LINGUISTIC GRAMMAR AS DYNAMIC LOGIC

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Abstract

Natural language is the primary vehicle of human communication. It involves a series of interlocked processes at various levels of aggregation. Proof theory and dynamic logic provide two paradigms for dealing with this variety. This paper gives an analysis of some main issues that arise in merging the two viewpoints towards this broader goal. Topics discussed include: proof as discourse, interfacing low-level fast and high-level slow systems, merging lambda dynamics and modal state dynamics, strategies that create feasible proof calculi. Overall prospects for broad 'dynamic architectures' look good.

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1 Connecting Proof and Communication

The programmatic title of the 1997 Rome Workshop was "proof-theoretical dimensions of communication processes". In my reading, this phrase calls for a meeting between two research areas, proof and communication, each with their own intellectual history. This is certainly an interesting juxtaposition, even though there is little linkage so far. Proof theory is an established branch of logic, but it has mainly studied static formal objects produced by one single thinker, rather than dynamic communication processes, which crucially involve actions of several agents, and discourse states that change over time. Processes of the latter kind, including many-person communication, are indeed coming up in contemporary logic, but then rather in the form of dynamic logics of natural language use, or distributed computation, or even information flow in general. [1] Nevertheless, proofs seem an excellent place for studying communication, because they may be seen as a first derivative of *argumentation*. The latter is presumably the source of logic in the first place, and it is an eminently social and communicative activity. [2] Moreover, being a relatively well-defined subspecies of discourse, argumentation may be a good pilot example for broader studies of communication. But, are we ready at this stage for *merging* the two realms? The purpose of this brief agenda paper is to discuss some different loci for dynamic phenomena in natural language use, and to identify some major logical issues that arise in their study. Our discussion does not cover all possible technical approaches to dynamic phenomena. In particular, for reasons of compactness, we phrase our issues in terms of dynamic logic and proof theory in their traditional guises. Thus, we must leave out the attractive interpretation of proofs as *logical games,* which is a rich alternative process paradigm for communication.

2 Dynamics in Logic and Language

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[1] For a general survey, cf. the 'Dynamics' chapter by R. Muskens et al. in the 1997 *Handbook of Logic, Language and Information*, or the monograph van Benthem 1996. For information flow between many agents, see the computer science-oriented Fagin, Halpern, Moses & Vardi 1995, and the linguistics-oriented Gerbrandy & Groeneveld 1997. These approaches are semantic in their main thrust. A prominent proof-theoretic paradigm with dynamic computational import is linear logic (Girard 1993, Schellinx 1998).

[2] Lorenzen's dialogue analysis in the 50s was a pioneering interpretation of proofs as two-person argumentation. In the 70s, Felscher rigorously proved Lorenzen's guiding intuition, that there is an effective correspondence between constructive proofs for intuitionistic formulas and winning strategies for the proponent of these statements in a dialogue game. By now, this interaction view has begun to flourish more broadly in the setting of linear logic: cf. the lecture notes Abramsky 1995.

Natural language use involves steady flow of information between speakers and hearers, writers and authors, or more symmetrically, the participants in a discussion. Thus, any faithful logical account of natural language must do justice to this dynamics. But in fact, *every* cognitive activity has a dynamic character – in particular, reasoning and evaluation, the key themes in traditional logic, which involve changing states of various actors. Therefore, recent trends in logic put cognitive activities and information flow at centre stage. For purposes of illustration, a sketch of one such paradigm will do. Processes can be modelled by 'process graphs' consisting of *states*, between which one can switch through *atomic actions*, viewed as binary transition relations between states, while more complex actions are defined over this basic repertoire through repeated use of *process constructions*. Examples are computer programs (changing memory states), games (changing collective states of participants by moves), belief revision (changing information states by updates), and general language use (changing complex discourse states of participants by successive assertions in context). A widely used repertoire of process constructions is Relational Algebra. It includes composition, converse, Boolean union, intersection, and complement, as well as iteration, or less familiar operations. [3] One can also make process graphs into game trees, with actions performed by different agents in a larger, perhaps distributed, process. Logical models like this are extremely abstract. They allow for studies of process constructions and dynamic inference, but do not address more specific issues. In real applications, a choice of appropriate concrete states and actions will be crucial. In particular, this holds true for natural language. Its dynamics encompasses many different processes. For instance, to a first approximation, one can think of making a statement as a means of updating the information state of one specific targeted interlocutor. But the more usual situation is when at least two persons engage in discourse, and exchange information, as happens even in a simple question-answer episode. Moreover, much more than just information is passed. Language conveys many subtle things, such as indications of people's formal or material preferences [4], or intentions, desires, other feelings, or even of some social hierarchy

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[3] A natural extension is Propositional Dynamic Logic, which merges relational algebra expressions R for actions with a modal language for defining propositions A about states, in a joint semantic format (1) $\mathbf{M}, s \models A$ (A is true of s in **M**), and (2) $\mathbf{M}, s_{in}, s_{out} \models R$ (executing R can take you from s_{in} to s_{out} in **M**). Cf. Harel, Kozen & Tiuryn 1995 for a modern text.

[4] Kameyama 1996, Jaspars & Kameyama 1998 study logico-computational architectures doing more justice to this. Cf. also Veltman 1996, or van Benthem, van Eyck & Frolova 1995 for the dynamics of preferences.

between the actors in communication - and all these may occur intertwined. In what follows, we shall be concerned only with information flow, even though the empirical fact of the matter seems to be that admixture of all these other elements seems to play some crucial role in efficient human communication, that is not yet understood.

3 Proof as Discourse

Before going into the mathematics of our subject, let us take the subject of this Workshop absolutely seriously. Proof is a form of *natural discourse* with participants, and an interesting one at that. Despite its vicinity to the hottest fires of formalization, its natural language aspect has withstood all historical attempts at its elimination. In this communicative role, proof has a primary sense of 'justification' which is indeed an activity with a clear social purpose. Proof serves to make oneself 'accountable', as all arguments are at the same time handles for subsequent objections and refutations by others. It may also serve to convince another person and direct her further thinking or action, or to help a whole profession organise hitherto isolated pieces of information...

The social aspect also shows in the basic moves of real proof action. Consider the two-person language employed in even formal mathematical proof. Control expressions like "assume", "let's"... steer the dynamics of derivation and definition. Far from being logically redundant stylistic ornaments, these are crucial to any effective presentation, and organizing not just a heap of propositions, but some pattern and overall purpose. But as it stands, current logical proof theory does not provide the necessary two-person communication models that would account for this. Hence we formulate our first

ISSUE I WHAT IS THE LOGICAL DYNAMICS OF ACTUAL PROOF?

The most congenial logical approach to date are the earlier-mentioned dialogue games. But these do not give the full picture – even though modern game logics are valuable 'laboratories' for testing effects of different conventions for roles of participants, scheduling of their actions, and storage of results. Additional cues might come from linguistic discourse semantics. [5] One major issue here should be to determine the

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[5] Cf. the work of computational linguists from the 80s surveyed in the Dynamics chapter of the *Handbook of Logic and Language*, as well as its chapter on Discourse Representation Theory (by Kamp & van Eyck).

important dynamic parameters that define a formal model of realistic proof structure. [6] Another would be useful many-person proof formats, describing joint argumentation.

4 Levels in Language Use: High and Low Dynamics

Natural language dynamics occurs across a whole range, from on-line understanding of the units of speech and text, through pronouncing or writing sentences, to planning and executing discourse stategies. This reflects a very general phenomenon in cognitive science, one of whose major challenges is to understand the spectrum of our cognitive performance between the poles of *conscious versus unconscious*, or *slow versus fast.* Likewise, we have both 'high' discourse dynamics and 'low' sentence or word dynamics to account for. In this paper, we use Categorial Grammar as our vehicle for discussing proof-theoretic approaches to natural language. [7] But one realizes at the outset that this linguistic approach is not about proof in the preceding sense, where the content of some (mathematical) assertion is at stake, rather than its mere grammatical structure. It rather analyses sentence processing, which is largely an unconscious fast process. Thus, we ask

ISSUE II HOW TO INTERFACE DISCOURSE AND SENTENCE DYNAMICS?

To this general question, there are no simple answers. For instance, no one knows yet how to integrate low-level image processing with high-level visual reasoning – even though humans are good at it. One possible bold answer may be stated (although it is bound to be over-simplified), as it illustrates the spirit of the Rome Workshops. Sentence dynamics and discourse dynamics use essentially *the same system*, with more carefree classical logics at the top, and more resource-economical linear logics at the bottom! One could call this *parametrization.* Logical systems should contain some 'buttons' that can be set differently for different tasks, while keeping the essential structure invariant. Such ideas have been proposed long ago for the architecture of universal linguistic theories by

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[6] One of these must surely be *dependence* between objects introduced in the course of arguments, as studied in van Lambalgen 1996, Meyer Viol 1995, continuing philosophical and linguistic work by Fine and others. Dependence is also becoming a key notion in general logical dynamics: cf. Chapter 9 of van Benthem 1996. Such additional parameters have profound consequences for natural 'Gentzen formats' and other central tools of current Proof Theory. More realistic formats may also lead to new logics. For 'higher' useful proof structures, cf. the more 'intelligent' theorem proving in Melis 1997, Siekmann, Kohlhase & Melis 1998.

[7] For a survey, cf. Moortgat and Buszkowski's chapters in the *Handbook of Logic and Language*, and for further logical theory, van Benthem 1991.

Noam Chomsky, and logic might profit from a similar set-up, too. This line may not be all that far-fetched. A form of 'parametrization' occurs in game-theoretical approaches, starting with Lorenzen's game conventions (supplementing the logical game moves of attack and defense), which can be set in many interesting ways.

Interactions between the two levels are of importance in their own right as a source of 'crossings'. E.g., human languages differ in their encoding of communicative features (such as 'definiteness' when referring to objects). Sometimes these are put explicitly in the syntax, sometimes implicitly in discourse conventions. We need to understand these things not just for logic, but also for any truly cross-linguistic Universal Grammar. Concrete instructions for crossings between syntactic coding and context coding may be found in the 'shift mechanisms' for indexical expressions discussed in recent work on the workings of context by John Perry and John McCarthy. [8] This would require proof systems whose languages can shift between possibly nested discourse contexts, dropping or acquiring argument positions for their predicates – perhaps using mechanisms like the context management found in Martin-Löf style type theories, or in more recent Gabbay-style labeled deduction. In a sense, these switches are familiar to mathematicians too, who will use the minimal formalization of their arguments that is compatible with their discourse environment, so as to save time and space.

5 Too Much Dynamics in Logic?

The next tension that we wish to discuss lies inside logic, rather than natural language. Notably, merging proof theory (which exemplifies the 'type-theoretical paradigm') with the world of dynamic logic requires overcoming a tension which may already be observed in intuitionistic logic. The latter's constructive character resides *both* in proof dynamics – witness the Brouwer-Heyting-Kolmogorov interpretation, backed up by the Curry-Howard isomorphism, *and* in update transitions between information states in semantic models (whether of Beth's or Kripke's variety). But the intuitive connection between the two remains unclear, and is not even discussed in major textbooks and surveys of the field. A similar tension may be observed in the standard exposition of categorial grammar. Curry-Howard dynamics derives annotated sequents of the form

$$\mathbf{x_1: A_1, ..., x_k: A_k \Rightarrow t: B}$$

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[8] J. van Benthem, 1998, 'Changing Contexts and Shifting Assertions', in A. Aliseda & R. van Glabbeek, eds., *Computing Natural Language*, CSLI Publications, Stanford.

where the final complex term t denotes a process. But on the other hand, we also have a relational view of e.g. the basic categorial Lambek Calculus, in the transition format

$$\mathbf{(s_1, s_2): A_1, ..., (s_k, s_{k+1}): A_k \Rightarrow (s_1, s_{k+1}) : B}$$

This is a dynamic logic view of categorial inference, involving successive actions induced by the premises, whose total effect is a state change executing the conclusion. The latter approach plausibly treats categorial product as relational composition, and the two slashes as its natural left- and right-inverses, i.e. operations in Relational Algebra. Andréka & Mikulas (1993) showed the significant mathematical result that the Lambek Calculus is complete for this interpretation – subsequently simplified in Kurtonina's 1995 dissertation *Frames and Labels.* But then, we have a third concern:

ISSUE III HOW TO RECONCILE LAMBDA & RELATIONAL DYNAMICS? [9]

Here, we have one more 'parametrization' to offer, be it in terms of a somewhat bleak primitive. There is at least a comprehensible general perspective that unifies both viewpoints. It uses a *ternary* core relation Rx, yz which may be interpreted as

syntactic concatenation (x = y*z) , or function application (x =y(z)) ,
or composition of two arrows y, z to form x .

The basic derivation rules for this notion are already known from Relevant Logic: [10]

- X, y:A, z: A–>B, Rs, yz |– s:B
- if X, y:A, Rs, yz |– s:B,
  then X |– z:A–>B
  (*modulo the usual conditions on freedom and bondage of* s, y, z)

Implementations might use labeled deductive systems, or translation into 'bounded' fragments of first-order logic (immediate from the above clauses; cf. Section 6). These rules provide a common format covering both lambda function dynamics and modal state

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[9] This may be a non-issue to fundamentalists of either creed, who see no sense to the opposite perspective. Some proof theorists might enjoy 'Löb's Principle': "semantics is a stopgap for holes in genuine proofs"...

[10] We copy these rules from J. van Benthem, 1996, 'Proofs, Labels and Dynamics in Natural Language', in H-J Ohlbach & U. Reyle, eds., *Festschrift for Dov Gabbay*, Kluwer, Dordrecht, to appear.

change dynamics. One can *extend* the standard lambda calculus to deal with this, allowing explicit *restriction* of choices for function values within suitable R-constraints. This will move from standard abstracts $\lambda y{:}A\bullet s{:}B$ to partial functions 'inside' R.

6 Feasible Dynamics

We conclude with a concrete technical illustration, in order to show that there is more to our theme than high philosophy. 'Dynamification' involves not only concrete decisions as to *ontology*, but also an explicit stand on *complexity* of the logics that we wish to design. After all, succesful natural language use and communication take place in real time... In this light, the above move to a ternary base relation turns out momentous. One of its underlying interpretations, being composition of two arrows into a new transition, makes the latter stand for *transitions* as dynamic objects in their own right. [11] This brings us to *Arrow Logic*, a family of decidable modal logics underneath classical relational algebra. Arrow logic systems remove the *associativity* of standard relational composition, indispensable for encoding algebraic word problems, and hence responsible for the undecidability of Relational Algebra and much of its offspring. The result is a modal language for composition triangles which contains all Booleans operation plus a converse modality, and the two categorial slashes – or alternatively, a triple of conjugated products. Its basic logic is decidable (indeed, PSPACE-complete). [12]

The general effect of introducing a primitive ternary composition may be understood via translation into first-order logic. Indeed, upon transcription of their semantic truth conditions in the obvious way, all categorial expressions will end up in the so-called *Guarded Fragment,* where all quantifiers occur bounded. [13] In particular, the above ternary relations will come to serve as atomic 'guards' bounding all occurrences of quantifiers and their following matrix statements. It is this *uniform relativization* which

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[11] Of course, one then still has to supply some plausible concrete view of the *primitive dynamic transitions* for Categorial Grammar, or other grammatical paradigms for that matter.

[12] Note that the Arrow Logic move corresponds to a switch to the so-called 'non-associative' Lambek Calculus – since the ternary relation semantics (unlike the earlier-mentioned binary one) is not automatically associative. For a quick introduction to Arrow Logic, see Venema's 'Crash Course on Arrow Logic' in the CSLI volume Marx, Masuch & Pólos 1996 on *Arrow Logic and Multi-Modal Logic*.

[13] Cf. Andréka, van Benthem & Németi, 1996, 'Modal Languages and Bounded Fragments of First-Order Logic', to appear in the *Journal of Philosophical Logic* 1997. Follow-up results on guarded first-order logic may be found in van Benthem 1997B. Grädel 1997 shows that GF is complete for doubly exponential time.

makes the 3-variable language of relational algebra *decidable*. Thus, from a general first-order perspective, categorial dynamics seems less expensive than full Relational Algebra. [14] But this pleasant outcome might still have to be improved. The reason is as follows.

Categorial logics tend to be much less complex than guarded fragments. They are often in (non-)deterministic *polynomial time*, and hence much more feasible in parsing as deduction. One reason is that they *lack Boolean disjunction and negation*, which avoids case-proliferation in proof search, and thereby stays inside 'feasible fragments' of full modal languages. [15] Thus, our 'decomplexification strategies' lead to a final general issue, of relevance if we want to understand the reliable on-line performance of natural language users – when climbing downward on the ladder from 'high' to 'low' dynamics as discussed in an earlier section:

ISSUE IV HOW TO TURN DECIDABLE LOGICS INTO FEASIBLE ONES?

One might drop the offending Booleans to achieve this effect. But a more principled route exists as well. One can *re-interpret* things so as to avoid unwarranted complexity. Our preference is to tame disjunction and negation in the latter fashion, much as happens in Urquhart's well-known 'information piece' semantics for Relevant Logic. In particular, disjunctions then become 'mixtures' of two cases, as happens also in quantum-logical interpretation of A-or-B via linear combination of A– and B-vectors. (Interestingly, this is like a linear logic interpretation of disjunction.) Thus, letting U be a new ternary 'mixing relation', and V a binary 'reversal relation', one would now put

$\mathbf{M}, s \models$ A-or-B iff *there exist* $t, u : Us, tu$ & $\mathbf{M}, t \models A$ & $\mathbf{M}, u \models B$
$\mathbf{M}, s \models$ not-A iff *there exists* $t : Vs, t$ & *not* $\mathbf{M}, t \models A$

The result is a landscape of propositional logics below classical two-valued logic, of varying complexity. Some classical laws will go through, others become optional 'extras'.

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[14] In this connection, the earlier Andréka & Mikulas completeness theorem says that for the original associative Lambek calculus, the difference between binary and ternary relational semantics does not matter. But this is a very restricted language, and additions will soon become too expressive for this match. Thus, the ternary semantics seems the more flxible framework over-all.

[15] There is a spate of recent results by Kurtonina, Moortgat, and de Rijke on their semantics and complexity. Kurtonina & de Rijke 1997 connects up with current work on low complexity 'terminological languages'.

Of course, this re-interpretation strategy for Boolean operations can also be applied to *arbitrary* further logics, say, to modal or first-order ones. [16] Once again, the move away from standard Booleans is also characteristic for the connectives found in Linear Logic. In particular, is there some deeper connection between linear phase models and the above ternary model semantics? This would draw together CG and LL lines at this Workshop.

7 Conclusion

Dynamic processes can be modelled inside logic. Most current theories of this sort have been semantic, with a view toward understanding update or communication algorithms, but proof theory provides a natural paradigm too. Some merge seems desirable – and prospects for communication-oriented proof theories for natural language look good. In particular, comparing our two main featured approaches, we conclude that

> *Categorial Grammar and Dynamic Logic do match:*
> *though their main off-spring so far are open problems.*

8 References

S. Abramsky, 1995, *Semantics of Interaction*, lecture notes, department of computing, University of Edinburgh.

H. Andréka, J. van Benthem & I. Németi, 1996, 'Modal Languages and Bounded Fragments of First-Order Logic', first version in *Bulletin of the Interest Group for Pure and Applied Logics* 3, August 1995, 685–720. London and Saarbruecken. Final version to appear in the *Journal of Philosophical Logic* 1998.

H. Andréka & S. Mikulas, 1993, 'The Completeness of the Lambek Calculus for Relational Semantics', *Journal of Logic, Language and Information* 3, 1 -37.

J. van Benthem, 1991, *Language in Action. Categories, Lambdas and Dynamic Logic*, North-Holland, Amsterdam.

J. van Benthem, 1996, *Exploring Logical Dynamics*, Studies in Logic, Language and Information, CSLI Publications, Stanford.

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[16] Van Benthem, 1996, *Exploring Logical Dynamics*, Ch. 8, has some samples, including 'correspondence results' for standard propositional tautologies. Spaan 1998 considers non-Boolean modal languages. Often, complexity goes down, but not always: as modalities plus conjunctions can mimick disjunctions. It would be of interest to tie this up with the treatment of highly regimented 'control modalities' in categorial languages due to Moortgat & Kurtonina (cf. Moortgat 1997).

J. van Benthem, 1997A, 'Proofs, Labels and Dynamics in Natural Language', in H-J Ohlbach & U. Reyle, eds., *Festschrift for Dov Gabbay*, Kluwer, Dordrecht, to appear.

J. van Benthem, 1997B, 'The Range of Modal Logic', Report ML–97–05, Institute for Logic, Language and Computation, University of Amsterdam. To appear in the *Journal of Applied Non-Classical Logics*.

J. van Benthem, 1998, 'Changing Contexts and Shifting Assertions', in A. Aliseda & R. van Glabbeek, eds., *Computing Natural Language*, CSLI Publications, Stanford, 51 - 65.

J. van Benthem, J. van Eyck & A. Frolova, 1993, 'Changing Preferences', Report CS-93-10, Centre for Mathematics & Computer Science, Amsterdam.

J. van Benthem & A. ter Meulen, eds., 1997, *Handbook of Logic and Language*, Elsevier Science Publishers, Amsterdam.

W. Buszkowski, 1997, 'Mathematical Linguistics and Proof Theory', in J. van Benthem & A. ter Meulen, eds., 683–736.

R. Fagin, J. Halpern, Y. Moses & M. Vardi, 1995, *Reasoning about Knowledge*, The MIT Press, Cambridge (Mass.).

J. Gerbrandy & W. Groeneveld, 1997, 'Reasoning About Information Flow', *Journal of Logic, Language and Information* 6:2, special TARK issue.

J-Y Girard, 1993, 'Linear Logic: A Survey', in L. Bauer & H. Schwichtenberg, eds., *Proceedings of the International Summer School of Marktoberdorff*, NATO Advanced Science Institutes, series F94, Springer, Berlin, 63 - 112.

E. Graedel, 1997, 'On the Restraining Power of Guards', department of mathematics and computer science, RWTH Aachen.

D. Harel, D. Kozen & J. Tiuryn, 1997, *Dynamic Logic*, department of computer science, Cornell University & Weizmann Institute, book in progress.

W. Hodges, 1998, A*n Invitation to Logical Games*, Queen Mary's College, University of London.

J. Jaspars & M. Kameyama, 1998, 'Discourse Preferences in Dynamic Logic', in A. Aliseda, R. van Glabbeek & D. Westerståhl, eds., *Computing Natural Language*, 67 – 96, CSLI Publications, Center for the Study of Language and Information, Stanford.

M. Kameyama, 1996, 'Indefeasible Semantics and Defeasible Pragmatics', in M. Kanazawa, Ch. Piñon & H. de Swart, eds., *Quantifiers, Deduction, and Context*, 111 – 138, CSLI Publications, Center for the Study of Language and Information, Stanford.

N. Kurtonina, 1995, *Frames and Labels*, dissertation, OTS Utrecht and ILLC Amsterdam.

N. Kurtonina & M. de Rijke, 1997, 'Expressiveness of First-Order Description Logics', Department of Computer Science, University of Warwick, Report RR-325.

M. van Lambalgen, 1996, 'Generalised Quantification as Substructural Logic', *Journal of Symbolic Logic* 61, 1006-1044.

E. Melis, 1998, 'AI-Techniques in Proof Planning', *European Conference on Artificial Intelligence* (ECAI-98), Kluwer, Brighton, 494-498.

W. Meyer Viol, 1995, *Instantial Logic*, dissertation, OTS Utrecht & ILLC Amsterdam.

M. Moortgat, 1997, 'Categorial Type Logics', in J. van Benthem & A. ter Meulen, eds., 93–177.

H. Schellinx, 1998, 'A Survey of Linear Logic', manuscript in preparation, Institute of Mathematics, Utrecht University.

J.H. Siekmann, M. Kohlhase & E. Melis, 1998, 'Omega: Ein mathematisches Assistenzsystem', to appear in *Kognitionswissenschaft*, vol. 7.

E. Spaan, 1998, Complexity of Non-Boolean Modal Languages, notes, Loyola College NY & ILLC Amsterdam.

F. Veltman, 1996, 'Defaults in Update Semantics', Journal of Philosophical Logic 25, 241 - 261.

Y. Venema, 1997, 'A Crash Course on Arrow Logic', in M. Marx, M. Masuch & L. Pólos, *Arrow Logic and Multi-Modal Logic*, CSLI Publications, Stanford, 3–34.