Semantical bounds for everyday language

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Abstract We consider the notion of everyday language. We claim that everyday language is semantically bounded by properties expressible in the existential fragment of second–order logic. Two arguments for this thesis are formulated. Firstly, we show that so–called Barwise's test of negation normality (Barwise, 1979) works properly only when assuming our main thesis. Secondly, we discuss the argument from practical computability for finite universes. Everyday language sentences are directly or indirectly verifiable. We show that in both cases they are bounded by second–order existential properties. Moreover, there are known examples of everyday language sentences which are the most difficult in this class ($NPTIME$–complete, see e.g. (Mostowski Wojtyniak, 2004)).

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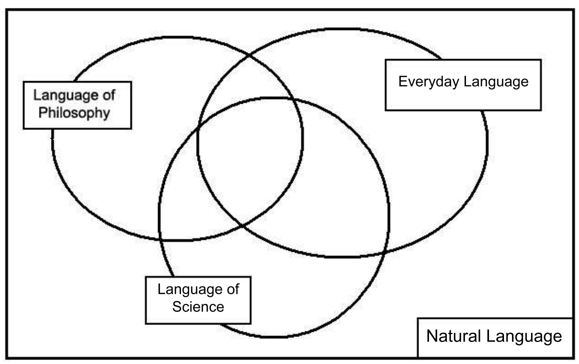
1 Introduction

There is a common and — from our point of view — controversial use of the term *natural language* as opposed not only to artificial languages but also to scientific language or technical jargons. A good example of such use is the term *natural language quantifiers*[1] as opposed to *logical quantifiers* (e.g. see (Keenan, 2002)). Obviously, *infinity* and *there is infinitely many* are natural language expressions just as *majority* or *many*. Nevertheless, we can see a natural intuition supporting a narrow use of the term *natural language*. However, in this narrow sense we prefer to use the term *everyday language*, instead. This is a fragment of natural language in which logicians communicate with bakers, students with postmen, quantum physicists with philologist, and so on. Everyday language is a pretheoretical part of a natural language, creating its basic and most common core[2]. The place of everyday language between different fragments of natural language can be illustrated in the following way:



We are looking for semantical bounds of everyday language. Firstly, we ask about the number of elements creating our universe of discourse. This is important because possible estimations of semantical power of everyday language heavily depend on semantical power of its quantifier constructions. Most authors considering semantics of natural language are interested only in finite universes; let us quote Dag Westerståhl:

'In general these cardinals can be infinite. However, we now lay down

[1]Let us observe that this phrase has essentially different presuppositions than the phrase *quantifiers in natural language*.

[2]We would even claim that it is the most biologically grounded part of natural language. However, it raises so many questions falling beyond the scope of this paper that we prefer not to go in this direction.

the following constraint:

(FIN) Only finite universes are considered.

This is a drastic restriction, no doubt. It is partly motivated by the fact that a great deal of the interest of the present theory of determiners comes from applications to natural language, where this restriction is reasonable' (Westerståhl, 1984).

This restriction seems reasonable because in typical communication situations we refer to relatively small finite sets of object. For example, intended interpretations of the following sentences are relatively small sets:

(1) Exactly five of my children went to the cinema.

(2) Everyone from my family has read *Alice's Adventures in Wonderland*.

Considering cardinalities of the universe of discourse we have three main possibilities:

1. small finite universes;
2. large finite universes;
3. infinite universes.

In many cases the restriction to finite interpretations essentially simplifies our theoretical considerations. Moreover, it is adequate for many communication situations. Neverthless, restricting to finite universes we omit many important cases. Some ideas can be easily formulated when we restrict ourselves to finite universes and their generalization for arbitrary universes would require subtle and technically difficult analysis (see the discussion of so called measure quantifiers in (Krynicki Mostowski, 1999)). In this work we consider arguments taking into account small finite universes and the general case covering all three mentioned kinds of universes.

2 A few examples

In this section we give a few examples of natural language sentences together with their semantical interpretations. We consider examples of sentences interpreted in the model $M = (U, V^M, T^M, H^M)$[3], where the universe U of M is the set of human beings, V^M is the set of villagers, T^M is the set of townsmen, and H^M is the relation

[3]Models are precise mathematical notions explicating possible worlds or possible interpretations of our language.

of hating each other. The corresponding predicates V, T, H are interpreted in M as: V^M, T^M, H^M, respectively.

We start with an easy sentence and its logical form:

(3) There are exactly two villagers.

(4) $\exists x \exists y[V(x) \wedge V(y) \wedge \forall z(V(z) \Rightarrow (z = x \vee z = y))]$

Therefore the logical form of sentence 3 can be given in terms of elementary logic[4] by formula 4.

The next sentence we are interested in is a bit more difficult. Consider the following pair a sentence and its logical form.

(5) Every other person is a townsman.

(6) $\exists P[\forall x \forall y(P(x,y) \Rightarrow (T(x) \wedge \neg T(y))) \wedge \forall x(T(x) \Rightarrow \exists y P(x,y)) \wedge$
$\wedge \forall y(\neg T(y) \Rightarrow \exists x P(x,y)) \wedge \forall x \forall y \forall y'((P(x,y) \wedge P(x,y')) \Rightarrow y = y' \wedge$
$\wedge \forall x \forall x' \forall y((P(x,y) \wedge P(x',y)) \Rightarrow x = x')]$

Formula 6 is not elementary because it starts with the second–order quantifier $\exists P$. The variable P runs through binary relations over the universe, in our case subsets of U^2. It is not equivalent to any elementary formula. It says that sets of townsmen and not townsmen have the same cardinality, because there is a one–to–one mapping P from one of these sets to another. In other words, every other element from U belongs to T^M. Therefore, formula 6 has the same truth–conditions as sentence 5. This is why formula 6 is a correct logical form for sentence 5.

Formula 6 has the form $\exists P \varphi(P)$, where P is a second–order variable and φ is a first order–formula with P as an additional binary predicate. The class of such existential second–order formulae is denoted by Σ^1_1. Formulae equivalent to Σ^1_1 formulae will also be called Σ^1_1 formulae.

Now let us consider a more complicated example:

(7) Most people live in a village.

(8) $\exists R[\forall x(V(x) \Rightarrow \exists y(\neg V(y) \wedge R(x,y))) \wedge$
$\wedge \forall x \forall y \forall y'((V(x) \wedge \neg V(y) \wedge \neg V(y') \wedge R(x,y) \wedge R(x,y')) \Rightarrow y = y') \wedge$
$\wedge \forall y(\neg V(y) \Rightarrow \exists x(V(x) \wedge R(x,y))) \wedge$
$\wedge \exists x \exists x' \exists y(V(x) \wedge V(x') \wedge x \neq x' \wedge \neg V(y) \wedge R(x,y) \wedge R(x',y))]$

Formula 8 is Σ^1_1. It says that there exists a function from V into $U - V^M$ which is surjective but not injective. Therefore, it says that most x from U belongs to V^M, then formula 8 is a proper logical form for sentence 7.

[4]Elementary logic — called also first–order logic — allows only quantifiers $\forall$ and $\exists$ binding individual variables

Essentially formula 8 defines the quantifier 'Most' of type (1). In what follows we need the quantifier MOST of type (1, 1)[5]. $\mathsf{MOST}x\ (\varphi(x), \psi(x))$ is defined by the following second–order formula:

(9) $\exists R[\forall x \exists y(\varphi(x) \wedge \psi(x) \wedge \varphi(y) \wedge \neg\psi(y) \wedge R(x,y)) \wedge$
$\wedge \forall x \forall y \forall y'(\varphi(x) \wedge \psi(x) \wedge \varphi(y) \wedge \neg\psi(y) \wedge \varphi(y') \wedge \neg\psi(y') \wedge R(x,y) \wedge R(x,y') \Rightarrow$
$y = y') \wedge$
$\wedge \forall y(\varphi(y) \wedge \neg\psi(y) \Rightarrow \exists x(\varphi(x) \wedge \psi(x) \wedge R(x,y))) \wedge$
$\wedge \exists x \exists x' \exists y(\varphi(x) \wedge \psi(x) \wedge \varphi(x') \wedge \psi(x') \wedge x \neq x' \wedge \varphi(y) \wedge \neg\psi(y) \wedge R(x,y) \wedge$
$R(x',y))]$

Now let us consider an example of a really hard sentence:

(10) Most villagers and most townsmen hate each other.

(11) $\exists A \exists B[\mathsf{MOST}x\ (V(x), A(x)) \wedge \mathsf{MOST}y\ (T(y), B(y)) \wedge$
$\wedge \forall x \forall y(A(x) \wedge B(y) \Rightarrow H(x,y))]$

Formula 11 is equivalent to a Σ^1_1–sentence. It says that there are sets A and B containing respectively most villagers and townsmen, such that every villager from A and every townsman from B hate each other. Formula 11 has the same truth–conditions as statement 10, thus it is the intended interpretation of sentence 10 in our model M.

Finally, we consider a sentence which is not expressible in the existential fragment of second–order logic.

(12) There are at most countably many entities.

(13) $\exists R[\forall x \neg R(x,x) \wedge \forall x \forall y(R(x,y) \vee R(y,x) \vee x = y) \wedge$
$\wedge \forall x \forall y \forall z(R(x,y) \wedge R(y,z) \Rightarrow R(x,z)) \wedge$
$\wedge \forall A(\exists x A(x) \Rightarrow \exists x(A(x) \wedge (\forall y R(y,x) \Rightarrow \neg A(y)))) \wedge$
$\wedge \forall x(\exists y R(y,x) \Rightarrow \exists z(R(z,x) \wedge \forall w(w \neq z \wedge R(w,x) \Rightarrow R(w,z))))]$

This sentence says that there exists a well–ordering such that each element in this ordering has a predecessor except for the least element. This is possible only in the case when the cardinality of the set is countable or finite.

Let us note that all previous quantifiers could be expressed in the existential fragment of second–order logic. In this case it is not possible, because of the fact that for existential fragment of second–order logic the Uppward Skolem–Löwenheim Theorem holds.

[5]For definition of generalized quantifiers and their types see (Lindström, 1966).

3 The main thesis

What follows is the main claim of our paper.

Main Thesis
Our everyday language is semantically bounded by the Σ^1_1–properties.

In other words, we claim that everyday language contains only these notions which can be defined in the existential fragment of second–order logic. If some property is not definable by any Σ^1_1–formula, then it falls outside the scope of everyday language. For example, quantifiers 'there exists', 'all', 'exactly two', 'at least four', 'every other' and 'most' belongs to everyday language. The counterexample is the notion 'there exists at most countably many' which is not definable by any Σ^1_1–formula. In the next two sections we give arguments for such upper bounds of everyday language.

Before discussing the arguments we present one of the consequences of the main thesis. First order–logic is closed on Boolean operations[6]. Σ^1_1 fragment of second–order logic is not closed on Boolean operations. Particularly, it is not closed on negation. However, this problem is open when we restrict interpretations to finite models. In this case Σ^1_1–notions are closed on Boolean operations if and only if $NP = coNP$ [7] which is one of the most difficult problems of computational complexity. Thus it is reasonable to suppose that everyday language — i.e. the fragment of natural language semantically bound by Σ^1_1–properties — is not closed on Boolean operations even on finite universes. It may be the case that a sentence belongs to everyday language, but its negation does not.

4 Argument from negation normality

It was observed by Jon Barwise that the negations of some simple quantifier sentences, i. e. sentences without sentential connectives different than 'not' before a verb, can easily be formulated as a simple quantifier sentences. In some cases it

[6]It means that if φ and ψ are elementary formulae, then also $\neg\varphi$, $\varphi \Rightarrow \psi$, $\varphi \vee \psi$ are elementary formulae.

[7]This problem seems to be equally difficult to the famous question $P = NP$?, which is worth at least the 1,000,000 $ prize offered by Clay Institute of Mathematics for solving one of the seven greatest open mathematical problems of our time (see e.g.(Devlin, 2002)). $P(PTIME)$ is the class of problems which can be computed by deterministic Turing machines in polynomial time. $NP(NPTIME)$ is the class of problems which can be computed by nondeterministic Turing machines in polynomial time. $coNP$ is the set of complements of the NP and we have a simple dependence: if $P = NP$, then $NP = coNP$.

is impossible (Barwise, 1979). Namely, the only way to negate some simple sentences is by prefixing them with the phrase 'it is not the case that' or an equivalent expression of theoretical character.

The sentences of the first kind are called negation normal. For example

(14) Everyone owns a car.

can be negated as follows:

(15) Someone doesn't own a car.

The sentences of the second kind are not negation normal. For example:

(16) Most relatives of each villager and most relatives of each townsman hate each other.

can only be negated in the following way:

(17) It is not the case that most relatives of each villager and most relatives of each townsman hate each other.

Barwise proposed the test of negation normality as a reasonable test for first–order definability (Barwise, 1979). The test is based on the following theorem which is a corollary from Craig's Interpolation Lemma[8].

Theorem
If φ is a sentence definable in Σ^1_1, the existential fragment of second–order logic, and its negation is logically equivalent to a Σ^1_1–sentence, then φ is logically equivalent to some first–order sentence.

The results of the negation normality test agree with our experience (see (Barwise, 1979), (Mostowski, 1994)). In other words, the test works only on the assumption that simple everyday sentences are semantically bounded by Σ^1_1–properties. This gives an argument in favour of our main thesis in arbitrary universes.

[8]The lemma states that: if φ and ψ are sentences such that $\varphi \Rightarrow \psi$ is a logically valid sentence, then there is sentence θ — called Craig interpolant of φ and ψ, such that:

1. $\varphi \Rightarrow \theta$ and $\theta \Rightarrow \psi$ are logically valid;
2. Every relation, function or constant symbol (excluding identity) which occurs in θ occurs in both φ and ψ (for proof see e.g. (Ebbinghaus et al., 1996)).

5 Argument from practical computability

The core sentences of everyday language are sentences which can be more or less effectively verifiable. In the case of small finite interpretations it means that their truth value can be practically computed (directly or indirectly).

Direct practical computability means that there is an algorithm which for a given finite interpretation computes the truth–value in a reasonable time. Our computational experience justifies the claim formulated by Jack Edmonds in (Edmonds, 1965).

Edmonds' Thesis
The class of practically computable problems is identical with PTIME class, that is the class of problems which can be computed by a deterministic Turing machine in a number of steps bounded by a polynomial function of the length of a query.

We take here Edmonds' thesis as granted. It follows that direct practical computability of the truth–value in small finite interpretations means that the problem of truth–value of a given sentence in finite interpretations is in $PTIME$.

In (Mostowski, 1994) it is observed that except referential meanings, we frequently understand sentences by their inferential meaning which is determined indirectly by inferential relations to easy sentences having well defined referential meanings. Let us consider the following three sentences:

(18) There were more boys than girls at the party.

(19) At the party every girl was paired with a boy.

(20) Peter came alone to the party.

We know that sentence 18 can be inferred from sentences 19 and 20. Then we can establish the truth–value of sentence 18 indirectly knowing that sentences 19 and 20 are true. Sentence 18 is easy in the sense that its truth–value is $PTIME$ computable (see e.g. (Immerman, 1999)).

However for some sentences the problem whether their truth–values are $PTIME$ computable is open [9]. Let us consider the following examples.

(21) Most villagers and most townsmen hate each other.

(22) Exactly half of all villagers and exactly half of all townsmen hate each other.

(23) At least one third of villagers and at least half of townsmen hate each other.

[9] The answer depends on the open problem, whether $P = NP$?

(24) Some relative of each villager and some relative of each townsman hate each other.

It is known that the problem of truth–value for each of these sentences is $NPTIME$–complete (see: (Mostowski Wojtyniak, 2004), (Sevenster, manuscript), (Mostowski Szymanik, manuscript)).

$NPTIME$ (for short NP) is the class of problems which can be solved by a nondeterministic Turing machine in a number of steps bounded by a polynomial function of the length of a query. Nondeterministic algorithms were defined for the first time by Alan Turing (Turing, 1936). The term *nondeterministic* is misleading. Originally, Turing used the term *with choice*. In the case of $NPTIME$ the nondeterministic behaviour can be described (see (Garey Johnson, 1979)) as follows:

> Firstly, choose a certificate of a size polynomially depending on the size of input. Then apply a $PTIME$ algorithm for finding the answer. The nondeterministic algorithm answers YES exactly when there is a certificate for which we get a positive answer.

Let us observe that such certificates are a kind of proofs. When we have a proof of a statement then we can easily check whether the sentence is true.

The logical relevance of the class $NPTIME$ follows from the Fagin's Theorem:

Theorem (Fagin, 1974)
A class of finite models is $NPTIME$ computable if and only if it is definable by a Σ^1_1–sentence.

Let us notice that all examples of natural language sentences considered in our work — which undoubtedly belong to everyday language — have Σ^1_1 logical forms.

$NPTIME$–complete problems are computationally the most difficult problems in the $NPTIME$ class. Particularly, it is known that $P = NP$ exactly when any $NPTIME$–complete problem is $PTIME$ computable. Therefore, on the ground of our current knowledge we can expect that $NPTIME$–complete problems are not practically computable. Nevertheless, similarly as all $NPTIME$ problems they can be practically justifiable. Let us consider an example.

Suppose that we have two predicate expressions A, B and the following true statements:

(25) Most villagers are A.

(26) Most townsmen are B.

(27) All A and all B hate each other.

From this sentences we can infer sentence 21:

Most villagers and most townsmen hate each other.

The predicate expressions A and B should be guessed. They are in a sense certificates or proofs of truth of sentence 21.

In this sense sentences with $NPTIME$ truth problem — or by Fagin's theorem Σ_1^1–expressible sentences — are indirectly verifiable. Moreover, $NPTIME$ seems to capture exactly indirect verifiability.

This concludes our second argument restricted to finite interpretations of our main thesis.

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