How to Make Friends: A Logical Approach to Social Group Creation *

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Abstract

This paper studies the logical features of social group creation. We focus on the mechanisms which indicate when agents can form a team based on the correspondence in their set of features (behavior, opinions, etc.). Our basic approach uses a semi-metric on the set of agents, which is used to construct a network topology. Then it is extended with epistemic features to represent the agents' epistemic states, allowing us to explore group-creation alternatives where what matters is not only the agent's differences but also what they know about them. We use tools of dynamic epistemic logic to study the properties of different strategies to network formations.

Keywords: social network; social network creation; similarity; middleman; epistemic logic; dynamic epistemic logic.

1 Introduction

It is commonly accepted that our social contacts affect the way we form our opinions about the world. Think, e.g., about socialization (inheriting and disseminating norms, customs, values and ideologies), conformity (changing our attitudes, beliefs and behaviors to match those of others), peer pressure and obedience. These phenomena have been studied not only by empirical sciences (e.g., sociology and social psychology: [10]) but also by theoretical computer science and economy [15]. Within the logic community, epistemic social phenomena have been studied with a diversity of logical tools. Since the birth of dynamic epistemic logic in the late eighties and ninetees, models were first designed to reason about agent's epistemic states in multi-agent environments, and the social dimension has gradually received more attention. As examples we mention the work on communication networks and protocols [24; 13; 22; 2; 1], belief change in social networks [19], the analysis of peer pressure [25], the study of informational cascades [5], priority-based peer influence [17], reflective social influence in [9] and the study of diffusion and prediction update in [6]. Still, while the structure of social groups plays an important role in these logical studies, the way the groups are created has till now received much less attention.

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This paper focusses on the logical structure behind the creation of social networks, the basic mechanism focussing on agents who become socially connected when the number of features in which they differ is small enough. In line with this idea we propose several group-creation policies, exploring the properties of the resulting networks. Section 2 introduces a similarity update operation which generates new reflexive and symmetric social networks. Then it discusses alternatives that produce irreflexive and not necessarily symmetric variations. After this, we introduce a version that asks for the agents not only to be 'close enough', but also for the existence of a middleman who can 'connect' them. Section 3 extends our setting with an epistemic dimension, as in real-life what matters its not only the actual situation, but also what the agents know about it. In this epistemic setting two new operations will be defined, both extending the similarity and middleman similarity operations by asking for the agents to have knowledge of the required condition. In both cases, the *de dicto* and *de re* variations of the epistemic conditions are further explored. The different logical settings in this paper make use of the techniques of *dynamic epistemic logic* (*DEL*; [4; 12; 7]) to represent group-creation actions, to define new languages to describe their effects, and to provide sound and complete axiom systems. Section 4 concludes with a list of topics for future work.

2 Modelling Social Networks

We adopt the basic setting of [6], which is a relational 'Kripke' model in which the domain is interpreted as the set of agents, the accessibility relation represents a social connection from one agent to another, and the atomic valuation describes the features (behavior/opinions) each agent has. Let A denote a finite non-empty set of agents, and P (with $\mathsf{A} \cap \mathsf{P} = \varnothing$) a finite set of features that each agent might or might not have:

Definition 2.1 (Social Network Model) A *social network model* (*SNM*) is a tuple $M = \langle \mathsf{A}, S, V \rangle$ where $S \subseteq \mathsf{A} \times \mathsf{A}$ is the *social* relation (Sab indicates that agent a is socially connected to agent b) and $V : \mathsf{A} \to \wp(\mathsf{P})$ is a *feature* function ($p \in V(a)$ indicates that agent a has feature p). ◀

Relation S is not required to satisfy any specific property (neither irreflexivity nor symmetry). Hence our social relation differs from the friendship relation in e.g. [19; 6; 9]. Given a social network model, we define a notion of 'distance' between agents based on the number of features in which they differ.

Definition 2.2 (Distance) Let $M = \langle \mathsf{A}, S, V \rangle$ be a SNM. Define the set of features distinguishing agents $a, b \in \mathsf{A}$ in M as $\text{msmtch}^M(a, b) := \big(V(a) \setminus V(b)\big) \cup \big(V(b) \setminus V(a)\big)$. Then, the distance between a and b in M is given by

$$\text{dist}^M(a, b) := |\text{msmtch}^M(a, b)|$$

Proposition 2.1 *Let $M = \langle \mathsf{A}, S, V \rangle$ be a SNM and take $a, b, c \in \mathsf{A}$. Then,*

- *Non-negativity:* $\text{dist}^M(a, b) \geq 0$.
- *Symmetry:* $\text{dist}^M(a, b) = \text{dist}^M(b, a)$.

- *Reflexivity:* $\text{dist}^M(a,a) = 0$.
- *Subadditivity:* $\text{dist}^M(a,c) \leq \text{dist}^M(a,b) + \text{dist}^M(b,c)$. ■

Here $\text{dist}^M(a,b)$ is a mathematical distance (satisfying non-negativity, symmetry and reflexivity) and also a semi-metric (a distance satisfying subadditivity).[1] It is not a metric as it does not satisfy *identity of indiscernibles*: $\text{dist}^M(a,b) = 0$ does not imply $a = b$, as two different agents may have exactly the same features.

Static Language $\mathcal{L}$. Following [6], social network models are described by a *propositional* language $\mathcal{L}$ with special atoms describing the agents' features and their social relationship. More precisely, formulas in $\mathcal{L}$ are given by

$$\varphi, \psi ::= p_a \mid \mathsf{S}_{ab} \mid \neg\varphi \mid \varphi \wedge \psi$$

with $p \in \mathsf{P}$ and $a, b \in \mathsf{A}$. We read p_a as *"agent a has feature p"* and S_{ab} as *"agent a is socially connected to b"*. Other Boolean operators ($\vee, \rightarrow, \leftrightarrow, \veebar$, the latter representing the exclusive disjunction) are defined as usual. Given a SNM $M = \langle \mathsf{A}, S, V \rangle$, the semantic interpretation of $\mathcal{L}$-formulas in M is given by:

$$\begin{array}{llll} M \Vdash p_a & \text{iff}_{\text{def}} \; p \in V(a), & M \Vdash \neg\varphi & \text{iff}_{\text{def}} \; M \nVdash \varphi, \\ M \Vdash \mathsf{S}_{ab} & \text{iff}_{\text{def}} \; Sab, & M \Vdash \varphi \wedge \psi & \text{iff}_{\text{def}} \; M \Vdash \varphi \text{ and } M \Vdash \psi. \end{array}$$

A formula $\varphi \in \mathcal{L}$ is valid (notation: $\Vdash \varphi$) when $M \Vdash \varphi$ holds for all models M. Since there are no restrictions on the social relation nor on the feature function, any axiom system for classical propositional logic is fit to characterize syntactically the validities of $\mathcal{L}$ over the class of social network models.

The remainder of this section deals with the creation of new networks by updating the network relation. In contrast to [6], which uses SNMs to study how the fixed network structure leads to changes in features, here we keep the agents' features fixed, focussing instead on the changes in the social structure.

2.1 Similarity Update

There are several ways in which new social relations can be defined. A natural option is to let two agents become friends when they are 'similar' enough. If a *threshold* $\theta \in \mathbb{N}$ is given, with $\theta < |\mathsf{P}|$ (recall: P is finite), then we can define a *similarity update* operation allowing agents to establish connections to others who differ in at most θ features.

Definition 2.3 (Similarity Update) Let $M = \langle \mathsf{A}, S, V \rangle$ be a SNM; take $\theta \in \mathbb{N}$. The similarity update of M generates a new SNM $M_{\odot_\theta} = \langle \mathsf{A}, S_{\odot_\theta}, V \rangle$ which differs from M only in its social relation, given by

$$S_{\odot_\theta} := \{(a,b) \in \mathsf{A} \times \mathsf{A} : \; \text{dist}^M(a,b) \leq \theta\}$$

Intuitively, each agent defines a circle of ratio θ with herself at the center, and her social contacts will be those agents falling inside it. The social relation of the updated model $M_{\odot_\theta}$ satisfies:

[1] See [11, Chapter 1] for more on mathematical distances.

Proposition 2.2 *Let $M = \langle \mathsf{A}, S, V\rangle$ be a* SNM*, with $M_{\odot_\theta} = \langle \mathsf{A}, S_{\odot_\theta}, V\rangle$ as in Definition 2.3. Then, $S_{\odot_\theta}$ is reflexive and symmetric.*

Proof. Each property follows from its namesake distance property (Proposition 2.1) and, in the case of reflexivity, from the fact that θ's lower bound is 0. ∎

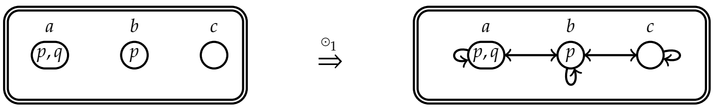
One can think of the social network that is generated by a similarity update as representing friends that have mutual access to each others feature-database, allowing every agent to access also her own database. Note that although reflexivity implies that every agent will have at least one friend (i.e., $S_{\odot_\theta}$ is serial), nothing guarantees an agent will have a friend other than herself (as the threshold may be 'too strict').

If a 'friendship' (irreflexive and symmetric) network is required, the update operation has to be adjusted to keep identity pairs out (e.g., $S'_{\odot_\theta} := \{(a, b) \in \mathsf{A} \times \mathsf{A} : a \neq b \text{ and } \mathrm{dist}^M(a, b) \leq \theta\}$). If, on the other hand, one requires a not necessarily symmetric network of 'informational access' (as in [8]), we can use a *personal threshold* for each agent $a \in \mathsf{A}$ (i.e., a function $\Theta : \mathsf{A} \to \mathbb{N}$). Then each agent can choose 'how different' others may be in order to add them to her social group, and the updated relation is given by $S'_{\odot_\Theta} := \{(a, b) \in \mathsf{A} \times \mathsf{A} : \mathrm{dist}^M(a, b) \leq \Theta(a)\}$. Note how, for example, the distance between a and b, say 2, may be good enough for a to consider b a social contact ($2 \leq \Theta(a)$), but not for b to consider a a social contact ($\Theta(b) < 2$).

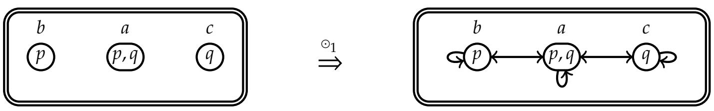
We illustrate briefly why other relational properties cannot be guaranteed.

Fact 2.1 *The relation $S_{\odot_\theta}$ needs to be neither transitive nor Euclidean.*

Proof. Transitivity can fail because, given agents a, b and c, what distinguishes a and b may be *only part of* what distinguishes a and c (i.e., $\mathrm{msmtch}^M(a, b) \subset \mathrm{msmtch}^M(a, c)$). For example, let $\theta = 1$ and consider the updated model below on the right, in which $S_{\odot_\theta}ab$ and $S_{\odot_\theta}bc$, but not $S_{\odot_\theta}ac$.



The relation may not be Euclidean because what distinguishes a and b may be *different from* what distinguishes a and c (i.e., $\mathrm{msmtch}^M(a, b) \cap \mathrm{msmtch}^M(a, c) = \varnothing$). For example, take $\theta = 1$: the updated model below on the right is such that $S_{\odot_\theta}ab$ and $S_{\odot_\theta}ac$, but neither $S_{\odot_\theta}bc$ nor $S_{\odot_\theta}cb$.



Dynamic Language $\mathcal{L}_{\odot_\theta}$. To express how a social network changes, we use the language $\mathcal{L}_{\odot_\theta}$ which extends the language $\mathcal{L}$ with a 'dynamic' modality $[\odot_\theta]$ to build formulas of the form $[\odot_\theta]\,\varphi$ (*"after a similarity update, φ is the case"*). The semantic interpretation of this modality refers to the similarity-updated model in Definition 2.3 as follows: Let M be a SNM, then

$M \Vdash [\odot_\theta]\,\varphi \quad \text{iff}_{\text{def}} \quad M_{\odot_\theta} \Vdash \varphi.$

Different from the well-known case of information updates under public announcements [21; 16], no precondition is required for a similarity update of a social network: the operation can take place in any situation. Because of this and the functionality of the model operation, the dual modality $\langle\odot_\theta\rangle\,\varphi := \neg\,[\odot_\theta]\,\neg\varphi$ is such that $\Vdash [\odot_\theta]\,\varphi \leftrightarrow \langle\odot_\theta\rangle\,\varphi$.

The following axiom system is build via the *DEL* technique of *recursion axioms*. First, note that, as P is finite, the following $\mathcal{L}$-formula is true in a model M if and only if agents a and b differ in exactly $t \in \mathbb{N}$ features:

$$\mathrm{Dist}^t_{ab} := \bigvee_{\{\mathsf{Q}\subseteq\mathsf{P}:\ |\mathsf{Q}|=t\}} \Big(\bigwedge_{p\in\mathsf{Q}} (p_a \veebar p_b) \wedge \bigwedge_{p\in\mathsf{P}\setminus\mathsf{Q}} (p_a \leftrightarrow p_b) \Big) \qquad [2]$$

Then, the following $\mathcal{L}$-formula is true in M if and only if a and b differ in at most $\theta \in \mathbb{N}$ features:

$$\mathrm{Dist}^{\leq\theta}_{ab} := \bigvee_{t=0}^{\theta} \mathrm{Dist}^t_{ab}$$

Hence, the following $\mathcal{L}_{\odot_\theta}$-formula characterizes the social relation in the similarity-updated model: a will consider b as a social contact if and only if, before the operation, a and b differed in at most θ features;

$$\Vdash [\odot_\theta]\,\mathsf{S}_{ab} \;\leftrightarrow\; \mathrm{Dist}^{\leq\theta}_{ab}$$

As only the social relation changes, the reduction axioms and the rules in Table 1 form, together with a propositional system, a sound and strongly complete axiom system characterizing the validities of $\mathcal{L}_{\odot_\theta}$. The here given syntax adapts the work of [6] for threshold-limited influence to the case of similarity update.

$\vdash [\odot_\theta]\,p_a \leftrightarrow p_a$	From $\vdash \varphi$ infer $\vdash [\odot_\theta]\,\varphi$
$\vdash [\odot_\theta]\,\mathsf{S}_{ab} \leftrightarrow \mathrm{Dist}^{\leq\theta}_{ab}$	From $\vdash \psi_1 \leftrightarrow \psi_2$ infer $\vdash \varphi \leftrightarrow \varphi\,[\psi_2/\psi_1]$ (with $\varphi\,[\psi_2/\psi_1]$ any formula obtained by replacing one or more occurrences of ψ_1 in φ with ψ_2).
$\vdash [\odot_\theta]\,\neg\varphi \leftrightarrow \neg\,[\odot_\theta]\,\varphi$	
$\vdash [\odot_\theta](\varphi \wedge \psi) \leftrightarrow ([\odot_\theta]\,\varphi \wedge [\odot_\theta]\,\psi)$	

Table 1: Axiom system for $\mathcal{L}_{\odot_\theta}$ over social network models ($a, b \in \mathsf{A}$).

If the mentioned 'irreflexive' version of similarity update is used, the axiom characterizing the new social relation should be restricted to cases with $a \neq b$, with a new axiom for the missing case:

$$\vdash [\odot_\theta]\,\mathsf{S}_{ab} \leftrightarrow \mathrm{Dist}^{\leq\theta}_{ab} \quad \text{for } a \neq b, \qquad\qquad \vdash [\odot_\theta]\,\mathsf{S}_{aa} \leftrightarrow \bot$$

If the 'personal threshold' option is chosen, then the axiom should state that, after the operation, a includes b as her social contact if and only if they differ in at most $\Theta(a)$ features:

$$\vdash [\odot_\Theta]\,\mathsf{S}_{ab} \;\leftrightarrow\; \mathrm{Dist}^{\leq\Theta(a)}_{ab}$$

[2] The formula states that there is at least one set of features Q, of size t, such that a and b differ in all features in Q and coincide in all features in $\mathsf{P}\setminus\mathsf{Q}$. There can be a most one such set, therefore the formula is true exactly when a and b differ in exactly t features.

2.2 Middleman Similarity Update

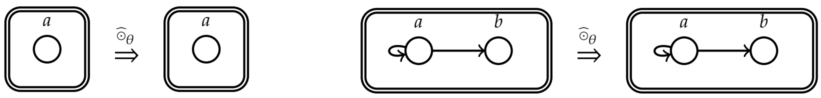
The network generated via a similarity update does not depend on the topology of the original network but only on the agent's current 'distance', regardless of whether they were earlier socially connected or not. Yet in most social scenarios, we see that the past network does play a role and that it takes a common acquaintance to introduce new friends to each other who are similar enough.

Definition 2.4 (Middleman Similarity Update) Let $M = \langle \mathsf{A}, S, V \rangle$ be a SNM; take $\theta \in \mathbb{N}$. The *middleman similarity updated* model SNM $M_{\widehat{\odot}_\theta} = \langle \mathsf{A}, S_{\widehat{\odot}_\theta}, V \rangle$ differs from M only in its social relation, which is given by

$$S_{\widehat{\odot}_\theta} := \{(a, b) \in \mathsf{A} \times \mathsf{A} : \ \mathrm{dist}^M(a, b) \leq \theta \text{ and } \exists\, c \in \mathsf{A} \text{ with } Sac \text{ and } Scb\}$$

In the new network, agent a will include agent b as a social contact if and only if they are similar enough and there is an agent c who belongs to a's social network and who includes b as one of her social contacts. Of course, the social requirements for the middleman c might vary. In some cases, a *symmetric* social relation between him and the two involved agents a and b might be required; while in other cases, an agent who has both a and b in her social network might be enough. Thanks to the formulae describing the social relation S in the syntax, our logical system is capable of dealing with all these cases, and other similar variations. Note also that, in line with our definition, the role of the middleman can be played by agent a or b themselves if they were already friends. Still, requiring a middleman changes the properties of the resulting network:

Fact 2.2 *If $M = \langle \mathsf{A}, S, V \rangle$ is a SNM, then $S_{\widehat{\odot}_\theta}$ needs to be neither reflexive nor symmetric. As the diagrams below show, a middleman who can establish new relations might not exist (no reflexivity on the left, no symmetry on the right):*



Just as there might be no middlemen for establishing new relations, there might be no middlemen for *preserving* old ones; thus, the middleman similarity update is not a monotone operation. To illustrate this, take a model with $S = \{(a, b)\}$. The middleman similarity update yields $S_{\widehat{\odot}_\theta} = \{\ \}$ regardless of a and b's similarities: neither Saa nor Sbb holds, hence neither a nor b can play the role of the middleman. Of course, one can always enforce monotonicity by defining the new social relation in an 'accumulative' way ($S_{\widehat{\odot}_\theta} := S \cup \cdots$), yet this is not always appropriate: in real scenarios, social connections can be created, but unfortunately (and in some cases, fortunately) they can also be lost. One advantage of not enforcing monotonicity is that it is possible to identify those situations that lead to it in a natural way. In our setting, a *reflexive* S guarantees that old social contacts will be preserved (modulo the agents' distance). In other words, for the agent to preserve her social connections, she should first consider herself 'worthwhile' as a friend.

The middleman similarity update operation *preserves* symmetry and moreover, if the initial M is fully symmetric and the update adds an edge from some a to some b, then it also adds its converse. Of course these preservation properties does not guarantee that $S_{\widehat{\odot}_\theta}$ will always be symmetric, as even when c plays the role of the middleman 'from left to right', lack of symmetry in the original M may make her unable to play the role 'from right to left'.[3]

Dynamic Language $\mathcal{L}_{\widehat{\odot}_\theta}$. We extend the above language $\mathcal{L}$ with a modality $[\widehat{\odot}_\theta]$ for describing the effect of the middleman similarity update. Thus, the resulting language $\mathcal{L}_{\widehat{\odot}_\theta}$ includes formulas of the form $[\widehat{\odot}_\theta]\,\varphi$ (*"after a middleman similarity update, φ is the case"*). Its semantics is as follows: let $M = \langle \mathsf{A}, S, V\rangle$ be a SNM, then

$$M \Vdash [\widehat{\odot}_\theta]\,\varphi \quad \text{iff}_{\text{def}} \quad M_{\widehat{\odot}_\theta} \Vdash \varphi.$$

Since both A and P are finite, the axioms and rules in Table 2 (plus a propositional axiom system) characterize the validities of $\mathcal{L}_{\widehat{\odot}_\theta}$ in SNMs. The difference w.r.t. Table 1 is the axiom characterizing the new social relation, asking now for the required middleman. Axiom systems for the variations mentioned before (keeping identity pairs out, personal thresholds) can be obtained as in the 'non-middleman' similarity update case (Page 5).

$\vdash [\widehat{\odot}_\theta]\,p_a \leftrightarrow p_a$	$\vdash [\widehat{\odot}_\theta](\varphi \wedge \psi) \leftrightarrow ([\widehat{\odot}_\theta]\,\varphi \wedge [\widehat{\odot}_\theta]\,\psi)$
$\vdash [\widehat{\odot}_\theta]\,\mathsf{S}_{ab} \leftrightarrow \big(\mathrm{Dist}^{\leq\theta}_{ab} \wedge \bigvee_{c\in\mathsf{A}}(\mathsf{S}_{ac} \wedge \mathsf{S}_{cb})\big)$	From $\vdash \varphi$ infer $\vdash [\widehat{\odot}_\theta]\,\varphi$
$\vdash [\widehat{\odot}_\theta]\,\neg\varphi \leftrightarrow \neg\,[\widehat{\odot}_\theta]\,\varphi$	From $\vdash \psi_1 \leftrightarrow \psi_2$ infer $\vdash \varphi \leftrightarrow \varphi\,[\psi_2/\psi_1]$

Table 2: Axiom system for $\mathcal{L}_{\widehat{\odot}_\theta}$ over social network models ($a, b, c \in \mathsf{A}$).

3 Epistemic Social Networks

The described approach for creating social networks connects agents that are similar enough. However, in real life, two 'identical souls' may never relate to each other, as they may not know about their similarities. Thus, a more realistic representation of social network creation should take into account not only the agents' similarities, but also the knowledge they have about them.

Definition 3.1 (Epistemic Social Network Model) An *epistemic social network model* (ESNM) is a tuple $M = \langle W, \mathsf{A}, \sim, S, V\rangle$ with a set $W \neq \varnothing$ of *possible worlds*, a set of agents A, an *epistemic* equivalence relation $\sim\colon \mathsf{A} \to (W \times W)$ for each $a \in \mathsf{A}$, and at each world the social relation $S : W \to \wp(\mathsf{A} \times \mathsf{A})$ and feature function $V : W \to (\mathsf{A} \to \wp(\mathsf{P}))$. ◀

An ESNM is a standard possible worlds model [18] in which each possible world represents a SNM (Definition 2.1) and the epistemic relation is an equivalence relation. Derived concepts, such as $\mathrm{msmtch}^M(\cdot,\cdot)$ and $\mathrm{dist}^M(\cdot,\cdot)$, can be defined as before *for each possible world $w \in W$*.

[3]Note that several further constraints can be imposed, for instance one can require that any agent c playing the middleman for a and b should be fully connected to the agents she will 'introduce' (Sac, Sca, Scb, Sbc).

Additional constrains can be imposed in the model. For instance, one can ask for the agents to *know themselves* (in this setting with *equivalence* epistemic relations, agent a knows herself at world w if and only if $w \sim_a u$ implies $V_w(a) \subseteq V_u(a)$) or to know who are her contacts (a knows who are her contacts at w if and only if $w \sim_a u$ implies $S_w[a] \subseteq S_u[a]$). For the sake of generality, here no such assumptions will be made.

Epistemic Language $\mathcal{L}^{\mathsf{K}}$. We follow [6] in designing an *epistemic* language $\mathcal{L}^{\mathsf{K}}$ with special atoms to describe the agents' features and social relationship. The formulas φ, ψ of $\mathcal{L}^{\mathsf{K}}$ are given by

$$\varphi, \psi ::= p_a \mid \mathrm{S}_{ab} \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathrm{K}_a\, \varphi$$

with $p \in \mathsf{P}$ and $a, b \in \mathsf{A}$. Formulas of the form $\mathrm{K}_a\, \varphi$ are read as *"agent a knows φ"*. Given a ESNM model $M = \langle W, \mathsf{A}, \sim, S, V \rangle$, the semantic interpretation of $\mathcal{L}^{\mathsf{K}}$-formulas is standard for Boolean operators and the epistemic modalities, with atoms p_a and S_{ab} interpreted relative to the point of evaluation.

$$(M, w) \Vdash p_a \;\text{ iff}_{\text{def}}\; p \in V_w(a), \qquad\qquad (M, w) \Vdash \mathrm{S}_{ab} \;\text{ iff}_{\text{def}}\; S_w ab.$$

The definition of (modal) validity ($\Vdash$) is as usual. We adopt here the well-known multi-agent $S5$ axiom system, as no extra restrictions are imposed in the model.

3.1 Knowledge-based Social Network Creation

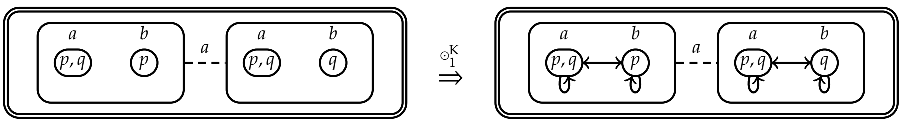
Definition 3.2 (Knowledge-based Similarity Update) Let $M = \langle W, \mathsf{A}, \sim, S, V \rangle$ be an ESNM; take $\theta \in \mathbb{N}$. The *knowledge-based similarity update* operation generates the ESNM $M_{\odot^{\mathsf{K}}_\theta} = \langle W, \mathsf{A}, \sim, S_{\odot^{\mathsf{K}}_\theta}, V \rangle$, differing from M only in the social relation at every $w \in W$, which is given by

$$(S_{\odot^{\mathsf{K}}_\theta})_w := \{(a, b) \in \mathsf{A} \times \mathsf{A} : \; \forall u \sim_a w \,,\, \mathrm{dist}^M_u(a, b) \leq \theta\}$$

This update operation is based on the operation in Definition 2.3 but asks for an additional epistemic requirement: in world w agent a will add b to her social network if and only if in this world a *knows* that b is similar enough (i.e., b is similar enough in all a's epistemic alternatives from w).

Note how, after this update, the social network at each possible world will be reflexive, even in those cases in which the agent 'does not know herself'.[4] On the other hand, social relations do not need to be symmetric, as the agents' knowledge might not have such property: at w agent a may know that she and b are similar enough (so $(S_{\odot^{\mathsf{K}}_\theta})_w ab$ will hold), but b may not know this (and thus $(S_{\odot^{\mathsf{K}}_\theta})_w ba$ will fail).

De dicto vs de re. The knowledge-based similarity update uses a *de dicto* approach: after the operation, a includes b in her network when a knows that b is close enough, even if she does not know exactly which are the features that she shares with b. Indeed, consider the ESNM below on the left:



[4]In any possible world, the distance between any agent and herself is 0.

Even though a does not know which features b has, she knows the 'distance' between them is just 1. Thus, after a knowledge-based similarity update with any $\theta \geq 1$, she will add b to her network (the ESNM on the right).

On the other hand, a *de re* approach would ask not for a to know that the number of differences between her and b is 'small enough', but rather for her to point out a 'large enough' set of features on which she and b coincide:

$$(S'_{\odot^{\mathrm{K}}_{\theta}})_w := \{(a,b) \in \mathsf{A} \times \mathsf{A} : \ \exists\, \mathsf{Q} \subseteq \mathsf{P} \text{ s.t. } \textbf{\textit{(i)}}\ |\mathsf{Q}| \geq |\mathsf{P}| - \theta \quad \text{and} \quad \textbf{\textit{(ii)}}\ \forall u \sim_a w\ \textbf{,}\ V_u(a) \cap \mathsf{Q} = V_u(b) \cap \mathsf{Q}\ \}$$

Thus, a will include b in her social network if and only if there is a set of features she knows she and b share, and this set is large enough for their number of differences to be smaller than θ.[5] This variation also highlights an alternative to the basic idea of this proposal: we have related agents when their differences are small enough, but a (perhaps more 'human') alternative is to relate them when their *similarities* are large enough.

Note also how the *de dicto* version asks for the agents to be 'close enough' in all epistemic possibilities, regardless of which are the features that distinguish them. This emphasizes that, for the agents, all features are equally important. But one can imagine a more realistic scenario in which certain features are more important than others: take an agent with features $\{p, q, r\}$ choosing an agent with $\{p\}$ over an agent with $\{q, r\}$ because, for her, p is more important than q and r together. Such a setting would require a *de re* approach.

Dynamic Epistemic Language $\mathcal{L}^{\mathrm{K}}_{\odot^{\mathrm{K}}_{\theta}}$. In order to express the way a knowledge-based similarity update affects a social network, a dynamic modality $[\odot^{\mathrm{K}}_{\theta}]$ is added to $\mathcal{L}^{\mathrm{K}}$ to yield language $\mathcal{L}^{\mathrm{K}}_{\odot^{\mathrm{K}}_{\theta}}$. This allows us to express that *"after a knowledge-based similarity update, φ is the case"*, $[\odot^{\mathrm{K}}_{\theta}]\,\varphi$. For its semantic interpretation, let M be an ESNM. Then,

$$(M, w) \Vdash [\odot^{\mathrm{K}}_{\theta}]\,\varphi \quad \text{iff}_{\text{def}} \quad (M_{\odot^{\mathrm{K}}_{\theta}}, w) \Vdash \varphi.$$

The axiom system is presented in Table 3. Note first the axiom describing the way the agents' knowledge changes: each epistemic modality K_a simply commutes with the dynamic modality, as the epistemic relation is not affected by the update operation. More importantly, the crucial reduction axiom states how, in order for a to 'add' b to her network, it is not enough for a and b to be 'similar enough': a should also know this. The unfolding of $\mathrm{Dist}^{\leq\theta}_{ab}$ in such an axiom makes explicit the *de dicto* approach: a only needs to know that $|\mathrm{msmtch}(a, b)|$ is smaller than θ.

$$\vdash [\odot^{\mathrm{K}}_{\theta}]\,\mathrm{S}_{ab} \leftrightarrow \mathrm{K}_a \bigvee_{t=0}^{\theta} \bigvee_{\{\mathsf{Q}\subseteq\mathsf{P}\,:\,|\mathsf{Q}|=t\}} \Big(\bigwedge_{p\in\mathsf{Q}} (p_a \veebar p_b) \wedge \bigwedge_{p\in\mathsf{P}\setminus\mathsf{Q}} (p_a \leftrightarrow p_b) \Big)$$

If the *de re* variation proposed above is chosen, the axiom becomes

$$\vdash [\odot^{\mathrm{K}}_{\theta}]\,\mathrm{S}_{ab} \leftrightarrow \bigvee_{t=|\mathsf{P}|-\theta}^{|\mathsf{P}|} \bigvee_{\{\mathsf{Q}\subseteq\mathsf{P}\,:\,|\mathsf{Q}|=t\}} \mathrm{K}_a \bigwedge_{p\in\mathsf{Q}} (p_a \leftrightarrow p_b)$$

[5] Both P and θ are commonly known, so *a knows* Q is enough to make her differences with b smaller than θ.

$\vdash [\odot_\theta^{\mathsf{K}}]\, p_a \leftrightarrow p_a$	$\vdash [\odot_\theta^{\mathsf{K}}]\, \mathrm{K}_a\, \varphi \leftrightarrow \mathrm{K}_a\, [\odot_\theta^{\mathsf{K}}]\, \varphi$
$\vdash [\odot_\theta^{\mathsf{K}}]\, \mathrm{S}_{ab} \leftrightarrow \mathrm{K}_a\, \mathrm{Dist}_{ab}^{\leq\theta}$	From $\vdash \varphi$ infer $\vdash [\odot_\theta^{\mathsf{K}}]\, \varphi$
$\vdash [\odot_\theta^{\mathsf{K}}]\, \neg\varphi \leftrightarrow \neg\, [\odot_\theta^{\mathsf{K}}]\, \varphi$	From $\vdash \psi_1 \leftrightarrow \psi_2$ infer $\vdash \varphi \leftrightarrow \varphi\, [\psi_2/\psi_1]$
$\vdash [\odot_\theta^{\mathsf{K}}](\varphi \wedge \psi) \leftrightarrow ([\odot_\theta^{\mathsf{K}}]\, \varphi \wedge [\odot_\theta^{\mathsf{K}}]\, \psi)$	

Table 3: Axiom system for $\mathcal{L}^{\mathsf{K}}_{\odot_\theta^{\mathsf{K}}}$ over social network models ($a, b \in \mathsf{A}$).

Note the differences with the *de dicto* axiom. The fundamental one is the position of the knowledge modality K_a, now under the scope of the disjuctions asking for the existence of the 'large enough' set of features Q (*"there is* Q *of size at least* $|\mathsf{P}| - \theta$ *such that agent a knows that ...*"). The other difference is that a does not need to know that Q is exactly what distinguishes her and b; it is enough for her to know that features in Q are common for them. (Note, again how the fact that P and θ are common knowledge implies that a knows Q is large enough.)

3.2 Middleman Knowledge-based Social Network Creation

In the epistemic setting one can ask for a middleman requirement. This again leads to a *de dicto* vs *de re* choice: either a knows there is someone who can link her with the 'similar enough' b (but she might not know who), or else there is someone a knows can link her with b. The definition below follows the *de re* alternative as, intuitively, a should know who this middleman is.

Definition 3.3 (Middleman Knowledge-based Similarity Update) Take M to be an ESNM $\langle W, \mathsf{A}, \sim, S, V\rangle$; take $\theta \in \mathbb{N}$. The *middleman knowledge-based similarity update* generates the ESNM $M_{\widehat{\odot}_\theta^{\mathsf{K}}} = \langle W, \mathsf{A}, \sim, S_{\widehat{\odot}_\theta^{\mathsf{K}}}, V\rangle$, which differs from M in its social relation at every $w \in W$. The new relation is given as follows:

$$(S_{\widehat{\odot}_\theta^{\mathsf{K}}})_w := \{(a,b) \in \mathsf{A} \times \mathsf{A} : \;\; \exists\, c \in \mathsf{A} \text{ s.t. } \forall u \sim_a w \,\boldsymbol{,}\;\; \textbf{\textit{(i)}}\ \mathrm{dist}_u^M(a,b) \leq \theta \;\text{ and }\; \textbf{\textit{(ii)}}\ S_u ac \text{ and } S_u cb\,\}$$

Of course, this is not the only possibility. Besides the alternatives for the social requirements of the middleman discussed above, another possibility (suggested by a reviewer) is to shift the epistemic burden to the middleman: a will add agent b to her social network if and only if they are 'close enough' and the middleman knows this (syntactically, $\bigvee_{c \in \mathsf{A}} \mathrm{K}_c(\mathrm{Dist}_{ab}^{\leq\theta})$).

The new relation will be reflexive for an agent a in a world w when the original relation for a was reflexive *in all worlds accessible from* w, but also when somebody else plays the middleman role in all a's epistemic possibilities from w. Moreover, if the original relation is reflexive in all possible worlds, then the operation preserves it (modulo the agents' distance). Finally, the new relationships may not be symmetric, as the agents' knowledge could be asymmetric.

A weaker *de dicto* requirement on the middleman condition, gives us:

$$(S'_{\widehat{\odot}_\theta^{\mathsf{K}}})_w := \{(a,b) \in \mathsf{A} \times \mathsf{A} : \;\; \forall\, u \sim_a w \,\boldsymbol{,}\;\; \textbf{\textit{(i)}}\ \mathrm{dist}_u^M(a,b) \leq \theta \;\text{ and }\; \textbf{\textit{(ii)}}\ \exists\, c \in \mathsf{A} \text{ s.t. } S_u ac \text{ and } S_u cb\,\}$$

$\vdash [\widehat{\odot_\theta}^{\mathrm{K}}]\, p_a \leftrightarrow p_a$	$\vdash [\widehat{\odot_\theta}^{\mathrm{K}}]\, \mathrm{K}_a\, \varphi \leftrightarrow \mathrm{K}_a\, [\widehat{\odot_\theta}^{\mathrm{K}}]\, \varphi$
$\vdash [\widehat{\odot_\theta}^{\mathrm{K}}]\, \mathrm{S}_{ab} \leftrightarrow \bigvee_{c\in\mathsf{A}} \mathrm{K}_a(\mathrm{Dist}^{\leq\theta}_{ab} \wedge \mathrm{S}_{ac} \wedge \mathrm{S}_{cb})$	From $\vdash \varphi$ infer $\vdash [\widehat{\odot_\theta}]\, \varphi$
$\vdash [\widehat{\odot_\theta}^{\mathrm{K}}]\, \neg\varphi \leftrightarrow \neg\, [\widehat{\odot_\theta}^{\mathrm{K}}]\, \varphi$	From $\vdash \psi_1 \leftrightarrow \psi_2$ infer $\vdash \varphi \leftrightarrow \varphi[\psi_2/\psi_1]$
$\vdash [\widehat{\odot_\theta}^{\mathrm{K}}](\varphi \wedge \psi) \leftrightarrow ([\widehat{\odot_\theta}^{\mathrm{K}}]\, \varphi \wedge [\widehat{\odot_\theta}^{\mathrm{K}}]\, \psi)$	

Table 4: Axiom system for $\mathcal{L}^{\mathrm{K}}_{\widehat{\odot_\theta}^{\mathrm{K}}}$ over social network models ($a, b, c \in \mathsf{A}$).

In this variation, it is enough for a to know there is someone linking her with b, even if she does not know exactly who this middleman is.

A dynamic epistemic language. The language $\mathcal{L}^{\mathrm{K}}_{\widehat{\odot_\theta}^{\mathrm{K}}}$ adds a modality $[\widehat{\odot_\theta}^{\mathrm{K}}]$ to $\mathcal{L}^{\mathrm{K}}$ in order to talk about what happens after a middleman knowledge-based similarity update. For its semantic interpretation, let M be an ESNM; then,

$$(M, w) \Vdash [\widehat{\odot_\theta}^{\mathrm{K}}]\, \varphi \quad \text{iff}_{\text{def}} \quad (M_{\widehat{\odot_\theta}^{\mathrm{K}}}, w) \Vdash \varphi.$$

The axiom characterizing the way the social network changes (see Table 4) reflects both the *de dicto* approach for the knowledge about the distance and the *de re* approach for the knowledge about the middle man. For the alternative *de dicto-de dicto* proposed above (i.e., only knowledge of the existence for both the distance and the middle man), this axiom becomes

$$\vdash [\widehat{\odot_\theta}^{\mathrm{K}}]\, \mathrm{S}_{ab} \leftrightarrow \mathrm{K}_a \left(\mathrm{Dist}^{\leq\theta}_{ab} \wedge \bigvee_{c\in\mathsf{A}} (\mathrm{S}_{ac} \wedge \mathrm{S}_{cb}) \right)$$

4 Conclusions

The present proposal explores a threshold approach to social network creation based on the agents' similarities, the key idea being that an agent will add someone to her social network if and only if the distance between them is smaller or equal than the given threshold. In this paper we have studied this idea as well as the *middleman* and *knowledge-based* variations; in each case, the properties of the resulting networks have been explored, and a sound a complete axiom system for the corresponding modality has been presented. The exploration can go deeper: for example, one can look for conditions guaranteeing that the resulting social network will have certain properties (reflexivity, seriality, symmetry, transitivity, Euclideanity). In the middleman case, one can also try to identify those situations in which the update operation will become idempotent, and thus further applications of it will not make a difference.

While this work is an initial exploration of the logical structure behind social group creation, our setting suggests several interesting alternatives. For example, as shown by the *de dicto* understanding of the knowledge requirement about the agent's distances, all features are equally important for all agents: what matters is the number of differences, and not what these differences are. In an alternative setting, we can treat certain features as more important than others, and we can let this 'priority ordering' among features differ from agent to agent. In a similar line, one can imagine situations in which not all features are relevant. Indeed, in [23] the authors use a game theoretic setting to define the agreement and disagreement of agents on a specific feature (or issue), which yields a way for them to update the social relation of agents with respect to one specific feature at a time. A similar idea can be worked out in our setting, which would require that only a subset of all features is relevant for each update operation. Such a structure can be useful when we want agents to control which one of their features becomes visible to other agents. A related alternative is to use an issue-dependent update to define different types of networks or social groups related to agent's different issues; after all, the network of football fans will be rather different from Lady Gaga's fan club.

There are also alternatives to the threshold similarity approach of this proposal. An interesting idea, arising from the cognitive science literature, is to take into account the size of the agent's 'social space'. In real life, agents may be willing to keep expanding their social network (even including people who are very different from them) as long as there is still enough space in their social environment. This is famously known as the *Dunbar's number*: a suggested cognitive limit to the number of people with whom one can maintain stable social relationships (see, e.g., [14]). Such a 'group-size' similarity approach produces different social relations, one example being the lack of symmetry: in situations in which most agents are 'closer' (i.e., more similar) to agent b than to agent a, while a may have 'enough space' to include b in her social group, b may be 'out of space' before she even considers a. This group-size approach can be used in combination not only with the presented epistemic models, but also with the above ideas requiring 'some features to be more important than others' or 'not all features to be visible'.

Both the threshold and the mentioned group-size approaches relate agents when they are similar enough, with this similarity based on a distance measure taken to be the standard Hamming distance between two sets of atomic formulae. It would be interesting to compare our obtained results to settings in which other notions of distances between sets are used (such as e.g. the Jaccard distance). An even bigger change is to consider the dual situation in which agents connect when they *complement each other*. In order to deal formally with this *complementary* idea, a more fine-grained setting is needed that takes into account not only the agents' features/behaviors, as in this paper, but also their doxastic state and their preferences (e.g., [3; 17]).

In our epistemic setting, an obvious next step is to study also 'knowledge changing operations' within the presented models (e.g., public and private announcements), focussing not only on the changes of the agents' knowledge about each other's features and social connections, but also on the interplay with knowledge-based social network changing operations. There are interesting situations in which agents can learn new facts about each other's features and about each other's social relations from both the knowledge they have about the group-formation rules and the way a network has changed.

Finally, an important topic is the interplay between the feature-based social-network-changing operations of this proposal, and the social-network-based feature-changing operations mentioned in the introduction. Both ideas deserve to be studied in tandem: the dynamics of one can affect the dynamics of the other, and our logical setting might be able to capture interesting properties about the interplay of these dynamic mechanisms.[6]

[6]In fact, one can see our proposal in this paper as a necessary first step towards that goal, as the

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formal grounds for both systems need to be settled before looking at their interaction.

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