Acts of Commanding and Changing Obligations

Tomoyuki Yamada

Graduate School of Letters, Hokkaido University
Nishi 7, Kita 10, Kita-ku, Sapporo, Hokkaido, 060-0810, Japan
yamada@LET.hokudai.ac.jp

Abstract. If we are to take the notion of speech act seriously, we must be able to treat speech acts as acts. In what follows, I will try to model changes brought about by various acts of commanding in terms of a variant of update logic. I will combine a multi-agent variant of the language of monadic deontic logic with a dynamic language to talk about the situations before and after the issuance of commands, and the commands that link those situations. Although the resulting logic inherits various inadequacies from monadic deontic logic, some interesting principles are captured and seen to be valid nonetheless. An outline of a proof of completeness and some interesting valid principles together with concrete examples will be presented.

1 Introduction

Suppose you are reading an article on logic in the office you share with your boss and a few other colleagues. While you are reading, the temperature of the room rises, and it is now above 30 degrees Celsius. There is a window and an air conditioner. You can open the window, or turn on the air conditioner. You can also concentrate on the article and ignore the heat. Then, suddenly, you hear your boss's voice. She commanded you to open the window. What effects does her command have on the current situation?

Her act of commanding does not affect the state of the window directly, except for making it vibrate a little, perhaps. Nor does it affect the number of alternatives you have. It still is possible for you to turn on the air conditioner, to ignore the heat, or to open the window, of course. But it has now become impossible for you to choose alternatives other than that of opening the window without going against your obligation. It is now obligatory upon you to open the window, although it was not so before.

If the notion of speech acts, or more specifically that of illocutionary acts, is to be taken seriously, it must be possible to see utterances not only as acts of uttering words but also as acts of doing something more. But speech acts do not seem to affect so called brute facts directly, except for those various physical and physiological conditions involved in the production and perception of sounds or written symbols. What differences can they bring about in our life?

In attempting to answer this question, it is important to be careful not to blur the distinction between illocutionary acts and perlocutionary acts. Since Grice (1948, 1957), many philosophers, linguists, and computer scientists have talked about utterers' intentions to produce various changes in the attitudes of addressees in their theories of communication. But utterers' intentions usually go beyond illocutionary acts by involving perlocutionary consequences, while illocutionary acts can be effective even if they do not produce intended perlocutionary consequences. Thus, in the above example, even if you refuse to open the window in question, that will not make her command void. Your refusal would not constitute disobedience if it could make her command void. Her command is effective in a sense even if she has failed to get you to form the intention to open the window. In order to characterize effects of illocutionary acts adequately, we need to be able to isolate them from perlocutionary consequences of utterances.

It is interesting to note, in this connection, that some illocutionary acts such as commanding, forbidding, permitting, and promising seem to affect our social life by bringing about changes in the deontic status of various alternative courses of actions. In the above example, before the issuance of your boss's command, none of your three alternatives were obligatory upon you, but after the issuance, one of them has become obligatory. In what follows, I will try to model changes acts of commanding bring about in terms of a new update logic. I will combine a multi-agent variant of the language of monadic deontic logic with a dynamic language to talk about the situations before and after the issuance of commands, and the commands that link those situations. Although the resulting language inherits various inadequacies from the language of monadic deontic logic, some interesting principles are captured and seen to be valid nonetheless.

The idea of update logic of commands is inspired by the update logics of public announcements and private information transmissions developed in Plaza (1989), Groeneveld (1995), Gerbrandy and Groeneveld (1997), Gerbrandy (1999), Baltag, Moss, & Solecki (1999), and Kooi & van Benthem (2004). In van Benthem (2000), the logics of such epistemic actions are presented as exemplars of a view of logic as "the analysis of general informational processes: knowledge representation, giving or receiving information, argumentation, communication", and used to show "how using a 'well-known' system as a vehicle, viz. standard epistemic logic, leads to totally *new issues* right from the start"(p.33). The basic idea of the update logic of acts of commanding is to capture the workings of acts of commanding by using deontic logic instead of epistemic logic as a vehicle. This may lead to a significant extension of the range of the kind of logical analysis advocated in van Benthem (2000), since acts of commanding exemplify a kind of speech acts radically different from those discussed in the logics of epistemic actions.

2 The static base language $\mathcal{L}_{\mathsf{MDL}^+}$

Consider the example above. In the situation before the command is given, it was neither obligatory upon you to open the window, nor was it so not to open it. But in the situation after your boss's act of commanding, it has become obligatory upon you to open it. In order to describe these situations, I use a language $\mathcal{L}_{\mathsf{MDL}^+}$, the Language of Multi-agent monadic Deontic Logic With an alethic modal operator, MDL^+. I represent the two situations by two models M and N with a world s for $\mathcal{L}_{\mathsf{MDL}^+}$. Thus, we can describe the difference between these situations as follows:

$$M, s \models_{\mathsf{MDL}^+} \neg O_a p \wedge \neg O_a \neg p \tag{1}$$

$$N, s \models_{\mathsf{MDL}^+} O_a p \ \ , \tag{2}$$

where the propositional letter p stands for the proposition that the window is open at such and such a time, say t_1. The operator O_a here is indexed by a given finite set $I = \{a, b, c, \ldots, n\}$ of agent, and the index a here represent you. A formula of form $O_i\varphi$ is to be read as meaning that it is obligatory upon the agent i that φ. Thus, I define:

Definition 1. *Take a countably infinite set* **Aprop** *of proposition letters and a finite set I of agents, with p ranging over* **Aprop** *and i over I. The multi-agent monadic deontic language $\mathcal{L}_{\mathsf{MDL}^+}$ is given by:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_i\varphi$$

The set of all well formed formulas (sentences) of $\mathcal{L}_{\mathsf{MDL}^+}$ is denoted by S_{MDL^+} and operators of the form O_i are called deontic operators. For each $i \in I$, we call a sentence i-free if no O_i's occur in it. We call sentence alethic if no deontic operators occur in it, and boolean if no modal operators occur in it. For each $i \in I$, the set of all i-free sentences is denoted by $S_{i\text{-free}}$. The set of all alethic sentences and the set of all boolean sentences are denoted by S_{Aleth} and S_{Boole} respectively.

$\bot$, $\vee$, $\rightarrow$, $\leftrightarrow$, and $\Diamond$ are assumed to be introduced by standard definitions. We also abbreviate $\neg O_i \neg\varphi$ as $P_i\varphi$, and $O_i\neg\varphi$ as $F_i\varphi$. Note that **Aprop** $\subset S_{\text{Boole}} \subset S_{\text{Aleth}} \subset S_{i\text{-free}} \subset S_{\mathsf{MDL}^+}$ for each $i \in I$.

Definition 2. *By an $\mathcal{L}_{\mathsf{MDL}^+}$-model, I mean a quadruple $M = \langle W^M, R_A^M, R_I^M, V^M \rangle$ where:*

(i) *W^M is a non-empty set (heuristically, of 'possible worlds'),*
(ii) *$R_A^M \subseteq W^M \times W^M$,*
(iii) *R_I^M is a function that assigns a subset $R_I^M(i)$ of R_A^M to each agent $i \in I$,*
(iv) *V^M is a function that assigns a subset $V^M(p)$ of W^M to each proposition letter $p \in$* **Aprop**.

We usually abbreviate $R_I^M(i)$ as R_i^M.

Note that for any $i \in I$, R_i^M is required to be a subset of R_A^M. Thus we assume that whatever is permitted is possible.

Definition 3. *Let M be an $\mathcal{L}_{\mathsf{MDL}^+}$-model and w a point in M. If $p \in Aprop$, φ, $\psi \in S_{\mathsf{MDL}^+}$, and $i \in I$, then:*

(a) $M, w \models_{\mathsf{MDL}^+} p$ *iff* $w \in V^M(p)$,
(b) $M, w \models_{\mathsf{MDL}^+} \top$,
(c) $M, w \models_{\mathsf{MDL}^+} \neg\varphi$ *iff* $M, w \not\models_{\mathsf{MDL}^+} \varphi$,
(d) $M, w \models_{\mathsf{MDL}^+} (\varphi \wedge \psi)$ *iff* $M, w \models_{\mathsf{MDL}^+} \varphi$ *and* $M, w \models_{\mathsf{MDL}^+} \psi$,
(e) $M, w \models_{\mathsf{MDL}^+} \Box\varphi$ *iff for every v such that* $\langle w, v \rangle \in R_A^M$, $M, v \models_{\mathsf{MDL}^+} \varphi$,
(f) $M, w \models_{\mathsf{MDL}^+} O_i\varphi$ *iff for every v such that* $\langle w, v \rangle \in R_i^M$, $M, v \models_{\mathsf{MDL}^+} \varphi$.

A formula φ is true in an $\mathcal{L}_{\mathsf{MDL}^+}$-model M at a point w of M if $M, w \models_{\mathsf{MDL}^+} \varphi$. We say that a set Σ of formulas of $\mathcal{L}_{\mathsf{MDL}^+}$ is true in M at w, and write $M, w \models_{\mathsf{MDL}^+} \Sigma$, if $M, w \models_{\mathsf{MDL}^+} \psi$ for every $\psi \in \Sigma$. If $\Sigma \cup \{\phi\}$ is a set of formulas of $\mathcal{L}_{\mathsf{MDL}^+}$, we say that ϕ is a semantic consequence of Σ, and write $\Sigma \models_{\mathsf{MDL}^+} \varphi$, if for every $\mathcal{L}_{\mathsf{MDL}^+}$-model M for every point w such that $M, w \models_{\mathsf{MDL}^+} \Sigma$, $M, w \models_{\mathsf{MDL}^+} \varphi$. We say that a formula φ is valid, and write $\models_{\mathsf{MDL}^+} \varphi$, if $\emptyset \models_{\mathsf{MDL}^+} \varphi$.

Note that it is not standard to relativize obligation to agents. In dealing with moral obligations, for example, it is natural to work with un-relativized obligations. But we are here trying to capture the effects of commands, and commands can be, and usually are, given to some specific addressees. In order to capture how such commands work in a situation where their addressees and non-addressees are present, it is necessary to work with a collection of accessibility relations relativized to various agents. In such multi-agent settings, we may need to talk about commands given to every individual agent in a specified group, as distinct not only from commands given to a single agent but also from commands meant for every agent, e.g. "Thou shalt not kill". And even among commands given to a group of agents, we may have to distinguish commands to be executed jointly by all the members of the group from commands to be executed individually by each of them. Although I will only consider commands given to a single agent in this paper, it doesn't seem impossible to extend my analysis to commands given to more than one agents.

A word about my choice of monadic deontic operators here may be in order. Monadic deontic logics are known to be inadequate to deal with conditional obligations and R. M. Chisholm's contrary-to-duty paradox; dyadic deontic logics are better in this respect. But there are still other problems which are unsolved even by dyadic deontic logics, and Åqvist (2002), for example, stresses the importance of temporal and quantificational machinery to viable deontic logics. My use of monadic deontic operators here do not reflect any substantial theoretical commitment. I am only trying to keep things as simple as possible as we are in such an early stage of the development. I will discuss some shortcomings resulting from the static nature of this language and the possibility of using different languages as vehicles later.

A word about the use of alethic modal operator may also be in order. It can be used to describe unchanging aspects of the changing situations. As we have seen in our example, even after your boss's act of commanding, it was still possible for you to turn on the air conditioner or to ignore the heat. Thus we have

$$M, s \models_{\mathsf{MDL}^+} \Diamond p \land \Diamond q \land \Diamond(\neg p \land \neg q) \tag{3}$$

$$N, s \models_{\mathsf{MDL}^+} \Diamond p \land \Diamond q \land \Diamond(\neg p \land \neg q) \ , \tag{4}$$

where p is to be understood as before, and q as meaning that the air conditioner is running at t_1. Note that the notion of possibility here is that of alethic (or metaphysical) possibility, and not that of epistemic possibility. Suppose, for example, you opened the window. Even after noticing it, one of your colleague might complain, without having noticed your boss's command, that if you hadn't opened the window, they would not be disturbed by the outside noises.

3 A dynamic Language $\mathcal{L}_{\mathsf{CL}}$

As is clear from the above example, formulas of $\mathcal{L}_{\mathsf{MDL}^+}$ can be used to describe the situation before and after the issuance of your boss's command. But note that your boss's act of commanding, which change M into N, is talked about not in $\mathcal{L}_{\mathsf{MDL}^+}$ but in the meta-language. In order to have an object language in which we can talk about acts of commanding, I now turn to the dynamification.

In order to represent the action type of commanding an agent i to see to it that φ, I introduce expressions of the form $!_i\varphi$ for each $i \in I$. Let a and p be understood as before. Then your boss's act of commanding was of type $!_a p$. The static base language $\mathcal{L}_{\mathsf{MDL}^+}$ shall be expanded by introducing new modalities indexed by expressions of this form. Thus, in the resulting language, the language $\mathcal{L}_{\mathsf{CL}}$, of Command Logic, we have formulas of the form $[!_i\varphi]\psi$, which should be read as meaning that after every successful act of commanding of type $!_i\varphi$, ψ holds. Thus, we define the language as follows:

Definition 4. *Take the same countably infinite set* **Aprop** *of proposition letters and the same finite set I of agents as before, with p ranging over* **Aprop** *and i over I. The language of command logic $\mathcal{L}_{\mathsf{CL}}$ is given by:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_i\varphi \mid [\pi]\varphi$$
$$\pi ::= !_i\varphi$$

Terms of the form $!_i\varphi$ and operators of the form $[!_i\varphi]$ are called command type terms and command operators, respectively. The set of all well formed formulas of $\mathcal{L}_{\mathsf{CL}}$ is referred to as S_{CL}, and the set of all the well formed command type terms as **Com**.

$\bot$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\Diamond$, P_i, F_i , and $\langle !_i\varphi\rangle$ are assumed to be introduced by definition in the obvious way. Note that $S_{\mathsf{MDL}^+} \subset S_{\mathsf{CL}}$.

Now, in order to give truth definition for $\mathcal{L}_{\mathsf{CL}}$, we need to specify how acts of commanding change models. As we have observed earlier, in the situation before the issuance of your boss's command, we have $\neg O_a p$ at s. This means that in M, at some point v such that $\langle s, v\rangle \in R_a^M$, $\neg p$ holds. Let t be such a point. Now in the updated situation N we have $O_a p$ at s, and this means that in N, there is no point w such that $\langle s, w\rangle \in R_a^N$ and $M, w \models_{\mathsf{MDL}^+} \neg p$. But since we have $M, t \models_{\mathsf{MDL}^+} \neg p$, we also have $N, t \models_{\mathsf{MDL}^+} \neg p$. As we have remarked, her command does not affect the state of the window directly. This means that in N, $\langle s, t\rangle$ is not in R_a^N.

A bit of terminology is of some help here. If a pair of points $\langle w, v\rangle$ is in some accessibility relation R, the pair will be referred to as the R-arrow from w to v. Thus we will talk about R_A^M-arrows, R_i^N-arrows, and so on. We will sometimes omit superscripts for models when there is no danger of confusion. Then the above consideration suggests that an act of commanding of the form $!_i\varphi$, if performed at w in M, eliminates from R_i^M every R_i^M-arrow that terminates in a world where φ doesn't hold. If a command works in this way, we say it is eliminative. As your boss's command in our example is eliminative, the updated model N differs from the original only in that it has $R_a^M - \{\langle w, v\rangle \in R_a^M \mid M, v \not\models_{\mathsf{MDL}^+} \varphi\}$, or equivalently $\{\langle w, v\rangle \in R_a^M \mid M, v \models_{\mathsf{MDL}^+} \varphi\}$, in place of R_a^M as the deontic accessibility relation for the agent a. [1]

In order to make this notion precise, the following definition will be useful:

[1] We can think of a more restricted, or local, variant of update operation, namely, that of replacing R_i^M with $R_i^M - \{\langle w, v\rangle \in R_i^M \mid w = s \text{ and } M, v \not\models_{\mathsf{MDL}^+} \varphi\}$ when an act of commanding of the form $!_i\varphi$ is performed at s in M. This operation leads to a logic which is slightly different from the logic to be discussed in this paper.

Definition 5. *Let* $M = \langle W^M, R_A^M, R_I^M, V^M \rangle$ *be an* $\mathcal{L}_{\mathsf{MDL}^+}$*-model, and* X *a subset of* R_A^M*. Then, the model obtained from* M *by replacing* R_i^M *with* X *is an* $\mathcal{L}_{\mathsf{MDL}^+}$*-model* $[R_i^M/X]M = \langle W^{[R_i^M/X]M}, R_A^{[R_i^M/X]M}, R_I^{[R_i^M/X]M}, V^{[R_i^M/X]M} \rangle$ *such that:*

(i) $W^{[R_i^M/X]M}$*,* $V^{[R_i^M/X]M}$ *and* $R_A^{[R_i^M/X]M}$ *are identical with* W^M*,* V^M *and* R_A^M*, respectively,*
(ii) $R_I^{[R_i^M/X]M}$ *is the function such that:*

(a) $R_I^{[R_i^M/X]M}(j) = R_I^M(j)$ *, for each* $j \in I$ *such that* $j \neq i$ *,*
(b) $R_I^{[R_i^M/X]M}(i) = X$ *.*

Then, the truth definition for the sentences of $\mathcal{L}_{\mathsf{CL}}$ that incorporates the above conception of an eliminative command can be given with reference to $\mathcal{L}_{\mathsf{MDL}^+}$-models.

Definition 6. *Let* $M = \langle W^M, R_A^M, R_I^M, V^M \rangle$ *be an* $\mathcal{L}_{\mathsf{MDL}^+}$*-model, and* $w \in W^M$*. If* $p \in$ **Aprop***,* $\varphi, \psi, \chi \in S_{\mathsf{CL}}$*, and* $i \in I$*, then:*

(a) $M, w \models_{\mathsf{ECL}} p$ *iff* $w \in V^M(p)$*,*
(b) $M, w \models_{\mathsf{ECL}} \top$*,*
(c) $M, w \models_{\mathsf{ECL}} \neg\varphi$ *iff* $M, w \not\models_{\mathsf{ECL}} \varphi$*,*
(d) $M, w \models_{\mathsf{ECL}} (\varphi \wedge \psi)$ *iff* $M, w \models_{\mathsf{ECL}} \varphi$ *and* $M, w \models_{\mathsf{ECL}} \psi$*,*
(e) $M, w \models_{\mathsf{ECL}} \Box\varphi$ *iff* $M, v \models_{\mathsf{ECL}} \varphi$ *for every* v *such that* $\langle w, v \rangle \in R_A^M$*,*
(f) $M, w \models_{\mathsf{ECL}} O_i\varphi$ *iff* $M, v \models_{\mathsf{ECL}} \varphi$ *for every* v *such that* $\langle w, v \rangle \in R_i^M$*,*
(g) $M, w \models_{\mathsf{ECL}} [!_i\chi]\varphi$ *iff* $[R_i^M/R_i^M \upharpoonright \chi^{\downarrow}]M, w \models_{\mathsf{ECL}} \varphi$*,*

where $R_i^M \upharpoonright \chi^{\downarrow} = \{\langle x, y \rangle \in R_i^M \mid M, y \models_{\mathsf{ECL}} \chi\}$*. The subscript "*ECL*" here stands for eliminative command logic. A formula* φ *is true in an* $\mathcal{L}_{\mathsf{MDL}^+}$*-model* M *at a point* w *of* M *if* $M, w \models_{\mathsf{ECL}} \varphi$*. We say that a set* Σ *of formulas of* $\mathcal{L}_{\mathsf{CL}}$ *is true in* M *at* w*, and write* $M, w \models_{\mathsf{ECL}} \Sigma$*, if* $M, w \models_{\mathsf{ECL}} \psi$ *for every* $\psi \in \Sigma$*. If* $\Sigma \cup \{\phi\}$ *is a set of formulas of* $\mathcal{L}_{\mathsf{CL}}$*, we say that* ϕ *is a semantic consequence of* Σ*, and write* $\Sigma \models_{\mathsf{ECL}} \varphi$*, if for every* $\mathcal{L}_{\mathsf{MDL}^+}$*-model* M *and every point* w *of* M *such that* $M, w \models_{\mathsf{ECL}} \Sigma$*,* $M, w \models_{\mathsf{ECL}} \varphi$*. We say that a formula* φ *is valid, and write* $\models_{\mathsf{ECL}} \varphi$*, if* $\emptyset \models_{\mathsf{ECL}} \varphi$*.*

The crucial clause in this definition is (g). The truth value of $[!_i\chi]\varphi$ at w in M is defined in terms of the truth value of φ at w in the updated model $[R_i^M/R_i^M \upharpoonright \chi^{\downarrow}]M$.

Note also that the remaining clauses in the definition reproduce the corresponding clauses in the truth definition for $\mathcal{L}_{\mathsf{MDL}^+}$. Obviously, the following condition is satisfied:

Corollary 1. *Let* M *be an* $\mathcal{L}_{\mathsf{MDL}^+}$*-model and* w *a point of* M*. Then for any* $\varphi \in S_{\mathsf{MDL}^+}$*:*

$$M, w \models_{\mathsf{ECL}} \varphi \text{ iff } M, w \models_{\mathsf{MDL}^+} \varphi \ .$$

The following corollary can be proved by induction on the length of ψ:

Corollary 2. *Let ψ be an i-free formula. Then, for any $\varphi \in S_{\mathsf{MDL}^+}$:*

$$M, w \models_{\mathsf{ECL}} \psi \text{ iff } [R_i^M / R_i^M{\upharpoonright}\varphi^{\downarrow}]M, w \models_{\mathsf{ECL}} \psi \ .$$

This means that acts of commanding will not affect deontic status of possible courses of actions of agents other than the addressee. This may be said to be a simplification. I will return to this point later.

Another thing the above corollary means is that acts of commanding will not affect brute facts and alethic possibilities in any direct way. Thus, in our example, if s in M is the actual world before the issuance of your boss's command, then s in $[R_i^M / R_i^M{\upharpoonright}p^{\downarrow}]M$ is the actual world after the issuance, and we have:

$$[R_i^M / R_i^M{\upharpoonright}p^{\downarrow}]M, s \models_{\mathsf{ECL}} \Diamond p \wedge \Diamond q \wedge \Diamond(\neg p \wedge \neg q) \ , \tag{5}$$

since we have $M, s \models_{\mathsf{ECL}} \Diamond p \wedge \Diamond q \wedge \Diamond(\neg p \wedge \neg q)$. But note that we also have:

$$M, s \models_{\mathsf{ECL}} [!_i p]O_i p \ . \tag{6}$$

Your boss's command eliminates all the R_i^M-arrows $\langle w, v\rangle$ such that $M, v \not\models_{\mathsf{ECL}} p$, and consequently we have $[R_i^M / R_i^M{\upharpoonright}p^{\downarrow}]M, s \models_{\mathsf{ECL}} O_i p$.

In fact this is an instantiation of the following principle:

Proposition 1 (CUGO Principle). *If $\varphi \in S_{i\text{-free}}$, then $\models_{\mathsf{ECL}} [!_i\varphi]O_i\varphi$.*

The restriction on φ here is motivated by the fact that the truth of φ at a point v in M does not guarantee the truth of φ at v in $[R_i^M / R_i^M{\upharpoonright}\varphi^{\downarrow}]M$ if φ involves deontic modalities for the agent i. For example, $[!_i P_i q]O_i P_i q$ is not valid. [2]

Example 1. Let $I = \{i\}$, and $M = \langle W^M, R_A^M, R_I^M, V^M\rangle$ where $W^M = \{s, t, u\}$, $R_A^M = \{\langle s, t\rangle, \langle t, u\rangle\}$, $R_I^M(i) = \{\langle s, t\rangle, \langle t, u\rangle\}$, and $V^M(q) = \{u\}$. Then we have $M, u \models_{\mathsf{ECL}} q$. Hence we have $M, t \models_{\mathsf{ECL}} P_i q$ but not $M, u \models_{\mathsf{ECL}} P_i q$. This in turn means that $\langle s, t\rangle$ is, but $\langle t, u\rangle$ is not, in $R_i^M{\upharpoonright}P_i q^{\downarrow}$. Thus we have $[R_i^M / R_i^M{\upharpoonright}P_i q^{\downarrow}]M, t \not\models_{\mathsf{ECL}} P_i q$, hence $[R_i^M / R_i^M{\upharpoonright}P_i q^{\downarrow}]M, s \not\models_{\mathsf{ECL}} O_i P_i q$. Therefore we have $M, s \not\models_{\mathsf{ECL}} [!_i P_i q]O_i P_i q$.

CUGO principle characterizes (at least partially) the effect of an act of commanding; though not without exceptions, commands usually generate obligations. The workings of an act of commanding of the form $!_i\varphi$ can be visualized by imagining $R_{!_i\varphi}$-arrows, so to speak. If an act of commanding $!_i\varphi$ is performed in M at a point w, it will take us to w in $[R_i^M / R_i^M{\upharpoonright}\varphi^{\downarrow}]M$ along an $R_{!_i\varphi}$-arrow. R_A-arrows, in contrast, only take us to points within M if it start from a point in M. While ordinary actions affect brute facts, acts of commanding affect deontic aspects of situations in our life. This difference is reflected in the difference between R_A-arrows and $R_{!_i\varphi}$-arrows. Different choices

[2] Let S_{CGO} be the set of sentences φ such that $\models_{\mathsf{ECL}} [!_i\varphi]O_i\varphi$. Since $O_i\varphi \to O_i\varphi \in S_{\mathrm{CGO}}$, we have $S_{i\text{-free}} \subset S_{\mathrm{CGO}} \subset S_{\mathrm{CL}}$. But exactly how large S_{CGO} is is an interesting open question.

of different alternative actions are represented by different worlds within one and the same $\mathcal{L}_{\mathsf{MDL}^+}$-model and these worlds are connected by R_A-arrows. In contrast, different $\mathcal{L}_{\mathsf{MDL}^+}$-models are used to represent situations differing from each other in deontic aspects, and only $R_{!_i\varphi}$-arrows connect those situations. It seems to me that this exemplifies the difference between usual acts and illocutionary acts. Illocutionary acts affects institutional facts while usual acts affect brute facts.

4 Towards the Logic of Eliminative Commands

The semantics defined in the previous section validates the following reduction axioms for eliminative commands:

(**RAt**)	$[!_i\varphi]p \leftrightarrow p \quad$ where $p \in Aprop$	(Atoms)
(**RVer**)	$[!_i\varphi]\top \leftrightarrow \top$	(Verum)
(**FUNC**)	$[!_i\varphi]\neg\psi \leftrightarrow \neg[!_i\varphi]\psi$	(Functionality)
(**RAnd**)	$[!_i\varphi](\psi \wedge \chi) \leftrightarrow ([!_i\varphi]\psi \wedge [!_i\varphi]\chi)$	($[!_i\varphi]$-Distribution)
(**RAleth**)	$[!_i\varphi]\Box\psi \leftrightarrow \Box[!_i\varphi]\psi$	(Alethic Modality)
(**RObl**)	$[!_i\varphi]O_i\psi \leftrightarrow O_i(\varphi \rightarrow [!_i\varphi]\psi)$	(Obligation)
(**RInd**)	$[!_i\varphi]O_j\psi \leftrightarrow O_j[!_i\varphi]\psi.$	(Independence)

Note that the form of **RObl** axiom is very closely similar to, though not identical with, that of the following axiom of the logic of public announcements:

$$[\varphi!]K_i\psi \leftrightarrow (\varphi \rightarrow K_i\,[\varphi!]\psi)\ .$$

This is interesting in view of the fact that, while public announcements affect informational states, commands affect obligations and permissions; they are radically different kinds of speech acts.

RAt and **RVer** enable us to eliminate any command operator prefixed to a propositional letter and $\top$ respectively, and other reduction axioms enable us to reduce the length of any sub-formula to which a command operator is prefixed step by step. Thus these axioms enables us to translate any sentence of $\mathcal{L}_{\mathsf{CL}}$ into a sentence of $\mathcal{L}_{\mathsf{MDL}^+}$ that is provably equivalent to it. This means that the logic of eliminative commands, ECL, is complete if the multi-agent monadic deontic logic with an alethic modal operator, MDL^+, is. We present the outline of the proof of the completeness for MDL^+ in section 5 and that for ECL in section 6.

5 Proof system for MDL^+

Now we define proof system for MDL^+.

Definition 7. *The proof system for* MDL^+ *contains the following axioms and rules:*

(Taut)	*all instantiations of propositional tautologies over the present language*	
($\Box$-Dist)	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$	*($\Box$-distribution)*
(O_i-Dist)	$O_i(\varphi \to \psi) \to (O_i\varphi \to O_i\psi)$ *for each* $i \in I$	*(O_i-distribution)*
(Mix)	$P_i\varphi \to \Diamond\varphi$ *for each* $i \in I$	*(Mix Axiom)*
(MP)	$\dfrac{\varphi \quad \varphi \to \psi}{\psi}$	*(Modus Ponens)*
($\Box$-Nec)	$\dfrac{\varphi}{\Box\varphi}$	*($\Box$-necessitation)*
(O_i-Nec)	$\dfrac{\varphi}{O_i\varphi}$ *for each* $i \in I$.	*(O_i-necessitation)*

An MDL^+*-proof of a formula* φ *is a finite sequence of* $\mathcal{L}_{\mathsf{MDL}^+}$*-formulas having* φ *as the last formula such that each formula is either an instance of an axiom, or it can be obtained from formulas that appear earlier in the sequence by applying a rule. If there is a proof of* φ*, we write* $\vdash_{\mathsf{MDL}^+} \varphi$*. If* $\Sigma \cup \{\varphi\}$ *is a set of* $\mathcal{L}_{\mathsf{MDL}^+}$*-formulas, we say that* ϕ *is deducible in* MDL^+ *from* Σ *and write* $\Sigma \vdash_{\mathsf{MDL}^+} \varphi$ *if* $\vdash_{\mathsf{MDL}^+} \varphi$ *or there are formulas* $\psi_1, \ldots, \psi_n \in \Sigma$ *such that* $\vdash_{\mathsf{MDL}^+} (\psi_1 \wedge \cdots \wedge \psi_n) \to \varphi$*.*

The above rules obviously preserves validity, and all the axioms are easily seen to be valid. Thus this proof system is sound.

To prove completeness, we take maximally consistent sets of formulas of $\mathcal{L}_{\mathsf{MDL}^+}$ and build a canonical model for MDL^+.

Definition 8 (The Canonical Model). *Given a set* I *of agents, the canonical model* $\mathfrak{M}^{\mathsf{MDL}^+}$ *is the quadruple* $\langle W^{\mathsf{MDL}^+}, R_A^{\mathsf{MDL}^+}, R_I^{\mathsf{MDL}^+}, V^{\mathsf{MDL}^+}\rangle$ *where:*

(i) W^{MDL^+} is the set of all maximally consistent sets of $\mathcal{L}_{\mathsf{MDL}^+}$-formulas,
(ii) $R_A^{\mathsf{MDL}^+} = \{\langle w, v\rangle \in W^{\mathsf{MDL}^+} \times W^{\mathsf{MDL}^+} \mid$ for all formulas ψ, $\Box\psi \in w$ implies $\psi \in v\}$,
(iii) $R_I^{\mathsf{MDL}^+}$ is the function that assigns to each $i \in I$ the set $R_i^{\mathsf{MDL}^+}$ where:
$R_i^{\mathsf{MDL}^+} = \{\langle w, v\rangle \in W^{\mathsf{MDL}^+} \times W^{\mathsf{MDL}^+} \mid$ for all formulas ψ, $O_i\psi \in w$ implies $\psi \in v\}$,
(iv) V^{MDL^+} is the valuation defined by $V^{\mathsf{MDL}^+}(p) = \{w \in W^{\mathsf{MDL}^+} \mid p \in w\}$.

Then the following two lemmas can be proved by familiar arguments:

Lemma 1. *The following two conditions hold for* $\mathfrak{M}^{\mathsf{MDL}^+}$*:*

(i) $\langle w, v\rangle \in R_A^{\mathsf{MDL}^+}$ iff for all formulas ψ, $\psi \in v$ implies $\Diamond\psi \in w$
(ii) $\langle w, v\rangle \in R_i^{\mathsf{MDL}^+}$ iff for all formulas ψ, $\psi \in v$ implies $P_i\psi \in w$, for all $i \in I$.

Lemma 2 (Existence Lemma). *The following two claim hold for* $\mathfrak{M}^{\mathsf{MDL}^+}$*:*

(i) For any state $w \in W^{\mathsf{MDL}^+}$, if $\Diamond\psi \in w$, then there is a state $v \in W^{\mathsf{MDL}^+}$ such that $\langle w, v\rangle \in R_A^{\mathsf{MDL}^+}$ and $\psi \in v$
(ii) For any state $w \in W^{\mathsf{MDL}^+}$, if $P_i\psi \in w$, then there is a state $v \in W^{\mathsf{MDL}^+}$ such that $\langle w, v\rangle \in R_i^{\mathsf{MDL}^+}$ and $\psi \in v$, for any $i \in I$.

Now, the following lemma can be proved by appealing to Mix Axiom Schema:

Lemma 3 (Verification Lemma). *$\mathfrak{M}^{\mathsf{MDL}^+}$ is an $\mathcal{L}_{\mathsf{MDL}^+}$-model.*

The following lemma can be proved by induction on the length of φ:

Lemma 4 (Truth Lemma). *For any formula $\varphi \in S_{\mathsf{MDL}^+}$, $\mathfrak{M}^{\mathsf{MDL}^+}, w \models_{\mathsf{MDL}^+} \varphi$ iff $\varphi \in w$.*

Then the completeness of MDL^+ can be proved by a familiar argument.

Theorem 1 (Completeness of MDL^+). *If $\Sigma \models_{\mathsf{MDL}^+} \varphi$ then $\Sigma \vdash_{\mathsf{MDL}^+} \varphi$.*

6 Proof system for ECL

Now we define proof system for ECL.

Definition 9. *The proof system for* ECL *contains all the axioms and all the rules of the proof system for* MDL^+ *and in addition the following reduction axioms and rules:*

(**RAt**)	$[!_i\varphi]p \leftrightarrow p$ where $p \in Aprop$	(Atoms)
(**RVer**)	$[!_i\varphi]\top \leftrightarrow \top$	(Verum)
(**FUNC**)	$[!_i\varphi]\neg\psi \leftrightarrow \neg[!_i\varphi]\psi$	(Functionality)
(**RAnd**)	$[!_i\varphi](\psi \wedge \chi) \leftrightarrow ([!_i\varphi]\psi \wedge [!_i\varphi]\chi)$	($[!_i\varphi]$-Distribution)
(**RAleth**)	$[!_i\varphi]\Box\psi \leftrightarrow \Box[!_i\varphi]\psi$	(Alethic Modality)
(**RObl**)	$[!_i\varphi]O_i\psi \leftrightarrow O_i(\varphi \rightarrow [!_i\varphi]\psi)$	(Obligation)
(**ROther**)	$[!_i\varphi]O_j\psi \leftrightarrow O_j[!_i\varphi]\psi$ where $i \neq j$	(Independence)
($[!_i\varphi]$-**Nec**)	$\dfrac{\psi}{[!_i\varphi]\psi}$ for each $i \in I$.	($[!_i\varphi]$-necessitation)

An ECL*-proof of a formula φ is a finite sequence of $\mathcal{L}_{\mathsf{ECL}}$-formulas having φ as the last formula such that each formula is either an instance of an axiom, or it can be obtained from formulas that appear earlier in the sequence by applying a rule. If there is a proof of φ, we write $\vdash_{\mathsf{ECL}} \varphi$. If $\Sigma \cup \{\varphi\}$ is a set of $\mathcal{L}_{\mathsf{ECL}}$-formulas, we say that ϕ is deducible in* ECL *from Σ and write $\Sigma \vdash_{\mathsf{ECL}} \varphi$ if $\vdash_{\mathsf{ECL}} \varphi$ or there are formulas $\psi_1, \ldots, \psi_n \in \Sigma$ such that $\vdash_{\mathsf{ECL}} (\psi_1 \wedge \cdots \wedge \psi_n) \rightarrow \varphi$.*

It is easy to verify that all these axioms are valid and the rules preserve validity on the semantics defined earlier in this article, and so the proof system for ECL is sound.

Obviously the following condition holds:

Corollary 3. *For any formula $\varphi \in S_{\mathsf{MDL}^+}$, if $\Sigma \vdash_{\mathsf{MDL}^+} \varphi$, then $\Sigma \vdash_{\mathsf{ECL}} \varphi$.*

Now, we define translation from $\mathcal{L}_{\mathsf{CL}}$ to $\mathcal{L}_{\mathsf{MDL}^+}$.

Definition 10 (Translation). *The translation function t that takes a formula from $\mathcal{L}_{\mathsf{CL}}$ and yields a formula in $\mathcal{L}_{\mathsf{MDL}^+}$ is defined as follows:*

$$
\begin{array}{llll}
t(p) & = p & t([!_i\varphi]p) & = p \\
t(\top) & = \top & t([!_i\varphi]\top) & = \top \\
t(\neg\varphi) & = \neg t(\varphi) & t([!_i\varphi]\neg\psi) & = \neg t([!_i\varphi]\psi) \\
t(\varphi \wedge \psi) & = t(\varphi) \wedge t(\psi) & t([!_i\varphi](\psi \wedge \chi)) & = t([!_i\varphi]\psi) \wedge t([!_i\varphi]\chi) \\
t(\Box\varphi) & = \Box t(\varphi) & t([!_i\varphi]\Box\psi) & = \Box t([!_i\varphi]\psi) \\
t(O_i\varphi) & = O_i t(\varphi) & t([!_i\varphi]O_i\psi) & = O_i(t(\varphi) \to t([!_i\varphi]\psi)) \\
 & & t([!_i\varphi]O_j\psi) & = O_j t([!_i\varphi]\psi) \textit{ where } i \neq j \\
 & & t([!_i\varphi][!_j\psi]\chi) & = t([!_i\varphi]t([!_j\psi]\chi)) \textit{ for any } j \in I.
\end{array}
$$

The following corollary can be proved by induction on the length of η:

Corollary 4 (Translation Effectiveness). *For every formula $\eta \in S_{\mathsf{CL}}$, $t(\eta) \in S_{\mathsf{MDL}^+}$.*

With the help of corollary 4 and reduction axioms, the following lemma is proved by induction of the length of η:

Lemma 5 (Translation Correctness). *Let M be an $\mathcal{L}_{\mathsf{MDL}^+}$-model, and w a point of M. Then for any formula η of $\mathcal{L}_{\mathsf{CL}}$, $M, w \models_{\mathsf{ECL}} \eta$ iff $M, w \models_{\mathsf{ECL}} t(\eta)$.*

Obviously the following condition holds:

Corollary 5. *Let M be an $\mathcal{L}_{\mathsf{MDL}^+}$-model, and w a point of M. Then for any formula η of $\mathcal{L}_{\mathsf{CL}}$, $M, w \models_{\mathsf{ECL}} \eta$ iff $M, w \models_{\mathsf{MDL}^+} t(\eta)$.*

Reduction axioms and Corollary 4 enable us to prove the following lemma by induction on the length of η:

Lemma 6. *For any formula $\eta \in S_{\mathsf{ECL}}$, $\vdash_{\mathsf{ECL}} \eta \leftrightarrow t(\eta)$.*

Then the completeness of ECL can be proved with the help of Corollary 3, Corollary 5 and Lemma 6.

Theorem 2 (Completeness of ECL). *For any set $\Sigma \cup \{\varphi\}$ of formulas of $\mathcal{L}_{\mathsf{CL}}$, if $\Sigma \models_{\mathsf{ECL}} \varphi$, then $\Sigma \vdash_{\mathsf{ECL}} \varphi$.*

7 Three Built-In Assumptions

The semantics of $\mathcal{L}_{\mathsf{CL}}$ defined in this paper incorporates a few assumptions. Firstly, as is mentioned earlier, the semantics incorporates the conception of an eliminative command. Thus commands are assumed to be always eliminative; we have $R_i^M \upharpoonright \varphi^{\downarrow} \subseteq R_i^M$. This might be said to be a simplification on the ground that some acts of commanding seem to add arrows. For example, suppose you are in a combat troop and now waiting for your captain's command to fire. Then you hear the command, and it has become obligatory upon you to fire. But before that, you were not permitted to fire. This forbiddance is now no longer in force. Thus it seems that after his command, you are permitted to fire at least in the sense of lack of forbiddance.

But the forbiddance in force before the issuance of your captain's command is not an absolute one; although you were forbidden to fire without his command, it was not forbidden that you should fire at his command. Unfortunately, we have no systematic way of expressing these facts in $\mathcal{L}_{\mathsf{CL}}$. Since a command type term of the form $!_i\varphi$ is not a sentence, it cannot be used to state the fact that you are commanded to see to it that φ, and no sentence we can build with $!_i\varphi$ and other expressions can be used to do so, either. Furthermore, a world in which you fire at his command is not simply a world in which he has commanded you to fire and you fire, but a world in which you fire because he has commanded you to do so. Thus even if we postulate that p and q express the proposition that your captain has commanded you to fire and the proposition that you fire, respectively, $p \wedge q$ doesn't fully characterize a world to be a world in which you fire at his command. But at least we can say that a world in which you fire at his command is also a world in which you fire. Thus at least one world in which you fire is among permissible possible worlds with respect to you in the current situation. This fact can be expressed in $\mathcal{L}_{\mathsf{CL}}$. Let M be your current situation, and t the current world. Then we have $M, t \models_{\mathsf{ECL}} P_i q$. Note that we also have $M, t \models_{\mathsf{ECL}} F_i(\neg p \wedge q) \wedge [!_i q]P_i q$. Moreover, we have:

Proposition 2. *For any* $\varphi \in S_{\mathsf{CL}}$, $\models_{\mathsf{ECL}} [!_i\varphi]P_i\varphi \rightarrow P_i\varphi$.

This principle closely parallels the above reasoning. This consideration suggests that whether an arrow-adding operation is necessary or not is not so clear as it may seem.

Secondly, as is noted in Corollary 2, commands of the form $!_i\varphi$ are assumed to have no effect on the deontic accessibility relations for any agents other than i. This might also be said to be a simplification. For example, suppose one of your colleague b was in your office in our first example. Let p and q be understood as in earlier discussions of this example. We have $M, s \models_{\mathsf{ECL}} P_b\, p \wedge P_b\, q \wedge P_b\,(\neg p \wedge \neg q)$. Then by our semantics, we have $[R_a^M / R_a^M \restriction p^{\downarrow}]M, s \models_{\mathsf{ECL}} P_b\, p \wedge P_b\, q \wedge P_b\,(\neg p \wedge \neg q)$. But if b turns on the air conditioner just after your boss's command, he would go against your boss's intention in a sense; his doing so will undermine the condition on which your opening the window would contribute to your boss's plan, and thereby prevent your boss's goal from being achieved as intended. Moreover, even b's opening the window could possibly be problematic in that it will preclude the possibility of your opening it.

One possible way of dealing with phenomena of this kind is to interpret your boss's command as meant to be heard by all the people in the office, and to obligate them to see to it that you see to it that the window is open. But again, we have no systematic way of expressing this. In order to represent commands of this kind, we need to extend our language by allowing deontic operators and actions terms to be indexed by groups of agents, and by introducing construction that enables us to have a formula which can expresses that you see to it that p. Although such an extension will be of much interest, it is beyond the scope of the present paper.

Thirdly, commands are assumed to have no preconditions. Although this assumption may be said to be unrealistic, it is not harmful. One natural candidate for the precondition for the act of commanding i to do A is the condition that the commanding agent has authority to do that. Such conditions will be of central importance, for example, when we try to decide, given an particular utterance of an imperative sentence by an agent in a particular context, whether a command is successfully issued in that utterance or not. But it is important to notice that there is another more fundamental question to ask, namely that of what a successfully issued command accomplishes. This question requires us to say what an act of commanding is. It is this question that $\mathcal{L}_{\mathsf{CL}}$ is developed to address, and when we use $\mathcal{L}_{\mathsf{CL}}$ to answer it, we can safely assume that the commands we are talking about are issued by suitable authorities.

Another natural candidate for the precondition for the act of commanding i to do A is the requirement that it should be possible for i to do A. But we have no direct way of requiring this, since we have no way of talking about actions other than commanding. Thus, the best we could do might be to require that $\Diamond\varphi$ holds, for example, as the precondition for the successful issuance of a command of the form $!_i\varphi$. In order to incorporate this condition, however, we have to modify the truth definition and the proof system, and even if we do so, there remains a real possibility of conflicting commands coming from different authorities. I will return to this point in the next section.

8 Some Interesting Validities and Non-validities

In addition to CUGO principle, our semantics validates the following principles:

(**DE**)	$[!_i(\varphi \land \neg\varphi)]O_i\psi$	(Dead End)
(**SC**)	$[!_i\varphi][!_i\psi]\chi \leftrightarrow [!_i(\varphi \land \psi)]\chi$ where $\psi \in S_{i\text{-free}}$	(Sequential Conjunction)
(**OI**)	$[!_i\varphi][!_i\psi]\chi \leftrightarrow [!_i\psi][!_i\varphi]\chi$ where $\varphi, \psi \in S_{i\text{-free}}$.	(Order Invariance)

Dead End principle states that contradictory commands lead to an obligational dead end. Since $\varphi \land \neg\varphi$ is not true at any world in any model, $R_i^M \restriction (\varphi \land \neg\varphi)^{\downarrow}$ is empty, and so for any model M, no possible world is a permissible possible world for the agent i in $[R_i^M / R_i^M \restriction (\varphi \land \neg\varphi)^{\downarrow}]M$. Sequential Conjunction principle states that commands given in a sequence usually, though not always, add up to a command with a conjunctive content. Unrestricted form of sequential conjunction principle is not valid because $(R_i^M \restriction \varphi^{\downarrow}) \restriction \psi^{\downarrow}$ can be distinct from $R_i^M \restriction (\varphi \land \psi)^{\downarrow}$. Similarly, unrestricted form of order invariance principle is not valid because $(R_i^M \restriction \varphi^{\downarrow}) \restriction \psi^{\downarrow}$ can be distinct from $(R_i^M \restriction \psi^{\downarrow}) \restriction \varphi^{\downarrow}$.

As a consequence of dead end principle, it is not possible for us to add the so-called D axiom, i.e. $O_i\varphi \to P_i\varphi$, to our proof system. For example, for any M and w, we have:

$$[R_i^M / R_i^M \restriction (p \land \neg p)^{\downarrow}]M, w \models_{\mathsf{ECL}} O_i(p \land \neg p) \land \neg P_i(p \land \neg p) \ . \tag{7}$$

Moreover, as an instance of sequential conjunction principle, we have

$$[!_i p][!_i \neg p]O_i(p \land \neg p) \leftrightarrow [!_i(p \land \neg p)]O_i(p \land \neg p) \ . \tag{8}$$

Although no boss might be silly enough to give you a command to see to it that $p \land \neg p$, you might have two bosses and after one of them give you a command to see to it that p, the other one might give you a command to see to it that $\neg p$. Unless both of them belong to the same hierarchy, neither command might be overridden by the other. Whichever command you may choose to obey, you will have to disobey the other.

If we require $\Diamond\varphi$ to hold as the precondition for successful issuance of a command of the form $!_i\varphi$, every command of the form $!_i(\psi \wedge \neg\psi)$ would be precluded. But even if we do this, there may be a situation M such that $\Diamond p \wedge \Diamond\neg p$ holds at w in M. In such a situation, a command of the form $!_i p$ can be issued. In the resulting situation, $\Diamond p \wedge \Diamond\neg p$ still holds at w, and hence it remains possible to issue a command of the form $!_i\neg p$.

One way of avoiding obligational dead end of this kind could be to require φ in $!_i\varphi$ to be in **Aprop**. But it would not be a real solution, and even if we did this, you might still find yourself in an obligational dead end. For example, after the boss of your department commanded you to attend an international one-day conference on logic to be held in São Paulo next month, your political guru might command you to join an important political demonstration to be held on the very same day in Tokyo. It is possible for you to obey either command, but it is transportationally impossible for you to obey both. Even after you decide which command to obey, you might still regret not being able to obey the other command. As we have not introduced the distinction of command issuing authorities into $\mathcal{L}_{\mathsf{CL}}$, your situation will be represented as an obligational dead end. Although no logical inconsistency is involved in the combination of obligations generated by these commands, no world is deontically accessible for you. But if we allow deontic accessibility relations to be indexed by the Cartesian product of the set of command issuing authorities and the set of addressees, then your situation can be represented not as an obligational dead end but as a situation which may be suitably called an obligational dilemma. Let a, b and c represent you, your boss and your guru, respectively. Then you are in a situation where there are $R_{\langle b,a\rangle}$-accessible worlds and $R_{\langle c,a\rangle}$-accessible worlds, but no world is both $R_{\langle b,a\rangle}$-accessible and $R_{\langle c,a\rangle}$-accessible. This modification also enables us to represent the situation you will be in if a command of the form $!_{\langle b,a\rangle}\neg p$ is issued after a command of the form $!_{\langle c,a\rangle}p$ is issued as an obligational dilemma. It will not be difficult to incorporate this modification into $\mathcal{L}_{\mathsf{CL}}$.

9 Related Works and Further Directions

As is noted in the introduction, the idea of ECL is inspired by the logics of epistemic actions developed in Plaza (1989), Groeneveld (1995), Gerbrandy and Groeneveld (1997), Gerbrandy (1999), Baltag, Moss, & Solecki (1999), and Kooi & van Benthem (2004). In the field of deontic reasoning, van der Torre & Tan (1999) and Žarnić (2003) extended the update semantics of Veltman (1996) so as to cover normative sentences and natural language imperatives, respectively. The main difference between their systems and ECL consists in that the former deal with the interpretation of sentences while the latter deals with the dynamics of acts of commanding. Broadly speaking, the relation between their systems and ECL is similar to that between Veltman's update semantics and the logics of epistemic actions.

In this respect, PDL based systems of Pucella & Weissman (2004), and Demri (2004) are closer to the present work in spirit. Their systems dynamified DLP of van der Meyden (1996). DLP is obtained from test-free PDL by introducing operators which have semantics that distinguish permitted (green) transitions from forbidden (red) ones.

The set of green transitions of each model is the so-called policy set. In Pucella & Weissman (2004), DLP is dynamified so that in the resulting system DLP_{dyn} the policy set can be updated by adding or deleting transitions, and in Demri (2004), DLP_{dyn} is extended to DLP^{+}_{dyn} by adding test operator "?" and allowing the operators for updating policy sets to be parameterized by the current policy set. One important difference between these PDL-based systems and ECL lies in the fact that in these PDL-based systems, we can talk about permitted or forbidden actions as well as obligatory state of affairs whereas we can only talk about permitted, forbidden or obligatory state of affairs in ECL.

Another interesting related work is stit theories of Belnap, Perloff, & Xu (2001) and Horty (2001). As the wording in this paper might have suggested, I believe that the contents of commands should be captured by agentives. But the language of monadic deontic logic lacks the resource for distinguishing agentives from non-agentives. This defect can be removed by using a language of stit theory. In order to do so, however, we have to rethink our update operation. In stit theory, we talk about "moments" in stead of possible worlds. Since moments are partially ordered in a tree like branching temporal structure, we have to take their temporal order into account. But the update operation of this paper is not sensitive to temporal order. Thus when we think of the points in our model as stages of some language game, for example, it might look problematic, since it can eliminate deontic arrows that connect stages earlier than the stage at which the command is issued. As it is possible to define different update operations even with respect to $\mathcal{L}_{\mathsf{MDL}^+}$-models, one immediate task for us is to examine the logics obtained by replacing the update operation in $\mathcal{L}_{\mathsf{CL}}$.

Finally, the most closely related work in this field is that of van Benthem & Liu (2006). They proposed "preference upgrade" as a counter part to information update (p.3). According to them, my "command operator for propositions *A* can be modeled exactly as an upgrade sending *R* to *R*;?*A* " in their system, and their paper "provides a much more general treatment of possible upgrade instructions"(p.20). Although their preference upgrade clearly has much wider application than the deontic update of this paper, the notion of preference upgrade seems to be connected with perlocutionary consequences, while the notion of deontic update is meant to be used to capture a differential feature of an act of commanding as a specific kind of illocutionary acts.

10 Conclusion

We have shown that commands can be considered as deontic updaters. Since the base language $\mathcal{L}_{\mathsf{MDL}^+}$ we dynamified is a variant of monadic deontic logic, our extended language $\mathcal{L}_{\mathsf{CL}}$ inherits various inadequacy of the language of monadic deontic logic. But the fact that even such a simple language can be used to capture some interesting principles may be said to suggest the possibility of further research, including dynamifying stronger deontic languages. Moreover, the possibilities of update logics of various other kinds of speech acts suggest themselves. For example, an act of promising can be considered as another updater of obligations, and an act of asserting as an updater of propositional commitments. We seem to be witnessing the beginning of the exploration into the vast area of the logical dynamics of communicative actions.

Acknowledgments

I thank Johan van Benthem, Fenrong Liu, John Perry and Keith Devlin for their comments on earlier versions of this paper. Special thanks go to Johan van Benthem for his constant encouragements.

References

Åqvist, L: "Deontic Logic", in Gabbay & Guentner (2002), 147–264

Baltag, A., Moss, L. S., Solecki, S.: "The Logic of Public Announcements, Common Knowledge, and Private Suspicions," Technical Report TR534, (Department of Computer Science (CSCI), Indiana University, 1999)

Belnap, N., Perloff, M., Xu, M.: *Facing the Future: Agents and Choices in Our Indeterminist World*, (Oxford, Oxford University Press, 2001)

van Benthem, J.: " Update Delights," (Amsterdam & Stanford, http://turing.wins.uva.nl/˜ johan/Update.Delights.ps, 2000)

van Benthem, J., Liu, F.: "Dynamic Logic of Preference Upgrade," to appear in *Journal of Applied Non-Classical Logics* (2006)

Demri, S.: "A Reduction from DLP to PDL," *Journal of Logic and Computation*, 15-5 (2005), 767–785

Gabbay, D. M., Guenthner, F.(eds.): *Handbook of Philosophical Logic*, 2nd Edition, Volume 8, (Dordrecht / Boston / London, Kluwer Academic Publishers, 2002).

Gerbrandy, J.: "Bisimulations on Planet Kripke," Ph. D. Thesis, ILLC Dissertation Series DS-1999-01, (University of Amsterdam, 1999)

Gerbrandy, J., Groeneveld, W.: "Reasoning about Information Change," Journal of Logic, Language, and Information, Vol.6 (1997), 147–169

Grice, H. P.: "Meaning," in Grice, H. P., *Studies in the Way of Words*, Cambridge, Massachusetts / London, England, Harvard University Press, 1989), 213–223

Groeneveld, W.: "Logical Investigations into Dynamic Semantics," Ph. D. Thesis, ILLC Dissertation Series DS-1995-18, (University of Amsterdam, 1995)

Horty, J. F.: *Agency and Deontic Logic*, (Oxford, Oxford University Press, 2001)

Kooi, B., van Benthem, J.: "Reduction Axioms for Epistemic Actions," in Schmidt, R., Pratt-Hartmann, I., Reynolds, M. and Wansing, H. (eds): *Preliminary Proceedings of AiML-2004*, 197–211, (Department of Computer Science, University of Manchester, 2004)

van der Meyden, R.: "The Dynamic Logic of Permission," *Journal of Logic and Computation*, Vol. 6, No. 3 (1996), 465–479

Plaza, J. A.: "Logics of public communications", in Emrich, M. L., Pfeifer, M. S., Hadzikadic, M., and Ras, Z. W. (eds.): *Proceedings of the 4th International Symposium on Methodologies for Intelligent Systems* (1989), 201–216

Pucella, R., Weissman, V.: "Reasoning about Dynamic Policies," in Walukiewicz (ed.), *FOSSACS* 2004, LNCS 2987 (2004), 453-467

van der Torre, L. W. N., Tan, Yao-Hua: "An Update Semantics for Deontic Reasoning," in McNamara, P. and Prakken, H. (eds.), *Norms, Logics and Information Systems*, New Studies on Deontic Logic and Computer Science, Volume 49 (1999), Frontiers in Artificial Intelligence and Applications, (IOS Press), 73–90

Veltman, F.: "Defaults in Update Semantics," *Journal of Philosophical Logic*, 25 (1996), 221–261

Žarnić, B.: "Imperative Change and Obligation to Do," in Segerberg, K. and Sliwinski (eds.), *Logic, Law, Morality: Thirteen Essays in Practical Philosophy in Honour of Åqvist*, Uppsala Philosophical Studies 51, (Uppsala, Department of Philosophy, Uppsala University, 2003), 79–95