Dynamic Montague Grammar*

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1 Introduction

In Groenendijk & Stokhof [1989] a system of dynamic predicate logic (*DPL*) was developed, as a compositional alternative for classical discourse representation theory (*DRT*). *DPL* shares with *DRT* the restriction of being a first-order system. In the present paper, we are mainly concerned with overcoming this limitation. We shall define a dynamic semantics for a typed language with λ-abstraction which is compatible with the semantics *DPL* specifies for the language of first-order predicate logic. We shall propose to use this new logical system as the semantic component of a Montague-style grammar (referred to as dynamic Montague grammar, *DMG*), which will enable us to extend the compositionality of *DPL* to the subsentential level. Furthermore, we shall extend this analysis also in this sense that we shall add new, dynamic interpretations for logical constants which in *DPL* were treated in a static fashion. This will substantially increase the descriptive coverage of *DMG*.

One of the motives behind the development of *DPL* was a concern for compatibility of frameworks. The days of the hegemony (or tyranny, if you will) of Montague grammar (*MG*) as *the* paradigm of model-theoretic semantics for natural language, are over. And rightly so, since the development of alternatives, such as *DRT*, clearly has been inspired by limitations and shortcomings of *MG*. These alternatives have enabled us to cover new ground, and they have brought us a new way of looking at our familiar surroundings. Yet, every 'expanding' phase is necessarily followed by one of 'contraction', in which, again, attempts are made to unify the insights and results of various frameworks. The present paper is intended as a contribution to such a unification.

It may seem conservative, though, to 'fall back' on the *MG*-framework. For several reasons we think that this is not the case. For one thing, a lot of adequate work has been done in the *MG*-framework which simply has not yet been covered in any of the alternatives. Some of it can be transferred in an obvious way, and thus offers no real challenge, but that does not hold for all of it. For example, the elegant and uniform treatment of various intensional phenomena that *MG* offers, will be hard to come by in some of the other frameworks. Also, we are convinced that the capacities of *MG* have not been exploited to the limit, that sometimes an analysis is carried out in a rival framework simply because it is more fashionable. But the most important reason of all concerns compositionality. The framework of

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MG is built on this principle, which, for philosophical and methodological reasons, we think is one of the most central principles in natural language semantics (see Groenendijk & Stokhof [1989] for some discussion). Consequently, confronting other frameworks with *MG*, trying to unify them, is also putting things to the test. Can this analysis be cast in a compositional mould? But also, what consequences does compositionality have in this case? (And, to be sure, some of the latter may cause us to reject it after all.)

So, we feel there is ample reason for the undertaking of this paper. But for those who are not really inspired by such grandiose considerations, we may add that, after all, in *DRT*, too, one would want to overcome the limitations of a first-order system in some way. Well, this paper offers one.

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We shall proceed as follows. In section 2, we motivate making a distinction between variables and discourse markers. In section 3, we present a system of dynamic intensional logic (*DIL*), along the lines of Janssen [1986]. In section 4, we indicate how *DIL* can be used as a means to represent the meanings of natural language sentences, and sequences thereof. Section 5 presents some definitions and facts. These will be used in section 6, where we turn to the task of defining a fragment of English and of showing that it can be interpreted in a rigorously compositional manner. Section 7 discusses some empirical limitations of the fragment and sketches how to overcome them by extending the dynamics.

Throughout, we shall suppose that the reader has a working knowledge of *MG*, and is familiar with the basics of *DRT* and/or *DPL*. Finally, we will not go into the relationship with the original work in the area of discourse representation (Kamp [1981,1983], Heim [1982,1983], Seuren [1985]), or with other attempts at a compositional formulation (Barwise [1987], Rooth [1987], Zeevat [1989]). For some discussion, see Groenendijk & Stokhof [1988,1989].

2 Variables and discourse markers

To appreciate the main arguments of this section, the reader needs to be aware of the following characteristics of *DPL*. In *DPL* a simple sequence of sentences such as (1) is translated into the formula (2), which in *DPL* is equivalent to (3):

(1) A man walks in the park. He whistles.

(2) $\exists x[\mathtt{man}(x) \wedge \mathtt{walk\_in\_the\_park}(x)] \wedge \mathtt{whistle}(x)$

(3) $\exists x[\mathtt{man}(x) \wedge \mathtt{walk\_in\_the\_park}(x) \wedge \mathtt{whistle}(x)]$

The dynamics of *DPL* makes sure that the occurrence of x in the last conjunct of (2), which is outside the scope of the existential quantifier, is still bound by that quantifier.

In a similar fashion, donkey-sentences like (4) and (5) translate into the *DPL*-formulae (6) and (7), respectively, which in *DPL* are equivalent to (8), their ordinary translation in predicate logic:

(4) If a farmer owns a donkey, he beats it.

(5) Every farmer who owns a donkey beats it.

(6) $\exists x[\mathtt{farmer}(x) \wedge \exists y[\mathtt{donkey}(y) \wedge \mathtt{own}(x,y)]] \rightarrow \mathtt{beat}(x,y)$

(7) $\forall x[[\mathtt{farmer}(x) \wedge \exists y[\mathtt{donkey}(y) \wedge \mathtt{own}(x,y)]] \rightarrow \mathtt{beat}(x,y)]$

(8) $\forall x \forall y[[\mathtt{farmer}(x) \wedge \mathtt{donkey}(y) \wedge \mathtt{own}(x,y)] \rightarrow \mathtt{beat}(x,y)]$

In *DPL* an existential quantifier inside the antecedent of a conditional can bind free variables in the consequent, with the effect of universal quantification over the implication as a whole.

Following usual practice, in *DPL* quantifying expressions of natural language are translated into expressions of the logical language that contain quantification over a certain variable, and pronouns are translated as variables. The essential feature of *DPL* is that it allows an existential quantifier to bind occurrences of variables outside its scope unless the quantifier is inside the scope of a negation and the occurrence of the variable outside.

If we extend *DPL* to a language which has a type structure and which contains λ-abstraction, we are dealing with a system in which from the viewpoint of natural language semantics, variables have (at least) two functions, instead of just one. If we use (static or dynamic) predicate logic to translate natural language sentences, variables are used in the represention of quantificational expressions, and of anaphoric relations between such expressions and pronouns. (We disregard free occurrences of variables, since we don't see a proper use for them from the natural language point of view. But if one insists, they may be interpreted as having yet a third, viz., a referring function.) However, once we switch to the use of a system that allows us to translate really compositionally, a different use of variables appears. For, λ-abstracts, and the variables occurring therein, play a different role: they are used to represent non-sentential, functional expressions, such as complex nouns and verbs. In such constructs, unless they originate from a quantificational expression, variables don't have a direct natural language counterpart. So, whatever reason there was to treat our logical quantifiers and variables dynamically that originates in the meaning of their natural language counterparts, does not automatically extend to these cases.

Consider the following example. The complex common noun phrase *farmer who owns a donkey* can be represented in a language with λ-abstraction as follows: $\lambda x[\mathtt{farmer}(x) \wedge \exists y[\mathtt{donkey}(y) \wedge \mathtt{own}(x, y)]]$. The occurrences of y derive directly from the quantificational expression *a donkey* as it occurs as the direct object of *own*. In the dynamic semantics of *DPL*, the existential quantifier may bind variables which are outside its scope, and that is the way in which we account, in a compositional manner, for the fact that the indefinite term may be the antecedent of pronouns in sentences to come. Now consider the occurrences of x. These do not have a natural language counterpart, and, at first sight at least, a dynamic approach to them seems to make no sense. From the viewpoint of natural language semantics, we might say that such variables are really artefacts, which as a consequence do not partake in any dynamic meaning of natural language expressions.

The point can perhaps be appreciated more clearly if we consider *direct* interpretation, i.e., interpretation without translation in a logical language, instead of the usual indirect approach. Stating directly the meaning of *farmer who owns a donkey*, for example as part of describing the meaning of the sentence *Every farmer who owns a donkey, beats it*, we would assign a dynamic meaning to the indefinite term *a donkey*. This carries over to the property which we associate with the entire phrase *farmer who owns a donkey*, but that is all. Being a property itself isn't anything dynamic, at least not as far as dynamics goes in the context of the present discussion, i.e., as far as anaphoric relations between quantificational expressions and pronouns are concerned. All we need to make sure is that the dynamics of the indefinite term is not blocked, i.e., we must allow properties to be dynamic in the secondary sense of being transparent to any intrinsically dynamic constituents they may have.

Returning to the perspective of indirect interpretation again, we notice that this argument extends to quantificational expressions in general. Although *some* occurrences of the existential quantifier need to be interpreted dynamically, viz., those which are used to represent the dynamic meaning of indefinite terms, it does not follow that *all* of them need to be handled this way. And in fact, once we transcend the simple, 'all in one fell swoop' way of translating that is our only choice if we use a first-order system, and we start doing some really compositional translating, we encounter lots of situations in which we would want to use the quantificational apparatus of our system in the standard, non-dynamic way. The case of the λ-operator is a particularly striking one, since it seems to defy any reasonably intuitive dynamic interpretation, at least in its ordinary uses such as the one indicated above. But also for the quantifiers, it is easy to come up with examples in which their ordinary interpretation is the most appropriate one. A good number of examples is provided by all kinds of instances of 'lexical decomposition' which takes place in the translation process.

To be sure, in a system like *DPL*, the ordinary, static meaning of quantifiers (and connectives) can be imitated. So, the line of reasoning just sketched, does not constitute an argument that is decisive in any respect. Yet, it does suggests rather strongly that our analysis will gain in perspicuity, if not in adequacy, if we make a distinction in our logical language between ordinary cases of the use of variables, quantifiers and λ-abstraction, and dynamic cases, which are those where we want to represent the dynamic semantics of quantificational structures of natural language and anaphoric relations.

This is indeed the line we shall take in what follows. We shall use variables and quantifiers in the ordinary, static way, and add to our logical language two new syntactic categories. One is that of so-called 'discourse markers', the other is that of so-called 'state switchers'. The former play the role of a new special kind of variables, the latter are a new binding device. Both are borrowed from Janssen [1986], where they are put to a different use. So, we free our usual quantificational apparatus from the task of dealing with the dynamic 'passing on' and 'picking up' of referents. As a consequence, assignments of values to variables are no longer the semantic locus in which the dynamics resides, in contradistinction to what is the case in *DPL*. The dynamics is now dealt with by means of the discourse markers and state switchers, which are interpreted in terms of a new semantic parameter, that of a 'state'.

The general idea is the following. Discourse markers are used in the translations of indefinite terms and pronouns. Part of the meaning of the indefinite term is to assign a value, one that is a witness of the term, to the discourse marker. Also, provisions are made to pass on this value, to further occurrences of the same discourse marker. This is taken care of, among other things, by the state switcher. The denotations of discourse markers are determined with reference to a state, and the state switcher will allow us to pick out certain states. More specifically, it will allow us to identify states in which discourse markers have certain values, by switching from the current state to such a new state. Finally, the meanings of sentences will be constructed in such a way that they determine that subsequent sentences are interpreted with respect to the most recently identified state. This they bring about by being themselves a kind of descriptions of states, as we shall see in section 4.

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Before turning to a statement of the system, and to its applications, a few words of warning are in order.

First of all, as we already indicated above, we do not claim that the distinction between variables and discourse markers, quantifiers and state switchers, assignments and states, is really forced upon us. There is no principled obstacle for a fusion of them, i.e., there is no reason to believe that we cannot deal with the dynamics of natural language by means of a typed system with λ-abstraction in which it are the variables and quantifiers which do the job (see Groenendijk & Stokhof [in prep] for some discussion). But we do feel rather strongly that the system to be given shortly, in which these distincions are made, is perspicuous, easy to learn and to use, and, very important, easy to extend.

That brings us to the second point. The system is referred to, unfortunately perhaps, as a system of dynamic *intensional* logic. This is justified because the formal apparatus of intensional logic is used. However, throughout this paper, intensionality in the ordinary sense of the word is ignored. The role of the states, which act as a parameter in the notion of interpretation, is strictly confined to assigning values to discourse markers, and to nothing else. Yet, we do use intensional terminology, such as 'proposition' and the like, in connection with them, we introduce the $^\wedge$-operator to express abstraction over them, and so on. We do this because these familiar terms have a clear meaning in the present context, too. But the reader should keep in mind the particular use that is made of the intensional vocabulary here.

Finally, in this paper we restrict our use of discourse markers to expressions which refer to individuals. Of course, in a later stage, we want to allow also discourse markers over other types of objects. See Groenendijk & Stokhof [in prep] for a system in which this restriction is lifted, and in which intensionality in the ordinary sense is incorporated as well.

3 Dynamic intensional logic

The system of dynamic intensional logic (henceforth, *DIL*) is a variant of the system of intensional type theory *IL*, which is used in Montague's *PTQ*. (For further details, see Janssen [1986].)

The types of *DIL* are the same as those of *IL*:

Definition 1 (Types) T, the set of types, is the smallest set such that:

1. $e, t \in T$
2. If $a, b \in T$, then $\langle a, b\rangle \in T$
3. If $a \in T$, then $\langle s, a\rangle \in T$

As usual, the syntax takes the form of a definition of ME_a, the set of meaningful expressions of type a. Given sets of constants, CON_a, and variables, VAR_a, for every type a, and a set of discourse markers DM, the definition runs as follows:

Definition 2 (Syntax)

1. If $\alpha \in CON_a \cup VAR_a$, then $\alpha \in ME_a$
2. If $\alpha \in DM$, then $\alpha \in ME_e$
3. If $\alpha \in ME_{\langle a,b\rangle}, \beta \in ME_a$, then $\alpha(\beta) \in ME_b$
4. If $\phi, \psi \in ME_t$, then $\neg\phi, [\phi \wedge \psi], [\phi \vee \psi], [\phi \rightarrow \psi] \in ME_t$
5. If $\phi \in ME_t, \nu \in VAR_a$, then $\exists\nu\phi, \forall\nu\phi \in ME_t$
6. If $\alpha, \beta \in ME_a$, then $\alpha = \beta \in ME_t$
7. If $\alpha \in ME_a, \nu \in VAR_b$, then $\lambda\nu\alpha \in ME_{\langle b,a\rangle}$
8. If $d \in DM, \alpha \in ME_e, \beta \in ME_a$, then $\{\alpha/d\}\beta \in ME_a$
9. If $\alpha \in ME_a$, then $^\wedge\alpha \in ME_{\langle s,a\rangle}$
10. If $\alpha \in ME_{\langle s,a\rangle}$, then $^\vee\alpha \in ME_a$
11. Nothing is in ME_a, for any type a, except on the basis of a finite number of applications of 1–10

New with respect to *IL* are clauses 2 and 8. According to 2, discourse markers are expressions of type e. Clause 8 introduces the state switchers: they turn an expression of arbitrary type a into an expression of the same type. The idea is that the expression to which the state switcher is applied, will be evaluated with respect to a state determined by the state switcher. Finally, notice that the ordinary intensional operators $\Box$ and $\Diamond$ do not occur (but they could easily be added). The $^\wedge$- and $^\vee$-operators are present, and will be seen to express abstraction over, and application to states, respectively.

Let us now turn to the semantics. Starting from two disjunct, non-empty sets D and S, of individuals and states respectively, D_a, the domain corresponding to type a, is defined in the familiar fashion:

Definition 3 (Domains)

1. $D_e = D$
2. $D_t = \{0, 1\}$
3. $D_{\langle a,b \rangle} = D_b^{D_a}$
4. $D_{\langle s,a \rangle} = D_a^S$

As will become apparent shortly, D_a can not be regarded generally as the domain from which all expressions of type a get their semantic value, the exception being D_e, since discourse markers, although expressions of type e, will be interpreted as 'individual concepts', i.e., as functions from S to D_e.

A model M is a triple $\langle D, S, F \rangle$, where D and S are as above, and F is a function which interprets the constants and the discourse markers. Specifically, if $\alpha \in CON_a$, then $F(\alpha) \in D_a$, and if $\alpha \in DM$, then $F(\alpha) \in D^S$. Further, G, the set of assignments, is the set of all functions g such that if $\nu \in VAR_a$, $g(\nu) \in D_a$. So, except for the discourse markers, all basic expressions are assigned extensional interpretations.

In order for the state switchers to be given their proper interpretation, we formulate two postulates which *DIL*-models should satisfy, which define certain properties of states:

Postulate 1 (Distinctness) If for all $d \in DM$: $F(d)(s) = F(d)(s')$, then $s = s'$

Postulate 2 (Update) For all $s \in S$, $d \in DM$, $\mathbf{d} \in D$ there exists an $s' \in S$ such that:

1. $F(d)(s') = \mathbf{d}$; and
2. for all $d' \in DM$, $d' \neq d$: $F(d')(s) = F(d')(s')$

Since the interpretation of the constants and variables is state-independent, these two postulates guarantee that, for each state s, discourse marker d and object $\mathbf{d}$, there exists a unique state s' which differs from s at most in this respect that the denotation of d in s' is $\mathbf{d}$. We refer to this state as $\langle d \leftarrow \mathbf{d} \rangle s$. This notation reminds one, of course, of the notation $g[\nu/\mathbf{d}]$ for assignments, and, indeed, states behave as assignments of values to discourse markers.

Now we state the semantics by defining the notion $[\![\alpha]\!]_{M,s,g}$, the interpretation of α with respect to M, s, and g, as follows:

Definition 4 (Semantics)

1. $[\![c]\!]_{M,s,g} = F(c)$, for every constant c
   $[\![\nu]\!]_{M,s,g} = g(\nu)$, for every variable ν
2. $[\![d]\!]_{M,s,g} = F(d)(s)$, for every discourse marker d
3. $[\![\alpha(\beta)]\!]_{M,s,g} = [\![\alpha]\!]_{M,s,g}([\![\beta]\!]_{M,s,g})$
4. $[\![\neg\phi]\!]_{M,s,g} = 1$ iff $[\![\phi]\!]_{M,s,g} = 0$
   $[\![\phi \wedge \psi]\!]_{M,s,g} = 1$ iff $[\![\phi]\!]_{M,s,g} = [\![\psi]\!]_{M,s,g} = 1$
   $[\![\phi \vee \psi]\!]_{M,s,g} = 1$ iff $[\![\phi]\!]_{M,s,g} = 1$ or $[\![\psi]\!]_{M,s,g} = 1$
   $[\![\phi \rightarrow \psi]\!]_{M,s,g} = 0$ iff $[\![\phi]\!]_{M,s,g} = 1$ and $[\![\psi]\!]_{M,s,g} = 0$
5. $[\![\exists\nu\phi]\!]_{M,s,g} = 1$ iff there is a $\mathbf{d} \in D_a$ such that $[\![\phi]\!]_{M,s,g[\nu/\mathbf{d}]} = 1$, where a is

the type of ν
$[\![\forall\nu\phi]\!]_{M,s,g} = 1$ iff for all $\mathbf{d} \in D_a$ it holds that $[\![\phi]\!]_{M,s,g[\nu/\mathbf{d}]} = 1$, where a is the type of ν

6. $[\![\alpha = \beta]\!]_{M,s,g} = 1$ iff $[\![\alpha]\!]_{M,s,g} = [\![\beta]\!]_{M,s,g}$
7. $[\![\lambda\nu\alpha]\!]_{M,s,g} =$ that function $h \in D_a^{D_b}$ such that $h(\mathbf{d}) = [\![\alpha]\!]_{M,s,g[\nu/\mathbf{d}]}$ for all $\mathbf{d} \in D_b$, where a is the type of α, and b the type of ν
8. $[\![\{\alpha/d\}\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,\langle d \leftarrow [\![\alpha]\!]_{M,s,g}\rangle s,g}$
9. $[\![{}^\wedge\alpha]\!]_{M,s,g} =$ that function $h \in D_a^S$ such that $h(s') = [\![\alpha]\!]_{M,s',g}$ for all $s' \in S$, where a is the type of α
10. $[\![{}^\vee\alpha]\!]_{M,s,g} = [\![\alpha]\!]_{M,s,g}(s)$

The notions of truth, validity, entailment and equivalence are defined in the usual way. We call ϕ *true with respect to M, s and g* iff $[\![\phi]\!]_{M,s,g} = 1$. We say that ϕ is *valid* iff ϕ is true with respect to all M, s, and g, and we write $\models \phi$. We set ϕ *entails* ψ iff for all M, s, and g: if $[\![\phi]\!]_{M,s,g} = 1$ then $[\![\psi]\!]_{M,s,g} = 1$, and we write $\phi \models \psi$. Finally, we call expressions α and β of the same type *equivalent* iff $\models \alpha = \beta$.

All clauses of definition 4 are completely standard, with the exception of 2 and 8. Clause 2 expresses, as was already announced above, that the interpretation of discourse markers is dependent on the state parameter. In clause 8, the semantics of the state switcher is given. The interpretation of $\{\alpha/d\}\beta$ with respect to a state s is arrived at by interpreting β with respect to a state s' which differs at most from s in that the denotation of the discourse marker d in s' is the object that is the denotation of the expression α in s. The two postulates on *DIL*-models guarantee that s' is unique, and, hence, that we can interpret state-switchers in this way. State switchers in fact operate in accordance with their name: they switch the state with respect to which an expression is evaluated to some uniquely determined, possibly different state. With a few exceptions, a state switcher $\{\alpha/d\}$ behaves semantically like the corresponding syntactic substitution operator $[\alpha/d]$, as is evident from the observations reported in fact 3 below. The operator $^\wedge$ is interpreted as abstraction over states. In effect, it functions as unselective abstraction over discourse markers. (Similarly, if $\Box$ and $\Diamond$ were added to the logical language, they could be looked upon as 'unselective adverbs of quantification'.)

The class of intensionally closed expressions is defined as follows:

Definition 5 *ICE*, the set of *intensionally closed expressions* is the smallest set such that:

1. $c, \nu \in ICE$, for every constant c and variable ν
2. ${}^\wedge\alpha \in ICE$, for every well-formed expression α
3. everything constructed solely from elements of ICE by means of functional application, negation, connectives, identity, quantifiers and/or λ-abstraction, is again an element of ICE

Notice that there are intensionally closed expressions which contain discourse markers, for example ${}^\wedge d$, and also that there are expressions which are not intensionally closed which do not contain them, for example ${}^\vee p$, where p is a variable of type $\langle s, t\rangle$.

It is easy to check that the following holds:

Fact 1 If $\alpha \in ICE$, then $[\![\alpha]\!]_{M,s,g} = [\![\alpha]\!]_{M,s',g}$, for all $s, s' \in S$

Now we observe the following fact concerning state switchers:

Fact 2 If $\beta \in ICE$, then $\{\alpha/d\}\beta$ is equivalent with β

It is easy to see that this holds: if $[\![\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,s',g}$, for all $s, s' \in S$, then certainly $[\![\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,\langle d \leftarrow [\![\alpha]\!]_{M,s,g}\rangle s,g}$, whence $[\![\{\alpha/d\}\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,s,g}$.

The following characterizes the behaviour of state switchers:

Fact 3 (Properties state switchers)

1. $\{\alpha/d\}c$ is equivalent with c, for every constant c
   $\{\alpha/d\}\nu$ is equivalent with ν, for every variable ν
2. $\{\alpha/d\}d$ is equivalent with α
   $\{\alpha/d\}d'$ is equivalent with d', for all $d' \in DM\colon d \neq d'$
3. $\{\alpha/d\}(\beta(\gamma))$ is equivalent with $\{\alpha/d\}\beta(\{\alpha/d\}\gamma)$
4. $\{\alpha/d\}(\phi \wedge \psi)$ is equivalent with $\{\alpha/d\}\phi \wedge \{\alpha/d\}\psi$ (analogously for negation and the other connectives)
5. $\{\alpha/d\}\exists\nu\phi$ is equivalent with $\exists\nu\{\alpha/d\}\phi$, if ν does not occur freely in α (analogously for $\forall\nu\phi$)
6. $\{\alpha/d\}(\beta = \gamma)$ is equivalent with $\{\alpha/d\}\beta = \{\alpha/d\}\gamma$
7. $\{\alpha/d\}\lambda\nu\beta$ is equivalent with $\lambda\nu\{\alpha/d\}\beta$, if ν does not occur freely in α
8. $\{\alpha/d\}{}^\wedge\beta$ is equivalent with ${}^\wedge\beta$
9. $\{\alpha/d\}{}^\vee\beta$ is *not* equivalent with ${}^\vee\beta$

Clauses 1 and 8 are instances of fact 2 above. In clause 2, the effect of application of a state switcher to a discourse marker is described. As for 9, it should be noticed that what is meant is that the equivalence does not hold for *all* β, although for some it does. A simple case is ${}^\wedge c$, for $\{\alpha/d\}{}^{\vee\wedge}c$ is equivalent with ${}^{\vee\wedge}c$. An example of a case in which the equivalence does not hold will be discussed shortly.

According to 3–7, the state switcher may always be pushed inside an expression, with the exception of those which begin with the extension operator ${}^\vee$, or another state switcher. State switchers distribute over function-argument structures (3), conjunctions, disjunctions, and implications (4), and identities (6). Also they may be pushed over negation (4), over the quantifiers (5) and the λ-operator (7), provided no binding problems occur. In combination with 1, 2 and 8, this allows us to 'resolve' state switchers in a large number of cases, though not in all. Generally, an occurrence of a state switcher in an expression may be moved inward until one of three cases obtains. It hits the corresponding discourse marker, in which case it is resolved in accordance with 2. It may end up in front of a constant, or a variable, or a different discourse marker, in which case it disappears. Or it may strand in front of an expression without a further move being possible. In this case we are dealing with an expression of the form ${}^\vee\beta$. As is stated in 9, $\{\alpha/d\}{}^\vee\beta$ can *not* always be reduced further. This role of 'stranding site' for state switchers that the extension operator ${}^\vee$ plays, will prove to be of prime importance in what follows. The following example illustrates why the reduction in question does not hold generally.

Consider the expresssion $\lambda p{}^\vee p$, in which p is a variable of type $\langle s,t\rangle$. The expression as a whole is of type $\langle\langle s,t\rangle,t\rangle$, and it denotes a set of propositions. With respect to a state s, the denotation of $\lambda p{}^\vee p$, $[\![\lambda p{}^\vee p]\!]_{M,s,g}$, is the set of those propositions $\mathbf{p}$ such that $\mathbf{p}$ is true in s. If we apply a state-switcher $\{\alpha/d\}$ to this expression, we get $\{\alpha/d\}\lambda p{}^\vee p$, which is equivalent to $\lambda p\{\alpha/d\}{}^\vee p$. In a state s, this expression denotes the set of propositions which are true in $\langle d \leftarrow [\![\alpha]\!]_{M,s,g}\rangle s$, the unique state which differs at most from s in that the denotation of the discourse marker d is the denotation of the expression α in the original state s. The latter set of propositions is only identical to the set denoted by our original expression $\lambda p{}^\vee p$ in s if $s = \langle d \leftarrow [\![\alpha]\!]_{M,s,g}\rangle s$. But of course, that need not be the case.

Notice that the following two familiar facts, concerning the interaction between

$^\wedge$ and $^\vee$, and concerning λ-conversion, go through:

Fact 4 ($^{\vee\wedge}$-elimination) $^{\vee\wedge}\alpha$ is equivalent with α

Fact 5 (λ-conversion) $\lambda\nu\alpha(\beta)$ is equivalent with $[\beta/\nu]\alpha$ if:

1. all free variables in β are free for ν in α
2. $\beta \in ICE$, or no occurrence of ν in α is in the scope of $^\wedge$ or a state switcher $\{\gamma/d\}$

In sections 5 and 6, we shall show how the *DIL*-system can be used to give the meanings of natural language sentences, and sequences thereof. Before doing so, we shall discuss in section 4 what type of *DIL*-object would be a proper one to be associated with sentences, and next, we show how translations which express this type of semantic object, can be obtained.

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We end this section by making some remarks which may help to illustrate the similarities between ordinary variables and discourse markers.

First of all, we observe that one could add to the language 'assignment switchers' which function analogously to state switchers:

$$\text{If } \nu \in VAR_a, \alpha \in ME_a, \beta \in ME_b, \text{ then } \{\alpha/\nu\}\beta \in ME_b$$

The semantics is as follows:

$$[\![\{\alpha/\nu\}\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,s,g[\nu/[\![\alpha]\!]_{M,s,g}]}$$

Of course, this does not really extend the language, for $\{\alpha/\nu\}\beta$ is equivalent to $(\lambda\nu\beta)(\alpha)$.

Secondly, we notice that we could introduce quantification over discourse markers and λ-abstraction:

$$[\![\exists d\phi]\!]_{M,s,g} = 1 \text{ iff there is a } \mathbf{d} \in D_e\colon [\![\phi]\!]_{M,\langle d\leftarrow\mathbf{d}\rangle s,g} = 1$$

$$[\![\lambda d\alpha]\!]_{M,s,g} \text{ that function } h \in D_a^{D_e}\colon h(\mathbf{d}) = [\![\alpha]\!]_{M,\langle d\leftarrow\mathbf{d}\rangle s,g} \text{ for all } \mathbf{d} \in D_e, \text{ where } a \text{ is the type of } \alpha$$

Again, this does not enrich the language, since quantification over discourse markers can be defined in terms of abstraction over them (and identity). And abstraction over discourse markers can be defined in terms of abstraction over variables and the state switchers:

$$\lambda d\alpha = \lambda x\{x/d\}\alpha, \text{ where } x \text{ is not free in } \alpha$$

We shall not make use of these possibilities in what follows, we just point them out to illustrate the fact that adding discourse markers and state switchers is really the same as adding a second kind of variable-and-binding device.

4 Interpreting and translating sentences

In *MG* an (indicative) sentence is translated into an expression of type t: the extension of a sentence is a truth value, and its intension a proposition. This is the explication that *MG* gives of the idea that the meaning of a sentence can be identified with its truth conditions. We shall see that this static notion of meaning is retained in *DMG*, but as a secondary one, which can be derived from the primary notion, which is essentially richer.

The general starting point of theories of dynamic semantics is that the meaning of a sentence resides in its information change potential. Sentences carry information, and the best way that this information can be characterized is by showing how it *changes* information, for example the information of someone who processes the sentence. This idea is quite general, and one way to capture part of it is by looking upon the meaning of a sentence as something which tells us which propositions are true after its contents have been processed. We may assume that this processing takes place in some situation, so that we can regard the extension of a sentence in a situation as consisting of those propositions which are true after the sentence has been processed in that situation. Notice that the situation in which the sentence is processed, may itself contribute information, i.e., which set of propositions results will in general also depend on the situation in which a sentence is processed. This informal description clearly displays the dynamic character of this view on meaning. And it also makes clear that we need a fairly complex type of objects if we are to give a formal explication of it.

At this point, we should remark, perhaps superfluously, that the present theory — like *DPL*, *DRT*, and others — accounts for just one aspect of this notion of meaning as information change potential. Here, 'information' is information about referents of discourse markers only. Analogously, a proposition in *DIL* is a function from states to truth values, and, as was already noticed earlier, this has nothing to do with intensionality in the ordinary sense. So no account is given of those aspects of meaning which concern updating of partial information about the world. In fact, in the present context the world can only be equated with the model in which interpretation takes places, which leaves no room for partiality of information in the intuitive sense of information about the world. See Groenendijk & Stokhof [1988, in prep] for more detailed discussion of these issues.

In *DMG*, sentences are translated into expressions of type $\langle\langle s,t\rangle,t\rangle$. In other words, in a state they denote a set of propositions. The meaning of a sentence is an object of type $\langle s,\langle\langle s,t\rangle,t\rangle\rangle$, i.e., a relation between states and propositions, or, equivalently, a function from states to sets of propositions.

Let us try to make this a bit more clear by exploiting the following analogy. The extension of a sentence in *DMG* is an object of type $\langle\langle s,t\rangle,t\rangle$, and such an object is a generalized quantifier over states, in the same way as objects of type $\langle\langle e,t\rangle,t\rangle$, the kind of semantic objects that function as the translations of quantified terms such as *the woman*, or *a man who was smoking*, are generalized quantifiers over individuals. A generalized quantifier over individuals denotes a set of properties (sets) of individuals, and a generalized quantifier over states denotes a set of properties (sets) of states, i.e., a set of propositions.

This means that we may paraphrase the extensions of sentences in more or less plain English, using such phrases as *a state which* ..., *the state such that* ..., and so on. A characteristic example is the following:

(9) a state which differs from s at most in this respect that the denotation of the discourse marker d is an object which belongs to the set of men and to the set of objects which walk in the park

Now, the extension of which English sentence is this the paraphrase of? First of all, we notice that (9) does not identify a state. It is an existentially quantified term, and it denotes a set of sets of states, the smallest elements of which are singleton sets, each containing a different state. Specifically, given some state s, for every object $\mathbf{d}$ which is a man and which walks in the park, $\{\langle d \leftarrow \mathbf{d}\rangle s\}$ is such a singleton. Secondly, we draw attention to the role played by the state s, remarking that everything which is true in s and which does not concern the denotation of d, remains true in each of the new states. So, the transition from s to this set of propositions characterizes a change of information about the denotation of this discourse marker d.

Let us now look at the following sentence:

(10) A man walks in the park.

Assume that indefinite terms are interpreted using discourse markers. What change of information does (10) bring about, if we interpret it with respect to some state s? Clearly, no unique state results, any state in which the relevant discourse marker denotes a man who walks in the park and which is like s in all other respects, will do. So, with regard to a fixed state s, the processing of a sentence like (10) should result in a set of states, each representing, so to speak, a possible way in which (10) could be true, given s. This set is of course no other than the union of the minimal elements of the set of sets denoted by (9). So, (9) is indeed a proper characterization of the information carried by (10) in s. And if we abstract over s in (9), we get a proper characterization of the information change potential, i.e., the meaning, of (10).

Of course, natural language sentences themselves don't contain discourse markers, nor do they refer to states explicitly, so we can't look upon (9) as the extension of (10). Rather, it is a paraphrase of the extension of its translation in *DIL*, and from (9) we can infer some characteristics of this translation. It will contain some discourse marker d, and it will, among other things, consist of assertions that express that the extension of d has the properties of being a man and of walking in the park: $\mathtt{man}(d) \wedge \mathtt{walk\_in\_the\_park}(d)$. The existential quantification that is part of the meaning of the indefinite term, will appear as an ordinary existential quantifier over elements of the domain: $\exists x$. A state switcher will relate the values of x to the discourse marker d: $\exists x\{x/d\}[\mathtt{man}(d) \wedge \mathtt{walk\_in\_the\_park}(d)]$. Now, this is not the actual translation of (10), since a very important element is still missing. (This is already obvious from the fact that this expression is of type t, and not of the required type $\langle\langle s,t\rangle,t\rangle$. Notice also that it is equivalent with $\exists x[\mathtt{man}(x) \wedge \mathtt{walk\_in\_the\_park}(x)]$.) What is not yet accounted for is the possibility inherent in (10) that the indefinite term serve as an antecedent for pronouns in following sentences. This means that its translation should be able in some way to pass on the information it carries about the discourse marker to other formulae, something in which the state switcher can be expected to play a crucial role, of course. Actually, how this is to be done is no other question than how sequences of sentences are to be interpreted. Before we address that question explicitly, however, two other observations are in order.

The first is that not all natural language sentences will involve 'state switching'. Consider (11):

(11) He whistles.

Suppose, again, that we interpret (11) with respect to some state s. Given some prior determination of which object the pronoun *he* refers to, we check whether s satisfies the condition that this object whistles. If it does, we continue to regard s as a possible state, if it doesn't, we discard it, i.e., we no longer take it into account. Notice that no other states than s itself are involved in this process. Information change potential and state switching are not to be confused: the change of information (about referents of discourse markers) takes place at the level of sets of sets of states, and it may, but need not, involve state switching, which takes place at the level of states. Nevertheless, we can describe what goes on, in the same format we used for (10). Consider (12):

(12) the state s, and the denotation of the discourse marker d in s is an object which whistles

If we translate the pronoun *he* in (11) by means of the discourse marker d, (12) can be regarded as giving the extension of (11) relative to s: (12) denotes the ultrafilter generated by s, if s satisfies the condition stated; and it denotes the empty set otherwise. Clearly, this neatly characterizes the information change brought about by (11) which we described informally above.

The second observation concerns the following. Above, we have said that the notions of the extension and the intension of a sentence with which we are familiar from the standard, static semantics, are retained as derivative notions. Now we show how. Our informal characterization of the dynamic meaning of a sentence was that the extension of a sentence in a state s consists of those propositions which are true after the sentence is processed in s. If the sentence can be succesfully processed, the resulting set of propositions will be non-empty, and should contain the tautologous proposition. A proposition can also be interpreted as a predicate over states, and the tautologous proposition corresponds to the predicate 'is a (possible) state'. If we apply the extension of a sentence in a state s to this predicate, what we get is an assertion which is either true or false in s. Consider (9). If we apply it to the predicate 'is a possible state', we end up with an assertion which can be phrased as follows:

(13) There is a state which differs from s at most in this respect that the denotation of the discourse marker d is an object which belongs to the set of men and to the set of objects which walk in the park

Since our postulates guarantee that to each state s a unique state s' corresponds which differs at most from s in that the denotation of the discourse marker d has certain properties, this means that (13) expresses the familiar truth conditions of (10): the bit about s, s', and d may be dropped, since it is guaranteed to hold, and we end up with:

(14) There exists an object which belongs to the set of men and to the set of objects which walk in the park

As is evident from (14), the truth or falsity of sentence (10) is completely independent of the denotations of discourse markers, and hence is also state independent. It depends solely on the model. But this does not hold for all sentences. An example is provided by (11). If we apply (12), the paraphrase of the extension of this sentence, to the predicate 'is a (possible) state', the following assertion results:

(15) The denotation of the discourse marker d in s is an object which whistles

In this case, the truth conditions are state dependent, since the truth or falsity of (15) depends on the value of the discourse marker d in s. This is, of course, nothing but a reflection of the fact that (11) contains the pronoun *he*, which makes its truth or falsity dependent on the state in which it is interpreted. For it is the state which determines the referent of the pronoun, by fixing the denotation of the discourse marker by means of which we translate it.

From these considerations two conclusions may be drawn. First, we can extract the ordinary truth conditions from the dynamic interpretation of a sentence by applying the latter to the tautologous proposition. And second, these truth conditions may be state dependent.

Now, we return to the question of how sequences of sentences are to be interpreted, and, in the wake of that, how the meanings of sentences are able to 'pass on' information to sentences to follow. We described the extension of a sentence in a state as a set of propositions, viz., those which are true after the sentence has been processed in this state. As we have seen above, this may have resulted in a state switch, but it need not have. Consider sentence (11), and the paraphrase of its extension in s, (12). Instead of (12), we could also have used:

(16) the set of propositions $\mathbf{p}$ such that the denotation of the discourse marker d in s is an object which whistles and such that $\mathbf{p}$ is true in s

This paraphrase is equivalent with (12): it characterizes s if the condition on d is fulfilled, and the empty set otherwise. Suppose we now process the following sentence, assuming it to be about the same individual as (11), i.e., interpreting it using the same discourse marker:

(17) He is in a good mood.

Like (11), this sentence involves no state switch, as (18) shows:

(18) the set of propositions $\mathbf{p}$ such that the denotation of the discourse marker d in s is an object which is in a good mood and such that $\mathbf{p}$ is true in s

Suppose (11) and (17) are both true in s. This implies that starting from s and interpreting, first (11), and next (17), we end up in s, having checked that it satisfies the conditions formulated by (11) and (17). In other words, the extension in s of the sequence:

(19) He whistles. He is in a good mood.

can be paraphrased as:

(20) the set of propositions $\mathbf{p}$ such that the denotation of the discourse marker d in s is an object which whistles and which is in a good mood and such that $\mathbf{p}$ is true in s

In getting to (20) from (16) and (18), we do something like the following. We construct from (18) a proposition which contains its truth conditional content (and something else to be explained later on), and to this proposition we apply (the characteristic function of) (16). This may look like a mere trick, but in fact it is completely in accordance with the description we have given of the extension of a sentence in a state as consisting of those propositions which are true in that state after the sentence has been processed. For a following sentence in a sequence, of course, claims to be just that: a sentence which is true in the situation with respect to which it is interpreted, which is the situation which results after processing its predecessor.

This means that we can also look upon the propositions which form the extension of a sentence as something giving the truth conditional contents of its possible continuations. This is not particularly useful if we just consider sentences which do not involve state switching. But in case a sentence does change the state with respect to which interpretation takes place, the abstraction over the contents of possible continuations gives us the means to pass on this information, i.e., to force the contents of sentences to come to be determined with respect to the changed state. Consider (10), and its paraphrase (9), again. The latter can also be phrased as:

(21) the set of propositions $\mathbf{p}$ such that there is a state s' which differs from s at most in this respect that the denotation of d is an object which is a man and which walks in the park and such that $\mathbf{p}$ is true in s'

The abstraction is now over propositions which are true in s', and this means that the truth conditional contents of sentences following (10) are to be determined with respect to s', and not with respect to the original s. If we first process (10), and then (11), the content of the latter will be determined with respect to s', a state in which d refers to a man which walks. And this means that (11) will be interpreted as saying of this individual that he whistles. Thus we get an account of the anaphoric relation between *a man* and *he* in:

(22) A man walks in the park. He whistles.

Two remarks are in order at this point. First, we notice once again that the change brought about by a sentence like (10) is relevant only for those sentences which follow it which contain the discourse marker d. For what is changed is the state parameter, which gives information about referents of discourse markers, not the situation which is being described, or, to put it differently, with respect to which sentences are evaluated. This remains fixed, being, in fact, embodied in the model in which interpretation takes place.

Second, we described the interpretation of a sequence of two sentences as the result of the application of the extension of the first sentence to a proposition constructed from the extension of the second sentence which 'contains its truth conditional content'. If we are interested in just the sequence itself, the truth conditional content, which can be acquired in the way described above, would suffice. However, we want to interpret longer sequences, too, and, more important, we want to interpret them in a compositional, step-by-step manner. And that is the reason why we need something more than just the truth conditional content of a sentence. For, unless we are sure that the sentence we are interpreting as part of a sequence is the last one, we need an 'angle' for the following sentences to hook on to, a 'place-holder' proposition for which the content of the following sentence can be substituted, which in its turn will contain another such place-holder, and so on. In other words, we need to make sure that if we calculate the extension in state s of a sequence consisting of a sentence ϕ followed by a sentence ψ, the result is again something which can be combined with another sentence χ. The extensions of sentences being functions, this means that sequencing of sentences is interpreted as intensional function composition. We apply the extension of the second sentence ψ to an arbitrary proposition, and abstract over the state. The result is a proposition which asserts the truth conditional content of ψ and the truth of this arbitary proposition. To this we apply the extension of the first sentence ϕ, which gives us the assertion of the truth conditional content of ϕ in conjunction with that of ψ and of the truth of the arbitrary proposition. If we abstract over the latter again, what we get is the set of propositions $\mathbf{p}$ such that $\mathbf{p}$ is true in the situation which results if first ϕ is processed in s, and next ψ is processed in the situation which results from that. Less informally, but more in accordance with the actual way in which sequences of sentences are translated in *DMG*:

(23) those propositions $\mathbf{p}$ such that after the processing of ϕ in s, it holds that $\mathbf{p}$ holds after the processing of ψ

Notice that the truth conditions of a sequence can be defined just as above: if the tautologous proposition is an element of this set of propositions, the sequence of the sentences ϕ and ψ is true in s.

Let us now show in some detail what actual translations of sentences into *DIL*-expressions look like, i.e., let us show how object language formulae can be obtained in a rather straightforward way by 'formalizing' so to speak the 'plain English' we have been using sofar.

If we use a logical notation for (9), the paraphrase of the extension in s of (10), we get the following formula of our meta-language:

(24) $\lambda \mathbf{p} \in D_{\langle s,t\rangle}\colon \exists s' \in S, \exists \mathbf{d} \in D\colon F(d)(s') = \mathbf{d} \;\&\; \forall d' \in DM\colon d' \neq d \Rightarrow F(d')(s) = F(d')(s') \;\&\; F(\mathtt{man})(\mathbf{d}) \;\&\; F(\mathtt{walk\_in\_the\_park})(\mathbf{d}) \;\&\; \mathbf{p}(s')$

Given our postulates we know that the s' in question is unique (for a fixed d) and that we may denote it as $\langle d \leftarrow \mathbf{d}\rangle s$, which means that we can reduce (24) to:

(25) $\lambda \mathbf{p}\colon \exists \mathbf{d}\colon F(\mathtt{man})(\mathbf{d}) \;\&\; F(\mathtt{walk\_in\_the\_park})(\mathbf{d}) \;\&\; \mathbf{p}(\langle d \leftarrow \mathbf{d}\rangle s)$

A *DIL*-formula which denotes this in state s is:

(26) $\lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk\_in\_the\_park}(x) \wedge \{x/d\}{}^{\vee}p]$

In a similar fashion we can write the interpretation of (11), as it was paraphrased in (12), as:

(27) $\lambda\mathbf{p}\colon F(\texttt{whistle})(F(d)(s))\ \&\ \mathbf{p}(s)$

A *DIL*-formula which has this denotation in state s is:

(28) $\lambda p[\texttt{whistle}(d) \wedge {}^{\vee}p]$

Now consider the denotation of a sequence of two sentences ϕ and ψ as described above in (23). Using a logical notation, we may write:

(29) $\lambda\mathbf{p}\colon [\![\phi]\!]_s(\lambda s'\colon [\![\psi]\!]_{s'}(\mathbf{p}))$

A corresponding *DIL*-formula is:

(30) $\lambda p[\phi({}^{\wedge}\psi(p))]$

If we substitute for ϕ and ψ the translations (26) and (28) of the two sentences (10) and (11) which make up sequence (22), the result is the following:

(31) $\lambda p[\lambda p\exists x[\texttt{man}(x) \wedge \texttt{walk\_in\_the\_park}(x) \wedge \{x/d\}{}^{\vee}p]({}^{\wedge}\lambda p[\texttt{whistle}(d) \wedge {}^{\vee}p](p))]$

After some conversion, we get:

(32) $\lambda p\exists x[\texttt{man}(x) \wedge \texttt{walk\_in\_the\_park}(x) \wedge \{x/d\}{}^{\vee\wedge}[\texttt{whistle}(d) \wedge {}^{\vee}p]]$

After ${}^{\vee\wedge}$-elimination and using the semantic properties of the state switcher stated in fact 3 in the previous section, this reduces to:

(33) $\lambda p\exists x[\texttt{man}(x) \wedge \texttt{walk\_in\_the\_park}(x) \wedge \texttt{whistle}(x) \wedge \{x/d\}{}^{\vee}p]$

The essential feature of this representation of the sequence of sentences (22) is that the binding of the indefinite term in the first sentence is passed on, by means of the state-switcher, to the second. Notice also that the fact that $\{x/d\}{}^{\vee}p$ can not be reduced plays an essential role here.

*

Let us end this section by pointing out the following. In the above we have referred to the meaning of a sentence as a function from states to generalized quantifiers over states. At first sight, this does not seem to fit the informal characterization, also referred to above, of the meaning of a sentence as its information change potential. According to the latter, the meaning of a sentence is a function from information states to information states. Bearing in mind that here we are only concerned with (partial) information about the reference of discourse markers, and observing that a state (completely) determines the latter, it is obviously the generalized quantifiers over states, and *not* the states themselves, which can be regarded as information states. Hence, the notion of meaning as information change potential suggests that meaning be formalized as a function from generalized quantifiers over states to generalized quantifiers over states. As a matter of fact, in the present context, such functions can be obtained from functions from states to generalized quantifiers over states as follows. Let I be an information state, i.e., a generalized quantifier over states. Then the information state which results after interpreting a sentence ϕ in I is the following generalized quantifier over states:

(34) $\lambda\mathbf{p}\colon I(\lambda s\colon [\![\phi]\!]_s(\mathbf{p}))$

As was to be expected, what the information change brought about by ϕ in I amounts to is taking ϕ in dynamic conjunction with I. (Cf. (29) above.)

In view of this, there is ample reason to use the simpler notion of a function from states to generalized quantifiers over states as our formal analogue of a sentence meaning. This notion is less complex, and the more intuitive one can be defined in terms of it. We point out that all this is particular to the present context in the following sense. First of all, under certain circumstances, we can make do with an even less complex notion, viz., that of a function from states to sets of states. This will be discussed in the next section. And secondly, incorporating other aspects of meaning, such as presuppositions, may very well necessitate the use of the more complex notion as our basic notion. Hence, the present analysis occupies one position in a space of possibilities that all fall under the general heading of dynamic semantics. (See Groenendijk & Stokhof [in prep] for more discussion.)

5 Some definitions and some facts

In this section, we introduce a few notation conventions which will facilitate the representation of translations of natural language expressions in *DMG* by providing an easy to read format.

Definition 6 (Uparrow) $\uparrow\phi = \lambda p[\phi \wedge {}^{\vee}p]$, where ϕ is an expression of type t, p a variable of type $\langle s,t\rangle$ which has no free occurrences in ϕ

The operator $\uparrow$ can be viewed as a type-shifting operation. If ϕ is true in state s, $\uparrow\phi$ denotes the set of all true propositions in s, if ϕ is false in s, then $\uparrow\phi$ denotes the empty set. The meaning of ϕ and $\uparrow\phi$ are hence one-to-one related. $[\![\uparrow\phi]\!]_{M,s,g}$ typically gives us the denotation of a sentence which does not have truly dynamic effects. For such sentences, it holds that their denotation in state s is always either the empty set, or the set of all propositions true in s.

Definition 7 (Downarrow) $\downarrow\Phi = \Phi({}^{\wedge}\texttt{true})$, where Φ is an expression of type $\langle\langle s,t\rangle,t\rangle$

An expression Φ of type $\langle\langle s,t\rangle,t\rangle$ is typically the kind of expression that functions as the translation of sentences in *DMG*. In our discussion of the interpretation of sentences in the previous section, we saw that application to the predicate 'is a possible state' of a generalized quantifier over states which gives the extension of a certain sentence, results in a proposition which represents the usual truth conditions of that sentences. Using $\texttt{true}$ as a constant of type t such that $F(\texttt{true}) = 1$, the expression ${}^{\wedge}\texttt{true}$ is a representation in *DIL* of the predicate 'is a possible state'. Application of the operator $\downarrow$ to the dynamic translation of a sentence hence results in a formula which represents its truth conditions, and in which all dynamic effects are cancelled. The formula $\downarrow\Phi$ is true in state s iff Φ can be succesfully processed in s. Its negation $\neg\downarrow\Phi$ boils down to the assertion that Φ cannot be processed succesfully in s, i.e., that after processing Φ no possible state results. We will refer to $\downarrow \Phi$ as the *truth conditional content* of Φ.

We notice the following fact:

Fact 6 ($\downarrow\uparrow$-elimination) $\downarrow\uparrow\phi = \phi$

Compare this with ${}^{\vee\wedge}$-elimination (fact 4 in section 4). What does not hold in general is that $\uparrow\downarrow\Phi = \Phi$, as similarly ${}^{\wedge\vee}\alpha = \alpha$ does not hold in general.

By way of example, consider the expression $\lambda p[\phi \wedge \{x/d\}{}^{\vee}p]$ of type $\langle\langle s,t\rangle,t\rangle$. Using definition 7, we can rewrite its 'lowering' $\downarrow\lambda p[\phi \wedge \{x/d\}{}^{\vee}p]$ to type t as $\lambda p[\phi \wedge \{x/d\}{}^{\vee}p]({}^{\wedge}\texttt{true})$, which reduces to $\phi \wedge \{x/d\}\texttt{true}$, by means of λ-conversion and ${}^{\vee\wedge}$-elimination. Since $\texttt{true}$ is a constant, and hence state independent, a further reduction to $\phi \wedge \texttt{true}$ is possible, an expression which is equivalent to ϕ. If we raise this expression of type t again to one of type $\langle\langle s,t\rangle,t\rangle$ by applying the operator $\uparrow$ to it, we get $\uparrow\phi$, which, by definition 6, can be written as $\lambda p[\phi \wedge {}^{\vee}p]$. But the latter expression is not equivalent to our original $\lambda p[\phi \wedge \{x/d\}{}^{\vee}p]$, from which fact we may conclude that $\uparrow\downarrow\Phi$ is not equivalent with Φ.

The operator $\downarrow$ may be looked upon as a kind of closure operator, one which closes off a (piece of) text. It reduces the meaning of a sentence to its truth-conditional content, 'freezing' so to speak any dynamic effects it may have had. The point is that once closed off, a piece of text remains that way, even if it is raised again to the higher type by means of $\uparrow$. We will refer to $\uparrow\downarrow \Phi$ as the *static closure* of Φ

Negation, also, is generally considered to be a static closure operation that blocks the dynamic effects of expressions inside its scope. Compare the following examples:

(35) It is not the case that a man walks in the park. He whistles.

(36) No man walks in the park. He whistles.

The pronoun in the second sentence of (34) and (35) cannot be interpreted as being anaphorically linked to the quantifying expressions in the first sentences of these examples. This means that if Φ is the translation of a sentence, its negation can be translated as $\uparrow\neg\downarrow\Phi$. The application of $\downarrow$ takes care of the static nature of negation, $\neg$ takes care of negation itself, and $\uparrow$ makes sure that in the end we get the right type of logical expression to translate sentences in the dynamic set-up. So, the result of negation is a static expression which either denotes the set of all true propositions in s, viz., in case Φ can not be processed succesfully in s, or the empty set, in case Φ can be so processed in s. This means that we may abbreviate negation as follows:

Definition 8 (Static negation) $\sim\Phi = \uparrow\neg\downarrow\Phi$

The double negation $\sim\sim\Phi$ is equivalent with $\uparrow\downarrow\Phi$: $\sim\sim\Phi = \uparrow\neg\downarrow\uparrow\neg\downarrow\Phi = \uparrow\neg\neg\downarrow\Phi = \uparrow\downarrow\Phi$. Since $\uparrow\downarrow\Phi$ is not equivalent with Φ, it follows that $\sim\sim\Phi$ is not equivalent with Φ. But notice that at the level of truth conditional content, double negation does hold: $\downarrow\sim\sim\Phi = \downarrow\Phi$.

Now, we turn to the most central dynamic notion, viz., conjunction:

Definition 9 (Dynamic conjunction) $\Phi\, ;\, \Psi = \lambda p[\Phi(^\wedge(\Psi(p)))]$,
where p has no free occurrences in either Φ or Ψ

An expression $\Phi\, ;\, \Psi$ represents the dynamic conjunction, or sequence, of two sentences. Notice that taking the dynamic conjunction of sentences amounts to taking the intensional composition of the functions that are denoted by the two sentences. From this it immediately follows that dynamic conjunction is associative, but not commutative. It is a truly sequential notion of conjunction.

Next, consider existential quantification:

Definition 10 (Dynamic existential quantifier) $\mathcal{E}d\Phi = \lambda p\exists x\{x/d\}(\Phi(p))$,
where x and p have no free occurrences in Φ

Notice that the quantifier $\mathcal{E}d$ itself could be written as $\lambda p\exists x\{x/d\}^\vee p$, and that the interpretation of $\mathcal{E}d\Phi$ then would amount to taking the intensional composition of the functions which are the interpretations of $\mathcal{E}d$ and Φ. Given the associativity of function composition it immediately follows that the dynamic existential quantifier has the following property:

Fact 7 $\mathcal{E}d\Phi\, ;\, \Psi = \mathcal{E}d[\Phi\, ;\, \Psi]$

This fact forms the basis of *DMG*'s account of cross-sentential anaphora.

In terms of the notions of negation, conjunction and existential quantification defined above, we can give definitions of notions of implication, disjunction and universal quantification which follow the usual pattern:

Definition 11 (Internally dynamic implication) $\Phi \Rightarrow \Psi = \sim[\Phi\, ;\, \sim\Psi]$

Definition 12 (Static disjunction) Φ **or** $\Psi = \sim[\sim\Phi\, ;\, \sim\Psi]$

Definition 13 (Static universal quantifier) $\mathcal{A}d\Phi = {\sim}\mathcal{E}d{\sim}\Phi$

From the definition of implication, we can see that it is 'internally dynamic': a dynamic existential quantifier in the antecedent can bind discourse markers in the consequent. But implication is not 'externally dynamic': no quantifiers inside either the antecedent or the consequent of an implication may bind discourse markers outside the implication. It is in line with this that the qualification 'dynamic' does not appear at all in definitions 12 and 13. These notions of disjunction and universal quantification, are neither internally nor externally dynamic. (In section 7, we shall give dynamic definitions of these constants.) Of course, it is the static character of negation that is all important here.

By simply applying the definitions, the following fact can be proved:

Fact 8 $\mathcal{E}d\Phi \Rightarrow \Psi = \mathcal{A}d[\Phi \Rightarrow \Psi]$

This equivalence is of paramount importance for a compositional interpretation of donkey-sentences.

The static nature of negation also blocks certain standard equivalences involving these notions. For example:

$$\mathcal{E}d{\sim}\Phi \neq {\sim}\mathcal{A}d\Phi$$

However, as is to be expected, these formulae do have the same truth conditional content, as is apparent from the following equivalence:

$$\downarrow\mathcal{E}d{\sim}\Phi = \downarrow{\sim}\mathcal{A}d\Phi$$

So, although the dynamic properties of these constants differ, at the truth conditional level the existential and the universal quantifier are related in the usual way.

The following equivalences will enable us to replace dynamic operators by their static counterparts at the level of truth-conditional content.

Fact 9

1. $\downarrow\uparrow\phi = \phi$
2. ${\sim}\uparrow\phi = \uparrow\neg\phi$
3. $[\uparrow\phi\,;\uparrow\psi] = \uparrow[\phi \wedge \psi]$
4. $[\uparrow\phi \Rightarrow \uparrow\psi] = \uparrow[\phi \rightarrow \psi]$
5. $[\uparrow\phi$ **or** $\uparrow\psi] = \uparrow[\phi \vee \psi]$

These equivalences will allow us to reduce translations of (sequences of) sentences in *DIL* in such a way that they contain only the usual operators in a large number of cases.

*

We end this section with an excurs on the relation between *DIL* and *DPL*. We shall show that there exist translations in both directions between the formulae of *DPL* and a fragment of *DIL*. We will refer to this fragment as DIL_1.

Let CON_1 be the set of first-order constants of *DIL*, i.e., $CON_1 = CON_e \cup CON_{\langle e,t\rangle} \cup CON_{\langle e,\langle e,t\rangle\rangle} \cup \ldots$ Let $ATOM_1$ be the set of formulae of type t that can be obtained from $CON_1 \cup DM$ by means of functional application. So, $ATOM_1$ is the set of first-order atomic formulae of *DIL* that do not contain variables, but consist of constants and discourse markers only. We now define DIL_1 as follows:

Definition 14 DIL_1 is the smallest subset of $ME_{\langle\langle s,t\rangle,t\rangle}$ such that:

1. if $\phi \in ATOM_1$, then $\uparrow\phi \in DIL_1$

2. if $\Phi \in DIL_1$, then $\sim\Phi, \mathcal{E}d\Phi, \mathcal{A}d\Phi \in DIL_1$
3. if $\Phi, \Psi \in DIL_1$, then $[\Phi\ ;\ \Psi], [\Phi \Rightarrow \Psi], [\Phi\ \mathbf{or}\ \Psi] \in DIL_1$

Next, we define a translation $\sharp$ from *DPL* to DIL_1:

Definition 15 (Translation $\sharp$ of *DPL* to DIL_1)

1. $\sharp x_n = d_n$
2. $\sharp c_n = c_{e,n}$
3. $\sharp P^k_n = P_{\langle e_1 \ldots \langle e_k, t\rangle \ldots\rangle, n}$
4. $\sharp Pt_1 \ldots t_k = \uparrow(\ldots(\sharp P(\sharp t_k))\ldots)(\sharp t_1)$
5. $\sharp\neg\phi = \sim\sharp\phi$
6. $\sharp[\phi \wedge \psi] = [\sharp\phi\ ;\ \sharp\psi]$, and similarly for the other connectives
7. $\sharp\exists x_n\phi = \mathcal{E}d_n\sharp\phi$, and similarly for the universal quantifier

We assume that the set of variables of *DPL* and the set of discourse markers of *DIL* are equinumerous, and that the same holds for the sets of constants of *DPL* and of DIL_1. Given this assumption, the clauses 1, 2, and 3 define straightforward mappings between the relevant sets of expressions. Notice that in clause 5 lifted versions of elements of $ATOM_1$ are used. The translation of the connectives and quantifiers is as is to be expected. Given our assumption of equinumerosity, a DIL_1 to *DPL* translation $\sharp'$ can be obtained by taking the inverse of $\sharp$.

In what sense are $\sharp$ and $\sharp'$ meaning preserving? This may not be immediately obvious. The meanings of *DPL*-formulae are relations between assignments. Identifying states and assignments, this means that the meanings of *DPL*-formulae correspond to objects of type $\langle s, \langle s, t\rangle\rangle$. The meanings of the expressions in DIL_1, however, are of type $\langle s, \langle\langle s, t\rangle, t\rangle\rangle$. However, as we shall see below, there is a one to one correspondence between the two.

Take a *DPL*-model M_{DPL} and a *DIL*-model M_{DIL}, based on the same set D, and with the interpretation functions F chosen so as to coincide on the individual constants and the relevant predicate constants. First, we observe the following. The variables of *DPL* and the discourse markers of *DIL* correspond one to one. This induces a one to one correspondence π between G_{DPL}, the set of *DPL*-assignments based on M_{DPL}, and S, the set of states in M_{DIL}, as follows:

$$\pi(g) = s \Leftrightarrow \text{ for all } n\colon g(x_n) = F(d_n)(s)$$

We get a correspondence between states and assignments by taking the inverse π' of π.

Next, we observe that the expressions of DIL_1, into which *DPL*-formulae translate, have some special properties.

First of all, they always denote *upward monotonic* generalized quantifiers over states. This notion is defined as follows:

Definition 16 (Upward monotonicity) A generalized quantifier Q is *upward monotonic* iff $\forall X, Y$: if $X \subset Y$ and $X \in Q$, then $Y \in Q$

And we can state the following fact:

Fact 10 Let $\Phi \in DIL_1$. Then $[\![\Phi]\!]_{M,s}$ is an upward monotonic quantifier over states

Notice that this means that if $[\![\Phi]\!]_{M,s}$ is non-empty, it will always contain as its largest element the set of all states S, which is denoted by the expression $^\wedge\mathtt{true}$. It is upward monotonicity that makes it possible to represent the truth conditional content of Φ as $\Phi(^\wedge\mathtt{true})$. Furthermore, we notice that upward monotonicity guarantees that the full set of propositions (sets of states) that is the extension of a

DIL_1-expression is already identified by its smallest elements (called its generators). We denote the set consisting of the smallest elements of $[\![\Phi]\!]_{M,s}$ as $\mathcal{G}[\![\Phi]\!]_{M,s}$.

Secondly, we observe that DIL_1-expressions have an even stronger property: the elements of $\mathcal{G}[\![\Phi]\!]_{M,s}$ are always singleton-sets, i.e., propositions which are true in exactly one state. This means that the set of propositions that is the extension of a DIL_1-expression can in fact already be identified by the union of the singleton-sets in $\mathcal{G}[\![\Phi]\!]_{M,s}$. The latter set we denote as $\mathcal{M}[\![\Phi]\!]_{M,s}$. So, the objects of type $\langle\langle s,t\rangle,t\rangle$ which serve as denotations of DIL_1 expressions are one to one related to objects of type $\langle s,t\rangle$:

Fact 11 Let $\Phi \in DIL_1$. Then: $\mathbf{p} \in [\![\Phi]\!]_{M,s} \Leftrightarrow$ there is an $s' \in \mathcal{M}[\![\Phi]\!]_{M,s}$: $s' \in \mathbf{p}$

Now we can formulate the following two facts concerning the meaning preservation properties of $\sharp$ and $\sharp'$:

Fact 12 Let $\phi \in$ *DPL*. Then: $\langle g,h\rangle \in [\![\phi]\!]_{M_{DPL}} \Leftrightarrow \pi(h) \in \mathcal{M}[\![\sharp\phi]\!]_{M_{DIL},\pi(g)}$

Fact 13 Let $\Phi \in DIL_1$. Then: $s' \in \mathcal{M}[\![\Phi]\!]_{M_{DIL},s} \Leftrightarrow \langle\pi'(s),\pi'(s')\rangle \in [\![\sharp'\Phi]\!]_{M_{DPL}}$

Together, these two facts, which we shall not prove here, imply that *DPL* and DIL_1 have the same logic. In Groenendijk & Stokhof [1989], we have discussed the logical properties of *DPL* in some detail. The mutual translatability of *DPL*-formulae and the formulae in DIL_1 guarantees that what was said there about *DPL* carries over to the DIL_1-fragment. We shall not go into details here, but just notice that, as was the case for *DPL*, three notions of entailment can be defined for DIL_1 as well. The first one is the notion of meaning inclusion:

$$\Phi \models_{mi} \Psi \Leftrightarrow [\![\Phi]\!]_{M,s,g} \subseteq [\![\Psi]\!]_{M,s,g}$$

The second notion corresponds to entailment on the level of truth-conditional content:

$$\Phi \models_{tc} \Psi \Leftrightarrow {\downarrow}\Phi \models {\downarrow}\Psi$$

The third notion is the real dynamic one:

$$\Phi \models_{dyn} \Psi \Leftrightarrow \text{ if } s' \in \mathcal{M}[\![\Phi]\!]_{M,s,g} \text{ then } [\![\Psi]\!]_{M,s',g} \neq \emptyset$$

It is the latter notion for which we have:

$$\Phi \models_{dyn} \Psi \Leftrightarrow\ \models_{dyn} \Phi \Rightarrow \Psi$$

Notice the following fact:

$$\Phi \models_{dyn} \Psi \Leftrightarrow \Phi \models_{mi} [\Phi\,;{\uparrow}{\downarrow}\Psi]$$

Finally, we observe that a fourth notion of entailment can be defined:

$$\Phi \models_{cons} \Psi \Leftrightarrow \Phi \models_{tc} [\Phi\,;{\uparrow}{\downarrow}\Psi]$$

This is a conservative notion of entailment, which corresponds to the conservative dynamic implication which is used in Chierchia [1988,1990].

6 Dynamic Montague grammar

In this section we shall outline how the expressions of a small fragment of English can be assigned a compositional dynamic interpretation through translation into *DIL*, using the definitions given in the previous section.

The fragment has as its basic categories the usual categories *IV* (intransitive verb phrases), *CN* (common noun phrases), and *S* (for (sequences of) sentences). Derived categories are of the form *A*/*B*, *A* and *B* any category. Employed in the fragment are *NP* (= *S/IV*, noun phrases), *Det* (= *NP/CN*, determiners), and *TV* (= *IV/NP*, transitive verb phrases). Furthermore, we shall make use of four syncategorematic constructions: sentence sequencing; formation of (indicative) conditionals; formation of restrictive relative clauses; and sentence negation.

First, we define a function f from categories to *DIL*-types:

Definition 17 (Category to type assignment)

1. $f(S) = \langle\langle s,t\rangle,t\rangle$; $f(CN) = f(IV) = \langle e,\langle\langle s,t\rangle,t\rangle\rangle$
2. $f(A/B) = \langle\langle s,f(B)\rangle,f(A)\rangle$

The relation with the ordinary *MG*-types will be obvious: t is replaced by $\langle\langle s,t\rangle,t\rangle$.

In the fashion of Partee & Rooth [1983] and Hendriks [1988], we might make our category-type relationship more flexible, allowing, for example, category S to be associated with type t, type $\langle\langle s,t\rangle,t\rangle$, and perhaps other types as well, all of which are related systematically by means of a small set of type-shifting principles. But in order not to introduce too much novelties at the same time, we stick to the orthodox, rigid category-type association. See Dekker [1990] for more discussion of the use of flexibility in dynamic semantics.

Among the syntactic rules of the fragment we find some rather obvious rules of functional application corresponding to the derived categories we employ. We won't spell these out here, but we only remark that the corresponding semantic rules exhibit the usual pattern: the translation of a functional expression is applied to the intension of the translation of its argument. Thus, letting α' denote the translation of α: if α is of category A/B, and β is of category B, then the result of applying functional application to them translates as $\alpha'({}^\wedge\beta')$. By the way, we notice just once more that the intension operator abstracts over states, i.e., over assignments of values to discourse markers, and not over possible worlds or other more familiar parameters.

Let us now turn to some examples of translations of basic expressions. In what follows x is a variable of type e, P and Q of type $\langle s,\langle e,\langle\langle s,t\rangle,t\rangle\rangle\rangle$, and $\mathcal{P}$ of type $\langle s,\langle\langle s,\langle e,\langle\langle s,t\rangle,t\rangle\rangle\rangle,\langle\langle s,t\rangle,t\rangle\rangle\rangle$; $\mathtt{j}$ is a constant of type e, $\mathtt{man}$ and $\mathtt{walk}$ are constants of type $\langle e,t\rangle$, and $\mathtt{love}$ of type $\langle e,\langle e,t\rangle\rangle$; the d_i are discourse markers.

Definition 18 (Translations of basic expressions)

1. $man \rightsquigarrow \lambda x{\uparrow}\mathtt{man}(x)$
2. $walk \rightsquigarrow \lambda x{\uparrow}\mathtt{walk}(x)$
3. $love \rightsquigarrow \lambda\mathcal{P}\lambda x[{}^\vee\mathcal{P}({}^\wedge\lambda y{\uparrow}\mathtt{love}(y)(x))]$
4. $a_i \rightsquigarrow \lambda P\lambda Q\mathcal{E}d_i[{}^\vee P(d_i)\,;\,{}^\vee Q(d_i)]$
5. $every_i \rightsquigarrow \lambda P\lambda Q\mathcal{A}d_i[{}^\vee P(d_i) \Rightarrow {}^\vee Q(d_i)]$
6. $no_i \rightsquigarrow \lambda P\lambda Q\mathcal{A}d_i[{}^\vee P(d_i) \Rightarrow {\sim}{}^\vee Q(d_i)]$
7. $he_i \rightsquigarrow \lambda Q[{}^\vee Q(d_i)]$
8. $John_i \rightsquigarrow \lambda Q[\{\mathtt{j}/d_i\}{}^\vee Q(d_i)]$

A few remarks. The translations 1–3 are remarkably like those we are familiar with from standard *MG*, the only difference being the occurrence of the operator $\uparrow$, which makes sure that a formula such as $\mathtt{man}(x)$, which is of type t, is raised in the appropriate way to $\lambda p[\mathtt{man}(x) \wedge {}^\vee p]$, of type $\langle\langle s,t\rangle,t\rangle$. As we have seen above, raising such a static formula makes no real difference, it just changes the type. The possible denotations of $\mathtt{man}(x)$ and $\lambda p[\mathtt{man}(x) \wedge {}^\vee p]$ are one-to-one related. So, for basic expressions of category *CN*, *IV*, and *TV*, the *DIL*-translations do not differ in any essential way from the usual ones. By the way, we also could produce the former from the latter by means of just one type-shifting rule.

As for determiners, things are slightly different. If we look at 4–6 above, the translations, thanks to our notation conventions, look pretty much like what we are used to, the main differences being that dynamic connectives occur in the place of static ones, and that we use dynamic quantifiers, introducing discourse markers, instead of the ordinary, static quantifiers. However, the most important point is that we need to chose a particular discourse marker in the translation, which is indicated by the index that occurs in the determiner itself. So, the present approach assumes that we do not translate sentences as such, but indexed structures, i.e., sentences in which determiners, pronouns, and proper names, are marked with indices. The function of the indexing mechanism is, of course, to stake out possible anaphoric relationships among constituents: if a pronoun is to be related anaphorically to an *NP*, a necessary (but not sufficient) condition is that both carry the same index.

In itself this is not entirely a new thing to do in *MG*. In more orthodox versions of *MG* such as *PTQ*, anaphoric relations are dealt with by means of quantification rules in the syntax. This involves deriving a structure which contains indexed syntactic variables, and subsequently 'quantifying in' one or more *NP*'s. And if an *NP* and a pronoun are to be related anaphorically, the underlying structure needs to contain (at least) two variables with the same index.

It may be remarked at this point that the quantification rules of orthodox *MG* serve two purposes. Beside being the mechanism with which anaphoric relations are dealt with, they are also used to account for scope ambiguities. The first role is taken over in *DMG* by the state-switchers and discourse markers. And the second role, too, can be played by a different mechanism, viz., that of type-shifting rules (see Hendriks [1988]), which means that in principle quantification rules can be dispensed with altogether. We shall leave these matters out of consideration here.

As is obvious from 8 above, translating a proper name also involves choosing an appropriate discourse marker in the translation. The value of this discourse marker is set to the individual that is denoted by the associated constant. Notice that this way of going about only deals with anaphoric relations. That proper names (like definite descriptions) sometimes also bear kataphoric relations to pronouns is not accounted for.

Finally, we turn to the translation rules which correspond to the four syncategorematic constructions we have introduced in our fragment:

Definition 19 (Translation of syncategorematic constructions)

1. Sentence sequencing: $\phi\ .\ \psi \rightsquigarrow \phi'\,;\psi'$
2. Conditional sentences: *If* ϕ*, then* $\psi \rightsquigarrow \phi' \Rightarrow \psi'$
3. Restrictive relative clauses: $\alpha_{CN} + \beta_S \rightsquigarrow \lambda x[\alpha'(x)\,;\beta']$
4. Sentence negation: *It is not the case that* $\phi \rightsquigarrow {\sim}\phi'$

Again, it may be noticed that these translations are exactly like their ordinary counterparts but for the occurrence of dynamic logical constants.

By way of illustration we shall work out a few examples in detail below. In doing so, we shall emphasize the reduction of the structurally simple and familiar translations that the rules provide, to 'basic' *DIL*-expressions, which exhibit the dynamic aspects of interpretation most clearly.

A man walks. He talks

The first example which we discuss, is the sequence of sentences *A man walks. He talks*, a slightly simplified version of our example (22), discussed in section 4. If we want to obtain a translation in which the pronoun in the second sentence is anaphorically related to the indefinite term occurring in the first one, we have to assign both the same index: A_1 *man walks.* He_1 *talks.* Functional application and some standard reduction produce the following translation of the first sentence:

$$\mathcal{E}d_1[\uparrow\mathtt{man}(d_1)\,;\uparrow\mathtt{walk}(d_1)]$$

By fact 9, $[\uparrow\phi\,;\uparrow\psi]$ is equivalent with $\uparrow[\phi\wedge\psi]$, so this can be reduced to:

$$\mathcal{E}d_1\uparrow[\mathtt{man}(d_1)\wedge\mathtt{walk}(d_1)]$$

Now we eliminate $\mathcal{E}d_1$ using definition 10:

$$\lambda p\exists x[\{x/d_1\}(\uparrow[\mathtt{man}(d_1)\wedge\mathtt{walk}(d_1)](p))]$$

Using definition 6, we replace $\uparrow[\mathtt{man}(d_1)\wedge\mathtt{walk}(d_1)]$ by $\lambda p[\mathtt{man}(d_1)\wedge\mathtt{walk}(d_1)\wedge{}^{\vee}p]$. Application of λ-conversion then gives us:

$$\lambda p\exists x[\{x/d_1\}[\mathtt{man}(d_1)\wedge\mathtt{walk}(d_1)\wedge{}^{\vee}p]]$$

Finally, we move the state switcher inwards, using the equivalences stated in fact 3 of section 3, replacing occurrences of d_1 by x. The state switcher strands in front of the last conjunct ${}^{\vee}p$:

$$\lambda p\exists x[\mathtt{man}(x)\wedge\mathtt{walk}(x)\wedge\{x/d_1\}{}^{\vee}p]$$

The ordinary first-order translation of *A man walks* can be deduced from this translation as follows. Using the operator $\downarrow$, we reduce this formula, which is of type $\langle\langle s,t\rangle,t\rangle$, to one of type t:

$$\downarrow\lambda p\exists x[\mathtt{man}(x)\wedge\mathtt{walk}(x)\wedge\{x/d_1\}{}^{\vee}p]$$

which, by means of definition 7, reduces to:

$$\lambda p\exists x[\mathtt{man}(x)\wedge\mathtt{walk}(x)\wedge\{x/d_1\}{}^{\vee}p]({}^{\wedge}\mathtt{true})$$

Of course, this formula no longer represents the full dynamic meaning of the sentence in question, but only its static meaning, i.e., its truth conditional content. This is easy to see, since λ-conversion, ${}^{\vee\wedge}$-elimination, and the state independent meaning of the constant `true` guarantee that this formula is equivalent to the usual translation:

$$\exists x[\mathtt{man}(x)\wedge\mathtt{walk}(x)]$$

Let us now turn to the second sentence in the sequence, *He talks.* This second sentence has as its reduced translation:

$$\uparrow\mathtt{talk}(d_1)$$

The translation rule for sentence sequences tells us that the translation of the entire sequence is:

$$\mathcal{E}d_1[\uparrow\mathtt{man}(d_1)\,;\uparrow\mathtt{walk}(d_1)]\,;\uparrow\mathtt{talk}(d_1)$$

From fact 7 in section 5, we know we can write this as:

$$\mathcal{E}d_1[\uparrow\mathtt{man}(d_1)\,;\uparrow\mathtt{walk}(d_1)\,;\uparrow\mathtt{talk}(d_1)]$$

But, for the sake of illustration, we work out this step in detail. First, we take the dynamic conjunction of the reduced representation of the first sentence and that of the second sentence:

$$\lambda p\exists x[\mathtt{man}(x)\wedge\mathtt{walk}(x)\wedge\{x/d_1\}{}^{\vee}p]\,;\uparrow\mathtt{talk}(d_1)$$

Eliminating the dynamic conjunction by definition 9, we get:

$$\lambda q[\lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}{}^{\vee}p]({}^{\wedge}(\uparrow \mathtt{talk}(d_1)(q)))]$$

Application of λ-conversion and ${}^{\vee\wedge}$-elimination gives:

$$\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}(\uparrow \mathtt{talk}(d_1)(q))]$$

Notice that λ-conversion is allowed in this case because the argument is intensionally closed. Elimination of $\uparrow$ and another application of λ-conversion gives:

$$\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}[\mathtt{talk}(d_1) \wedge {}^{\vee}q]]$$

Moving the state switcher inside the conjunction we get as a final result:

$$\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \mathtt{talk}(x) \wedge \{x/d_1\}{}^{\vee}q]$$

Again, if we apply this formula to ${}^{\wedge}\mathtt{true}$, we get the ordinary first-order translation of this sequence. However, we did obtain this result in a completely compositional way.

If a man walks, he talks

As our second example, we choose the donkey-conditional which consists of the two sentences which constitute our first example. Using the reductions made above, and the translation rule for conditional sentences, we get:

$$\lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}{}^{\vee}p] \Rightarrow \lambda p[\mathtt{talk}(d_1) \wedge {}^{\vee}p]$$

We rewrite this using definition 11, which defines $\Phi \Rightarrow \Psi$ as $\sim[\Phi\,;\sim\Psi]$. First we take a look at the negation of the consequent:

$$\sim\lambda p[\mathtt{talk}(d_1) \wedge {}^{\vee}p]$$

Using the fact that $\sim\Phi$ is equivalent with $\lambda p[\neg\Phi({}^{\wedge}\mathtt{true}) \wedge {}^{\vee}p]$, and applying some standard reductions, this can be rewritten as:

$$\lambda p[\neg\mathtt{talk}(d_1) \wedge {}^{\vee}p]$$

Taking this result in dynamic conjunction with the antecedent, we arrive at:

$$\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \neg\mathtt{talk}(x) \wedge \{x/d_1\}{}^{\vee}q]$$

All we need to do now, is to form the negation of this subresult. Rewriting the negation according to definition 8 gives us:

$$\lambda p[\neg[\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \neg\mathtt{talk}(x) \wedge \{x/d_1\}{}^{\vee}q]({}^{\wedge}\mathtt{true})] \wedge {}^{\vee}p]$$

The constant $\mathtt{true}$ can be eliminated, and the final result then is:

$$\lambda p[\neg\exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \neg\mathtt{talk}(x)] \wedge {}^{\vee}p]$$

Notice that the state switcher has disappeared altogether. So, should we continue this conditional with a sentence containing the pronoun he_1, the latter would not be anaphorically related to the indefinite term in the conditional. This seems to borne out by the facts, for a sequence such as:

(37) If a man walks, he talks. He waves.

can not be interpreted in such a way that the pronoun in the second sentence is anaphorically linked to the indefinite term in the first sentence. (However, see section 7 for some more discussion of this matter.)

Finally, we remark that the translation given above is equivalent with:

$$\lambda p[\forall x[[\mathtt{man}(x) \wedge \mathtt{walk}(x)] \rightarrow \mathtt{talk}(x)] \wedge {}^{\vee}p]$$

Consequently, the truth conditional content of our example is represented in the standard way, which we obtain by application of the $\downarrow$-operator:

$$\forall x[[\mathtt{man}(x) \wedge \mathtt{walk}(x)] \rightarrow \mathtt{talk}(x)]$$

It is easily checked that the sentence *Every man who walks, talks*, which has its direct translation:

$$\mathcal{A}d_1[[\uparrow\mathtt{man}(d_1)\,;\uparrow\mathtt{walk}(d_1)] \Rightarrow \uparrow\mathtt{talk}(d_1)]$$

reduces to the same formula, and that hence the equivalence of this sentence with the example we have just treated, is accounted for.

Every farmer who owns a donkey, beats it

Although the previous example has already shown how donkey-type anaphora are dealt with in *DMG*, we shall also show how the classical donkey-sentence *Every farmer who owns a donkey, beats it* is derived. We use the following indexing: *Every*$_1$ *farmer who owns a*$_2$ *donkey, beats it*$_2$.

Applying some reductions, we get as the translation of *own a*$_2$ *donkey*:

$$\lambda x \mathcal{E}d_2[\uparrow\mathtt{donkey}(d_2)\,;\uparrow\mathtt{own}(d_2)(x)]$$

Using our translation rule for restrictive relative clauses, we get as translation of the complex CN *farmer who owns a donkey*:

$$\lambda x[\uparrow\mathtt{farmer}(x)\,;\mathcal{E}d_2[\uparrow\mathtt{donkey}(d_2)\,;\uparrow\mathtt{own}(d_2)(x)]]$$

The reduced translation of the entire subject term *every*$_1$ *farmer who owns a*$_2$ *donkey* then is:

$$\lambda Q \mathcal{A}d_1[[\uparrow\mathtt{farmer}(d_1)\,;\mathcal{E}d_2[\uparrow\mathtt{donkey}(d_2)\,;\uparrow\mathtt{own}(d_2)(d_1)]] \Rightarrow {}^{\vee}Q(d_1)]$$

The intransitive verb phrase *beat it*$_2$ translates simply as:

$$\lambda x[\uparrow\mathtt{beat}(d_2)(x)]$$

Combining the translation of the subject term with that of the verb phrase by functional application, gives the following translation for the sentence as a whole:

$$\mathcal{A}d_1[[\uparrow\mathtt{farmer}(d_1)\,;\mathcal{E}d_2[\uparrow\mathtt{donkey}(d_2)\,;\uparrow\mathtt{own}(d_2)(d_1)]] \Rightarrow \uparrow\mathtt{beat}(d_2)(d_1)]$$

As we know from fact 8 in section 5, an occurrence of $\mathcal{E}d_i$ in the antecedent of a conditional, can be replaced by an occurrence of $\mathcal{A}d_i$ which has scope over the conditional as a whole. This means that we get:

$$\mathcal{A}d_1\mathcal{A}d_2[[\uparrow\mathtt{farmer}(d_1)\,;\uparrow\mathtt{donkey}(d_2)\,;\uparrow\mathtt{own}(d_2)(d_1)] \Rightarrow \uparrow\mathtt{beat}(d_2)(d_1)]$$

The occurrences of $\uparrow$ can be moved outwards (fact 9), changing the dynamic connectives into static ones:

$$\mathcal{A}d_1\mathcal{A}d_2\uparrow[[\mathtt{farmer}(d_1) \wedge \mathtt{donkey}(d_2) \wedge \mathtt{own}(d_2)(d_1)] \rightarrow \mathtt{beat}(d_2)(d_1)]$$

Elimination of $\uparrow$ results in:

$$\mathcal{A}d_1\mathcal{A}d_2\lambda p[[[\mathtt{farmer}(d_1) \wedge \mathtt{donkey}(d_2) \wedge \mathtt{own}(d_2)(d_1)] \rightarrow \mathtt{beat}(d_2)(d_1)] \wedge {}^{\vee}p]$$

Now we use the equivalence of $\mathcal{A}d\Phi$ with $\lambda p[\forall x\{x/d\}\Phi({}^{\wedge}\mathtt{true}) \wedge {}^{\vee}p]$, and reduce this, through a number of standard reductions, to:

$$\lambda p[\forall x \forall y\{x/d_1\}\{y/d_2\}[[\mathtt{farmer}(d_1) \wedge \mathtt{donkey}(d_2) \wedge \mathtt{own}(d_2)(d_1)] \rightarrow \mathtt{beat}(d_2)(d_1)] \wedge {}^{\vee}p]$$

Moving the two state-switchers inwards gives us as a final result:

$$\lambda p[\forall x \forall y[[\mathtt{farmer}(x) \wedge \mathtt{donkey}(y) \wedge \mathtt{own}(y)(x)] \rightarrow \mathtt{beat}(y)(x)] \wedge {}^{\vee}p]$$

Notice that the last conjunct ${}^{\vee}p$ was *not* in the scope of the two state-switchers. Consequently, these have disappeared, and so have the two discourse markers. Hence, the terms in this sentence will not be able to bear any anaphoric relations to pronouns in sentences to come. This seems to be borne out by the facts. (But, again, we refer to the next section for some discussion.)

This example concludes our exposition of the fragment. What we have shown in detail is that the two central phenomena of cross-sentential anaphora and donkey-sentences, can be treated in *DMG* in an adequate and completely compositional way. This means that *DMG* is indeed a semantic theory that unifies important insights from *MG* and *DRT*.

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The version of *DMG* presented above, has roughly the same empirical coverage as classical *DRT* (and *DPL*). The main difference is that in *DMG* the relevant phenomena are dealt with in a compositional manner. The fragment does not cover all of classical *MG (PTQ)*, since it lacks intensionality in the ordinary sense, and, unlike *PTQ*, it uses the higher-order facilities of *DIL* only in building up translations of sentences in a compositional way. The (sequences of) sentences in our fragment are all first-order and extensional: their translations are (equivalent to) expressions of the DIL_1-fragment discussed at the end of section 5. The higher-order nature of *DIL* serves compositionality only, and the intensional nature of *DIL* serves only the dynamics of the semantics.

Extending the fragment to cover all of *PTQ* is a standard exercise. What is less standard, is to extend the quantificational apparatus and anaphoric reference of *DMG* beyond type e. Extensions along these lines will not be pursued here (see Groenendijk & Stokhof [in prep]). But, in the next section we shall indicate that, even restricting ourselves to the first-order level of quantification and anaphoric reference, *DMG* is potentially more than just the sum of *MG* and *DRT*.

7 Extending the dynamics

In this final section, we shall explore the potential of *DMG* a little further. The reader should be warned that we regard much of what follows as rather tentative. The analyses proposed are meant as illustrations of the possibilities inherent in *DMG* rather than as final analyses of the phenomena in question.

A feature that the version of *DMG* presented above inherits from *DRT* and *DPL* is that negation is treated in a static way. Given the notion of negation defined in definition 8, it holds that once a sentence is negated, all anaphoric links between terms occurring in the sentence and pronouns 'outside' are blocked.

Now, it seems that this is corroborated by the facts. Consider the following examples:

(38) It is not the case that a man walks in the park. He whistles.

(39) No man walks in the park. He whistles.

In these examples, it is not possible, indeed, to interpret the pronoun in the second sentence as being anaphorically linked to the term in the first sentence.

However, certain observations can be made which seem to point in a different direction. Consider double negation. Under the present notion of negation, double negation does not restore any dynamic effects a sentence may have had. Again, this seems warranted by the facts. If we negate the first sentences in (38) and (39) once more, it is still not possible to interpret the pronoun in the second sentence as being anaphorically linked to the term in the first sentence:

(40) It is not the case that it is not the case that a man walks in the park. He whistles.

(41) It is not the case that no man walks in the park. He whistles.

Yet, other examples of double negation point in another direction: (42) and (43) do seem to express the same thing:

(42) It is not the case that John doesn't own a car. It is red and it is parked in front of his house.

(43) John owns a car. It is red and it is parked in front of his house.

These examples suggest that at least in cases such as these, a sentence and its double negation are fully equivalent, i.e., that they do not just have the same truth conditions, but also the same dynamic properties. This seems to call for a non-static notion of negation for which the law of double negation does hold.

It may be worthwhile, though, to briefly point out another line of reasoning, which does not strike us as altogether untenable. It seems possible to argue that in (42) the pronoun *it* is not directly anaphorically related to the indefinite term *a car* in the preceding sentence, but only indirectly, mediated through a unique object, the existence of which can be inferred from the previous discourse and the context. Uniqueness is supposed to be all important here. For the example in question, uniqueness can be supposed to be guaranteed by the context, viz., by some kind of default assumption that associates a unique car with every person. That uniqueness is necessary may be argued by pointing out other examples, such as (40) and (41) presented above, or the following example, which is structurally the same as (42):

(44) It is not the case that John doesn't own a book. It is lying on his desk.

Observe that an interpretation of the pronoun *it* in the second sentence as being anaphorically linked to the indefinite term *a book* in the first one, seems possible only if we assume that we are talking about a specific book, say the text-book used in a particular course, of which, again by default, we may assume that every participant has a unique copy. In a neutral context, however, an anaphoric interpretation does not seem to be possible, and this can be explained by pointing out that a default assumption that associates a unique book with each person is rather unrealistic.

Although an account along these lines is attractive, it remains to be seen whether it will stand up in the end. Clearly, it needs to be investigated further. However, in the remainder we shall investigate the other, more direct way of tackling these problems.

As was already remarked above, disjunction, implication and universal quantification inherit the static character of negation, in terms of which they are defined: they all become externally static, and disjunction becomes internally static as well. In these cases, too, positive empirical evidence may be adduced, but, as we shall see, there is also evidence to the contrary. We shall discuss examples of each for the various cases, and we shall propose new dynamic definitions of these expressions. In each case, the old definition is obtained by taking the static closure of the new one. This will extend the dynamics of the fragment in a conservative way, resulting in a system which has greater empirical coverage. However, we shall also see that we need to impose restrictions in order to keep it from overgenerating.

Consider the sequence:

(45) Every man walks in the park. He whistles.

The pronoun *he* in the second sentence can not be interpreted as an anaphor which is bound by the universal term *every man* in the first sentence. And the externally static character of the universal quantifier and implication nicely accounts for this.

However, there are certain kinds of discourses in which anaphoric relations of the indicated kind are possible. Consider the following example:

(46) Every player chooses a pawn. He puts it on square one.

We can give a translation of this sequence in *DIL* as it stands which has the intended interpretation. However, such a translation can not be arrived at in a compositional way. The following formula expresses the right meaning:

(47) $\mathcal{A}d_1[\texttt{player}(d_1) \Rightarrow [\mathcal{E}d_2[\texttt{pawn}(d_2)\,;\texttt{choose}(d_1,d_2)]\,;$
$\texttt{put\_on\_square\_one}(d_1,d_2)]]$

The universal and the existential existential quantifier bind the rightmost occurrences of the discourse marker alright, but this translation is *not* the dynamic conjunction of the translations of the two sentences in the sequence. Hence, this analysis is not adequate. It constitutes a flagrant violation of compositionality.

Compositionality dictates that we translate sequence (46) by forming the dynamic conjunction of the translations of the two component sentences, which would give us:

(48) $\mathcal{A}d_1[\texttt{player}(d_1) \Rightarrow \mathcal{E}d_2[\texttt{pawn}(d_2)\,;\texttt{choose}(d_1,d_2)]]\,;$
$\texttt{put\_on\_square\_one}(d_1,d_2)$

But given the definitions of universal quantification and implication we have used sofar, formula (48) will not receive a correct interpretation: the discourse markers in the second conjunct are not by the quantifiers in the first conjunct. This leaves us with the task of finding new externally dynamic interpretations of implication and universal quantification which would make (48) equivalent with (47).

We should remark that once we have found such interpretations, we can account for sequences of sentences such as (46), but we can not at the same time explain why an example such as (45) does not allow for such an analysis. In a correct translation of (45) we would have to stick to the externally static notions of implication and universal quantification defined in section 5, blocking the intended anaphoric reference. Intuitively, the different behaviour with respect to antecedent-anaphor relations of examples such as (45) and (46), which are structurally the same, has something to do with the fact that they constitute different types of discourses. Although this is certainly not the whole story, one can observe that (46) is most naturally interpreted as an instructive discourse, whereas (45) is not. If we do impose an instructive interpretation on (45), e.g., by reading it as a stage direction, the intended anaphoric reference, indeed, becomes possible. However, in the present paper we are not concerned with the issue of what properties of discourse are relevant in this respect. Our purpose here is only to develop a semantic theory which has enough expressive power to handle both kinds of cases.

There are also well-known examples which suggest that disjunction should receive a dynamic interpretation:

(49) Either there is no bathroom here, or it is in a funny place.

Notice that this disjunction is equivalent with the conditional:

(50) If there is a bathroom here, it is in a funny place.

Now, (50) is a donkey-type conditional, which, as was shown above, is dealt with adequately in the framework as it stands, but remarkably enough, the equivalent donkey-disjunction (49) is not.

To sum up, although in many cases a static treatment of negation, implication, disjunction and universal quantification seems to be correct, there are other cases which seem to require a dynamic treatment of these expressions. In the limited set-up of *DRT* and *DPL* it is far from clear how such a treatment can be achieved without major alterations of the frameworks involved. However, in *DMG* a treatment of these cases is easily forthcoming.

For, up to now, we have used the dynamic potential of *DIL* only to a limited extent. For example, in *DIL* it is easy to define, besides the dynamic existential quantifier, a dynamic universal quantifier as well. The following definition naturally suggests itself:

Definition 20 (Dynamic universal quantifier) $\mathcal{A}d\Phi = \lambda p \forall x[\{x/d\}(\Phi(p))]$

We notice the following two facts:

Fact 14 $\uparrow\downarrow\mathcal{A}d\Phi = \lambda p[\forall x[\{x/d\}(\Phi({}^\wedge\mathtt{true}))] \wedge {}^\vee p]$

Fact 15 $\mathcal{A}d\Phi \,;\, \Psi = \mathcal{A}d[\Phi \,;\, \Psi]$

The first fact expresses that if we take the static closure $\uparrow\downarrow\mathcal{A}d\Phi$ of $\mathcal{A}d\Phi$ as defined in definition 20, we end up with the old definition 13, given in section 5. So, the old static version of the universal quantifier can be obtained from the new dynamic one by taking its standard static closure. This is important, for it means that only one notion of universal quantification is involved. Leaving it dynamically open or taking its static closure gives us the two variants which we need in different types of discourse.

The second fact expresses the dynamic binding properties of this notion of universal quantification. Like its existential counterpart, the universal quantifier can bind pronouns which are outside its syntactic scope.

We note in passing, that if we add the new dynamic universal quantifier to the fragment DIL_1 of *DIL* as it was defined in section 5, it will remain the case that every expression in DIL_1 denotes an upward monotonic generalized quantifier over states, but it no longer holds that its generators are always singleton-sets. The latter fact means that by adding dynamic universal quantification, the close connection between DIL_1 and *DPL* is lost. Dynamic universal quantification lies essentially outside the expressive power of *DPL*, but it is there in *DIL*, ready to be used.

The connection between the dynamic existential quantifier defined in definition 11 in section 5, and the dynamic universal quantifier defined above, can be expressed in terms of negation in the usual way, if we give negation, too, a dynamic interpretation. In the following definition, negation is defined standardly as complementation:

Definition 21 (Dynamic negation) $\sim\Phi = \lambda p \neg(\Phi(p))$

Here, too, it holds that the old definition 8 of static negation can be obtained from the dynamic one, by taking its standard static closure:

Fact 16 $\uparrow\downarrow\sim\Phi = \uparrow\neg\downarrow\Phi$

Further, the law of double negation holds, which enables us to account for the equivalence of examples such as (42) and (43):

Fact 17 $\sim\sim\Phi = \Phi$

Given the old static notion of negation, static universal quantification can be defined in terms of existential quantification, but not the other way around. In terms of the new dynamic notion of negation, they are fully interdefiniable in the customary way:

Fact 18 $\mathcal{E}d{\sim}\Phi = {\sim}\mathcal{A}d\Phi$

There is another feature of the notion of dynamic negation that we have to point out. It can easily be seen that the following holds:

Fact 19 ${\sim}\Phi \mathbin{;} \Psi = {\sim}[\Phi \mathbin{;} \Psi]$

Obviously, this is not something that we want in general. It predicts that if we translate sentence negation as dynamic negation, the negation in that sentence extends to sentences that follow it in the discourse. In order to prevent this, we should close off a negated sentence, i.e., we should use static negation. On the other hand, if the sentence that is negated is itself a negation, we do have to be able to treat both negations dynamically, for otherwise we can't get a correct account of double negation examples such as (42).

This poses the following problem. Why, and when, should ${\sim}\Phi$ be ruled out as a correct translation of a negated sentence? In other words, why, and when, do we have to choose its static closure $\uparrow\downarrow{\sim}\Phi$? An answer is suggested by the following observations.

In section 4, we saw that the denotation of a sentence, being an object of type $\langle\langle s,t\rangle,t\rangle$, can be viewed as a generalized quantifier over states, or, equivalently, as a set of propositions. In a state s, the denotation of a sentence consists of those propositions which are true after the sentence has been processed in s. Intuitively, if a proposition is true of the situation which results after a sentence has been processed, then any weaker proposition will be true in that situation as well. In particular, if any proposition is true in the resulting state, then the proposition $^\wedge\mathtt{true}$ will be true in it, which expresses that a possible state results after processing. What this suggests is that if a generalized quantifier over states is to serve as a possible denotation for a sentence, it is to be upward monotonic.

Another way of looking at it is the following. For any proper translation Φ of a sentence at the discourse level, it should hold that the truth conditional content of Φ continued with Ψ, is at least as strong as that of Φ itself: $\downarrow[\Phi \mathbin{;} \Psi] \models \downarrow\Phi$. This amounts to requiring that every step in a discourse is always a real update, and can never constitute a weakening of the truth conditional content of the discourse up to that point.

What these observations suggest is that we impose the following general constraint: the translation of each sentence which constitutes a separate step in the discourse should denote an upward monotonic quantifier over states.

Imposing this requirement has the desired effects in the cases considered above, as can be seen as follows. The notion of dynamic negation defined above reverses the monotonicity of its argument. If Φ is upward monotonic, then ${\sim}\Phi$ is downward monotonic. Its double negation ${\sim}{\sim}\Phi$ is upward monotonic again. On the other hand, the static closure of its negation $\uparrow\downarrow{\sim}\Phi$ is upward monotonic. Closure always results in an upward monotonic quantifier, even if its argument is downward monotonic. Generally, ${\sim}\Phi$ will be a proper translation of a step in the discourse only if Φ is downward monotonic.

Notice that we impose the monotonicity requirement not generally on all sentence translations, since if we would, we could not account for the double negation facts. On the other hand, it seems that we need to apply the monotonicity constraint sometimes also below the discourse level, i.e., to sentential constructions which are a proper subpart of a step in a discourse. An example is the sentential expression which is used to form a relative clause. Such a sentence must be required to denote an upward monotonic quantifier, since otherwise a negation occurring inside a relative clause might get wide scope over expressions outside of it. Another example is the consequent of a conditional sentence, a case which will be discussed below.

This suggests that the monotonicity constraint can be regarded as as a construction sensitive constraint on translations. Certain constructions, notably discourse sequencing, but also relative clause formation, and the formation of conditional sentences, invoke the constraint. (We note that the fragment given in the previous section fully complies with the monotonicity constraint, since the resulting translations of its (sequences of) sentences are all in the DIL_1-fragment.)

Let us now consider dynamic implication. Unlike the notion of implication defined in definition 11 in section 5, the notion defined below is not only internally dynamic, but also externally dynamic:

Definition 22 (Dynamic implication) $\Phi \Rightarrow \Psi = \sim[\Phi\,;\sim\Psi]$

As was the case for the new notions of negation and universal quantification, the old static version of implication can be obtained by static closure of the new dynamic one. Further we note the following fact, which illustrates the externally dynamic character of implication given the new definition:

Fact 20 $[\Phi \Rightarrow \Psi]\,;\Upsilon = \Phi \Rightarrow [\Psi\,;\Upsilon]$

In view of this fact, it holds that a sequence consisting of a conditional followed by another sentence can be interpreted in such a way that the sentence following the conditional, is interpreted as a conjunct of the consequent of the conditional. This is what is needed to deal with examples like the following:

(51) If a client comes in, you treat him politely. You offer him a cup of coffee.

Notice that the discourse in this example is again of an instructive nature.

Concerning monotonicity, we note the following. Consider the following example:

(52) If a client comes in, you don't put him off. You offer him a cup of coffee.

The consequent of the first sentence is the negation of an atomic sentence. If we would translate the negation dynamically, its scope would extend to the second sentence in the discourse, which is obviously incorrect. We can avoid this by requiring that the translation of the consequent of a conditional sentence comply with the monotonicity constraint, i.e., that it denote an upward monotonic quantifier over states. Consequently, the translation of (52) should be of the form: $[\Phi \Rightarrow \uparrow\downarrow\sim\Psi]\,;\Upsilon$. This has the required effects. In view of fact 20, the second sentence of (52) semantically still turns up as a conjunct inside the consequent of the first sentence, but not inside the negation that occurs there.

In conjunction with fact 15, fact 20 implies that we have:

Fact 21 $\mathcal{A}d[\Phi \Rightarrow \Psi]\,;\Upsilon = \mathcal{A}d[\Phi \Rightarrow [\Psi\,;\Upsilon]]$

And on this fact an account of examples such as (46) can be based.

Considerations concerning monotonicity are also relevant for an example such as:

(53) No player leaves the room. He stays were he is.

If we want to account for the coreference and at the same time comply with the monotonicity requirement, we need a translation of the form $\mathcal{A}d[\Phi \Rightarrow \uparrow\downarrow\sim\Psi]\,;\Upsilon$.

Using the new dynamic notion of negation, we can also define a new dynamic notion of disjunction:

Definition 23 (Dynamic disjunction) **either** Φ **or** $\Psi = {\sim}[{\sim}\Phi \,;\, {\sim}\Psi]$

In this case, it does not hold that the old version of disjunction defined in definition 12 in section 5 can be obtained by static closure of the new notion defined above. (This is why we introduce a new phrase to denote this notion.) Below, we shall see that there is an alternative notion of disjunction for which this does hold.

We note that the following equivalences hold:

Fact 22 **either** Φ **or** $\Psi = {\sim}\Phi \Rightarrow \Psi$

Fact 23 **either** ${\sim}\mathcal{E}d\Phi$ **or** $\Psi = \mathcal{E}d\Phi \Rightarrow \Psi = \mathcal{A}d[\Phi \Rightarrow \Psi]$

This means that the extended dynamic semantics is also able to deal with a donkey disjunction such as (49), and in such a way that it is equivalent with the donkey conditional (50). It can further be noted that this notion of dynamic disjunction is like dynamic implication in this respect that sentences which follow it, are in fact conjoined with its second argument:

Fact 24 [**either** Φ **or** Ψ] $;\, \Upsilon =$ **either** Φ **or** $[\Psi \,;\, \Upsilon]$

This is needed for a treatment of examples such as:

(54) Either there is no bathroom here, or it is in a funny place. In any case, it is not on the ground floor.

So, the dynamic disjunction defined in definition 23 enables us to account for anaphoric relations between terms in the first disjunct and pronouns in the second, and between terms in the second disjunct and pronouns in subsequent sentences. It is both internally and externally dynamic.

Beside the notion of dynamic disjunction just defined, there is another obvious candidate, which is the union operation on the level of type $\langle\langle s,t\rangle,t\rangle$. It is this second notion of dynamic disjunction for which it holds that its static closure is equivalent to the old static notion of disjunction. Notice also that disjunctions of this kind need to take upward monotonic arguments to meet the monotonicity requirement:

Definition 24 (Externally dynamic disjunction)
Φ **or** $\Psi = \lambda p[\Phi(p) \vee \Psi(p)]$

In a similar fashion, we may define an alternative notion of conjunction, which comes down to intersection:

Definition 25 (Externally dynamic conjunction)
Φ **and** $\Psi = \lambda p[\Phi(p) \wedge \Psi(p)]$

The notions of conjunction and disjunction which 'underly' the dynamic universal and existential quantifiers, are these externally dynamic notions.

Unlike the dynamic disjunction defined in definition 23, the notion defined in definition 24 is only externally dynamic. It does not allow dynamic quantifiers in the first disjunct to bind discourse markers in the second disjunct. But it does allow quantifiers in either disjunct to bind discourse markers in subsequent formulae. Using the externally dynamic disjunction, sentences which follow are in fact treated as conjoined with each of the disjuncts:

Fact 25 $[\Phi$ **or** $\Psi] \,;\, \Upsilon = [\Phi \,;\, \Upsilon]$ **or** $[\Psi \,;\, \Upsilon]$

The following sequence exhibits this pattern:

(55) A professor or an assistant professor will attend the meeting of the university board. She will report to the faculty.

So, there seems to be a rôle to play for externally dynamic disjunction as well.

The externally dynamic notion of conjunction behaves like the externally dynamic disjunction, as the following fact shows:

Fact 26 $[\Phi \textbf{ and } \Psi] ; \Upsilon = [\Phi ; \Upsilon] \textbf{ and } [\Psi ; \Upsilon]$

We do not provide examples in this case, since that would mean that we would have to introduce plurality, which we want to avoid here.

The reader may now have lost oversight over the various notions introduced and their interrelations, so we sum up what is available systematically. We have defined the following logical operators: $\mathcal{E}$, $\mathcal{A}$, $\sim$, ; , $\Rightarrow$, **either**. . . **or**. . . , **or**, and **and**. A minimal set which allows us to derive all of these, by means of standard definitions, should contain: one of the quantifiers $\mathcal{E}$ and $\mathcal{A}$; $\sim$-negation; one of the connectives ; , $\Rightarrow$, and **either**. . . **or**. . . ; and one of the connectives **or** and **and**. Each of these dynamic logical constants can be turned into a static one by means of static closure with $\uparrow\downarrow$. For $\mathcal{A}$, $\sim$, $\Rightarrow$, and **or**, what we get are the notions defined in section 5.

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The extended dynamics presented above enables us to account for a wide range of facts. Two observations need to made however.

First of all, as we already remarked above, the system as such does not explain why in certain contexts we can (or must) use the dynamic versions of the various operators, whereas in other cases we need to use their static closures. It remains to find out which textual and contextual factors determine this. But clearly, this is a well-defined, by and large empirical question which further investigation may very well answer.

Second, we have proposed to control the dynamics of the resulting system by subjecting its use in the translation of natural language to the monotonicity constraint. For several reasons we regard this as a temporary measure. It needs to be investigated further whether the constraint itself is sufficiently strong. A more principled objection is that one would prefer to restrict the logical system itself rather than the use that is made of it. This requires, at least, a different notion of negation which by itself would ensure that the monotonicity effects come out as they should. In fact, such an analysis is available, it is presented in Dekker [1990].

Finally, it will not have escaped the reader's notice that the discussion in this paper has been directed towards the development of adequate tools, rather than towards their employment in the development of extensive and empirically adequate analyses. For one thing, we have discussed only a carefully restricted set of facts. Also, no attempt has been made to say anything about the incorporation of definites, about the role of plurals, and so on, and so forth. However, some substantial work on these matters has been carried out in (modifications of) the framework presented here (see, e.g., van de Berg [1989], Dekker [1990], Chierchia [1988,1990]). We take this as indicating that it is the present paper that is limited in its empirical scope, rather than the ideas it has tried to convey.

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