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THE SEMANTICS OF PLURAL NOUN PHRASES

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0 Introduction

In this paper, two major approaches to plural NPs shall be reviewed in some detail: Scha (1981) and Link (1984;1987). We aim to show that their analysis of sentences like:

(1) Two girls ate five sandwiches
(2) All boys crossed the street

can be improved upon. Disregarding the issue of scope ambiguity — which is a matter quite separate from finding a proper semantics for plural NPs, and which we shall thus ignore whenever possible, — both Scha and Link attach different readings to (1): Scha nine, and Link four. In contrast, our semantics takes it to be unambiguous. We are prepared to speak about a collective and a distributive reading, but only as a conceptually handy way to distinguish between two types of situation which, among other types, may verify sentence (1). In our opinion, (1) is vague as to being collective, distributive or otherwise.

Scha will be discussed in section 1, Link in section 2, and in section 3 we shall present our own semantics for plural NPs. It is characterized by the following properties:

- Sentences get a unique meaning in a compositional way in which the subject NP has the object NP within its scope;
- The semantics works for arbitrary NPs;
- All NPs are of the same type and no NP is structurally ambiguous between a distributive and a collective reading;

- We shall restrict ourselves to NPs containing count nouns, which allows us to stick to standard type-logical techniques rather than using an algebraic semantics along the lines of Link (1983).

As said, we shall not deal with quantifier scope ambiguities and neither shall we treat nouns like *group* or *committee*, which ask for a more general treatment (cf. Landman 1990).

1 Calculations on Scha

The well-deserved reputation of Scha (1981) still suffers from widespread misinterpretation, or so we think. Part of this may be due to the non-standard means used by Scha to express his Montegovian views. In this section, we shall have a dialogue with Scha in order to attain a precise characterization of his semantics for plural NPs.[1]

Scha operates on the basis of the following three assumptions:

- NPs like *four men* and *three tables* denote sets of sets of sets. Typelogically they are of type $<<<e,t>,t>,t>$;
- the scoping of quantifiers corresponds to the order of the NPs in the surface structure of a sentence;[2]
- an NP like *four men* has three readings: a distributive reading D and two collective readings C1 and C2.

Thus, sentences of the form NP_1 V NP_2 such as

(3) Four men lifted three tables

are predicted to have nine readings as shown in Figure 1.

C1C1	C1C2	C1D
C2C1	C2C2	C2D
DC1	DC2	DD

Figure 1

Besides these nine, there is a tenth, so-called cumulative reading in which the interpretation of subject and object NP are mutually independent.[3] Note furthermore that the proliferation of readings is induced in particular by discerning among C1 and C2.

The C1 reading of four men is given in (4):[4]

(4) $\lambda P_{<<e,t>,t>}.\exists X_{<e,t>} \in \{Y_{<e,t>} \subseteq \cup men^*: |Y| = 4\}: P(X)$

Here X and Y are variables of type $<e,t>$: their values are sets, just like the denotation of *men*; the variable P is of type $<<e,t>,t>$: denoting a set of sets. Consequently, the NP represented by (4) is of type $<<<e,t>,t>,t>$: a set of sets of sets. The set men* = $\{\{m\} \mid m \in men\}$, the set of all singletons containing a man. So $\cup$men* is identical to the set *men*. Applied to an expression α of type $<<e,t>,t>$, (4) yields $\exists X \in \{Y \subseteq \cup men^*: |Y| = 4\}: \alpha(X)$ by lambda-conversion. This says that there is a four-membered set X of men to which α applies. In particular, *walk* in *Four men walked* should be treated as denoting a semantic object of type $<<e,t>,t>$ (cf. Bartsch 1973, Bennett 1974, Hausser 1974, Verkuyl 1981, Hoeksema 1983, among others for a similar approach). Modelling groups by means of sets, the C1 reading of *Four men walked* will then be that a group of four men walked.

The C2 reading of four men is given in (5):

(5) $\lambda P_{<<e,t>,t>}.|\cup\{Y_{<e,t>} \subseteq \cup men^*: P(Y)\}| = 4$

This represents *four men* in an interpretation of sentence (3) in which, e.g., three men lifted two tables as did one man. If (5) is applied to an expression α of type $<<e,t>,t>$, the resulting expression is interpreted as: the union of the sets of men for which the predicate α holds, has four members. Link (1984:21) calls this the partitional reading, which is inaccurate, for the sets of men involved may plainly overlap.

The distributive reading of *four men* is given in (6):

(6) $\lambda P_{<<e,t>,t>}. |\cup\{X_{<e,t>} \in men^*: P(X)\}| = 4$

Now the application to an $<<e,t>,t>$-typed expression α yields: there are exactly four 'man-membered' singletons for which α holds. Since singletons are intuitively identified with their members, one obtains the standard distributive reading.

It is quite illustrative to represent Scha's strategy in the form of the diagram in Figure 2.

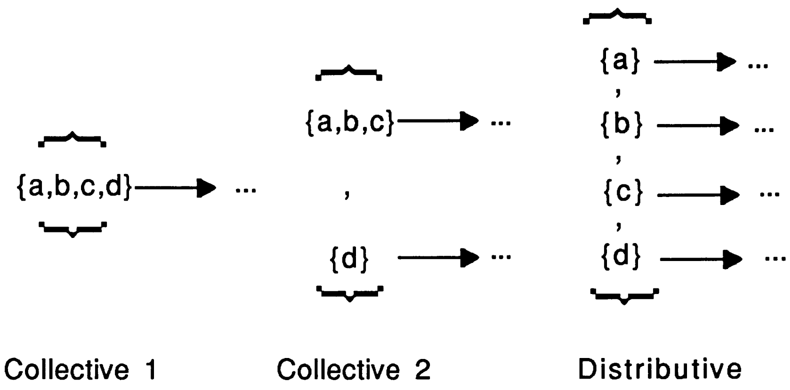

Figure 2

The arrow is used to indicate which sets are involved in a particular predication. Figure 2 makes plain that the distributive (D) and the Collective 1 (C1) predications are extreme cases of the Collective 2 (C2) predications: C1-predication selects only the unit set, D-predication takes singletons, while C2-predication accounts for these two and several other intermediate situations. One of the intermediate situations is actually given in Figure 2. All in all, we obtain the following nine readings for sentence (3).

C1C1	$\exists X \in \{U \subseteq \cup men^*: \lvert U \rvert = 4\}: \exists Y \in \{V \subseteq \cup table^*: \lvert V \rvert = 3\}: lift'(X,Y)$
C1C2	$\exists X \in \{U \subseteq \cup men^*: \lvert U \rvert = 4\}: \lvert \cup \{Y \subseteq \cup table^*: lift'(X,Y)\} \rvert = 3$
C1D	$\exists X \in \{U \subseteq \cup men^*: \lvert U \rvert = 4\}: \lvert \cup \{Y \in table^*: lift'(X,Y)\} \rvert = 3$
C2C1	$\lvert \cup \{X \subseteq \cup men^*: \exists Y \in \{V \subseteq \cup table^*: \lvert V \rvert = 3\}: lift'(X,Y)\} \rvert = 4$
C2C2	$\lvert \cup \{X \subseteq \cup men^*: \lvert \cup \{Y \subseteq \cup table^*: lift'(X,Y)\} \rvert = 3\} \rvert = 4$
C2D	$\lvert \cup \{X \subseteq \cup men^*: \lvert \cup \{Y \in table^*: lift'(X,Y)\} \rvert = 3\} \rvert = 4$
DC1	$\lvert \cup \{X \in men^*: \exists Y \in \{V \subseteq \cup table^*: \lvert V \rvert = 3\}: lift'(X,Y)\} \rvert = 4$
DC2	$\lvert \cup \{X \in men^*: \lvert \cup \{Y \subseteq \cup table^*: lift'(X,Y)\} \rvert = 3\} \rvert = 4$
DD	$\lvert \cup \{X \in men^*: \lvert \cup \{Y \in table^*: lift'(X,Y)\} \rvert = 3\} \rvert = 4$

As stated previously, Scha does not use scopal change to obtain readings. But if he had done so, the number of readings would have increased drastically. A simple transitive sentence such as the one under consideration, would have had at most 18 readings. And in case of bitransitive verbs, the situation is much worse still. Poor children, forced to master their language! Perhaps Figure 3 —which contains possible situations covered by the nine readings,— will help them.

C1C1	C1C2	C1D
$\{m_1,m_2,m_3,m_4\}\Rightarrow\{t_1,t_2,t_3\}$	$\{m_1,m_2,m_3,m_4\}$ $\Rightarrow\{t_1,t_2\}+\{t_3\}$	$\{m_1,m_2,m_3,m_4\}$ $\Rightarrow\{t_1\}+\{t_2\}+\{t_3\}$
C2C1	**C2C2**	**C2D**
$\{m_1,m_2\}\Rightarrow\{t_1,t_2,t_3\}$ $\{m_3\} \Rightarrow \{t_4,t_5,t_6\}$ $\{m_4\} \Rightarrow\{t_7,t_8,t_9\}$	$\{m_1,m_2\}\Rightarrow\{t_1,t_2\}+\{t_3\}$ $\{m_3\}\Rightarrow \{t_4\} + \{t_5,t_6\}$ $\{m_4\}\Rightarrow\{t_7\}+\{t_8\}+\{t_9\}$	$\{m_1,m_2\}\Rightarrow\{t_1\}+\{t_2\}+\{t_3\}$ $\{m_3\} \Rightarrow\{t_4\}+\{t_5\}+\{t_6\}$ $\{m_4\} \Rightarrow\{t_7\}+\{t_8\}+\{t_9\}$
DC1	**DC2**	**DD**
$\{m_1\}\Rightarrow\{t_1,t_2,t_3\}$ $\{m_2\}\Rightarrow\{t_4,t_5,t_6\}$ $\{m_3\}\Rightarrow\{t_7,t_8,t_9\}$ $\{m_4\}\Rightarrow\{t_{10},t_{11},t_{12}\}$	$\{m_1\}\Rightarrow\{t_1,t_2\}+\{t_3\}$ $\{m_2\}\Rightarrow\{t_4\}+\{t_5,t_6\}$ $\{m_3\}\Rightarrow\{t_7\}+\{t_8\}+\{t_9\}$ $\{m_4\}\Rightarrow\{t_{10},t_{11},t_{12}\}$	$\{m_1\}\Rightarrow\{t_1\}+\{t_2\}+\{t_3\}$ $\{m_2\}\Rightarrow\{t_4\}+\{t_5\}+\{t_6\}$ $\{m_3\}\Rightarrow\{t_7\}+\{t_8\}+\{t_9\}$ $\{m_4\}\Rightarrow\{t_{10}\}+\{t_{11}\}+\{t_{12}\}$

Figure 3

The variation on the right-hand side of the arrows indicates that the object NP applies to one set (C1) or to a number of sets separated by '+' (D or C2). The symbol '+' is used to reduce the number of rows within a column. For example, $\{m_1\} \Rightarrow \{t_1\}+\{t_2\}+\{t_3\}$ is short for: $\{m_1\} \Rightarrow \{t_1\}$ and $\{m_1\} \Rightarrow \{t_2\}$ and $\{m_1\} \Rightarrow \{t_3\}$. In the columns with headings of the form DX, one finds four rows recording the fact that the subject NP is interpreted distributively. Under headings of the form C1X one finds just one row, since here the subject NP is interpreted collectively. In short, the rows under the heading of a reading indicate predications assigned to members of the subject NP denotation. In temporal terms, C1C1 comes close to optimal collectivity: four men lifted three tables on one occasion. Likewise, the situation in DD comes close to the other extreme: for every man there are three different tables which each of them must have lifted. As in Figure 1 one sees that C1C1 and DD are extremes, and that the other readings are intermediate. Under C1D a situation is given in which the subject NP is collective, while the direct object NP is distributive, in C2D the subject NP denotation is less collective than in C1D, etc.

Note that all the situations in Figure 3 could count as a C2C2-situation. This also becomes clear in Figure 4. The column *Situations* contains an example for the "basic configuration" which allows for overlap between C2 on the one hand, and C1 and D on the other. The column *Readings* gives the readings covering these situations.

	Situations	Readings
C1C1:	$\{m_1,m_2,m_3,m_4\}\Rightarrow\{t_1,t_2,t_3\}$	C1C1, C1C2, C2C1, C2C2
C1D:	$\{m_1,m_2,m_3,m_4\}\Rightarrow\{t_1\}+\{t_2\}+\{t_3\}$	C1D, C2D, C1C2, C2C2
DC1:	$\{m_1\}\Rightarrow\{t_1,t_2,t_3\}$ $\{m_2\}\Rightarrow\{t_4,t_5,t_6\}$ $\{m_3\}\Rightarrow\{t_7,t_8,t_9\}$ $\{m_4\}\Rightarrow\{t_{10},t_{11},t_{12}\}$	DC1, DC2, C2C1, C2C2
DD:	$\{m_1\}\Rightarrow\{t_1\}+\{t_2\}+\{t_3\}$ $\{m_2\}\Rightarrow\{t_2\}+\{t_3\}+\{t_1\}$ $\{m_3\}\Rightarrow\{t_3\}+\{t_1\}+\{t_2\}$ $\{m_4\}\Rightarrow\{t_1\}+\{t_2\}+\{t_3\}$	DD, DC2, C2D, C2C2

Figure 4

The C1C1-reading in which four men collectively lifted three tables is also covered by C2C2, C2C1 and C1C2. Likewise, the DD-reading in which four men individually lifted the same three tables separately can be obtained by C2C2, C2D and DC2. Note that C2C2 is the one reading which occurs four times in the righthandside column.

Figures 2 and 4 may be taken to reveal that there is some redundancy in Scha's system. The distributive reading uses the set of atoms (singletons) while C1 applies to a unit set containing part of the denotation of the common noun, whereas C2 is defined to allow these cases as special instances.

The observation is also made by Link (1984:22), but in our view he asked the wrong question, namely: "Why does Scha distinguish between C1 and C2?". As we shall see shortly, his answer was to drop C2 altogether and to use D and C1 only. According to us, the proper question would have been: "Why use D and C1? Why discern among readings which are embraced by the one C2?". The same sort of question can be posed to Scha: "Your 1981-article is an eye-opener, but what precisely is the notion of reading used? Are C1 and D not just conceptually convenient labels which may tag different kinds of verifying situations covered by C2?".

Here one touches "on a methodological point of quite a general nature in linguistics: Where exactly does the line of demarcation run between proper readings and mere models realizing a reading?"(Link 1984:23). In this respect our stance in the semantics of plural NPs is quite radical. In an atemporal framework, there is no meaningful way to discern among collective and distributive readings of sentences, if they do not contain lexical items triggering these readings explicitly.

2 Calculations on Link

In this section, we shall discuss the treatment of collective and distributive quantification in Link (1984;1987). To fully understand Link, one has to know that his domains of interpretation have lattice structure (cf. Link 1983). For convenience, we give a simple lattice structure which has the three elements a, b and c.

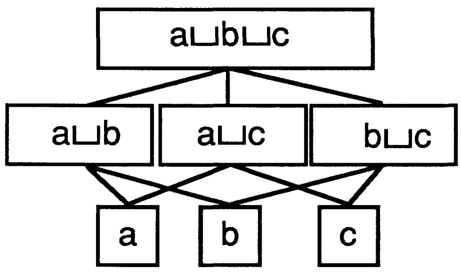

Figure 5

In Link's system, one may turn a one-place predicate P into a pluralized predicate *P. So, if the set $[\![\text{table}']\!] = \{a,b,c\}$, then the set $[\![{*}\text{table}']\!] = \{a, b, c, a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}$. It is the set of all so-called i-sums constructed on the basis of the atomic individuals a, b and c. The supremum or top element in this set is the uniquely specified three-membered i-sum $a \sqcup b \sqcup c$. In models with more than three tables, $a \sqcup b \sqcup c$ would be just one of the three-membered i-sums. But Figure 5 pictures a model in which *three tables* and *the three tables* happen to have the same extension. An i-sum $a \sqcup b \sqcup c$ is an individual which would be the denotation of the collective NP *three tables*. To obtain a distributive reading of *three tables*, Link needs an operation to "get at" the atoms of the i-sum, the a-,b- and c- part of the i-sum $a \sqcup b \sqcup c$. To this end, he introduces a distributive operator D which —when attached to a predicate— allows one to quantify over atomic individuals.[5]

2.1 *Eight readings*

We shall now investigate the logical representations assigned by Link (1984;1987) to sentences like (3) *Four men lifted three tables*. We do so in some detail, in order to be able to characterize his notion of a reading.

Link assumes that every plural NP gives rise to either a collective or distributive reading, and in contrast to Scha, he also uses scopal change to differentiate interpretations. Thus, two parameters are involved: (a) quantification over atomic individuals and quantification over sets (or i-sums) of individuals; (b) the relative scope of quantifiers. In this way, eight different logical forms (LFs) are predicted, as shown in Figure 6.

Wide scope for NP_1			**Narrow Scope for NP_1**		
	NP_1	NP_2		NP_2	NP_1
LF1	Coll	Coll	LF5	Coll	Coll
LF2	Coll	Distr	LF6	Coll	Distr
LF3	Distr	Coll	LF7	Distr	Coll
LF4	Distr	Distr	LF8	Distr	Distr

Figure 6

In the following, we shall discuss LF1 - LF8. The label MT indicates that the NP *four men* (M) has wide scope over the NP *three tables* (T), TM indicates the reverse situation. C is short for Coll, D for Distr.

The first logical form is LF1, which is logically equivalent with LF5. It is the doubly collective reading.

LF1: $\exists 4x[*man'(x) \wedge \exists 3y[*table'(y) \wedge lift'(x,y)]]$ MT/CC
$\Leftrightarrow$ $\exists 4x \exists 3y[*man'(x) \wedge *table'(y) \wedge lift'(x,y)]$
LF5: $\exists 3y[*table'(y) \wedge \exists 4x[*man'(x) \wedge lift'(x,y)]]$ TM/CC
$\Leftrightarrow$ $\exists 3y \exists 4x[*table'(y) \wedge *man'(x) \wedge lift'(x,y)]$

LF2 accounts for a collective subject-NP and a distributive object-NP.

LF2: $\exists 4x \exists 3y[*man'(x) \wedge *table'(y) \wedge {}^{\cdot D}Lift'(x,y)]$ MT/CD
$\Leftrightarrow$ $\exists 4x \exists 3y[*man'(x) \wedge *table'(y) \wedge \forall u[u {}^{\cdot}\Pi y \rightarrow Lift'(x,u)]]$

To the left of the distributive operator D a dot says that the predicate is only distributive in its second argument place.[6] Π is the 'part of'-relation. The dot in front of the Π means that the values for u need to be atoms. LF2 says that each table is related individually to the group of four men.

In LF3 the subject-NP having wide scope is taken distributively, whereas the direct object-NP is taken collectively. According to Link, the distributive operator D applies to the one-place lambda-predicate corresponding to *to lift* *three tables*, which is applied to the bound variable x, the result being universal quantification over the atomic members of the set man'.[7]

$$\text{LF3: } \exists 4x[{*}\text{man}'(x) \wedge {}^{D}\lambda u \exists 3y[{*}\text{table}'(y) \wedge \text{lift}'(u,y)](x)] \qquad \text{MT/DC}$$
$$\Leftrightarrow \quad \exists 4x[{*}\text{man}'(x) \wedge \forall u[u{\cdot}\Pi x \rightarrow \exists 3y[{*}\text{table}'(y) \wedge \text{lift}'(u,y)]]$$

In this case, each individual man relates to three tables. Thus, LF 3 may capture an interpretation in which twelve different tables were lifted. This may or may not be done on the same occasion. LF3 also represents the situation in which the same three tables were lifted by each of the men, evidently on different occasions to maintain distributivity.

LF4 represents the doubly distributive reading with wide scope for the subject-NP:

$$\text{LF4: } \exists 4x[{*}\text{man}'(x) \wedge {}^{D}\lambda u \exists 3y[{*}\text{table}'(y) \wedge {}^{\cdot D}\text{lift}'(u,y)](x)] \qquad \text{MT/DD}$$
$$\Leftrightarrow \quad \exists 4x[{*}\text{man}'(x) \wedge \forall u[u{\cdot}\Pi x \rightarrow \exists 3y[{*}\text{table}'(y) \wedge \forall v[v{\cdot}\Pi y \rightarrow \text{lift}'(u,v)]]$$

Here the total number of tables may vary from 3 to 12. Intuitively, this variation involves time structure: if there are three tables, constraints are put on the order in which the tables are lifted. If there are twelve tables, the four men could lift their tables at the same interval.

LF6 represents the reading in which the object-NP (taken collectively) has wide scope over the subject-NP (taken distributively):

$$\text{LF6: } \exists 3y[{*}\text{table}'(y) \wedge \exists 4x[{*}\text{man}'(x) \wedge {}^{D\cdot}\text{lift}'(x,y)] \qquad \text{TM/CD}$$
$$\Leftrightarrow \quad \exists 3y \exists 4x[{*}\text{table}'(y) \wedge {*}\text{man}'(x) \wedge \forall u[u{\cdot}\Pi x \rightarrow \text{lift}'(u,y)]]$$

That is, each of the men lifted the members of the set $\mathcal{T} = \{t_1,t_2,t_3\}$ such that no two men lifted $\mathcal{T}$ together. For each of the men it is the case that he lifted all members of $\mathcal{T}$.[8]

LF 7 represents the reading in which the distributive object-NP has wide scope over the collective subject-NP:[9]

$$\text{LF7: } \exists 3y[{*}\text{table}'(y) \wedge {}^{D}\lambda u \exists 4x[{*}\text{man}'(x) \wedge \text{Lift}'(x,u)](y)] \qquad \text{TM/DC}$$
$$\Leftrightarrow \quad \exists 3y[{*}\text{table}'(y) \wedge \forall u[u{\cdot}\Pi y \rightarrow \exists 4x[{*}\text{man}'(x) \wedge \text{Lift}'(x,u)]]$$

There is a group of three tables and for each of its atomic members there is a group of four men.

Finally, the doubly distributive reading with wide scope for the object-NP is represented by LF8.

LF8: $\exists 3y[{*}\text{table}'(y) \wedge {}^{D}\lambda w \exists 4x[{*}\text{man}'(x) \wedge {}^{D}\cdot\text{lift}'(x,w)](y)]$ TM/DD
$\Leftrightarrow$ $\exists 3y[{*}\text{table}'(y) \wedge \forall v[v\cdot\Pi y \rightarrow \exists 4x[{*}\text{man}'(x) \wedge \forall u[u\cdot\Pi x \rightarrow \text{lift}'(u,v)]]$

In LF8, the number m of men may vary between 4 and 12. If m = 12, then Link is forced to assume that there are 12 different liftings though intuitively there may be only four intervals at which the liftings take place: at interval i_1 t_1 is lifted by m_1, t_2 by m_5 and t_3 by m_9, etc. So, the number of intervals at which the liftings take place may also vary between 4 and 12. If m = 4, then it takes some organizational skill to minimalize the number of intervals.

In Link (1984:22), the number of readings of a simple transitive sentence is justified by means of a simple calculation. In Scha's terminology, adopted by Link, an NP can be either D, C1 or C2, and a transitive sentence contains two NPs each of which allow the three options. This gives nine readings, and, as we already remarked, eighteen if scopal change is recognized. Link reduces this number to eight by disregarding that NPs can be C2. Hence, there are four readings without and eight readings with scopal change. For particular kinds of quantifier, this number may be reduced due to logical equivalence.

At other places, Link speaks of ‘Distributive vs. Collective Predication’ (1984:17-21) and of distributivity as a lexical feature of the verb (1983:310) which should be marked by use of D-operators. But this is a factor not used in the above multiplication. So, there seems to be some vagueness as to his real position.[10] Apart from this, there are also more concrete observations leading to some reserve. As in the case of Scha, these concern the coverage of the "readings".

3. Covers, partitions, and pseudo-partitions

Link seems only to allow the extreme cases of "each individually" (D) or "all together" (C1), while the intermediate situations covered by C2 are expelled. This is not to say that Link would be strongly opposed to these readings, for in Link 1984 he suggests that they may be seen as special instances of C1. It remains unclear, however, how this could be so.

That the readings covered by C2 exist, is also argued by Gillon (1987), who uses (7) to make his point:[11]

(7) Hammerstein, Hart and Rodgers wrote musicals

This sentence may well be true if, as in fact, Rodgers wrote both with Hammerstein and Hart. And a most straightforward analysis of this would be that (7) may be used in a situation where to write musicals holds of two overlapping subsets of the set denoted by *Hammerstein, Hart and Rodgers*. More precisely, Gillon states that the truth conditions of (7) are:

> (7) is true iff there is a minimal cover of the set denoted by the NP which is a subset of the VP.

According to Gillon, this should be understood as saying that each minimal cover will yield a separate reading of (7), where the notion of minimal cover is defined by:

DEFINITION (MINIMAL-COVER) A set $Y \subseteq \wp(X)$ is a minimal cover of X, iff Y covers X, i.e. $\bigcup Y = X$, but Y does not contain covers as a real part. ($\neg\exists Z$: Z covers X & $Z \subset Y$)

That is, each element in a minimal cover Y of X contributes at least one element in covering X not contributed by any of the other elements of Y.

Although we do agree with Gillon (and Scha for that matter) that intermediate interpretations exist, we reject the idea that they induce a combinatorial explosion in the number of readings.[12] As it stands, (7) is simply vague as to which minimal cover will verify it, a vagueness which may be eliminated in a particular context of use. Apart from this, there are two further objections against Gillon. Commonly it is understood that NPs denote sets of VP extensions and not just sets. So, how does Gillon's proposal extend to arbitrary NPs, and how does it deal with multiple quantification? Moreover, is the notion of minimal cover general enough?

From 1984 onwards, the first author has been arguing that NP-denotations involved in terminative aspect should be taken in terms of partitions.

DEFINITION (PARTITIONS) A set $Y \subseteq \wp(X)$ is a partition of X, if $\bigcup Y = X$ and for all U,V $\in Y$: U $\cap$ V = $\varnothing$

A partition is a minimal cover whose elements do not overlap. The reason why one should like to have them is that in sentences like

(8) Three children crossed the street

the children may have crossed together, one by one or in a 1-2 or 2-1 configuration. In order to account for the underdeterminedness, Verkuyl (1988) proposed to use partitions; guided by the empirical observation that if one extends this sentence with ... *and they did it one after the other*, we should be able to relate the temporal structure assumed by the use of *one after the other* to the representation of (8) by relating the cells of the partition to time units. Similarly if one had proceeded with *...first Judith crossed it, and then David and Jessica*, or with ... *together.*

More formally, the use of partitions in the analysis of sentences like (8) is inspired by the set-theoretical theorem which says:

> every set X can be mapped canonically onto the quotient set X/E, where E is an equivalence relation.

For if the thematic relation, i.e. the way an argument relates to the verb, is conceived of as the equivalence relation 'being involved in the verbal predication at the same time', it is natural to use the theorem. That is, the above set of three children may be partitioned on the ground of the equivalence relation 'having crossed the street at the same time'. In this way, a partition of the set of three children into {{Judith}, {David, Jessica}} expresses the fact that the two cells of the partition are related to different time intervals (which constitute an event described by (8)).

However, nwo we believe that (8) can be understood in the following way: Judith crossed the street first, she observes David and Jessica at the other side, she walks back and then the three of them crossed it all together. That (8) can be applied to this situation cannot be denied and plainly in an atemporal semantics this would result in a non-partitional set of sets. To take another (real life) example, Max, Henk and Hans are playing chess in following way: Max and Henk play together against Hans and so do Hans and Henk against Max, all at the same time. So *Max, Henk and Hans played two chess games* is a perfect sentence for this situation, though it describes many others as well.

Why not simply replace the notion of partition by the most general notion of a cover? It is observed in Van der Does (1991) that Scha's C2 also makes use of covers, for the C2-denotation of a numeral *n* can be written equivalently as:

$$\lambda X \lambda Y.\ \exists Z \subseteq X\ [|Z| = n \wedge \exists W(W \text{ covers } Z \wedge W = Y \cap \wp(X))]$$

He goes on to argue that it is precisely this fact which makes C2-readings unsuitable for giving the meaning of (3) *Four men lifted three tables*. On the C2C2-reading, (3) can be true if there is a set W of collections formed out of a set Z of four men such that each of those collections has the property of lifting three tables. Since the largest such set, $\wp(Z)$, contains sixteen items, this would mean that 48 tables may be involved in a situation verifying (3). This runs counter to the "law" formulated in Verkuyl (1988), which captures the common intuition that in a sentence of the form:

$[_{NP}\ k\ N_1]\ V\ [_{NP}\ m\ N_2]$

the number of N_2's V-ed may vary between $1 \times m$ and $k \times m$. In particular, if four men lifted three tables, the number of tables lifted may vary between 3 and 12. In this way, one and the same table may have been lifted by different men. Given the temporal structure of sentences it might be necessary to distinguish $\langle t_1,i\rangle$ (a particular table lifted at time i) from $\langle t_1,j\rangle$ (the same table lifted at a different time j) and to apply quantification to the pairs rather than to the tables.

The above considerations lead us to incorporate the numerical upper-bound given by the atemporal NPs, especially by the external argument NP_1, into the definition of the sort of cover which is going to be used.

DEFINITION (PSEUDO-PARTITION) A set $Y \subseteq \wp(X)$ is a pseudo-partition of X, if $\bigcup Y = X \ \&\ |Y| \leq |X|$.

A pseudo-partition Y of X will exhaust all the elements in X, but the members of Y itself may overlap. That X is pseudo-partitioned by Y will be written as: X pp Y, where pp is a relation between a set and a set of sets.[13]

In Van der Does (1991) it is proved that minimal covers are pseudo-partitions, but not conversely. The notions discussed here relate to one another as follows:

partitions $\subseteq$ minimal covers $\subseteq$ pseudo-partitions $\subseteq$ covers

Summarizing, it seems as if interpretations in which we have to assume partitioning are often more normal than the other forms, for example, in sentences containing VPs like *eat five sandwiches*, *kill three soldiers*, *kiss three men*, etc., Both lexical and temporal information might be relevant for a particular choice.

In view of this, it is necessary to make sure that one does not reject a priori the analysis of NPs in terms of partitions, or (minimal) covers or pseudo-partitions. Due to the aspectual properties of sentences expressing time structure, we have adopted pseudo-partitions, but it might be so that atemporal sentences require other sorts of structures on sets. Our choice is mainly empirically motivated and it should be possible to reject or support it on the basis of facts (cf. Verkuyl (to appear) for a more detailed analysis).

4 A scalar approach to quantification

In this section, we shall present a scalar approach to quantification in sentences with plural NPs, which tries to circumvent the proliferation of readings in the proposals under discussion. In fact, we shall grant each sentence but one meaning. As said in the introduction, it may be convenient to have names for two situations covered by this meaning: the purely distributive and the purely collective interpretation of a sentence, corresponding to a distinction between quantification over singletons and quantification over a unit set. In linguistic theory the distinction between the "all-individuals-taken-as-a-set" interpretation and the "all-individuals-taken-apart" interpretation may even be a universal. Still, we would like to claim that often the use of NPs is genuinely vague as to which of these or other interpretations are intended, and that one should therefore strive to remain neutral in that respect within a logical semantics. This does not preclude, of course, that sometimes lexical items take their interpretations at one end of the scale. For example *every* always denotes a distributive quantifier.

Technically, our approach involves type-lifting of the NP since it is necessary to assume a slightly more complex internal structure. This will bring us in the "adjectival" tradition in which e.g. numerals are treated as adjectival elements. In section 4.1, we shall discuss two of these adjectival analyses and reject them. Here we shall also argue that the problems of the adjectival and of the scalar approach may be solved by assuming a more complicated determiner structure. In section 4.2, we are getting formal; the formal machinery is put at work in the sections 4.3 - 4.5.

In the literature based on Link, the relation symbols are not interpreted in a set of individuals (treated as being non-compound), but in a Complete Boolean

Algebra (CAB). That NPs with Count Nouns may be handled as well in an extensional type-theory, with basic types *e* (pre-Link individuals) and *t* (truth values), has already been shown by Bartsch (1973) and Scha (1981), among others. However, the use of an extensional type theory does require some deviations from the Montegovian practice. According to Scha, individuals and sets should be of the same type: individuals are seen as singletons (atoms), a limiting case of sets. Consequently, one sometimes has to introduce meaning postulates to ensure that the extension of, e.g., nouns consists of atoms only.[14] We prefer to work in an extensional type theory in which objects of a higher type are used only if they are called for empirically.

4.1 *The adjectival approach to numerals*

In line with the above observations, one finds several proposals which treat the NP as being of type $<<<e,t>,t>,t>$, i.e. as denoting sets of sets of sets, rather than being of type $<<e,t>,t>$, i.e. sets of sets of individuals.[15] This results from the fact that NPs are sets of VP-extensions and that VPs are now of type $<<e,t>,t>$: properties of groups (here just sets). In order to attain the proper type for an NP like *the three children*, one could take the numeral as an adjective of type $<<e,t>,<<e,t>,t>>$, which assigns a set of subsets to the extension of the head noun of the NP. *The* is then a determiner of type $<<<e,t>,t>$, $<<<e,t>,t>,t>>$, while *three children* is taken as [ø [three children]]. Here ø — the empty determiner— brings about the type-lift of the N' *three children*, as in Verkuyl (1981:589). We shall show that this approach has some shortcomings, which can be solved by locating *three* in the determiner rather than in N'.

Semantically, the adjectival view on determiners consists in the observation that quite generally determiners of type $<<e,t>,<<e,t>,t>>$ can be turned into "adjectives" of the same type by means of the modifier ADJ:

(8) $\text{ADJ} =_{df} \lambda D_{<<e,t>,<<e,t>t>>} \lambda X_{<e,t>} \lambda Y_{<e,t>} [Y \subseteq X \wedge D(X)(Y)]$

Using the standard determiner denotations (cf. Barwise & Cooper 1981), this will yield, for instance, the following N'-interpretations:[16]

(9)
a. $\text{ADJ}([\![\text{some}]\!])(A) = \lambda Y_{<e,t>} [Y \subseteq A \wedge Y \neq \varnothing]$
b. $\text{ADJ}([\![\text{all}]\!])(A) = \lambda Y_{<e,t>} .Y = A$
c. $\text{ADJ}([\![\text{n}]\!])(A) = \lambda Y_{<e,t>} [Y \subseteq A \wedge |Y| \geq n]$
e. $\text{ADJ}([\![\text{most}]\!])(A) = \lambda Y_{<e,t>} [Y \subseteq A \wedge |A/Y| < |Y|]$

Above we have observed that VPs will be of type $<<e,t>,t>$; they denote sets of sets. Consequently, we cannot take ADJ(⟦three⟧)(⟦children⟧) as the extension of the NP three children, for it is of type $<<e,t>,t>$ too. A possible solution to this problem would be to introduce an empty determiner of type $<<<e,t>,t>, <<<e,t>,t>,t>>$, which turns ADJ(⟦three⟧)(⟦children⟧) into a set of VP-extensions, an object of type $<<<e,t>,t>,t>$. Then the task is to find an empty determiner which gives sentences their proper truth conditions.

A popular proposal is to define the empty determiner ø as the non-empty intersection of two sets of sets:

(10) $\emptyset \;=_{df}\; \lambda X_{<<e,t>,t>}\lambda Y_{<<e,t>,t>}.[X \cap Y \neq \emptyset]$

which amounts to existentially introducing a set W which is both a member of X and Y. This solution works quite well for the quantifiying expressions in (9), but it is not general enough. Van Benthem (1986:52) points out that there are problems with non-monotonic numerals (or more generally for non-monotone increasing determiners, see Van der Does 1991). For example: using (10) one would have to assign to (11a) the truth conditions of (11b):

(11) a. Exactly three children made music
 b. At least three children made music

In order to show this, we shall have a closer look at the syntactic and the type-logical structure of (11a) along these lines in Figure 7.[17]

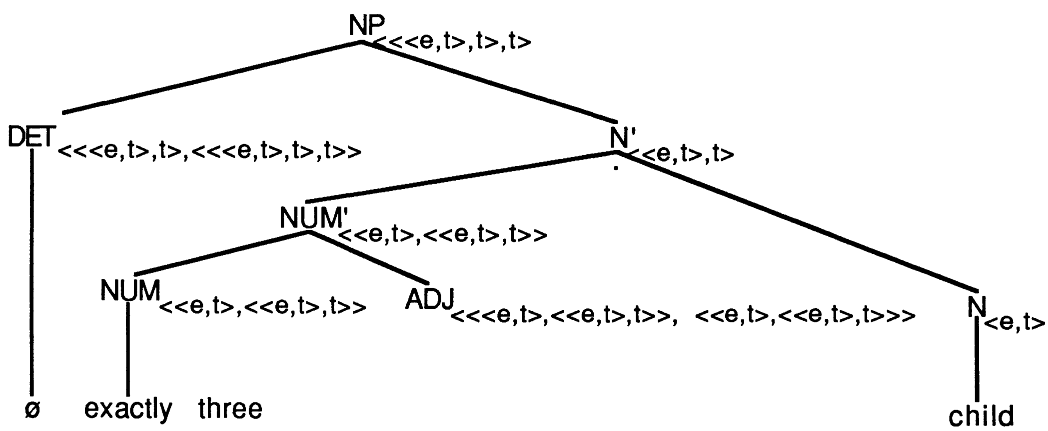

Figure 7

By means of the ADJ-operation the standard meaning of the determiner *exactly three* is turned into an adjective of type $<<e,t>,<<e,t>,t>>$ which takes *chil-*

dren to form *exactly three children*. The denotation of this N' is the set of three membered sets of children. After being subjected to the ø-operation, the NP [ø[*exactly three children*]] would be interpreted as:

(12) $\lambda Y_{<<e,t>,t>}.\text{ADJ}([\![\text{exactly three}]\!])([\![\text{children}]\!]) \cap Y \neq \varnothing$

Hence the truth conditions of (11a) become:

(13) $\exists X[X \in \text{ADJ}([\![\text{exactly three}]\!])([\![\text{children}]\!]) \wedge X \in [\![\text{made\_music}]\!]]$

But (13) states the truth conditions of (11b) incorrectly, as the existence of a set with exactly three music making children does not preclude there also being a set of more than three music making children. So, this way is blocked.

One may try to remedy this by taking [[exactly]] as modifying [[three children]]; that is, as modifying the set of all triples of children. Now, the problem is to find the proper denotation of *exactly*. Observe that in the distributive case (14a) and (14b) are equivalent:

(14) a. $|[\![\text{children}]\!] \cap [\![\text{walk}]\!]| = 3$
b. $\exists! X[X \in \text{ADJ}([\![\text{three}]\!])([\![\text{children}]\!]) \wedge X \subseteq [\![\text{walk}]\!]]$

This says that *exactly* picks out a unique triple set of children such that all of them walk. Likewise, *at most* in *at most three children walk* would select at most one triple set of walking children. In the present setting this suggest as the denotation for *exactly*:

(15) $[\![\text{exactly}]\!] =_{\text{df}} \lambda X_{<<e,t>,t>}\lambda Y_{<<e,t>,t>}.|X \cap Y| = 1$

However nice this solution may seem at first glance, it suffers from being incorrect. The sentence *Exactly three children made music* would become true if it were the case that exactly one group of three children made music, even though there were also a group of two or six children making music. The proposed denotation of *exactly* only determines a unique group of three children as belonging to the intersection [[children]] ∩ [[made music]], but it does not exclude groups of children of other cardinalities from that intersection.

The minimal requirement for obtaining an NP from an N would be that such an operation work for all NPs, regardless of the monotonicity behaviour of the original determiners. The following proposal, based on the structure given in Figure 8, seems to do just that.

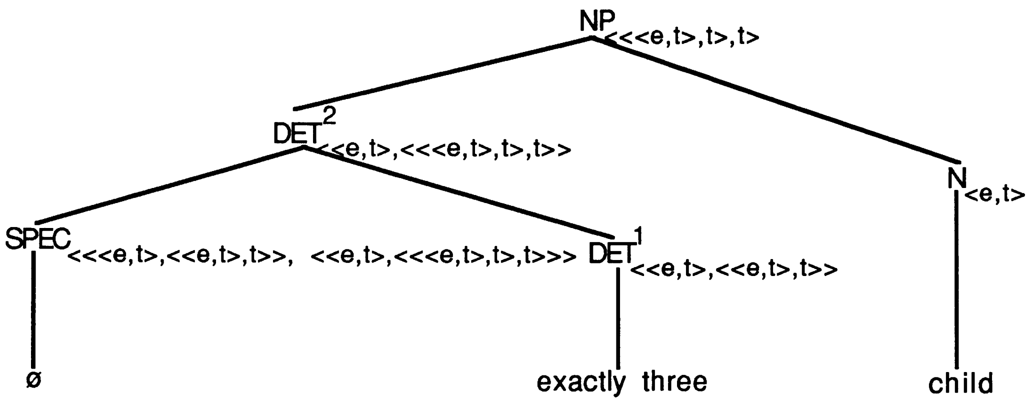

Figure 8

In view of our wish to stay as closely as possible to accepted principles of X-bar grammar in GB-syntax, the Specifier node SPEC modifies a DET^1-node yielding a complex determiner DET^2. The underlying idea is that DET^1, transmitting quantificational information, may be linked with the VP as soon as SPEC, which contains "referential" information introducing a set or identifying it, turns DET^1 into a determiner DET^2 of the proper (lifted) type. Thus, we obtain a division of labour in the determiner DET^2. The information contained by ADJ above is now incorporated in SPEC. This means that there is no need to distinguish DET^1 from DET^0, so the former will be DET^1 of the X-bar grammar involved. We did not include DET^0 in Figure 8 as there is no semantic difference between DET^0 and DET^1 in the sentences under analysis. For syntactic reasons one might distinguish between them, e.g. in view of an NP like [[the[two following]] children], where *following* can be taken as the complement of DET^0 making a DET^1 *two following*.[18]

The determiner structure of NPs will now be investigated with a view on the contributions made by DET^1 and SPEC. Given the definition of pseudo-partition, we are in the position to define SPEC in a structure of the form $[_{\text{DET}2}[\text{SPEC DET}^1]]$. It makes use of an operator $|$ restricting type $<<e,t>,t>$-objects to objects of type $<e,t>$:

$$(16)\quad | =_{df} \quad \lambda X_{<e,t>} \lambda Y_{<<e,t>,t>} \lambda Z_{<e,t>}\ \exists W \in Y[Z = W \cap X]$$

We shall write $Y|_X$ rather than $|(X)(Y)$. In set notation, the definition amounts to:

(17) $Y|_X = \{X \cap Z \mid Z \in Y\}$

Thus, for $Y = \{\{a_1,a_2,c_1\}, \{a_3,c_2,c_3\}\}$ and $[\![\text{child}]\!] = \{c_1,c_2,c_3\}$, $Y|_{[\![\text{child}]\!]} = \{\{c_1\},\{c_2,c_3\}\}$. If Y is thought of as the denotation of a VP (a set of sets), then $Y|_X$ contains exactly those parts of the sets in Y which consists of X's. [19] The definition of the empty SPEC is now:

(18) $[\![\text{SPEC } \emptyset]\!] =_{df} \lambda D\lambda X\lambda Y.\exists W[W \subseteq X \wedge D(X)(W) \wedge W \text{ pp}Y|_X]$

In (18), D is a variable of type $<<e,t>,<<e,t>,t>>$, which makes SPEC itself of type $<<<e,t>,<<e,t>,t>>,<<e,t>,<<<e,t>,t>,t>>>$. Note that this type is quite different from the type of the empty determiner in Figure 7 which was of type $<<<e,t>,t>,<<<e,t>,t>,t>>$. Underlying this change in type is the idea that a specifier should modify a determiner of type $<<e,t>,<<e,t>,t>>$ rather than an N' of type $<<e,t>,t>$. This also enables us to incorporate into SPEC the information, previously associated with ADJ, as (18) is equivalent to:

(19) $[\![\text{SPEC } \emptyset]\!] =_{df} \lambda D\lambda X\lambda Y.\exists W[\text{ADJ}(D)(X)(W) \wedge W \text{ pp}Y|_X]$

In this way we obtain a determiner structure in which the cardinality information expressed by an NP is located in the DET, whereas other contextually determined information is located in SPEC. In fact, SPEC is the bridge between propositional information expressed by the sentence and the context in which this sentence will be used. To attain this, we shall differentiate between the indefinite and definite use of SPEC below.

Before getting formal, syntactically and semantically, we shall demonstrate the present set up with the help of example (11a) *Exactly three children made music*. The following steps show how the denotation of *exactly three* is obtained from the lexical denotation of *three*.[20]

(20) a. $[\![\text{three}]\!] \quad =_{df} \quad \lambda X\lambda Y.\ |X \cap Y| = 3$
b. $[\![\text{SPEC } \emptyset]\!]([\![\text{three}]\!])$
$= \quad \lambda X\lambda Y.\exists W[W \subseteq X \wedge |W| = 3 \wedge W \text{ pp}Y|_X]$

Applied to $[\![\text{child}]\!]$ this yields the extension of the NP *three children*:

$[\![\text{SPEC } \emptyset]\!]([\![\text{three}]\!])([\![\text{child}]\!])$
$= \lambda Y.\exists W[W \subseteq [\![\text{child}]\!] \wedge |W| = 3 \wedge W \text{ pp}Y|_{[\![\text{child}]\!]}]$

For convenience, this formula will sometimes be shortened to:

$[\![\text{SPEC } \emptyset]\!]([\![\text{three}]\!])([\![\text{child}]\!])$
$= \lambda Y.\exists W \in [\![\text{three children}]\!][W \text{ pp} Y|_{[\![\text{child}]\!]}]$

The makes plain that we have introduced SPEC categorematically here by assuming that the NP *exactly three children* be analysed as [ø[exactly three children]], where the SPEC ø receives the interpretation given in (19). Combining this NP with the VP *made music* yields:

$[\![\text{three children}]\!]([\![\text{make music}]\!]) =$
$\exists W \in [\![\text{three children}]\!][W \text{ pp } [\![\text{make music}]\!]|_{[\![\text{child}]\!]}]$

This means: inspecting the sets containing music making children—i.e. $[\![\text{made music}]\!]|_{[\![\text{child}]\!]}$—, one finds that all these sets are formed out of a particular group consisting of exactly three children.

Note that in contrast to (10), these truth conditions do not give rise to the Van Benthem problem mentioned earlier, due to the more sophisticated relation between the NP and the VP expressed by the determiner. In particular, the simple $W \in Y$ is replaced by the pseudo-partition relation W pp $Y|_X$.[21]

The SPEC used above applies to indefinite NPs, but for definite determiners like *the* we will use the context sets introduced by Westerståhl (1984):

$[\![\text{the}]\!] =_{df} \lambda D\lambda X\lambda Y.D(X\cap C)([\![\text{thing}]\!]) \wedge X\cap C \text{ pp} Y|_{X\cap C}]$

In this definition of $[\![\text{the}]\!]$, the context set C, a contextually give part of the domain, is built in in its representation as a restriction on the head noun denotation. Hence $[\![\text{the}]\!]$ is a three-place relation between DET, a contextually restricted head noun set and the VP, which expresses that this restricted head noun set is pseudo-partitioned so that the resulting cells are involved in the VP predication. In this way, using the syntactic analysis (SPEC(PL))(boy)(make music), *The boys made music* would become:

$|[\![\text{boy}]\!]\cap C| > 1 \wedge [\![\text{boy}]\!]\cap C \text{ pp } [\![\text{make music}]\!]|_{[\![\text{boy}]\!]\cap C}$

This says that the context provides for more than one boy and that the contextually given set of boys can be formed into groups which are precisely those that made music.

4.2 *Getting formal*

The expressions of the formal language are generated by means of an X-bar PLUral Grammar (PLUG), which is chosen so that the syntactic structure of a formal expression is virtually identical to the syntactic structure of the corresponding English expression.

PLUG

Grammar:

1. $S : [\![NP]\!]([\![VP]\!]) \quad \Leftarrow \quad NP : [\![NP]\!]\ ,\ VP : [\![VP]\!]$
2. $\begin{cases} VP : [\![V1]\!] \\ VP : \lambda X.\ [\![NP]\!](\lambda Y.[\![V_2]\!](Y)(X)) \quad \Leftarrow \quad V_2 : [\![V_2]\!]\ ,\ NP : [\![NP]\!] \end{cases}$
3. $NP : [\![DET^2]\!]([\![N]\!]) \quad \Leftarrow \quad DET^2 : [\![DET^2]\!]\ ,\ N :\ [\![N]\!]$
4. $DET^2 : [\![SPEC]\!]([\![DET^1]\!]) \quad \Leftarrow \quad SPEC : [\![SPEC]\!]\ ,\ DET^1 : [\![DET^1]\!]$

Lexical entries are of the form:

5. $SPEC\ : [\![SPEC_i]\!]$
6. $DET^1 : [\![D_i]\!]$
7. $N :\ [\![P_i]\!]$
8. $V_1 : [\![T_i]\!]$
9. $V_2 :\ [\![R_i]\!]$

The following notational conventions are used:

Variables	*Constants*	*Types*
x,y,z	-	*e*
X,Y,U,V,W,Z	P_i, T_i	$<e,t>$
-	R_i	$<e,<e,t>>$
X,Y,U,W,Z	-	$<<e,t>,t>$
D	D_i	$<<e,t>,<<e,t>,t>>$

The expressions P_i, T_i, R_i, D_i ($i \in \mathbf{N}$) are treated as lexical constants receiving their interpretation from an underlying model $\mathbf{M} = \langle E\ ,[\![-]\!]\rangle$ left implicit in the sequel. Furthermore, $E = [\![thing]\!]$.

The rules of PLUG are stated using the format:

$$SYN : F(SEM_1, \ldots, SEM_n) \quad \Leftarrow \quad SYN_1 : SEM_1\ , \ldots ,\ SYN_n : SEM_n$$

Here SYN_i denotes the syntactic category and SEM_i its semantic denotation. The symbol '$\Leftarrow$' indicates that SYN is built from $\mathrm{SYN}_1 \ldots \mathrm{SYN}_n$ and that its semantic denotation is $F(\mathrm{SEM}_1, \ldots, \mathrm{SEM}_n)$. If SYN_i is basic (i.e. a lexical category, as in rules 5 - 9), the rules reduce to SYN : SEM, where SEM is given by the lexical definition. Most of the times F is simply functional application, but for convenience F in: $VP \Leftarrow V_2 \;\; NP$, in rule 2 is a more complex though standard operation.

In the formulation of the semantic analysis of a syntactic expression of the form [V NP] an asymmetry is expressed between the internal argument and the external argument of the verb. That is, an NP in the direct object position is treated differently from an NP in the subject position. Impressionistically, an NP is "sensitive" for the position in which it occurs. The fusion of a verb and its internal argument (in the sentences under analysis occurring as a direct object NP) requires a semantic operation of matching the $<<e,t>,<<e,t>,t>>$-type of the V_2 and the $<<<e,t>,t>,t>$-type of the NP. This is now done syncategorematically by rule 2, along familiar lines, but it can also be done categorematically by the type-lifting operator

$$\lambda V_{2<<e,t>,<<e,t>,t>>} \lambda NP_{<<<e,t>,t>,t>} \lambda X_{<e,t>}.NP(\lambda Y_{<e,t>}.V_2(Y)(X))$$

As this operation does not play a role in the analysis of the quantificational structure of all NPs, we leave the matter here, but in the analysis of the temporal structure built up from the verb and its internal argument, it is not unreasonable to think of the verb as contributing such an operator. Syntactically the type-shift operation, say INT, might be located in the verb, say at the place where the combination with the internal argument NP takes place.[22] An analysis along these lines is given in Verkuyl (to appear).

Rule 3 says that, in general, NPs are of the form DET^1 N. Actually, we could have written DET^{max} rather than DET^2 to underscore the X-bar nature of PLUG. Rule 4 represents the splitting up of quantificational information (DET^1) and referential information (SPEC). The lexicon will contain entries like the following:

Lexicon:
SPEC
$[\![\text{the}]\!]$: $\lambda D \lambda X \lambda Y.[D(X \cap C)([\![\text{thing}]\!]) \wedge X \cap C \text{ pp } Y|_{X \cap C}]$
$[\![\emptyset]\!]$: $\lambda D \lambda X \lambda Y.\exists W[W \subseteq X \wedge D(X)(W) \wedge W \text{ pp } Y|_X]$

DET^1
$[\![\text{SG}]\!]$: $\lambda X_{<e,t>} \lambda Y_{<e,t>}. [|X \cap Y| = 1]$

$[\![\text{PL}]\!]$: $\lambda X_{<e,t>}\lambda Y_{<e,t>}.\ [|X \cap Y| > 1]$
$[\![\text{n}]\!]$: $\lambda X_{<e,t>}\lambda Y_{<e,t>}.\ [|X \cap Y| = n]$
$[\![\text{most}]\!]$: $\lambda X_{<e,t>}\lambda Y_{<e,t>}.\ [|X \cap Y| > 1/2 \bullet |X|]$[23]
$[\![\text{INC}]\!]$ $\lambda X_{<e,t>}\lambda Y_{<e,t>}.\ [|X \cap Y'| = 0]$

In Verkuyl (1981) *the*, *these* and *all* received the same entries, their difference being accounted for by differences in the degree of contextual identification (e.g. by requiring different context sets). In the present paper, however, *all* is defined as ø(INC). We shall come back to this below.

The following derivations for the NPs *the child* and *the children* are yielded by PLUG:

- $[\![\text{the}]\!]$: $\lambda D\lambda X\lambda Y.[D(X\cap C)([\![\text{thing}]\!]) \wedge X\cap C \text{ pp } Y|_{X\cap C}]$
- $[\![\text{SG}]\!]$: $\lambda X\lambda Y.\ [|X \cap Y| = 1]$
- $[\![\text{the}]\!]([\![\text{SG}]\!])$
  $= \lambda D\lambda X\lambda Y.[D(X\cap C)([\![\text{thing}]\!]) \wedge X\cap C\text{pp } Y|_{X\cap C\text{the}}](\lambda X\lambda Y.[|X\cap Y| = 1])$
  $= \lambda X\lambda Y.[|X\cap C| = 1 \wedge X\cap C \text{ pp } Y|_{X\cap C}]$
- $[\![\text{the}]\!]([\![\text{SG}]\!])([\![\text{child}]\!])$
  $= \lambda X\lambda Y.[|X\cap C| = 1 \wedge X\cap C \text{ pp } Y|_{X\cap C}]([\![\text{child}]\!])$
  $= \lambda Y.[|[\![\text{child}]\!]\cap C| = 1 \wedge [\![\text{child}]\!]\cap C \text{ pp } Y|_{[\![\text{child}]\!]\cap C}]$

Similarly:

$[\![\text{the}]\!]([\![\text{PL}]\!])([\![\text{child}]\!])$
$= \lambda Y.[|[\![\text{child}]\!]\cap C| > 1 \wedge [\![\text{child}]\!]\cap C \text{ pp } Y|_{[\![\text{child}]\!]\cap C}]$

Note once more that the definiteness of the NPs is given directly in terms of a context set. The cardinality of this set intersected is constrained by PL and SG. In this way, the SPEC *the* expresses definiteness (givenness) rather than cardinality.One might wish to evade abstract syntactic elements like Ø, SG and PL, but there are some reasons for not doing so. Firstly, there are languages in which singularity and plurality are marked. Secondly, assuming one syntactic form *the* which is ambiguous between our DET^2-meanings $[\![\text{the}]\!]([\![\text{SG}]\!])$ and $[\![\text{the}]\!]([\![\text{PL}]\!])$, would cost two determiners. Thirdly, one would miss a generalization which makes plain how different DET^2-meanings are systematically related to each other.

4.3 *Multiple quantification.*

How does PLUG provides the syntax and semantics of sentences with multiple quantification without specifying the multitude of readings discerned by Link and Scha? We will answer this question by showing in detail how PLUG deals with (1)

(1) Two girls ate five sandwiches

The syntactic structure of (1) is given in Figure 9.

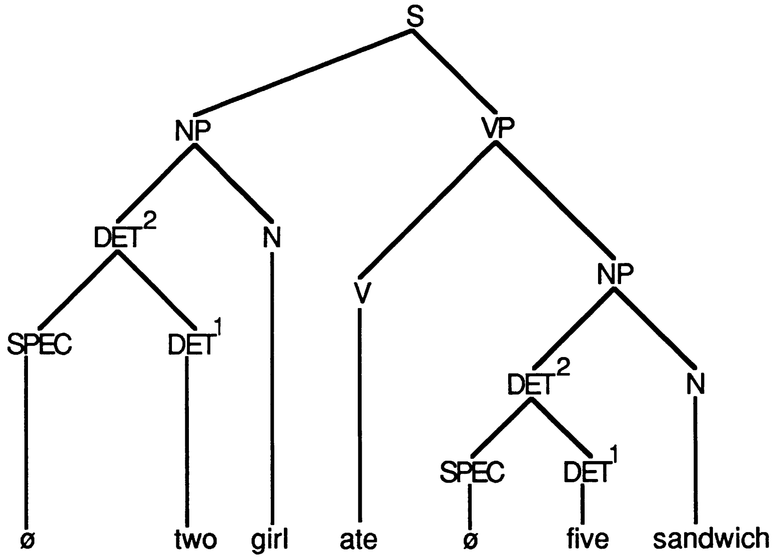

Figure 9

As to the semantics of (1), we have the following bottom-top derivation:

V: $\llbracket \text{eat} \rrbracket$

NP: $\llbracket ø \rrbracket(\llbracket \text{five} \rrbracket)(\llbracket \text{sandwich} \rrbracket)$
$= \lambda Y.\exists W[W \subseteq \llbracket \text{sandwich} \rrbracket \wedge |W|=5 \wedge W \text{ pp } Y|_{\llbracket \text{sandwich} \rrbracket}]$
shortened as:
$= \lambda Y.\exists W \in \llbracket \text{five sandwiches} \rrbracket [W \text{ pp } Y|_{\llbracket \text{sandwich} \rrbracket}]$

VP: $\llbracket \text{eat five sandwiches} \rrbracket$
$= \lambda X.\lambda Y.\exists W \in \llbracket \text{five sandwiches} \rrbracket \ [W \text{ pp } Y|_{\llbracket \text{sandwich} \rrbracket}](\lambda Y.\llbracket \text{eat} \rrbracket(Y)(X))$
$= \lambda X.\exists W \in \llbracket \text{five sandwiches} \rrbracket \ [W \text{ pp } \lambda Y \llbracket \text{eat} \rrbracket(Y)(X)|_{\llbracket \text{sandwich} \rrbracket}]$
written as:
$= \lambda X.\exists W \in \llbracket \text{five sandwiches} \rrbracket \ [W \text{ pp } \{U \cap \llbracket \text{sandwich} \rrbracket | \ \llbracket \text{eat} \rrbracket(U)(X)\}]$

NP: $[\![\emptyset]\!]([\![\text{two}]\!])([\![\text{girl}]\!])$
$= \lambda X.\exists Z\in [\![\text{two girls}]\!][Z \text{ pp } X|_{[\![\text{girl}]\!]}]$

S: $[\![\emptyset]\!]([\![\text{two}]\!])([\![\text{girl}]\!])([\![\text{eat five sandwiches}]\!])$
$= \lambda X.\exists Z\in [\![\text{two girls}]\!][W \text{ pp } X|_{[\![\text{girl}]\!]}]([\![\text{eat five sandwiches}]\!])$
$= \exists Z\in [\![\text{two girls}]\!][Z \text{ pp } [\![\text{eat five sandwiches}]\!]|_{[\![\text{girl}]\!]}]$
$= \exists Z\in [\![\text{two girls}]\!][Z \text{ pp } \{V \cap [\![\text{girl}]\!] \mid [\![\text{eat five sandwiches}]\!](V)\}]$
$= \exists Z\in [\![\text{two girls}]\!][Z \text{ pp } \{V \cap [\![\text{girl}]\!] \mid$
$\exists W\in [\![\text{five sandwiches}]\!][W \text{ pp } \{U \cap [\![\text{sandwich}]\!] \mid [\![\text{eat}]\!](U)(V)\}]\}]$

This says that there is set Z of two girls which can be subdivided into subcollections $V \cap [\![\text{girl}]\!]$ —the members of a pseudo-partition X—, and for each of the sets V there is a set W of five sandwiches which is subdivided by Y into subsets $U \cap [\![\text{sandwich}]\!]$ such that the set U is eaten by V.

The collective extreme obtains in a model in which a set $\{girl_1, girl_2\}$ induces the (pseudo-) partition $X = \{\{girl_1, girl_2\}\}$. In this case, $\{girl_1, girl_2\}$, say G_2, is assigned a set $\{s_1, s_2, s_3, s_4, s_5\}$, say S_5, of five sandwiches which in turn induces the partition $\{S_5\}$, so that G_2 stands in the eat-relation to S_5. The distributive extreme occurs in a model in which the set $\{girl_1, girl_2\}$ induces the partition $\{\{girl_1\}, \{girl_2\}\}$. And to each of the sets $\{girl_1\}$ and $\{girl_2\}$ is assigned a set S_5^1 and a set S_5^2, respectively, which induce the partitions $\{\{s_1\}, \{s_2\}, \{s_3\}, \{s_4\}, \{s_5\}\}$ and $\{\{s'_1\}, \{s'_2\}, \{s'_3\}, \{s'_4\}, \{s'_5\}\}$ such that $girl_1$ eats s_i ($1 \leq i \leq 5$) and $girl_2$ eats s'_i ($1 \leq i \leq 5$). Our semantics even allows for a situation like the one depicted in Figure 10.

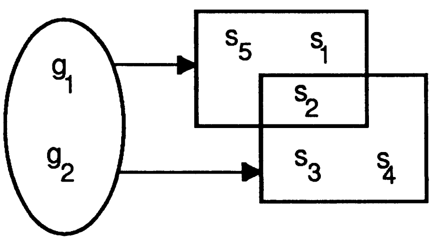

Figure 10

One way to make sense of this is to interpret Figure 10 as the situation in which the girls had two sandwiches each and shared the fifth one. In other words, if the situation in Figure 10 holds, one would expect that the situation of either Figure 11a or Figure 11b holds, too.

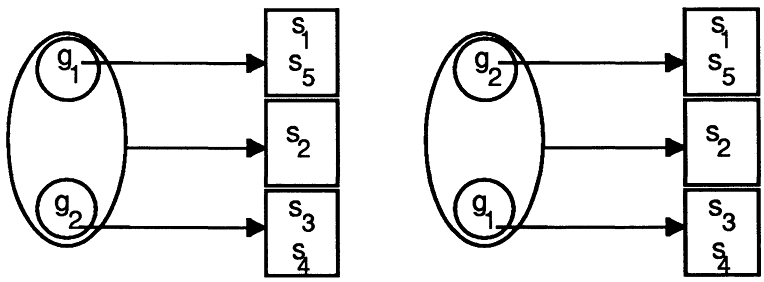

Figure 11

We may safely assume that all two-place predicates satisfy this regularity, except for those verbs which are to be marked as strictly distributive in one of their arguments, but we do not know such verbs in English or Dutch. In general, this expresses the intuition that two-place verbs remain vague as to whether the individuals which make up their arguments are involved strictly individually or in groups. Even the intransitive *die* might be argued to be vague in this respect: one need not to have traveled on the Orient Express to realize that a killing may be collective.

Along the same lines, our analysis of (3) results in:

(3') $\exists Z \in [\![\text{four men}]\!][Z \text{ pp } \{V \cap [\![\text{man}]\!] \mid \exists W \in [\![\text{three tables}]\!]$
$[W \text{ pp } \{U \cap [\![\text{tables}]\!] \mid [\![\text{lift}]\!](U)(V)\}]\}]$

This formula captures all the so-called readings of Scha and Link at once. We can even go one better than that, as exemplified in Figure 12, which makes (3') and hence sentence (3) true. It plainly shows that among the lifting sets there may be entities other than men and among the sets lifted entities other than tables.

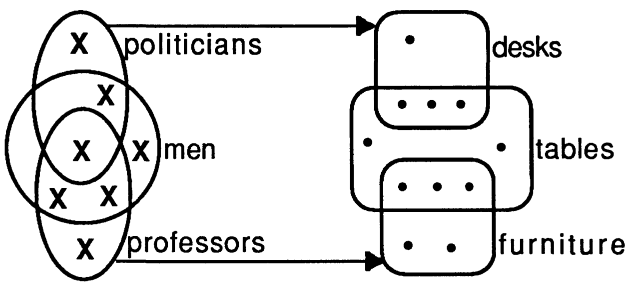

Figure 12

In other words, if three politicians lifted four desks (and there are three tables among the desks) and if four professors lifted five pieces of furniture (among which three tables) and if two of the politicians and three of the professors are male (one being a politician and one a professor), (3) will be a true sentence.

As it turns out, the core of this example, given in Figure 13, may give rise to an interesting debate.

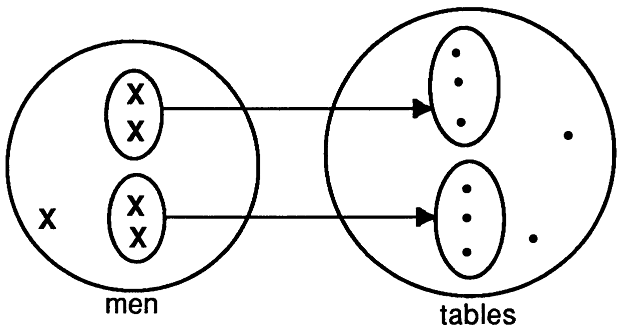

Figure 13

Could (3) be true when two of the four men lifted three tables, while the remaining two lifted two other ones? It is plain from the outset that this use of (3) will be rare and highly dependent on the particular time-structure involved. But this is not the issue here, wondering whether this use exists at all. To be honest, we ourselves have different opinions on this matter. According to the first author, (3) could be used in situations in which for each of the individuals it is checked whether he was involved in the lifting of three tables. Imagine, e.g., a removal company requiring a minimum duty for each man. The second author, however, has a less flexible imagination. He would grant situations like these only on a cumulative reading, where the subject and object NP do not have scope over each other (cf. Van der Does (1991)). E.g., the situation in Figure 13 would verify the cumulative reading of *Four men lifted six tables.*[24]

4.4 *All* and *each* and *every* and *a*

In this section, we will consider PLUG's capacity to solve problems one may have with the elimination of distributive and collective readings. Our basic point is that these two notions are not structural in the sense that that a sentences is either distributive or collective. Sentences like (3b) are neither collective nor distributive, whereas a sentence like (3a) is distributive as far as the subject NP is concerned:

(3a) Each men lifted three tables
(3b) All men lifted three tables

However, we ascribe this sort of distributivity to the lexical item *each*, which, being a SPEC, is able to penetrate deeply into the sentential structure.

Of course, the difference between the distributive (3a) and the non-distributive (3b) is due to the use of *each* and *all*, respectively. The rules in PLUG reduce the semantics of plural determiners to the semantics of their lexical items, one of the corollaries being that the distinction between *each* and *all* is not made in the rules of grammar. Without further specifications, PLUG would not be able to give a satisfactory treatment of (21) and (22). In particular, it would not distinguish between (21a) and (21b).

(21) a. All men dispersed
 b. *Every man dispersed
(22) a. All men wore trousers
 b. Each man wore trousers

We prefer a solution in which inherently distributive determiners can be uniformly distinguished from their default unmarked counterparts. Instead of having a uniform lift from $<<e,t>,<<e,t>,t>>$ to $<<e,t>,<<<e,t>,t>,t>>$, as given by the SPEC $\emptyset$, one could make a distinction between lifts for distributive and neutral determiners. The latter, among which *all*, are treated as in PLUG: they are assigned the node SPEC given above. But if a language has determiners with only a distributive interpretation, such as *each* and *every*, then they denote their usual relation (i.e. inclusion), but in a restricted way. In order to define this we need a function AT of type $<<e,t>,<<e,t>,t>>$:

(23) $\text{AT} =_{df} \lambda X_{<e,t>} \lambda Y_{<<e,t>,t>}. \forall Z_{<e,t>} [Z \in Y \leftrightarrow |Z \cap X| = 1]$

So, AT(X) is the set of all atoms which can be formed out of X: it is Scha's *-operator. We now define a function $[\![\text{SPEC}_d]\!]$ of type $<<<e,t>,<<e,t>,t>>, <<e,t>,<<<e,t>,t>,t>>>$ as follows:

(24) $[\![\text{SPEC}_d]\!] =_{df} \lambda D \lambda X \lambda Y. \exists W[W \subseteq X \wedge D(X)(W) \wedge W \text{ pp } \text{AT}(X) \cap Y]$

We propose to interpret all *distributive* NPs by use of the lexical format $[\![\text{SPEC}_d]\!]$, dependent on whether or not a language has this sort of constituents.

As said, this is a matter of the lexicon rather than of syntax itself. If a language contains, say SPEC_c-elements, which express collectivity only, one has to provide for an appropriate lexical definition. *All* in English and *alle* in Dutch do not belong to SPEC_d: they range over collective and distributive interpretations and so they capture (24) as just one of their possible interpretations. To our knowledge, English and Dutch do not have simple SPEC_c-determiners, i.e. no determiners where in all models the pseudo-partition Z necessarily contains just one member Z where $|Z| > 1$. In contrast, both English and Dutch contain SPEC_d-determiners, just like some languages have a dualis, whereas the other languages have more general forms.

Simplifying, it may be assumed that the intransitive verbs *disperse* and *wear trousers* respectively satisfy the meaning postulates:

(25) a. $X \in [\![\text{disperse}]\!] \Rightarrow |X| > 2$
b. $X \in [\![\text{wear trousers}]\!] \Rightarrow X$ is an atom

Thus, the truth conditions of (21) and (22) are respectively:

(21') a. $[\![\text{man}]\!]\ \text{pp}\ [\![\text{disperse}]\!]|_{[\![\text{man}]\!]}$
b. $\text{AT}([\![\text{man}]\!]) \subseteq [\![\text{disperse}]\!]$
(22') a. $[\![\text{man}]\!]\ \text{pp}\ [\![\text{wear trousers}]\!]|_{[\![\text{man}]\!]}$
b. $\text{AT}([\![\text{man}]\!]) \subseteq [\![\text{wear trousers}]\!]$

Plainly, (21'b) cannot obtain, for (25) stipulates $[\![\text{disperse}]\!]$ to be atomless. On the other hand, (21'a), (22'a) and (22'b) are satisfiable.

It is worth our noting that in (22'a), we again have distributivity as an "extreme" case. The pseudo-partition stated to exist is forced by (26) to be the unique finest possible. Consequently, we see that for all distributive predicates D we have $[\![\text{each N D}]\!] = [\![\text{all N D}]\!]$. It is also nice to observe that the desirable result is accomplished without imposing ambiguity on the NPs involved.

There is a subtlety in the definition of SPEC_d which may be overlooked: when this determiner is combined with an N, it yields a set of sets rather than a set of atoms. Consequently, a sentence in which, say, *each man* is combined with a non-distributive VP, may be satisfiable as long as the interpretation of the VP contains the relevant atoms.

As far as the indefinite article is concerned, one could analyze the Dutch NP *een kind* (a child) as (26a) whereas *een kind* (*een* pronounced as *één* (= *one*)) would be analyzed as (26b):

(26)
a. $[\![\mathrm{SPEC_d}]\!]([\![\mathrm{SG}]\!])([\![\mathrm{child}]\!])$
$= \lambda Y.\exists W[W \subseteq [\![\mathrm{child}]\!] \wedge |W| = 1 \wedge W \text{ pp } \mathrm{AT}([\![\mathrm{child}]\!]) \cap Y]$

b $[\![\mathrm{spec}\emptyset]\!]([\![\mathrm{one}]\!])([\![\mathrm{child}]\!])$
$= \lambda Y.\exists W[W \subseteq [\![\mathrm{child}]\!] \wedge |W| = 1 \wedge W \text{ pp } Y|_{[\![\mathrm{child}]\!]}]$

In contrast to (26b), (26a) allows the presence of more than one child, be it that the other children present should be part of non-atomic collections. However, one could also reconsider the definition of SG by allowing ≥1 rather than =1. Then *één kind* (one child) would correspond to the situation in which there is exactly one child, whereas *een kind* (a child) would allow for more children.

4.5 *Negation*

For several reasons we are interested in the interaction between quantification and negation. One of them is that the aspectual behaviour of sentences having terminative aspect changes under negation. Thus, there is a crucial difference between sentences in (27) on the one hand and sentences in (28) - (30) on the other:

(27) a. Two girls ate five sandwiches
b. The three men lifted four pianos
(28) a. Two girls ate no sandwich
b. The three men lifted no pianos
(29) a. No girl ate five sandwiches
b. Nobody lifted four pianos
(30) a. Two girls did not eat five sandwiches
b. The three men did not lift four pianos

This difference shows up if one tries to combine the latter with durational adverbials like *for an hour*: (27) is terminative because a combination with *for an hour* results into a queer sort of repetition, whereas the other sentences have a durative reading; they pertain to a state or an unbounded situation.

The second reason is the observation in Verkuyl (1987) that the intuitive distinction between collective and distributive readings is neutralized under negation. In our terms: the two extremes of the scale no longer seem to matter under negation. Above, the collective and distributive extremes in (27b) were characterized in terms of an opposition between $\{\{man_1,man_2,man_3\}\}$ and the set containing only singletons, $\{\{man_1\}, \{man_2\}, \{man_3\}\}$ for the subject NP.

It makes no sense at all to do that for (28b) nor for (29b). The intuitive reason for this is: if no(ne of the) men in a model lifted pianos, why should one be interested in structuring the set of men? And, if two girls ate no sandwich, why should one be interested in the way they did not eat them.

Neither Scha (1981) nor Link (1983;1987) paid attention to this phenomenon. More in general, in the literature on collective and distributive quantification virtually no attention has been given to the link with aspect and negation. The connection is that in sentences like (27) both the nature of its aspect and the nature of quantification change, dependent on the sort of negation. Lønning (1989) argued that Verkuyl (1987) wrongly opposed sentences like (27b) to (29b) rather than (30b) to make its point on the blurring of the distinction between collectivity and distributivity. However, it is very hard to determine the meaning of a negative sentence like (30b) without taking into account intonation, because the negation element may operate dependent on the place where contrastive stress is put. The blurring is visible if the sentences in (29) are chosen not so much in terms of a contrary or dual opposition to (27) but rather in terms of their proper paraphrases. The sentences in (29) form the clearest examples in which an NP does not express a quantity and they do not like to be paraphrased with an appeal to the notion of distributivity or collectivity. An NP without reference to a specified quantity makes it impossible for the sentences in (27) to express terminative aspect.[25] Likewise, NPs like *no girl* or *nobody* do not pertain to a collectively or distributively taken empty set. Finally, if a sentence like (30a) is read non-contrastively (if that is possible at all), we do not feel forced to assume that there is a set of distributive and collective readings. The presence of *not* immediately affects the function covering the scale between the two extremes. Thus, there will be no partition of the set of no girls such that for each of its members there is a set of five sandwiches etc. Likewise, (30b) should not have e.g. a reading in which the three men each did not lift four different pianos on different occasions. If they did not lift or eat, how do we know how they did not do this? Or are we to speak about intentions in dealing with quantification?

The present PLUG-approach allows for *two girls* in (27) and (30) to be pseudo-partitioned. As observed, the sentences in (30) may be used contrastively, as in (31):

(31) a. ... but they ate FOUR sandwiches
b. ... but they lifted four HARPSICHORDS

The continuations in (31) make (30) terminative, as pointed out in Verkuyl (1987), where the negation theory of Jacobs (1982) was used.

Sentences like (28) - (30) form a litmus test for the present analysis (cf. also Lønning 1989). Therefore we shall give their derivations in some detail. We shall start out with (29a).

VP : $[\![\text{eat five sandwiches}]\!]$
$= \quad \lambda X.\exists W\in [\![\text{five sandwiches}]\!][W \text{ pp } \{U \cap [\![\text{sandwich}]\!] \mid [\![\text{eat}]\!](U)(X)\}]$

DET¹ : NO
$=_{df} \quad \lambda U_{<e,t>}\lambda V_{<e,t>} | U \cap V | = 0$

DET² : $[\![\text{no}]\!] = [\![\emptyset]\!](\text{NO})$
$= \quad \lambda D\lambda X\lambda Y.\exists W[W\subseteq X \wedge D(X)(W) \wedge W \text{ pp } Y|_X](\lambda U\lambda V.|U\cap V| = 0)$
$= \quad \lambda X\lambda Y.\exists W[W\subseteq X \wedge |X\cap W| = 0 \wedge W \text{ pp } Y|_X]$
$= \quad \lambda X\lambda Y.\exists W[W\subseteq X \wedge |W| = 0 \wedge W \text{ pp } Y|_X]$
$= \quad \lambda X\lambda Y.\{\varnothing\} = Y|_X]$

NP : $[\![\text{no}]\!]([\![\text{girl}]\!])$
$= \quad \lambda Y.\{\varnothing\} = Y|_{[\![\text{girl}]\!]}]$

$[\![\text{no}]\!]([\![\text{girl}]\!])([\![\text{eat five sandwiches}]\!])$
$= \quad \{\varnothing\} = [\![\text{eat five sandwiches}]\!]|_{[\![\text{girl}]\!]}$
$= \quad \{\varnothing\} = \{V\cap[\![\text{girl}]\!] \mid \exists W\in [\![\text{five sandwiches}]\!]$
$\qquad\qquad [W \text{ pp } \{U \cap [\![\text{sandwich}]\!] \mid [\![\text{eat}]\!](U)(V)\}]$

That is: looking for girls in the sets which eat five sandwiches you find none.

As to (28a), we obtain:

VP : $[\![\text{eat no sandwich}]\!]$
$= \quad \lambda X.\{U \cap [\![\text{sandwich}]\!] \mid [\![\text{eat}]\!](U)(X)\} = \{\varnothing\}$

NP : $[\![\emptyset]\!]([\![\text{two}]\!])([\![\text{girl}]\!])$
$= \quad \lambda U.\exists Z\in [\![\text{two girls}]\!][Z \text{ pp } U|_{[\![\text{girl}]\!]}]$

S : $[\![\text{NP}]\!]([\![\text{VP}]\!])$
$= \quad \lambda U.\exists Z\in [\![\text{two girls}]\!][Z \text{ pp } U|_{[\![\text{girl}]\!]}]([\![\text{eat no sandwich}]\!])$
$= \quad \exists Z\in [\![\text{two girls}]\!][Z \text{ pp } [\![\text{eat no sandwich}]\!]|_{[\![\text{girl}]\!]}]$

$= \quad \exists Z \in [\![\text{two girls}]\!][Z \text{ pp } \{V \cap [\![\text{girl}]\!] \mid [\![\text{eat no sandwich}]\!](V)\}]$
$= \quad \exists Z \in [\![\text{two girls}]\!][Z \text{pp}\{V \cap [\![\text{girl}]\!] \mid \{U \cap [\![\text{sandwich}]\!] \mid [\![\text{eat}]\!](U)(V)\} = \{\varnothing\}\}]$

This says that there is a set of two girls such that if you look at the collection of things eaten by them, either collectively or individually or both, you will find no sandwiches.

The relevant part of the derivation of (29b) is:

VP : $[\![\text{eat five sandwiches}]\!]$
$= \quad \lambda X.\exists W \in [\![\text{five sandwiches}]\!][W \text{ pp } \{U \cap [\![\text{sandwich}]\!] \mid [\![\text{eat}]\!](U)(X)\}]$
NP : $[\![\text{nobody}]\!]$
$= \quad \lambda Y.\{\varnothing\} = Y|_{[\![\text{human}]\!]}$

S : $[\![\text{nobody}]\!]([\![\text{eat five sandwiches}]\!])$
$= \quad \{\varnothing\} = [\![\text{eat five sandwiches}]\!]|_{[\![\text{human}]\!]}$
$= \quad \{\varnothing\} = \{V \cap [\![\text{human}]\!] \mid \exists W \in [\![\text{five sandwiches}]\!]$
$[W \text{ pp } \{U \cap [\![\text{sandwich}]\!] \mid [\![\text{eat}]\!](U)(V)\}]$

The bottom-line of the derivation of the non-contrastive (30a) is:

$\exists Z \in [\![\text{two girls}]\!][Z \text{ pp } \{V \cap [\![\text{girl}]\!] \mid \neg\exists W \in [\![\text{five sandwiches}]\!]$
$[W \text{ pp } \{U \cap [\![\text{sandwiches}]\!] \mid [\![\text{eat}]\!](U)(V)\}]\}]$

If we make use of Jacobs (1982) "correction predicate" CORR, the contrastive reading of (30a)+(31a) would become something like:

$\exists Z \in [\![\text{two girls}]\!][Z \text{ pp } \{V \cap [\![\text{girl}]\!] \mid \neg\text{CORR}\exists W \in [\![\text{five sandwiches}]\!]$
$[W \text{ pp } \{U \cap [\![\text{sandwiches}]\!] \mid [\![\text{eat}]\!](U)(V)\}]\}] \wedge \text{CORR}\exists W \in [\![\text{four sandwiches}]\!]$
$[W \text{ pp } \{U \cap [\![\text{sandwiches}]\!] \mid [\![\text{eat}]\!](U)(V)\}]\}]$

which says that there are two girls for which it is not correct to say that they ate five sandwiches but that it is correct to say that they ate four.[26]

In all these cases the terminative and the durative nature of the sentences involved is correctly predicted. That is, in (29a) and (29b) and (28) the cardinality of the negative NPs is zero. In the non-contrastive interpretation of (30a), the cardinality information in *five sandwiches* is affected by the negation, but in (30a)+(31a), the cardinality information expressed by the NPs is not affected as the negation operates on the predicate CORR (cf.Verkuyl 1987, to appear for a more detailed analysis).

Summarizing, our proposal is to interpret both the singular and the plural form of *no* by means of the SPEC ø as ø(no). One may wonder whether this yields truth conditions which are too strong.[27] For instance, taking numerals in their at least meaning, are sentences like (32) and (33) satisfiable simultaneously?

(32) Two girls ate three sandwiches
(33) No girl(s) ate four sandwiches

Of course, they should be. But consider the distributive extreme of (32) where each of the two girls ate exactly three sandwiches (while nothing else happened). Is this not also a situation in which (34) is true, contradicting (33)?

(34) Two girls ate six sandwiches

The idea would then be to grab the six sandwiches together, yielding a pseudo-partition, and then to distribute them over the girls. But note that this is not sufficient for the truth of (34) because this requires that there be at least one collection containing a girl which stands in the eat-relation to collections containing six sandwiches. But there are no such collections here.

5 Conclusion

As quoted above, Link (1984:23) observed that “we touch on a methodological point of quite a general nature in linguistics: Where exactly does the line of demarcation run between proper readings and mere models realizing a reading?” In this respect our stance in the semantics of plural NPs is quite radical. In an atemporal framework, as the ones discussed above, there is no meaningful way to discern among collective and distributive readings of sentences which do not contain lexical items triggering these readings explicitly. We have presented an atemporal, compositional semantics for arbitrary plural and singular NPs, called PLUG, in which NPs are dealt with as being neutral or vague in this respect. Consequently, the numbers of readings of a transitive sentence reduces to one.

In the Theory of Generalized Quantification relatively little attention is paid to the internal structure of determiners. All of them would correspond to an analysis on the level of DET^2 in the above PLUG-representations. We do not deviate from this view, but we have provided for a more detailed analysis of determiner structure. One might regard the "area" between DET^0 and DET^2 as a matter of the lexicon rather than of syntax, but it seems rewarding to apply syntactic techniques to a systematic investigation of the determiner structure. At any rate, the analysis of DET^2 in terms of SPEC(DET^1) is compositional.

Van der Does (1991) contains more technical details of the present set up and formal comparisons with other proposals, among other things. Verkuyl (to appear) contains a temporalized version, PLUG$^+$. In Van der Does and Verkuyl (in preparation), we hope to confront the notions of partition, pseudo-partition and cover with temporal data in order to see whether the transition from an atemporal treatment of quantification differs substantially from a treatment in which the interaction between temporal and atemporal structure is taken into account. It should show how the temporal information conveyed by a verb influences the type of verifying situations.

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[1] We thank Remko Scha for having read this section very carefully.

[2] Scha builds in the possibility of having scope ambiguity by including the optional rule F4 (1981:506). This is to provide for readings that are marked, e.g. by intonation or by information in the discourse. Including F4 would bring the number of readings for Scha to eighteen.

[3] Actually, Scha (pers. comm.) says that there are more readings not derivable from the first nine, but he does not indicate how many more there are supposed to be.

[4] In this section we use Scha's notation. Using a more common notation, e.g. (4) could equivalently be written as $\lambda P \exists X[X \subseteq \text{men} \ \& \ |X| = 4 \ \& \ P(X)]$.

[5] Different though they may appear at first sight, it is argued by Landman (1989) that logically speaking Scha's and Link's models amount to the same thing. This point is also made in Lønning 1989. However, as pointed out in Van der Does (1991), their treatments of distributivity do differ.

[6] ${}^{\bullet D}\mathrm{Lift}' := \lambda v \lambda w \forall u[u^{\bullet}\Pi w \rightarrow \mathrm{Lift}'(v,u)]$.

[7] There is no dot because the lambda-expression turns the relation into a one-place predicate.

[8] ${}^{D\bullet}\mathrm{Lift}' := \lambda v \lambda w \forall u[u^{\bullet}\Pi v \rightarrow \mathrm{Lift}'(u,w)]$.

[9] In the preceding version, we assumed that Link was correct in assuming that LF2 and LF 7 are logically equivalent, as stated in Link (1984). However, Jan Tore Lønning (pers. communication) pointed out that LF 7 is not equivalent to LF2, in spite of Link's own contention.

[10] The vagueness is also observed by Roberts (1987) and Lønning (1989), who argue that NPs always have a C1 reading. According to them, the different readings are obtained by the use of a distributive operator at the VP level. However, see Van der Does (1991) for a discussion of this position.

[11] Neither Gillon nor Lasersohn observe that these readings are allowed by Scha's C2. Moreover, the issue of the existence of intermediate readings has resulted in a debate of Lasersohn against Gillon (see Gillon (1987, 1989) and Lasersohn (1989)).

[12] See Lasersohn (1989) for arguments against intermediate readings, and Gillon (1989) for a reply.

[13] The relation pp is the converse of the relation p used in Verkuyl (to appear). Note that $W\ \mathrm{pp}Y|_X$ is equivalent to $\exists Z[Z\mathrm{p}W \ \& \ Z = Y|_X]$, which points to a proper analysis of thematic relations expressed by the argument structure, because some of them require a subset relation rather than identity.

[14] Recall that we do not treat group nouns.

[15] Cf. Scha (1981), Verkuyl (1981), Hoeksema (1983), among others.

[16] In Verkuyl (1981) *some* and *all* are treated as determiners, whereas the other quantifiers in (9) are introduced as NUM' as in Figure 7. However, the objections against the treatment of the empty determiner holds against this proposal as well.

[17] In the generative syntactic literature, the assumption of an empty determiner for NPs without an overt determiner has been argued for many times.

[18] In generative grammar it would be standard to assume in Figure 8 a DET^0 as the only daughter node of a nonbranching DET^1. Cf. Verkuyl (to appear) for a more detailed analysis of the internal determiner structure in terms of X-bar syntax. The present analysis is compatible with the so-called DP-analysis of Abney (1987), in which det is taken to be the head of the NP, but it would also allow to take N as the head of the NP. A choice might be determined by the syntax (of agreement) of a specific language, so some languages may take the DET as the head of the NP, other languages might take the N itself.

[19] This is more general than the restriction used implicitly in Scha's C2: $Y \text{ restr } X =_{df} Y \cap \wp(X)$, which expresses a restriction to those groups in *Y* which consist of X's only (Cf. Van Benthem, 1991, and the discussion in section 3 above).

[20] Here we treat the difference between *exactly three* and *three* basically a matter of the lexicon. Of course one can also treat *exactly* as the identity relation between numbers if we treat *three* itself as: $[\![\text{three}]\!] = \lambda r \lambda X \lambda Y.\ |X \cap Y|\ r\ 3$ (Cf. Scha, 1981).

[21] Note that the two-place relation $\in$ is replaced by a three-place relation also involving the noun extension. In Van der Does (1991) it is shown how this is related to conservativity in type $\langle\langle e,t\rangle,\langle\langle\langle e,t\rangle,t\rangle,t\rangle\rangle$.

[22] Cf. Verkuyl (1987; 1988) for the asymmetry inherent in aspectual construal.

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[23] Here, only of the (at least) four meanings of many is treated. Cf. Westerståhl (1985).
[24]Examples like these were at the heart of the debate of Lasersohn against Gillon mentioned above.
[25] Verkuyl (1988) uses the notion of [+SQA] to refer to NPs which denote a Specified Quantity of A. In Verkuyl (to appear) this notion is defined in terms of the Theory of Generalized Quantification. Roughly, an NP is [+SQA] iff it expresses cardinality information about the set W in representations as discussed in the present section. An NP is [-SQA] if it denotes the empty set or if it does not contain any cardinality information at the right type-logical level (i.e. about W).
[26] For Jacobs, CORRα is true iff α is true and α has a sufficient adequacy value *ad* (which is determined by the context).
[27]This issue was raised by one of the referees.

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