Quantification and Predication

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1 Introduction

In this paper, we will consider sentences like (1) and (2) from the point of view of quantification and predication.

(1) Three girls mailed a letter

(2) Three girls mailed four letters

As to the issue of quantification, Verkuyl & Van der Does 1991 tried to reduce the large numbers of readings often assigned to these sentences to just one by adopting a so-called scalar approach. This approach is based on the following observation.

Scha 1981 stipulated that NPs are ambiguous between a distributive (D) reading and two collective readings (C_1 and C_2). In sentences with two NPs, combinations of these three readings lead to at least nine readings for (2): DD, DC_1, ..., C_2C_2. For example, on the DD-reading of (2), each of the girls mailed four letters, each letter on a different occasion. On the C_1C_1-reading, the girls mailed the four letters together on one occasion. The C_1C_2-reading would say that the three girls as a group mailed four letters, say on one occasion one letter, and one day later the three other letters. Observe that C_2 allows both 1+3- and 2+2-configurations of the set of four letters. In fact it also comprises D and C_1. On the C_1C_2- reading just mentioned, C_2 allows also the 4- and the 1+1+1+1-configuration. Here the idea of a scale comes up quite naturally, but this idea was not taken up by Scha himself, nor did Link 1984 pay attention to it.

Verkuyl & Van der Does 1991 decided to take a strengthening of the C_2-reading as basic, in fact as the only reading that can be attached to (1) and (2). They chose (3) as the format for the analysis of the denotation of NPs like *three girls* in sentences such as (1) and (2):

(3) $\lambda\mathbf{P}.\exists X[X \subseteq [\![\mathrm{girl}]\!] \wedge |X| = 3 \wedge \exists\mathbf{Q} cv_p X[\mathbf{Q} = \mathbf{P}|_{[\![\mathrm{girl}]\!]}]]$

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It says that there is a set X of three girls that can be partitioned into a collection $\mathbf{Q}$ which is the predicate $\mathbf{P}$ restricted to the set of girls.[1] In this way, we obtain Scha's D just in case $\mathbf{Q}$ is partitioned into three singletons whereas (an instance of) Scha's C_1 is obtained when $\mathbf{Q} = \mathbf{P}|_{[\![\mathrm{girl}]\!]} = \{X\}$.

The leading idea of the scalar approach is the empirical fact that sentences like (1) and (2) do not give away which configuration is actualized, and that the variant of C_2 in (3) seems the right way of expressing this. It includes the whole range between and including the extremes D and C_1. However, although (3) has the virtue of scalarity and thus captures the underdeterminedness of information inherent in (1) and (2), it still has some shortcomings. For one, it does not capture the so-called cumulative reading of these sentences. In case of (2) this reading would say that the total number of girls mailing letters is three and that the total number of letters being mailed is four. This cannot be obtained by (3) because the predicate $\mathbf{P}|_{[\![\mathrm{girl}]\!]}$ will always contain information about the second argument NP, whereas the cumulative reading requires the scope of the two NPs to be independent.

At this point, the second conjunct of the title of the present paper comes in: one cannot have a theory of (collective) quantification without a theory of (collective) predication which tells us how exactly the argument denotations and their members are involved in the predication itself; i.e., we want to account for the possible ways in which the members of the argument NP satisfy the predicate. The problem is—informally speaking—that in (1) we use the predicate *mail a letter* whereas in fact we might speak about mailing three letters. Sentences like (4) show the problem even more clearly.

(4) Four men lifted three tables

They allow us to speak about between three and twelve tables which were lifted, whereas the predicate is 'lift three tables'. This means that in order to maintain our scalarity thesis we need the formal means to provide underdeterminedness as to how the different tables in (4) have been involved in the predication, and to establish how this affects the quantification expressed by the NP.

It is this perspective which will be worked out in detail here. We will formalize such notions as VP-predication, Path, 'mode' within the framework of generalized quantification, but with the explicit purpose of integrating in it some points of view from the linguistic tradition called *localism*. Sentences like (1) express a change of state which in a localistic approach is

[1] In (3), cv_p stands for partitioning, a form of covering. For a discussion on other forms of covering see Verkuyl & Van der Does 1991, Van der Does 1993.

analyzed in terms of an abstract "movement" from a point of origin to a point at which the predicate is satisfied. In

(5) Mary mailed a letter

one may say that Mary is "going through" a Path as a way of saying that she underwent the predication. Similar things can be said of the set of three girls, in the sense that we need to pay attention to the way in which each of the girls undergoes the predication, i.e., has an individual Path. In particular we are interested in the way the individual Paths of the girls may interfere. In Verkuyl 1988 it was argued that only two modes are available in this respect: either the Paths are totally disjoint or they are essentially one. It is obvious that the metaphoric way of introducing the intuitive notion of localism—change in time expressed by a verb like *mail* can be dealt with in terms of a cumulative structure built up from an origin to an end—is to be replaced by a precise formalism. This has been done in Verkuyl 1993 and a simplified version will be used in section 3.1. We shall show that the incorporation of localistic insights into the model-theoretic approach of generalized quantification makes it possible to reduce ambiguities.

This paper has the following structure. In section 2 we give an overview of our earlier attempt to reduce the ambiguity of plural sentences, and also of the problems this gave rise to. Next, section 3 introduces the linguistic tradition of localism and its core concept of a Path. After formalizing this notion, we focus on two 'modes' namely the one in which the relevant members of the external object have distinct verbal Paths, and the one in which they share the same verbal Path. These extremes are called the π-injective and the π-constant modes. In section 4, we prove that the π-constant mode, which captures a special kind of collectivity, gives an impressive collapse of plural as well as of polyadic readings. We therefore suggest to capture the readings by means of the two π modes combined with iterated neutral plural quantification, but to use no further representation within the semantics.

2 The Scalar Approach

In this section we formally characterize our earlier attempts to reduce the ambiguity of sentences like (1) and (2). To this end, section 2.1 first discusses the quantifier liftings in Van der Does 1993, which generalize the treatment of numerals in Scha 1981. In section 2.2 we give a short overview of the considerations which led to the attempt in Verkuyl and Van der Does 1991 to reduce ambiguities, and of the subsequent discussion it gave rise to. Finally, in section 2.3 we discuss some problems we and others have with that proposal. So, the present section paves the way for a further development of our theory in section 3 and section 4.

2.1 Quantifier liftings

Determiners as they are studied in the theory of generalized quantification live in type $(et)(ett)$; they are relations among sets. These relations have to satisfy some constraints. In particular, most natural language determiners are conservative, and have extension and isomorphy.

Definition 1 A determiner $\mathbf{D}$ is a functor which assigns to each non-empty domain E an element from $\wp(\wp(E) \times \wp(E))$. $\mathbf{D}$ is *conservative* (CONS) iff for each E and all $A, B \subseteq E$: $\mathbf{D}_E AB \Leftrightarrow \mathbf{D}_E A A \cap B$. $\mathbf{D}$ has *extension* (EXT) iff for all $E, E' \supseteq A, B$: $\mathbf{D}_E AB \Leftrightarrow \mathbf{D}_{E'} AB$. $\mathbf{D}$ has *isomorphy* (ISOM) iff for all bijections $\pi : E \longrightarrow E'$: $\mathbf{D}_E AB \Leftrightarrow \mathbf{D}_{E'} f[A] f[B]$, where $f[X] := \{f(x) : x \in X\}$. $\mathbf{D}$ is a *quantifier* iff $\mathbf{D}$ has CONS, EXT, and ISOM.

Due to EXT we may forget about E and stipulate that $\mathbf{D}AB$ iff for some E: $\mathbf{D}_E AB$. Also, for a quantifier $\mathbf{D}$ the truth of $\mathbf{D}AB$ only depends on the two cardinals: $|\{a \in A : a \notin B\}|$ and $|\{a \in A : a \in B\}|$. In this article, we shall often use *positive* determiners which require their arguments to be non-empty: $\mathbf{D}AB$ implies $A \neq \emptyset \neq B$.

The above treatment of determiners does not cover the phenomena typical of plural noun phrases. Following up on a suggestion in Van Benthem 1991, Van der Does 1992,1993,1994 studies different approaches to plural quantification by means of liftings from type $(et)((et)t)$ to type $(et)(((et)t)t)$. In the latter type the verbal part of a quantifier can be taken to hold of sets instead of just atoms, which makes quantification over these sets possible. The relevant liftings, given in (6), turn out to be generalizations of the numeral denotations in Scha 1981.[2]

(6)
$$\begin{array}{lll} \text{D} & \text{D}(\mathbf{D})X\mathbf{Y} & \Leftrightarrow & \mathbf{D}X\{d \mid \{d\} \in \mathbf{Y}\} \\ \text{C} & \text{C}(\mathbf{D})X\mathbf{Y} & \Leftrightarrow & \exists Z[\mathbf{D}XZ \ \wedge \ X \cap Z \in \mathbf{Y}] \\ \text{N} & \text{N}(\mathbf{D})X\mathbf{Y} & \Leftrightarrow & \mathbf{D}X \bigcup(\mathbf{Y} \cap \wp(X)) \end{array}$$

The names of the liftings are mnemonic for 'distributive', 'collective', and 'neutral', respectively. On the conceptual side they are based on the three perspectives we seem to employ in describing collections. We either quantify over 'atomic' partless individuals (D), or over genuine collections (C, Scha's C_1), or over the atoms which take part in certain collections (N, Scha's C_2)). In the latter situation we remain neutral as to the precise structure and size of these collections.[3]

[2] Scha (1981) discerns two forms of collective quantification instead of collective and neutral quantification.

[3] It is perhaps misleading to talk about distributive quantification as pointed out in Verkuyl 1994. The quantification is over atoms (i.e., the elements of type e or, equivalently, the singletons in type (et)), so 'atomic quantification' seems more accurate (Van der Does 1992, 1993). A *predicate* of sets can be distributive in that it is closed under non-empty subsets. Atomic predicates, which only contain singletons, are trivially distributive in this sense. Yet, the two notions should not be confused. E.g., a predicate

Disregarding scopal ambiguities and non-iterative forms of quantification, the determiner readings in (6) make (4) *Four men lifted three tables* nine times ambiguous. The readings are summarized in (7).

Nine readings is a bit much for the simple (4), and it is tempting to try to do with less. In the next subsection, we give an overview of our earlier proposal in that direction.[4]

(7)

NP_1/NP_2	D	N	C
D	DD	DN	DC
N	ND	NN	NC
C	CD	CN	CC

2.2 The One-Reading Hypothesis

The proposal in Verkuyl and Van der Does 1991 grew out of Verkuyl's idea to reduce the number of readings of (4) to one by taking the neutral reading of numerals as their plural denotation. The distributive and the collective readings could then be seen as depending on the nature of its verbal argument in a particular context. The verbal argument takes its value on a scale ranging from sets of singletons at the one end via several intermediate cases to singletons of sets at the other. The endpoints yield the distributive and collective 'reading', which, however, need not be represented at logical form.[5] As proposition 1 shows, the liftings in (6) can be used to formalize and generalize these intuitions.

Proposition 1 (FIN) *Call* $\mathbf{Y}$ positive *iff* $\emptyset \notin \mathbf{Y}$. *For all positive* $\mathbf{Y} \cap \wp(A)$ *it holds that* $\forall X \in \mathbf{Y} \cap \wp(A)[|X| = 1]$ *iff for all* $\mathbf{D}$, X: $\mathrm{N}(\mathbf{D})X\mathbf{Y}$ *and* $\mathrm{D}(\mathbf{D})X\mathbf{Y}$ *are equivalent. For all* $\mathbf{Y}$ *it holds that* $|\mathbf{Y} \cap \wp(A)| = 1$ *iff for all conservative* $\mathbf{D}$ *and all* X: $\mathrm{N}(\mathbf{D})X\mathbf{Y}$ *and* $\mathrm{C}(\mathbf{D})X\mathbf{Y}$ *are equivalent.*

PROOF. It is plain that for the appropriate $\mathbf{Y}$ $\mathrm{N}(\mathbf{D})$ is either equivalent to $\mathrm{D}(\mathbf{D})$ or to $\mathrm{C}(\mathbf{D})$. For the other directions we need FIN.[6] By way of example we show the distributive case. Assume that $\mathbf{Y} \cap \wp(A)$ contains a Z which is not a singleton. Since $\mathbf{Y} \cap \wp(A)$ is positive $|Z| > 1$. By FIN

can also be distributive up to a certain size, if it is closed under subsets of this size and larger. Still the concepts are intimately related: each atomic predicate gives rise to a unique predicate which is closed under non-empty subsets and arbitrary unions. Van der Does calls such sets of sets strictly distributive as opposed to the partly distributive ones, which are closed under subsets of size greater than one. A quantifier $\mathrm{D}(\mathbf{D})$ is equivalent to a relation between sets and (strictly) distributive sets of sets, and it derives its name from the latter (cf. Van der Does *ibidem*).

[4] We do not discuss Link's proposal not to discern between C and N 'for methodological reasons' (Link 1991). The logical differences between the denotations do not sustain this strategy (cf. Van der Does 1993 for details).

[5] Of course the proposal is limited to the NPs which allow the readings. Inherently distributive (i.e. atomic) NPs, such as *every man*, are obtained via the D-denotation of its determiner.

[6] 'FIN' indicates that we assume the models to have a finite domain.

$|\{d \in A : \{d\} \in \mathbf{Y}\}| < |\bigcup(\mathbf{Y} \cap \wp(A))|$. Set $n = |\bigcup(\mathbf{Y} \cap \wp(A))|$. Then $\mathrm{N}(\mathbf{n})A\mathbf{Y}$ but not $\mathrm{D}(\mathbf{n})A\mathbf{Y}$. □

Given proposition 1 it seems natural to give (2) only its NN reading and leave the other interpretations to context. However, there are some problems with this strategy.

2.3 Problems

The problems concern neutral readings of the external argument NP in a sentence with a complex VP. They are followed by suggested solutions.

Problem I In his reaction to an earlier version of Van der Does 1993, Lønning 1991 pointed out that the neutral reading allows a splitting of the collections quantified over. For example, on the neutral reading of *four men* in (4) the sentence may be true if there is a single man who lifted three tables besides a set of three men who also lifted three tables. That this is so, is best seen by means of (8), which is equivalent to $\mathrm{N}(\mathbf{D})X\mathbf{Y}$ for conservative $\mathbf{D}$:

(8) $\exists Y \subseteq X[\mathbf{D}XY \;\wedge\; \exists \mathbf{Z} \text{ cv } Y : \mathbf{Z} = \mathbf{Y} \cap \wp(X)]$

Here the relation $\mathbf{Z}$ cv Y—$\mathbf{Z}$ *covers* Y,—is defined by $\bigcup \mathbf{Z} = Y$. For (4), $\mathbf{Y}$ is the set of Z such that 'Z lifted three tables'. On the subject neutral reading (8) says: there is a set Y of four men and a cover $\mathbf{Z}$ of Y which is identical to the set of 'men-parts' of the collections Z which lifted three tables. But such a cover could be of the form $\{\{m_1\}, \{m_2, m_3, m_4\}\}$. The judgements whether or not these truth-conditions are correct differ widely.

Problem II Van der Does 1993 observes that the application of neutral quantifiers is also limited for another reason. Call a quantifier *bounded* iff there is an n such that for all A, B: if $\mathbf{D}AB$ then $|A \cap B| \leq n$ (cf. Westerståhl 1989). Numerals are prime examples of bounded quantifiers. In sentence (4), which iterates two numerals n and m, one expects under the normal scoping that the number of tables described lies between m and $n * m$. But neutral plural quantification allows an upper bound of $2^n * m$. In the truth-conditions of (4) as given by (8) $\mathbf{Z}$ may vary over the poorest cover $\{Y\}$ of Y via intermediate alternatives to its richest cover $\wp(Y)$. Thus, a neutral reading of the subject in (4) allows the number of tables to range from three to $2^4 * 3$.

2.4 Suggested Solutions

Problem II can be solved by strengthening the notion of cover in (8) to that of partition, mimimal cover, or pseudo-partition (cf. Verkuyl and Van der Does 1991, Van der Does 1993, for details). On the localistic analysis advocated in Verkuyl 1993, and introduced in section 3, partitions are a natural choice. As we shall point out in sections 3.1 and 4.1 the subject NP should then vary over the partition which arises from the equivalence relation 'to share the same Path'. However, this strategy offers no solution to Problem I.

As an alternative solution to Problem II, one could use a referential rather than a quantificational treatment of the NP, as suggested by Van der Does 1993,1994. To be precise, (8) could be replaced by (9).[7]

(9) $\mathbf{D}X\bigcup\mathbf{P} \ \wedge\ \mathbf{P}\ \textit{partitions} \bigcup\mathbf{P} \ \wedge\ \mathbf{P} = \mathbf{Y} \cap \wp(X)$

Here $\mathbf{P}$ is a contextually given set of sets. On this view the meaning of a determiner is a Kaplanian character; it needs contextual information to yield a denotation. In uttering (4) $\mathbf{P}$ remains underspecified, but (4) will be false with respect to any situation that does not comply with its structure. Note that this solution, too, leaves Problem I unsolved. The partitional reading still allows the spliced subject NP noted by Lønning. Verkuyl (1992, 1994) has some examples where such a split is not unlikely, and there may be pragmatical principles which rule out the remaining odd cases.

Van der Does 1993 feels uncomfortable with the spliced subjects. He seeks a semantical solution by holding that they only occur in case of non-iterative forms of quantification, such as the cumulative reading. On this reading arbitrary covers are allowed, for it leaves the quantificational elements of the internal argument NPs outside the scope of the external one. In the case of iterations there is an asymmetry between the external and the internal argument NP (cf. section 3.1). The internal argument NP favours a neutral reading—again allowing for covers,—while the external argument may be either distributive or collective. On this view a simple transitive sentence can have one of the following three readings:

(10)
$$\begin{array}{ll} \text{DN} & \mathrm{D}(\mathbf{D}_1)A\mathrm{N}(\mathbf{D}_2)B\mathbf{R} \\ \text{CN} & \mathrm{C}(\mathbf{D}_1)A\mathrm{N}(\mathbf{D}_2)B\mathbf{R} \\ \text{NN} & \mathrm{N}(\mathbf{D}_1)A\,\textsc{dom}(\mathbf{R} \cap \wp(A) \times \wp(B)) \\ & \wedge\ \mathrm{N}(\mathbf{D}_2)B\,\textsc{rng}(\mathbf{R} \cap \wp(A) \times \wp(B)) \end{array}$$

Van der Does does not explain why these readings are realized and no others. Below we point out that the first two readings are strict analogues of Verkuyl's π_{inj} and π_{con} modes (cf. section 3.2 and 4.2). Here π is the localistic Path-function accounting for the participancy of the members of the NP-denotation. The next section gives a detailed exposition of a formalization of this localistic notion. This prepares the ground for clearing up how Verkuyl's localistic approach figures within the wider landscape of plural and polyadic quantification.

[7] Recall that a set of sets $\mathbf{Y}$ *partitions* a set X iff (i.e., if and only if) the following three requirements are met: (i) $\bigcup\mathbf{Y} = X$, (ii) $\forall X, Y \in \mathbf{Y}[X \neq Y \Rightarrow X \cap Y = \emptyset]$, (iii) $\emptyset \notin \mathbf{Y}$.

3 Two modes of Predication

3.1 The π-function

Linguists have a long tradition of analyzing sentences like (1) and (11) into a NP VP structure, as in figure 1. In generative grammar, there are two independent lines of thought which lead to the idea of a certain asymmetry between the two NPs in figure 1.

(11) John loves Mary

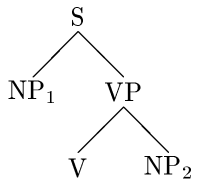

FIGURE 1 *NP VP*

The first line is syntactic and it shows up in the issue about whether or not languages are configurational, the leading hypothesis being that this is universally the case. Around 1980 it became standard to distinguish in sentences like (11) between the external NP$_1$-argument *John* and an internal NP$_2$-argument *Mary*. This means that the basic format of the sentences of natural language is the one in figure 1. The second line is the localistic one as it has developed in the generative framework. In Jackendoff (1978;1983;1990) and in Verkuyl (1978;1987;1993), the internal argument has closer ties with the verb than the external argument, at least when temporal structure is taken into account. Jackendoff (1972) still analyzed change expressed in sentences like

(12) a. John went to New York
b. John became angry

in terms of the 3-place format GO(X,Y,Z), where in (12a) X = John, Y = some point of departure and Z = New York, whereas in (12b) X = John, Y = some peace of mind, and Z = the state of being angry, but in his later work he comprised the two arguments of the GO-predicate into a Path obtaining asymmetry. Verkuyl (1972:106) accepted the NP VP asymmetry in view of the composition of aspectuality and has maintained it.

For the purpose of this paper, the notion of compositionally formed aspectuality is best discussed in terms of features assigned to the verb and its arguments. These features have a precise model-theoretic interpretation which is given in Verkuyl 1993. It would carry too far to discuss the formal machinery dealing with aspectuality here. It suffices to observe that sentences like (5) are called terminative, i.e. have terminative aspectuality: they pertain to a bounded event as opposed to sentences like *Mary mailed* *letters*, *Mary mailed no letter* and *Nobody mailed a letter.* These are called durative: they pertain to events that can be prolonged indefinitely or to non-events. One of the standard litmus test is given in (13):

(13) *a.* #For hours Mary mailed a letter
b. For hours Mary mailed letters
c. For hours nobody mailed a letter

Sentence (13a) is odd: it enforces a queer sort of repetition. In any case, it blocks the one-event reading of (5). The other two sentences in (13) have a normal interpretation: they pertain to the same sort of event or non-event as the sentences without the durational adverbial. In Verkuyl's compositional theory (simplified here to an algebra of features) the terminative aspectuality in (13), in (12) and in (1) and (2) are due to the fact that these sentences all satisfy two conditions: (a) their verb is a [+ADD TO]-verb, i.e. a nonstative verb, which builds up a cumulative index structure;[8] (b) the arguments of the verb are [+SQA]-NPs. The abbreviatory label SQA stands for Specified Quantity of A, where A is the head noun denotation. The idea behind the model-theoretic definition is that in a type-logical analysis of the NP as a semantic object of type (*ettt*) the basic format of the representation is $\ldots \exists X[X \subseteq A \wedge |X| = k \ldots$. In *four letters* $k = 4$, in *a letter* $k = 1$, in *some letters*, $k \geq 1$, etc. Compositionally, for the verb the aspectually relevant information of NPs (with Count Nouns) is the cardinality information, either explicitly expressed (numerals, *both*) or implicitly given (*some*, *many*, *few*), in order to establish terminative aspectuality.

In the theory of aspectuality of Verkuyl 1993 VP-aspectuality is essentially different from sentential aspectuality. Intuitively, the terminativity in sentences like (1) and (2) must be dealt with exactly in the way we deal with the quantificational structure of these sentences: if the girls each mailed a letter, then aspectually we must end up with three terminative events, whereas if they mailed just one letter we should end up with one terminative event. Localistically, this is just another way of saying that the Path of each of the girls is assigned at the VP-level. Note that the Path in (1) is bounded, whereas in *Three girls mailed letters*, which is durative, the Path is unbounded.

The above informal introduction of the aspectual asymmetry is our point of departure for explicating the functions associated with the information expressed by figure 1, which are depicted in figure 2. Intuitively, two functions, π and ℓ, are involved in the interaction between the temporal ([+ADD TO]) information expressed by verbs like *mail*, *lift*, *eat*, (but

[8] Nonstative verbs, i.e. verbs expressing change, like *walk*, *talk*, *eat*, etc. are distinguished from stative verbs like *hate*, *love*, by their being able to invoke an interpretation in which their arguments participate in a temporal structure. One may call the nonstative verbs [+ADD TO] and the stative verbs [−ADD TO] to distinguish them lexically. To say that a verb compositionally contributes to the formation of the VP is taken to say that it contributes semantic information to the VP by interacting with the NP_2-information.

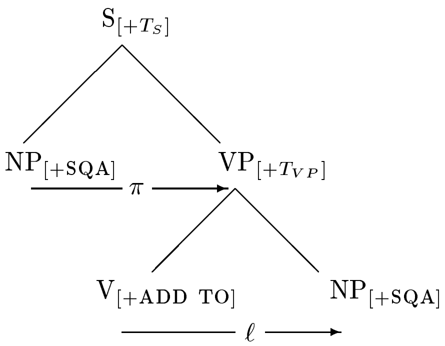

FIGURE 2 *Basic structure of aspect construal.*

not *love* and *believe*), and the atemporal ([+SQA]) information expressed by their arguments. In terms of the informal notions used above: ℓ is the function defining a Path, whereas π determines how the members of the external argument NP_1 are mapped onto the Path assigned to them by the predication.

This intuition can be formalized as follows. For the sake of the present exposition, let verbs take their denotation in type $e(i(et))$, with i the type of indices. Then π is the function from $\textsc{dom}([\![V]\!] \cap [\![N_1]\!] \times [\![N_2]\!])$ to the function space

$$\mathbf{D}_i \longrightarrow \wp(\mathrm{rng}([\![V]\!] \cap [\![N_1]\!] \times [\![N_2]\!])$$

where the ℓ's take values. To be precise, for each $a \in \textsc{dom}([\![V]\!] \cap [\![N_1]\!] \times [\![N_2]\!])$ $\pi(a)$ is the ℓ_a defined by: $\lambda i \lambda d' \in [\![N_2]\!].[\![V]\!]aid'$. So, $\ell_a(i)$ is the set of $[\![N_2]\!]$'s a is $[\![V]\!]$-related to at i. For (4), *Four men lifted three tables*, this says that the function π assigns to each of the relevant men m a function ℓ_m, which is a map from indices to subsets of the tables. E.g., the function π could be spelled out as in (14) with the situation sketched in (15) as a possible outcome of its application (only non-zero values of ℓ_n are depicted).

(14) $\mathrm{m}_1 \mapsto \ell_1 \quad \mathrm{m}_2 \mapsto \ell_2$
$\mathrm{m}_3 \mapsto \ell_3 \quad \mathrm{m}_4 \mapsto \ell_4$

(15) $\mathrm{m}_1 \mapsto \{\langle i, \{\mathrm{t}_1, \mathrm{t}_2\}\rangle, \langle j, \{\mathrm{t}_3\}\rangle\} \quad \mathrm{m}_2 \mapsto \{\langle j, \{\mathrm{t}_4, \mathrm{t}_5, \mathrm{t}_6\}\rangle\}$
$\mathrm{m}_3 \mapsto \{\langle j, \{\mathrm{t}_1\}\rangle, \langle m, \{\mathrm{t}_7, \mathrm{t}_8\}\rangle\} \quad \mathrm{m}_4 \mapsto \{\langle i, \{\mathrm{t}_1\}\rangle, \langle k, \{\mathrm{t}_9, \mathrm{t}_{10}\}\rangle\}$

In fact the notion of Path in Verkuyl 1993 is much richer, as it involves the use of structured indices in an essential way. In particular, a [+ADD TO] -verb V is interpreted over intervals of natural numbers, which model progress in time among other things. Moving from the beginpoint of an interval, an object a makes successive steps to go through its V-Path. Each possible step from the beginpoint to the endpoint of an interval contributes a cell of a partition of the set $\ell_n(a)$. The partition is warranted on the basis of the equivalence relation 'being lifted at the same index as'.[9] In this way, ℓ as part of the definition of π ensures that in a sentence like (4) the number of tables may vary from 3 to 12, and that a large number of different situations are captured, among which (15).

One of the purposes of the present paper is to investigate how far we can get without using indices. Our wish to abstract away from indices is to provide for a common ground for the treatment of [+ADD TO]-verbs like *lift* and [−ADD TO]-verbs like *love*, for which the π-function has not been defined. After this, it may be easier to account for the difference between these verbs in domains with temporal structure.

There are two ways of disregarding indices, which become plain by considering an intermediate step of abstraction where they are structureless. At this level, a transitive verb can be of type $e(i(ett))$ or of type $e(i(et))$. In the first case, the way in which the set of objects comes about while going through an index, is represented within the second argument as a set of sets. In the second case only the set itself is given. To start with, we concentrate on the first option. Without indices, it leads us to consider a transitive verb R as of type $e(ett)$, so that $\pi_R(a)$ is a set of sets for each $a \in \text{DOM}(R)$. It should be kept in mind, however, that intuitively such an a is R-related to $\bigcup \pi_R(a)$; i.e., the set obtained after 'processing' the entire interval.

3.2 Two Modes

Verkuyl (1988) proposed to put an empirically motivated restriction on π by assuming that it should either be an injection or a constant function. This amounts to holding that the elements in the domain of π_V either have their own Path (injective) or all share the same Path (constant).[10] In (15) π_{lift} is an injection, due to the fact that none of the men has the same Path, even when sometimes the same tables have been lifted at different indices. In particular, $\langle i, \{\mathrm{t}_1\}\rangle \neq \langle i, \{\mathrm{t}_1, \mathrm{t}_2\}\rangle \neq \langle j, \{\mathrm{t}_1\}\rangle$ because t_1 occurs in different index-set pairs. But π_{lift} can also map all the originals to one and the same image, as in (16):

(16)
$$\begin{aligned}
\mathrm{m}_1 &\mapsto \{\langle i, \{\mathrm{t}_1, \mathrm{t}_2, \mathrm{t}_3\}\rangle\}\\
\mathrm{m}_2 &\mapsto \{\langle i, \{\mathrm{t}_1, \mathrm{t}_2, \mathrm{t}_3\}\rangle\}\\
\mathrm{m}_3 &\mapsto \{\langle i, \{\mathrm{t}_1, \mathrm{t}_2, \mathrm{t}_3\}\rangle\}\\
\mathrm{m}_4 &\mapsto \{\langle i, \{\mathrm{t}_1, \mathrm{t}_2, \mathrm{t}_3\}\rangle\}
\end{aligned}$$

---

[9] Recall that partitions are closely related to equivalence relations (i.e., two-place relations which are reflexive, symmetrical, and transitive). For if $\mathbf{Y}$ partitions X, then Rxy iff $\exists Y \in \mathbf{Y}[x, y \in Y]$ is an equivalence relation. In section 4.1 we give the canonical way to turn an equivalence relation into a partition.

[10] A function $f : A \longrightarrow B$ is an injection iff for all $a, a' \in A$: if $f(a) = f(a')$, then $a = a'$. The function is constant iff for all $a, a' \in A$: $f(a) = f(a')$. In section 4 it will appear that Verkuyl's intuition should be formalized by: *relative to an index i* the function π^i_R, defined by $\lambda d \lambda d'.\pi_R did'$, should be either constant or injective.

The function π_{lift} is now constant. At this point the notion of kolkhoz-collectivity comes in. At the index i, the men are not just related to the same collection of three tables but this collection is required to be unique.[11] None of the men can be said to have his own Path, so it is impossible for the relevant members of the N_1-denotation to be 'held responsible' for the satisfaction of the predicate 'lift two tables'. The following sentences show that this is in fact a frequently used mode of predication:

(17) a. The twelve passengers killed that horrible man
b. Hans and Uwe wrote a book about DRT
c. 500 Dutch firms own 6000 American computers

In (17a) none of the passengers may claim that he or she killed that horrible man. The purpose of the sentence seems to be to evade such a claim. In (17b) neither of the two men may claim 'I have written a book about DRT'. The essence of the information is that they did it together, blurring the individual contribution to the satisfaction of the VP-predicate. In a sense, Scha's famous (17c) appears to fall under the notion of kolkhoz-collectivity as well. It can be seen as a claim about a unique set of Dutch firms and a unique set of American computers which somehow stand in the own-relation to each other (cf. section 4.3).

In section 4, we will argue that the distinction between the constancy and injectivity of π underlies two basic modes of predication. Moreover, these modes should provide for a considerable reduction of ambiguity of sentences like (1) and (2). Rather than specifying many different readings in terms of NP denotations or quantificational structures, we aim to show that to a large extent these 'readings' are encompassed by the modes of π. In particular we want to argue that the difference between the distributive and the totalizing collective use of a sentence is a matter of mode rather than of representation.

In the above, the two modes of handling verbal information are made dependent on indices. The function ℓ has indices in its domain, and the distinction between the two forms of π is based on indices as well. In section 4 we show that the two modes are available without any appeal to indices. This does not imply that indices are not necessary. Rather it implies that the intuition on which the modes are based are independent of them.

Now, we want to clarify how the modes of π are related to the readings in terms of plural NP denotations. As a first step in that direction we give an

[11]We use the term 'collection' to remain neutral with respect to their precise nature (sets, sums, groups). In this article, we use sets. Also, Verkuyl (1994) took kolkhoz-collectivity to be about collections which are minimal within the VP. But now we hold that this notion should be strengthened to uniqueness (cf. Van der Does' (1994) discussion of kolkhoz-collectivity in terms of maximized, minimized, and referential collective readings). The use of maximized, not necessarily unique sets in collective readings is also an important theme in Van den Berg 1995 (and earlier).

explicit account of the intuitions concerning Kolkhoz-collectivity. We do so by means of liftings and lowerings of the basic relations, and characterize the higher level relations which arise in this manner (section 4.1). It is shown what reduction in readings is effected if attention is restricted to these relations (section 4.2), and how they can be used to give a connection between plural and polyadic quantification (section 4.4).[12] In this way we make precise how these readings relate to the modes of predication.

4 The Present Framework

Localistically, a standard model can be seen to give what is the case at a certain index (interval, event, ...). In the previous section we noted that on this view it is natural to take a two-place relation as carrying information concerning maximal, even unique sets. On the one hand there is for each $a \in A$ in the domain of the relation the unique set of B's standing in the R-relation to it (i.e., the set of Bs on a's R-path). On the other hand, there are the unique sets of A's which stand in the R-relation to the same B's (i.e., the set of A's which share the same R-path as restricted to B).[13] As soon as the totalizing nature of this is made explicit at the level of relations among sets, it suggests that at the higher level not all relations are admissible. We now characterize the relations which are obtained in this manner. Intuitively, only those transitive verb denotations are allowed which relate two unique sets per index. In fact, we prove a slightly more general result to enable a discussion of the options concerning transitive verbs in type $e(et)$ and in type $e(ett)$ discerned above.

4.1 Lifting Relations

Let R be a two-place relation of type $\alpha(\beta t)$. Define

$$\pi_R : \textsc{dom}(R) \longrightarrow \wp(\textsc{rng}(R))$$

by: $\pi_R(a) \mapsto R_a$ with $R_a := \{b : Rab\}$, and define $\langle \textsc{dom}(R), \sim_R \rangle$ by:

$$a \sim_R a' \text{ iff } \pi_R(a) = \pi_R(a')$$

Clearly $\sim_R$ is an equivalence relation, which induces a partition of $\textsc{dom}(R)$. The cells of this partition are:

$$[a]_R := \{a' \in \textsc{dom}(R) \mid a' \sim_R a\}$$

Using these notions, we define a lifting $\uparrow$ from type $\alpha(\beta t)$ to type $(\alpha t)(\beta tt)$ as follows:

(18) $\uparrow_\pi(R)XY$ iff $\exists a \in \textsc{dom}(R)[X = [a]_R$ and $Y = \pi_R(a)]$

The subscript π is dropped if no confusion is likely. Notice that we have R_a as the value of $\uparrow(R)$ on $[a]_R$: $\uparrow(R)([a]_R) = R_\mathrm{a}$. In fact, the relations

[12] Such a connection is first noted in the appendix of Hoeksema 1983. The question is often raised by Van Benthem (cf. the issues for further study in Westerståhl 1994).

[13] Another view on models takes each tuple in a relation as a unique 'atomic' event. With explicit indices, this would satisfy: $Rx_1 \dots x_n i$ and $Rx_1 \dots x_n i'$ implies $i = i'$.

within the image of $\uparrow$ can be characterized as the partitional injections in type $(\alpha t)(\beta tt)$; i.e., the injections $\mathbf{R} : \textsc{dom}(\mathbf{R}) \longrightarrow \textsc{rng}(\mathbf{R})$ with $\textsc{dom}(\mathbf{R})$ partitions $\bigcup \textsc{dom}(\mathbf{R})$.

Proposition 2 *The partitional injections* $\mathbf{R}$ *in type* $(\alpha t)(\beta tt)$ *are precisely those of the form* $\mathbf{R} = \uparrow(R)$ *for an* R *of type* $\alpha(\beta t)$.

PROOF. It is clear from its definition that $\uparrow(R)$ is a function with a partitioned domain. To see that it is even an injection, assume for $X,\ X' \in \textsc{dom}(\uparrow R)$ that $X \neq X'$. There are $a,\ a' \in \textsc{dom}(R)$ such that $X = [a]_R$ and $X' = [a']_R$. Since $[a]_R \neq [a']_R$, also $\pi_R(a) \neq \pi_R(a')$. That is, $\uparrow R(X) \neq \uparrow R(X')$.

In order to show that any partitional injection $\mathbf{R}$ is of the form $\uparrow(R)$, we introduce the lowering $\downarrow$ defined by:

(19) $\downarrow(\mathbf{R})xy$ iff $\exists XY[\mathbf{R}XY$ and $x \in X$ and $y \in Y]$

That is, $\downarrow\mathbf{R} := \bigcup\{X \times Y \mid \mathbf{R}XY\}$. Our proof is complete if we can show that for all the $\mathbf{R}$'s under consideration $\uparrow(\downarrow\mathbf{R}) = \mathbf{R}$. So, let $\mathbf{R}$ be an partitional injection. Then, $\downarrow(\mathbf{R}) = \bigcup\{X \times \mathbf{R}(X) \mid X \in \textsc{dom}(\mathbf{R})\}$. Here the X's are pairwise disjoint and $\mathbf{R}(X)$ is only assigned to X. Consequently, for all $a \in \textsc{dom}(\downarrow(\mathbf{R}))$: $\pi_{\downarrow\mathbf{R}}(a) = \mathbf{R}(X)$, where X is the unique $X \in \textsc{dom}(\mathbf{R})$ with $a \in X$. But then for all $a \in \textsc{dom}(\downarrow(\mathbf{R}))$: $[a]_{\downarrow\mathbf{R}}$ is that X as well. Thus, $\mathbf{R}XY$ iff $\exists a \in \textsc{dom}(\downarrow\mathbf{R})[X = [a]_{\downarrow\mathbf{R}}\ \&\ Y = \pi_{\downarrow\mathbf{R}}(a)]$ iff $\uparrow(\downarrow\mathbf{R})$; as required. $\Box$

Proposition 2 highlights once more the notion of kolkhoz-collectivity. First, the sets of objects which share the same Path occur uniquely within the domain of $\mathbf{R}$. Second, these unique sets are $\mathbf{R}$-related to a unique set in the range of $\mathbf{R}$.

A next step is to determine the specific form of $\uparrow(R)$ in case π_R has special properties. It is at this point that clear connections with collective quantification emerge. The properties of π we are interested in are given in the formulation of proposition 3, which can be proved along the same lines as proposition 2.

Proposition 3 *The atomic injections* $\mathbf{R}$ *in type* $(\alpha t)(\beta tt)$ *– i.e., the injections with a domain which consists of singletons – are precisely those of the form* $\mathbf{R} = \uparrow_\pi(R)$ *for an* R *of type* $\alpha(\beta t)$ *and* π *an injection. The singletons* $\mathbf{R}$ *in type* $(\alpha t)(\beta tt)$ *are precisely those of the form* $\mathbf{R} = \uparrow_\pi(R)$ *for an* R *of type* $\alpha(\beta t)$ *and* π *constant.* $\Box$

The above observations are preliminary to showing how iterative quantification at the lower level is related to a form of iterative quantification at the level of sets. It is this relation which allows us to establish in what way the readings at the higher level vary with the nature of a Path function π.

4.2 Plural Quantification

In this section we take transitive verbs to be of type $e(ett)$ (cf. section 3.1). These denotations are obtained from their denotation in type $e(i(ett))$ by fixing a certain index i. In Verkuyl 1993 the indices are totally ordered. The Path-function π_{R} of a verb R of type $e(ett)$ assigns to each individual in its domain a set of sets, which intuitively comes about by going trough the ordered index i. For example, one could have the injective (20) or the constant (21):

(20) $m_1 \mapsto \{\{t_1, t_2\}\},\ m_2 \mapsto \{\{t_3\}, \{t_4\}\},\ m_3 \mapsto \{\{t_3\}, \{t_5\}\}$

(21) $m_1, m_2, m_3 \mapsto \{\{t_1, t_2\}\}$

To be able to adapt to this diversity, the internal argument NP should receive a neutral plural interpretation. As to the plural interpretation of the external argument NP, the lifting used in section 4.1 may yield an arbitrary partition as the domain of the shifted verbal denotation. Therefore, the plural form of this NP should be neutral as well. The basic connection between the atomic and the plural version of the external argument is given by proposition 4.

Proposition 4 *Let A and B be sets of type (et), and R a relation of type $e(ett)$. One has for all $\mathbf{D}_1$ and all positive $\mathbf{D}_2$:*

$$\mathrm{N}(\mathbf{D}_1)A\{X \mid \mathrm{N}(\mathbf{D}_2)B{\uparrow}(R)(X)\} \Leftrightarrow \mathbf{D}_1 A\mathrm{N}(\mathbf{D}_2)BR$$

N.B. We write ${\uparrow}(R)(X)$ rather than ${\uparrow}(R)_X$. The latter is a set of sets of sets, which is one level too high.

PROOF. It follows from the positivity of $\mathbf{D}_2$ that a–c are true:

a. $\{X : \mathrm{N}(\mathbf{D}_2)B{\uparrow}(R)(X)\}$
$= \{X \in \textsc{dom}({\uparrow}(R)) \mid \mathrm{N}(\mathbf{D}_2)B{\uparrow}(R)(X)\}$

b. $\bigcup\{X \in \textsc{dom}({\uparrow}(R)) \mid \mathrm{N}(\mathbf{D}_2)B{\uparrow}(R)(X)\}$
$= \{a \in \textsc{dom}(R) \mid \mathrm{N}(\mathbf{D}_2)BR_{\mathrm{a}}\}$

c. $\{a \in \textsc{dom}(R) \mid \mathrm{N}(\mathbf{D}_2)BR_a\}$
$= \{a \mid \mathrm{N}(\mathbf{D}_2)BR_a\}$

Given these identities, the required equivalence is almost immediate. □

Observe that the plural forms of the NPs show the NN configuration. This was also our point of departure in section 2.2, but it was found problematic in section 2.3 since not all neutral NPs can take scope over each other. In the present situation it the NN configuration is unproblematic, because ${\uparrow}(R)$ is not just any relation between sets and sets of sets. In particular, proposition 2 shows its domain to be a partition. As we have seen in section 2.4, moves like this eliminate the unwanted effects of a wide-scope N reading.

Proposition 5 determines the effects of the π-modes in terms of plural NP denotations.

Proposition 5 (FIN) *Let A, B be of type (et), and R of type $e(ett)$. We write $R\lceil_{A,B}$ for $R \cap A \times \wp(B)$.*

i) $\pi_{R\lceil_{A,B}}$ *is an injection iff for all positive* $\mathbf{D}_1$ *and* $\mathbf{D}_2$ $\mathbf{D}_1 A\mathrm{N}(\mathbf{D}_2)BR$ *is equivalent to* $\mathrm{D}(\mathbf{D}_1)A\{X : \mathrm{N}(\mathbf{D}_2)B\uparrow(R)(X)\}$.

ii) $\pi_{R\lceil_{A,B}}$ *is constant iff for all positive* $\mathbf{D}_1$ *and* $\mathbf{D}_2$ $\mathbf{D}_1 A\mathrm{N}(\mathbf{D}_2)BR$ *is equivalent to* $\mathrm{C}(\mathbf{D}_1)A\{X : \mathrm{N}(\mathbf{D}_2)B\uparrow(R)(X)\}$.

PROOF. We prove the characterization of $\pi_{R\lceil_{A,B}}$ injective. Since $\mathbf{D}_1$ and $\mathbf{D}_2$ are conservative, $\mathbf{D}_1 A\mathrm{N}(\mathbf{D}_2)BR$ is equivalent to $\mathbf{D}_1 A\mathrm{N}(\mathbf{D}_2)BR\lceil_{A,B}$. By proposition 4 this, in turn, is equivalent to

a) $\mathrm{N}(\mathbf{D}_1)A\{X : \mathrm{N}(\mathbf{D}_2)B\uparrow(R\lceil_{A,B})(X)\}$

Further, $\mathbf{D}_2$ is positive, so (a) is equivalent to:

$$\mathrm{N}(\mathbf{D}_1)A\{X \in \textsc{dom}(R\lceil_{A,B}) : \mathrm{N}(\mathbf{D}_2)B\uparrow(R\lceil_{A,B})(X)\}$$

Since $\mathbf{D}_1$ is positive too, the set

$$\{X \in \textsc{dom}(\uparrow(R\lceil_{A,B})) \mid \mathrm{N}(\mathbf{D}_2)B\uparrow(R\lceil_{A,B})(X)\}$$

is non-empty. In case $\pi_{R\lceil_{A,B}}$ is an injection, proposition 3 with $\alpha = e$ and $\beta = (et)$ says that the domain of $\uparrow(R\lceil_{A,B})$ is atomic. So proposition 1 in combination with the above observations shows that $\mathbf{D}_1 A\mathrm{N}(\mathbf{D}_2)BR$ and $\mathrm{D}(\mathbf{D}_1)A\{X : \mathrm{N}(\mathbf{D}_2)B\uparrow(R)(X)\}$ are equivalent.

As to the converse direction of (i), assume that $\pi_{R\lceil_{A,B}}$ is not an injection. Then there are $a, a' \in \textsc{dom}(R\lceil_{A,B})$ with $\pi_{R\lceil_{A,B}}(a) \neq \pi_{R\lceil_{A,B}}(a')$. Set $n = |\pi_{R\lceil_{A,B}}(a)|$. Since $a \in \textsc{dom}(R\lceil_{A,B})$, $\mathbf{n}$ is positive. Next, set $m = |\bigcup\{X \in \textsc{dom}(\uparrow R\lceil_{A,B}) : \mathrm{N}(\mathbf{n})B\uparrow(R\lceil_{A,B})(X)\}|$. Since

$$\mathrm{N}(\mathbf{n})B\uparrow(R\lceil_{A,B})([a]_{R\lceil_{A,B}})$$

$\mathbf{m}$ is positive, too. Also, $\mathrm{N}(\mathbf{m})A\{X : \mathrm{N}(\mathbf{n})B\uparrow(R\lceil_{A,B})(X)\}$, and so with proposition 4 and the conservativity of the numerals: $\mathbf{m}A\mathrm{N}(\mathbf{n})BR$. But by FIN *not* $\mathrm{D}(\mathbf{m})A\{X : \mathrm{N}(\mathbf{n})B\uparrow(R\lceil_{A,B})(X)\}$, for the set of singletons in $\{X \in \textsc{dom}(\uparrow R\lceil_{A,B}) : \mathrm{N}(\mathbf{n})B\uparrow(R\lceil_{A,B})(X)\}$ is strictly smaller than the number of elements in the union of this set. For one, the cel $[a]_{R\lceil_{A,B}}$ is in this set but is not a singleton. The characterization of $\pi_{R\lceil_{A,B}}$ constant is proved along similar lines. □

Proposition 5 proves the external argument to be distributive iff the relevant A's have their own Path. They share the same Path iff the argument is collective. As to the internal argument, recall that the sets of sets within the image of π represents how the set of elements to which an element j is V-related comes about while going through a well-ordered index. Therefore, this set has information on the nature of the internal argument NP at a particular index. If the set is given one atom after the other the internal argument is used distributively; if the set is given in one go its use is collective; etc.

According to proposition 5 there is a strict analogue between the readings (10DN) and (10CN) proposed by Van der Does (1993), and the present formalization of the injective and constant mode of π in Verkuyl 1988. These treatments are essentially the same but for their treatments of transitive verbs. In Van der Does (1993) these verbs are relations among sets, whereas on the present interpretation Verkuyl (1988) uses relations among individuals and sets of sets. This difference is eliminated by means of the lift operation.

The status of the cumulative reading (10NN) remains. Here we do not find such characterizations as the above. But we shall show that the π-constant mode is a strong assumption, which makes several readings collapse. To this end, we concentrate on polyadic quantification in section 4.3, and then on a connection between polyadic and collective quantification in section 4.4.

4.3 Polyadic Quantification

In this section we give examples of cases where the readings of sentences 'collapse' given specific information concerning the lowest level of predication. In particular we show that the iterative, the cumulative, and a branching reading of a transitive quantificational sentence are equivalent as soon as π is a constant function for the transitive verbs of type $e(et)$.

For the next two sections we first concentrate on the relation between iterative and cumulative quantification, and then on the relation between cumulative quantification and some simple versions of branching quantification. We observe that they are all equivalent iff $\pi_{R\lceil_{A,B}}$ is constant.

4.3.1 Iterative and Cumulative Quantification

The most familiar notion of quantification is iterative. It corresponds to subsequently combining the internal argument NP with the transitive verb and then combining the result of this with the external argument NP to yield a sentence. Formally:

(22) $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR \equiv \mathbf{D}_1 A\{a : \mathbf{D}_2 BR_a\}$

Cumulative quantification is due to Scha (1981), and is defined by:

(23) $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR \equiv \mathbf{D}_1 A\textsc{dom}(R \cap A \times B) \wedge \mathbf{D}_2 B\textsc{rng}(R \cap A \times B)$

In case of (4) it says that the total number of men who lifted tables is 4 and that the total number of tables lifted by men is 3. $\mathbf{D}_1$ and $\mathbf{D}_2$ are used to determine the size of two unique sets: the domain and range of $R \cap A \times B$. Lemma 6 describes the logical relation between $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR$ and $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$. Its proof is close to that of the product decomposition lemma's of Keenan (1992) and Westerståhl (1993).

Lemma 6 (FIN) *For all A, B, R: $R \cap A \times B$ is a non-empty product iff for all positive $\mathbf{D}_1$, $\mathbf{D}_2$: $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR$ and $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$ are equivalent.*

PROOF. [$\Rightarrow$:] First note that if $R \cap A \times B$ is the non-empty product $X \times Y$, one has:

i) $\forall a \in X : (R \cap A \times B)_a = Y$
ii) $\{a \mid \mathbf{D}_2 B(R \cap A \times B)_a\} = X$, if $\neg \mathbf{D}_2 B\emptyset$ and $\mathbf{D}_2 BY$.

Assume $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$, i.e.: $\mathbf{D}_1 A\textsc{dom}(R \cap A \times B)$ and $\mathbf{D}_2 B\textsc{rng}(R \cap A \times B)$. Since $R \cap A \times B = X \times Y$, also $\mathbf{D}_1 AX$ and $\mathbf{D}_2 BY$. Given (i) and (ii), it follows that $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR \cap A \times B$. But $\mathbf{D}_1$ and $\mathbf{D}_2$ are conservative, so this is equivalent to $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR$.

Conversely, assume $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR$ and that $R \cap A \times B = X \times Y \neq \emptyset$. Conservativity gives $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR \cap A \times B$, i.e., $\mathbf{D}_1 A\{a \mid \mathbf{D}_2 B(R \cap A \times B)_a\}$. $\mathbf{D}_1$ is positive, so there is a $a \in \textsc{dom}(R \cap A \times B)$ with $\mathbf{D}_2 B(R \cap A \times B)_a$. But $(R \cap A \times B)_a = Y$ for all $a \in \textsc{dom}(R \cap A \times B)$, so $\mathbf{D}_2 BY$. Since $\mathbf{D}_2$ is positive as well, it follows from (ii) that:

$$\{a \mid \mathbf{D}_2 B(R \cap A \times B)_a\} = X$$

so $\mathbf{D}_2 AX$. All in all, we see that $\mathbf{D}_1 A\textsc{dom}(R \cap A \times B)$ and $\mathbf{D}_2 B\textsc{rng}(R \cap A \times B)$. That is, $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$.

[$\Leftarrow$:] Let $R \cap A \times B$ fail to be a product, and let $n = |\textsc{dom}(R \cap A \times B)|$ and $m = |\textsc{rng}(R \cap A \times B)|$. Since $n, m > 0$, the quantifiers (exactly) $\mathbf{n}$ and $\mathbf{m}$ are positive, and clearly $\mathrm{CM}(\mathbf{n}, \mathbf{m})ABR$. But not $\mathrm{IT}(\mathbf{n}, \mathbf{m})ABR$. For this would imply that $|R \cap A \times B| \geq n * m$, which in a finite model only holds if the relation is a product. $\square$

It is an almost immediate consequence of this lemma that iteration and cumulation are indistinguishable as soon as π is constant for the given relation and sets. Indeed, the remaining step consists in observing the simple truth of lemma 7.

Lemma 7 *For all A, B, and R: π_R is constant iff $R \cap A \times B$ is a product.*

PROOF. Plainly, if $R \cap A \times B = X \times Y$ $\pi_R(a)$ is constant. Assume for a contradiction that $R \cap A \times B$ is no product. Then there is a $\langle d, d' \rangle \in (\textsc{dom}(R \cap A \times B) \times \textsc{rng}(R \cap A \times B)) \setminus R \cap A \times B$. This implies that $d' \notin \pi_R(d)$. But also that there is a d'' with $d' \in \pi_R(d'')$. So, $\pi_R(d) \neq \pi_R(d'')$, i.e., π_R is not constant. $\square$

Combining the lemma's 6 and 7 we get a proof of the following proposition:

Proposition 8 (FIN) *For all A, B, and R: the function*

$$\pi_{\mathrm{R}} : \textsc{dom}(R \cap A \times B) \longrightarrow \wp(\textsc{rng}(R \cap A \times B))$$

is constant iff: $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR$*, and* $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$ *are equivalent, for all positive* $\mathbf{D}_1$, $\mathbf{D}_2$. $\square$

Along these lines we obtain a logical reconstruction of the claim in Verkuyl 1994 that the cumulative reading is brought about by π constant. Proposition 8 says that in this case iteration and cumulation collapse. As a corollary we see that under these circumstances the issue of scope ambiguity does not arise:

Corollary 9 (FIN) *For all A, B, and R: if π_{R} is constant then for all positive $\mathbf{D}_1$, $\mathbf{D}_2$ (i) and (ii) are equivalent.*

i) $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR$
ii) $\mathrm{IT}(\mathbf{D}_2, \mathbf{D}_1)BAR^{-1}$

PROOF. Since $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$ is equivalent to $\mathrm{CM}(\mathbf{D}_2, \mathbf{D}_1)BAR^{-1}$, the corollary is immediate from proposition 8. □

Of course we should not conclude from proposition 8 that in general iterations and cumulations are identical. As is well-known, there logical behaviour differs widely. Verkuyl's claim is rather an empirical one: cumulation is only used when π is constant. This is an issue which is open for further discussion. As an argument in favour of it, Verkuyl (1994) highlights the notion of totalization, which also plays a crucial role in the way we lifted relations. In section 4.2, on plural quantification, we have already seen that the constancy of π is closely tied up with this notion.

This ends our discussion of the relationship between iterative and cumulative quantification. We now turn to similar observations concerning cumulative and branching quantification.

4.3.2 Branching quantification

Hintikka (1973) claimed that natural language quantification is sometimes branching. Barwise (1979) found convincing arguments to support this claim by considering generalized quantifiers (rather than just first-order ones). He considered the most prominent readings of (24).

(24) $\begin{bmatrix} \text{Few} \\ \text{Two} \\ \text{Most} \end{bmatrix}$ of these girls and $\begin{bmatrix} \text{at most four} \\ \text{three} \\ \text{quite a few} \end{bmatrix}$ of those boys
all dated each other

E.g., for the monotone increasing *most* and *quite a few*, (24) comes to mean: there are sets X and Y containing most boys and quite a few girls such that the product $X \times Y$ is part of the relation denoted by *to date*. Similarly for the other cases. Formally, we have the schemes:

Definition 2 [branching quantification]

i) Monotone decreasing: $\mathrm{BR}^{\mathrm{md}}(\mathbf{D}_1, \mathbf{D}_2)ABR \equiv$
$\exists XY[\mathbf{D}_1 AX \wedge \mathbf{D}_2 BY \wedge R \cap A \times B \subseteq X \times Y \cap A \times B]$
ii) Non-monotone: $\mathrm{BR}(\mathbf{D}_1, \mathbf{D}_2)ABR \equiv$
$\exists XY[\mathbf{D}_1 AX \wedge \mathbf{D}_2 BY \wedge X \times Y \cap A \times B = R \cap A \times B]$
iii) Monotone increasing: $\mathrm{BR}^{\mathrm{mi}}(\mathbf{D}_1, \mathbf{D}_2)ABR \equiv$
$\exists XY[\mathbf{D}_1 AX \wedge \mathbf{D}_2 BY \wedge X \times Y \cap A \times B \subseteq R \cap A \times B]$

These schemes are partial, in that they apply depending on the monotonicity behaviour of the determiners.[14] The schemes for monotone quantifiers are due to Barwise (1979), and that for non-monotone quantifiers to Van Benthem (cf. Westerståhl 1987, 274).[15] A more general definition for continuous determiners is in Westerståhl 1987.[16] Recently there is a renewed attention for the phenomena. For instance, Sher (1990) and Spaan (1993) argue that branching quantification involves a notion of maximality which is lacking in the earlier proposals. Here we concentrate on the simpler notions of branching given above.

We want to know how the schemes relate to other forms of quantification, and in particular to cumulative quantification. In case of $\mathrm{BR}^{\mathrm{md}}$ we can be quick, since Westerståhl (1987, 285) observes that for monotone decreasing determiners $\mathrm{BR}^{\mathrm{md}}$ and CM are equivalent. The scheme for non-monotone quantifiers is more interesting. Although the observation below holds for the intended class, we shall in fact treat it as a general scheme. Lemma 10 describes the logical relationship between BR and cumulative quantification for arbitrary quantifiers.

Lemma 10 *For all A, B, R: $R \cap A \times B$ is a product iff for all quantifiers $\mathbf{D}_1$, $\mathbf{D}_2$ $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$ is equivalent to $\mathrm{BR}(\mathbf{D}_1, \mathbf{D}_2)ABR$.*

PROOF. [$\Rightarrow$:] First note that branching quantification is stronger than cumulative quantification. Conversely, if we know $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$ and in addition that $R \cap A \times B$ is a product, then $\mathrm{BR}(\mathbf{D}_1, \mathbf{D}_2)ABR$. For in this case, $R \cap A \times B = \textsc{dom}(R \cap A \times B) \times \textsc{rng}(R \cap A \times B)$.

[$\Leftarrow$:] It is sufficient to observe that if $R \cap A \times B$ is not a product, $(\textsc{dom}(R \cap A \times B) \times \textsc{rng}(R \cap A \times B)) \setminus R \cap A \times B \neq \emptyset$. So, $\mathrm{CM}(\mathbf{some}, \mathbf{some})ABR$, but not $\mathrm{BR}(\mathbf{some}, \mathbf{some})ABR$. □

Branching for monotone increasing quantifiers remains. As it happens, we can use the previous proof to give the same kind of characterization here.[17]

Lemma 11 (FIN) *For all A, B, R: $R \cap A \times B$ is a product iff for all monotone increasing $\mathbf{D}_1$, $\mathbf{D}_2$ $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$ and $\mathrm{BR}^{\mathrm{mi}}(\mathbf{D}_1, \mathbf{D}_2)ABR$ are equivalent.*

PROOF. For monotone increasing $\mathbf{D}_1$ and $\mathbf{D}_2$, $\mathrm{BR}^{\mathrm{mi}}$ implies CM. On the other hand, if $R \cap A \times B$ is a product, it follows from lemma 10 that CM is equivalent to BR for all quantifiers. But BR implies $\mathrm{BR}^{\mathrm{mi}}$, and this

[14] A determiner $\mathbf{D}$ is (right) monotone decreasing iff for all A, B, C: if $\mathbf{D}AC$ and $B \subseteq C$ then $\mathbf{D}AB$. $\mathbf{D}$ is (right) monotone increasing iff for all A, B, C: if $\mathbf{D}AB$ and $B \subseteq C$ then $\mathbf{D}AC$.

[15] In fact we use slight variations in order to get a better fit with the notion of collective quantification in (6).

[16] A determiner $\mathbf{D}$ is *continuous* iff for all A, B, C, D: if $\mathbf{D}AB$, $B \subseteq C \subseteq D$, and $\mathbf{D}AD$, then $\mathbf{D}AC$.

[17] Checking the proof one notes that lemma 11 also holds in case $\mathrm{BR}^{\mathrm{mi}}$ quantifies over products which are maximal with respect to the inclusion relation, as in Sher 1990.

holds in particular for monotone increasing quantifiers. Finally, whenever $R \cap A \times B$ is not a product, we can adapt the proof of lemma 6 to show that

$$\mathrm{CM}(\textbf{at least n}, \textbf{at least m})ABR$$

while not $\mathrm{BR}^{\mathrm{mi}}(\textbf{at least n}, \textbf{at least m})ABR$ for certain n and m. □

Let us take stock. We have shown that as soon as $R \cap A \times B$ is a product, branching is indistinguishable from cumulation. As a consequence we have:

Corollary 12 (FIN) *For all A, B, R: $R \cap A \times B$ is a non-empty product (that is: $\pi_{R\lceil_{A,B}}$ is constant) iff for all positive $\mathbf{D}_1$, $\mathbf{D}_2$: $\mathrm{IT}(\mathbf{D}_1, \mathbf{D}_2)ABR$, $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)ABR$, and $\mathrm{BR}(\mathbf{D}_1, \mathbf{D}_2)ABR$ are equivalent.* □

With a view to the reduction of ambiguity, corollary 12 suggests to consider to treat (2ii) as a general scheme. This would have the drawback that $R \cap A \times B$ is required to be a product, which seems too strict, even in case of non-monotonic quantifiers. However, in order to enforce a branching reading one often has to resort to such linguistic means as reciprocals. And it might well be possible to interpret the reciprocal used so as to turn a transitive verb into a product; e.g., by selecting a contextually salient or maximal product part of its denotation. Cf. Schwarzchild 1992. For the present case one might have:

(25) $[\![\mathrm{EO}]\!](R, A, B) \in \{X \times Y : X \times Y \subseteq R \cap A \times B\}$

Consequently, the meaning of (24) can either be written as (26a) or as (26b).

(26) a. $\mathrm{CM}(\mathbf{D}_1, \mathbf{D}_2)AB[\![\mathrm{EO}]\!](R, A, B)$
b. $\mathrm{BR}(\mathbf{D}_1, \mathbf{D}_2)AB[\![\mathrm{EO}]\!](R, A, B)$

If so, the linguistic use of branching could be dispensed with in favour of cumulative quantification. And if the quantifiers are positive, even to 'standard' iterative quantification (cf. lemma 6). Needless to say that this observation disregards the many subtleties concerning the other meanings of reciprocals. Cf. Dalrymple et al. 1994.

This finishes our discussion of polyadic quantification. In the next section we shall use the present observations to give a connection between polyadic and collective quantification.

4.4 Polyadic vs. Collective Quantification

It remains to clarify the status of the cumulative and other non-iterative forms of quantification in case π_{R} is constant, for R a plural verb. To this end, recall that on the localistic view we took such verbs to be of type $e(ett)$ in order to represent the main effect of 'passing trough' a structured index. Now, if π_{R} is constant, this set of sets is the same for all $a \in \mathrm{DOM}(R)$. This correctly predicts that a collective use of (4), which more or less corresponds to a use of (27), allows for some variation with respect to the internal argument NP.

(27) Three men lifted two tables together

The men may have jointly lifted two different tables, or they may have lifted two tables in one go. That is, in case of (27) the constancy of π_{R} is compatible with such configurations as in (28).

(28)
$$\begin{array}{lcl} m_1, m_2, m_3 & \mapsto & \{\{t_1, t_2\}\} \\ m_1, m_2, m_3 & \mapsto & \{\{t_1\}, \{t_2\}\} \end{array}$$

It should be observed, however, that the underlying set of tables the men turn out to be related to is the same.

Formally, one may abstract from the process of generation by means of the type-shift ABS:

$$\text{ABS} := \lambda R_{\mathrm{e(ett)}} \lambda x_e \lambda y_e . \exists Z_{(\mathrm{et})}(RxZ \wedge y \in Z)$$

Then the above observation becomes: for all $R_{\mathrm{e(ett)}}$, if π_{R} is constant, so is $\pi_{\mathrm{abs(R)}}$. The converse need not be true: at an index i one can ultimately be R-related to the same set ($\pi_{\mathrm{abs(R)}}$ is constant), even though this set is arrived at in different ways within i (π_{R} is not).

It might be clear now how the constancy of π_{R} within type $e(ett)$ connects collective with polyadic quantification. In that circumstance two unique sets are related to each other. The collective use takes these sets as they come, while the polyadic forms of quantification uses their product. Proposition 15 has the details, but first some lemma's.

Lemma 13 *Let* $\mathbf{R}$ *be a relation between sets, and* $\mathbf{D}_1$ *and* $\mathbf{D}_2$ *positive determiners. If* $|\mathbf{R}| = 1$ *then (i) and (ii) are equivalent.*

i) $\mathrm{N}(\mathbf{D}_1)A\mathrm{N}(\mathbf{D}_2)B\mathbf{R}$
ii) $\mathrm{C}(\mathbf{D}_1)A\mathrm{C}(\mathbf{D}_2)B\mathbf{R}$

PROOF. In case $\mathbf{R}$ is a singleton, it follows from the positivity of $\mathbf{D}_2$ and proposition 1 that:

$$\{X \mid \mathrm{N}(\mathbf{D}_2)B\mathbf{R}_X\} = \{X \mid \mathrm{C}(\mathbf{D}_2)B\mathbf{R}_X\}$$

Since $\mathbf{D}_1$ is positive: $|\{X \mid \mathrm{C}(\mathbf{D}_2)B\mathbf{R}_X\}| = 1$. So the equivalence follows by applying proposition 1 once more. $\square$

The converse of this lemma is false. For take $\mathbf{R} \subseteq \wp(A) \times \wp(B)$ to be: $\{\langle\{a\},\{b\}\rangle, \langle\{a\},\{c\}\rangle, \langle\{d\},\{c\}\rangle\}$. Then, $\mathrm{C}(\mathbf{1})A\mathrm{C}(\mathbf{1})B\mathbf{R}$ is equivalent to $\mathrm{N}(\mathbf{1})A\mathrm{N}(\mathbf{1})B\mathbf{R}$, but $\mathbf{R}$ is no singleton. Lemma 14 proves a similar equivalence between the doubly collective and the branching reading.

Lemma 14 *Let* $\mathbf{R}$ *be a relation between sets. If* $|\mathbf{R}| = 1$ *then (i) and (ii) are equivalent.*

i) $\mathrm{C}(\mathbf{D}_1)A\mathrm{C}(\mathbf{D}_2)B\mathbf{R}$
ii) $\mathrm{BR}(\mathbf{D}_1, \mathbf{D}_2)AB{\downarrow}(\mathbf{R})$

PROOF. First define $\mathbf{R}\lceil_{A,B} := \mathbf{R} \cap \wp(A) \times \wp(B)$. If $|\mathbf{R}| = 1$, $\downarrow(\mathbf{R}\lceil_{A,B}) = \downarrow(\mathbf{R}) \cap A \times B$. Using this, we reason as follows:

$$\begin{array}{ll} & \mathrm{C}(\mathbf{D}_1)A\mathrm{C}(\mathbf{D}_2)B\mathbf{R} \\ \Leftrightarrow & \exists XY[\mathbf{D}_1 AX \ \wedge \ \mathbf{D}_2 BY \ \wedge \ \mathbf{R}X \cap AY \cap B] \\ \Leftrightarrow & \exists XY[\mathbf{D}_1 AX \ \wedge \ \mathbf{D}_2 BY \ \wedge \ X \times Y \cap A \times B = \downarrow(\mathbf{R}\lceil_{A,B})] \\ \Leftrightarrow & \exists XY[\mathbf{D}_1 AX \ \wedge \ \mathbf{D}_2 BY \ \wedge \ X \times Y \cap A \times B = \downarrow(\mathbf{R}) \cap A \times B] \\ \Leftrightarrow & \mathrm{BR}(\mathbf{D}_1, \mathbf{D}_2)AB\downarrow(\mathbf{R}) \end{array}$$

□

Again the converse of lemma 14 is false. The constancy of π_R with R of type $e(ett)$ can now be shown to have the following effect.

Proposition 15 *Let A and B be sets, R a relation of type $e(ett)$, and $\mathbf{D}_1$ and $\mathbf{D}_2$ positive determiners. If π_R is constant, $(i - iv)$ are equivalent.*

i) $\mathrm{N}(\mathbf{D}_1)A\{X : \mathrm{N}(\mathbf{D}_2)B\uparrow(R)(X)\}$
ii) $\mathrm{N}(\mathbf{D}_1)A\{X : \mathrm{N}(\mathbf{D}_2)B\uparrow(\textsc{abs}(R))\}$
iii) $\mathrm{C}(\mathbf{D}_1)A\{X : \mathrm{C}(\mathbf{D}_2)B\uparrow(\textsc{abs}(R))\}$
iv) $\mathrm{BR}(\mathbf{D}_1, \mathbf{D}_2)AB\textsc{abs}(R)$

According to theorem 8 similar equivalences hold with respect to the iterative and cumulative reading.

PROOF. Observe that for all $X \in \textsc{dom}(R)$: $\bigcup(\uparrow R(X)) = (\uparrow\textsc{abs}(R))_X$. The equivalence of (i) and (ii) now follows from the positivity of $\mathbf{D}_2$. Moreover, if π_{R} is constant so is $\pi_{\mathrm{abs(R)}}$. Proposition 3 with $\alpha = \beta = e$ gives that $\pi_{\mathrm{abs(R)}}$ is a singleton. Therefore, lemma 13 and 14 can be used to obtain the remaining equivalences. □

On this view, the differences in readings merely result from a shift in perspective on the underlying collections. The crudest view pertains in case of the kolkhoz-collective use, which states that at an index two unique collections are related to each other. The doubly neutral use, on the other hand, relates the same two collections, but employs the way the second collection is generated within the index. Finally, branching takes the members of the two collections to be all related to each other, which yields perhaps the finest perspective possible. For the present investigations it is important to note that these uses cannot be distinguished in terms of truth-conditions. We therefore suggest to capture them by means of the π_{inj}- and the π_{con}-mode of the verbal components combined with iterated neutral quantification, but to use no further representation within the semantics.

5 Conclusions

In 1991 the present authors agreed upon the need to reduce the ambiguity of sentences like (1) and (2) and decided to embark on an enterprise to end up with just one reading to these sentences rather than the usual six, eight, nine or more.

Verkuyl 1988 offered the intuition to bring back the number of readings to just two by means of the two modes of the π-function. This reduction was possible due to the integration of an important linguistic tradition called *localism* with the theory of generalized quantification. At the time of our common enterprise the constant and the injective functions appeared to be two sides of a coin, so our optimism seemed to be justified, as we suggested in Verkuyl and Van der Does 1991.

In separate studies—Van der Does 1993,1994 and Verkuyl 1993, 1994—it became clear that there were some problems. Even if it were possible to reduce Scha's nine reading to one on the basis of the scale approach, the cumulative reading required a special treatment. Van de Does 1993 ended up with the three readings in (10) as the minimal number of readings to be assigned to sentences like (1) and (2), and Verkuyl 1994 ended up with one reading for the combinations predicted by a scalar approach and one reading which was called kolkhoz-collectivity.

In the present paper, the number of readings is reduced to two by analyzing the relation between kolkhoz-collectivity and Scha's cumulativity. Whenever π is constant, a transitive doubly collective sentence is equivalent to a cumulative one. To show this, among other things, we introduced an abstract notion of Path. The main insight is that the notion of kolkhoz-collectivity, i.e., the π constant mode, can be used to connect the different forms of quantification.

Kolkhoz-collectivity arises naturally within the localist framework, but it is also of independent interest. Plainly, this notion requires a strong appeal to context, and we suggest to use indices for this purpose. It is shown that the two modes of predication, which correspond to natural restrictions on the Paths provided by a verb, covers the 'readings' discerned in the literature. We would wish to maintain that it is less natural to speak about two readings rather than modes; the difference between the reading corresponding to the $\pi_{injective}$-function and the kolkhoz-collective $\pi_{constant}$-function is not visible in the logical form. Of course, the restriction on the denotation of the verb can be seen in terms of a representational clue, but linguistically it makes more sense to continue to think about this matter in terms of modes.

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