Semantic bounds for everyday language

MARCIN MOSTOWSKI and JAKUB SZYMANIK

Abstract

We consider the notion of everyday language. We claim that everyday language is semantically bounded by the properties expressible in the existential fragment of second–order logic. Two arguments for this thesis are formulated. Firstly, we show that so–called Barwise's test of negation normality works properly only when assuming our main thesis. Secondly, we discuss the argument from practical computability for finite universes. Everyday language sentences are directly or indirectly verifiable. We show that in both cases they are bounded by second–order existential properties. Moreover, there are known examples of everyday language sentences which are the most difficult in this class (NPTIME–complete).

Keywords: everyday language; natural language; semantics; second–order logic; finite models; computational complexity.

1. Introduction

There is a common and – from our point of view – controversial use of the term *natural language* as opposed not only to artificial languages but also to scientific language or technical jargons. A good example of such a use is the term *natural language quantifier*[1] as opposed to *logical quantifiers*, see e.g. (Keenan 2002). Obviously, *infinity* and *there are infinitely many* are natural language expressions just as *majority* or *many*. Nevertheless, we can see a natural intuition supporting the narrow use of the term *natural language*. However, in this narrow sense we prefer to use the term *everyday language*, instead. This is a fragment of natural language in which logicians communicate with bakers, students with postmen, quantum physicists with philologist, and so on. Everyday language is a pre–theoretical part of natural language, creating its basic and most common core[2].

We are looking for semantic bounds of everyday language. Firstly, we ask about the number of elements creating our universe of discourse. This is important because the possible estimations of the semantic strength of everyday language heavily depend on the expressive power of its quantifier constructions. Most authors considering semantics of natural language are interested only in finite universes; let us quote Dag Westerståhl:

'In general these cardinals can be infinite. However, we now lay down the following constraint:

(FIN) Only finite universes are considered.

This is a drastic restriction, no doubt. It is partly motivated by the fact that a great deal of the interest of the present theory of determiners comes from applications to natural language, where this restriction is reasonable' (Westerståhl 1984).

This restriction seems reasonable because in typical communication situations we refer to relatively small finite sets of objects. For example, in the interpretations of the following sentences relatively small sets are involved.

(1) Exactly five of my children went to the cinema.

(2) Everyone from my family has read ‘Alice's Adventures in Wonderland’.

Considering cardinalities of the universe of discourse we have three main possibilities:

- small finite universes;
- large finite universes;
- infinite universes.

In many cases the restriction to finite interpretations essentially simplifies our theoretical considerations. Some ideas can easily be formulated when working only with finite universes and their generalization for arbitrary models would require subtle and technically difficult analysis[3]. Moreover, this restriction is adequate for analyses of many communication situations. Nevertheless, with the restriction to finite universes we omit many important cases. Therefore, in this work we consider arguments taking into account small finite universes as well as the general case covering all the mentioned cases.

2. A few examples

In this section we give a few examples of natural language sentences together with their semantic interpretations. We consider examples of sentences interpreted in the model[4] $\mathbf{M}=(U, V^M, T^M, H^M)$, where the universe U of **M** is the set of all human beings, V^M is the set of all villagers, T^M is the set of all townsmen, and H^M is the relation of hating each other. The corresponding predicates V, T, H are interpreted in **M** as: V^M, T^M, H^M, respectively.

We start with an easy sentence and its logical form:

(3) There are exactly two villagers.

(4) $\exists x \exists y\, [V(x) \wedge V(y) \wedge x \neq y \wedge \forall z\, (V(z) \Rightarrow (z=x \vee z=y))]$

Therefore, the logical form of sentence (3) can be given in terms of elementary logic[5] by formula (4).

The next sentence we are interested in is a bit more difficult. Consider the following pair consisting of sentence and its meaning representation.

(5) Every other person is a townsman.

(6) $\exists P[\forall x \forall y\, (P(x, y) \Rightarrow (T(x) \wedge \neg T(y))) \wedge \forall x\, (T(x) \Rightarrow \exists y\, P(x, y)) \wedge \forall y\, (\neg T(y) \Rightarrow \exists x\, P(x, y)) \wedge \forall x \forall y \forall y'\, ((P(x, y) \wedge P(x, y')) \Rightarrow y=y' \wedge \forall x \forall x' \forall y\, ((P(x, y) \wedge P(x', y)) \Rightarrow x = x')]$

Formula (6) is not elementary because it starts with the second–order quantifier $\exists P$. The variable P runs through binary relations over the universe, in our case subsets of U^2. It is not equivalent to any elementary formula. It states that the set of townsmen and not–townsmen have the same cardinality because there is a one–to–one mapping P from one of these sets to another. In other words, every other element from U belongs to T^M. Therefore, formula (6) has the same truth–conditions as sentence (5). This is why formula (6) is a correct logical form for sentence (5).

Formula (6) has the form $\exists P\varphi(P)$, where P is a second–order variable and φ is a first order–formula with P as an additional binary predicate. The class of such existential second–order formulae is denoted by Σ^1_1. Formulae equivalent to Σ^1_1–formulae will also be called Σ^1_1–formulae.

Now, let us consider more complicated example:

(7) Most people live in a village.

(8) $\exists R\, [\forall x\, (V(x) \Rightarrow \exists y\, (\neg V(y) \wedge R(x, y))) \wedge$
$\forall x \forall y \forall y'\, ((V(x) \wedge \neg V(y) \wedge \neg V(y') \wedge R(x, y) \wedge R(x, y')) \Rightarrow y=y')$
$\wedge \forall y\, (\neg V(y) \Rightarrow \exists x\, (V(x) \wedge R(x, y))) \wedge$
$\exists x \exists x' \exists y\, (V(x) \wedge V(x') \wedge x \neq x' \wedge \neg V(y) \wedge R(x, y) \wedge R(x', y))]$

Formula (8) is Σ^1_1. It says that there is a function from *V* into $U - V^M$ which is surjective but not injective. Therefore, it says that most *x* from *U* belong to V^M. Then formula (8) is a proper logical representation of sentence (7).

Essentially, formula (8) defines the quantifier 'Most' of type (1). In what follows we need the quantifier MOST of type (1, 1)[6].

MOST x ($\varphi(x)$, $\psi(x)$) is defined by the following second–order formula:

(9) $\exists R[\forall x \exists y\, (\varphi(x) \wedge \psi(x) \wedge \varphi(y) \wedge \neg\psi(y) \wedge R(x, y)) \wedge$
$\forall x \forall y \forall y'\, (\varphi(x) \wedge \psi(x) \wedge \varphi(y) \wedge \neg\psi(y) \wedge \varphi(y') \wedge \neg\psi(y') \wedge$

$R(x, y) \wedge R(x, y') \Rightarrow y=y') \wedge \forall y\, (\varphi(y) \wedge \neg\psi(y) \Rightarrow \exists x\, (\varphi(x) \wedge \psi(x) \wedge R(x, y))) \wedge \exists x \exists x' \exists y\, (\varphi(x) \wedge \psi(x) \wedge \varphi(x') \wedge \psi(x') \wedge x \neq x' \wedge \varphi(y) \wedge \neg\psi(y) \wedge R(x, y) \wedge R(x', y))]$

Now, let us consider an example of a really hard sentence:

(10) Most villagers and most townsmen hate each other.

(11) $\exists A \exists B\, [\mathsf{MOST}\ x\, (V(x), A(x)) \wedge \mathsf{MOST}\ y\, (T(y), B(y)) \wedge \forall x \forall y\, (A(x) \wedge B(y) \Rightarrow H(x, y))]$

Formula (11) is equivalent to a Σ^1_1–sentence. It says that there are sets A and B containing, respectively, most villagers and most townsmen such that every villager from A and every townsman from B hate each other. Formula (11) has the same truth–conditions as statement (10), thus it is the intended interpretation of sentence (10) in our model **M**.

Finally, we consider a sentence which is not expressible in the existential fragment of second–order logic.

(12) There are at most countably many entities.

(13) $\exists R\, [\forall x\, \neg R(x, x) \wedge \forall x \forall y\, (R(x, y) \vee R(y, x) \vee x=y) \wedge$
$\forall x \forall y \forall z\, (R(x, y) \wedge R(y, z) \Rightarrow R(x, z)) \wedge$
$\forall A\, (\exists x\, A(x) \Rightarrow \exists x\, (A(x) \wedge (\forall y\, R(y, x) \Rightarrow \neg A(y)))) \wedge$
$\forall x\, (\exists y\, R(y, x) \Rightarrow \exists z\, (R(z, x) \wedge \forall w\, (w \neq z \wedge R(w, x) \Rightarrow R(w, z))))]$

This sentence says that there exists a well–ordering such that each element in this ordering has a predecessor except for the least element. This is possible only in the case when the cardinality of the set is countable or finite.

Let us note that all the previously mentioned quantifiers can be expressed in the existential fragment of second–order logic. In the case of (13) it is impossible because for existential fragment of second–order logic the Upward Skolem–Löwenheim Theorem holds.

3. The main thesis

What follows is the main claim of the paper.

Main Thesis *Everyday language is semantically bounded by the Σ^1_1–properties.*

In other words, we claim that everyday language contains only notions which can be defined in the existential fragment of second–order logic. If some property is not definable by any Σ^1_1–formula, then it falls outside the scope of everyday language. For example, quantifiers 'there exists', 'all', 'exactly two', 'at least four', 'every other' and 'most' belong to everyday language. A counterexample is the notion 'there exists at most countably many' which is not definable by any Σ^1_1–formula. In the next two sections we give arguments for such upper bound for everyday language[7].

Before discussing the arguments, we present one of the consequences of the main thesis. First order–logic is closed on Boolean operations[8] as opposed to Σ^1_1 fragment of second–order logic. Particularly, it is not closed on negation. However, this problem is open when we restrict interpretations to finite models. In this case Σ^1_1–notions are closed on Boolean operations if and only if NP = co–NP[9] which is one of the most difficult problems of computational complexity theory.
Thus, it is reasonable to assume that everyday language, i.e., the fragment of natural language semantically bounded by Σ^1_1–properties, is not closed on Boolean operations even on finite universes. Therefore, it may be the case that a sentence belongs to everyday language but its negation does not.

4. Argument from negation normality

It was observed by Jon Barwise (1979) that negations of some simple quantifier sentences, i.e., sentences without propositional connectives different than 'not' before a verb, can easily be formulated as simple quantifier sentences. For some sentences it is impossible. Namely, the only way to negate them is by adding the prefix 'it is not the case that' or an equivalent expression of a theoretical character.

The sentences of the first kind are called negation normal. For example, consider the following sentence:

(14) Everyone owns a car.

It can be negated as follows:

(15) Someone doesn't own a car.

The sentences of the second kind are not negation normal. For instance, consider the following proposition:

(16) Most relatives of each villager and most relatives of each townsman hate each other.

It can only be negated in the following way:

(17) It is not the case that most relatives of each villager and most relatives of each townsman hate each other.

Barwise has proposed the test of negation normality as a reasonable criterion for first–order definability. The results of the negation normality test agree with our experience (see (Barwise 1979) and (Mostowski 1994)). The test is based on the following theorem which is a corollary from Craig's Interpolation Lemma (see e.g. Ebbinghaus et al. 1996):

Theorem 1 *If φ is a sentence definable in the existential fragment of second–order logic, and its negation is logically equivalent to a Σ^1_1–sentence, then φ is logically equivalent to some first–order sentence.*

In other words, the test works only with the assumption that simple everyday sentences are semantically bounded by Σ^1_1–properties. This gives an argument in favor of our main thesis in arbitrary universes.

5. Argument from practical computability

The core sentences of everyday language are sentences which can be effectively verifiable. In the case of small finite interpretations it means that their logical–value can be practically computed (directly or indirectly).

Direct practical computability means that there is an algorithm which for a given finite interpretation computes the logical–value in a reasonable time. Our computational experience justifies the claim formulated by Jack Edmonds (1965).

Edmonds' Thesis *The class of practically computable problems is identical with PTIME class that is the class of problems which can be computed by a deterministic Turing machine in a number of steps bounded by a polynomial function of the length of a query.*

We take here Edmonds' thesis for granted. It follows that a sentence's logical–value is directly practically computable in small finite interpretations, whenever the problem of logical–value of this sentence in finite interpretations is in PTIME[10].

In understanding everyday language sentences we not only use their referential but also inferential meaning. The latter is determined indirectly, by inferential relations with other sentences having well–defined referential meanings. To see this, let us consider the following three sentences:

(18) There were more boys than girls at the party.

(19) At the party every girl was paired with a boy.

(20) Peter came alone to the party.

We know that sentence (18) can be inferred from sentences (19) and (20). Hence, we can establish the logical–value of sentence (18) indirectly knowing that sentences (19) and (20) are true.

Sentence (18) is easy in the sense that its logical–value is PTIME computable. However, for some sentences the problem whether their logical–values are PTIME computable is open[11]. Let us consider the following examples.

(21) Most villagers and most townsmen hate each other.

(22) At least one third of villagers and at least half of all townsmen hate each other.

(23) Most of the parliament members hate each other.

(24) Some relative of each villager and some relative of each townsman hate each other.

It is known that the problem of checking the logical–value for each of these sentences is NPTIME–complete, see (Sevenster 2006), (Mostowski and Wojtyniak 2004), and (Szymanik 2007)[12].

NPTIME (for short NP) is the class of problems which can be solved by a nondeterministic Turing machine in a number of steps bounded by a polynomial function of the length of a query. Nondeterministic algorithms were defined for the first time by Alan Turing (1936). The term *nondeterministic* is misleading. Originally, Turing used the term *with choice*. In the case of NPTIME the nondeterministic behavior can be described as follows:

'Firstly, choose a certificate of a size polynomially depending on the size of input. Then apply a PTIME algorithm for finding the answer. The nondeterministic algorithm answers YES exactly when there is a certificate for which we get a positive answer.' (Garey and Johnson 1979).

Let us observe that such certificates are a kind of proofs. When we have a proof of a statement, then we can easily check whether the sentence is true.

The logical relevance of the class NPTIME follows from the Fagin's Theorem (see Fagin 1974):

Theorem 2 *A class of finite models is NPTIME computable if and only if it is definable by a* Σ^1_1*–sentence.*

Let us notice that all examples of natural language sentences considered in this paper – which undoubtedly belong to everyday language – have Σ_1^1 expressible meanings.

NPTIME–complete problems are computationally the most difficult problems in the NPTIME class. Particularly, it is known that P=NP if any NPTIME–complete problem is PTIME computable. Therefore, on the ground of our current knowledge we can expect that NPTIME–complete problems are not practically computable. Nevertheless, similarly as all NPTIME problems they can be practically justifiable. Let us consider an example.

Suppose that we have two predicate expressions A, B and the following true statements:

(25) Most villagers are A.

(26) Most townsmen are B.

(27) All A and all B hate each other.

From these sentences we can infer the following statement:

(28) Most villagers and most townsmen hate each other.

The predicate expressions A and B should be guessed. They are in a sense certificates (proofs) for truth of sentence (28).

In this sense sentences with NPTIME logical–value checking problem – or by Fagin's theorem, Σ_1^1–expressible sentences – are indirectly verifiable. Moreover, NPTIME seems to capture exactly indirect verifiability.

This concludes second argument for our main thesis.

Notes

[1] Let us observe that this phrase has essentially different presuppositions than the phrase *quantifiers in natural language*.

[2] We would even claim that it is the most biologically grounded part of natural language. However, it raises so many questions falling beyond the scope of this paper that we prefer not to go in this direction.

[3] E.g., see discussion of so-called measure quantifiers in (Krynicki and Mostowski 1999).

[4] Models are precise mathematical notions explicating possible worlds or possible interpretations of our language.

[5] Elementary logic — called also first–order logic — allows only quantifiers $\exists$ and $\forall$ binding individual variables.

[6] For definition of generalized quantifiers and their types see (Lindström 1966). For a recent monograph on generalized quantifiers consult (Peters and Westerståhl 2006).

[7] Notice that our claim is similar to the methodological NP–completeness thesis formulated by Ristad (1993). He claims that the complexity of natural language semantics is bounded from below and above by non–deterministic polynomial time.

[8] It means that if φ and ψ are elementary formulae, then also $\neg\varphi$, $\varphi \Rightarrow \psi$, $\varphi \vee \psi$, and $\varphi \wedge \psi$ are elementary formulae.

[9] This problem seems to be equally difficult to the famous question P=NP?, which is worth at least the prize of 1,000,000 $ offered by Clay Institute of Mathematics for solving one of the seven greatest open mathematical problems of our time, see e.g. (Devlin 2002). P (PTIME) is the class of problems which can be computed by deterministic Turing machines in polynomial time. NP (NPTIME) is the class of problems which can be computed by nondeterministic Turing machines in polynomial time. Co–NP is the set of complements of the NP and we have a simple dependence: if P=NP, then NP=co–NP.

[10] Notice that Edmods' Thesis has its counterpart in cognitive sciences, so-called P–Cognition Thesis, saying that human cognitive

(linguistic) capacities are constrained by polynomial–time computability. As far as we know, it was explicitly formulated for the first time by Frixione (2001).

[11] The answer depends whether P=NP.

[12] Other examples of NPTIME–complete natural language sentences, involving anaphora, might be found in (Ristad 1993) and (Pratt-Hartmann 2004).

References

Barwise, J. (1979). On Branching Quantifiers in English, *Journal of Philosophical Logic*, 8 (1979), 47–80.

Devlin, K. (2002). *The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time*, Basic Books 2002.

Ebbinghaus, E.D., Flum, J., Thomas, W. (1996). *Mathematical Logic*, Springer 1996.

Edmonds, J. (1965). Paths, trees, and flowers, *Canadian Journal of Mathematics*, 17 (1965), 449–467.

Fagin, R. (1974). Generalized First–Order Spectra and Polynomial Time Recognizable Sets. In *Complexity of Computation*, Karp, R. (ed.), 43–73 , SIAM-AMS Proceedings 7 (1974).

Frixione, M. (2001). Tractable Competence, *Minds and Machines*, 11 (3) (2001), 379–397.

Garey, M.R., Johnson, D.S. (1979). *Computers and Intractability. A Guide to the Theory of NP–Completness*, W.H. Freeman and Company 1979.

Immerman, N. (1999). *Descriptive complexity*, Springer Verlag 1999.

Keenan, E. (2002). Some Properties of Natural Language Quantifiers, *Linguistics and Philosophy*, 25 (2002), 627–654.

Krynicki, M., Mostowski, M. (1999). Ambiguous quantifiers. In *Logic at work*, Orłowska, E. (ed), 548–565, Heidelberg 1999.

Lindström, P.(1966). First order predicate logic with generalized quantifiers, *Theoria*, 32 (1966), 186–195.

Mostowski, M. (1994). Kwantyfikatory rozgałęzione a problem formy logicznej. In *Nauka i język*, Omyła, M. (ed.), 201–242, Biblioteka Myśli Semiotycznej 1994.

Mostowski, M., Wojtyniak, D. (2004). Computational Complexity of the Semantics of Some Natural Language Constructions, *Annals of Pure and Applied Logic*, 127 (2004), 219–227.

Peters, S., Westerståhl, D. (2006). *Quantifiers in Language and Logic*, Clarendon Press 2006.

Pratt-Hartmann, I. (2004). Fragments of Language, *Journal of Logic, Language and Information*, 13 (2) (2004), 207–223.

Ristad, E. S. (1993). *The Language Complexity Game,* The MIT Press 1993.

Sevenster, M. (2006). *Branches of imperfect information: logic, games, and computation*, PhD Thesis, University of Amsterdam 2006.

Szymanik, J. (2007). A Strong Meaning Hypothesis from a computational Perspective. In *Proceedings of the Sixteenth Amsterdam Colloquium*, Aloni, M., Dekker, P., and Roelofsen, F. (eds.), 211–216 University of Amsterdam 2007.

Turing, A.(1936). On Computable Numbers, with an application to the Entscheidungsproblem, *Proceedings of the London Mathematical Society*, 42 (1936), 230–265.

Westerståhl, D. (1984). Some Results on Quantifiers, *Notre Dame Journal of Formal Logic*, 25 (1984), 152–169.