesis will not explicitly define the notions 'Group Knowledge Base' and 'Individual Knowledge Base', because they do not add value to this thesis. However the knowledge and beliefs of the agents are presented in the models and can be expressed by the logical language, hence it would not be difficult to define such notions if one wishes.

3.3 Intuitive protocols for gossip with an epistemic goal

Gossip thus captures an interplay between agents, they all have partial knowledge of, and beliefs about the world. The interplay of internal reasoning and communication will change their knowledge and beliefs at least until the gossip ends. Previously we have seen the different goals gossip might have, two of those are epistemic: either to get to know an explanation of the surprising observation or to get to know whether one particular explanation is true. Those two goals give rise to a particular set of sequences of an observation, abductive inferences and announcements. This section aims to make those sequences explicit.

3.3.1 When a surprising observation will provoke gossip

Gossip starts with a surprising observation. We say that ϕ is a surprise if and only if the agent did not believe ϕ before the observation, but does believe ϕ after the observation. Note that this includes the case where the agent believes $\neg\phi$ before the observation. However, not every surprising observation will start gossip. In order to provoke gossip, the surprising observation has to be an observation about an agent. As long as the observation is about a person and surprising, the observer can choose whether or not to start a gossip scenario. Whether or not the agent will start gossiping depends partly on her goals. However, even if the agent wants to achieve a goal that can be achieved by gossip, she does not necessarily have to achieve this goal by gossiping. The epistemic goals of gossip can sometimes also be achieved by other means. If the goal is, for example, to get to know an explanation of a surprising event about agent a, most of the time agent a knows an explanation of the surprising event. Hence it might be easiest just to ask agent a. The same reasoning holds when the goal is to get to know the truth value of certain possible explanation for the surprising event. If the goal of the agent is different, or if there is no possibility to just ask for an explanation, the agent might start to gossip.

3.3.2 The interpretation of announcements and different kind of surprises

This thesis will use the notion 'announcement' to denote just the act of speaking and not yet the attitude of the listener nor the revision of the beliefs of the agents. What we call an announcement thus is followed by an action of the receiver(s) who decides what to do with this new information and, as a consequence, might change their beliefs.

In any conversation the listener first has to make a choice, does the agent express a statement of belief or knowledge with his announcement? Sometimes this is easy to determine, for example when an agent explicitly announces whether it is knowledge or a belief or when the agent expresses how he got to know what he just announced. For simplicity we assume that an agent can only announce things he at least believes to be true. Further, this thesis assumes that the receivers know that the agent who makes the announcement at least believes the announcement to be true. The announcement of a proposition by an agent might thus give rise to a surprising observation in two ways: the proposition itself can be a surprise, but the fact that the announcer believes (or knows) the proposition can be also surprising. If it is the belief or knowledge of an agent that surprises the receiver, it is often easier to get to know the agent's explanation by just asking for it instead of by starting a gossip. Once the agent decides to interpret the announcement as a belief or as knowledge, the agent will get to know the fact that agent a believes or knows what she just announced (this action will be modelled as a fair game upgrade). Furthermore, the listener might himself (privately) upgrade his beliefs and come to believe or know the announced proposition.

3.3.3 The structure of a gossip conversation

The figures below show the structure of gossip. The total diagram is split into smaller diagrams. The starting node of the first diagram is denoted by a double rectangle. The final node of each diagram is also depicted as the starting node of the subsequent diagram. The blue arrows represent the possible path of actions that will occur if agent a has the goal to know whether the explanation she has for ϕ is true or false. The green arrows represent the possible path of actions that will occur if agent a has the goal to get to know an explanation for ϕ. In this first part of the full diagram this distinction might seem irrelevant; in the next diagrams, however, this distinction becomes relevant. Gossip typically starts with a surprising observation of ϕ. Hence the start node of the diagram states: 'Agent a observes ϕ'. Now there are two possibilities:

1. agent a already believed ϕ, in this case the observation was no surprise and the observation does not give rise to gossip;

2. agent a did not believe yet that ϕ was true (i.e. ϕ was a surprise).

This thesis thus uses a natural definition of a 'surprising observation', i.e. the observation of ϕ is a surprising observation if before the observation the agent did not yet believe ϕ. Note that this also covers the case in which the agent believed the negation.

If the observation was a surprise agent a comes to believe a particular best explanation (denoted by χ^a_{best}, where a stands for the agent) and the gossip starts, otherwise the protocol ends and the observation does not provoke gossip. The agent now can have two epistemic goals:

1. to get to know whether χ^a_{best} is true (represented by blue arrows in the diagram);

2. to get to know an explanation of ϕ (represented by green arrows in the diagram).

In both cases agent a has to choose whether or not to start a gossip. If the agent chooses to start a gossip she announces χ^a_{best} to a subgroup G of all agents A (typically all agents he has a social connection with, but not the agent the gossip is about).

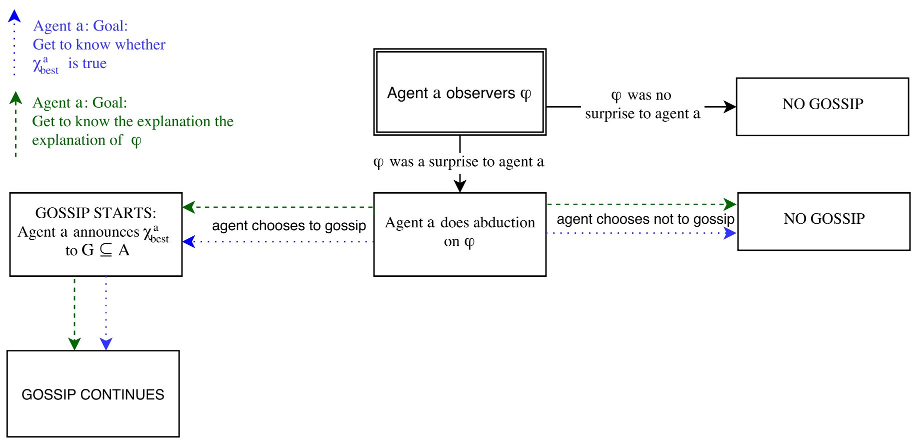

Figure 3.1: First part of the diagram

The gossip started with the announcement of χ^a_{best} by agent a. For the next step we introduce a set B. At the start of a gossip, the set B consist of just agent a. This set is introduced to define which agents might respond to the announcement of χ^a_{best}. All of the agents in $G \setminus B$ (i.e. the set-theoretic difference of G and B) now might possibly respond, the order depends on their personality and other factors. This work does not model this and hence we assume a random order. We introduced the set B. At the start of the gossip, it simply adds agent a to this set. Then, it picks an agent $b \in G \setminus B$ and let him respond. After his response (assuming that the gossip has not ended yet) agent b is added to set B and we pick again an agent in $G \setminus B$ (and add b to B) until $G = B$ or the gossip ended for other reasons. There are some points later in the flow diagram where B is emptied and a is added to the set B. For example in the case where agent a gets to believe a new non-trivial best explanation. We empty the set B in order to enable all agents to respond to this new belief.

Each agent b will be, at his turn, in one of the following cases:

1. agent b knows χ^a_{best};
2. agent b knows $\neg\chi^a_{best}$;
3. agent b knows neither χ^a_{best} nor its negation (i.e. $\neg\chi^a_{best}$) and it was a surprise to agent b that agent a believes χ^a_{best};
4. agent b knows neither χ^a_{best} nor its negation (i.e. $\neg\chi^a_{best}$) and it was no surprise to agent b that agent a believes χ^a_{best}.

We will now discuss what happens in each of those cases.

In case:

1. Agent b knows χ^a_{best}: The agent b will announce χ^a_{best}. In the case where the goal of agent a was to get to know the truth value of χ^a_{best}, this announcement will stop the gossip, since the goal is achieved. In the case where the goal of agent a was to get to know an explanation of ϕ, the goal is achieved (but only if the explanations are mutual exclusive). In real life, those explanations are not necessarily exclusive. Therefore, the gossip might continue. And it will continue, but only if it was a surprise to agent b that agent a believed χ^a_{best}. Agent b will perform abductive reasoning on the proposition that agent a believes χ^a_{best} (denoted by $B_a\chi^a_{best}$).

2. Agent b knows $\neg\chi^a_{best}$: The agent b will announce $\neg\chi^a_{best}$. In the case where the goal of agent a was to get to know the truth value of χ^a_{best} this might stop the gossip, since the goal is achieved. However, the gossip might continue if it was a surprise to agent b that agent a believed χ^a_{best}; agent b might then do abduction on $B_a\chi^a_{best}$.

   In the case where the goal of agent a was to get to know an explanation of ϕ, the goal is not achieved. Agent a has new information about the truth value of the possible explanations (she now knows that χ^a_{best} is false). Hence there will be a new best explanation χ'^a_{best}. This might give a new goal: get to know whether χ'^a_{best} is true or false. The agent again has the choice whether or not to start gossiping about this new best explanation. If she chooses so, the protocol starts over from the announcement of χ^a_{best} onwards with each χ^a_{best} substituted by χ'^a_{best}.

   If the goal was to get to know an explanation of ϕ the protocol is repeated from the node with the announcement of χ^a_{best} onwards, again with χ^a_{best} substituted by χ'^a_{best}.

3. Agent b knows neither χ^a_{best} nor its negation (i.e. $\neg\chi^a_{best}$) and it was a surprise to agent b that agent a believes χ^a_{best}: Agent b will do abduction on $B_a\chi^a_{best}$.

4. Agent b knows neither χ^a_{best} nor its negation (i.e. $\neg\chi^a_{best}$) and it was no surprise to agent b that agent a believes χ^a_{best}: Agent b did not receive any new information from the announcement of agent a, hence it makes no sense for this agent to change his beliefs. Agent b does not know whether χ^a_{best} is true or false. Hence it makes no sense to announce any information. We will pick a new agent.

If $G = B$, all agents in G got their turn and cannot add anything valuable to the gossip, hence the protocol ends (possibly without achieving a goal).

As soon as agent b did abduction on χ^a_{best} he is able to just ask agent a the best explanation. Note that agent b might also choose to start a new gossip, which brings us at the very start of the protocol again, with a new gossip. Let us assume agent b decides to ask agent a for the agent's explanation of her belief. We will denote this by an announcement of his disbelief by agent b. This announcement brings us by the next part of the protocol diagram.

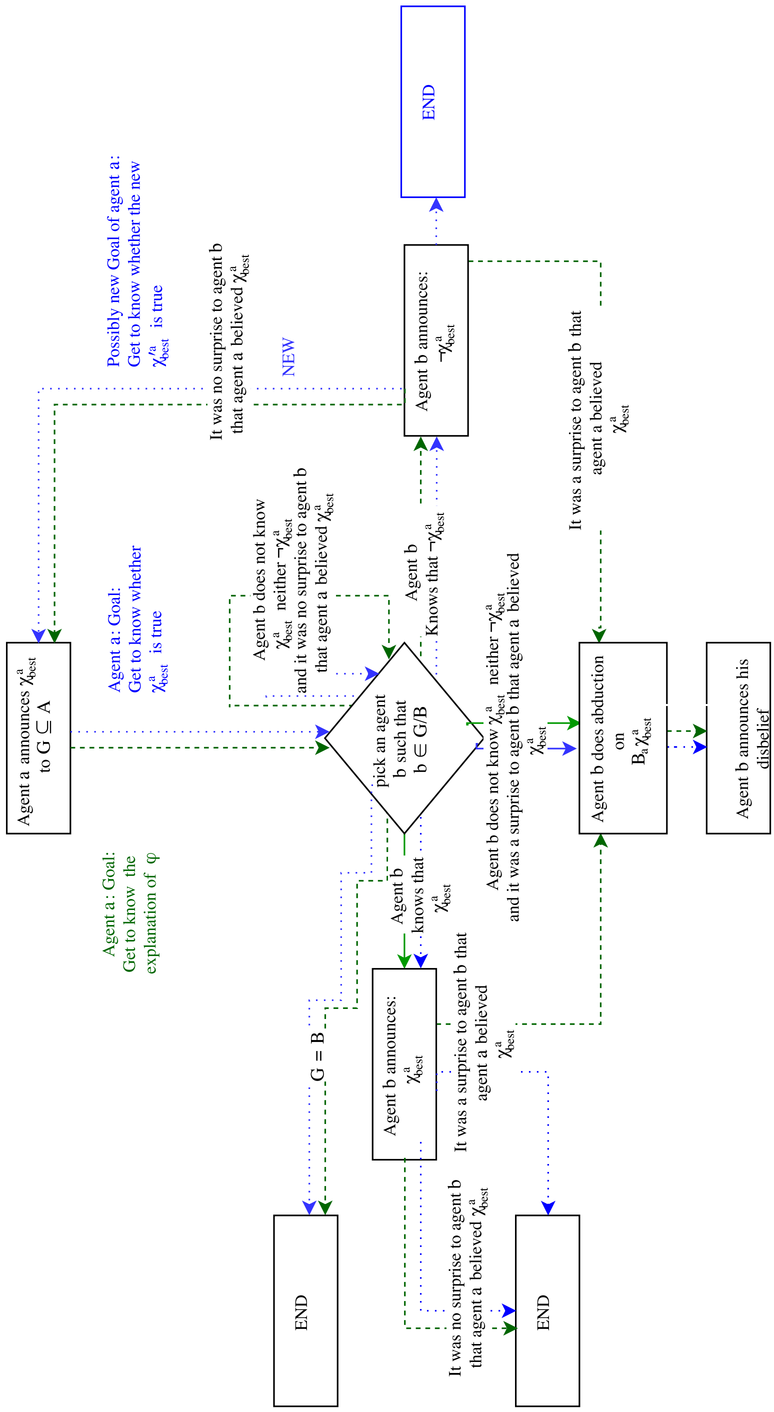

Figure 3.2: Second part of the diagram

As a response to the announcement of disbelief by agent b, agent a will typically announce ϕ, i.e. the agent's explanation of the fact that she believes χ^a_{best}. Now there are again four cases to distinguish, note that these cases are different from the four cases after the announcement of χ^a_{best},

1. agent b knows ϕ;

2. agent b knows $\neg\phi$;

3. agent b does neither know ϕ nor its negation (i.e. $\neg\phi$) and did not yet believe ϕ;

4. agent b does neither know ϕ nor its negation (i.e. $\neg\phi$) but did already belief ϕ.

In case:

1. Agent b knows ϕ; agent b will announce ϕ, and his best explanation for ϕ: χ^b_{best}. We assume that since b already knows ϕ he either knows an explanation of ϕ or he did, when he got to know ϕ, abduction on ϕ and therefore already believes some best explanation.

2. Agent b knows $\neg\phi$; agent b will announce $\neg\phi$. And the gossip ends. This case can only happen if the surprising event caused the belief of ϕ in agent a, hence the observation was not a source of knowledge.

3. Agent b knows neither ϕ nor its negation (i.e. $\neg\phi$) and did not yet believe ϕ; the agent now will do abduction on ϕ and share his own best explanation.

4. Agent b knows neither ϕ nor its negation (i.e. $\neg\phi$) but already believed ϕ; in this case ϕ is no surprise, and agent b will already have a best explanation for ϕ. Hence, he will share his best explanation.

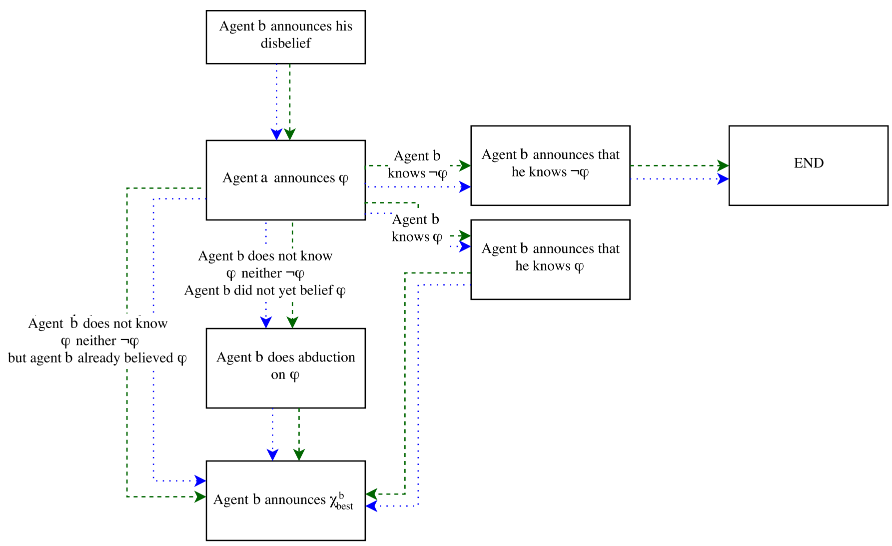

Figure 3.3: Third part of the diagram

Agent b announced his best explanation for ϕ, denoted by χ^b_{best}. Now either the fact that agent b believes χ^b_{best} after the abduction of agent b on ϕ is or is not a surprise to agent a. If this is no surprise at all, this announcement does not have any epistemic value for agent a, hence we will move on and pick another agent in $G \backslash B$. If it was a surprise to agent a that after the abduction on ϕ agent b believes χ^b_{best}, agent a will proceed by doing abduction on $B_b\chi^b_{best}$. Agent a now will announce her disbelief after which agent b announces his reason for believing χ^b_{best} (this thesis denotes this by γ). This typically will be the announcement of some of the background knowledge that agent b has and agent a lacks. Agent a now upgrade his beliefs with γ, either by a private upgrade or by a fair game upgrade. This might give rise to a new best explanation for agent a, denoted by χ'^a_{best}. If the goal of agent a was to get the know the truth value of χ^a_{best}, the protocol continues with the next agent. If the goal was to find an explanation of ϕ, we will start over from the point where agent a announces χ^a_{best}, and χ^a_{best} has to be substituted with χ'^a_{best} in the schema. Of course from the new belief of χ'^a_{best} a new goal might arise, namely to get to know the truth of χ'^a_{best}, in which case we also continue at the point in the diagram where agent a announces χ^a_{best} and substitute the old best explanation by the new one. In those two last scenarios the set B will be made empty again, hence every agent can have a new turn.

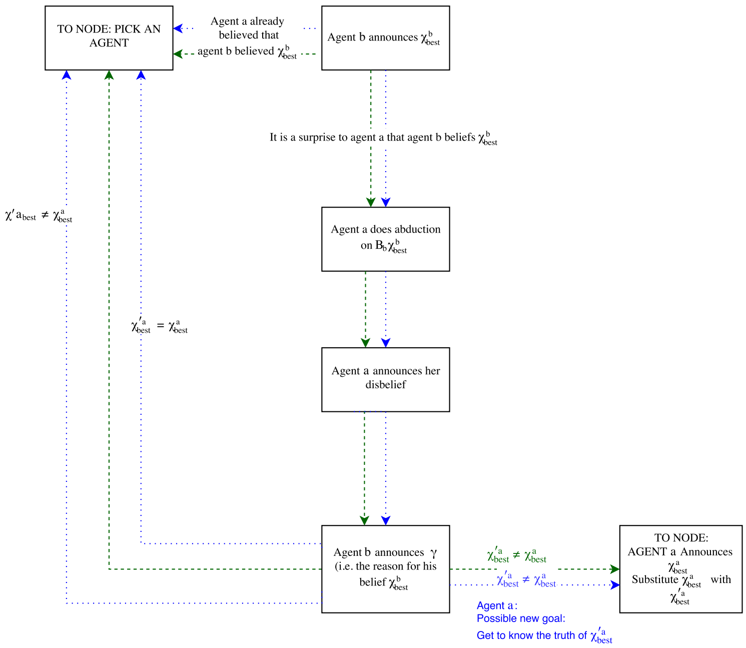

Figure 3.4: Fourth part of the diagram

In the diagrams above we draw the states that connect two diagrams twice. Further, some of the diagrams above contain states referring to sates in previous diagrams. The next diagram shows the transitions between the previous diagrams. In the diagram below we only show the states that appear in two diagrams and the states that appear in one diagram and to which is referred in another. Further, we only draw the arrows that connect a state in one diagram to a state in another diagram.

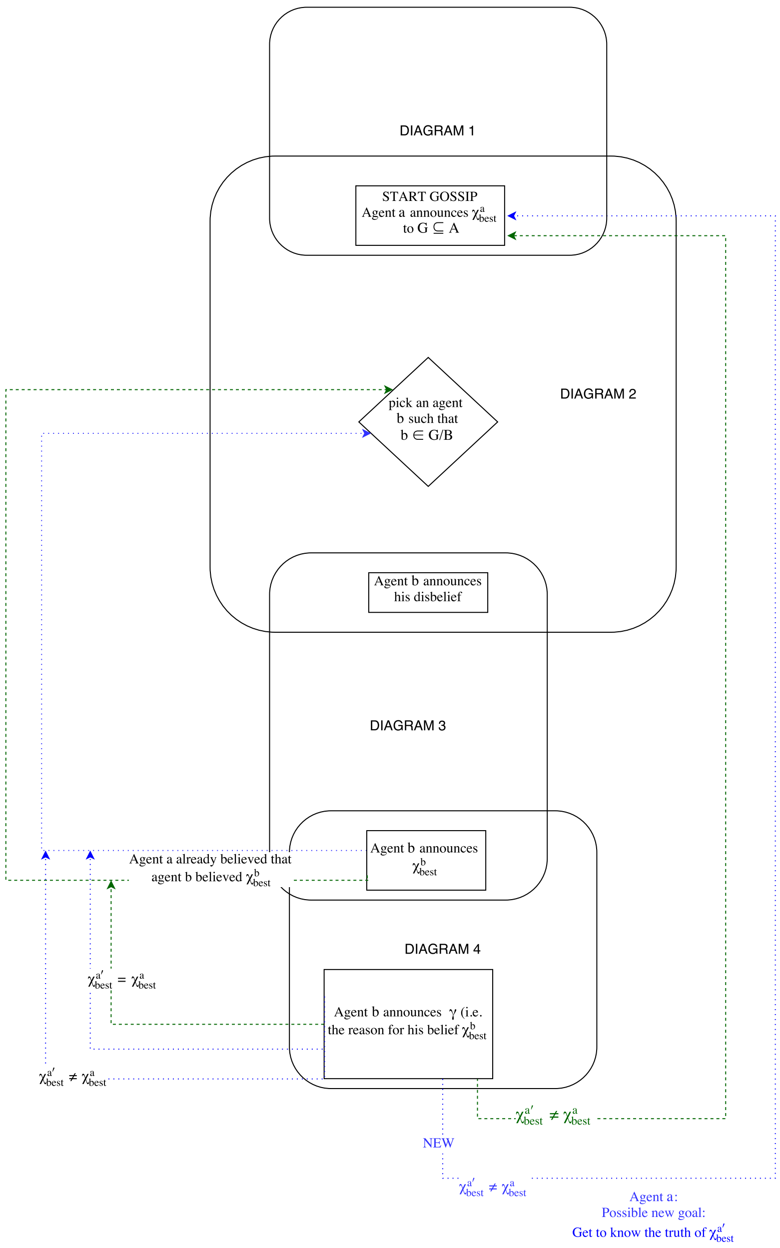

Figure 3.5: The transition between the diagrams

3.4 Changes in the social network relations and intuitive protocols

Apart from the changes in the knowledge and beliefs of the agents, gossip might cause changes in the social network of the agents. The goal of gossip might be to exclude someone from or include someone in a social group or to strengthen or weaken the group bonds. This thesis argues that the bond between two agents is strong if more people believe (and even stronger if they know) about this bond. Likewise, the more people believe (or even know) two people not to be socially connected, the weaker the social relation is.

3.4.1 The notion of a social group

The goal of gossip may be to include an agent in a social group, or to exclude an agent from a social group. Another goal might be to strengthen the bondings within a social group. We understand a social group as any set of agents such that each pair of agents in this social group is (indirectly) connected to the other agent. A group might thus consist of four agents a, b, c and d, with a connected to b and b to c and c to d. In this scenario, however agent a and d are not directly connected by the network relation, agent a and d are in the same social group. This is possible since they are indirectly connected by social network relation relations.

3.4.2 Positive and negative gossip

Gossip is not only always about a person, it also always has an (implicit) moral value. Having an affair is generally seen as something negative. Therefore, when one gossips about an agent stating that he or she has an affair, the gossip is seen as negative. We say that the proposition: 'person a has an affair 'is a negative proposition. The proposition: 'person a works as a volunteer in a retirement home'is an example of a positive proposition. Since this is generally seen as something good. Those valuations depend on the social group that gossips. We will not formally define the notion of a positive or negative proposition. However, those notions will be used in the informal protocols.

The notions of positive and negative propositions about a person are important for the informal protocols of gossip with a social goal. We say that a gossip is negative if the announced best explanation is a negative proposition about a person. Likewise we say that a gossip is positive if the announced best explanation is a positive proposition about a person.

This work assumes that someone will only negatively gossip about an agent if there is no social bond between both agents. Intuitively this fits the scenario where two agents a and b are part of the same social group, but are not direct friends of each other. Now one of them decides that he wants his friends to break the social connection with the other agent. If one wants to exclude some agent b from a group, we may assume that he will start by breaking his own connection with agent b. This is also the case when one wants to weaken the social relations between an agent and a group. Likewise and for similar reasons, this thesis assumes that someone will only positively gossip about someone if it is one of his friends.

Gossip can have several social goals. The diagram below expresses the intuitive gossip protocol for excluding someone from a group and for including someone in a group. This thesis assumes that an agent can add someone to a group only if; 1. this person is a friend of him and 2. he himself is part of the group. Likewise, an agent can only exclude someone from a group if this person is not a friend of the agent and if the agent himself is part of the group.

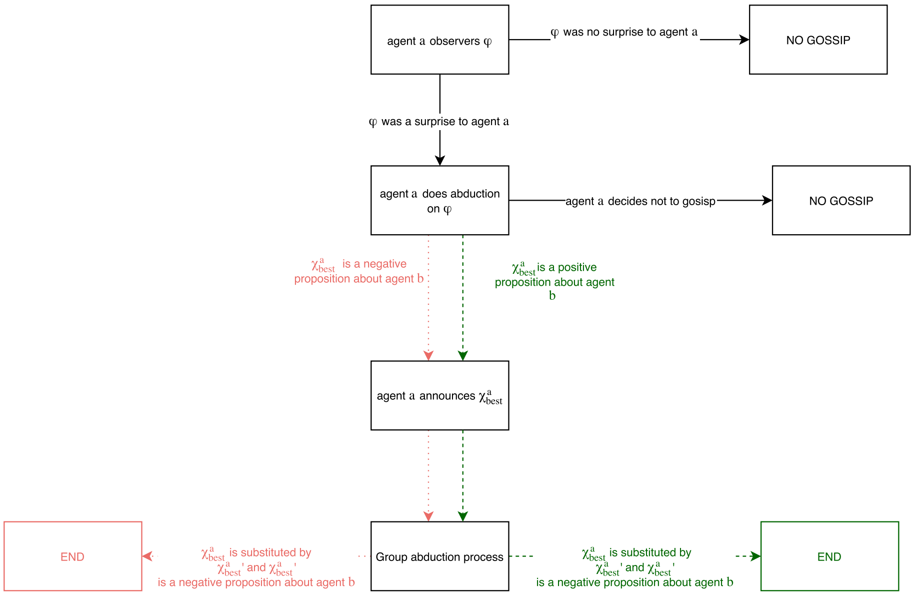

Figure 3.6: Social diagram

Gossip with a social goal starts in a way similar to gossip with an epistemic goal. It will only be able to start if it is provoked by a surprising event. Even if some agent wants to exclude another agent from a group, such a surprising event is necessary in order to start a gossip scenario. Further agent a has, after abduction on the surprising event, to decide whether or not to start a gossip. If $a's$ best explanation is a negative proposition about agent b, this can help agent a to exclude agent b from a certain group. But of course a will not start an act of gossip if she wants to include b instead of excluding him. Likewise, if a's best explanation is a positive proposition about agent b this can help agent a to include agent b in a group, but it would not help to exclude him. If chosen to start the gossip, the agent will announce her best explanation. The other agents will now respond exactly as in the case of the epistemic goal of getting to know an explanation of the surprising observation (ϕ). Only now the gossip will terminate at different points in the gossip protocol. It will still terminate when $G = B$, i.e. when no agent has any information to add to the conversation. But it will also terminate when the best explanation of agent a changes into an explanation with the opposite moral value. The gossip will not terminate when agent a knows the best explanation, since in that case the agent wants everyone to know this best explanation, and only then it will terminate.

There are two remaining social goals, i.e. strengthening and weakening the bondings within a group. For both of those goals we will discuss the following three cases:

1. strengthen/weaken the bonding between oneself and one other agent;
2. strengthen/weaken the bonding between an other agent and the group;
3. strengthen/weaken the bondings in between all agents in a group.

When the goal is to strengthen or weaken the bonds of agent b and the other agents in the group, we follow the protocol for respectively including and excluding an agent from a group.

Strengthening the bonding between oneself (agent a) and another agent (agent b) will also follow the protocol of including an agent in the group. By announcing a positive proposition about someone, the other agents will come to believe or even know that there is a friendship connection between agent a and b. By definition of strength of a network relation (i.e. strength of the social bond) this will strengthen the relation between agent a and b. By announcing a negative proposition about someone, the other agent will come to believe that the announcer does not like the agent the gossip is about, and thus that there is no friendship connection; by the definition of the weakness of a bonding this will weaken the friendship relation between agent a and agent b.

When the social goal is to strengthen the bondings of a certain group the protocol is a bit different, as this goal can be reached by positive and negative gossip about an outsider. By gossiping together, the agents in a group will get to know who is part of the group. As soon as someone makes an announcement, the agents in the group will come to believe that this person is part of the group (i.e. connected to (most) the other participants of the gossip with a friendship relation). By definition of the strength of a friendship relation participating in a gossip will strengthen the friendship relations between the agent and the other agents in the group. In this case the gossip ends when either $G = B$, or the agent knows an explanation for ϕ. See the next diagram.

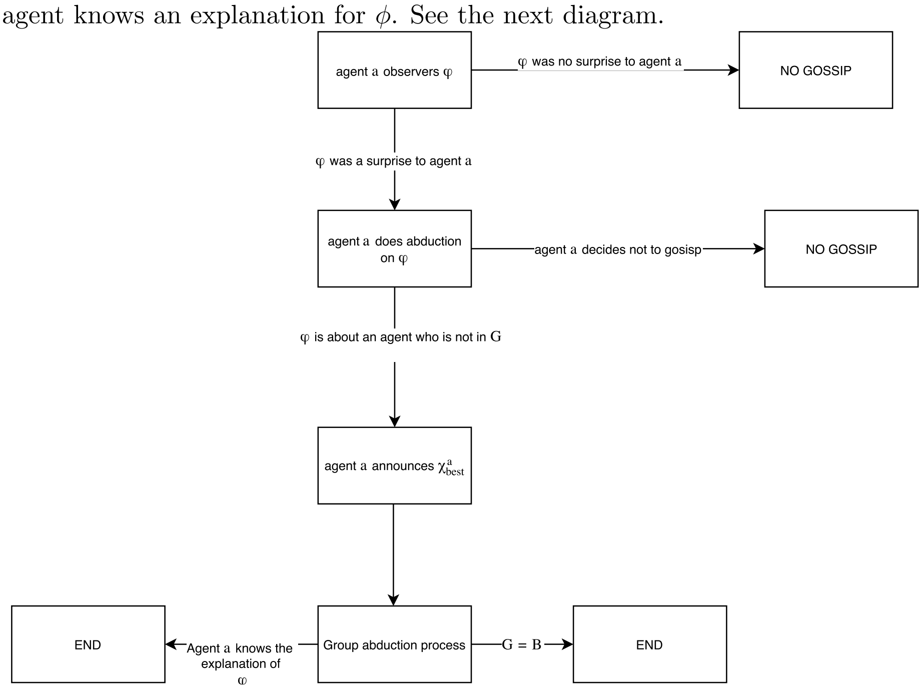

Figure 3.7: Strengthen the bonding of a group

This section described for each of the goals of gossip the typical structure a gossip conversation has. In the next sections we will give a formal logic that is able to describe and model the changes in the epistemic and doxastic states of the agents. Later on, we extend the logic such that it can also express and model the network relation between agents, is able to deal with the changes of those relations and can express and model the knowledge and beliefs of the agents about the network structure. Once the formal language is given, this thesis uses the diagrams above to define for each of the goals gossip can fulfil formal protocols for a gossip conversation. The logic will not model announcement, but only the consequences of the announcements on the epistemic states of the agents. Each announcement might cause changes in the beliefs or knowledge of an agent, those changes will be dealt with in the logic.

Chapter 4

Epistemic dimension of protocols

4.1 Multi-agent dynamic epistemic logic

The previous sections defined gossip, described the goals gossip can fulfil and discussed the informal epistemic dynamics of gossip with an epistemic goal. Further, the previous part showed the abductive nature of gossip. Velázquez-Quesada et al. (2013) and Nepomuceno-Fernández et al. (2013) propose to use Dynamic Epistemic logic (as presented by Baltag and Smets (2008)) to model abductive reasoning. This thesis uses this logical language in order to model the epistemic and doxastic dynamics gossip gives rise to.

4.1.1 Static part

Syntax

As the static part of the language we use a logic for knowledge and safe belief $K\Box$ as presented by Baltag and Smets (2008).

Given a non-empty set of atomic propositions P, let A be a finite non-empty set of agents; the formulas ϕ of the language $\mathcal{L}$ are given by recursion:

Definition 6. *Multi-agent language $K\Box$ (Baltag and Smets, 2008, p.37)*
$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid \Box_a\phi \mid K_a\phi$
with $p \in P$ and $a \in A$.

The other boolean operators are defined as usual. Further this language includes a $S4.3$ modality $\Box_a$ for every agent $a \in A$ that expresses safe, non-negatively introspective belief and a $S5$ knowledge modality K_a that expresses the fully introspective notion of knowledge. The diamond modality for K is defined as usual and denoted by $\tilde{K}_a\phi$. (i.e. $\tilde{K}_a\phi \coloneqq \neg K_a\neg\phi$), $\top$ denotes some tautological sentence (e.g. $\neg(p \wedge \neg p)$). In this language, belief and conditional belief are defined as follows:

Definition 7. *Belief and Conditional Belief (Baltag and Smets, 2008, p.37)*
$B_a^{\phi}\psi \coloneqq \tilde{K}_a\phi \rightarrow \tilde{K}_a(\phi \wedge \Box_a(\phi \rightarrow \psi))$,
$B_a\phi \coloneqq B_a^{\top}\phi$

Intuitively $K_a\phi$ expresses that in all states which are for agent a epistemically indistinguishable from the real state, ϕ is the case. $B_a\phi$ states that in the (according to agent a [1]) (set of) most plausible states, ϕ is true. $\Box_a\phi$ expresses that in all state that are (according to agent a) as least as plausible as the real state, ϕ is true.

The proof system for this logic is given by Baltag and Smets (2008) on page 38.

Semantics

Definition 8. *Epistemic State Models (also called Epistemic Plausibility Models)*

An Epistemic Plausibility Model is a model $\mathcal{M} = (S, \leq_a, \sim_a, V)_{a\in A}$ where:

1. *A is a finite set of agents;*
2. *S is a finite set of states;*
3. *$\leq_a$ is a family of plausibility relations $\sim_a$ a family of equivalence relations, both are labelled by the agents $a \in A$. We assume that:*
   - *$\leq_a$-comparable states are $\sim_a$-indistinguishable (i.e. $s \leq_a t$ implies $s \sim_a t$);*
   - *the restriction of each plausibility relation $\leq_a$ to each $\sim_a$-equivalence class is a locally connected and well-founded reflexive and transitive relation (i.e. a well-preorder).*
4. *$V : \Phi \to \mathcal{P}(S)$ is a valuation map that maps every element of a given set Φ of atomic propositions to a subset of the set of all states S.*

We denote the real world in S by $s*$. In the figures of those models in section 4.4 we will indicate those states by a dashed line

Note that the epistemic indistinguishably relation can be recovered from the plausibility relations following $s \sim_a t$ iff $s \leq_a t$ or $t \leq_a s$. Therefore we will denote the Epistemic Plausibility Models without denoting the family of equivalence relations; $\mathcal{M} = (S, \leq_a, V)_a \in A$.

We read $s \leq_a t$ as: 'for agent a state t is as least as plausible as s '. $s \sim_a t$ has to be interpreted as: 'for agent a state s and t are epistemically indistinguishable'.

Further, the semantic interpretation is defined as usual:

Definition 9. *Semantic interpretation*
Let $\mathcal{M} = (M, \leq_a, V)_{a\in A}$ be an Epistemic State Model and $s \in S$ a state. Then:
$\mathcal{M}, s \Vdash p$ *iff* $s \in V(p)$
$\mathcal{M}, s \Vdash \neg\phi$ *iff* $\mathcal{M}, s \nVdash \phi$
$\mathcal{M}, s \Vdash \phi \vee \psi$ *iff* $\mathcal{M}, s \Vdash \phi$ *or* $\mathcal{M}, s \Vdash \psi$
$\mathcal{M}, s \Vdash K_a\phi$ *iff for all* $t \in S$ *such that* $(s,t) \in \sim_a$, $\mathcal{M}, t \Vdash \phi$
$\mathcal{M}, s \Vdash \Box_a\phi$ *iff for all* $t \in S$ *such that* $s \leq_a t$, $\mathcal{M}, t \Vdash \phi$.
We write: $\mathcal{M} \Vdash \phi$ *iff for all* $s \in S$, $\mathcal{M}, s \Vdash \phi$

[1] According to agent a means that her beliefs are what she considers most plausible given the knowledge that she has.

4.1.2 Dynamic part

The static part of the logic is defined above. We use this logic to describe and model the knowledge and beliefs of the agents at a certain moment in the gossip. This section defines the dynamic part of the logic, which enables the model to change. Action models represent the epistemic actions that provoke the agents to change their beliefs and knowledge. From the Epistemic State Model together with the Epistemic Action Model the logic will be able to generate upgraded Epistemic State Models that model the situation after the action. This part is also adopted from the language proposed by Baltag and Smets (2008). Epistemic Action Models are defined as follows:

Definition 10. *Epistemic Action Models, based upon (Baltag and Smets, 2008, p.40)*
An Epistemic Action Model is a model $\mathcal{N} = (\Sigma, \leq_a, Pre)_{a\in A}$ where:

1. *Σ is a set of (basic doxastic) actions, also called events, σ. Those represent the deterministic belief-revision actions of a particularly simple nature.*
2. *$\leq_a$ is, like above, a family of plausibility relations, labelled with the agents.*
3. *$Pre : \Sigma \to \Phi$ is a function that maps each element of Σ to some doxastic proposition pre_σ. We call pre_σ the precondition of the action σ.*

Intuitively, the precondition defines the domain of applicability of action σ: it can be executed on a state s iff s satisfies its preconditions. The relations $\leq_a$ give the agents' beliefs about which actions are more plausible than others.

On page 37 we denote the real event in Σ by $\sigma*$ and denote this special state in the figures by a dashed line.

The new model is generated from the old Epistemic State Model $\mathcal{M} = (S, \leq_a, V)_{a\in A}$ and an Epistemic Action Model $\mathcal{N} = (\Sigma, \leq_a, Pre)_{a\in A}$ acting on the plausibility model. Using the action priority upgrade, which is explained below, we compute the new upgraded Epistemic State Model which is denoted $\mathcal{M} \otimes \mathcal{N}$.

Definition 11. *The upgraded Epistemic State Model, based upon (Baltag and Smets, 2008, p.44-55)*

Let $\mathcal{M} = (S, \leq_a, V)_{a\in A}$ be an Epistemic State Model and $\mathcal{N} = (\Sigma, \leq_a, Pre)_{a\in A}$ an Epistemic Action Model. The upgraded Epistemic State Model $\mathcal{M} \otimes \mathcal{N} = (S', \leq'_a, V')_{a\in A}$ is such that:

- *the set of states is a subset of the Cartesian product, $S' := \{(s, \sigma) : s \Vdash pre(\sigma)\}$,*
- *the upgraded plausibility order is given by the Action priority Rule, with $(s, \sigma) \leq'_a (s', \sigma')$ iff either $\sigma <_a \sigma'$ and $s \sim_a s'$, or else $\sigma \tilde{=}_a \sigma'$ and $s \leq_a s'$, with $\tilde{=}_a$ defined as $\sigma \tilde{=}_a \sigma'$ iff $\sigma \leq_a \sigma'$ and $\sigma' \leq_a \sigma$,*
- *the upgraded valuation is given by the original valuation from the input-Epistemic State Model: for all $V'(p) := \{(s, \sigma) \in S' : s \in V(p)\}$.*

Further, if $s*$ was the actual world in $\mathcal{M}$ and $\sigma*$ was the actual event in $\mathcal{N}$, then $(s*, \sigma*)$ represents the actual state in the upgraded Epistemic State Model.

The motivation for this definition of the upgraded plausibility relation can be found in the work of Baltag and Smets (2008).

4.2 Types of Epistemic Action Models

This section illustrates some specific Epistemic Action Models. Baltag and Moss (2004) explain different action signatures, van Ditmarsch (2000, 2002) discusses the action models that correspond to several kinds of game announcements. Gossip typically includes two types of epistemic actions, which we can intuitively describe as fair game upgrades by an aware subgroup and private upgrades by an aware subgroup. We will discuss those two notions in the subsequent sections. A fair game upgrade is generally called a fair game announcement and was first introduced by van Ditmarsch (2000, 2002). This thesis however uses the name 'fair game upgrade' instead of 'fair game announcement'. The reason for this change is that calling this action an 'announcement' can be confusing. The notion of announcement as often used in standard DEL refers to the act or event itself of the announcement, as well as to the knowledge and beliefs of agents about the announcement that is happening [2] However, in the previous chapter we used the term 'announcement' to denote just the act of the speaker, not yet including the attitude of the listener.

This thesis illustrates the actions (fair game upgrade by an aware subgroup, private upgrade by an aware subgroup and in a later chapter some actions that change the network relations) for a set of agents $G \subseteq A$. This set G is never empty and can also consists of one single agent. Further we include a second set of agents in the definitions: $F \subseteq A$. This set consists of the agents that are aware that the action takes place. For all actions this thesis assumes that $G \subseteq F$. The agents that change their beliefs or knowledge will be aware of those changes. Further, this thesis gives multiple State Models and Action Models, the plausibility relation is transitive and reflexive but the reflexive and transitive arrows are not drawn in the figures.

4.2.1 Fair game upgrade

In case of a fair game upgrade to an aware subgroup, we want to express what happens if an agent (or a group of agents) changes his beliefs because of an observation or based upon an announcement. In gossip such an upgrade often happens when an agent observes the surprising event ϕ. Most often, after the observation of ϕ the agent will know that ϕ. Most of the time the other agents are not aware of the fact that agent a came to know ϕ. This upgrade will also occur if an agent makes an announcement. When agent a announces ϕ to agent b agent b will get to know that $K_a\phi$ or $B_a\phi$ and might upgrade his epistemic state with ϕ. All agents that were unaware of this announcement will not know that agent b changed his epistemic state.

Definition 12. *Epistemic Action Model for a fair game upgrade ϕ to an (aware) subgroup $G \subseteq A$ noticed by all agents in F. Note that $G \subseteq F \subseteq A$. This action is denoted by $\phi!^F_G$, where G stands for the set of agents who upgrade their belief*

[2] The update of information states in standard DEL is related to earlier studies in the area of dynamic semantics including the work on update logic. See e.g. Nouwen et al. (2016).

and F stands for the set of agents who notice this upgrade. [3]

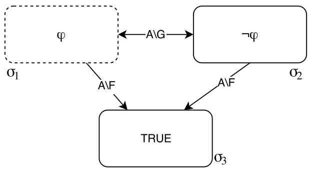

Figure 4.1: Epistemic Action Model. for fair game upgrade ϕ to an (aware) subgroup $G \subseteq A$) noiced by all agents in $F \subseteq A$

This Epistemic Action Model[4] *The model expresses that after this action each agent in G can distinguish between the states where ϕ is true and the states where ϕ is false. The other agents (i.e. the agents in $A \setminus G$ do not learn that ϕ is true. The agents in $A \setminus F$ are not aware that this action took place, so they consider it possible but believe that nothing happened. The upper left event $\sigma*$, represents that agents in G get to know that ϕ is the case.*

When we upgraded the Epistemic State Model with states $S = \{s_1, ...s_n\}$ we will obtain a new Epistemic State Model which consist of two parts. The first part consists of the worlds $((s_1, \sigma_3), ..., (s_n, \sigma_3))$ and is bisimular to the original Epistemic State Model. The second part consists of all states (s, σ_1) such that $(\mathcal{M}, s) \Vdash \phi$ and all states (s, σ_2) such that $(\mathcal{M}, s) \Vdash \neg\phi$ and is like a copy of the first without the $\leq_a$ relation for all $a \in G$ that connect a state where ϕ is true and with a state where $\neg\phi$ is true. These two parts of the model are connected with $\leq_a$ relations for all agents in F in a way such that if $s = s'$ then $(s, \sigma) \leq_a (s', \sigma')$ together with all relations betweens states that we obtain by transitivity.

*We used a dashed line in the figures and an * in the text to denote a special state in the Epistemic Action Model. In each Action Model we will mark one special state σ as σ^*. This state $\sigma* \in \Sigma$ will be used for the evaluation of a formula in a state after an action. Further, if $s*$ is the state in an Epistemic State Model that represents the real world, $(s*, \sigma*)$ will represent the real world in the upgraded Epistemic State Model.*

Note that the Epistemic Action Model for a fair game upgrade to an aware subgroup G noticed by all agents in F of $\neg\phi$ looks like the Epistemic Action Model above. The only difference is that in the Epistemic Action Model for a fair game upgrade to an aware subgroup G of $\neg\phi$, the upper right world will be the $\sigma*$.

In order to express the beliefs and knowledge of agents after an upgrade with this specific type of Epistemic Action Model, we introduce a modality for every set of agents G, every set of agents F and every formula ϕ to our language. This expresses that all agents in G applied a fair game upgrade to their Epistemic State Model and that all agents in F are aware of this action:

[3] The agents in F are aware of the fact that the event happens in the sense of 'knowing' that the event happens, they do not consider other options (e.g. that nothing happens) possible.

[4] We adopt the convention not to draw the reflexive arrows in the models. Each state (in all state and action models drawn in this thesis) will have an reflexive arrow for all agents (i.e. they all have the label A)

Definition 13. *Modality for an fair game upgrade (to an aware subgroup)*
We define $[\phi!_G^F]$ *such that:*

$\mathcal{M}, s \Vdash [\phi!_G^F]\psi$ *iff* $\mathcal{M}, s \Vdash \phi$ *implies* $\mathcal{M} \otimes \Sigma, (s, \sigma*) \Vdash \psi$.

Where Σ *is the Epistemic Action Model of the action* $\phi!_G^F$.

The dual is defined as usual (i.e. $\langle\phi!_G^F\rangle\psi := \neg[\phi!_G^F]\neg\psi$).

This work assumes that $G \subseteq F$, an agent will be aware of the upgrade of his own beliefs. Furthermore, if $F = G$ this thesis uses $\phi!_G$ as an abbreviation for $\phi!_G^G$.

4.2.2 Private upgrade

Another type of action occurs in gossip when an agent announces ϕ to another agent and the receiver might come to believe that ϕ is true. The announcer might or might not come to believe that the receiver upgraded his beliefs. Further if the announcement is an announcement to the group, each agent might or might not come to believe that ϕ is true. We let G be the set of agents that actually upgraded their beliefs and F the set of agents that is aware of this upgrade. This work assumes that $G \subseteq F$, an agent will be aware of his own upgrade, and of the upgrade of the other agents' beliefs.

Definition 14. *Private upgrade of* ϕ *by an aware subgroup* G *noticed by all agents in* F*:*

We let $G \subseteq A$ *be the set of agents that (privately) upgrades their belief with* ϕ*. We let* $F \subseteq A$ *be the set of agents that are aware of the announcement, hence they know that the agents in* G *applied a private upgrade of their belief with* ϕ*. We assume that* $G \subseteq F$*; an agent that privately upgrades his belief with* ϕ *will be aware of this. We denote this type of actions with* $\phi \Uparrow_G^F$*, where* G *denotes the set of agents that upgrades their beliefs, and* F *the set of agents that is aware of this upgrade. If* $F = G$ *this thesis uses* $\phi \Uparrow_G$ *as an abbreviation for* $\phi \Uparrow_G^G$*. This type of action has the following Epistemic Action Model:*

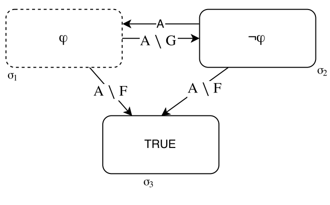

Figure 4.2: Epistemic Action Model for private upgardes by an aware subgroup G noticed by F

Also for this action we introduce for every set of agents G, every set of agents F and every formula ϕ in our language a modality $[\phi \Uparrow_G^F]$ defined as follows:

Definition 15. *Modalities for private upgrades (by an aware subgroup)*
$\mathcal{M}, s \Vdash [\phi \Uparrow_G^F]\psi$ *iff* $\mathcal{M}, s \Vdash \phi$ *implies* $\mathcal{M} \otimes \mathcal{N}, (s, \sigma*) \Vdash \psi$.

Also for this modality, the dual is defined as usual (i.e. $\langle \phi \Uparrow^F_G \rangle \psi := \neg[\phi \Uparrow^F_G]\neg\psi)$. Also for the private upgrade this work assumes that $G \subseteq F$, an agent will be aware his own upgrade. Further, if $F = G$ this thesis uses $\phi \Uparrow_G$ as an abbreviation for $\phi \Uparrow^G_G$.

The syntax of the proposed language is defined as follows:

Definition 16. *Let P be a set of propositions, the formulas ϕ of the language $\mathcal{L}$ by using double recursion defined by:*
$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid K_a\phi \mid \Box_a\phi \mid [\pi]\phi$
$\pi ::= \phi!^F_G \mid \phi \Uparrow^F_G$ *with $p \in P$, $a \in A$ and $G \subseteq F \subseteq A$.*

We let Φ denote all formulas in the language.

4.2.3 Abduction

Previously this thesis discussed the abductive nature of gossip. Nepomuceno-Fernández et al. (2013) and Velázquez-Quesada et al. (2013) propose to model abductive reasoning in a Dynamic Epistemic Logic setting. This thesis adopts the definition of an abductive problem in terms of beliefs, based upon the proposal of Nepomuceno-Fernández et al. (2013, p.949).

As mentioned before, abduction is typically said to be caused by a surprising observation. A surprising observation however can be defined in several ways. Some formula ϕ can be said to be a surprising observation when the agent does not believe ϕ. Sometimes the restrictions can be stronger and the agent has to believe $\neg\phi$ in order for ϕ to be a surprising observation. We will use the former ideas and follow Velázquez-Quesada et al. (2013) in their definition of a Abductive problem:

Definition 17. *Definition Abductive problem*
Let $(\mathcal{M}, s)$ be a pointed plausibility model. at $(\mathcal{M}, s)$, a formula χ can become an abductive problem if and only if it is not believed and will be believed after observing ψ, that is, if and only if:
$(\mathcal{M}, s) \Vdash \neg B_a\chi \wedge [\psi!^F_a]B_a\chi$ *or*
$(\mathcal{M}, s) \Vdash \neg B_a\chi \wedge [\psi \Uparrow^F_a]B_a\chi$.

In this definition ψ represents the surprising observation, and χ the abductive problem. Those notions are not necessarily the same in the setting of epistemic logic, because of the logical omniscience problem. By a surprising observation the agent can get to know (or believe) not only the formula that expresses the surprising observation but also other propositions that are consequences of this observation. This thesis however will assume those two formulas to be the same and use the notions of surprising observation to mean abductive problem. Further note that we defined an abductive problem in terms of beliefs; Velázquez-Quesada et al. (2013) also discuss similar definitions in terms of knowledge or both (beliefs and knowledge).

The definition of an abductive solution is defined adopted from Nepomuceno-Fernández et al. (2013) follows:

Definition 18. *Abductive solution by Nepomuceno-Fernández et al. (2013, p.949)*
Let (M, s) be a pointed plausibility model, if at (M, s) the formula χ can become an abductive problem, then η will be an abductive solution if and only if the

agent knows that η implies χ, that is, if and only if:
$(M, s) \Vdash K_a(\eta \to \chi)$.

We will denote the set of abductive solutions by Σ_χ where χ is the abductive problem. We will (as proposed by Nepomuceno-Fernández et al. (2013)) syntactically restrict the set of abductive solutions to the non-trivial ones. If e is a surprising event, clearly $e \to e$ holds, and all agents will know this after the observation, the same holds for $\perp \to e$. We will therefore exclude all contradictions and all formulas that are logically equivalent to the formula that expresses the abductive problem. This thesis thus restricts the set of abductive solutions to non-trivial explanations. Further if f is an explanation of e also $f \wedge f$ and for any g, $f \vee g$ will be explanations. Also those explanations will not count as abductive solutions. This thesis therefore restricts the set of abductive solutions to literals. If the surprising formula is expressing the belief or knowledge of an agent, the possible explanation might also be a belief. Therefore we also allow the abductive solutions the be of the form $B\phi$ or $K\phi$ where ϕ is a literal.

Above we defined the set of abductive solutions. The question is now how to select one best explanation from this set. Also here we follow the proposal of Nepomuceno-Fernández et al. (2013) and Velázquez-Quesada et al. (2013) based on the idea of lifting the plausibility ordering from worlds to sets of worlds (i.e. formulas). Using this 'lifted 'the agent can select one best explanation from the set of abductive solutions. van Benthem et al. (2009) discusses eight different ways of defining this binary plausibility orders among formulas: $\leq_{\exists\exists}$,$\leq_{\forall\exists}$,$\leq_{\exists\forall}$, $\leq_{\forall\forall}$,$<_{\exists\exists}$,$<_{\forall\exists}$,$<_{\exists\forall}$, $<_{\forall\forall}$. The definitions of these orders are intuitive. For example, for the multi-agent plausibility models $\leq_{a,\exists\exists}$ will get the following definition: $\mathcal{M} = (S, \leq_a, V)$, then $\phi \leq_{a,\exists\exists} \psi$ if and only if there there exists a state $s \in S$ and a state $t \in S$ such that $M, s \Vdash \phi$, $M, t \Vdash \psi$ and $s \leq_a t$. Likewise $\phi \leq_{a,\forall\exists} \psi$ if and only if for all states $s \in S$, if $M, s \Vdash \phi$ then there exists a state $t \in S$ such that $M, t \Vdash \psi$ and $s \leq_a t$.

This thesis uses the $\leq_{AE}$ preference relation to define the ordering among the set of abductive solutions.

Given these definitions this work now follows Nepomuceno-Fernández et al. (2013) by defining a modality for abductive reasoning. In contrast to Nepomuceno-Fernández et al. (2013), the current work defines the abduction modality as initiated by either a fair game upgrade or a private upgrade:

Definition 19. $(\mathcal{M}, s) \Vdash [Abd^F_G\chi]\phi$ *iff*

- $(\mathcal{M}_{\chi!^F_G}, s) \Vdash B_a\chi$ *and* $(\mathcal{M}, s) \Vdash \neg B_a\chi$ *and*
- $((\mathcal{M}_{\chi!^F_G})_{\bigvee \Sigma_\chi(\Uparrow)^F_G}, s) \Vdash \phi$

or

- $(\mathcal{M}_{\phi(\Uparrow)^F_G}, s) \Vdash B_a\chi$ *and* $(\mathcal{M}, s) \Vdash \neg B_a\chi$
- $((\mathcal{M}_{(\phi\Uparrow)^F_G})_{(\bigvee \Sigma_\chi\Uparrow)^F_G}, s) \Vdash \phi$

Where Σ_χ is the set of abductive solutions [5] for χ, i.e. $\Sigma_\chi = \{\eta \mid (\mathcal{M}, s) \Vdash K_a(\eta \rightarrow \chi)\}$. (Nepomuceno-Fernández et al., 2013)

Note that the only difference between the first and the second disjunct is the kind of upgrade the agents use after the surprising observation.

Observe that abduction is seen as a two step process and therefore the model of abductive reasoning consists of three parts. First the initial situation before the surprising observation. Then the surprising observations changes the model into an intermediate state. Then the real abduction takes place, modelled by an upgrade with all possible, non-trivial explanations. The assumption is that those explanations are non-trivial, and literals. Further one should note that the abductive modality is evaluated in the second state of the model and depends on the first state. Once the fair game upgrade with the surprising event took place, the model changed. The proposed language is not capable of talking about the previous states of the models since it hence it is impossible to talk in a later state about a previous state of the model, since it does not include a backward looking modality (as proposed by Yap (2011)). Instead, keeping the definition of abduction in mind; this thesis uses an upgrade with ϕ followed by an upgrade with all possible non-trivial explanations of ϕ (where ϕ is a literal), to model this abductive process.

4.3 Formal protocols

This section is based upon the diagrams in Chapter 3 together with the defined logic. The intuitive protocol diagrams in the previous section show the order in which the announcements and acts of abductive reasoning are to take place. Each announcement and reasoning inference upgrades the beliefs of some agent(s). The formal protocols in this section translate the diagrams into sequences of acts as defined in the logic. Note that these protocols do not include the announcements of agents but just the upgrades of the beliefs caused by the announcements. We thus present the possible order in which those acts take place. We start with any Epistemic State Model $\mathcal{M} = (S, \leq_a, V)_{a \in A}$, and the state $s* \in S$ that represents the real world. We represent the order in which actions $(\pi_1, ..., \pi_n)$ typically will occur in a gossip scenario by a sequence of dynamic modalities in the language. This order is based upon the diagrams in the previous section. The application of act π_1 followed by act π_2 is represented as $[\pi_1][\pi_2]$. This representation enables us to talk about the Epistemic State Models after the application of the sequence of actions, since we can now express that the Epistemic State Model, makes the formula ϕ true after the application of act π_1 followed by the application of act π_2: $\mathcal{M}, s \Vdash [\pi_1][\pi_2]\psi$.

We let $\mathcal{M} = (S, \leq_a, V)_{a \in A}$ be an Epistemic State Model and $s* \in S$ the real world.

When the gossip starts, typically the following actions occur:

$$[Abd_a\phi]\tau$$

Where ϕ stands for the surprising observation, a for the agent that makes the observation and starts the gossip and τ for a formula that we characterize

[5] Nepomuceno-Fernández et al. (2013) describes two different ways to upgrade the initial plausibility order after the upgrade with the surprising formula. We use the specific case where the agent applies an upgrade with the set of all possible explanations

below. We will first explain more about the use of this formula τ and how we proceed in defining the protocols.

The protocol consists of several steps. Each step indicates how to proceed the next step of the protocol or it indicates that the protocol terminates. In order to keep track of the total sequence of acts we start with the formula $[Abd_a\phi]\tau$ where τ is characterized below.

Further we use $[\pi\natural]$ as a syntactical abbreviation the sequence of previous (denoted) acts. This enables us to express whether some formula is true in a state of an Epistemic State Model after the application of the total sequence of acts (denoted by $[\pi\natural]$). Some steps of the protocol involve if-cases (decisions). The truth value of the antecedents of those if-cases depend on the initial Epistemic State Model and the initial (real) state as well as on the previous actions that were applied to this model. More explicit, the antecedents are formulas $\mathcal{M}, s* \Vdash [\pi\natural]\psi$, where $\mathcal{M} = (S, \leq_a, V)_{a\in A}$ is the initial model, and $s* \in S$ the real event. ψ is the formula that defines the if-case.

Recall that the protocol begins with:

$$[Abd_a\phi]\tau$$

Where ϕ expresses the surprising formula, a the observing agent and τ the formula that will be defined in the next step. We let χ^a_{best} denote the best explanation of agent a for ϕ. χ^a_{best} thus stands for the formula that agent a came to believe by abductive reasoning.

The reason for an agent to start to gossip might be to get to know the truth value of χ^a_{best}, in which case the sequence of actions will be: (With $\{a, b\} \subseteq G \subseteq F \subseteq A$).

If $G = B$, the protocol ends (this case matches the scenario where no agent has any relevant information to add to the conversation.)

Otherwise, pick an agent b and add b to B. Now:

1. if $\mathcal{M}, (s) \Vdash [\pi\natural]K_b\chi^a_{best}$ then:
   $\tau = [\chi^a_{best}!^F_a]K_a\chi^{best}_a$ [6]. It is now the case that $M, s \Vdash [\pi\natural]K_a\chi^a_{best}$.
   Hence, agent a achieved her goal and the protocol **ends**[7] .

2. if $\mathcal{M}, (s) \Vdash [\pi\natural]K_b\neg\chi^a_{best}$ then:
   $\tau = [\neg\chi^a_{best}!^F_a]K_a\neg\chi^a_{best}$
   It is now true that $M, s \Vdash [\pi\natural]K_a\neg\chi^a_{best}$. Hence, agent a achieved her goal and the protocol **ends**.

3. if $\mathcal{M}, (s) \Vdash [\pi\natural](\neg K_b\chi^a_{best}) \wedge (\neg K_b\neg\chi^a_{best})$ then:
   $\tau = [Abd_b(B_a\chi^a_{best}!^F_b)]\rho$ with ρ as defined below:

   - if $\mathcal{M} \Vdash K_b\neg\phi$ then:
     $\rho = [\neg\phi!^F_a]K_a\neg\phi$. Hence,

[6] Notice that this indeed holds, since the best explanation χ can only be either a literal or of the form $B\phi$ or $K\phi$ where ϕ is a literal. (Or in the case of higher order beliefs a sequence of B and K operators and a literal). In this way we can avoid Moore sentences (Holliday and Icard (2010)) which would require an adjustment of our setting.

[7] Note that this sequence of actions denoted by $[\pi\natural]$ differs form the previous sequence of actions $[\pi\natural]$. This is because after the previous instance of $[\pi\natural]$ we applied the program $[\chi^a_{best}!^F_a]$. Hence this $[\pi\natural]$ denotes the sequence of acts denoted by the previous $[\pi\natural]$ followed by $[\chi^a_{best}!^F_a]$.

$\mathcal{M} \Vdash [\pi\natural]K_a\neg\phi$. Since χ^a_{best} implies ϕ (and the agent knows this), we also have $\mathcal{M} \Vdash [\pi\natural]K_a\neg\chi^a_{best}$. Hence the agent achieved her goal and the gossip (protocol) **ends**.

- if $\mathcal{M} \Vdash K_b\phi$ then:
  $\rho = [\phi!^F_a]\zeta$, with ζ defined below, continue with **3b**.
- if $\mathcal{M} \Vdash (\neg K_b\neg\phi) \wedge (\neg K_b\phi) \wedge (B_b\phi)$ then:
  $\rho = \zeta$, with ζ defined below, continue with **3b**.
- if $\mathcal{M} \Vdash (\neg K_b\neg\phi) \wedge \neg K_b\neg\phi$ then:
  $\rho = [Abd_b\phi]\zeta$, with ζ defined below, continue with **3b**.

3b. Let γ be any other belief of any other agent that gave that agent b the belief that χ^b_{best}, now either:

- if $\mathcal{M} \Vdash \neg B_aB_b\chi^b_{best}$ then:
  $\zeta = [B_b\chi^b_{best}!^F_a][\gamma!^F_a]\tau'$, with τ' defined later, now there are two cases:
  - If $B = G$, the protocol **ends**, $\tau' = \neg K_a\chi^a_{best} \wedge \neg K_a\neg\chi^a_{best}$. As a consequence, it will now be the case that $\mathcal{M}, s \Vdash [\pi\natural]\neg K_a\chi^a_{best} \wedge \neg K_a\neg\chi^a_{best}$: the agent did not achieve her goal.
  - If $B \neq G$, pick a new agent b, continue with **1**, with τ in case 1 substituted by τ', ρ by ρ' and ζ by ζ'. (We do not substitute the formulas that are already defined, but only those denoted in the cases such that they can serve as new undefined formulas.)
- if $\mathcal{M} \Vdash [\pi\natural]B_aB_b\chi^b_{best}$:
  $\zeta = \tau'$, now either
  - If B=G the protocol **ends**, we have: $\tau' = \neg K_a\chi^a_{best} \wedge \neg K_a\neg\chi^a_{best}$. We then have $\mathcal{M}, s \Vdash [\pi\natural]\neg K_a\chi^a_{best} \wedge \neg K_a\neg\chi^a_{best}$: the agent did not achieve her goal.
  - If $B \neq G$: pick a new agent b, continue with **1**, with τ substituted by τ', ρ by ρ' and ζ by ζ'.

4. if $\mathcal{M}, s \Vdash [\pi\natural]\neg K_b\chi^a_{best} \wedge \neg K_b\neg\chi^a_{best} \wedge B_bBa\chi^a_{best}$, then:

   - If $G = B$, the protocol **ends**, and we have $\tau = \neg K_a\chi^a_{best} \wedge \neg K_a\neg\chi^a_{best}$. We then have $\mathcal{M}, s \Vdash [\pi\natural]\neg K_a\chi^a_{best} \wedge \neg K_a\neg\chi^a_{best}$: the agent did not achieve her goal.
   - If $G \neq B$, pick an agent $b' \in G \setminus B$ and add b' to B. Start over from step 1, and substitute b by b'.

Another reason to start gossip might be to get to know an explanation for ϕ, in which case the sequence of actions will be:
(With $\{a, b\} \subseteq G \subseteq F \subseteq A$).

1. if $\mathcal{M} \Vdash [\pi\natural]K_b\chi^a_{best}$, then:

   - if $\mathcal{M}, (s) \Vdash [\pi\natural]B_bB_a\chi^a_{best}$, then:
     $\tau = [\chi^a_{best}!^F_a]K_a\chi^a_{best}$.
     We now have that $\mathcal{M}, s \Vdash [\pi\natural]K_a\chi^a_{best}$. Hence agent a achieved her goal and the protocol **ends**.

- if $\mathcal{M}, (s) \Vdash [\pi\natural]\neg B_b B_a \chi^a_{best}$, then:
  $\tau = [\chi^a_{best}!^F_a]\rho$, with ρ defined below, continue with 3a.

2. if $\mathcal{M} \Vdash [\pi\natural]K_b\neg\chi^a_{best}$, then:

   - if $\mathcal{M} \Vdash B_b B_a[\pi\natural]\chi^a_{best}$, then:
     $\rho = \tau'$ $\tau = [\chi^a_{best}!^F_a]\tau'$ and:
     - if $G = B$, the protocol **ends** [8]
     - if $G \neq B$ otherwise, start over from **1**, with τ substituted by τ' and χ^a_{best} substituted by χ'^a_{best} with $B = G \setminus a$
   - if $\mathcal{M} \Vdash \neg B_b B_a \chi^a_{best}$, then:
     $\tau = [\chi^a_{best}!^F_a]\rho$, with ρ defined below, continue with case **3a**.

3. if $\mathcal{M} \Vdash (\neg K_b \chi^a_{best}) \wedge (\neg K_b \neg\chi^a_{best})$, then: $\tau = \rho$, with ρ defined below continue with case **3a**.

   (a) $\rho = [Abd_b(B_a \chi^a_{best}{}^F_b)]\zeta$
   - if $\mathcal{M} \Vdash [\pi\natural]K_b\neg\phi$, then:
     $\rho = [\neg\phi!^F_a]K_a\neg\phi$
     Now $\mathcal{M}, (s) \Vdash [\pi\natural]\neg\phi$. Hence there cannot be an explanation for ϕ (and the agent knows this). Therefore the protocol **ends**.
   - if $\mathcal{M} \Vdash K_b\phi$, then:
     $\zeta = [\phi!^F_a]\nu$, with ν defined below, continue with **3b**.
   - if $\mathcal{M} \Vdash (\neg K_b\neg\phi) \wedge (\neg K_b\phi) \wedge (B_b\phi)$, then:
     $\zeta = \nu$, with ν defined below, continue with **3b**.
   - if $\mathcal{M} \Vdash (\neg K_b\neg\phi) \wedge \neg K_b\neg\phi$, then:
     $\zeta = [\phi!^F_b]\nu$ or $\zeta = [\phi \Uparrow^F_b]\nu$, with ν defined below, continue with **3b**.

   Let γ be any other belief of other agents that gave that agent b the believes that χ^b_{best}.

   (b) if $\mathcal{M} \Vdash [\pi\natural]\neg B_a B_b \chi^b_{best}$, then:
   $\nu = [B_b\chi^b_{best}!^F_a][\gamma!^F_a]\mu$ with τ' defined below, continue with one of the two cases:
   - if $\chi'^a_{best}{=}\chi^a_{best}$, the either:
     - if $B = G$, the protocol **ends**
     - if $B \neq G$, pick a new agent $b' \in G \setminus B$, add b' to B, continue with **1**. and substitute b by b' and τ by τ'.
   - if $\chi'^a_{best} \neq \chi^a_{best}$ then either:
     - if $G = B$, the protocol **ends**
     - if $B \neq G$, start over from **1**, let $B = G \setminus a$ and substitute each χ^a_{best} by χ'^a_{best} and τ by τ'.

4. if $\mathcal{M}, s \Vdash [\pi\natural]\neg K_b\chi^a_{best} \wedge \neg K_b\neg\chi^a_{best} \wedge B_b Ba\chi^a_{best}$, then:

[8] Note that at this point (as well as at some other points in the protocol) we have not yet provided the characterization of τ'. In order to make the formula (indicating the actions that have taken place) complete, one can take $\tau' = \top$

- If $G = B$, the protocol **ends**
- If $G \neq B$, pick an agent $b' \in G \setminus B$ and add b' to B. Start over from step **1**, and substitute b by b'.

4.4 Examples

The simple example below models the sequence of epistemic actions (i.e. a surprising observation followed by abductive reasoning) that make an agent to start a gossip.

4.4.1 Example 1, abduction and the initial situation

During an experiment done by DiFonzo et al. (2013) Students were asked to choose the most plausible cause of a given situation. One example was the following:

"A professor you know who made fun of students was found dead this morning (e)."

Possible explanations:

1. He was shot by an angry student (a).
2. He was robbed (b).
3. He accidentally shot himself while cleaning his gun (c).

(ibid. p384)

Clearly those examples include abductive reasoning. The students will first take e into account (which is modelled by a fair game upgrade of the model with e). Then they will select the best explanation for e (modelled by a private upgrade of the agents models with the disjunction of all non-trivial explanations χ such that $\chi \to e$. Since a as well as b and c imply e, the students will (after this upgrade) believe either a, b or c. Which of these three propositions the student will believe after the upgrade depends on the student's epistemic state before the surprising observation.

Let Frederick be a student. Assume that he was among the students that chose option a in the survey. He thinks it is most plausible that the professor was shot by an angry student and least plausible that he accidentally shot himself while cleaning his gun. Modelling this example the way Nepomuceno-Fernández et al. (2013) and Velázquez-Quesada et al. (2013) suggest to model abductive reasoning gives the following models:

In all models below $A = f$ where f represents Frederick. Further the arrow represents the plausibility relation $\leq_f$, the reflexive and transitive arrows are not drawn. We assume that s_1 is the real state.

The initial situation is given in the following model $\mathcal{M}_1$:

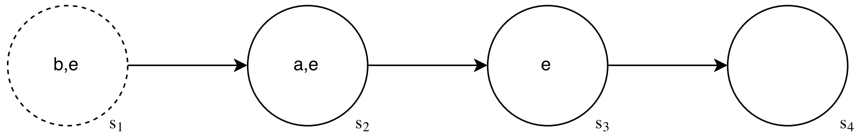

Figure 4.3: Epistemic State Model $\mathcal{M}_1$, model of the inital situation

This model shows the initial situation. It shows that Frederick believes that the professor is not dead $B_f \neg e$. Now Frederick is told that a professor she knows who made fun of students was found dead this morning. Under the assumption that Frederick takes this as hard evidence, his model has to be upgraded by a fair game upgrade with e (i.e. $e!_f$). This gives the following Epistemic Action Model $\mathcal{M}_1$:

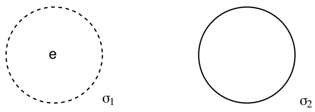

Figure 4.4: Epistemic Action Model $\mathcal{N}_1$, modeling $e!_f$

$\mathcal{M}_1 \otimes \mathcal{N}_1$ gives:

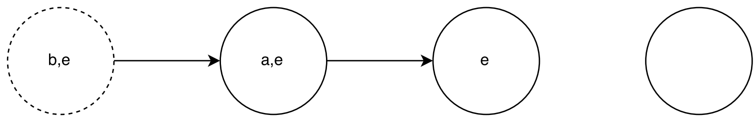

Figure 4.5: Second Epistemi State Model $\mathcal{M}_2$, the result of $\mathcal{M}_1 \otimes \mathcal{N}_1$

Frederick now knows e: $\mathcal{M}_2 \Vdash K_f e$. Frederick now does a private upgrade of his the model representing his epistemic states with all non-trivial explanations for e, i.e. $a \vee b$. This corresponds with the following Epistemic Action Model $\mathcal{N}_2$:

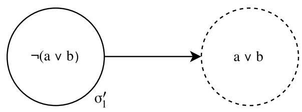

Figure 4.6: Epistemic Action Model $\mathcal{N}_2$, represents the private upgrade with all non-trivial explanations of e

$\mathcal{M}_2 \otimes \mathcal{N}_2$ gives $\mathcal{M}_3$:

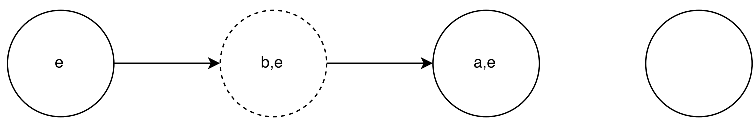

Figure 4.7: The last Epistemic State Model $\mathcal{M}_3 = \mathcal{M}_2 \otimes \mathcal{N}_2$

In this last Epistemic State Model it is indeed the case that Frederick believes that the professor was shot by an angry student: $\mathcal{M}_3 \Vdash B_f a$

4.4.2 The initial model

Another thing to discuss is the initial plausibility order. The plausibility order of the world where e, a is true and the world where e, b is true is obtained by the background knowledge of the agent. It is also obvious why the worlds where e is true are less plausible compared to the worlds where e is false. Before the observation, one simply does not expect his professor to be dead out of a sudden. The order between the world where e is true, but a, b and c are false and each of the worlds where e is true and a, b or c is true is less obvious. This order allows us to model abduction as a two step reasoning process, first the upgrade, or private upgrade with e, secondly a private upgrade with all possible (non-trivial) explanations. One might question what the world where e is the only true proposition expresses. It might seem weird that it is considered more plausible that e is true without any of the explanations being the case compared to the world where e is true together with the most plausible explanation. My claim is however that the initial order is indeed correct. Of course an agent will consider it more likely that e has a (non-trivial) explanation than that e has non explanation at all. But there is no specific explanation that e considered to be more likely than $\neg e$. The reason that the agents expect the world where only e and non of its explanations is true represents the conservativeness of the agent. The agent believes that e, a, b end c are all false in the current world. She simply believes that is is more plausible that she is wrong about one of those propositions than about two. Only by abduction the agent will realise that it does not makes sense that the professor died without any cause. Therefore the agents upgrades her model with all possible, not-trivial explanations of e. This actually meets our daily experience. After a long day of work you finally got home, it is dark inside so you switch the light switch on, but it stays dark. You realise that the light does not work. My intuition and experience is that, one will just keep switching the switch a couple of times before they come up with a possible explanation. This might seem irrational, but it seems to be the case indeed. Further if you ask someone to imagine that his light does not work, and you ask for the most plausible explanation an agent needs some time. However they do believe that there is an explanation for the broken light, but they do not believe one specific explanation yet.

4.4.3 Combining multiple single-agent models

The previous example only involved the actions typically occur before a gossip scenario starts. There are, as the protocols above define, different ways of how such a gossip can continue. In order to model a whole gossip scenario, the empirical and doxastic states of all agents involved are simultaneously modelled below by different single-agent dynamic epistemic models.

Those single agent models however are unable to model the beliefs about and knowledge of agents about the beliefs and knowledge of other agents. In real life lots of announcements (e.g. 'Petra has an affair! ' or 'I saw Petra hugging someone ') will be interpreted as statements about the beliefs and knowledge of the announcer (e.g. 'I believe that Petra has an affair' and 'I know that Petra was hugging someone '). Both, the knowledge and belief statements might be a surprise to the listener. However, only beliefs can be the conclusion of abductive reasoning. Counter abduction of the listener thus only makes sense in the case of an announcement that is interpreted merely as a belief (and especially not as knowledge) of the announcer. A later part of the thesis uses multi-agent models that allow to model beliefs about and knowledge of the beliefs and knowledge of other agents. For now this work will just point out whether an announcement of ϕ is interpreted by the receiver as believed or known by the speaker, this justifies whether the receiver gets just to believe or will also know that ϕ is true (i.e. modelled by respectively the application of a private upgrade or a fair game upgrade) and also whether the receiver gets to know that the announcer respectively believes or knows ϕ (which is modelled by a fair game upgrade).

4.4.4 Example 2, discussing the initial analysis of Bertolotti and Magnani.

Jason: ‘Guess what! Petra must be having an affair!’
Lynda: ‘No way! How do you know?’
Jason: ‘I saw her this morning at the station, holding passionately a man that was not her husband...’
John: ‘Oh no, that was her brother. She muttered something about not being able to come to the corporate picnic yesterday because he was visiting.’
Patricia: ‘That’s impossible. My aunt and Petra’s mother were very close friends, Petra is an only child’
(Bertolotti and Magnani, 2014)[p.4042].

According to Bertolotti and Magnani (2014) the first part of this dialogue is abductive reasoning:

1. Petra was holding passionately a man (who is not her husband) at the station. (The surprising fact C is observed).

2. But if Petra was having an affair, then she would be holding passionately a man who is not her husband. (But if A were true, C would be a matter of course.)

3. Hence Petra must be having an affair. (Hence there is reason to suspect that A is true).

(Bertolotti and Magnani, 2014, p.4052).

As soon as one wants to model this dialogue, one will have to deal with two implications which seems to be non-classical, one in the object-language and one in the meta-language. The implication in the meta-language (1 and 2 implies 3) is what this thesis calls the abduction implication, which is consider to be indeed non-classical. This implication is the result of abductive reasoning, that the current work defined based upon the proposal of Nepomuceno-Fernández et al. (2013).

However, the other non-classical implication, the implication in 2, is not just the implication abduction deals with. The proposed logic consists of a classical implication. This might seem to be a problem. My claim is however, that it is not because of the nature of the dialogue that the implication appears to be non-classical, but just because of the phrasing off this reasoning process. The proposal is that the non-classical implication in the example is a result of neglecting several classical implications (i.e. deductive reasoning steps). My aim is to model this dialogue using a language in which the implication is classical. Therefore, in contrast to Bertolotti and Magnani (2014), the current thesis suggest the following analysis of the dialogue:

1. Petra was holding passionately a man (who is not her husband) at the station this morning. (The surprising fact C is observed)
2. If Petra was holding her lover passionately at the station this morning, then she would be passionately holding a man who is not her husband at the station this morning. (But if A were true C would be the matter of course).
3. Hence Petra must have hold passionately her lover at the station this morning. (Hence there is a reason to suspect that A is true.
4. If Petra was holding her lover passionately at the station this morning, she has an affair. (Deduction)
5. Therefore Petra must have an affair (Conclusion).

The claim thus is that this example is a combination between abductive and deductive reasoning. Since this thesis simplifies the agents to omniscient agents, we will neglect the deduction of this example. Note that, following the analysis above, all implications are classical implications. This enables us to use the proposed language to model this dialogue.

The following propositions will be used:
e: evidence (Petra was holding passionately a man (who is not her husband) at the station this morning)
a: Petra was holding her lover passionately at the station this morning
b: Petra was holding her brother passionately at the station this morning.

The initial situation
The next model shows the initial situation, before the gossip even started.

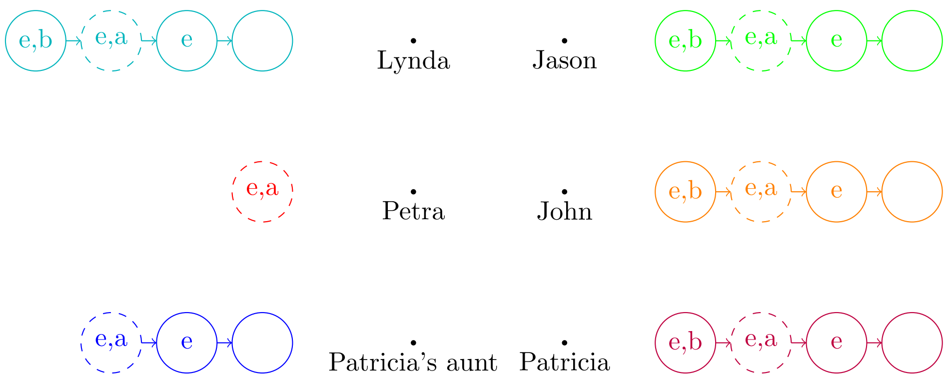

Figure 4.8: The initial situation

Model 1, the initial situation

The next step is to model the actions that happened after the initial situation. Since each action changes only the information of one single agent, all those actions can be modelled simultaneously. This thesis therefore will do the following upgrades at once. The Epistemic Action Models are not drawn.

$(\neg b)!_{Patricia}$: The aunt of Patricia tells Patricia that Petra is an only child, hence we apply a fair game upgrade to the Epistemic State Model of Patricia with $\neg b$.

$e!_{Jason}$: Jason observes Petra passionately holding a man (who is not her husband) at the station (this morning).

$(\neg e \vee (e \wedge \neg a \wedge \neg b) \vee (e \wedge b)) \Uparrow_{John}$: Petra tells John that she will not be at the company lunch because her brother is visiting. One might wonder why we upgrade the model with $(\neg e \vee (e \wedge \neg a \wedge \neg b) \vee (e \wedge b))$ and not simply with b. Keep in mind that b expresses: 'Petra was holding her brother passionately at the station this morning'. The announcement that Petra wont be at the company lunch because her brother is visiting does indeed slightly increase the probability that Petra was holding her brother passionately at the station this morning. John however initially does not believe that Petra was passionately holding a man at the station this morning. And knowing the fact that her brother is visiting will not suddenly make him do so. This announcement only changes what Bertolotti and Magnani (2014) call the "base" of the agent. One possibility would be to upgrade with $e \rightarrow b$, in this case, after the fair game upgrade with $e \rightarrow b$ John will have the belief that if Petra was indeed holding someone at the station this morning, it probably would be her brother (since he is visiting). This however is not a belief one simply gets after the announcement that the brother of Petra is visiting. Only after getting to know that Petra was passionately holding some man (not her husband) at the station this morning, i.e. after some abductive reasoning, John will get to believe that Petra was passionately holding her brother at the station this morning. Therefore such an announcement that changes the base of an agent, is modelled by the upgrade with a formula which is true exactly in those worlds where either of the following cases holds:

1. the formula expressing the surprising event is false;

2. the surprising event is true but all non-trivial, possible explanations are false;

3. the surprising event is true and one specific explanation (which after deduction will become the best explanation) is true. This one specific explanation in this case is b.

This gives:

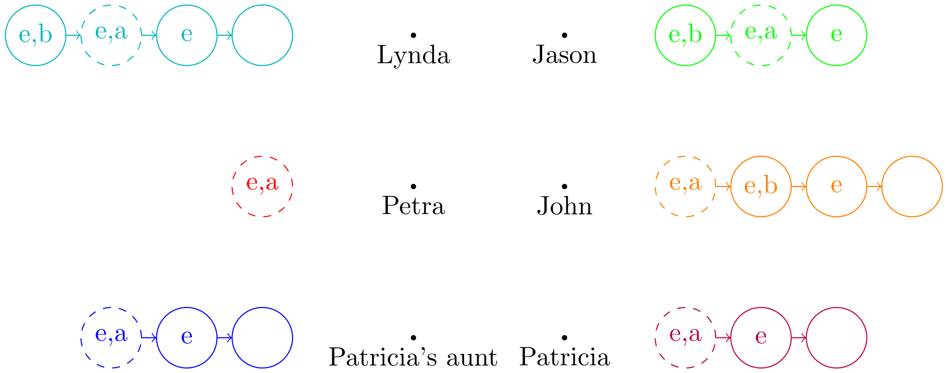

Figure 4.9: Model $\mathcal{M}_2$

This models nicely shows how the different agents have a different Individual Knowledge Base (and different beliefs). e was a surprise to Jason which causes him to perform abductive reasoning. The model of Jason is upgraded with the disjunction of all possible, non-trivial explanations for the surprising formula e. This gives the following upgrade:

$(\bigvee \Sigma\chi) \Uparrow_{Jason}$ equals in this case $(a \vee b) \Uparrow_{Jason}$

This gives us the following model:

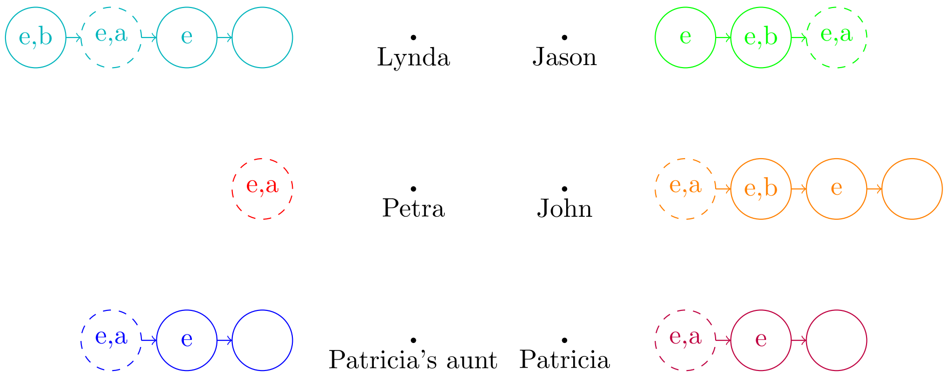

Figure 4.10: Model $\mathcal{M}_3$

This model shows the situation just before the actual conversation (gossip) begins.

After the upgrade, Jason believes that Petra has an affair. Jason, for some epistemological or social reason (e.g. excluding Petra from the social network, getting to know who Petra was holding passionately at the station this morning or just because he wants to know whether or not Petra indeed has an affair) announces a to Lydia, Jason, John and Patricia. Before this announcement Lynda, John and Patricia clearly did not yet believe that Petra has an affair. That is why Lynda is surprised when Jason states that Petra has an affair. The announcement 'Petra must be having an affair', does not expresses knowledge, but a belief. Lynda wants to know why Jason believes that Petra has an affair (this is part of the counter-abduction as described by Bertolotti and Magnani

(2014)). In this case Lynda is able to ask Jason the reason for his belief. For reasons given above, this step will not be represented in the model, but the response is represented; Jason then shares the information that he saw the event e (i.e. Petra holding a man passionately at the station this morning) with Lynda, John and Patricia.

Since we do not yet model the beliefs and knowledge about beliefs and knowledge of other agents, the only announcement that changes the first order beliefs and knowledge of any of the agents is Jason's announcement of e. This is a statement expressing knowledge. Therefore we will model this by a fair game upgrade of the model of Lynda, John and Patricia with e:

$e!_{Lynda,John,Patricia}$

Which gives:

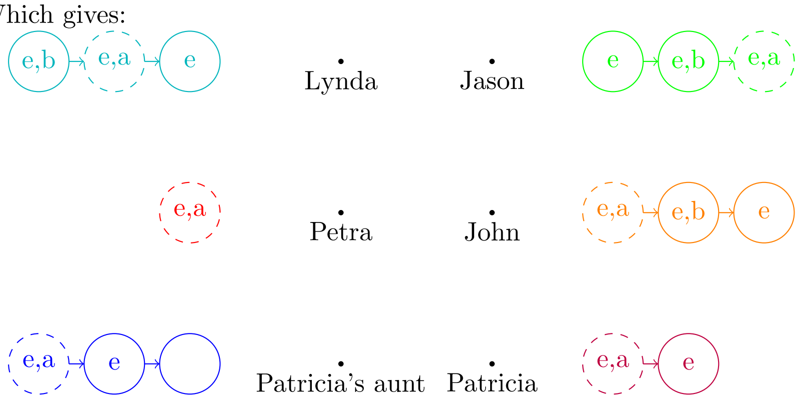

Figure 4.11: Model $\mathcal{M}_4$

After which Lynda, John and Patricia can use abduction to upgrade their model with all non-trivial possible explanations.

$(\Sigma \bigvee \chi) \Uparrow_{Lynda,John,Patricia}$ which, in this case, equals $(a \vee b) \Uparrow_{Lynda,John}$ and $(a) \Uparrow_{Patricia}$.

Hence the model now becomes:

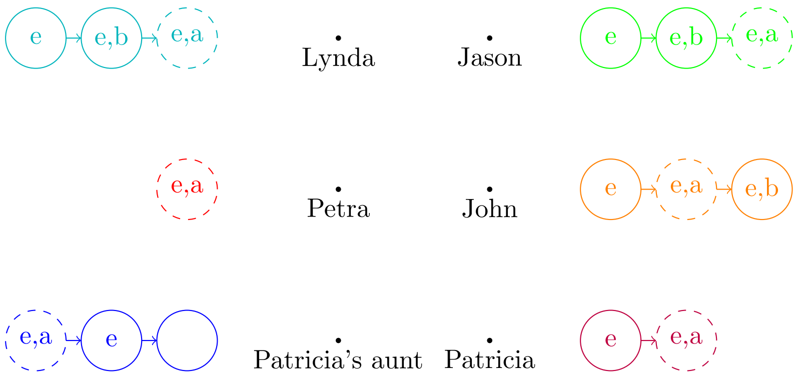

Figure 4.12: Model $\mathcal{M}_5$

After this upgrade, John has the believe that Petra was passionately holding her brother at the station, not her lover. He will share this belief with the rest of the group. This announcement does not lead to an upgrade of the beliefs of the other agents, since it conflicts the knoweldge of Patricia; she knows that Petra does not have a brother. She announces this piece of knowledge, and the other agents (i.e. Lynda, Jason and John) will get to know that $\neg b$. We will model this by a fair game upgrade of their beliefs with $\neg b$.

$\neg b!_{Lynda,Jason,John}$ which gives the following model:

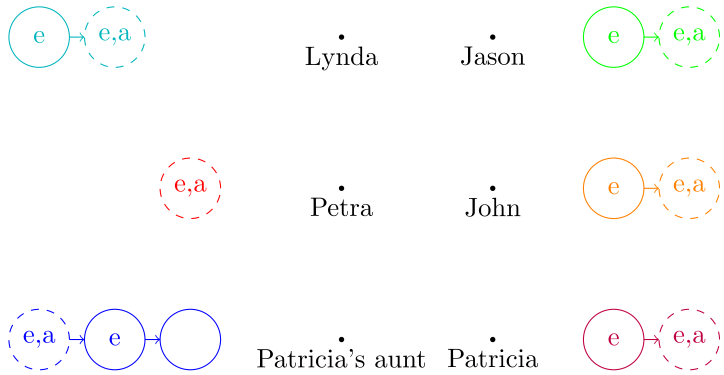

Figure 4.13: Model $\mathcal{M}_6$

As this last model shows, all agents (except from the aunt of Patricia, who was not among the agents gossiping about Petra) end up with the belief that Petra has an affair.

4.4.5 Multi-agent models

The previous section uses a combination of single-agent models to model the dialogue. This section uses multi-agent models, such that also the beliefs of agents about the epistemic and doxastic state of other agents can be modelled. In order to keep the models readable, we will reduce the gossip scenario a bit by restricting it to Jason, John and Patricia. In the models the real world is denoted by a dashed line.

The model includes the following propositions:
'a' for Petra was passionately holding her lover at the station this morning
'b' for Petra was passionately holding her brother at the station this morning
'e' for Petra was passionately holding a man who is not her husband at the station this morning.

The model includes the following agents:
'j' for Jason, 'k' for John, 'p' for Patricia and 'A' for all agents.

The figures below show the dynamic multi-agent models for the gossip scenario.

The Epistemic State Model just before the gossip starts is already quite complex. Therefore, this work starts with the initial situation instead. The first Epistemic State Model represents the state of the world before the agents got any information concerning this scenario. It thus pictures the situation before

Jason saw Petra at the station, before Petra told John about her brother, and before the aunt of Patricia told Patricia that Petra is only child:

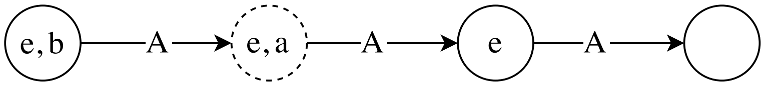

Figure 4.14: The first state of the model

In this state all agents have the same (relevant) information, hence they have the same beliefs about the truth of the propositions. Further they all know that no one has distinct knowledge or beliefs.

Let's now consider the situation where Patricia's aunt tells Patricia that Petra is only child. Assume that this gives rise to a fair game upgrade, Patrica knows that her aunt is a good friend of Petra her mother.

This gives $\mathcal{M}_2 = \mathcal{M}_{1_{b!^{Petra}_{Petra}}}$

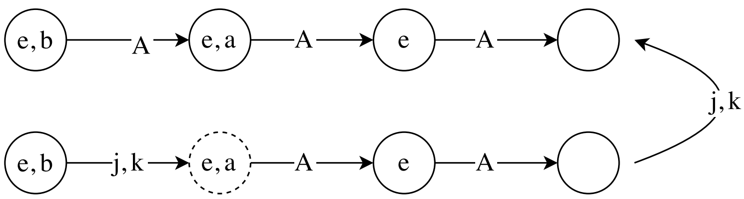

Figure 4.15: Epistemic State Model $\mathcal{M}_2$

The arrow at the left of the diagram abbreviates a set of plausibility arrows that connect the states with same propositional variables, (in the direction of the arrow at the left, for the agents denoted at the arrow at the left). The same will be the case for the arrows at the left side of the subsequent models.

Next, Petra tells John that she will not be at the company lunch because her brother is visiting. Hence John comes to believe that Petra has a brother. Hence, John upgrades his model with $(\neg e \wedge \neg a \wedge \neg b) \vee (e \wedge \neg a \wedge \neg b) \vee (e \wedge b \wedge \neg a)$:

$\mathcal{M}_3 = \mathcal{M}_{2_{(\neg e \wedge \neg a \wedge \neg b) \vee (e \wedge \neg a \wedge \neg b) \vee (e \wedge b \wedge \neg a) \Uparrow^{John}_{John}}}$
Which gives:

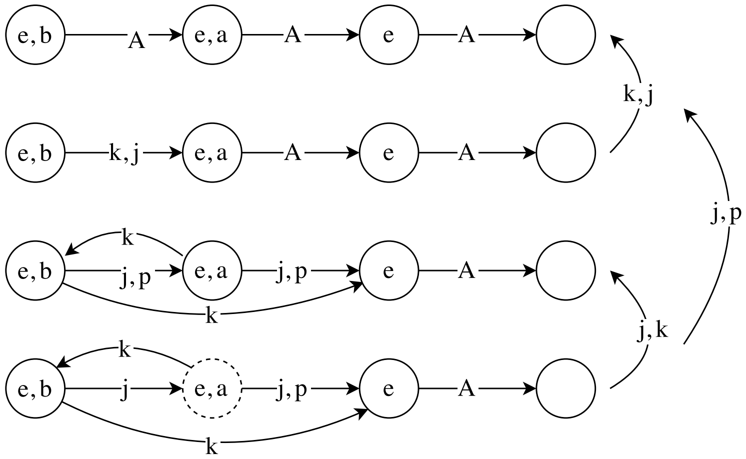

Figure 4.16: Epistemic State Model $\mathcal{M}_3$

Now Jason observes Petra passionately holding a man (who is not her husband) at the station this morning. We will model this by $e!_{Jason}$, hence: $\mathcal{M}_4 = {\mathcal{M}_3}_{e!^{Jason}_{Jason}}$

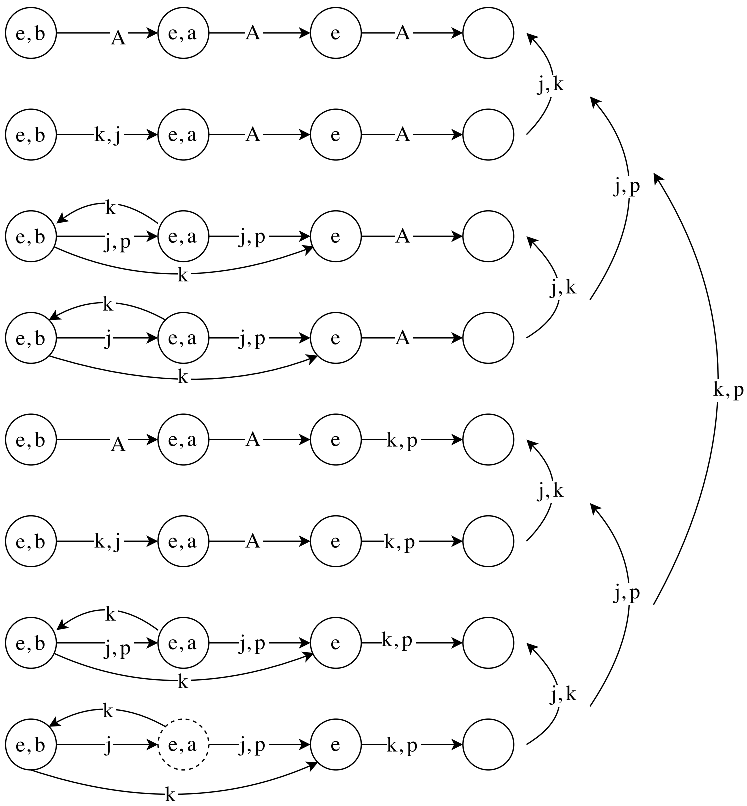

Figure 4.17: Epistemic State Model $\mathcal{M}_4$

Next, Jason upgrades his beliefs with all possible non-trivial η such that $\eta \rightarrow e$, in his case $a \vee b$ (modelled by a private upgrade). Hence:
$\mathcal{M}_5 = \mathcal{M}_{4_{\Sigma_e \Uparrow^{Jason}_{Jason}}}$

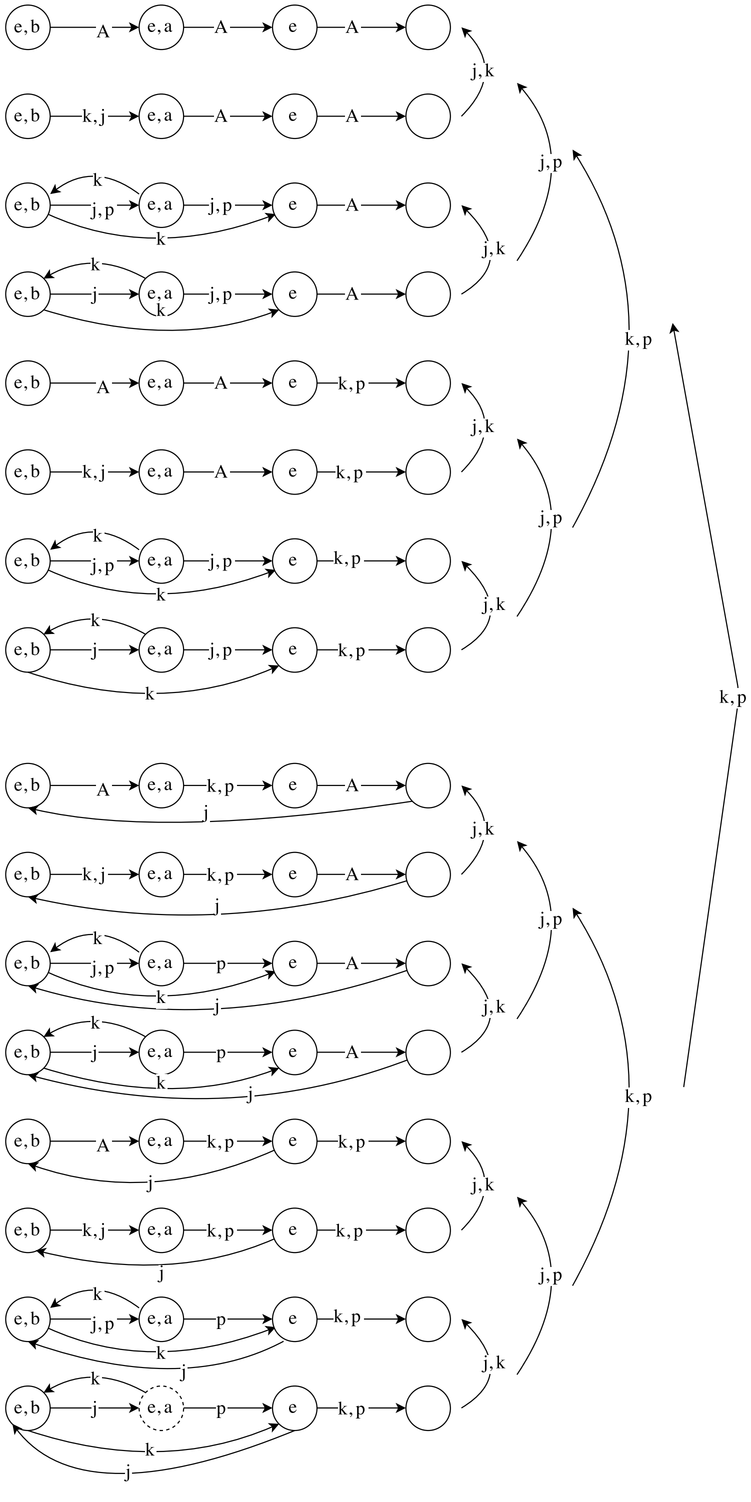

Figure 4.18: Epistemic State Model $\mathcal{M}_5$

Jason now makes the announcement that Petra must have an affair. The other agents will now get to know that Jason believes that Petra has an affair, hence:

$\mathcal{M}_6 = \mathcal{M}_{5_{B_j a \Uparrow^A_{p,k}}}$

Which gives:

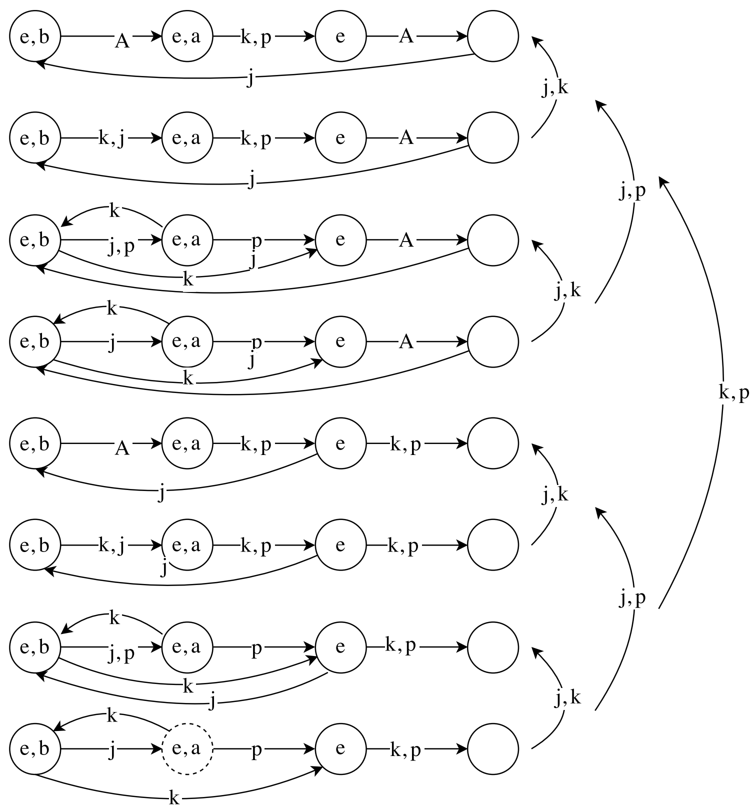

Figure 4.19: Epistemic State Model $\mathcal{M}_6$

Since before this upgrade the Patricia and John did not yet believe that Jason had the belief that Petra has an affair, they will ask Jason the reason for his belief. As a response he announces that he saw e. In this case, after Jason's announcement that Petra has an affair, the agents already concluded that Jason believes e, but not yet that he knows e. Hence, a fair game upgrade with $K_j e$ for all agents except Jason follows. This upgrade however has as a consequence that the agents also come to know themselves that e is true. Hence a fair game upgrade for Patricia and John $K_j e$ includes a fair game upgrade with e.

$\mathcal{M}_7 = \mathcal{M}_{6_{e!^A_{p,k}}}$

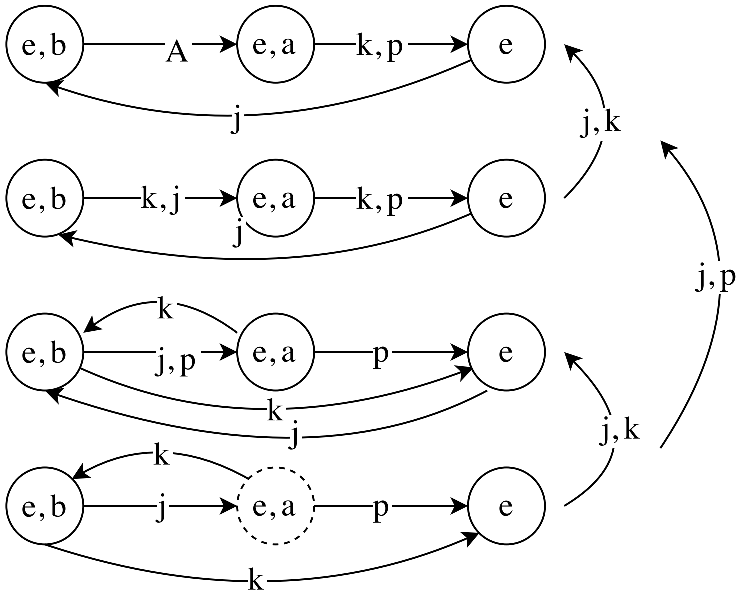

Figure 4.20: Epistemic State Model $\mathcal{M}_7$

Next, Patricia and John will do abduction on e. Both thus will upgrade their model will all non-trivial explanations (i.e. $a \vee b$ for John and a for Patricia): $\mathcal{M}_8 = \mathcal{M}_{7_{a \vee b \Uparrow^A_k a, \Uparrow^A_p}}$

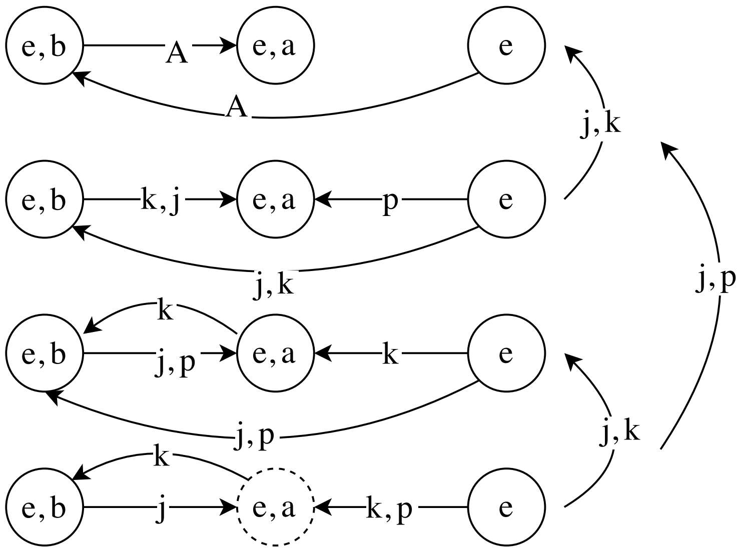

Figure 4.21: Epistemic State Model $\mathcal{M}_8$

Now John will announce his belief that Petra was hugging her brother at the station this morning: $\mathcal{M}_9 = \mathcal{M}_{8_{B_k b!^A_{j,p}}}$

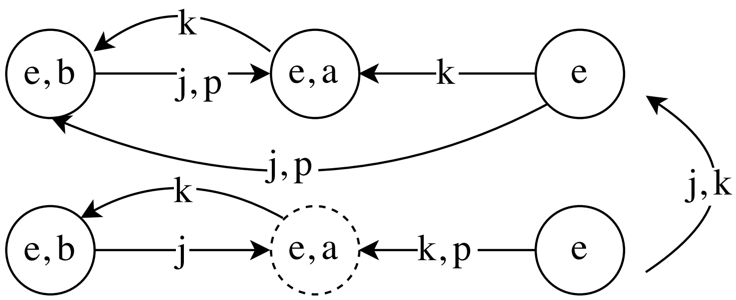

Figure 4.22: Epistemic State Model $\mathcal{M}_9$

After which Patricia will announce that she knows that Petra does not have a brother. Which causes a fair game upgrade with $K_p\neg b$:
$\mathcal{M}_{10} = \mathcal{M}_{9_{K_p\neg b!^A_{j,k}}}$

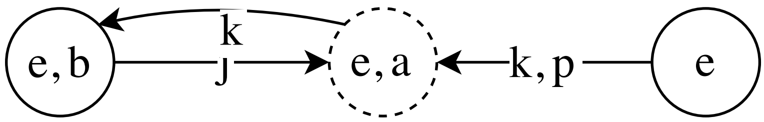

Figure 4.23: Epistemic State Model $\mathcal{M}_{10}$

Followed by a fair game upgrade with $\neg b$: $\mathcal{M}_{11} = \mathcal{M}_{10_{\neg b!^A_{j,k}}}$
:

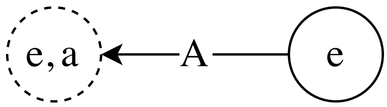

Figure 4.24: Epistemic State Model $\mathcal{M}_{11}$

Chapter 5

Social dimension of protocols

Until now we only considered the epistemic changes during a gossip scenario. This section adds the changes of the social network relations as well as the knowledge and beliefs of the agents about this network to the picture. We will use a multi-dimensional dynamic epistemic and doxastic logic to describe and model these changes.

5.1 Epistemic dynamic network language

5.1.1 Static part

The static part of the language is defined as follows:

Definition 20. *Dynamic Epistemic Network Logic.*
Let P be a set of atomic propositions p, N a set of atomic network propositions $N_{a,b}$ and A a set of agents,
$\phi ::= p \mid N_{a,b} \mid \neg\phi \mid (\phi \vee \phi) \mid K_a\phi \mid \Box_a\phi$
with $p \in P$, $N_{a,b} \in N$ and $a, b \in A$

Compared to the multi-agent dynamic language discussed in the previous section, we add for each two agents $a, b \in A$ an atom $N_{a,b}$ to the language. $N_{a,b}$ expresses that agents a and b are socially connected (e.g. they are friends via a social network relation). Note that, even though $N_{a,b}$ is an atomic proposition, it is not an element of P. A similar set of atoms can also be found in the language proposed by Christoff (2016); Baltag et al. (2016).

Semantics

Definition 21. *Epistemic Network plausibility models (Epistemic Network State Models)*
Let S be a set of epistemic states and A a finite set of agents. A model is defined as:
$\mathcal{L} = \langle S, A, N, \leq_a, V\rangle$. *$S, \leq_a$ and V are defined as in the Epistemic State Models. The only difference is the set N. $N(s)$ is a function that maps each state to*

a function that maps each agent to a subset of all agents. The function $N(s)$ thus defines the network relations among agents in each state of the model. We assume this relation to be symmetric, hence $b \in N(s)(a)$ iff $a \in N(s)(b)$. The real world in S is again denoted by $s*$. In the drawings we indicate this state by a dashed line.

The semantic interpretation is recursively defined as follows:

Definition 22. *Semantic interpretation*
$\mathcal{L}, s \Vdash \bot$ *never*
$\mathcal{L}, s \Vdash p$ *iff* $s \in V(p)$
$\mathcal{L}, s \Vdash N_{a,b}$ *iff* $b \in N(s)(a)$
$\mathcal{L}, s \Vdash \neg\phi$ *iff* $\mathcal{L}, s \nVdash \phi$
$\mathcal{L}, s \Vdash \phi \vee \psi$ *iff* $\mathcal{L}, s \Vdash \phi$ *or* $\mathcal{L}, s \Vdash \psi$
$\mathcal{L}, s \Vdash K_a\phi$ *iff for all* $t \in S$ *such that* $(s,t) \in \sim_a$, $\mathcal{L}, t \Vdash \phi$
$\mathcal{L}, s \Vdash \Box_a\phi$ *iff for all* $t \in S$ *such that* $s \leq_a t$, $\mathcal{L}, t \Vdash \phi$.
We write: $\mathcal{L} \Vdash \phi$ *iff for all states* $s \in S$, $\mathcal{L}, s \Vdash \phi$

5.1.2 Dynamics

Definition 23. *Epistemic Network Action Models based upon (Baltag and Smets, 2008, p.40)*
An Epistemic Network Action Model: $\mathcal{R} = (\Sigma, A, N^+, N^-, \leq_a, Pre)_{a\in A}$ *, where* $\Sigma, A, \leq_a$ *and* Pre *are defined as in the case of Epistemic Action Models.* N^+ *is a function that maps each state* σ *to a subset of* $A \times A$ *and represents a set pairs of agents* (a,b) *who became friends by the action. Likewise* N^- *is function that maps each state in* Σ *to a subset of* $A \times A$ *and represents the set of pairs of agents* (a,b) *which broke their friendship relation by the action. The set* N^+ *will be used in the definition for adding friends to one's network, the set* N^- *in the action of deleting friends from one's network. We denote the real event in* Σ *by* $\sigma*$*; in the figures we indicate this state by a dashed line.*

The new model is generated from the old Epistemic Network State Model $\mathcal{L} = (S, A, N, \leq_a, V)_{a\in A}$ and an Epistemic Network Action Model $\mathcal{R} = (\Sigma, A, N^+, N^-, \leq_a, Pre)_{a\in A}$ acting on the Epistemic Network State Model. Using the action priority upgrade, which is explained below, we compute the new upgraded Epistemic Network State Model which is denoted by $\mathcal{L} \otimes \mathcal{R}$.

Definition 24. *Upgrade set of state, upgraded valuation, upgraded plausibility order and upgraded network relation.*

The set of states of the upgraded Epistemic Network State Model are defined exactly the same as in the case of an upgrade Epistemic State Model, also the upgraded plausibility order is defined exactly the same as before. Only the upgraded valuation is defined slightly different:

The upgraded valuation is essentially given by the original valuation from the input Epistemic Network State Model: for all $(s,\sigma) \in S \otimes \Sigma$ *we put* $(s,\sigma) \Vdash p$ *iff* $s \Vdash p$ *for* $p \in P$ *(as before). The valuation of the atomic network propositions are defined as:* $(s,\sigma) \Vdash N_{a,b}$ *iff* $(a,b) \in N^+(\sigma)$ *or* $s \Vdash N_{a,b}$ *and* $(a,b) \notin N^-(\sigma)$.

Note that the valuation of atomic proposition of the form $N_{a,b}$ changes with the action. We dealt with this in a way similar to van Ditmarsch and Kooi (2008), who make use of so called "post-conditions".

5.2 Specific social scenarios

For fair game upgrades and private upgrades the Epistemic Network Action Models look almost the same as in the multi-agent language discussed before. The only difference is that we have to add $N^- = \emptyset$ and $N^+ = \emptyset$ to each of the nodes in the Epistemic Action Model. Only when an action involves the change of the network relation between agents, the Epistemic Network Action Models will look different. Two basic examples are given below:

5.2.1 Deleting a friend from one's network

Suppose that all agents in group $G \subseteq A$ want to unfriend agent c. This includes the case where G consists of one single agent. Suppose that $F \subseteq A$ know about this. (We assume that $G \subseteq F$, an agent knows if he/she breaks a friendship relation). We denote this action by $N^{-,F}_{(c,G)}$. This action is given by the following Epistemic Network Action Model:

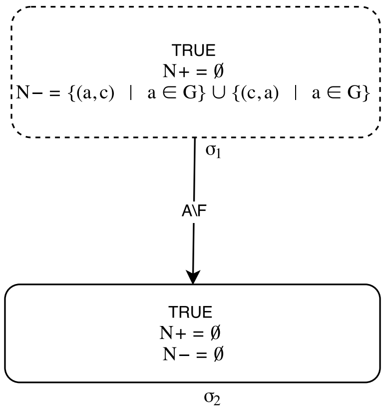

Figure 5.1: Deleting a friend

5.2.2 Adding a friend to one's network

Suppose that all agents in group $G \subseteq A$ want to befriend agent c. This includes the case where G consists of one single agent. Suppose that $F \subseteq A$ know about this. (We assume that $G \subseteq F$, an agent knows if he/she befriends someone). We denote this action by $N^{+,F}_{(c,G)}$. This action is given by the following Epistemic Network Action Model:

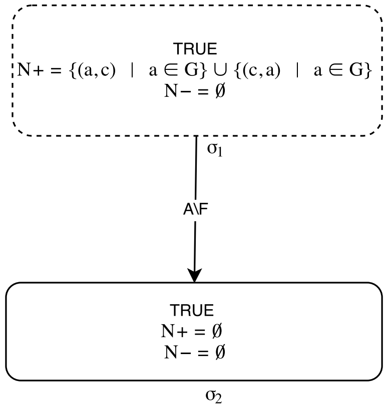

Figure 5.2: Adding a friend

Strength of bonds between two agents

Let b and c be two distinct agents. Now either:
(1) b and c are connected:

(a) Everyone knows for all $a \in A$: $K_a N_{b,c}$. This is the strongest connection.

(b) Everyone believes, but at least one agent does not know for sure: for all $a \in A$: $B_a N_{b,c}$ and there exists an agent d such that $\neg K_d N_{b,c}$. The higher the number of agents that do not know about this, the weaker the connection.

(c) Some agents do believe there is no connection: There exists a d such that $B_d \neg N_{b,c}$. The larger the number of agents that disbelieve the connection to exist, the weaker the connection.

There is no connection between b and c:

(a)Every agent knows that b and c are not connected, hence: forall $A \in a$: $K_a \neg N_{b,c}$. This is the strongest non-connection that can exist between agents. Think about two agents which are enemies and told everyone they are.

or:

(b)It also might be the case that at least every agent believes that b and c are not connected, hence:
for all $A \in a$: $B_a \neg N_{b,c}$. This is already a bit weaker, but at least one agent does not know this for sure, there exist an agent e such that $\neg K_e \neg N_{b,c}$. The more agents believe but do not know that there is no connection between b and c the less weak the social relation.

or: (c) There are agents that do believe that there is a connection between both and agents that do not:
There exist an agent e such that $\neg B_e \neg N_{b,c}$ and there exist an agent f such that $\neg B_e N_{b,c}$. The higher the number of agents that believe that there is a connection between b and c, the less weak the social relation.

Strength of the bond of a group of agents $G \subseteq A$

The social bonding of a group is the strongest if everyone inside de group is connected, and all agents inside and outside know. For all $a, b \in GN_{a,b}$ and for all a, b, c such that $a, b \in G$, $c \in A$: $K_a N_{a,b}$.

The social bonding is a bit less if more people do not know the existence of a social connection between two agents in the group. And even weaker if there are also agents who believe that two agents inside the group are not connected.

It is even weaker if some agents within the group are indeed not connected. Further, the higher the number of agents that know about this disconnection the weaker the bonding is.

We follow this reasoning down to the weakest scenario: There are no $a, b \in G$ such that there is a connection between those, and everyone knows this.

Hence: Strengthening the bonds between two agents can be modelled by a fair game upgrade or private upgrade with $N_{a,b}$. Likewise, weakening the bonds between two agents can be modelled by an upgrade with $\neg N_{a,b}$.

5.2.3 Dynamic syntax

Similar to the Epistemic Doxastic Dynamic syntax we discussed before, we will add for every formula ϕ, every agent a, every finite set of agents G and every finite set of agents F, a modality $[\phi!_a]^F_G$ and $[\phi_{\Uparrow a}]^F_G$ to the language, as defined before. Further we add for every agents $a \in A$, every finite set of agents $G \subseteq A$ and every finite set of agents $G \subseteq F$, two modalities to the language: $[N^{+,F}_{(a,G)}]$ and $[N^{-,F}_{(a,G)}]$, such that:

Definition 25. *Modalities for adding and deleting friendship relations*

$\mathcal{L}, s \Vdash [N^{+,F}_{(a,G)}]\phi$ *iff* $\mathcal{L} \otimes \mathcal{R}, (s, \sigma*) \Vdash \phi$.

Where $\sigma*$ is the $*$ indicated state in the Epistemic Network Action Model of $N^{+,F}_{(a,G)}$. Likewise:

$\mathcal{L}, s \Vdash [N^{-,F}_{(a,G)}]\phi$ *iff* $M, (s, \sigma*) \Vdash \phi$.

This time, $\sigma*$ is the $*$ indicated state in the Epistemic Network Action Model of $N^{-,F}_{(a,G)}$. Note that a denotes an agent and G a group of agents such that for each $b \in G$ the connection between a and b is either added or deleted. Further F is the set of agents which is aware of this action. This work assumes that $G \subseteq F$. $N^{-}_{(a,G)}$ is used as an abbreviation for $N^{-,G}_{(a,G)}$. Note that a is not necessarily in G and hence not necessarily in F; It thus might be the case that an agent a is not aware of the fact that another agent b breaks the bond between a and b.

This gives the following syntax:

Definition 26. *Let P be a set of propositions, the formulas ϕ of the language $\mathcal{L}$ by using double recursion defined by:*
$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid K_a\phi \mid \Box_a\phi \mid [\pi]\phi$
$\pi ::= \phi!^F_G \mid \phi \Uparrow^F_G \mid N^{-,F}_{(a,G)} \mid N^{+,F}_{(a,G)}]$ *with $p \in P$, $a \in A$ and $G \subseteq F \subseteq A$.*

Let Φ denote the set of all formulas in the language.

5.3 Formal protocols

In paragraph 4.3 this thesis gave the possible protocols for gossip with an epistemic goal. Apart from epistemic goals, gossip can also be used to achieve social goals. The main structure of the protocols for gossip with a social goal is similar to the structure of the protocols for gossip with an epistemic goal. If the goal is to include or exclude an agent from the group or to strengthen or weaken the bondings, the protocol of gossip where the goal is to get to know an explanation for the surprising event can be used. The only differences are in the conditions that have to be met in order to start and terminate the gossip. In the case of inclusion of an agent c or strengthening the bondings with an agent c, χ^a_{best} has to be about agent c and seen as a morally positive proposition. In the case of the exclusion of agent c or the weakening of the bondings with agent c the proposition χ^a_{best} has to be about agent c and seen as morally negative. In all four cases, apart from the termination conditions given in the protocol when the goal of gossip is to get to know an explanation, the protocol now also terminate as soon as χ^a_{best} is substituted by a proposition χ'^a_{best} with the opposite moral valuation. Note that after such a substitution it's agent a's turn to make an announcement. She can simply refuse to announce the new best explanation and may try to change the topic of the conversation instead. Of course other agents with different goals, might wish to continue the conversation. In that case the conversation might continue, but the protocol starts over as if a new gossip started. It is another agent with another goal that plays the role of 'agent a'. We therefore say that the protocol terminates and a new protocol starts.

If the goal is to strengthen the bondings within a group, one can simply use the protocol of gossip where the goal is to get to know an explanation for the surprising event. The only difference are the conditions to start the gossip. I.e. the gossip needs to be about an outsider.

5.4 Example

This section shows how gossip can lead to exclusion. This example is based upon the previous example, and involves the agents Petra, Jason and Lynda. We assume network relations between Petra and Lynda and Jason and Lynda. Jason does not like Petra and has the goal to break the network relation between Petra and Lynda. He observes Petra passionately holding with a man who is not her husband at the station and obtains the belief that she must have an affair (i.e. that she was passionately holding her lover at the station this morning). The Epistemic Network Action Models are not drawn. The actual world is denoted with a dashed line. In this example the set of agents A consists of Petra, Jason and Lynda. Normally the model would also include the plausibility relations for Petra, however, in order to keep the models readable, we neglect the plausibility relations for Petra.

a denotes: Petra was passionately holding her lover at the station this morning.
b denotes: Petra was passionately holding her brother at the station this morning.
e denotes: Petra was passionately holding a man (who is not her husband) at

the station this morning.
l denotes: Lynda.
j denotes: Jason.
p denotes: Petra.

The first model shows the initial situation:

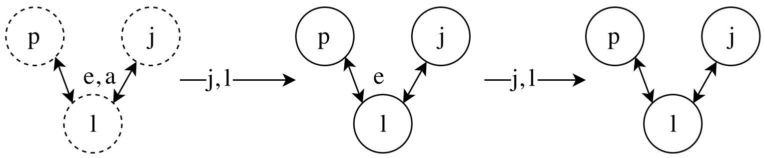

Figure 5.3: The first Epistemic Network State Model $\mathcal{L}_1$

Now Jason (privately) observes Petra passionately holding a man (who is not her husband) at the station this morning. This action is modelled by a fair game announcement of e to Jason. And only Jason is aware of this action: $e!^j_j$, which gives:

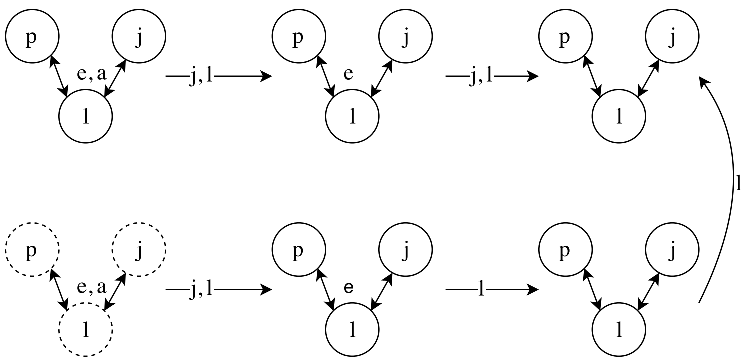

Figure 5.4: The second Epistemic Network State Model $\mathcal{L}_2$

This is directly followed by abductive reasoning, i.e. by an upgrade with all non-trivial explanations for e. In this case a, b.
$a \vee b \Uparrow^j_j$ gives:

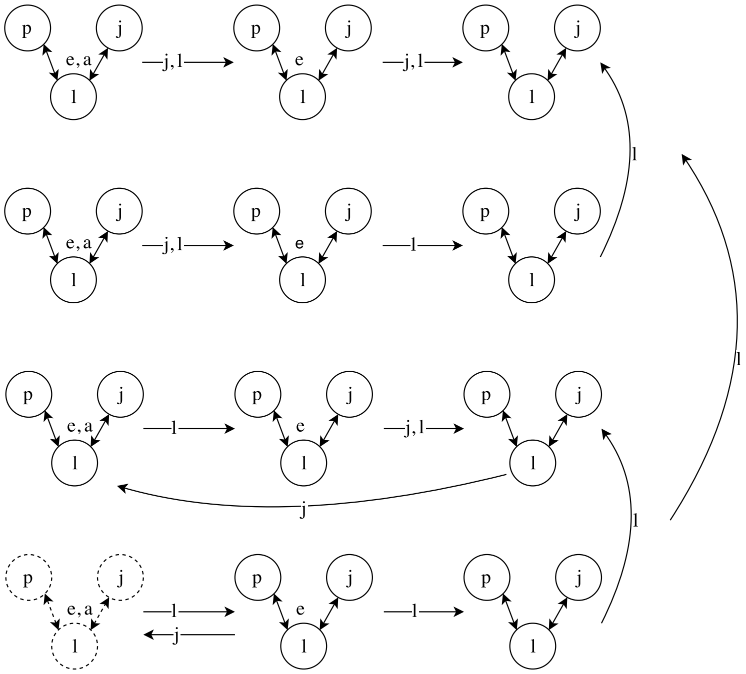

Figure 5.5: Epistemic Network State Model $\mathcal{L}_3$

Jason now believes a and shares this belief with Lynda. This action is modelled by a fair game announcement to Lynda. Both Lynda and Jason are aware of this action: $B_j a!_l^{l,j}$:

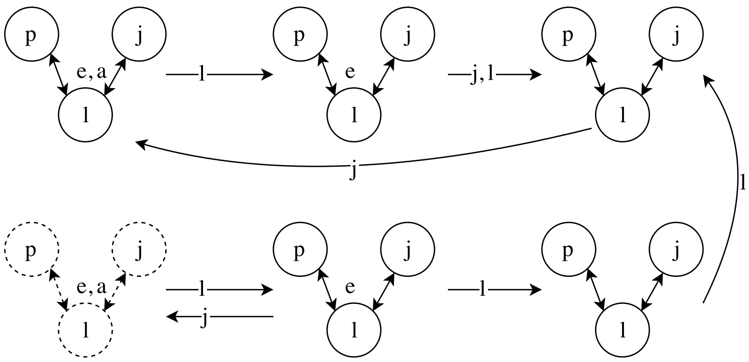

Figure 5.6: Epistemic Network State Model $\mathcal{L}_4$

Note that Lynda now believes that Jason believes e. This makes sense, if one believes a (i.e. Petra was passionately holding her lover at the station this morning), he also believes e (i.e. Petra was passionately holding a man (who is not her husband) at the station this morning). However Lynda does not yet know that Jason knows e.

It was a surprise to Lynda that John believed a, therefore, she will ask John why he believes this. As a response he will announce that he knows e. The next step is that Lynda gets to know that $K_j e$, this is modelled by a fair game upgrade. Note that after this upgrade Lynda herself knows that e is true.

Jason will know that Lynda does so. Hence we model this action by the following fair game announcement:
$K_j e!_{l,j}$ gives the following model:

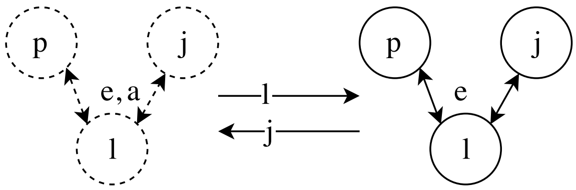

Figure 5.7: Epistemic Network State Model $\mathcal{L}_5$

Now Lynda herself will upgrade her model with all possible non-trivial explanations for e (i.e. a, b).
$a \vee b \Uparrow_l^{l,j}$

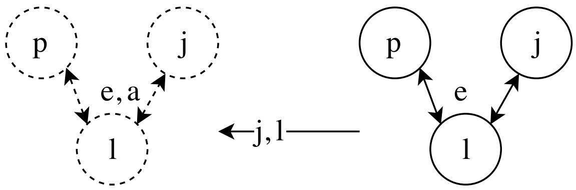

Figure 5.8: Epistemic Network State Model $\mathcal{L}_6$

After this upgrade, Lynda comes to believe that Petra has an affair. As a response she will break the friendship relation with Petra: $N^{-,\{l,j\}_{p,l}}$, this gives:

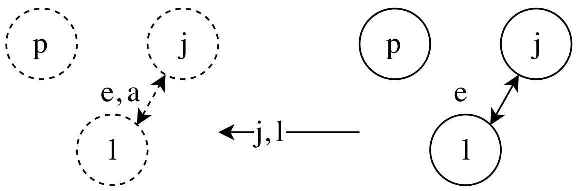

Figure 5.9: Epistemic Network State Model $\mathcal{M}_7$

Chapter 6

Conclusion and discussion

This thesis started with a famous quote that compared the gossiper with the genius. Both were said to have great intellectual curiosity, excellent recall, and the ability to make novel and original connections between events. This thesis was not about the genius, but had to do with the gossiper instead. We gave a formal logical analysis of gossip; a social phenomenon with an interesting epistemic structure. This analysis was based upon (our interpretation of) theories about the structure of gossip. The aim was to gain further insights about the structure of gossip as well as the epistemic and social dynamics that arises from it by studying these theories in a formal framework. The first two chapters of this thesis studied gossip and its structure from a social and epistemic perspective. The similarities between the gossiper and the genius where shown to be huge. It is the use of abductive reasoning that connects the gossiper with the genius. And it is exactly that kind of reasoning inference, that makes gossip a fascinating phenomenon to study from an epistemic point of view. For the reason that abduction is seen as the only inference that generates new ideas. It is abductive reasoning that makes that both, the genius and the gossiper, can achieve their goals; since both depend on new ideas. The epistemic analysis showed that gossip has an abductive nature. Gossip starts with the observation of a surprising event that initiates the abductive search for the best explanation; first by the initial observer, then by the other participants. By communication, the group members share their knowledge and aim together for a best explanation. But if gossip were just an epistemic phenomenon to enable groups of agents to find a best explanation, gossip would not have provoked so much ethical discussion. Apart from the fact that one agent can not gossip on his own -the gossiper needs a group to gossip with- it is the content of gossip that brings its social importance. The content of gossip always involves a person. Gossip is evaluative negative or positive talk about an (absent) agent. This is why gossip can be used to reach not just epistemic, but also social goals. By means of gossip one can exclude or include an agent from a group. Further, gossip can be used to strengthen or weaken the bondings within a social group. This thesis proposes to define the strength of a social bonding as a function of the agents' knowledge of and beliefs about the social network structure. All different goals that one can reach by means of gossip bring their own possible communication structures. The last part of the second chapter uses flow diagrams to describe the possible communication structures of a gossip scenario.

Those first two chapters are the foundation for the formal logical analysis this thesis aims at. The third chapter uses a dynamic doxastic and epistemic logic that is able to express and model the changes of agents' beliefs and knowledge during gossip. The fourth chapter extends this logic in a way such that it is able to describe and model not just the doxastic and epistemic dynamics, but also the social dynamics of gossip. This language is able to describe the beliefs and knowledge of each agent, the network relations between those agents and the beliefs and knowledge agents have about the network structure. The logic is a dynamic logic that makes use of action models in its semantics and includes modalities for each action in the language. Hence it is able to express and model the network structure and the beliefs and knowledge of agents after the occurrence of two types of epistemic actions (the fair game upgrades and the private upgrades) or social actions (adding a network relation or deleting a network relation). Based on the flow diagrams of the communication structure of gossip, as given in the second chapter, this thesis defined the possible orderings of the social and epistemic actions in a gossip scenario. This thesis thus gave a formal analysis of the epistemic, doxastic and network dynamics of gossip.

In future work it would be interesting to enable the logic to keep track of the past state models, this can be done as proposed by Yap (2011). Even more interesting would be to enable the logic to express the beliefs and knowledge that agents had in a previous state. That is, to express after a certain action took place, the agent's beliefs and knowledge before the action. This would enable agents to express the fact that an observation was a surprise. Another expansion of the current work would be to include the moral value of propositions to the language. This could be done in two ways. First, each proposition could obtain a general positive or negative value. In that case, formally one can split the set of proposition into a set of positive and a set of negative propositions. Another possibility is to include the moral valuation for each agent and each propositional variable individually. In that case the logic needs a function for each agent that maps each propositional variable to either the agents' set of positive propositions or the agents' set of negative propositions. Once the second option is chosen, one could define the dynamics of the moral values of the agents. The agents could influence each others moral valuation of a proposition. General doxastic influence is modelled by Christoff (2016); Baltag et al. (2016). This logical framework could be useful to model the dynamics of the moral valuation. Furthermore, the current formal protocols are meant to be descriptive. They try to make the intuitions and theories that are presented in the literature formally explicit. It would be nice to prove some properties about those protocols and possibly improve the protocols based upon those properties.

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