Unrestricted Fusion and Unrestricted Quantification: Mereological Essentialism and the Universe

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ABSTRACT

Salvatore Florio and Øystein Linnebo have recently put forward an argument against unrestricted comprehension in plural logic using plausible assumptions. Since mereology and plural logic are similar from a formal point of view, the question naturally arises whether this argument can be replicated in mereology with the conclusion that Unrestricted Fusion has to be restricted. This thesis investigates whether the assumptions can be motivated in the case of mereology by taking an in-depth look at the literature and by arguing against Florio and Linnebo's claim regarding the modal profile of mereological fusions. It is shown, that one of the assumptions is difficult to motivate and that, without it, the argument does not work. However, if the assumption is granted, the argument succeeds and shows that, contrary to defenders of Unrestricted Fusion, that there is a well-motivated way to restrict Unrestricted Fusion.

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1 INTRODUCTION

Naïve set theory has the intuitive assumption that every property determines a set. As is well known, this assumption leads to paradoxes with the most famous being the Russell's paradox. In order to avoid these paradoxes, axiomatic set theory ZF(C) was developed.

Still, the intuitive pull of this idea is great and two other formal theories, namely classical mereology and plural logic, have analogues of this axiom – Unrestricted Fusion and unrestricted plural comprehension – according to which every property determines a fusion or a plurality, respectively. That is, given any property, there is a fusion or a plurality that has all and only those things as parts or members which satisfy the property. Neither of these theories faces an analogue of Russell's paradox (or other paradoxes), so they are consistent.

While classical mereology has initially been developed as a nominalistically acceptable foundation of mathematics to replace set theory, the focus of this thesis is on its use as a (tool in) formal ontology. In this context, Unrestricted Fusion is quite controversial since it entails the existence of numerous fusions whose existence is counter intuitive. Defenders of Unrestricted Fusion appeal to the idea that a fusion is no further ontological commitment once one has accepted the existence of its parts and also that there is no well-motivated way to restrict Unrestricted Fusion.

Recently, there has been an argument by Salvatore Florio and Øystein Linnebo that unrestricted plural comprehension has to be restricted given certain assumptions. They also take their argument to supply them with good reason to do so. Since mereology and plural logic are quite similar from a formal point of view, the question naturally arises whether this argument can be replicated in the case of classical mereology.

To achieve this, I highlight the assumptions Florio and Linnebo, 2021 make and present a schematic version of their argument that allows one to easily see which assumptions are used at each step. Furthermore, I identify the mereological analogues of these assumptions. In some cases, like unrestricted plural comprehension, this is straightforward, while other cases are more difficult. I also argue for the plausibility of these mereological analogues with the exception of one which is rather difficult to argue for and which is even explicitly rejected in the literature.

Since the assumptions of the argument are used as a guide throughout the thesis, it is worthwhile to briefly state them here:

(UnrQuan) Unrestricted quantification, i. e. there is an all-encompassing domain,

(UnrComp) Unrestricted plural comprehension, i. e. for any condition there is a plurality consisting of all and only those objects that satisfy the condition (entails the existence of a universal plurality),

(UniSing) Universal singularization, i. e. there is a 'set of' function that maps any given plurality to an object (its set),

(Rigidity) Pluralities are rigid, i. e. they have the same members at every possible world at which they exist.

Thus, the aim of this thesis is to investigate whether these assumptions can be argued for in the case of mereology and whether the argument can be replicated to the effect that Unrestricted Fusion has to be restricted.

The answer to this will be negative unless a strong assumption between the correspondence of sets and mereological fusions is made, namely (UniSing). Still, this result is interesting since it shows, contrary to defenders of Unrestricted Fusion, that there is a well-motivated way to restrict Unrestricted Fusion given this correspondence between sets and mereological fusions.

1.1 STRUCTURE

The structure of the thesis is as follows. In chapter two (2), plural logic is briefly introduced before moving on to Florio and Linnebo, 2021's argument against unrestricted plural comprehension. Connected to this, the concepts of (plural) rigidity and extensional definiteness/indefiniteness or circumscribability are discussed as well. Lastly, unrestricted quantification and the two dominant positions, relativism and absolutism, are introduced.

In chapter three (3), classical mereology is introduced as a (tool in) formal ontology. Furthermore, an axiomatization of it is given and the philosophical motivation behind Unrestricted Fusion, a controversial axiom of classical mereology, is discussed. I then argue that as a mereologist one should strive for absolutism. In terms of the above assumptions, I address (UnrQuan), (UnrComp), and (UniSing). The latter only briefly, I return to it in chapter five (5).

In chapter four (4), the corresponding notion of plural rigidity in the mereological context is introduced. It is shown that this parallel arises naturally since it corresponds to the position of Mereological Essentialism. The philosophical arguments in favor of it are discussed and evaluated. Lastly, an important distinction by Jubien, 2001 is used to argue against Florio and Linnebo, 2021 claim that parthood is not rigid and that it is far from clear what the formal parts of many fusions should be like. In terms of the above assumptions, I address (Rigidity).

In chapter five (5), it is then investigated whether an analogous argument to the one by Florio and Linnebo against unrestricted plural comprehension can be given in mereology. The assumptions in each case are discussed and it is shown how they are used in the argument. It is argued that in the case of mereology one crucial assumption, (UniSing), is difficult to motivate and without it, the argument is unsuccessful. If one grants the assumption, however, the argument succeeds and shows that there is a well-motivated way to restrict Unrestricted Fusion. In terms of the above assumptions, I address (UniSing) again in more detail.

In chapter six (6), the main findings of each chapter are listed and summarized.

1.2 A NOTE ON TERMINOLOGY AND FORMATTING

Sadly, classical mereology has no standard terminology, but I follow Cotnoir and Varzi, 2021's terminology and use of symbols. Unless otherwise stated, I always mean classical mereology even if I only write mereology. When quoting authors who differ in their terminology, such as using the term 'compose' instead of 'fuse', I put the corresponding term I use in square brackets. Furthermore, the formatting (italics, boldface, etc.) of quotes is taken over.

In my own writing, the following formatting conventions are used. The distinction between use and mention is indicated by single quotes around the term, as was done above. Single quotes are also used around sentences for informal glosses of symbols. When writing about kinds, the kind terms are formatted in *italics*. I also use *italics* when important concepts are introduced. Titles of papers and books are formatted in SMALL CAPS. Lastly, the full name of a person is used the first time they are mentioned and afterwards only their last name is used.

For the mereological analogues of the above assumptions, I use the same names with an added asterisk, i. e. (UnrQuan*) for (UnrQuan), except for (UnrComp) where I use (UnrFus*) as this makes more sense mnemonically.

2 PLURAL LOGIC, RIGIDITY AND UNRESTRICTED QUANTIFICATION

In this chapter, plural logic is briefly introduced (2.1). Afterwards, an introduction to the topic of unrestricted quantification is given (2.2) before the central concepts of *(plural) rigidity* (2.3, 2.3.2) and *extensional definiteness/indefiniteness or circumscribability* (2.4) are presented. Florio and Linnebo, 2021 use these concepts in their argument that in some domains, so-called extensionally indefinite ones, there cannot exist a universal plurality and that, consequently, plural comprehension has to be restricted (2.5).

2.1 PLURAL LOGIC

Since plural logic is not the main concern of this thesis, only a gloss of it is given here without further explanation, except for the unrestricted plural comprehension scheme as this is the axiom Florio and Linnebo take issue with. I follow their presentation of traditional plural logic (plural first-order logic with plural predicates), which is the most common version of plural logic in the literature (cf. Florio and Linnebo, 2021, p. 15). One starts off with standard first-order logic and adds the following:

- Plural variables xx, yy etc., and plural constants aa, bb etc.;
- Quantifiers for plural variables, $\forall xx, \exists yy$;
- A binary predicate '$\prec$' for plural membership between objects and pluralities, i. e. the first argument expression is a singular term and the second argument expression is a plural term. The predicate should be read as 'is one of' or 'is among';
- Symbols for collective plural predicates 'P', 'Q'.

Thus, the language has two types of variables: singular and plural (cf. Florio and Linnebo, 2021, pp. 15–17). Furthermore, a many-many relation of plural inclusion '$xx \preccurlyeq yy$' is defined as:

$$\forall z(z \prec xx \rightarrow z \prec yy).$$

Using this definition, the notion of plural identity is defined as mutual member inclusion or coextensiveness, i. e.:

$$xx \approx yy := (xx \preccurlyeq yy \wedge yy \preccurlyeq xx).$$

Hence, two pluralities are identical if and only if they have the same members (cf. Florio and Linnebo, 2021, p. 18). Just as sets are identical if and only if they have the same elements.

Lastly, some axioms and rules are introduced which govern the formation and indiscernibility of pluralities.[1] The first axiom states that every plurality is non-empty and hence there is no empty plurality corresponding to the empty set. Since pluralities are taken to be 'nothing over and above' their members, this makes sense.

(Non-empty) $\forall xx \exists y \; y \prec xx.$

The second one is an axiom scheme that is the plural analogue of Leibniz's law stating that two coextensive pluralities satisfy all the same formulas.

(Indisc) $\forall xx \forall yy (xx \approx yy \rightarrow (\varphi(xx) \leftrightarrow \varphi(yy))).$

The third axiom is also a scheme which is called unrestricted plural comprehension. It states that if there exists some thing satisfying a formula, then there exists a plurality consisting of all and only those things that satisfy the formula.

(Plural Comprehension) $\exists x \varphi(x) \rightarrow \exists xx \forall x (x \prec xx \leftrightarrow \varphi(x)).$

So there are no requirements on when some things form a plurality given that there is at least one thing that satisfies the condition φ (cf. Florio and Linnebo, 2021, pp. 19–20). As we will see in (3.2), the mereological analogue, called Unrestricted Fusion, also places no requirements on when some things form a fusion.

Florio and Linnebo object to unrestricted plural comprehension, however, and argue that it has to be restricted. They develop an argument that aims to show that there is a non-arbitrary and motivated way of doing so. This argument will be presented in (2.5). In order to understand it, the topic of unrestricted quantification and the concepts of plural rigidity (2.3.2) and extensional definiteness/indefiniteness or circumscribability (2.4) have to be introduced first.

2.2 UNRESTRICTED QUANTIFICATION

We now turn to the topic of unrestricted quantification and the two opposing views of *relativism* and *absolutism*. The topic of unrestricted quantification is connected to plural logic because unrestricted plural comprehension entails the existence of a universal plurality which has everything as a member. The immediate question now is: How should 'everything' be understood?

1 The axioms and rules of the first-order fragment are the usual ones and the plural quantifiers adhere to analogues of the first-order quantifiers.

To put it differently: What is the domain of quantification over which the quantifiers range?[2] Is it absolutely unrestricted or is it, in some sense, restricted? The former view is called absolutism and accepts that it is possible to quantify over absolutely everything. The latter view is called relativism and denies that it is possible to quantify over absolutely everything (cf. Florio, 2014, pp. 442–443).

In connection to this, it is useful to distinguish between the metaphysical and the availability question of such a domain. The former is a question about whether or not there is an all-encompassing domain and what its ontological status is, i. e. is it an object? A property? The latter question is about whether or not this domain is ever salient for the interpretation of our linguistic practice (cf. Uzquiano, 2009, pp. 301–302). If the answer to the metaphysical question is negative, then the linguistic question changes and turns into a question about the availability of this restricted domain. Thus, the metaphysical question is more fundamental and also the one of interest to this thesis.[3]

In the next subsections, the positions are introduced in more detail and the main problems they face are discussed.

2.2.1 *Relativism*

There is an immediate challenge to relativism which stems from everyday language use. It intuitively seems like people sometimes quantify over everything such as when they state that 'no pig talks', $\forall x(Px \to \neg Tx)$. If the quantifier only ranges over a restricted domain, then this universal generalization does not rule out all counterexamples: there could be a talking pig in an extended domain (cf. Florio, 2014, p. 443).

The second challenge stems from the fact that the relativist seems to lack the adequate resources to properly state their view. Consider the naive way of stating it as follows: 'It is not possible to quantify over absolutely everything'. Now either 'everything' ranges over absolutely everything or it does not. In the former case, the position is inconsistent. In the latter case, the sentence does not adequately express the view since 'everything' is used in a restricted sense which is not the intended meaning of the relativist position. So what are the other options?

Suppose that the relativist describes their view in the following way: 'Given any domain d, there is an x such that x is not in d'. Let

2 While domains are usually conceived of as sets in model-theory, this is not assumed here since in ZF(C), for example, there exists no universal set. Due to this, some people prefer to use pluralities or (Fregean)concepts to represent domains see Florio and Linnebo, 2021.

3 Of course the linguistic question is not limited to the discipline of linguistics, but in order to even engage with the metaphysical one, one has to affirm the linguistic question. That is, one has to assume that there are contexts where the all-encompassing domain is salient. This is a substantial assumption which is unavoidable.

d^+ be the domain over which the quantifier ranges. If the sentence quantifies over every domain, it includes d^+. Hence, it implies that there is an x such that it is not in d^+ where the existential quantifier ranges over d^+ too. So there is something in d^+ such that it is not in d^+, this is an outright contradiction. If the sentence does not quantify over every domain, it is not ruled out that there might be an all-encompassing domain. So again, this is not the intended meaning of the relativist position (cf. Williamson, 2003, pp. 427–428; Florio, 2014, p. 443). That is, the relativist faces the same issues as before.

Faced with these problems, relativists often resort to express their view in a schematic way. The idea goes back to Russell's notion of *typical ambiguity* and uses free variables to get a version of absolutely general quantification (cf. Parsons, 2006, p. 217; Florio and Linnebo, 2021, p. 247). Florio and Linnebo, 2021's way of stating it, uses an operation that, when applied to any interpretation I, yields an extended interpretation I^+ such that:

$$I \subset I^+$$

Since both I and I^+ are arbitrary, this encapsulates the relativists thought that for any domain, there is another domain which strictly extends it. While this might seem promising at first, the schematic formulation is very limited since the "[s]chematic statements cannot be negated and cannot be freely combined in other truth-functional ways" (Florio and Linnebo, 2021, p. 247).[4]

Another approach to express relativism is to use the modal notions of necessity '$\Box$' and possibility '$\Diamond$'. Using these resources, the position can now be expressed as:

> Necessarily, for any interpretation I, there could exist an extended one I^+.

Put formally:

$$\Box\forall I \Diamond \exists I^+ (I \subset I^+).$$

The idea is that, no matter what the domain is for any interpretation, it could be extended to an interpretation whose domain is a proper superset of the former. The first question that has to be addressed now is, what kind of modality is used here. It can hardly be the usual metaphysical interpretation, since mathematics is thought to be necessary and therefore also the objects necessarily exist.

> [T]he existence of the relevant objects, such as pure sets, is often assumed to be metaphysically necessary, which

4 For further discussion of these limitations see Williamson, 2003, pp. 438–440 and Studd, 2019, pp. 131–135.

> rules out any variation of the domain of such objects across metaphysical possibilities. (Florio and Linnebo, 2021, p. 248)

Even if there is a suitable interpretation available for the modality in question, there is a further worry: might this formulation of the view end up too strong? Florio and Linnebo argue that this is the case because the modal operators allow one to quantify "not just over everything in the range of the quantifiers as *currently* interpreted, but over everything in their range on *any possible* interpretation" (Florio and Linnebo, 2021, p. 248).

Thus, the relativist ends up asserting that it is possible to quantify over absolutely everything, contrary to the actual meaning of their view. As we have seen, the relativist struggles to express their view properly in each case. Having presented the two main challenges relativists face, I now turn to discuss the alternative view of absolutism and the challenges it faces.

2.2.2 *Absolutism*

The most pressing arguments against absolutism are the well-known set-theoretic paradoxes, i. e. the Russell's paradox and the Burali-Forti paradox. The former will be briefly discussed since it raises two issues which are important for the thesis.

First, how can one motivate a non-arbitrary restriction on admissible conditions for the comprehension principle in naïve set theory. Second, the paradox can also be understood as being about the Dummettian notion of an *indefinitely extensible concept*, which is important for Florio and Linnebo's argument I discuss in (2.5).

Turning to Russell's paradox. The comprehension scheme used in naïve set theory asserts that for any formula φ, there exists a set that contains all and only those x that satisfy the formula:

$$\text{(Comprehension)} \quad \forall z_0, \dots, z_n \exists x \forall y (y \in x \leftrightarrow \varphi(y, z_0, \dots, z_n)).$$

Where φ ranges over the set-theoretic formulas and n ranges over the corresponding number of parameters. Now let $\varphi = y \notin y$ be the condition to define a set. By comprehension we get that there exists a set $x = \{y | y \notin y\}$, i. e. there exists a set that contains all and only those sets that are not elements of themselves. Now assume for contradiction that $x \in x$, by definition we get that $x \notin x$. Assume for contradiction again that $x \notin x$, by definition we get that $x \in x$. Hence, x cannot be a set in which case comprehension has to be restricted since it entails the existence of such a set. Hence, naïve set theory is inconsistent.

To avoid this set-theoretical paradox in the absolutist setting, one has to give a principled distinction between which conditions define a set and others which do not. Axiomatic set theory, ZF(C), was developed to avoid this paradox. To lend its axioms intrinsic justification, the iterative conception of set is often used.[5] For our purposes, it is not important to know the details of this theory. The upshot is that, without a good story, a restriction on comprehension seems arbitrary and unjustified and the iterative conception is used resolve these issues. Florio and Linnebo, 2021 take their argument against unrestricted plural comprehension to show that there is a non-arbitrary and motivated way of restricting plural comprehension as well. That is, of doing (some of) the work the iterative conception does for ZF(C).

As mentioned before, another way to resolve the paradox is to give up on absolutism and take the paradox to show that the concept of 'set' is indefinitely extensible. Here it is helpful to cite Michael Dummett in full.

> An indefinitely extensible concept is one such that, if we can form a definite conception of a totality all of whose members fall under that concept, we can, by reference to that totality, characterize a larger totality of all whose members fall under it. (Dummett, 1996, p. 441)

Such an indefinitely extensible concept is associated with a principle of extension "which given a definite conception of a totality of objects falling under a concept enables one to form a conception of an object which intuitively falls under the concept but is not in the totality" (Incurvati, 2022).

In the case at hand, this means the following. Given the condition $y \notin y$, one can form the set x of all the things falling under this concept. x cannot be a member of itself, in which case it satisfies the condition and falls under the concept. If one now considers the members of x together with x itself, one has specified "a more inclusive totality than [x] all of whose members fall under the concept of ['set not a member of itself']" (Dummett, 1996, p. 441). Therefore, 'set that is not a member of itself' is an indefinitely extensible concept and consequently also the concept of 'set'.

The relativist can straightforwardly explain this phenomenon by claiming that the universal quantifier is always restricted and the new set x lies outside of the domain of the universal quantifier. That is, consistency is restored by allowing to extend the initial domain to a bigger one that includes the new set x (cf. Florio, 2014, p. 445).

The similar notion of an *extensionally indefinite domain* plays a central role in Florio and Linnebo, 2021 argument for rejecting the universal plurality. Because of this, unrestricted plural comprehension has to be restricted which is discussed in detail in (2.5).

5 For an overview see Incurvati, 2020, chapter 2. For the classical papers see Boolos, 1971, Parsons, 1977, and Boolos, 1989.

Importantly, Florio and Linnebo take their argument to show that there is a well motivated restriction on unrestricted plural comprehension. This is also relevant to mereology because one argument in favor of Unrestricted Fusion is the thought that there is no well-motivated way to restrict it. So if an analogous argument can be made in mereology, it would show that there is a well-motivated way to restrict Unrestricted Fusion.

As was shown, relativism and absolutism face problems and the question which one should be chosen is not easy. Nevertheless, Florio and Linnebo, 2021 are proponents of absolute generality. The connection between unrestricted quantification and mereology is discussed in (3.4). I also argue that in the case of mereology there is an additional reason one should be an absolutist.

2.3 RIGIDITY

An important part of Florio and Linnebo, 2021's argument against unrestricted plural comprehension is *plural rigidity*. They state that:

> A plurality has a rigid membership profile: it has the very same members at any possible world at which it exists. (Florio and Linnebo, 2021, p. 228)

To put it slightly differently, this means that pluralities are modally rigid: they neither gain nor lose members at different possible worlds where they exist. Due to this, tracking a plurality across possible worlds is trivial since it is just a matter of tracking its members. Thereby one also tracks the plurality, i. e. pluralities are tracked extensionally across possible worlds.

Florio and Linnebo split the modal rigidity of pluralities into two principles: *RGD*$^+$ and *RGD*$^-$. *RGD*$^+$ asserts that if something x is a member of a plurality yy, then it is a member of yy in every possible world where yy exists; this can be put formally, using 'E' as an existence predicate, as:

$$RGD^+ \quad \Box\forall x\forall yy(x \prec yy \to \Box(Eyy \to x \prec yy)).$$

RGD$^-$ asserts that if something x is not a member of a plurality yy, then it is not a member of yy in any possible world; put formally:

$$RGD^- \quad \Box\forall x\forall yy(x \not\prec yy \to \Box(x \not\prec yy)).$$

Pluralities being modally rigid distinguishes them from groups and also from mereological fusions. The latter two do not have their members necessarily according to Florio and Linnebo (cf. Florio and Linnebo, 2021, pp. 206–207, 213–214, 219, 228). This difference cannot be explained by appealing to the fact that a plurality is many things taken as many, whereas a group or a mereological fusion are many things taken as one. A set is also many things taken as one, but it has its members necessarily as well (cf. Florio and Linnebo, 2021, pp. 205–207, 214). The case of mereological fusions is discussed at length in (4) where I also argue that Florio and Linnebo are too quick to dismiss the rigidity of fusions (4.3.1).

They resort to explain the difference between pluralities and groups or mereological fusions by claiming that "a plurality is nothing over and above its members and is thus fully specified when we have circumscribed its members" (Florio and Linnebo, 2021, p. 206). As we will see in (3.3), however, this idea is also used in mereology to argue that a mereological fusion does not commit one to an additional object given one accepts the existence of its parts.[6]

2.3.1 *Set Rigidity*

Florio and Linnebo start by considering an argument for the rigidity of sets and then construct a similar one for pluralities. An interesting part of this argument is that they claim an analogous argument for mereology does not work. Hence, mereological fusions are not rigid according to them. As already mentioned, this claim will be contested in (4.3.1). Now the argument for sets will be presented.

The logic in the background of their argument is the modal logic T with Brouwer's axiom, where the accessibility relation R is reflexive and symmetric.[7] Furthermore, they take Leibniz's law:

$$\text{(Leibniz)} \quad \Box\forall x\forall y(x = y \to (\varphi(x) \leftrightarrow \varphi(y)))$$

for granted and assume that identity is necessary, i. e.:

$$\Box(x = x).$$

One can then derive from $\Box(x = x)$ that if two things are identical, then they are necessarily identical:

$$\Box\forall x\forall y(x = y \to \Box x = y).$$

Furthermore, given Brouwer's axiom, one can derive from the above that if two things are distinct, then they are necessarily distinct (cf. Florio and Linnebo, 2021, pp. 210–211).

6 This is called the Composition as Identity (CAI) thesis in the mereological literature. See Cotnoir and Baxter, 2014 for an overview of the issue and different ways to understand the view. One way to understand CAI is in a strict sense, i. e. the fusion is identical with its parts. Another way to understand CAI is in a loose sense, i. e. the fusion is in some sense identical to its parts.

7 That is, R is characterized by $\Box p \to p$ and $p \to \Box\Diamond p$ and we have necessitation (if p is a theorem, then so is $\Box p$) and the K axiom $\Box(p \to q) \to (\Box p \to \Box q)$.

$$\Box\forall x\forall y(x \neq y \to \Box x \neq y)$$

Using the fact that sets obey the extensionality axiom:

$$\text{(Set-Ext)} \quad \forall x\forall y(x = y \leftrightarrow \forall z(z \in x \leftrightarrow z \in y))$$

one can then derive with Leibniz's law that if two sets are coextensive, i. e. have the same elements, then they are necessarily coextensive (cf. Florio and Linnebo, 2021, pp. 211–212):

$$\text{(Set-Cov)} \quad \forall x\forall y(\forall z(z \in x \leftrightarrow z \in y) \to \Box\forall z(z \in x \leftrightarrow z \in y)).$$

This result highlights an important difference between sets, which are extensionally tracked across possible worlds, and groups, which are intensionally tracked across possible worlds. Therefore, the former are modally rigid, but the latter are not. The difference is due to the fact that sets obey the principle of extensionality, whereas groups do not.

Furthermore, any reason to accept the extensionality of sets is also a reason to accept that set-membership is rigid (cf. Florio and Linnebo, 2021, p. 216). In the case of sets, having the same members is a necessary and sufficient for identity. For groups, on the other hand, having the same members is only a necessary, but not a sufficient condition for identity. Florio and Linnebo, 2021 claim that the reason for this is the following:

> [S]ets, unlike groups, are constituted by their members. A set is fully characterized by specifying its members. . . . By contrast, a group has additional features that go beyond its members, which means that having the same members need not suffice for identity." (Florio and Linnebo, 2021, p. 212)

This means that sets are modally rigid just like pluralities. Thus, they obey the analogues of the plural rigidity principles mentioned above. That is, sets do not lose elements:

$$\textit{Set-RGD}^{+} \quad \Box\forall x\forall y(x \in y \to \Box(Ey \to x \in y)).$$

Sets also do not gain elements:

$$\textit{Set-RGD}^{-} \quad \Box\forall x\forall y(x \notin y \to \Box(x \notin y)).$$

Florio and Linnebo end their argument by noting that these considerations give rise to a dilemma concerning "any . . . notion of collection" (Florio and Linnebo, 2021, p. 213). For any such notion of collection, one either has to give up the principle of extensionality for the collection in question or one has to accept that the collection is modally rigid.

2.3.2 *Plural Rigidity*

After this aside on the rigidity of sets, I now turn to Florio and Linnebo, 2021's argument that pluralities are rigid.

Their argument for the rigidity of pluralities starts with the indiscernibility of pluralities (Indisc) introduced in (2.1).

(Indisc) $\Box\forall xx\forall yy(xx \approx yy \to \varphi(xx) \leftrightarrow \varphi(yy))$

where '$\approx$' means that the two pluralities are coextensive, i. e. have the same members as defined in (2.1). Florio and Linnebo now derive an analogue of the necessity of identity from (Indisc), basic facts about plural logic and the modal logic T in the background. They call this principle covariation (Cov). It asserts that, necessarily if two pluralities are coextensive, then they are necessarily coextensive; in symbols:

(Cov) $\Box\forall xx\forall yy(xx \approx yy \to \Box(xx \approx yy))$

Given Brouwer's axiom, they also derive that if two pluralities are not coextensive, then they are necessarily so; in symbols:

$\Box\forall xx\forall yy(xx \not\approx yy \to \Box(xx \not\approx yy))$

This is just the analogue of the necessity of distinctness for sets. Similarly to the case of sets, where (Leibniz) and (Set-Ext) entail (Set-Cov), (Indisc) entails (Cov) in the case of pluralities. While (Cov) is compatible with the non-rigidity of pluralities – just as (Set-Cov) is compatible with the non-rigidity of sets – it is far more plausible given that pluralities are rigid. More significantly, any reason for (Cov) is also a reason to accept that plural membership is rigid. This parallels the relationship of extensionality and rigidity of sets.

The thought behind (Cov) is that pluralities are 'nothing over and above their members' and hence tracking them across possible worlds comes down to tracking their members. According to Florio and Linnebo, this is precisely how mereological fusions differ from sets and pluralities since tracking fusions across possible worlds also involves tracking their formal parts. This makes tracking them non-trivial. Florio and Linnebo's argument is discussed in detail in (4.1.1) and (4.3.1).

Because tracking pluralities is a trivial matter, one again faces the dilemma mentioned in (2.3.1): one has to either reject (Indisc) or accept that pluralities are rigid. Rejecting the former is not up for debate and hence Florio and Linnebo conclude that pluralities are rigid (cf. Florio and Linnebo, 2021, pp. 217–218).

2.4 extensional definiteness/indefiniteness or circumscribability

Since the notion of *extensional definiteness/indefiniteness* or *circumscribability* figure prominently in the argument by Florio and Linnebo to restrict unrestricted plural comprehension, it is worth to look at it in detail.[8] In the following I sometimes drop the 'circumscribed' as pluralities are, by definition, circumscribed.

Florio and Linnebo take the intuitive understanding of this notion as primitive and do not provide any formal explications of it. However, the terminology and the idea behind it is reminiscent of Dummett, 1996 where he introduces the notion of an "indefinitely extensible concept" (Dummett, 1996, p. 441) and discusses how this relates to domains of quantification in mathematics. The first concern here will be about pluralities and only in (2.5.2) about domains of quantification.[9]

Florio and Linnebo, 2021 start off by considering what things can obviously be circumscribed and then consider plausible ways one can get new pluralities by already existing ones. That is, they add axioms that allow one to build further (circumscribable) pluralities from already existing ones and supply intuitive justification for these axioms (cf. Florio and Linnebo, 2021, pp. 280–283). They call the resulting system "critical plural logic" (Florio and Linnebo, 2021, p. 278) since it differs from traditional plural logic as introduced in (2.1). Most of the axioms of *critical plural logic* are familiar looking as they are plural analogues of the usual set-theoretic ones.

Clearly, any single object can be circumscribed. Therefore, there are pluralities made up of only single objects, i. e. singleton pluralities.

(Singleton) $\forall x \exists yy \forall z (z \prec yy \leftrightarrow z = x)$.

Adding another object to a plurality again yields a plurality. Hence, they accept a principle of adjunction, i. e. adding a thing x to a plurality yy yields the new plurality $yy + x$ defined by:

(Adjunction) $\forall z (z \prec yy + x \leftrightarrow z \prec yy \lor z = x)$.

Given that one has a circumscribed plurality and a clear criterion to distinguish the members of this plurality as either satisfying the

8 It seems that they use these terms interchangeably as they write, for example, "every plurality must be extensionally definite, or properly circumscribed" (Florio and Linnebo, 2021, p. 12), "the notion of being properly circumscribed will play an important role and will be analyzed under the label of *extensional definiteness*" (Florio and Linnebo, 2021, p. 64) and "[w]e thus ask what it is for a collection to be circumscribed or extensionally definite" Florio and Linnebo, 2021, p. 280. Thus, I will also do so.

9 These are very closely connected, though, since pluralities are sometimes used as domains instead of sets when one wants to represent universal domains. See Florio and Linnebo, 2021, pp. 130–135.

criterion or not, one gets a plurality of all and only those objects that either satisfy the criterion or those that do not. That is, given a plurality xx and a condition $\varphi(x)$ such that there is at least one member in xx which satisfies φ, there is a plurality yy consisting of those members of xx that satisfy it. On these grounds, Florio and Linnebo also accept a plural separation principle.

(Separation) $\exists x(\varphi(x) \wedge x \prec xx) \rightarrow \exists yy \forall z(z \prec yy \leftrightarrow z \prec xx \wedge \varphi(z)).$

The next axiom ensures that two pluralities can be, so to speak, put together, i. e. it is a plural binary union axiom. The justification for it comes from an intuitively plausible idea. Suppose one has two circumscribed pluralities, putting them together forms a single plurality which is again circumscribed.[10]

(Union) $\forall xx \forall yy \exists zz \forall z(z \prec zz \leftrightarrow z \prec xx \vee z \prec yy).$

So far, these principles do not entail the existence of any infinite plurality and axiom of infinity is added. As Florio and Linnebo themselves admit, it is far from clear whether an infinite collection can be circumscribed or not. Recall that only if it can be circumscribed, there is a corresponding plurality.

In order to give some sort of justification for this axiom, Florio and Linnebo start out by considering the natural numbers. They use the fact that every natural number immediately precedes another.

$\forall x \exists y \mathtt{Pred}(x, y).$

They then wonder whether the natural numbers as a whole can be circumscribed or not. That is, are there some objects xx such that they contain 0, are closed under $\mathtt{Pred}$ and can be circumscribed? Put differently, they wonder if the following axiom should be adopted.

(Infinity) $\exists xx(0 \prec xx \wedge \forall x \forall y(x \prec xx \wedge \mathtt{Pred}(x, y) \rightarrow y \prec xx)).$

While accepting that such a plurality exists is a substantial assumption, Florio and Linnebo accept this analogue of the set-theoretic axiom of infinity because of its great success in mathematics. So for this axiom they rely entirely on abductive grounds for its justification and not on any intuitive justification.[11]

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10 A generalized plural union principle is adopted as well (cf. Florio and Linnebo, 2021, p. 281).

11 They note that one could take issue with the principle above as it is specifically about the natural numbers and therefore lacks the topic neutrality usually associated with logic. Their response is that a similar argument can be given that is topic neutral. So nothing depends on the specific nature of the natural numbers (cf. Florio and Linnebo, 2021, pp. 282–283).

To complete the set of axioms, an analogue of the axiom of replacement is adopted. The intuitive justification for it is that, given a circumscribed plurality, one can change or keep the members of this plurality. By doing this, one gets a circumscribed plurality again.

$$\text{(Replacement)} \quad \forall xx(\forall x(x \prec xx \rightarrow \exists! y\psi(x,y)) \rightarrow \exists yy\forall y(y \prec yy \leftrightarrow \exists x(x \prec xx \wedge \psi(x,y))).$$

Putting all of this together shows which constraints a collection must fulfil in order to be a plurality or, to put it differently, shows one how to build pluralities from already existing ones.

The most striking difference between traditional plural logic (2.1) and critical plural logic is that unrestricted plural comprehension has been rejected and with it also the universal plurality.

Before moving on to Florio and Linnebo, 2021's argument against unrestricted plural comprehension, it is helpful to take stock of what has been done so far in this chapter. After the brief presentation of plural logic (2.1), we moved on to the topic of unrestricted quantification (2.2) and I established a connection between plural logic and unrestricted quantification. In (2.3), we then looked at the rigidity principles of sets and of pluralities. These principles express the thought that a set or a plurality have exactly the same elements or members at each possible world where they exist. This was contrasted with groups and mereological fusions which do not have their members or parts by necessity. The difference was explained by appealing to the fact that sets and pluralities are constituted by their elements or members, i. e. they are nothing over and above them.

We then moved on to first consider an argument for the rigidity of sets (2.3.1) and afterward the argument for the rigidity of plurals was discussed (2.3.2). In both cases the conclusion was that one should accept the respective rigidity principles. In this section the notion of extensional definiteness/indefiniteness or circumscribability was clarified. With all of this in place, I now present Florio and Linnebo, 2021's argument against a universal plurality. This is tantamount to rejecting unrestricted plural since it entails the existence of the universal plurality.

2.5 UNIVERSAL PLURALITY

One of Florio and Linnebo's aims in the book, is to use pluralities to better understand sets. They state that "every plurality is properly circumscribed and can thus figure as an argument of the 'set of' operation" (Florio and Linnebo, 2021, p. 72). This 'set of' operation, (SO), maps a plurality, xx, to its corresponding set, $\{xx\}$. That is, $xx \mapsto \{xx\}$ for any plurality.

The need or want for this kind of singularization, i. e. collecting many things together into one thing, is motivated two things. First, by the just mentioned desire to understand sets via pluralities. Second, by taking into account the theoretical success of such singularizations as evidenced by set theory or the linguistic analysis of plural and mass terms using mereology (cf. Florio and Linnebo, 2021, pp. 3, 62–63, 66, 85–90).

So Florio and Linnebo accept that an absolute interpretation of the quantifiers is possible and also accept that universal singularizations are possible. They reject, as mentioned before, unrestricted comprehension. They cannot have the three of them together, as it would lead to an inconsistent theory. One obvious reason is that unrestricted comprehension entails the existence of the universal plurality and by (SO) it would entail the existence of a universal set.

Furthermore, the *Plural Cantor* theorem, which is discussed in (2.5.1), also poses a problem. Their view circumvents the problem posed by the Plural Cantor theorem since they restrict what pluralities there are, as discussed in (2.4). In particular, the existence of the universal plurality is not entailed by their view.

In addition to (SO), they have two bridge principles which establish a connection between set theory and plural logic (cf. especially Florio and Linnebo, 2021, pp. 60–64). The first one establishes a relation between the identity of sets and coextensive pluralities. It states that two sets are identical just in case the pluralities they contain are coextensive:

$$\{xx\} = \{yy\} \text{ if and only if } xx \approx yy$$

The second one establishes a relation between 'being an element of' in set theory and 'being a member of' in plural logic. It states that something is an element of a set just in case that thing is a member of the plurality the set contains:

$$a \in \{xx\} \text{ if and only if } a \prec xx$$

With these principles in place, the connection between pluralities and sets now is a very close one. The main difference being that the former are about many things, whereas the latter are only about one thing.

As mentioned, one singularization that is impossible is that of a universal plurality as the corresponding universal set would lead to a contradiction.[12] By rejecting unrestricted plural comprehension, the existence of the universal plurality is not entailed. But now the issue is to "explain why there are no pluralities that are so large that they cannot be singularized" (Florio and Linnebo, 2021, p. 244). This is where Florio and Linnebo's argument comes in, but before, we take a closer look at the Plural Cantor theorem.

12 If there were a universal set in ZF(C), one would get the Russell's paradox by applying separation with the usual condition $y \notin y$ to it.

2.5.1 *Plural Cantor*

Another result that restricts what sort of singularizations are possible is the Plural Cantor theorem. As the name suggests, it is the analogue of *Cantor's Theorem* for plural logic. I. e. :

> For any plurality xx with two or more members, the subpluralities of xx are strictly more numerous than the members of xx. (Florio and Linnebo, 2021, p. 41)

This can be rephrased as the claim that there is no surjective function f from xx to its subpluralities.[13] With the aid of new variables standing in for functions from objects to pluralities, this function can be formally defined as:[14]

$$\forall yy(yy \preccurlyeq xx \rightarrow \exists x(x \prec xx \wedge f(x) \approx yy))$$

No such function can exist in the case of the universal plurality (cf. Florio and Linnebo, 2021, pp. 43–44). Suppose the universal plurality exists, then this result entails that there are more pluralities than there are objects: there are $2^n - 1$ pluralities, but only n objects. This is problematic since "it is impossible to assign to each plurality a distinct object" (Florio and Linnebo, 2021, p. 42).[15]

The motivation behind this requirement is, as mentioned before, that one wants to be able to assign single objects to pluralities in order to also talk about them as a single thing (as sets or mereological fusions, for example) in order to retain the theoretical benefits and respect the scientific practice in mathematics and linguistics (cf. Florio and Linnebo, 2021, pp. 1, 59–60 75).

The Plural Cantor theorem only poses a problem when one assumes the following three things:

(1) Absolute generality,

(2) Traditional plural logic, specifically the unrestricted plural comprehension scheme,

13 Since there is no empty plurality, we know that any plurality xx has $2^n - 1$ subpluralities, where n is the number of members of the plurality.

14 The newly introduced function takes an object and maps it to one or more objects, i. e. pluralities.

15 A similar problem has already been pointed out by Yablo, 2006, pp. 149–150 who analyzes the issue of a universal set and writes that

> [t]he problem is that U [the universal set] is an example of a *type* of set *all* of whose instances should belong to U. I refer, of course, to U's subsets. U being universal ought to contain all of these. But it can't, because a set has more subsets than members ... This is the logical core of Cantor's Theorem, and it gives what I take to be the real problem with a universal set: it would have to contain a distinct object for each plurality of objects (viz. the set of objects), and that is logically impossible.

(3) Universal singularizations.

The first two assumptions entail the existence of the universal plurality and the third one entails the existence of a surjective function from pluralities to objects, which is impossible.

If one rejects absolute generality, then one only gets the universal plurality for some restricted domain d^* which turns out to not be universal. Hence, the objects outside the restricted domain d^* can be used as proxies serving as assignments for the pluralities that have not yet been assigned an object. So in this case, there can exist a surjective function from pluralities to objects, assuming that there are enough objects outside of d^*.

Given Florio and Linnebo's view, however, this way of resolving the inconsistency is blocked and hence they have to give an argument as to why the universal plurality does not exist.

2.5.2 *Definite and Indefinite Domains*

One initial question that might arise is the following: Why is an argument even needed to restrict plural comprehension? Why not just restrict it? The main issue here is that any such restriction seems arbitrary and unmotivated. Recall how in (2.2.2) this charge in the case of set theory was resolved by appealing to the iterative conception of set. As we will see in the case of mereology (3.3), restrictions on *Unrestricted Fusion* are also faced with the charge of being arbitrary or not doing justice to our intuitions. This is where Florio and Linnebo, 2021's argument comes in: they take it to show that in the case of pluralities, there is a well-motivated and non-arbitrary way of restricting unrestricted plural comprehension.

Their approach derives from the thought that domains of quantification can be extensionally indefinite. In such a case, a condition which is satisfied by every object whatsoever, like self-identity, yields the universal plurality given traditional plural logic. The issue now is that the universal plurality is also extensionally indefinite since, by definition, every element of the domain is a member of it. If it were definite, then it could figure as an argument of the 'set of' operation and would be mapped to a set which did not exist before. Hence, not every member of the domain would be a member of the universal plurality, contradicting its universality.

The fact that the universal plurality is extensionally indefinite is contrary to the claim that every plurality is extensionally definite or circumscribable. I. e. the universal plurality in this case violates plural rigidity according to which pluralities cannot vary in membership. Therefore, the assumption that the universal plurality exists leads to a contradiction and is rejected. Hence, unrestricted plural comprehension has to be restricted and the rigidity of pluralities provides a reason for this restriction.

An example of such an extensionally indefinite domain is an absolutely general one, i. e. an all-encompassing one which is "of particular interest to philosophers" (Florio and Linnebo, 2021, p. 295). So according to them, absolute generality makes the range of the quantifiers extensionally indefinite (cf. Florio and Linnebo, 2021, pp. 244–245). The question that has to be addressed now is the following: What does it mean for a domain to be extensionally indefinite?

Since their idea derives from Stephen Yablo, it is worth citing his informal description of a genie constructing sets according to a single simple instruction:

> The set of Xs is always additional to the Xs. So in the set case, *the genie's work is never done.* This is not because set-creation is so intrinsically time consuming, but because whatever you might propose as the stopping point affords the genie materials for adding something new. The instruction more fully stated is
>
> (*) whenever you have made some things, form their set, *continuing forever.*
>
> ... 'Continuing on forever' means, and you can consider this stipulative, 'anything a faster-moving genie *could* make, you eventually *do* make.' (Yablo, 2006, p. 153)

Clearly, an absolutely general domain should, in particular, include all sets. Given the instruction that whenever some things are made, the genie also forms their set means that this process will never end. Hence, the domain can always be extended in this sense: it is extensionally indefinite or not circumscribable. This is very similar to the Dummettian notion of an indefinitely extensible concept introduced in (2.2.2).

Florio and Linnebo, 2021's idea is closely related to the above because they postulate that to every plurality there is a corresponding set, as mentioned in (2.5). While a plurality is 'nothing over and above' its constituent objects, the objects still have to be circumscribed for the plurality to exist. Given some metaphysical views, reality as a whole cannot be circumscribed. Consequently, there cannot be a universal plurality as this "would require circumscribing something uncircumscribable. It follows that the plural comprehension scheme must be restricted" (Florio and Linnebo, 2021, pp. 268–269). Let us now take a look at their argument that shows how a restriction on unrestricted plural comprehension can be motivated.

> To define a plurality, we need to circumscribe some objects. But when we circumscribe some objects, we can use these objects to define yet another object, namely their set, in a way that would not be possible were the objects in question not circumscribed. And since yet another object

> can be defined, it follows that the circumscribed objects cannot have included *all* objects. Thus, reality as a whole cannot be circumscribed: there is no universal plurality. Consequently, the plural comprehension scheme needs to be restricted. (Florio and Linnebo, 2021, p. 276)

Florio and Linnebo, 2021, p. 279 write that this argument "hinges on the idea that every plurality is circumscribed" which is correct, but hides the fact that the bridge principles between pluralities and sets is just as important.[16] Therefore it is better to say that it relies on at least two crucial assumptions, (UniSing) and (Rigidity):[17] first, to every circumscribed plurality there is a corresponding set that is the result of the 'set-of' function (2.5); second, every plurality is circumscribed or extensionally definite (2.3.2), i. e. cannot vary in membership.

Without (UniSing), there would not be a new object when circumscribing a plurality and thus the universal plurality would contain everything there is.[18] Without (Rigidity), the constituent objects of a plurality would not have to be circumscribed in order for the plurality to exist. Consequently, (UniSing) would not apply since the 'set-of' function only applies to circumscribed pluralities. In both cases, no new objects come into existence that contradict the definition of the universal plurality as something that has everything as a member.

Note how similar this idea is to that of an indefinitely extensible concept: in constructing a universal plurality, new objects come into existence by the above principles. These new objects also have to be members of the universal plurality by definition, but they cannot be as they were not available before. Hence, the universal plurality does not contain all objects which is a contradiction.

Furthermore, Florio and Linnebo take this argument as support for their claim that an absolutely general domain is extensionally indefinite since it cannot be definite by the above reasoning (cf. Florio and Linnebo, 2021, p. 277).

The argument also shows why unrestricted plural comprehension has to be restricted. Suppose some domain d is absolutely general and hence extensionally indefinite. Unrestricted plural comprehension entails the existence of a universal plurality such that all objects of d are

16 This is also the reason why an analogous argument in mereology cannot easily be made since such a bridge principle, (UniSing), is a substantial assumption. This will be discussed in detail in (5).

17 The third assumption, (UnrQuan), is in the background as well. This ensures that the universal plurality should also include all sets, so in particular the new set as well.

18 The step from a definition to existence for mathematical objects is due to Florio and Linnebo, 2021's liberal view of definitions for which they cite Cantor, Hilbert, and Poincaré as precursors. Florio and Linnebo state that "it suffices for a mathematical object to exist that an adequate definition of it can be provided" (Florio and Linnebo, 2021, p. 273). For an explanation of what an adequate definition consists in see Florio and Linnebo, 2021, pp. 272–275.

members of it. Since the domain is extensionally indefinite, the universal plurality is extensionally indefinite as well because it would not be universal otherwise. Recall, though, that every plurality is extensionally definite and therefore cannot vary in membership. Hence, there cannot exist a universal plurality if the domain is absolutely general. To block the existence of a universal plurality, unrestricted plural comprehension is restricted and instead the axioms introduced in (2.4) are adopted (cf. Florio and Linnebo, 2021, pp. 278–284).

Lastly, the argument provides a well-motivated and non-arbitrary restriction on unrestricted plural comprehension. The restriction is motivated by the thought that domains can be extensionally indefinite. Unrestricted plural comprehension entails the existence of a universal plurality that is extensionally indefinite. This contradicts the fact that pluralities are extensionally definite or circumscribable, i. e. (Rigidity).

To sum up, Florio and Linnebo, 2021 take the argument to show three important things:

(1) An absolutely general domain is extensionally indefinite;

(2) In an absolutely general domain, a universal plurality would also be extensionally indefinite contradicting the fact that pluralities are extensionally definite;

(3) Unrestricted plural comprehension has to be restricted on grounds of (2) which provides a motivated and principled way of doing so.

We will now move away from plural logic and focus on mereology and the necessary steps it takes to reconstruct the above argument in that setting.

This reconstruction is guided by the four assumptions Florio and Linnebo make and the identification of the mereological analogues of them. While we encountered them already in (1), it is convenient to list them here again:

(UnrQuan) Unrestricted quantification, i. e. there is an all-encompassing domain,

(UnrComp) Unrestricted plural comprehension, i. e. for any condition there is a plurality consisting of all and only those objects that satisfy the condition (entails the existence of a universal plurality),

(UniSing) Universal singularization, i. e. there is a ‘set of’ function that maps any given plurality to an object (its set),

(Rigidity) Pluralities are rigid, i. e. they have the same members at every possible world at which they exist.

In the next chapter, (UnrQuan), (UnrComp), and (UniSing), are addressed in the case of mereology. (UniSing) is only briefly discussed and I return to it in (5).

3 MEREOLOGY AND UNRESTRICTED QUANTIFICATION

In this chapter, mereology as a formal theory is introduced with a focus on its role in ontology and not as an alternative to set theory (3.1, 3.2). Furthermore, the philosophical motivation behind its most controversial axiom, Unrestricted Fusion, is explained by looking at some of its prominent defenders (3.3). Lastly, a connection between the absolutist position regarding unrestricted quantification and mereology is established (3.4).

3.1 MEREOLOGY AS A (FORMAL) THEORY

In very general terms, mereology is about parthood relationships, i. e. that of part to whole and also of parts within a common whole. While such considerations can be traced back to the beginning of philosophy, the important development for this thesis happened at the beginning of the twentieth century (cf. Cotnoir and Varzi, 2021, p. 2).

This marks the first time, the parthood relation itself came to the forefront and was investigated in a systematic fashion. This development is usually attributed to Edmund Husserl and Stanisław Leśniewski, although mereology was employed for different aims by them. Husserl used it as a tool in formal ontology, whereas Leśniewski used it as a nominalistically acceptable alternative to set theory eschewing its abstract notion of 'set'.

3.1.1 *Ontology and Set Theory*

For Husserl, mereology was an important aspect in his formal ontology understood by him as "lay[ing] bare the formal structure of what there is *no matter what it is*" (Cotnoir and Varzi, 2021, p. 6). So irrespective of whether an entity is concrete or abstract (or anything else), it still has to exhibit certain features and is subject to general laws. The task of formal ontology is to find out what these features and laws are. One such law for Husserl is the transitivity of parthood, i. e. if x is part of y and y is part of z, then x is part of z. He did not, however, develop a full account of mereology, but he envisaged it as a "complete law-determined survey of the *a priori* possibilities of complexity in the form of wholes and parts, and an exact knowledge of the relations possible in this sphere" (Husserl, 1900–1901, p. 484). It is this sense – mereology as a (tool in) formal ontology – that is important for this thesis and not in the sense Leśniewski understood it.

Leśniewski, on the other hand, developed mereology to serve as a nominalistically acceptable alternative to set theory in order to get rid of the abstract notion of 'set'. That is, he envisaged mereology to serve as a foundation for mathematics in the same way set theory does, i. e. all mathematics should, in theory, be reducible to it. For this reason, Leśniewski pursued an axiomatic approach according to which parthood is asymmetric, transitive, and thus also irreflexive.[1] Hence, parthood constitutes a strict partial order just like the strict subset-relation in set theory. Furthermore, he also proposed axioms that expressed identity in terms of parts and defined the fusion of an object (cf. Leśniewski, 1916, p. 131). Leśniewski also proved analogues of several set-theoretic results in his mereological system such as Cantor's Theorem – although it has an additional condition which is important for considerations of a universal object (5.1.2) – and that Russell's paradox does not arise.

The sense in which mereology is considered in this thesis is to pursue Husserl's aim, i. e. that of formal ontology, by ways of Leśniewski, i. e. an axiomatic approach.[2] So considerations whether or not mereology can serve as a foundation for mathematics are not of importance here.[3]

Mereology has been a very widely used tool in contemporary metaphysics and ontology, although many of its axioms and implications have been challenged.[4] In order to avoid any confusion that arises within natural language and to stay true to the axiomatic approach, classical mereology is introduced in the formal setting of first-order logic with identity in the next section.

3.2 AN AXIOMATIZATION OF CLASSICAL MEREOLOGY

Since there are many different, but formally equivalent, axiomatizations of mereology, I will use the one which is most widespread in the literature.[5] Before the axioms are introduced, a quick aside on notation: I abbreviate improper parthood by 'P', proper parthood by 'PP', overlap by 'O', etc. and not use any non-alphabetic symbols for

1 Leśniewski's axioms, and even the primitive notions, differed from publication to publication. As we will see, these differences result in the same mereology considered formally.

2 Interestingly enough, these different aims of Husserl and Leśniewski are paralleled by those of Simons, 1987 and Lewis, 1991.

3 Although the general consensus is that Leśniewski's mereology is too weak to serve as a foundation for mathematics since every model of mereology can be transformed to a Boolean algebra by adding a zero element. See Urbaniak, 2014 for Leśniewski especially. For an interpretation of mereology within set theory, see Hamkins and Kikuchi, 2016.

4 For the use of mereology within ontology see, for example, Simons, 1987, Meixner, 1997, Sider, 2001, and Koslicki, 2008.

5 This axiomatization corresponds to Hovda, 2009's First Way. His paper also offers some other possible ways of axiomatizing classical mereology.

mereological relations. I take improper parthood as a primitive and then define the other notions like overlap, disjointness, etc. in terms of it. Lastly, Pxy should be read as 'x is an (improper) part of y'.

As mentioned before, the improper parthood relation in mereology is taken to have to formal properties of a partial order. That is, improper parthood satisfies the following three axioms:

(A1) $\forall x Pxx$

(A2) $\forall x \forall y((Pxy \wedge Pyx) \rightarrow x = y)$

(A3) $\forall x \forall y \forall z((Pxy \wedge Pyz) \rightarrow Pxz)$

Thus, everything is part of itself by (A1). By (A2), two objects sharing all their parts are identical. So this is an extensionality principle formulated in terms of parthood (cf. Cotnoir and Varzi, 2021, p. 73). Lastly, (A3) specifies that if something x is part of another thing y and y is part of some thing z then x is also part of z. While anti-symmetry and transitivity have been challenged by different authors, in classical mereology these are taken to be constitutive of improper parthood.[6]

With that in place, the following predicates can be defined:

(D1) $PPxy := Pxy \wedge x \neq y$

(D2) $Oxy := \exists z(Pzx \wedge Pzy)$

(D3) $Dxy := \neg\exists z(Pzx \wedge Pzy)$

$PPxy$ states that if something x is part of another thing y and they are not identical, then x is a proper part of y.[7] Oxy asserts that if there is some thing z that is part of both x and y, then these things overlap. Lastly, Dxy expresses that if there is no thing z that is both part of x and y, then these things are disjoint. (D3) is just the negation of (D2), but it makes for easier readability of the definition of fusion I adopt and hence is included. Although more predicates can be defined, this constitutes the basic mereological vocabulary for us.

Going forward, one has to decide on what kind of supplementation principle should hold for the objects in question. These principles concern the decomposition of objects, i. e. when does an object have proper parts? What is the relation of the proper parts to the whole? And so forth. Furthermore, some sort of supplementation principle

6 For arguments against anti-symmetry see Tillman and Fowler, 2012, Cotnoir and Bacon, 2012 and for arguments against transitivity see Rescher, 1955, Winston et al., 1987, and Fiorini et al., 2014.

7 This is the so-called *non-identity conception* of proper parthood. There is also another definition of proper parthood called the *outstripping conception* (D1′) $PPxy := Pxy \wedge \neg Pyx$. In the presence of (A2), these two conceptions coincide, however. To see this assume (D1), then by contraposition of (A2) we know that $\neg Pxy \vee \neg Pyx$. By assumption the first disjunct is false and hence the second has to be true. Thus, we have $Pxy \wedge \neg Pyx$ as desired. Assume (D1′), then y has a part, namely itself, which is not a part of x, hence $x \neq y$ by Lebniz's law. Thus, we have $Pxy \wedge x \neq y$ as desired. Cf. Cotnoir, 2018.

is needed to rule out unwanted models. The two most prominent of these supplementation principles in the literature are *Weak Supplementation* and *Strong Supplementation*:

$$(WSP) \quad \forall x \forall y (PPyx \to \exists z (Pzx \wedge Dzy))$$
$$(SSP) \quad \forall x \forall y (\neg Pxy \to \exists z (Pzx \wedge Dzy))$$

(WSP) asserts that if something x has a proper part y then it has another part z that is disjoint from y. (SSP) asserts that if something x does not have something as a part y, then it has a part z that is disjoint from y. Given (A1)–(A3), (SSP) implies (WSP).[8] I adopt (SSP) here which, together with the partial ordering axioms, entails that two composite objects having the same parts are identical from the viewpoint of mereology:

$$(\mathrm{Ext}) \quad \forall x (\exists w PPwx \to \forall y (\forall z (PPzx \leftrightarrow PPzy) \to x = y))$$

Now one further axiom is needed to arrive at classical mereology that states that fusions exist. Before, we first need to define what a fusion is. There are different definitions, but in order to get classical mereology given our prior decisions, one has to adopt the following:[9]

$$(D4) \quad F_{\varphi} z := \forall y (Oyz \leftrightarrow \exists x (\varphi x \wedge Oyx))$$

Thus, a fusion z of φ's is something that overlaps exactly those things that overlap at least one φ. By extensionality, a fusion is unique (cf. Cotnoir and Varzi, 2021, p. 189 for this definition of fusion). So it makes sense to talk of the fusion of φ's.

It is worth mentioning that fusion is a singularization operation. I. e. the fusion operation takes the many as one just like set theory does.[10] So it seems that this is the mereological analogue of (UniSing). However, as I will discuss in (5.1) and (5.2.1), this is not strong enough for Florio and Linnebo's argument to work in the mereological setting. That is, the 'set of' operation is needed in mereology as well for the argument to work. Therefore, it would be misleading to identify the fusion operation as the mereological analogue of (UniSing).

The last thing needed is an axiom scheme that states that whenever some φ's exist, there is a fusion of them:

$$(A4) \quad \exists x \varphi x \to \exists z F_{\varphi} z$$

8 To see this, assume (SSP) and the antecedent of (WSP), then by $PPyx$ and asymmetry of 'PP' we get $\neg PPxy$. Furthermore, since $x \neq y$ we get that $\neg Pxy$. Thus, $\exists z (Pzx \wedge Dzy)$ as desired. Without (A2) one does not get that 'PP' is asymmetric which is needed in the proof. Thus, when (A2) is rejected (SSP) does not entail (WSP).

9 See Cotnoir and Varzi, 2021, p. 109 for an explanation of this claim. The definition of fusion goes back to Goodman, 1977 and is what Cotnoir and Varzi, 2021 call Fusion".

10 With the exception of the fusion of one object, of course.

Classical mereology does not place any requirements on when some things, given that they exist, have a fusion. So it is like unrestricted plural comprehension in that regard. Consequently $(A4)$ is called *Unrestricted Fusion* or *Unrestricted Composition* in the literature.[11] This, then, is the mereological analogue of (UnrComp).

Unrestricted Fusion is not uncontroversial, however, and the question under which conditions things have a fusion is known as the *Special Composition Question*.[12] The answer offered by classical mereology is that, as long as the thing(s) exist, there is a fusion of them. This position is known as *Universalism*.[13]

By substituting something for φ that is satisfied by everything, such as self-identity, Unrestricted Fusion entails that there is a fusion z of everything that satisfies this formula. Since everything satisfies it, z is the universal fusion which is usually abbreviated by u. This fact will be important for the argument in (5) since the existence of u is rejected and therefore also Unrestricted Fusion has to be restricted.

To sum up, the axiomatization of classical mereology I adopt consists of five axioms: the partial order axioms $(A1)$–$(A3)$, the supplementation axiom (SSP), and the Unrestricted Fusion axiom $(A4)$.[14] Because Unrestricted Fusion and the fact that it entails the existence of the universal fusion u is of central importance for this thesis, the philosophical motivation behind it is explained in the next section.

3.3 THE PHILOSOPHICAL MOTIVATION FOR UNRESTRICTED FUSION (UF)

Since Unrestricted Fusion is – as mentioned before – quite controversial and the mereological analogue of (UnrComp), it is illuminating to look at how philosophers have motivated it. For this reason, I look at Nelson Goodman, David Lewis, Achille Varzi, and Maegan Fairchild and John Hawthorne.

3.3.1 *Goodman on UF*

Goodman developed a system that he called the 'calculus of individuals'. This calculus is "formally indistinguishable" (Leonard and Goodman, 1940, p. 46) from Leśniewski's mereology. Note that Leśniewski's mereology and the calculus of individuals are equivalent to classical

11 I will use the former throughout the thesis and also write that some things fuse something instead of some things compose something.

12 Critics of Unrestricted Fusion include van Inwagen, 1990, Bøhn, 2009, and Korman, 2015 among others.

13 This view is sometimes also called *Conjunctivism* (van Cleve, 1986).

14 It is worth mentioning that classical mereology does not take a stance on whether or not reality ultimately consists of atoms or not. Therefore, either an axiom can be added that asserts reality is atomic or an axiom can be added that reality is not atomic. For a discussion of this see Cotnoir and Varzi, 2021, pp. 142–158.

mereology and only differ in their choice of the primitive notions and axioms (cf. Cotnoir and Varzi, 2021, pp. 21, 45–47, 47–49). In the paper A WORLD OF INDIVIDUALS he explicitly addresses an objection levelled against Unrestricted Fusion and writes:

> To keep the rule of nominalism by generating wholes, rather than classes, of individuals costs as much as it pays; for it often means forcing the imagination to accept as individuals some scattered or heterogeneous conglomerations that are never in practice recognized as single units[.] (Goodman, 1972, p. 165)

Goodman's reply to this objection is simply that the terminology of a system (in his case, 'part', 'whole', and 'individual') are irrelevant. Consequently, any intuitions that hinge on the use of these words with regards to what is and what is not acceptable as a whole are insubstantial.

One can replace these terms by whatever one feels comfortable with and still end up with the system Goodman envisages. It only has to satisfy his nominalistic principle that "no two distinct entities have ... exactly the same atoms" (Goodman, 1972, p. 166). Here, an atom is something that has no proper part in the sense defined above, i. e. $Ax := \neg\exists y(PPyx)$.

3.3.2 *Lewis on UF*

Moving on to Lewis who defended Unrestricted Fusion in ON THE PLURALITY OF WORLDS and PARTS OF CLASSES. In the former, he boldly states that "mereological composition [fusion] is unrestricted" (Lewis, 1986b, p. 211) and makes it clear in a footnote that he takes it to be "*absolutely* unrestricted" (Lewis, 1986b, p. 212). This directly establishes a connection with the topic of unrestricted quantification as discussed in (2.2) and I explore it further in (3.4).

Lewis' argument for Unrestricted Fusion is indirect since he considers restrictions on Unrestricted Fusion which, according to him, lead to an absurd conclusion and hence he keeps Unrestricted Fusion.

3.3.2.1 *On the Plurality of Worlds*

It is worth looking at the argument in more detail. Lewis claims that Unrestricted Fusion "cannot be restricted in accordance with our intuitions" (Lewis, 1986b, p. 212). This is so because "[i]t is a vague matter whether a given class satisfies our intuitive *desiderata* for composition [fusion]" (Lewis, 1986b, p. 212). According to Lewis, the desiderata used in order to decide whether a class of objects fuses something are similarity with other objects in the class, being adjacent to each other, sticking together, and acting together (cf. Lewis, 1986b, p. 211).

Given a class of objects, it is a vague matter to decide if they satisfy the desiderata or not. Consequently, it is vague whether there is a fusion of these things and hence restricting Unrestricted Fusion according to our intuitions amounts to a vague restriction. Thus, given that Unrestricted Fusion obeys a vague restriction, fusion is itself vague, i. e. "it must sometimes be a vague matter whether composition [fusion] takes place or not. And that is impossible" (Lewis, 1986b, p. 212). It is impossible for Lewis because, for him, vagueness is to be located in language and thought and not in ontology.

Furthermore, certain parts of language are not vague and, crucially, this part includes the things necessary to settle the question whether fusion takes place or not. Hence, this question cannot have a vague answer: there either is a fusion or there is not and because of this, no restriction on Unrestricted Fusion can be vague. This is at odds with the intuitively compelling assumption that it is a vague matter whether something satisfies our desiderata for fusion or not and, consequently, "no restriction on composition [fusion] can serve the intuitions that motivate it" (Lewis, 1986b, p. 213).

So any such restriction on Unrestricted Fusion would be arbitrary. Recall that the issue of arbitrariness has already come up in (2.2.2) in the case of set theory. Furthermore, if Unrestricted Fusion were to be restricted by vague conditions, it would be "indeterminate what exists" (Lando, 2017, p. 183). For that reason Lewis concludes that fusion is unrestricted (cf. Lewis, 1986b, pp. 212–213).

3.3.2.2 *Parts of Classes*

Five years later in PARTS OF CLASSES Lewis states that "there is no good independent reason to restrict composition [fusion]" (Lewis, 1991, p. 19) which goes back to his argument described above, but he also advances a new argument in favor of Unrestricted Fusion. The new argument is closely connected to the idea that 'the fusion is nothing over and above its parts' (Cf. Lewis, 1991, p. 80).[15] Thus, in order to adequately describe a fusion it is enough to describe its parts because "[i]ts character is exhausted by the character and relations of its parts." (Lewis, 1991, p. 80)

Lewis admits that some fusions one is committed to are quite different from ordinary objects, but this has no bearing on their existence. Being a fusion of something is "coextensional with existence" (Lando, 2017, p. 179) and since its parts exist, so trivially does the fusion. He then repeats the argument from ON THE PLURALITY OF WORLDS that any restriction of Unrestricted Fusion is vague, but existence does not come in degrees. Thus, the vague line between weird fusions and or-

15 This idea is called the ontological innocence of mereology in the literature and has been contested by various people. See e. g. van Inwagen, 1994 and Yi, 1999.

dinary ones and the sharp line between existence and non-existence cannot coincide (cf. Lewis, 1991, pp. 80–81).[16]

Important for this thesis is the fact that in both books, Lewis then goes on to write that one should "[r]estrict quantifiers, not composition [fusion]" (Lewis, 1986b, p. 213) and similarly "[i]f. . . you quantify subject to restrictions, then you can leave it [the weird fusion] out" (Lewis, 1991, p. 80). That is, he accepts Unrestricted Fusion and restricts quantifiers to salient domains in a context (cf. Lewis, 1986b, p. 213).

Hence, Lewis keeps classical mereology and restricts quantifiers in a given case because, he claims, the restriction on quantifiers can be vague unlike that on Unrestricted Fusion (cf. Lewis, 1991, p. 81).

3.3.3 *Varzi on UF*

Varzi's defense of Unrestricted Fusion takes his conviction as its starting point that the universe, the universal fusion u, exists as an individual. He then considers what kind of ontological category this individual belongs to. Unless one assumes that there only is one ontological category, answering this question is not straightforward. The individual parts of the universe belong to different ontological categories, so to which of these categories should the universe belong to?

According to Varzi, this question occurs for many more fusions than just the universal one. Given Unrestricted Fusion, there are many "transcategorial sums [fusions]" (Varzi, 2006, p. 111). I. e. fusions whose parts are from different ontological categories. These transcategorial fusions then belong to a different category than any of its (single-category) parts, though this new category might not be as natural as the common ones.

Varzi briefly considers the option of taking *mereological fusion* as an ontological category, but dismisses it. Rather, he opts for the idea that the fusion is 'nothing over and above' its parts (cf. Varzi, 2006, pp. 110–112). His argument is similar to the one by Lewis we have just seen from Parts of Classes as Varzi writes that:

> [A] thing is identical with the mereological sum [fusion] of its constituent parts. I also share the view that a thing is 'nothing over and above' its parts, hence I don't think the question of the existence of a mereological sum [fusion] makes much sense in case we already agree on the existence of the pieces . . . I hold this to be true of 'natural', categorially homogeneous sums [fusions], such as the sum [fusion] of Chisholm's left foot and the rest of his body, as

16 These arguments are not uncontroversial. Especially in cases like inter-world or abstract-concrete fusions, no vagueness seems to be present and hence composition could be restricted in a non-arbitrary way in these cases (cf. Lando, 2017, p. 187.

> well as hybrid transcategorial sums [fusions], such as the sum [fusion] of Chisholm's left foot and Sebastian's stroll. (Varzi, 2006, pp. 111–112)

Consequently, the existence of a fusion is no further ontological commitment given one accepts the existence of its parts. Furthermore, Varzi also echoes Lewis' thought that, while one might think certain fusions are not salient or natural, this has no bearing on the ontological question of their existence (cf. Varzi, 2006, p. 113).

3.3.4 *Fairchild and Hawthorne on UF*

Fairchild and Hawthorne's defense of Unrestricted Fusion appears in the context of a more general argument against conservative positions within metaphysics. They characterize conservative positions as accepting the existence of "'ordinary objects' like trees, dogs, and snowballs, but deny[ing] the existence of 'extraordinary objects', like composites of trees and dogs ('trogs')" (Fairchild and Hawthorne, 2018, p. 45).

They themselves are permissivists and endorse Unrestricted Fusion, which they call Universalism, and offer three lines of arguments. I will only present the first two of them, however, since the third argument takes issue with a particular feature of Korman, 2015's account of objects, namely the role intentions play, and hence only applies to similar positions (cf. Fairchild and Hawthorne, 2018, pp. 47–48, 68, 73).

Their first argument is interesting since they explicitly state that "our central complaint here is not a 'vagueness' argument" (Fairchild and Hawthorne, 2018, p. 66). They criticize Korman's idea that "making a physical object requires substantial physical alteration" (Korman, 2015, p. 155) on the grounds that the category of *substantial physical alteration* cannot serve as a marker of an ontological divide. While Korman admits that it is difficult to specify what sort and threshold of alteration is required for a new object to exist, this is not what Fairchild and Hawthorne take issue with. I. e. their criticism is not that a certain criterion, like that of *substantial physical alteration* for example, might be vague and thus the question of whether or not an object exists becomes a vague question as well. So they clearly differ from Lewis in this respect.

Their complaint should rather be understood in the following way: The criterion that is used for the ontological divide, in this case *substantial physical alteration*, is "obviously not the kind of thing to constitute a special criterion of existence generation" (Fairchild and Hawthorne, 2018, p. 67). That is, the difference between being *substantially physically altered* and not being *substantially physically altered* is not a metaphysically special line according to which the existence of an object can be decided (cf. Fairchild and Hawthorne, 2018, pp. 66–67).

Fairchild and Hawthorne's second argument takes a naturalistic stance regarding objects as its starting point. They propose to take modern physics, and modern science in general, into consideration when one rejects extraordinary objects on the basis that they are scattered or causally disconnected. When considering ordinary things like tables, it seems that they are continuous, but given a microscopic image of a table, one can see that this is an illusion: the table is, in fact, also scattered (cf. Fairchild and Hawthorne, 2018, pp. 69–70).

The general argument is that many, but not all, of the often invoked contrasts between ordinary and extraordinary objects lose their force when one considers modern science. These contrasts then show themselves to be a matter of degree rather than of quality. Therefore, this kind of argument put forward by detractors of Unrestricted Fusion loses its force since the contrast between ordinary and extraordinary objects will be too weak to base ontological considerations on it (cf. Fairchild and Hawthorne, 2018, p. 71). So both arguments question whether certain criteria can do the ontological work that is usually attributed to them.

We have seen that Unrestricted Fusion enjoys support in the mereological literature. Furthermore, a number of different arguments are used to defend it. Because of this, Unrestricted Fusion is still widely accepted even given its controversial consequences. Thus, there is enough support for the mereological analogue (UnrFus*) of unrestricted plural comprehension (UnrComp).

3.4 MEREOLOGY AND ABSOLUTISM

After we have looked at how different authors have motivated Unrestricted Fusion, we now turn to the topic of how unrestricted quantification and mereology are related.

The connection is very similar to the case of plural logic (2.2) since Unrestricted Fusion entails the existence of the universal fusion u just as unrestricted plural comprehension entails the existence of the universal plurality. Similarly, the universal fusion is such, that everything is a part of it. Therefore, I argue that if one employs mereology within ontology, one should strive to be an absolutist. That is, I argue for the case of unrestricted quantification in the case of mereology (UnrQuan*).

Without delving into the history of the field, it is helpful to briefly remind oneself that ontology is usually characterized as being about everything there is, the most fundamental entities, and the relations among them (cf. Hofweber, 2023, p. 13). The fact that philosophers famously disagree about what there actually is, should not be understood as them also thereby contesting what ontology is about. Rather, they (mostly) agree that it is about everything there is, but disagree about what exists (cf. Gruszczyński and Varzi, 2015, section 2).

Since mereology is used as a formal (tool in) ontology it should also be about everything there is. To elucidate this thought more clearly, it is helpful to use the taxonomy by Thomas Hofweber from Logic and Ontology. Here, mereology is categorised as formal ontology which is characterized by the following three properties (cf. Hofweber, 2023, pp. 26–28):

(I) The study of artificial languages (axioms and theorems of classical mereology),

(II) The study of what there is (kinds of entities),

(III) The most general features of these entities and how they relate to each other in the most general way (parthood relation and mereological fusion);

The fact that mereology is categorised this way, does not rule out different understandings of it. The main point here is that the most straightforward understanding of mereology as a formal (tool in) ontology is the one which strongly favors an absolutist position.[17] That is, to endorse the view that it is possible to quantify over absolutely everything since this is how ontology is generally understood.

So it is in the interest of a mereologist to be an absolutist unless they have a good story to tell why they favor another understanding of mereology.[18] Furthermore, if mereology is not about everything, the universal fusion u would also not be truly universal. In such a case, there would be things outside of the domain of mereology since the position amounts to some form of relativism. Hence, some things would also not be part of the universal fusion u.

It is interesting to recall that Lewis explicitly states to "[r]estrict quantifiers, not composition [fusion]" (Lewis, 1986b, p. 213) and similarly "[i]f... you quantify subject to restrictions, then you can leave it [the weird fusion] out" (Lewis, 1991, p. 80). These remarks should not be taken to be about the domain of mereology, but rather are about ways to resolve the tension between the weird fusions whose existence is entailed by Unrestricted Fusion and the folk belief about

17 Lando, 2017 is an example who restricts the domain of mereology to concrete entities. He writes in the introduction that "mereological monism [the thesis that mereology as presented in (3.2) is the general theory of parthood and composition] *is not absolutely general and absolutely topic neutral*: the categorial divide between abstract and concrete entities makes a lot of difference for mereological monism. My defense of mereological monism is focused on concrete entities[.]" (Lando, 2017, p. 10).

18 See also Gruszczyński and Varzi, 2015, section 2 for a discussion of this. Lewis, for example, did not think that universals have a mereological structure and hence the parthood relation cannot be applied to them. However, the existence of universals themselves depends on a metaphysical assumption which nominalists, in the old meaning of the word, reject.

these fusions.[19] That is, Lewis takes ordinary talk to exclude weird fusions since the quantifiers in this setting are restricted to salient things (given a context) (cf. Lewis, 1986b, p. 213. He does not take mereology as a theory to be restricted in this sense. This is quite clear when he writes that:

> [W]e seldom admit it [a weird fusion] to our domains of restricted quantification. It is very sensible to ignore such a thing [a weird fusion] in our everyday thought and language. But ignoring it won't make it go away. (Lewis, 1986b, p. 213)

So even though it might initially seem like Lewis is restricting the domain of mereology, it turns out that he only takes our ordinary talk to be restricted. So this does not concern the domain of mereology.[20] The fact comes out quite clearly in the above quote since the ontological question of whether a fusion exists or not is unaffected by the restriction of ordinary language. Hence, mereology extends beyond the restricted domain that is used in our everyday talk.

To sum up, if one uses mereology as (a tool in) formal ontology, one should also be in favor of unrestricted quantification since the domain of ontology is traditionally taken to be about everything there is. So, there is a reason to assume that the domain of mereology should be an all-encompassing one. Hence, this is additional support for unrestricted quantification in the case of mereology and therefore (UnrQuan*) is warranted.[21]

In the next chapter, I identify the mereological analogue of (Rigidity). This is more difficult to argue for since Florio and Linnebo argue against the rigidity of mereological fusions. Furthermore this claim is, in general, quite controversial. Nevertheless, I argue that one can make a case for (Rigidity*).

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19 This sort of strategy to resolve the issues between Unrestricted Fusion and folk belief is not without detractors. See Korman, 2008 for example.

20 In Lewis, 1986a he explicitly states that universals cannot be mereologically structured and they therefore lie outside the domain of application of mereology (cf. Lewis, 1986a, p. 34). So in this sense, Lewis restricts the domain of mereology in the end. On the other hand, Florio and Linnebo specifically highlight that these all-encompassing domains are of particular interest to philosophers (cf. Florio and Linnebo, 2021, p. 295.

21 Uzquiano, 2006 contains an interesting discussion regarding issues between ZFCU and classical mereology if both are taken to be absolutely general. However, these issues appear only if one also assumes an atomistic classical mereology and a Maximality Principle for ZFCU stating that "[t]here is a $1-1$ map from the entire universe into the pure sets (cf. Uzquiano, 2006, pp. 149–151).

4 MEREOLOGICAL ESSENTIALISM AND RIGIDITY

In this chapter, the mereological analogue of plural rigidity, called *Mereological Essentialism*, is introduced (4.1) and Florio and Linnebo's argument against it is presented (4.1.1). Afterwards, the motivation behind Mereological Essentialism is discussed by looking at its proponents (4.2). The distinctions introduced by Jubien (4.2.4) are then used to argue that Florio and Linnebo are too fast when they dismiss Mereological Essentialism. Furthermore, it is argued that Mereological Essentialism has intuitive force for many fusions, including the universal one (4.3.2).

4.1 MEREOLOGICAL ESSENTIALISM (ME)

Since plural logic and mereology are similar from a formal standpoint, mereological analogues of the plural rigidity principles RGD^+ and RGD^- from (2.3) can easily be reformulated in mereology as ME^+ and ME^-, respectively.[1] ME^+ then states that if something x is part of another thing y, then it is part of y in every possible world where y exists. This can be put formally, again using 'E' as an existence predicate, as:

$$ME^+ \quad \Box\forall x\forall y(Pxy \rightarrow \Box(Ey \rightarrow Pxy)).$$

Similarly, ME^- states that if something x is not part of another thing y, then it is not part of y in any possible world. Put formally:

$$ME^- \quad \Box\forall x\forall y(\neg Pxy \rightarrow \Box(\neg Pxy)).$$

So taken together these principles imply that a mereological fusion neither gains nor loses parts or, to put it differently, it has exactly the same parts in every possible world. If this is assumed, tracking a mereological fusion across possible worlds amounts to tracking its parts. I. e. the fusion is then tracked extensionally just as sets and pluralities are. This parallel between the rigidity of pluralities and that of mereological fusions is not an arbitrary one, but rather arises naturally since the claim that mereological fusions are rigid is known as Mereological Essentialism in the literature.

Because mereology is used to reason about ordinary objects like humans, tables, etc., this position seems patently false. These ordinary objects can, intuitively, gain and lose (at least some) parts while still

1 See Florio and Linnebo, 2021, pp. 81–83, 101–103 for a comparison between atomistic mereology and one-sorted plural logic.

being the same object. A number of philosophers, however, are proponents of Mereological Essentialism.[2] Since plural rigidity (Rigidity) is a crucial assumption in the argument against a universal plurality (2.5), the hope of developing a similar argument in the case of mereology depends on Mereological Essentialism, (Rigidity*). Because of this, it is therefore instructive to look at how some proponents defended it against the common objections. In doing so, it will also become clear where the controversy regarding Mereological Essentialism is located.

The aim here is not to give a novel argument for Mereological Essentialism, but rather argue that the position cannot be as easily dismissed as it usually is.

4.1.1 *Mereological Rigidity*

I now present Florio and Linnebo's argument against the rigidity of fusions. For the sake of argument assume that two things, x and y, share all their parts, i. e. $\forall z(Pzx \leftrightarrow Pzy)$. Given that 'P' is a partial-order, hence reflexive and anti-symmetric, x and y are identical. Furthermore, necessarily x shares all of its parts with itself and therefore one gets by Leibniz's law that necessarily x and y have all the same parts.

This seems compatible with parthood being non-rigid which would call into question the conclusion of the set-theoretic argument, namely that set membership is rigid (2.3.1). Florio and Linnebo's way out is to deny that the two cases are analogous. In the case of sets, any reason to accept extensionality is also a reason to accept that set membership is rigid. For fusions this is not the case: there is a reason to accept that having the same parts is sufficient for identity which is not a reason to accept that parthood is rigid (cf. Florio and Linnebo, 2021, pp. 213–214).

To make sense of what Florio and Linnebo, 2021, p. 214 call "contingent parthood", they suggest to think of objects as having matter

2 Furthermore, anyone who holds that the fusion is 'nothing over and above its parts' in a strict sense, i. e. every proponent of the strict reading of Composition as Identity (CAI) that asserts that composition is identity, is committed to mereological essentialism. This has been first made explicit by Merricks, 1999 who writes that "composition as identity implies that [some specific object] O – and, of course, every other composite object – must, in every world in which it exists, be composed [fused] of the parts that actually compose [fuse] it. Composition as identity entails mereological essentialism" (Merricks, 1999, p. 193). He uses this fact as an argument against (CAI) in the end since he rejects mereological essentialism and therefore also rejects (CAI). According to Merricks, Lewis is committed to mereological essentialism (cf. Merricks, 1999, footnote 3). This is debatable, however, since Lewis only states that composition is like identity and lists five aspects they have in common (cf. Lewis, 1991, pp. 82, 85–86).

and form, i. e. a sort of Aristotelian hylomorphism.[3] To make it more concrete, they give the following example:

> [A] molecule that is part of you might not have been so because tracking you across possible worlds involves more than merely tracking your matter. (Florio and Linnebo, 2021, p. 214)

So on this view, tracking an object involves tracking both its matter and its form. Given this conception, parthood is sensitive to matter and form. Consequently, tracking something across possible worlds is more than just tracking the matter of the specific thing. x and y sharing all their parts means they share all of the material and formal parts and this ensures that x and y are identical.

This explains why having all the same parts is sufficient for identity without it also being a reason to accept that parthood is rigid. As just mentioned, this is compatible with objects involving form and hence them being tracked non-trivially across possible worlds unlike sets and pluralities. That is, objects are not tracked purely extensionally across possible worlds since there is more to them, namely their formal parts.

Recall that to track sets or pluralities across possible worlds it is enough to just track their elements or members. In the case of mereological fusions, on the other hand, one has to track their formal parts in addition to their material parts. Therefore, Florio and Linnebo conclude that "the principle that sameness of parts ensures identity admits of an explanation that does not support the rigidity of parthood" (Florio and Linnebo, 2021, p. 214). So one is not forced to accept the rigidity of parthood if one accepts the extensionality for fusions, whereas this is the case for the rigidity of sets and pluralities: any reason to accept extensionality is also a reason to accept the rigidity claim (cf. Florio and Linnebo, 2021, p. 214).

What if one does not accept their hylomorphic picture and thinks that if two objects x and y have the same material parts it is enough for their identity? They briefly address this issue in a footnote where they write that "[a] better analogue of the set-theoretic principle of extensionality is the principle that sameness of *material* parts ensures identity" (Florio and Linnebo, 2021, p. 214). Now, any reason to accept mereological extensionality is also a reason for the rigidity of material parthood.

While they do not say anything about abstract parts, this opens up a possible way to resist their argument against the rigidity of parthood in the case of objects which intuitively lack formal parts. This line of argument will be explored in (4.3) where I challenge Florio and Linnebo's conclusion by using Jubien, 2001 distinctions.

3 See Koslicki, 2008 for a recent defense of this sort of view.

4.2 ARGUMENTS FOR MEREOLOGICAL ESSENTIALISM

As with Unrestricted Fusion, Mereological Essentialism runs counter our intuitions in such a way that it is important to look at how it has been defended as a philosophically viable position. To do this, I discuss and evaluate prominent examples from the literature. I argue then that in all of the cases, Mereological Essentialism is taken to straightforwardly hold for mereological fusions, physical objects, or masses of matter. These have in common that they are identified in terms of their parts, i. e. sharing all the same parts is necessary and sufficient for their identity. Mereological Essentialism is only contested for mereological fusions when considered as ordinary objects, i. e. artifacts, persons, etc., since their identity seems to be governed by additional principles.

4.2.1 *Chisholm on ME*

The starting point for Mereological Essentialism in the contemporary literature is Roderick Chisholm and hence it is ours as well. He argues for this position by introducing a distinction between different kinds of objects. The conclusion of his argument is that there is no real conflict of intuitions, but only an apparent one (cf. Chisholm, 1973, p. 584).

Chisholm expresses mereological essentialism in the following way:[4]

> The principle [of mereological essentialism] may be formulated by saying that, for any whole x, if x has y as one of its parts then y is part of x in every possible world in which x exists. (Chisholm, 1973, pp. 581–582)

He then goes on to offer different formulations of the principle:

> The principle may also be put by saying that every whole has the parts that it has necessarily, or by saying that if y is part of x then the property of having y as one of its parts is essential to x. (Chisholm, 1973, p. 582)

He continues with the claim that if Mereological Essentialism is true, it entails that "if y is ever part of x, y will be part of x as long as x exists" (Chisholm, 1973, p. 582). This principle is sometimes called *Mereological Constancy* in the literature.[5] Mereological Constancy will neither be discussed nor defended here since Florio and Linnebo, 2021

4 This view, or at least one similar to it, has been held by philosophers such as Boethius, Abelard, and Leibniz before Chisholm. See Arlig, 2019, chapter 4 for an overview.

5 See Nicolas, 2009 and Cotnoir and Varzi, 2021 who use *Mereological Constancy*. Another name for the principle is *Mereological Changelessness* by Plantinga, 1975 but Chisholm, 1975, pp. 481–482 takes issue with that name since, according to him, there are four types of mereological change even if mereological essentialism is true.

do not consider the notion of change over time in their discussion of plural rigidity and only focus on different possible worlds. Consequently, my discussion of Mereological Essentialism ignores the temporal dimension and issues connected to it.

Furthermore, I use the first formulation of ME for two reasons. First, it is the one taken up by the literature.[6] Second, it is the formulation that is most similar to the way plural rigidity was introduced in (2.3).

Chisholm's position, though, seems to only encapsulate the thought behind ME^+ and not that behind ME^-. However, I take him to also accept ME^-, as he would otherwise have to admit that a mereological fusion gaining additional parts is still the same fusion. This would make his position very one-sided regarding the relationship between the parts of a fusion and its identity. He also cites Abelard's thought that "no thing has more or less parts at one time than at another" (Henry, 1972, p. 120) immediately afterwards which captures both ME^+ and ME^-, although in a temporal sense and not the modal one.

Chisholm is very much aware that Mereological Essentialism is at odds with how we normally talk and think about ordinary objects and how they can undergo (at least a certain amount of) change while still being the same object. For him, this conflict between our intuitions is only an apparent one, however, and he takes himself to resolve it (cf. Chisholm, 1973, pp. 583–584). Nevertheless, he starts with an example where our intuition seems to be in favor of Mereological Essentialism:

> Let us picture to ourselves a very simple table, improvised from a stump and a board. Now one might have constructed a very similar table by using the same stump and a different board, or by using the same board and a different stump. But the only way of constructing precisely *that* table is to use that particular stump and that particular board. (Chisholm, 1973, p. 583)

It is, I think, rather clear why this example works in favor of Mereological Essentialism: both the stump and the board are relevant to the

6 See Plantinga, 1975, p. 468 who cites Chisholm's definition. van Cleve, 1986, p. 141 states that "[i]f x is part of y, then x is part of y in every possible world in which y exists" and also claims that this formulation "implies that a whole cannot survive the *addition* of a part, since it implies that a whole with a new part could not have existed as that very whole previously" van Cleve, 1986, p. 154. So according to van Cleve, Chisholm's formulation captures both ME^+ and ME^-. Jubien, 2001, pp. 7–8 changes Chisholm's definition slightly and is explicit about a thing not gaining a part "the thing could not have existed without having precisely the parts it actually does have." Similarly, Wallace, 2014, p. 111 who writes that "for any composite object, O, O is composed of (all and only) its parts $O_1, \dots O_n$, in every possible world in which O exists. Lastly, Cotnoir and Varzi, 2021, p. 246 use Chisholm's original definition without the 'for any whole' condition "[i]f x is part of y, then it is part of y in every possible world in which y exists".

function of the table and therefore seem, in some sense, to be essential to it. Already Alvin Plantinga considers where our intuitions lie when one focuses on smaller parts of the table. He writes that "[i]f we think of the stump and board as themselves composed of molecules of wood, let's say, we are disinclined to think that we get a new stump just by knocking off a molecule or two" (Plantinga, 1975, p. 470). The thought here is that the the molecules of wood do not contribute to the function of the table and are, literally, very small parts of it. Thus, they do not seem in any way essential to the table. We are disinclined to say that the stump is different and thereby also disinclined to say we have a different table.

So this example on its own is rather weak support for Mereological Essentialism and even a bit misleading, since it focuses on parts of the table that are also relevant to its function as a table. Chisholm also gives a general argument that is quite elaborate and involves a number of technical terms and distinctions. Luckily, not all of them are relevant for the purposes at hand and only the relevant ones are now introduced.

The first distinction is a linguistic one between "'part' in its ordinary, or loose and popular, sense, and 'S-part' or 'part' in its strict and philosophical sense" (Chisholm, 1973, p. 586). S-part corresponds to the notion of a proper part introduced in (3.2). So it is irreflexive, asymmetric and transitive, whereas part in ordinary talk is not governed by any axioms. He then restates Mereological Essentialism using 'S-part' instead of 'part' and takes it as a basic principle of his theory (cf. Chisholm, 1973, p. 587).

The second distinction is an ontological one between primary and ordinary (nonprimary) objects. Here, Chisholm, 1973, pp. 597–598's definitions are rather confusing and I adopt Plantinga, 1975's definitions of the same concepts. Primary objects are defined as "an object that has parts and all of whose parts are S-parts" (Plantinga, 1975, p. 471). So all parts of primary objects obey the axioms of strict-partial orders. An ordinary object on the other hand, "has some parts that are not S-parts" (Plantinga, 1975, p. 472). Consequently, an ordinary object has at least one part in the loose and popular sense. On Chisholm's account, the primary objects constitute (in a technical sense) the ordinary objects the details of which are unimportant for us.

With these distinctions in place, he then proceeds to argue that ordinary objects, like cars, can gain or lose parts in the loose and popular sense, but they cannot lose or gain S-parts. Mereological Essentialism does not hold for ordinary objects, but only for primary objects. The usual counterexamples and objections stemming from our everyday experience, mix up these two different kinds of objects (cf. Chisholm, 1973, pp. 591–593). So Chisholm's way of resolving our conflicting intuitions and arguing for Mereological Essentialism, is to make an ontological distinction between primary and ordinary (nonprimary) objects and a linguistic one between 'S-part' and 'part'.

This is unsatisfying for two reasons. First, only a limited class of objects obey Mereological Essentialism and other objects do not. Second, the proliferation of objects on Chisholm's account forces him to define two ways of talking about how many objects there are. There is a loose and popular one and a strictly philosophical one, so that it corresponds to our ordinary talk (cf. Chisholm, 1973, p. 589). In general, Chisholm's ontological distinction leads to issues which go against our common sense and should be regarded only as a last resort.

Nevertheless, the general idea behind it, i. e. distinguishing between different kinds of objects or fusions, has proven to be very influential in the literature arguing in favor of Mereological Essentialism as will become clear in the course of this section.

4.2.2 *Van Cleve on ME*

Moving on to James van Cleve who adopts Chisholm, 1973's definition of Mereological Essentialism. While he does not work with Chisholm's ontological distinction between primary and ordinary objects, he uses a similar one and states that:

> There is at least one class of entities to which the application of mereological essentialism . . . is not in much dispute – namely, mereological sums [fusions], or what Locke calls masses of matter. Mereological essentialism in regard to such entities is highly intuitive. After all, if one particle in a mass of matter is removed, how can it be the very same mass that remains? What is controversial (and usually controverted) is the application of mereological essentialism to entities of other kinds – artifacts, living creatures, and (especially) persons. (van Cleve, 1986, p. 147)

Note that in the above quote, van Cleve uses the mass term 'matter' to appeal to our intuition that Mereological Essentialism clearly holds in such cases, but also identifies mereological fusions with masses of matter. This is especially interesting since mass terms are plural by nature, that is, they take the many as many, like plural logic does. Mereology and set theory, though, take the many as one (a mereological fusion or a set) so they are not plural by nature.

Therefore, the appeal to intuition should rather be stated as 'if one particle in a mereological fusion is removed, how can it be the very same fusion that remains?' Here the conflict with our intuition seems to show up again, depending on how we view the mereological fusion. On the one hand, if the mereological fusion is identified with an ordinary object like a table, it seems that we still want to say it is the same table after all. If, on the other hand, the mereological fusion is identified with a mass of matter as in the above example, it seems van Cleve gets what he wants regarding our intuitive judgment.[7] This highlights an important difference regarding our intuitions: they heavily depend on how we view the object in question.

The main takeaway from this discussion is twofold. First, van Cleve does not even consider it necessary to argue for Mereological Essentialism in the case of mereological fusions which he identifies with masses of matter. Here I remarked, that only because he uses this identity and conflates it in a crucial way, do our intuitions favor Mereological Essentialism. Second, how we view an object greatly impacts the intuitive plausibility of Mereological Essentialism: if we consider a mereological fusion as a table, it seems implausible; if we consider it as a mass of matter, it seems plausible.

After having seen two proponents of Mereological Essentialism, we now look at a critic of it as it further shows where the controversy about the position is located.

4.2.3 *Willard on ME*

Dallas Willard takes issue with the dialectic of Chisholm, 1973, the examples he uses to argue for Mereological Essentialism, and his move from Mereological Essentialism about ordinary objects to restricting it to only primary objects (cf. Willard, 1994, pp. 125–128, 131–135). My focus is on the latter two since Willard's comments on these issues further help to understand the fundamental difference between Mereological Essentialism for ordinary objects and for mereological fusions.

Recall that Chisholm uses a very simple improvised table as an example to argue in favor of Mereological Essentialism. This table consists only of a stump and a board. Willard's first complaint is about this example and he states it thus:

> [W]e hardly have a *table* at all. An *improvised* table is, precisely, not a table but something arranged to serve *as* a table in given circumstance. Exactly how the board and stump are united to form a whole is not indicated. (Willard, 1994, p. 127)

The issue is that, because the described table is so simple and, crucially, we do not know in what way the parts are put together, that it is not what we would ordinarily call a table. Rather, it is an aggregate or some collection of objects, its parts. Hence, the simple table lacks some sort of structure that one usually attributes to ordinary objects.

7 Some support for the identification of mereological sums with matter comes from the use of mereology in linguistics. There it is used to give a uniform analysis of mass terms. On one view, mass terms denote mereological sums (cf. Steen, 2022, pp. 15–22).

This is echoed by Willard, 1994, p. 128 when he writes that "[t]here is much more to them [tables] than parts in spatial arrangement".

He does, however, grant Chisholm that Mereological Essentialism seems true for wholes if one thinks of them as aggregates. On such a view, the whole is nothing more than its parts. Therefore, the only thing that can ensure the identity is the sameness of parts. Willard only contests Mereological Essentialism for ordinary objects, but is happy to accept it for aggregates which have no structure (cf. Willard, 1994, pp. 128, 142–143). Willard's second complaint is directly tied to this observation. Chisholm's initial claim seems to be an interesting and shocking claim that ordinary objects obey Mereological Essentialism, but he reformulates it to the claim that only primary objects do (cf. Willard, 1994, pp. 132–134).

So again, what emerges is that Mereological Essentialism is only controversial for ordinary objects, while for primary objects or aggregates or mereological fusions it is not. In the case of the latter, even Willard, a critic of Mereological Essentialism, agrees that it is an intuitively plausible view. One major issue that remains, however, is that on Chisholm's view there are different objects in our ontology, namely the ordinary ones and the primary ones and how they relate to one another.

Here Michael Jubien offers an interesting view. He reconstrues Chisholm's ontological distinction as an epistemic one and shows how this distinction can be used to take the force out of the usual counterexamples against Mereological Essentialism. The distinction is a systematic treatment of what I noted in the discussion of van Cleve's position: how we view an object changes the plausibility of Mereological Essentialism.

4.2.4 *Jubien on ME*

Jubien, 2001 remarks that there are three tendencies of ordinary thought which are at the reason why Mereological Essentialism is controversial, namely:

(1) The parts divide;

(2) The arrangement divide;

(3) Object fixation.

According to the parts divide, how we think of an object greatly influences the importance of the object's parts and its identity. If we think of an object as a familiar kind, say a table, the parts of it do not play a big part for its identity. I. e. we are happy to speak of the same table even if some parts of it are replaced by different ones.[8] If, on the other

8 Where exactly to draw the line is, of course, very difficult and goes back to the thought-experiment of Theseus' ship.

hand, we think of the same object, the table, as a physical object that is nothing more than the matter it is made out of, its parts are crucial for its identity. I. e. if a part of matter is replaced with some different part, it is more likely to say that it is not the same physical object anymore. This is similar to what we have seen in this chapter so far. Note that both Mereological Essentialism and the counterexamples to it gain support from these (differing) intuitions.

The arrangement divide is similar to the parts divide, but concerns the relationship between the arrangements of parts and the object's identity. That is, parts do not get replaced but rearranged. As before, the importance of the arrangement of parts for the identity of the object depends on how we think of the object. If we think of an object as a familiar kind, say a clay statue of David, the way the parts are arranged is very important since the sculptor could swap David's left arm with his left leg and thereby create a new statue. I. e. we would not say that it is the same statue in that case. If, on the other hand, we think of the statue as a physical object that is nothing more than the clay it is made out of, the arrangement of its parts does not seem to matter much, if at all. I. e. if some portion of the clay is swapped with a different portion of the same clay, it seems reasonable to say that it is still the same clay.

Lastly, object fixation is the tendency to think that, since ordinary objects are physical objects, anything that is true of the ordinary object qua ordinary object is also true of the physical object qua physical object. But the two divides above show that this is not so. While we might happily accept that replacing a part of a table still leaves us with the same table, this is not true of the physical object. Thus, not keeping these distinctions in mind leads to tension because our intuitions clash (cf. Jubien, 2001, pp. 4–5).

We have seen one way to resolve these clashing intuitions regarding Mereological Essentialism already, namely Chisholm's ontological distinction between primary and ordinary (nonprimary) objects according to which there actually are two objects. Jubien sees this option only as a last resort and does not pursue it since it clashes with common sense (cf. Jubien, 2001, p. 5). Instead, he thinks the issue is object fixation and not keeping the parts divide in mind. Here is his diagnosis of the problem:

> The counterexamples [to Mereological Essentialism] are always about objects of familiar kinds, like boats. It's simple common sense that a boat could have had at least some different parts. After all, this is just the familiar-kind of the parts divide. Since a given *boat* could have had a different sail, we're supposed to conclude that there's at least one *thing* that could have had a different part, and hence that mereological essentialism is refuted. (Jubien, 2001, p. 8)

His diagnosis starts with the observation that the counterexamples to Mereological Essentialism always ask us to consider objects of familiar kinds, i. e. ordinary objects. And, as we have seen, these objects can survive replacement of (some of their) parts. Thus, there is at least one object that could have had different parts and therefore Mereological Essentialism is false. It is the last step in the argument Jubien objects to since it is a case of object fixation: one takes a truth about the ordinary object to also be a truth about the physical object. In the case of physical objects, though, we have seen that they could not have had different parts and still be the same physical object. So this move is illegitimate because it is at odds with our judgment and the counterexample only works because it conflates the two notions.

To resolve the conflict, Jubien argues that there is a specific ordinary object of the kind *boat*. Considered this way, it could have had different parts, but still be the same boat. This is so because we consider it as being on the familiar-kind side of the part divide. Considering the same object as a physical object, it could not have had different parts and still be the same object. This is so because we consider it as being on the physical-object side of the part divide. Therefore, if we keep the distinction in mind, the counterexample turns out to pose no problem to Mereological Essentialism (cf. Jubien, 2001, p. 8).

So similarly to the arguments for and against Mereological Essentialism we have seen so far, the way Jubien argues for it is to introduce a distinction. Unlike the one by Chisholm, though, it is not an ontological one: there are not actually two different objects, primary and ordinary (nonprimary) ones, which are related to each other by a certain relation. Rather, it is an epistemological one: there is one object that can be viewed as either an object of a familiar kind (ordinary object) or as a physical object (mass of matter, primary object, mereological fusion). Depending on how one views the object, intuitions regarding Mereological Essentialism differ. Counterexamples to it make the illegitimate move of attributing truths about the object viewed one way to also attribute them to the object viewed the other way, i. e. object fixation.

Furthermore, classical mereology by itself as introduced in (3.2) does not contain any locative notions, i. e. the predicates definable in this theory only concern the part-whole relation and nothing about the location of the parts. So from the standpoint of mereology, two mereological fusions are identical if and only if they have the same parts by (A2). Nothing about how the parts are arranged is expressed by this axiom so it is misguided to ascribe any additional (structural) properties to mereological fusions as is done by the numerous counterexamples. That is, I agree with Jubien that counterexamples to Mereological Essentialism start from considerations that are reasonable when objects are thought of as ordinary objects, but then make the illegitimate move to claim that the same holds for the objects considered as masses of matter or mereological fusions.[9]

There are two main takeaways from this discussion. First, Jubien reconstrues Chisholm's ontological distinction as an epistemological one. Because of this, Jubien neither has to posit the existence of additional objects nor does he have to explain how they relate to the ordinary objects. Second, his distinction makes sense from the standpoint of classical mereology since it cannot, as I noted, say anything about additional structure an ordinary object might have. Consequently, sameness of parts is the only possible criterion of identity in classical mereology as introduced in (3.2). The question whether there are also formal parts in addition to material parts depends on a further metaphysical assumption which is independent of classical mereology.

4.3 PUTTING THE DISTINCTIONS TO WORK

I now use these distinctions to show two things. First, I argue that Florio and Linnebo are too quick in their dismissal of mereological rigidity (4.1.1) and that on the view that emerged in this chapter there might be an argument in favor of Mereological Essentialism instead. Second, I argue that even if our intuitions regarding Mereological Essentialism are conflicting in many cases, this is not the case for the universal fusion u and many other fusions entailed by Unrestricted Fusion. This is so because object fixation does not apply in these cases. I. e. we view the universal fusion u as something such that everything is part of it and there is no ordinary object which could correspond to this. Hence, there is no risk of us slipping into object fixation and because of this Mereological Essentialism has intuitive plausibility for these mereological fusions.

4.3.1 *Florio and Linnebo's Argument Against ME Reconsidered*

For convenience, I briefly restate Florio and Linnebo's argument from (4.1.1) towards the conclusion that mereological fusions are not rigid. The assumption was that two things, x and y, share all their parts, i. e. $\forall z(Pzx \leftrightarrow Pzy)$. Given that 'P' is a partial-order, x and y are identical. Furthermore, necessarily x shares all of its parts with itself and therefore one gets by Leibniz's law that necessarily x and y have all the same parts.

9 In connection to this, a helpful taxonomy is offered by Moore, 2015. He starts by correctly stating that "[a]ccording to classical mereology, mereological summation [mereological fusion] is unstructured" (Moore, 2015, p. 74). Since Moore argues that there are also mereological fusions that have structure, he calls these fusions "Maximally Unstructured Mereological Summations" (Moore, 2015, p. 74). These fusions cannot change their parts without thereby also becoming different fusions (cf. Moore, 2015, p. 81).

This seems compatible with parthood being non-rigid which would call into question the conclusion of the set-theoretic argument that set membership is rigid. Florio and Linnebo, 2021 claimed that the two cases are not analogous. For sets, any reason to accept extensionality is also a reason to accept that set membership is rigid. For fusions this is not the case: there is a reason to accept that having the same parts is sufficient for identity which is not a reason to accept that parthood is rigid (cf. Florio and Linnebo, 2021, pp. 213–214).

Florio and Linnebo suggested to think of objects as having matter and form, i. e. a sort of Aristotelian hylomorphism and gave the following example to motivate it:

> [A] molecule that is part of you might not have been so because tracking you across possible worlds involves more than merely tracking your matter. (Florio and Linnebo, 2021, p. 214)

On this view, tracking an object involves tracking both its matter and its form. Consequently, parthood is sensitive to matter and form and thus tracking something across possible worlds is more than just tracking the matter of the specific object. That is, objects are not tracked purely extensionally across possible worlds since there is more to them, namely their formal parts. Note that Florio and Linnebo do not at all talk about mereological fusions, but only about objects although their argument is against the claim that mereological fusions are rigid. So they seem to identify mereological fusions with ordinary objects.

In (4.1.1) it was already hinted at that this relies on a hylomorphic conception of objects. In their example, the object they consider is a person. Using the distinctions from this chapter, this is an ordinary object or an object of a familiar kind. As we have seen, intuitions greatly differ depending on how we view an object. In the case of a person viewed as an ordinary object, it might seem reasonable to assume that they have formal parts and that they can undergo some change while still being the same person. If the person is viewed as a physical object or mereological fusion, however, this is not so clear. A physical object or a mereological fusion just is the matter it is made out of so there seems to be no reason to assume it additionally has formal parts. Because of this there seems to be no other criterion of identity for physical objects or mereological fusions than sharing all the same parts. Consequently, they cannot undergo any change while still being the same object.

Florio and Linnebo do not seem to make this distinction since their motivating example is that of an ordinary object from which they then generalize to objects in general, including mereological fusions. So they are guilty of object fixation. They go from attributing formal parts to objects considered as ordinary objects, to also attributing formal parts to physical objects or mereological fusions. Linnebo and Florio do not do this explicitly, but implicitly since they do not discuss this distinction. For objects considered as physical objects or mereological fusions, though, this is not intuitive as their only criteria of identity is the matter they are made out of, i. e. their (material) parts.

Since they do not pay attention to these important distinctions, they are too quick to just dismiss the mereological case when they take their argument to show that one is not forced to accept the rigidity of parthood if one accepts the extensionality for fusions. Their example only shows that objects viewed as ordinary objects can lose parts and thus parthood is not rigid in such a case. This is common sense and even Jubien, a proponent of Mereological Essentialism, accepts this, as we have seen. Furthermore, for objects viewed as physical objects or mereological fusions, the principle that "sameness of *material* parts ensures identity" (Florio and Linnebo, 2021, p. 214) seems quite intuitive which would lend Mereological Essentialism additional support. As this would restore the analogy between extensionality in set theory and mereology and now any reason to accept mereological extensionality is also a reason for the rigidity of (material) parthood (cf. Florio and Linnebo, 2021, p. 214).[10]

4.3.2 *Weird Fusions and ME*

As was discussed in this chapter, the counterexamples to Mereological Essentialism involve ordinary objects. This is also the reason for their intuitive plausibility. It is common-sense that objects considered as ordinary objects can change (some of) their parts while intuitively still being the same object. One way to argue against these counterexamples is to use Chisholm's ontological distinction between primary and ordinary (nonprimary) objects where Mereological Essentialism only applies to the former. Here I sided with Jubien who offers another way to resist these counterexamples by using an epistemological distinction. Counterexamples to Mereological Essentialism make the illegitimate move from attributing truths about ordinary objects also to physical objects or mereological fusions (and vice versa). This is what Jubien calls object fixation.

Since we are working with Unrestricted Fusion, there are many fusions which cannot be viewed as anything but as a fusion.[11] That is, there is no ordinary object that the fusion could be viewed as. To make it more concrete, consider the fusion $fomu$ consisting of a

10 In the case of abstract objects (which arguably do not have any material parts), one needs to consider their abstract parts. What exactly these parts are, differs from case to case. For sets one could use Lewis, 1991 treatment, for example.

11 Note that this is legitimate since we are working within classical mereology where Unrestricted Fusion is one of the axioms. Here we are using Jubien's distinction to help us explain our intuitions regarding Mereological Essentialism when faced with these fusions.

fork and a mug. If we consider the parts individually, there are two ways to view each object, namely as an ordinary one and a physical one. Consequently, intuitions differ with regards to how plausible Mereological Essentialism is as explained in (4.2.4). If we consider them together as a fusion, there is no way to consider it as anything but a physical object or mereological fusion since there is no ordinary object it could be viewed as. Consequently, intuitions cannot differ in the usual way and remain in favor of Mereological Essentialism.

If one were to replace either the fork or the mug, clearly the fusion $\mathfrak{fomu}$ would also be different. That much is clear just by virtue of $\mathfrak{fomu}$ having two parts and replacing one seems to be a quite substantial change. What if $\mathfrak{fomu}$ loses or gains a molecule?[12] I claim that then we also end up with a different fusion than $\mathfrak{fomu}$. Since we can only consider it as a physical object, in our case a mereological fusion, the only criterion of identity is that of sharing all the same parts and this is not the case here. Furthermore, this also makes sense since we are forced to view the mereological fusion as a physical object so we should also consider its parts as physical objects. And in both cases Mereological Essentialism seems reasonable. Thus, for fusions to which no ordinary object corresponds to, our intuitions are not lead astray by object fixation.

Now, where do our intuitions lie regarding the universal fusion $\mathfrak{u}$ and Mereological Essentialism?[13] By definition it is something such that everything is part of it. In (3.4) it was argued that 'everything' should really be understood in the absolutely unrestricted way. So the universal fusion $\mathfrak{u}$ should also have abstract objects, like sets, as parts if they exist. That there is no object of a familiar kind (i. e. no ordinary object is such that everything is part of it) which can correspond to this, is even more clear than in the above case.

Even if one grants that ordinary objects can have abstract parts, it seems that there is just no ordinary object that could fulfil this role. Thus, there is no risk of slipping into object fixation. Hence, our intuitive judgment of Mereological Essentialism regarding the universal fusion $\mathfrak{u}$ does not suffer from the usual difficulties. Furthermore, the intuition that Mereological Essentialism is plausible for $\mathfrak{u}$ is even stronger since what other criterion of identity could there possibly be except sharing all the same parts? So losing or gaining a part leads to a different universal fusion $\mathfrak{u}'$. Therefore, I think that Mereological Essentialism has intuitive appeal in the case of the universal fusion

12 The case of replacement of molecules is more difficult since my intuitions are not clear in that case. Furthermore, I follow the tradition in mereology of talking about objects losing molecules. See Plantinga, 1975, p. 470.

13 Since we are working with classical mereology, the existence of the universal fusion $\mathfrak{u}$ is entailed by Unrestricted Fusion when a trivial condition is used.

and also for any mereological fusion which cannot be viewed as an ordinary object.[14]

4.4 CONCLUSION

Since we have covered a lot in this chapter, it is worth summarizing the main findings. Mereological Essentialism is controversial and even seems outright false when one considers objects as ordinary objects. In such cases the object can change some of its parts while still being the same object according to common sense. Mereological Essentialism is rather uncontroversial, however, when one considers objects as physical objects or masses of matter or mereological fusions. In these cases, the object cannot change any of its parts and still remain the same object. Mereology as a theory only says something about mereological fusions since they do not have any additional structure besides the parthood structure.[15] If one assumes that ordinary objects are more than just the fusion of their parts, classical mereology as introduced in (3.2) is ill-suited as a tool and should be extended to also include locative notions, for example.

With the distinction offered by Jubien, it is possible to resolve the issues posed by the counterexamples since they make an illegitimate move between objects considered as ordinary objects and considered as physical objects or masses of matter or mereological fusions. I argued that, if one keeps this distinction in mind, Mereological Essentialism seems plausible for mereological fusions. Furthermore, the distinction was used to show that Florio and Linnebo are guilty of object fixation in their argument against the rigidity of mereological fusions (4.3.1). So their argument is no real threat for Mereological Essentialism. Lastly, it was argued (4.3.2) that for many mereological fusions, including the universal one, we have no other way than to view them as fusions as there are no ordinary object they could be

14 In cases like the above is also not immediately clear how they fit in a hylomorphic conception as Florio and Linnebo, 2021 use in their counterexample. Consider the case of $\mathtt{fomu}$: should it have the individual formal parts of its parts? Can an object have multiple formal parts or just one which exhausts its formal features? If its the former, what about prima facie contradictory formal parts such as 'being a sphere' and 'being a cube'? If its the latter, which object should trump the other(s) and why? These issues are not decisive in any way, but have to be answered first if one wants the fusion to have some relation to the formal parts of its parts. It seems that most of the issues discussed by Cotnoir and Varzi, 2021, pp. 219–220 regarding the category of the universal fusion (or any transcategorial fusion for that matter) arise in this context as well and these issues are not easily answered.

15 If one assumes that the structure of an object is a formal part of it, then this changes of course. In the case of physical object or masses of matter it is not obvious what the formal parts. One option would be to claim that the object has to be arranged in the way it actually is. This is not really a satisfying answer, however, since each object would have its specific formal part as a sort of identifier. This is not to say that there might not be another, satisfying, way of resolving this, but a further investigation is outside the topic of this thesis.

viewed as. So there is no risk of object fixation in these cases and intuitions are in favor of Mereological Essentialism.

Not only can Florio and Linnebo's argument be resisted, but Mereological Essentialism itself has quite some support when keeping the important distinctions in mind. Therefore, (Rigidity*) is warranted.

In the next chapter, I discuss the case of the mereological analogue of (UniSing) in detail. As it will turn out, it is rather difficult to argue for it. Furthermore, I take stock of all the assumptions, reconstruct Florio and Linnebo's argument in mereology, and highlight where the assumptions are used in the mereological argument.

5 AN ARGUMENT AGAINST UNRESTRICTED FUSION

In this chapter, it is investigated whether an analogous argument to the one made by Florio and Linnebo in plural logic to restrict unrestricted comprehension also works in the case of mereology to the extent that Unrestricted Fusion has to be restricted. To this end, the assumptions in each case will be considered (5.1), before moving on to the argument itself (5.2). The mereological argument does not work unless a strong assumption about the correspondence between mereological fusions and sets is made (5.2.2). In this case, the argument goes through and has the conclusion that Unrestricted Fusion has to be restricted.

5.1 ASSUMPTIONS

Before trying to make an argument along the lines of Florio and Linnebo, 2021's against unrestricted plural comprehension against Unrestricted Fusion, it is helpful to explicitly look at what the assumptions were in the plural case and see to what assumptions they correspond to in the mereological case.

As it turns out, one assumption, (UniSing), is hard to motivate in the case of mereology. Mereological fusions are singular by nature unlike pluralities which are plural. So mereological fusions are like sets in that they take the many as one and not like pluralities which take the many as many. Recall that one motivation for universal singularization by Florio and Linnebo was to also have singular objects corresponding to pluralities (2.5).

5.1.1 *The Plural Case*

Florio and Linnebo assume the following:

(UnrQuan) Unrestricted quantification, i. e. there is an all-encompassing domain,

(UnrComp) Unrestricted plural comprehension, i. e. for any condition there is a plurality consisting of all and only those objects that satisfy the condition (entails the existence of a universal plurality),

(UniSing) Universal singularization, i. e. there is a 'set of' function that maps any given plurality to an object (its set),

(Rigidity) Pluralities are rigid, i. e. they have the same members at every possible world at which they exist.

Florio and Linnebo argue for the first assumption by considering the alternative of relativism as unattractive and riddled with too many problems as was discussed in (2.2.1). The relativist does not seem to be able to even coherently articulate their position without either undermining it or not fully capturing its intended meaning.

The second assumption is part of traditional plural logic introduced in (2.1). This is the one which Florio and Linnebo reject based on their argument from (2.5.2) that domains of quantification can be extensionally indefinite and no universal plurality exists in such cases. They replace unrestricted plural comprehension with a restricted version in their critical plural logic and also adopt other axioms (2.4).

They argue for the third assumption in two ways. First, by considering the linguistic and set-theoretical practice which involves singular objects. Second, by their desire to understand sets via pluralities. As was discussed in (2.5), this is achieved by the 'set of' function and two bridge principles which establish a connection between the identity of pluralities and sets and also between the 'being an element of' relation and the 'being a member of' relation.

The fourth assumption is motivated by the idea that a plurality is 'nothing over and above' its members. This leads Florio and Linnebo to the conclusion that a plurality can be trivially tracked across possible worlds by just tracking its members, i. e. extensionally. Since this is also the only criteria of identity for pluralities, they can neither gain nor lose members as was discussed in (2.3.2).

Another background assumption they make is about the existence of mathematical objects: all it takes for them to exist is a consistent definition and the ability to understand the new object in terms of already existing ones. This is their liberal view of definitions. I am not listing this assumption separately because it is tightly connected to (3). The acceptance of the 'set of' function relies on the idea that the existence of the set is entailed by this function and that we can understand the set via already existing objects, namely the plurality. I. e. there is nothing more to the existence of that set than a consistent definition in terms of the already existing plurality.

5.1.2 *The Mereological Case*

Where does one stand in the case of mereology?

Ad (UnrQuan): The focus of the thesis is on the use of mereology as a formal tool in ontology or even as a formal ontology itself. Because of this, it was argued in (3.4) that one should also be in favor of unrestricted quantification since the domain of ontology is traditionally taken to be about everything there is. So this domain should be all-encompassing. Hence, there is a reason to assume that the domain of mereology should be an all-encompassing one (UnrQuan*), i. e. one should assume unrestricted quantification is possible.

Ad (UnrComp): Classical mereology as introduced in (3.2) features the Unrestricted Fusion axiom which entails that there is a fusion of all and only those objects that satisfy some condition. The only requirement is that the objects have to exist and then Unrestricted Fusion entails that there is a fusion of these objects. While this axiom is not uncontroversial, we have seen how it is usually motivated and defended in (3.3). Its proponents argue that any restriction on Unrestricted Fusion would be arbitrary or vague, a mereological fusion is no further ontological commitment given one has already accepted the existence of its parts, and that the usual restrictions cannot do the work they are commonly thought to do. Because of this, Unrestricted Fusion still enjoys widespread acceptance even given its somewhat controversial consequences. Thus, there is enough support to assume the mereological analogue of unrestricted plural comprehension (UnrFus*).

Ad (UniSing): One of the most obvious differences between the mereological fusion of some objects and the plurality of the same objects is that the former is a single object, whereas the latter is not. Mereological fusions are already singular (3.2), so there is no need for an additional singularization operation like the 'set of' one that maps them to their corresponding set. Florio and Linnebo motivate the need for such an operation by two considerations. First, singular objects like sets and mereological fusions are widespread in mathematics and linguistics, respectively, and they want to respect these practices. Second, they employ pluralities in order to understand sets better. The first of these considerations cannot be motivated in the same way since fusions are already singular objects. Hence, an argument from practice is more difficult to develop for mereology.

Ad (Rigidity): In (4.1.1) we saw the argument against the rigidity of parthood put forward by Florio and Linnebo. They argue that the set-theoretic case and the mereological case are different and hence no analogous argument can be made to the effect that mereological fusions are rigid. This conclusion was challenged in (4.3.1) using Jubien's epistemological distinction between ordinary objects and physical objects. Given this distinction, an object can be viewed as either one and this greatly influences our intuitions. I argued that Florio and Linnebo are too fast when they dismiss mereological rigidity since their argument is a case of object fixation: they view an object as an ordinary one and take an intuitive truth about it to also be true of the object considered as a physical object or a mereological fusion. Hence, their argument can be resisted this way. Furthermore, there is an analogous position to plural rigidity within the mereological literature called Mereological Essentialism. This position asserts that fusions have exactly the same parts in every possible world at which they exist. Hence, they are tracked extensionally across possible worlds and therefore rigid. So (Rigidity*) is warranted.

To sum up, I have argued that in the case of mereology the following assumptions should also hold:

(UnrQuan*) Unrestricted quantification, i. e. there is an all-encompassing domain,

(UnrComp*) Unrestricted Fusion, i. e. for any condition there is a fusion consisting of all and only those object that satisfy the condition (entails the existence of a universal fusion),

(Rigidity*) Mereological fusions are rigid, i. e. they have the same parts at every possible world at which they exist.

Universal singularization is a given although in a very different sense than with the 'set of' operation. Therefore, it would be misleading to consider it as an analogue of (UniSing). The analogue of it should rather be:

(UniSing*) Universal singularization, i. e. there is a 'set of' function that maps any given fusion to an objects (its set).

Mereological fusions are already singular. The fusion of some objects is singular just like the set of those objects is. However, the plurality of some objects is still plural. Hence, there is no need for a singularization operation just to get singular objects, whereas this is the case for plural logic.

The crucial difference between the fusion operation and the 'set of' operation is discussed in (5.2.1). There, I also show that the former is an unproblematic singularization from an ontological standpoint, whereas the latter is not. The fact that mereological fusions are singular also makes it more difficult to argue for the need of the 'set of' function in mereology as compared to plural logic.

5.2 THE ARGUMENT AGAINST UF

It is now time to see whether the argument by Florio and Linnebo against unrestricted plural comprehension can be replicated in mereology to the effect that Unrestricted Fusion has to be restricted. If the argument succeeds, it would show that there is a principled and well-motivated way of doing so – contrary to the arguments and claims we have seen in (3.3).

For the purpose of this, it is helpful to look at Florio and Linnebo's argument in terms of premises and conclusion as we have seen their full argument already in (2.5.2). Furthermore, this makes it easier to see where the assumptions are used at each step.

(P1) To define a plurality, some objects have to be circumscribed.

(P2) Circumscribed objects can be used to define their set.

(C1) The set did not exist before the objects were circumscribed.

(P3) The set is not a member of the universal plurality, but it should be.

(C2) Therefore, the universal plurality did not contain all objects which is a contradiction and hence it does not exist.

Now the straightforward mereological analogue of the argument looks like this:

(P1*) To define a mereological fusion, some objects have to be circumscribed.

(P2*) Circumscribed objects can be used to define their set.

(C1*) The set did not exist before the objects were circumscribed.

(P3*) The set is not part of the universal fusion, but it should be.

(C2*) Therefore, the universal fusion did not have all objects as parts which is a contradiction and hence it does not exist.

Let's see how far we can get with our current assumptions from (5.1.2).

In order to define a mereological fusion, some objects have to be specified. This makes sense given (Rigidity*) since the fusion of some objects is tracked extensionally across possible worlds so there is nothing more to the fusion than its parts. Hence, they have to be specified in order to track their fusion.

Already at (P2*), however, we encounter an issue in the mereological setting. Florio and Linnebo argue for (P2) since they want to understand sets via pluralities. To achieve this, they use two bridge principles which were introduced in the beginning of (2.5). These bridge principles ensure that the identity of sets and set-membership relation can be understood in terms of pluralities. Together with the 'set of' function, (UniSing), this ensures the existence and understanding of a set that corresponds to the plurality from (P1). Support for the 'set of' function came from practice based considerations stemming from linguistic and mathematics practice as discussed in (2.5 and their reliance on singularization. Given these bridge principles and the 'set of' function, there is an additional object which did not exist before circumscribing the plurality. As already mentioned, the problem for the mereological case now is that it is hard to motivate the 'set of' function, (UniSing), in a similar way since mereological fusions already are singularizations. I. e. they map some objects to a single object.

Here it is helpful to look at Lewis, 1991 since it is arguably the most worked out proposal to understand sets using mereology (and plural logic). To this extent he proposed some principles that explain sets and set-membership using mereological notions.[1] Since Lewis' treatment is quite complex and extensive, I will only state some of these principles as examples and not go into too much detail about individuals etc.

First Lewis states that "[a] *class* is any fusion of singletons" (Lewis, 1991, p. 16) and goes on to write that "a *set* is either the null set or else a class that is not a proper class" (Lewis, 1991, p. 4). Consequently, a set is a fusion of singletons as well, with the additional requirement that it is small, in a technical sense of the word (cf. Lewis, 1991, p. 89). Furthermore, he also states a principle that explains set-membership in terms of parthood: "x is a *member* of y iff y is a class and the singleton of x is part of y" (Lewis, 1991, p. 16). There is another principle about the identity of fusions, and since sets are fusions of singletons, it is about their identity as well: "*Uniqueness of Composition*: It never happens that the same things have two different fusions" (Lewis, 1991, p. 74).

While these principles seem quite similar to the bridge principles proposed by Florio and Linnebo, there is an important difference: they take pluralities and their bridge principles (and their critical plural logic more generally) to offer an explanatory account of sets. Lewis, however, takes sets to be mereological fusions of singletons. So Lewis is interested in an ontological reduction, whereas Florio and Linnebo are interested in an explanatory one.

He also explicitly considers the 'set of' function, which he calls singleton formation, but restricts it and keeps Unrestricted Fusion. Here is what Lewis writes:

> We dare not allow a *set* of all sets that are non-self-members, but there are two alternative ways to avoid it. One way would be to restrict composition [fusion] . . . [b]ut there is no good independent reason to restrict composition [fusion]. Mereology *per se* is unproblematic and not to blame for the set-theoretical paradoxes; . . . The better remedy, which I have adopted, is to restrict not composition [fusion], but rather the making of singletons. We *can* fuse all the singletons of sets that are non-self-members, thereby obtaining a proper class of sets, but this proper class does not in turn have a singleton. (Lewis, 1991, pp. 17–18)

By following Florio and Linnebo we take the way Lewis rejects, but also have a good reason to restrict Unrestricted Fusion, namely the

1 I specifically do not use the term 'bridge principles' here since Lewis does not use mereological fusions and the proposed principles to understand sets the way Florio and Linnebo use pluralities and their bridge principles to understand sets.

distinction between extensionally definite and extensionally indefinite domains.

Even though Lewis proposes principles that explain sets, their identity, and set-membership in terms of mereological notions, the issue is that he explicitly rejects that anything along the lines of the 'set of' function should always be applicable to fusions. Thus, new sets do not always come into existence even given these principles since the 'set of' function is restricted in some way. In particular, no proper class has a singleton by definition and also cannot be a member of anything (cf. Lewis, 1991, p. 98).

5.2.1 *Mereological Fusion as an Unproblematic Singularization Operation*

We have just seen that Lewis explicitly argues against the universal applicability of the 'set of' function in the case of mereology. But does one even need the 'set of' function to apply to every fusion? Is the fusion operation not enough to get the argument to work? This line of reasoning will be discussed now.

Consider the following idea someone might have: Mereological fusions are already singular, so when one circumscribes some objects, their mereological fusion exists by (UnrFus*) in addition to those objects. Therefore, there is an additional object which can then be used to get a new mereological fusion just like with the 'set of' function.

This line of reasoning is mistaken. A common understanding of mereological fusions asserts that the fusion of some objects is no further ontological commitment given that one has already accepted the objects which are part of it.[2] So, the fusion is no new object in an ontological sense unlike the set of these objects.

Furthermore, the fusion function differs from the 'set of' function in a very important way. Repeated application of the 'set of' function generates new objects, sets, at each step since some object is mapped it to its singleton. The singleton of that object is different from the object itself by the axiom of extensionality. Consider starting with x, then after the first application we get the singleton of x, $\{x\}$, after the second application we get the singleton of the singleton of x, $\{\{x\}\}$, and so forth. By extensionality, these sets are different as they do not have the same elements.

Repeated application of mereological fusion operation, on the other hand, does not work this way. The fusion operation is idempotent. I. e. repeated application leaves us with the same object we started with. If we apply fusion to some object x, we just get the same object x back. That is, the fusion of some object is not a new object in an ontological sense. This goes back to Goodman, 1977's principle of

2 Lewis and Varzi also appeal to this understanding when they defend Unrestricted Fusion as we have seen in (3.3).

nominalism. So no new object is generated this way unlike in the set-theoretic case. Or as Cotnoir and Varzi nicely put it:

> And if fusions are unique [which they are in classical mereology] . . . , this means we can never generate anything but the object itself when we apply the composition [fusion] operation to it; all we can get out of a given entity z is z itself. (Cotnoir and Varzi, 2021, pp. 203–204)

Therefore, the fusion operation as an analogue of the 'set of' operation does not generate new objects and thus does not lead to an ever growing ontology, i. e. an extensionally indefinite domain, although it is also an operation of singularization. Thus, not just any singularization operation is enough for the argument to work. One also needs that the operation entails the existence of new objects in an ontological sense and thereby leads to an ever growing domain.

Going back to the argument, this means that the universal fusion u has every object as part since the fusion operation does not generate any new objects. Thus, u actually is universal in the desired sense and does not vary in its parts.

5.2.2 *Adding (UniSing*)*

Even if (UniSing*) might be difficult to motivate and it was explicitly rejected in the literature, what happens if one assumes it? I. e. we grant that the 'set of' function can be motivated and assume it in the mereological context, contrary to Lewis.

By adding (UniSing*) to our assumptions, we get the second premise (P2*) in the mereological argument since we can apply the 'set of' function to any mereological fusion. (Rigidity*) is important here as well, since this assumption ensures that sets and mereological fusions have the same modal profile: both are modally rigid. They are trivially tracked across possible worlds by their elements or parts, respectively, i. e. extensionally. In particular, we can also apply the 'set of' function to the universal fusion u whose existence is entailed by (UnrFus*) and get the singleton of u, $\{u\}$. Therefore, we get the first conclusion (C1*) since this set did not exist before.

Note that in the case of the universal fusion u, the corresponding set we get is the universal set which does not exist in ZF(C) since it leads to inconsistency.[3] To resolve this, we can either restrict the

3 Mereology does not face an analogue of Cantor's Theorem. The closest one can get to a similar result has two additional conditions. The first one is the analogue of the one in plural logic, namely that the fusion has to have more than one object. The second one is new and states that the objects of the fusion must not overlap. Thus, the Mereological Cantor is as follows: "[I]f there is more that one φ, and the φ's are pairwise disjoint, then there are fewer (*mniej*) φ's than collective classes thereof" (Cotnoir and Varzi, 2021, p. 218). *Collective classes* is the term Leśniewski used for mereological fusions (cf. Cotnoir and Varzi, 2021, p. 216).

application of the 'set of' function to certain mereological fusions or restrict Unrestricted Fusion. The former is ruled out since one of our assumptions is (UniSing*), so we have to restrict Unrestricted Fusion. We now proceed with the argument that then shows how such a restriction can be non-arbitrary and motivated.

By the first conclusion (C1*), we know that there is an object, namely a set, which is not part of the universal fusion u. It should be part of it, however, since we assumed (UnrQuan*) and this gives us the third premise (P3*). Consequently, the universal fusion u does not have everything as part. This is a contradiction to its assumed universality and hence the existence of the universal fusion u has to be rejected. I. e. we get the second conclusion (C2*).

The idea of getting a new universal fusion u', by also including the singleton of the old universal fusion, $\{u\}$, does not work. In constructing u', one can again apply the 'set of' function. Then the argument works the same way and shows that u' does not include everything as part. Because of this, the domain over which we unrestrictedly quantify is extensionally indefinite. In constructing the universal fusion, new object(s) come into existence.

The universal fusion u, would also have to be extensionally indefinite to be truly universal. But given (Rigidity*) we know that mereological fusions are rigid and thus cannot vary regarding their parts. That is, the universal fusion u cannot have everything as part in an extensionally indefinite domain as this would violate (Rigidity*). This then also shows that there is a good reason to restrict Unrestricted Fusion.

The existence of the universal fusion u is entailed by Unrestricted Fusion. Since u cannot exist by the above argument, Unrestricted Fusion has to be restricted.[4] For example, it could be replaced by the mereological analogue of the axiom of separation:

$$\text{(Separation)} \quad \exists x \varphi x \to (\exists z F_\varphi z \leftrightarrow \exists y \forall x(\varphi x \to Pxy)).$$

This axiom only commits one to subfusions of some fusion. Just like the axiom of separation in ZF(C), it only commits one to subsets. This does not entail the existence of the universal fusion u anymore (cf Cotnoir and Varzi, 2021, p. 215).

To conclude, if we grant (UniSing*), and thereby also adopt Florio and Linnebo's view that a consistent definition of a mathematical objects is enough for its existence, then the argument also works in the mereological case.[5]

Furthermore, the argument also shows that there is a principled and well-motivated way to restrict Unrestricted Fusion, namely the

4 In theory, one could, of course, also reject one of the other assumptions, but the goal was to see if the argument can be used to make a case why Unrestricted Fusion (UnrFus*) should be rejected.

5 In order to fix the identity of sets and the membership-relation, one could adopt mererological analogues of Florio and Linnebo's bridge principles.

distinction between extensionally definite and extensionally indefinite domains. In extensionally indefinite domains, Unrestricted Fusion would lead to fusions that vary in what parts they have, contradicting (Rigidity*). While the additional assumption (UniSing*) is substantial, it shows that there is, after all, a way to restrict Unrestricted Fusion contrary to the claims we have seen in (3.3).[6]

To sum up, we have seen that the argument does not work with the fusion operation as an analogue of the 'set of' operation. The reason for this is that the fusion operation does not lead to new objects when applied repeatedly. If one assumes (UniSing*), i. e. that the 'set of' operation applies to every fusion, then the argument succeeds and shows that Unrestricted Fusion has to be rejected. As just mentioned, it also supplies a good reason to do so which makes the restriction non-arbitrary and motivated.

6 On another note, u's existence is also connected to some problems. For example, to which category should u belong to? What should be done when objects have u as a proper part such as propositions? (cf. Cotnoir and Varzi, 2021, pp. 218–220). For a discussion of the former see Simons, 2003 and the reply by Varzi, 2006. For the latter see Tillman and Fowler, 2012 who reject anti-symmetry (A2) and keep Unrestricted Fusion. Depending on how serious one takes these issues to be, this could be seen as indirect support for (UniSing*). But these considerations lie outside of the scope of the thesis.

6 CONCLUSION

The aim of this thesis was to investigate whether the argument against unrestricted plural comprehension, put forward by Florio and Linnebo, 2021, can be replicated in classical mereology to the extent that Unrestricted Fusion has to be rejected. As I showed in (5), the answer is negative unless one assumes that the 'set of' operation applies to every mereological fusion, i. e. (UniSing*) is a necessary assumption.

Since the four assumptions (UnrQuan), (UnrComp), (UniSing), and (Rigidity) served to also structure the thesis, I use them again here to highlight the main findings.

In chapter three (3), the mereological analogues of the first three assumptions, (UnrQuan*), (UnrFus*), and (UniSing*), were addressed although the latter only briefly. First, I identified Unrestricted Fusion in mereology with unrestricted plural comprehension in plural logic and, by looking at proponents of the former, showed that it is supported by different arguments in the mereological literature. Second, I argued that (UnrQuan*) is warranted as well if one uses mereology as a (tool in) formal ontology. The reason is that the domain of ontology is traditionally taken to be about everything there is. Consequently, it is sensible to assume that the domain of mereology is all-encompassing. Third, I claimed that identifying the mereological fusion operation with (UniSing) would be misleading since it differs from the 'set of' operation in an important aspect.

In chapter four (4), the mereological analogue of the third assumption, (Rigidity*) was addressed. First, I identified Mereological Essentialism in mereology with plural rigidity in plural logic. This makes sense since the former states that mereological fusions have the same parts at every world at which they exist which exactly parallels plural rigidity. Second, since Mereological Essentialism is a controversial position, I located the basis of disagreement and evaluated different arguments in favor of it. Third, I used the epistemological distinction by Jubien, 2001 to challenge Florio and Linnebo's conclusion that mereological fusions are not rigid and to argue for the plausibility of Mereological Essentialism for fusions.

In chapter five (5), the mereological analogue of the third assumption, (UniSing*), was addressed in detail and the mereological argument was reconstructed. First, I argued that this is a substantial assumption for which it is difficult to argue with respect to classical mereology. In the case of plural logic, this was motivated by the desire to have singular objects corresponding to the pluralities. However, mereological fusions are already singular objects. Second, I showed that the fusion operation alone is not enough for the argument to work. The reason is that it is an unproblematic singularization operation from an ontological viewpoint. That is, repeated application of it does not yield new objects unlike the 'set of' operation which is needed for the argument to work. Third, I demonstrated that the argument succeeds in the mereological case if (UniSing*) is granted. Hence, the universal fusion u and thereby also Unrestricted Fusion have to be rejected.

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