Validity of practical inference

Berislav Žarnić

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Abstract

The paper develops a dynamic framework suitable for analysis of practical reasoning. The application of functional approach in dynamic semantics confirms the results obtained in philosophical analyses of practical inference, regarding, in particular, defeasibility, undetachability and distinctive quality of conclusion. Contradictory results of proposed tests of validity and conflicting intuitions on valid forms of practical inference are reconciled.

The paper is divided in four parts. In Introduction the notion of practical inference is defined. The second part displays the syntax and semantics of language of practical $Fiat/Est$ logic. The third part gives extensive elaboration of reasons for proposed formal semantical solutions. In the fourth part, the notion of validity of practical inference is defined and the decision procedure is given.

1 Practical inference

Aristotle discovered that practical inference is a type of inference different from the theoretical one. Although he did not give an explicit account on the nature of 'practical syllogism', its attributes are implicitly defined. The first attribute seems obvious: in practical inference at least one premise sets a goal. The existence of a goal-sentence in the premise set is the minimal condition to be included in *definiens* of practical inference, and, it may be reasonably argued, a sufficient one.

The clear way of describing the process of (instrumental) practical reasoning in Aristotle's *Nichomahean Ethics* (1112b, 11-14) deserves our attention:

> We deliberate not about ends but about means. For a doctor does not deliberate whether he shall heal, nor an orator whether he shall persuade, nor a statesman whether he shall produce law and order, nor does any one deliberate about his end. They assume the end and consider how and by what means it is to be attained; and if it seems to be produced by several means they consider by which it is most easily and best produced, while if it is achieved by one only they consider how it will be achieved by this and by what means this will be achieved till they come to the first cause, which in the order of discovery is last...And if we come on an impossible, we give up the

search, e.g. if we need money and this cannot be got; but if a thing appears possible we try to do it.

1.1 Modelling deliberation process

An agent's knowledge increment is usually described as a reduction of uncertainty: having more information means leaving fewer possibilities open. In the folklore account, an agent's initial state (zero-information) is a state in which no possibility is excluded, the set of possibilities shrinks when new information is received. Take a simple example, the agent comes to know that (1) $p \rightarrow q$ and that (2) $p \vee q$; a kind of 'possibility elimination table' may serve as a substitute for truth tables:

possibility	p,q	p	q	$\emptyset$
eliminated by		1		2

The logical consequence of (1) and (2) is any proposition holding in non-eliminated possibilities.

The similar approach may be adopted in modeling deliberation which may be represented as a process of evolution of an agent's motivational state, leading to states with lesser uncertainty regarding acceptability of *goals.* The zero-information state is the one in which 'anything could be a case', similarly, the initial motivational state would be a state where 'anything may become a goal'.

The semantics for goal sentences requires introduction of an ordering relation on possibilities since the least condition for a state of affairs being a goal lies in the fact that occurrence of the state is preferred to its non-occurrence. That primitive notion suggests that a structure and its reductive development could provide a model for goal-sentences and deliberation process. In deliberation, as the typical form of practical reasoning, descriptive[1] (factual) and proscriptive (goal) sentences interact. A structure whose reductive evolution should model the process of deliberation has to do two jobs simultaneously: it must account for informational as well as motivational changes.

The dynamic semantics for goal sentences may be given according to general prescription "the meaning of a sentence is the change an utterance of it brings about" (see [20]). For that purpose a hierarchy of interpretations

[1] In the course of this paper 'factual', 'descriptive', '*Est*', and 'indicative' sentences; and 'goal', 'proscriptive', '*Fiat*', and 'imperative' sentences are being used as synonyms.

must be constructed, starting from valuations of propositional letters, and ending with model variations (see Section2), since 'propositional attitude' sentences consist of attitude ('direction of fit', mood, tropic) indicator and propositional content (phrastic). Kenny [26] used '*Fiat*' and '*Est*' as indicators of two principal attitudes. Since both sentence types may occur in practical inference, the variations which should represent their meanings must be different variations of the same structure.

The basic problem of giving a 'dynamic semantics' for a language consists in defining the notion of 'a sentence being *accepted* in a *state*', and, for 'update semantics' as a special case, the notion of 'a sentence being *acceptable* in a *state*'. The notion of change implies the difference between two states. If the meaning of the sentence is the change it brings about, then its meaning equals the difference between a state in which it is not accepted and the state in which it is. The meaningfulness of a sentence is relative: it depends on the agent's accepting state whether a sentence is relatively meaningless or not, and it depends on class of pairs of states whether a sentence is absolutely devoid of meaning. The former (relativistic) view on meaning provides rationale for functional approach with sentences being interpreted as functions, taking a state as an argument and delivering a state in which that sentence is accepted as a value. The second (absolutistic) view favors relational approach, interpreting a sentence φ as binary relation consisting of pairs of states $\langle \sigma, \sigma' \rangle$ which satisfy two conditions (i) sentence φ is accepted in the state σ', and (ii) pair $\langle \sigma, \sigma' \rangle$ is an element of an ordering relation, intuitively understood as minimal change on σ resulting with state in which φ is accepted. The dynamic notion of the 'acceptance' must be correlated to a corresponding static notion; and for the $Fiat/Est$ logic that is the notion of 'a sentence being verified by a model'(Definition 4). The 'update semantics' is 'a forward looking system', in which 'uncertainty increment' is not representable. In that framework we encounter the notion of acceptability as a pure dynamic notion, definable only in terms of possible evolution of a state (see Definition 9). The 'update semantics' is close to classical logic in so far as its primary purpose lies in defining a notion of validity, and not in modelling cognitive or motivational advances and retreats of a rational agent.

The additional expressive power of dynamic semantics is gained in virtue of having at our disposal four semantic values: a sentence is not only accepted or not accepted in a state, but it is also acceptable or not acceptable. In virtue of that fact a dynamic interpretation may go further than the static

one since it allows for finer grained semantic values. The list of motives for opting for dynamic semantics may be enlarged: besides the needs for dynamic connectives or dynamic notions of inference [31], there is also a need for dynamic semantic values.

1.2 Goal sentences

The sentences in optative or in imperative mood assign the goal-status to a particular state of affairs (or an act involved in its realization) or they exhibit a desirable ('goal-worthy') character of a type of states. Disregarding their pragmatic differences, we might say that those sentences express a *pro*-attitude of an agent. It seems that the term 'goal sentence' can cover different types of sentences characterized by having specific 'direction of fit with the world'. Under the term 'goal sentence' we may subsume a large class of sentence types, ranging from those describing a (desirable, obligatory, valuable, etc.) property of a type of actions, like 'Promises are to be kept', through those expressing the preference relations between states, like 'She prefers Mozart to Bartok', up to those which assign to a particular state the status of a goal or preferred state, like 'Open the door!' or 'I wish it was not raining.' All mentioned types of sentences have something in common: their acceptability is not dependent on the world in the way acceptability of indicative sentences is; broken promises, listening to Bartok, closed doors and falling rain do not affect the semantic value of those sentences.

1.2.1 Moods and preferences

Two principal ways of analyzing goal-sentences have been pursued: one takes preference relation as primitive while the other finds the goal-status to be a *residuum* of analysis. For instance, Davidson followed the 'preference approach' and took the universal form of a *pro*-attitude to be of the form

$$'prima\ facie\,(\forall x \forall y\,((Ax \wedge \neg Ay) \rightarrow x \succ y))\,'$$

with variables ranging over action events, predicates designating action types, and '$\succ$' expressing relative relation of preference, relativity being indicated by *prima facie* sentential operator. Singular goal-sentence expresses the desirability of an action event a under description A :

$$prima\ facie\,(a \succ b, (prima\ facie \forall x \forall y\,((Ax \wedge \neg Ay) \rightarrow x \succ y)) \wedge (Aa \wedge \neg Ab))$$

with *prima facie* functioning this time as sentential operator indicating that singular sentence ($a \succ b$) may not be detached from its inferential basis. The preference approach took von Wright in defining a desirable state s (under the circumstances C_i):"...*s would be* a good thing (to have) in the world w shall mean that $sP\neg s$ when $\neg s \wedge C_i = w$' [37] (p. 165.); or, in a unrefined way, by 'p is *good* (desirable) iff $pP\neg p$'. The other approach takes desirability, rather than preference, as a primitive notion. For instance, Kenny represents the difference between goal-sentences (imperative and optative) and factual-sentences (indicative) by introducing mood indicators $Fiat$ and Est, respectively. The idea is that mood indicators correspond to different 'directions of fit' between propositional content and the 'world'. For, instance the imperative or optative sentence of the form $Fiat(p)$ and indicative sentence $Est(p)$ have the same descriptive content (phrastic, propositional content) and different attitude towards it (tropic). The two approaches in formalization of goal sentences, relational and modal, are connected to main branches of practical logic: the former to the preference logic and the latter to deontic logic and to the logic of commands. Those approaches to formalization may be reconciled by retaining mood indicators for syntactic purposes, and using preference relation in their interpretation.

2 The language of *Fiat/Est* logic

2.1 Syntax of $L_{F/E}$

1. Primitive symbols of $L_{F/E}$ are the following:

   (a) Mood indicators: '$Fiat$', 'Est'.

   (b) Propositional letters: 'p', 'q',... .

   (c) Connectives: '$\neg$', '$\vee$'

   (d) Modality signs: '$\Box$', '$\Diamond$' .

   (e) Auxiliary symbols: '(', ')'.

2. The rules of formation:

   - All propositional letters $p, q, r, ...$ are atomic sentences of descriptive language L_D

- If φ and ψ are sentences of L_D then $\neg\varphi$, $\varphi \wedge \psi$ are compound sentences of L_D.
- Other operators in L_D are defined in standard way:$(\varphi \vee \psi)$ as $\neg(\neg\varphi \wedge \neg\psi)$, $(\varphi \rightarrow \psi)$as $(\neg\varphi \vee \psi)$
- All non-tautological sentences of L_D prefixed with mood indicator are *elementary sentences* of language $L_{F/E}$; if $\varphi \in L_D$ then $Fiat(\varphi)$, $Est(\varphi) \in L_{F/E}$
- If $\varphi, \psi \in L_D$ then $Fiat(\Diamond\varphi)$, $Est(\Diamond\varphi) \in L_{F/E}$
- Modal operator '$\Box$' is defined as '$\neg\Diamond\neg$'.
- If φ and ψ are sentences of $L_{F/E}$ then $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \rightarrow \psi$, $\diamond\varphi$ are sentences of $L_{F/E}$.

2.2 Semantics

2.2.1 Static semantics

Static semantics for L_D

- D is set of all propositional letters of L_D.
- Possible world w is an element of powerset of D.
- Valuation of a propositional letter p in a possible world w is a characteristic function $\mathcal{V}_w$ such that $\mathcal{V}_w(p) = \begin{cases} \top, \text{ if } p \in w \\ \bot, \text{ if } p \notin w \end{cases}$
- Valuation of compound sentences of L_D is defined in the standard way: $\mathcal{V}_w(\neg\varphi) = \top$ iff $\mathcal{V}_w(\varphi) = \bot$; $\mathcal{V}_w(\varphi \wedge \psi) = \top$ iff $\mathcal{V}_w(\varphi) = \top$ and $\mathcal{V}_w(\psi) = \top$

Static semantics for $L_{F/E}$ Elementary sentences of $L_{F/E}$ are valuated in a relational structure $\sigma = \langle P, R\rangle$.

Definition 1 *A relational structure σ is a an ordered pair $\langle P, R\rangle$, of which*

- P is a set of possible worlds, $P \subseteq W$ and $W = \mathcal{P}(A))$
- R is an accessibility relation, $R \subseteq P \times P$

Valuation $\mathcal{V}_{(\sigma,w)}$ of elementary sentence of $L_{F/E}$ in possible world w given model σ depends on valuation of their propositional content (*i.e.* sentences of L_D) and accessibility relation R.

Definition 2 *Valuation of elementary goal sentence* $Fiat(\varphi)$ *in possible world* w *given the model* σ

$$\mathcal{V}_{(\sigma,w)}(Fiat(\varphi)) = \top \text{ iff } \forall v\,((wRv \rightarrow \mathcal{V}_v(\varphi) = \bot) \wedge (vRw \rightarrow \mathcal{V}_v(\varphi) = \top))$$

Definition 3 *Valuation of elementary factual sentence* $Est(\varphi)$ *in possible world* w *given the model* σ

$$\mathcal{V}_{(\sigma,w)}(Est(\varphi)) = \top \text{ iff } \forall v(wRv \rightarrow \mathcal{V}_v(\varphi) = \top)$$

Definition 4 *A model* σ *verifies an elementary sentence* φ *of* $L_{F/E}$ *iff* φ *is true in every possible world* w*;*

$$\sigma \models \varphi \text{ iff } \forall w\mathcal{V}_{(\sigma,w)}(\varphi)) = \top$$

or, in a shorthand notation, $\sigma \models \varphi$ iff $\forall w(w \models \varphi)$

2.2.2 Dynamic semantics of $L_{F/E}$.

Dynamic interpretation is a function $[\cdot]$ which assigns to each sentence $\varphi \in L_{F/E}$ an operation $[\varphi]$ on the set Σ of model variations. Sentence function $[\varphi]$ takes model σ as argument and it delivers minimal model variation σ' that verifies φ as its value.

Definition 5 *Model* $\sigma' = \langle P', R' \rangle$ *is a minimal model variation on* $\sigma = \langle P, R \rangle$ *with respect to sentence* φ *iff*

- σ' differs from σ *at most* in its value for R, and
- $\sigma' \models \varphi$ and there is no $\sigma'' = \langle P'', R'' \rangle$ such that $\sigma'' \models \varphi$ and $R' \subset R'' \subset R$

or

- σ' differs from σ in its value for P, and
- $\sigma' \models \varphi$ and there is no $\sigma'' = \langle P'', R'' \rangle$ such that $\sigma'' \models \varphi$ and $P' \subset P'' \subset P$

The notation for R-variations is '$\sigma =_R \sigma'$'; the notation for P-variations is '$\sigma =_P \sigma'$'.

Definition 6 *Dynamic interpretation of an elementary sentence $\varphi \in L_{F/E}$ is a function $[\varphi] : \Sigma \to \Sigma$, such that $[\varphi]\,\sigma = \sigma'$ iff*

- $\sigma' \models \varphi$
- σ' is a minimal model variation.

Remark 1 *The first part of the above definition provides the bridge between static and dynamic semantics since: $[\varphi]\,\sigma = \sigma'$ only if $\sigma' \models \varphi$. The corresponding dynamic proposition is a stronger one: $[\varphi]\,\sigma = \sigma'$ iff $\sigma' = [\varphi]\,\sigma'$.*

Definition 7 *Set Σ of model variations: $\Sigma = \{\sigma \mid \sigma = 0 \vee \sigma \in \mathcal{F} \vee \sigma \in \mathcal{I}\}$*

- Unrestricted set of model variations

$$\Sigma^{\mathbf{0}} = \{\langle P, R \rangle \mid P \subseteq W \wedge R \subseteq P \times P\}.$$

- Σ is restricted set of states or model variations: $\Sigma \subset \Sigma^{\mathbf{0}}$;
- Initial state 0 is model $0 = \langle W, W \times W \rangle$.
- Class of final states $\mathcal{F} = \{\langle P, R \rangle \mid P = \emptyset \vee R = \emptyset\}$.
- Class of intermediate states $\mathcal{I}$

$$\mathcal{I} = \{\langle P, R \rangle \notin \mathcal{F} \mid ((wRv \wedge vRw) \to (vRv \wedge wRw)) \wedge ((wRv \wedge vRz) \to wRz)\}.$$

- Class of motivational states $\mathcal{M}$

$$\mathcal{M} = \{\langle P, R \rangle \in \mathcal{I} \mid wRv \to (\neg vRv \wedge \neg vRw)\}$$

Proposition 8 *Sentence functions are eliminative:*

$\forall\sigma\forall\sigma'\left([\varphi]\,\sigma = \sigma' \rightarrow (P' \subseteq P \wedge R' \subseteq R)\right)$, where $\sigma = \langle P, R\rangle$, $\sigma' = \langle P', R'\rangle$

Definition 9 *Relative semantic values of a sentence* φ

- A sentence φ is acceptable in model σ iff $[\varphi]\,\sigma \notin \mathcal{F}$.
- A sentence φ is accepted in model σ iff $[\varphi]\,\sigma = \sigma$

Remark 2 *Corresponding static definitions are: a sentence* φ *is acceptable in model* σ *iff* $[\varphi]\,\sigma \models \varphi \wedge [\varphi]\,\sigma \notin \mathcal{F}$*; a sentence* φ *is accepted in model* σ *iff* $\sigma \models \varphi$.

- In a final state $\sigma \in \mathcal{F}$ every statement is accepted and no statement is acceptable.

$$\forall\varphi_{\in L_{F/E}}\left(\sigma \in \mathcal{F} \rightarrow [\varphi]\,\sigma = \sigma\right)$$

- A sentence φ may have one of the following semantic values relative to a state σ:
  (a) φ is accepted and acceptable in σ : $[\varphi]\,\sigma = \sigma \wedge \sigma \notin \mathcal{F}$
  (b) φ is accepted and not-acceptable in σ: $[\varphi]\,\sigma = \sigma \wedge \sigma \in \mathcal{F}$
  (c) φ is not-accepted and acceptable in σ: $[\varphi]\,\sigma \neq \sigma \wedge [\varphi]\,\sigma \notin \mathcal{F}$
  (d) φ is not-accepted and not-acceptable in σ : $[\varphi]\,\sigma \neq \sigma \wedge [\varphi]\,\sigma \in \mathcal{F}$

Initial sentence functions

Definition 10 *Initial sentence functions are dynamically interpreted elementary sentences of* $L_{F/E}$*. Initial sentence functions are functions whose values differ from their arguments at most in* R.

Proposition 11 *If* φ *is an elementary Fiat-sentence then* $[\varphi]\,\sigma \in \mathcal{M}$ *or* $[\varphi]\,\sigma \in \mathcal{F}$.

Recursive sentence functions

Definition 12 *$P-$variation Est-sentence functions*

- $[Est(\Box(\varphi))]$ is P-variation sentence function:$[Est(\Box(\varphi))]\sigma =_P \sigma'$
- Taking M as general mood indicator, a P-variation sentence function may be recursively defined by:

$$[Est(\Box(\varphi))]\sigma = \sigma' \text{ iff } \forall\sigma''(\sigma'' =_R \sigma' \rightarrow [M(\neg\varphi)]\sigma'' \in \mathcal{F})$$

and that equals

$$\begin{aligned}[Est(\Box(\varphi))]\sigma &= \sigma', \text{ where } \sigma = \langle P, R\rangle, \sigma' = \langle P', R'\rangle, \text{ and} \\ P' &= \{w \in P \mid w \models \varphi\}, \text{ and} \\ R' &= \{(w,v) \in R \mid w \in P' \wedge v \in P'\}\end{aligned}$$

Definition 13 *Modal $Fiat$-sentences*

- for $\varphi \in L_D$:

$$[Fiat(\Diamond\varphi)]\sigma = \begin{cases} \sigma, \text{ if } \sigma \in \mathcal{M} \wedge [Fiat(\varphi)]\sigma \notin F \wedge [Fiat(\neg\varphi)]\sigma \in F \\ \langle\emptyset,\emptyset\rangle, \text{ otherwise}\end{cases}$$

Definition 14 *Simple test sentence functions*

- For $\varphi \in L_{F/E}$: $[\Diamond\varphi]\sigma = \begin{cases} \sigma, \text{ if } [\varphi]\sigma \notin \mathcal{F} \\ \langle\emptyset,\emptyset\rangle, \text{ otherwise}\end{cases}$

Definition 15 *Recursive R-variation sentence functions:*

- $For\varphi \in L_D : [\neg Fiat(\varphi)]\sigma = \begin{cases} \sigma' \text{ , if } [Fiat(\varphi)]\sigma' \in \mathcal{F} \wedge \\ \wedge\ \sigma' \in \mathcal{M} \wedge [Fiat(\neg\varphi)]\sigma' \notin \mathcal{F} \\ \langle\emptyset,\emptyset\rangle, \text{ otherwise}\end{cases}$
- For $\varphi \in L_D$: $[\neg Est(\varphi)]\sigma = [Est(\neg\varphi)]\sigma$
- For $\varphi, \psi \in L_{F/E}$: $[\varphi \wedge \psi]\sigma = [\psi]([\varphi]\sigma)$
- For $\varphi, \psi \in L_D$: $[Est(\varphi) \rightarrow Fiat(\psi)]\sigma = \begin{cases} [Fiat(\psi)]\sigma, \text{ if } [Est(\varphi)]\sigma = \sigma \\ \sigma, \text{ otherwise}\end{cases}$
- For $\varphi, \psi \in L_D$: $[Fiat(\varphi) \vee Fiat(\psi)]\sigma = [Fiat(\varphi \vee \psi)]\sigma$
- For $\varphi, \psi \in L_D$: $[Est(\varphi) \vee Est(\psi)]\sigma = [Est(\varphi \vee \psi)]\sigma$

3 Dynamic semantics and practical logic

3.1 States, sentences and models

Practical reasoning is usually represented as the process of transmission of the motive force from the end to the means [27]. Taking means-end reasoning as a typical case, we may represent practical reasoning as a sequence of state transitions starting with a motivational state in which an elementary goal-sentence $Fiat(\varphi)$ is accepted and ending with an extended motivational state.

Definition 16 *Extended motivational state is a state in which an elementary goal-sentence* $Fiat(\varphi)$ *and a modal goal-sentence* $Fiat\,(\Diamond(\psi))$ *are accepted, provided that resulting state is not a final one and that* φ *is not equivalent to* ψ.

For example, $[Fiat(p \wedge q)]\,0$ is an extended motivational state (in a weaker sense), since $Fiat(\Diamond p)$ is accepted in it. More interesting are strongly extended motivational states in which an elementary goal-sentence $Fiat(\varphi)$ and a modal goal-sentence $Fiat\,(\Diamond(\psi))$ are accepted, and in which φ *does not entail* ψ. Strongly extended motivational states are characteristic of means-end reasoning.

The class of states in which a genuine goal is accepted is not identical to class of states in which a possible subgoal is accepted. In philosophical literature the similar distinction may be found, such as the difference between 'unmotivated desire' and 'motivated desire' [35], or difference between 'reason-giving force of intentions' and 'desiderative reasons without reason giving-force' [5].

motivational state	goal accepted	state
pure	goal φ	$[Fiat(\varphi)]\,0$ where $\varphi \in D$
extended	goal φ possible subgoal ψ	$[Fiat(\varphi)]\,0 =$ $= [Fiat\,(\Diamond\psi)]\,([Fiat(\varphi)]\,0)$ where φ does not entail ψ

In the context of practical inference ends and means have different phenomenology, the first being genuine goal, while the latter is relative and possible goal. The conflicting intuitions on the validity of forms of practical inference arouse from the failure to observe the fact that semantics for

genuine goal sentences (ends) must differ from the semantics for derived goal sentences expressing relative and possible goal status (possible means). For example, Devlin ([12] p.203) writes:

> In general, if agent A believes/desires/sees/see that $\varphi \wedge \psi$, then A believes/desires/sees/see that φ.

On the contrary, Geach ([17] p.79.) claims:

> A more surprising result of Kenny's theory is that in practical reasoning the fiat $Fiat(p \wedge q)$ is not deductively equivalent to the pair $Fiat(p)$, $Fiat(q)$...This is not really a paradox; the equivalence supposed would lead to an absurd result. For by a parity of reasoning the set $Fiat(p)$, $Fiat(q)$, $Fiat(r)$... would be deductively equivalent to $Fiat(p \wedge q \wedge r \wedge ...)$. But this last fiat could be fulfilled only by a policy guaranteed to satisfy all our wants at once...

The distinction between two types of states and two types of goal sentences can reconcile conflicting intuitions (see p.4): '$Fiat\,(\,\varphi \wedge \psi)$, therefore $Fiat(\varphi)$' is not a valid piece of practical reasoning, but similar '$Fiat\,(\,\varphi \wedge \psi)$, therefore $Fiat(\Diamond\varphi)$' is a valid one.

3.2 Genealogy and satisfaction

There are two main types of intermediate states ($\mathcal{I}$) an agent traverses in process of deliberation: motivational states ($\mathcal{M}$) comprising pure motivational and extended motivational states, and not-motivational cognitive states ($\mathcal{I} \setminus \mathcal{M}$). The other non-motivational states include initial state (0) and final states ($\mathcal{F}$). Depending on its genealogy the final state $\langle P, \emptyset \rangle$ may be interpreted as a state of satisfaction or as a state of incoherence. Final state is reached when motivational cycle ends with a non-motivational state. The class of non-motivational states includes initial state indexed with '0', final states of the type $\langle P, \emptyset \rangle$ and absurd state $\langle \emptyset, \emptyset \rangle$. The subtle difference can be made in the way the ending point of motivational extension is reached. Both satisfaction and incoherence result with a final state. The difference lies in its genesis: if $\sigma \in \mathcal{M}$ and $[Est\,(\varphi)]\,\sigma = \langle P, \emptyset \rangle$ then $\langle P, \emptyset \rangle$ is a state of satisfaction if reached by accepting sentence $Est\,(\varphi)$ in the state σ, similarly, if $\sigma \in \mathcal{I} \setminus \mathcal{M}$ and $[Fiat\,(\varphi)]\,\sigma = \langle P, \emptyset \rangle$ than $\langle P, \emptyset \rangle$ is a state of satisfaction if reached by accepting sentence $Fiat\,(\varphi)$ in the state σ. The corollary for

architecture of rational agents follows, stating that *only agents with capacity of remembering may be satisfied*.

Two principles summarize the importance of state genealogy:

- by Humean '*desire-in, desire out* principle' extended motivational state requires that at least one of previous states was pure motivational. The term is taken from Wallace who [35] fixes its meaning by:

  "[.] ..processes of thought which give rise to a desire (as 'output') can always be tracked back to a further desire (as 'input')..."(p.170)

- by principle of '*reflective satisfaction*' reaching the final state of satisfaction presupposes the difference in the 'direction of fit' of accepted sentences since

  - a final state $\sigma' = \langle P, \emptyset \rangle$ is a state of satisfaction only if a previous state σ is a motivational one and σ' is reached by accepting $Est\,(\varphi)$, or previous state σ is a non-motivational intermediate state and σ' is reached by accepting $Fiat\,(\varphi)$.

3.3 Two kinds of intentional states or restricted evolution of a state?

The standard approach in the analysis of propositional attitudes, i.e. intentional states, uses the distinction between *noema* and *noesis* or 'direction of fit' and 'propositional content' . The difference in 'direction of fit' is partially reflected in the language as difference between moods of a sentence, therefore it seems natural to formalize propositional attitude sentences by introducing mood indicators. At the most general level intentional states are divided into states with which world must fit and states which must fit with the world, justifying division of propositional attitude sentences into $fiats$ and $ests$. The difference between two basic types of intentional states Smith [32]describes (p.54.):

"For the difference between beliefs and desires in terms of their direction of fit comes down to the difference between the counterfactual dependence of a belief and desire that p, on a perception that *not* p: roughly, a belief that p is a state that tends to go of existence in

the presence of a perception that *not* p, whereas a desire that p is a state that tends to endure, disposing the subject in that state to bring about that p.'

The dynamic functional interpretation of $Fiat/Est$ language reflects mentioned properties of intentional states by assigning a final state/model for successive accepting of contradictory *est* sentences:

$$\forall\sigma\,[Est\,(\neg\varphi)]\,([Est\,(\varphi)]\,\sigma) \in \mathcal{F}$$

and by assigning a motivational state/model for successive accepting of sentences with different mood indicator and with contradictory propositions in their *phrastic*:

$$[Est\,(\neg\varphi)]\,([Fiat\,(\varphi)]\,0) \notin \mathcal{F}$$

$$\forall\sigma\,([Fiat\,(\varphi)]\,\sigma \notin \mathcal{F} \rightarrow [Est\,(\neg\varphi)]\,([Fiat\,(\varphi)]\,\sigma) = [Fiat\,(\varphi)]\,\sigma)$$

Still, the idea that there are two basic kinds of intentional states (supposing that direction of fit is taken as *principium divisionis*) together with semantical assumption that agent's states are models leads to unacceptable consequence that models for *fiat* and *est* sentences should be different. For the purpose of defining the validity of practical inference with mixed premises we must suppose that the same structure may do static modelling job for both types of sentences. A *fiat* sentence may be verified only by a model of motivational type (relational structures $\mathcal{M}$), while an *est* sentence may be verified also by a model of cognitive type (structures $\mathcal{I} \setminus \mathcal{M}$). Therefore, instead of two basic types of intentional states, there is one family of generated model variations (Definition7) and one set of restrictions imposed on reductive evolution of an initial model, reflecting the differences between two types of sentences.

From a standpoint of dynamic semantics an agent's state can not be individuated by propositional content of her beliefs and desires. For example, attribution of belief to an agent does not imply individuation of an informational state, although it does imply characterization.. When we know that agent A believes that p we do not know in which state she is, but we know the class to which her state belongs. The philosophical insight into 'holism of the mental field' [11] becomes accounted for by difference between the

accepted sentence and the state in which it is accepted. The same sentence may be accepted by different states, therefore the predictive power of rationalizing explanations depends on the number of accepted sentences taken into account.

The peculiar nature of interpretation of $Fiat/Est$ language lies in the fact that it covers two, semantically different types of sentences. No matter whether static or dynamic approach is adopted, the semantics for mixed language requires one model for both sentence types. That methodological fact forces us into philosophical thesis that an agent's motivational state is also a cognitive one. In that spirit, Cross [10] uses one static model with two accessibility relations, one holding between equally desirable possible worlds, the other holding between the possible worlds indistinguishable on the basis of available information. Kenny in [26], in a non-modal approach attempted a reduction of goal sentences by their conditions of satisfaction[2]. Here proposed approach is similar to Cross' [10] in treating propositional attitude sentences as modal and similar to Kenny's inasmuch as it relies on twofold divison of sentence types.

Motivational state can be described as 'a state in which a $fiat$ sentence is accepted', while pure cognitive state may be described as 'a state in which an est sentence is accepted and in which no $fiat$ is accepted'. Motivational states are also cognitive since (by Definitions 2 and 4) acceptance of a goal implies believing its not realized (Proposition 26). It is obvious that under these assumptions 'motivation extension' is understood only in practical sense as 'having a reason to accept a $fiat$ sentence', and not in theoretical sense of 'having a reason to make sure whether something is the case'(see Definition27).

[2]"Fiats contain descriptions of possible states os affairs whose actualization satisfies the desire expressed by them." ([26]p.71). We can define 'satisfactoriness' in terms of 'satisfaction', "A is satisfactory relative to set G iif if A is satisfied every member of G is satisfied." [26] (p.73). The idea of reduction is given by the sequence 'satisfactory-satisfied-actualized'

3.4 Goals and possibilities

It seems that there is a common agreement that a goal may not be actual[3] and that it must be possible. Take for example Devlin's generalization ([12] p.210):

$intend(A) \Longrightarrow desire(A) \wedge beleive(\Diamond A)$[4]

The difference between non-actual/possible and non-actual/impossible states of affairs must be representable in language of practical logic. The rules eliminate some states of affairs as being both non-actual and impossible, while facts exclude only non-actual states. The generality of a rule may be preserved in dynamic propositional modal logic by assigning a different type of model variation to conditionals (see Definition12). Such a reading of conditional is akin in spirit to Lewis' '$\neg \diamond (\varphi \wedge \neg\psi)$'. The structural transformations must reflect the norm by which it is rational to desire something which is not the case, while it is not rational to desire the impossible. For this reason, the notion of epistemic possibility must be supplemented with the notion of practical possibility.

Definition 17 *A state of affairs in which φ holds is epistemically possible for an agent in a state σ if an elementary sentence $Est(\varphi)$ is acceptable, $[\Diamond Est\,(\varphi)]\,\sigma \notin \mathcal{F}$ (see also Definition14).*

Definition 18 *A state of affairs in which φ holds is practically possible for an agent in a state σ if the sentence $Fiat(\varphi)$ is acceptable, $[Est\,(\Diamond\varphi)]\,\sigma \notin \mathcal{F}$ (see also Definition12).*

Epistemic possibility implies practical (Proposition 19), but the converse does not hold (Proposition 20); practical possibility is 'immune' to facts (Proposition22), while epistemic is not (Proposition21). The practically impossible state may not become a goal (Proposition24)

Proposition 19 $\forall\sigma_{\in\Sigma}\,([\Diamond Est\,(\varphi)]\,\sigma \notin \mathcal{F} \rightarrow [Est\,(\Diamond\varphi)]\,\sigma \notin \mathcal{F})$

[3] We are here opting for strict treatment of *Fiat* sentences, so a genuine goal must be possible and not actual ('desire in the sense of goal/belief incompatibility' in Cross'[10] terminology).

[4] The canonical form of this generalization is: $Fiat(\varphi)$, therefore $Fiat(\Diamond\varphi) \wedge Est(\Diamond\varphi)$, if 'desire' is taken in the weaker sense, as 'possible and motivated desire'. Alternatively the generalization may be noted as $[Fiat(\Diamond\varphi) \wedge Est(\Diamond\varphi)]\,([Fiat(\varphi)]\,\sigma)$
$= [Fiat(\varphi)]\,\sigma$

Proposition 20 $\exists\sigma_{\in\Sigma}\left(\left[\Diamond Est\left(\varphi\right)\right]\sigma \in \mathcal{F} \wedge \left[Est\left(\Diamond\varphi\right)\right]\sigma \notin \mathcal{F}\right)$

Proposition 21 $\forall\sigma_{\in\Sigma}\left[Est\left(\neg\varphi\right)\right]\left(\left[\Diamond Est\left(\varphi\right)\right]\sigma\right) \in \mathcal{F}$

Proposition 22 $\exists\sigma_{\in\Sigma}\left[Est\left(\neg\varphi\right)\right]\left(\left[Est\left(\Diamond\varphi\right)\right]\sigma\right) \notin \mathcal{F}$

The structure whose 'reductive evolution' represents changes induced by *fiat* and *est* sentences may be changed in two ways: by elimination of points in the structure and by elimination of relations between them. Those changes may be termed 'variations on a Kripkean model'(see [8]). The points in the structure (frame) are not structureless: each point represents one possible world that can be distinguished in the part of the language under consideration. Hintikka's division of all possible worlds into "those possible worlds which are in accordance with the attitude in question and into those which are incompatible with it"[23] (p.91) is represented by accessibility relation R. For example, Hintikka's 'approximate paraphrase' for 'a believes that p' is:

> "a believes that p = in all the possible worlds compatible with what a believes, it is the case that p"([23], p.92)

In the framework of $Fiat/Est$ logic, the set $\{w \mid \neg\exists v\ vRw\}$ represents possible worlds compatible with what agent a desires, and the set $\{w \mid \neg\exists v\ wRv\}$ does the same for beliefs.

3.5 Conditional semantics: a special kind of model variation

An agent's belief that something is possible may be understood in a twofold way: as not being sure whether something is the case, or as not excluding a possibility by accepted lawlike propositions. The conditional is usually used for expressing lawlike relations and that fact qualifies it as a general proposition contrary to its singular phenomenology. Implicit generality makes propositional translation of a conditional implausible[5]. Assigning a specific semantic for rule expressing conditionals seems to be the natural solution for propositional logic. Veltman [33] takes default rules to be representable

[5] Jeffrey's example [24] (p.78.) can illustrate the point: "I'll break my leg today. I know that's true because I know it's false that if I break my leg today, I'll ski tomorrow."

in propositional dynamic logic and gives specific semantics for conditionals: they introduce preference relations among possibilities, an 'expectation pattern'.The difference between singular proposition (fact) and general proposition (deafault rule) is reflected in the kind of model variation they introduce: the former eliminates some possibilities, while the latter changes their ordering[6].

We must account for the generality of a conditional in practical propositional logic: accepting a deterministic law expressed by a conditional affects the acceptability both of *fiat* and *est* sentences.. From an agent's point of view, a state of affairs excluded on the basis of accepted deterministic laws may not become a goal and it is not actual. The conditionals introduce P-variations and that fact reflects their general character in the sense that acceptance of rule affects the acceptability of both sentence types (Definition 12). In deontic logic there is a principle '*ad impossible nemo obligatur*', on the other hand, '$\Box p \rightarrow p$' is not valid with '$\Box$' interpreted as 'it is obligatory that $___$'. Those facts force us to distinguish between impossibility of a state and its not obtaining, or on epistemic side between impossibility as a matter of accepted laws, and ignorance as matter of accepted facts. No one is obliged to do the impossible, but everyone may be obliged to bring about something that is not the case.

Therefore, we opt for a solution that assigns different semantic impact to facts and rules. Factual sentences introduce R-variations, rule sentences introduce P-variations. Still, one can think of conditional induced P-variations as a general restriction imposed on R-variations. The definition for modal *est* sentence (Definition 12) provides a bridge between functional and relational approach. Taking M as general mood indicator, we may characterize conditional P-variations on the basis of R-variations:

$$[Est\,(\Box\,(\varphi \rightarrow \psi))]\,\sigma \in \{\sigma \mid \forall \sigma'\,([M\,(\varphi)]\,\sigma' \in \mathcal{F} \rightarrow [M\,(\psi)]\,\sigma' \in \mathcal{F})\}$$

Non-actual state may acquire status of a goal (Proposition 23), while, the impossible may not (Proposition 24). A goal state must be practically possible (Proposition 25) and believed not to be realized, *i.e.* epistemically impossible(Proposition 26).

[6] For example, inference 'Students normally are not employed. John is a student. Therefore, presumably John is not employed.' Veltman formalizes as '$(p \rightsquigarrow \neg q) \wedge p$, therefore, *presumably* $\neg q$', obviously presupposing that a rule may enter the language of modal propositional logic, as '*normally*(If John is a student than he is unemployed'([33], p.250).

Proposition 23 $\exists\sigma_{\in\Sigma}\left(\left[Est\left(\varphi\right)\right]\sigma\notin\mathcal{F}\wedge\left[Fiat\left(\varphi\right)\right]\sigma\notin\mathcal{F}\right)$

Proposition 24 $\forall\sigma_{\in\Sigma}\left(\left[Est\left(\Box\varphi\right)\right]\sigma\in\mathcal{F}\rightarrow\left[Fiat\left(\varphi\right)\right]\sigma\in\mathcal{F}\right)$

Proposition 25 $\forall\sigma_{\in\Sigma}\left(\left[Fiat\left(\varphi\right)\right]\sigma\notin\mathcal{F}\rightarrow Est\left(\Diamond\varphi\right)\sigma\notin\mathcal{F}\right)$

Proposition 26 $\forall\sigma_{\in\Sigma}\left(\left[Fiat\left(\varphi\right)\right]\sigma\notin\mathcal{F}\rightarrow\left[\Diamond Est\left(\varphi\right)\right]\sigma\in\mathcal{F}\right)$

3.6 A comparison

Cross [10] gives 'the discrepancy model' with two binary relations: $M=\langle W,R_1,R_2,\mathcal{V}\rangle$ for four types of modal sentences: $1.\triangle p$ being true in a state of desire where goal p is believed to be unrealized, $2.\oplus p$ being true in a state of satisfaction with goal p believed to be realized , $3.\bigtriangledown p$ being true in a state where the realization of goal p is unknown , $4.\odot p$ being true in state of motivational indifference where an agent knows that p is the case but neither p, nor $\neg p$ is her goal. The status of genuine goal state in dynamic practical logic has only state 1. of 'goal-belief discrepancy in the incompatibility sense' while the other states are defined in terms of acceptance and acceptability of sentences of $L_{F/E}$. It is hard to imagine the natural language sentences that correspond to modalities '$\oplus$', '$\bigtriangledown$' and '$\odot$': for example a conditionalized command 'Close the door if it is not closed!' has the form '$\bigtriangledown p$' in 'modal logic of discrepancy', while in $Fiat/Est$ logic it reads '$Est(\neg p)\rightarrow Fiat(p)$' . It seems that theoretical aim of 'modal logic of discrepancy' lies in modelling the cognitive-motivational states rather then in giving a semantics for goal sentences. Still, an important kind of state is missing from the picture, namely the 'extended motivational state'.

> Cross defines modality '$\triangle$' [10] (p.148) by: $\mathcal{V}_{(M,w)}(\triangle\varphi)=\top$ iff $\forall w'\left(wR_1w'\rightarrow\mathcal{V}_{(M,w')}(\varphi)=\top\right)\wedge$
> $\wedge\,\forall w''\left(wR_2w''\rightarrow\mathcal{V}_{(M,w'')}(\varphi)=\bot\right)$
> and that is, expect for one accessibility relation instead of two, similar to our Definition2: $\mathcal{V}_{(\sigma,w)}(Fiat(\varphi))=\top$ iff
> $\forall v\left((wRv\rightarrow\mathcal{V}_v(\varphi)=\bot)\wedge(vRw\rightarrow\mathcal{V}_v(\varphi)=\top)\right)$

The problematic issue concerns the relation between models and agent's states: Cross takes non-modal propositions $\{w'\mid\langle w,w'\rangle\in R_1\}$ and $\{w'\mid\langle w,w'\rangle\in R_2\}$ to be representations of agent's goal and belief state, respectively, at a world

w ([10], p.148). In the dynamic perspective models represent cognitive-motivational states, while accepted sentences characterize those states.

For example, Cross identifies 'goal/belief discrepancy in the incompleteness sense" (i.e. a state in which an agent accepts a goal and yet is not sure whether it is actualized) with possible world w such that $\bigtriangledown\varphi$ is true in w:

$$\mathcal{V}_{(M,w)}(\bigtriangledown\varphi) = \top \text{iff}$$

$$\forall w' \left(wR_1w' \rightarrow \mathcal{V}_{(M,w')}(\varphi) = \top\right) \wedge$$
$$\exists w' \exists w'' \left(wR_2w' \wedge wR_2w'' \wedge \mathcal{V}_{(M,w')}(\varphi) = \top \wedge \mathcal{V}_{(M,w'')}(\varphi) = \bot\right)$$

In dynamic perspective the corresponding notion is the notion of conditionalized $fiat$: a procedure rather than a state/model verifying mixed conditional (see 27). There is no motivational effective state verifying the sentence 'Let p be the case if p is not the case' (see Proposition 28 below) different from the state veriying $Fiat(p)$. Cross tacitly recognizes the non-motivational status of that state by saying that in such case "the agent has a reason to make sure that p".

Definition 27 $[Est(\neg\varphi) \rightarrow Fiat(\varphi)]\,\sigma = \begin{cases} [Fiat(\varphi)]\,\sigma, \textit{ if } [Est(\neg\varphi)]\,\sigma = \sigma \\ \langle P, \emptyset\rangle, \textit{ otherwise} \end{cases}$

Proposition 28 $\forall \sigma_{\notin \mathcal{F}}\, [Est(\neg\varphi) \rightarrow Fiat(\varphi)]\,\sigma \neq \sigma$

It is not surprising that Cross interprets the modality $\oplus p$ negatively as 'having no reason to act to bring p about', while other modalities are informally interpreted as having a reason to - bring it about that p ($\bigtriangleup p$), or as having a reason - to make sure that p ($\bigtriangledown p$),. When the process of extension of motivational influence reaches a final state, it happens either because there was no need for deliberation since the original goal had already been realized, or because information was incoherent or goals incompatible. Final states are not motivational and there is no need for characterizig its subclass by special modality. It is only the state genesis that settles the question whether a final state is a state of satisfaction (see *principle of satisfaction* above, p.14).

The dynamic explanation of the postponed motivational extension may be extended to include practical reasoning aimed at preserving a desirable state of affairs. In that case an agent presupposes that the state of the world

is going to change in such a way that her goal will no longer be realized, so she reasons under the presupposition that actual and satisfying state does not obtain. The validity of an inference does not depend on the fact whether some premise is presupposed or not.

3.7 Modal Fiats

The semantics of 'modal logic of discrepancy' can not provide a model for a motivational state in which $Fiat(\varphi)$ is acceptable and $Fiat(\neg\varphi)$ is not acceptable (Definition 13). In initial state, with no goals or information given, any goal sentence is acceptable; therefore, arriving at the state in which some restrictions are imposed on the acceptability of goal sentences indicates a motivational change, provided an important condition is met: being in an extended motivational state implies having a goal, *i.e.* accepting an elementary goal sentence (see p.14). Therefore, accepting a possible *subgoal* φ is not the same as $Fiat(\varphi)$ being acceptable, since there must be a reason for the acceptability of a possible subgoal. We use notation $Fiat(\Diamond\varphi)$ to indicate that φ may become a subgoal. That kind of $fiat$ sentences has an important role in instrumental reasoning and explains non-monotonic nature of practical reasoning. The difference between 'motivated desire' and 'unmotivated desire' (see Wallace [35] p.364.) lies in the type of psychological explanation they admit:

> [...] motivated desires also (and *necessarily*) admit of a different type of psychological explanation, in which the propositional content of the desire is shown to be justified or by the content of other of the person's attitudes.

The distinction between motivated and unmotivated desires is not visible in natural language sentences. A possible explanation of that fact may lie in the fact that a conclusion of a practical inference is typically a modal $fiat$, while a $fiat$ that corresponds to an intention is a pure $fiat$. For example, the conclusion following from premises '$Fiat(p)$' and '$Est\,(\neg\Diamond(p \wedge \neg q))$' is '$Fiat\,(\Diamond q)$', but '$Fiat\,(\Diamond q)$' is a test-sentence. It has no 'reason giving force', it must be upgraded to '$Fiat(q)$' in order to become a motivated desire (to become motivationally effective). Therefore, we may and we may not accept Cross' thesis in [10] p.165.:

Even for an ideal agent, motivation does not transfer from ends to means *as a matter of logic.*

The upgrading of '$Fiat\,(\Diamond q)$' to '$Fiat\,(q)$' *is not* a matter of logic because modal $fiat$ indicates only the possibility of q acquiring a goal status (acceptability of $Fiat\,(q)$) and the inclination towards goal q (non-acceptability of $Fiat\,(\neg q)$). On the other hand, the derivation of '$Fiat\,(\Diamond q)$' *is* a matter of logic.

Goals and states:

p is a goal?	description of state/model σ
yes	$\sigma \notin \mathcal{F} \wedge [Fiat(p)]\,\sigma = \sigma$
possiblly and relatively	$\sigma \in \mathcal{M} \wedge [Fiat(p)]\,\sigma \notin \mathcal{F} \wedge$ $\wedge\, [Fiat(\neg p)]\,\sigma \in \mathcal{F} \wedge [Fiat(p)]\,\sigma \neq \sigma$
no	$\sigma = 0 \vee ([Fiat(p)]\,\sigma \notin \mathcal{F} \wedge [Fiat(\neg p)]\,\sigma \notin \mathcal{F})$
no	$\sigma \in \mathcal{F}$

3.7.1 Acceptability and negation

We have defined the negation of a sentence φ as an operation leading to the state in which sentence φ is not acceptable (see definitions under number 15). A sentence $Fiat(\varphi)$ is not acceptable in $\sigma = \langle P, R\rangle$ if

$$R \subseteq \{(w, v) \in P \times P \mid \mathcal{V}_w(\varphi) = \bot \vee \mathcal{V}_v(\varphi) = \top\}$$

Therefore, we can find three different states σ such that $[[Fiat(\varphi)]]\,\sigma \in F$, For instance, such are states with $R^F \subseteq \{(w, v) \in P \times P \mid \varphi \notin w\}$ and with $R^E \subseteq \{(w, v) \in P \times P \mid \varphi \in v\}$. Suppose that negation of a $fiat$ leads to a state with R^F; in theat case negation functions like a prohibition. A prohibition makes some goals not acceptable, it does not bring in the information whether that state is actual or not. On the other hand, if negation of a $fiat$ leads to a state with R^E than negation functions like a factual est statement what threatens the original assumption about different semantics for two types of sentences. The choice of R^F for a model/state that verifies a negation of a $fiat$ allows for incorporation of deontic logic in the framework in dynamic practical propositional logic.

4 Validity of practical inference

There are some points of convergence in philosophical analyses of practical reasoning: a number of authors recognizes the fact that practical inference does not have the usual properties of classical inference, like (a) detachment or (b) monotonicity, and that (c) a conclusion of a practical inference is not a sentence corresponding to an intention. Take some examples! *ad* a) Davidson [11] used *prima facie* sentential operator in order to indicate that a goal sentence in conclusion of practical inference *may not be detached* from its inferential basis. *ad* b) Geach [17] explained non-monotonicity by absence of relevant *fiat*:

> But a piece of practical reasoning from a set of premises can be invalidated thus: your proponent produces a fiat you have to accept, and the addition of this to the fiats you have already accepted yields a combination with which your conclusion is inconsistent.

ad c) Davidson claimed that the conclusion of a practical inference does not correspond to an action:

> 'Prima facie judgements cannot be directly associated with actions, for it is not reasonable to perform an action merely because it has a desirable characteristic...The judgement that corresponds to, or perhaps is identical with, the action cannot, therefore be a prima facie judgement; it must be an all-out or unconditional judgement which, if we were to express it in words, would have a form like 'This action is desirable." ([11], 98.ff.)

. *ad* c) Bratman [5] suggested that forming an intention to do an act provides an additional reason giving force that correspondent evaluative judgement did not have before.

The dynamic logic of practical reasoning confirms philosophical intuitions. Dynamic semantics may account for a) undetachment by interpreting sentences as test-sentence functions, b) for non-monotonicity by inclusion of initial state in the definition of the validity, and c) for distinctive quality of the conclusion by emphasizing that conclusion of practical inference is usually a relative goal sentence, while a sentence corresponding to an intention is unrelativized.

The tests of validity of practical inference may be divided in two groups: those attempting at reducing practical logic to assertoric logic and those trying to find a specific way to asses validity. Unfortunately different test types often lead to contradictory results. It has been claimed that the validity of inference consisting solely of commands can be assessed in the light of the validity of the purely assertoric syllogism composed of the corresponding termination statements. By that criterion (i) '$Fiat(p)$, therefore $Fiat(p \vee q)$' is qualified as valid since '$p \rightarrow (p \vee q)$' is a tautology .In the same vein Cross' lemma 27. "If $\vDash p \supset q$ then $\vDash \triangle p \supset (\triangle q \vee \bigtriangledown q \vee \oplus q)$' classifies (i) as valid, although in a modified way, since from 'Let p be the case' it follows something like 'Let $(p \vee q)$ be the case, no matter whether $(p \vee q)$ is realized'. On the other hand, Kenny used specific criterion of inverted assertoric logic: $Fiat(p)$ is satisfactory for $Fiat(q)$ iff q being satisfied implies p being satisfied.

> In general: if CQP is a logical law, then $\mathfrak{F}P \vdash \mathfrak{F}Q$ in the logic of satisfactoriness.([26]p.74.)[7]

By that criterion '$Fiat(p)$, therefore $Fiat(p \vee q)$' is not a valid inference schema since '$(p \vee q) \rightarrow p$' is not a tautology. The similar conflicts arise for inferences with mixed premises. Take for example, inference schema (ii) '$Fiat(p)$, $Est(p \rightarrow q)$, therefore $Fiat(q)$'. Von Wright (in [36]) takes it to be elementary form of practical inference, namely 'A intends to make p happen. A believes $\neg p$, unless a. A does a'. Although von Wright's conclusion differs from the one in scheme (ii), it is reasonable to suppose that an action has a corresponding $Fiat$. H.-N. Castañeda (in [6] p.454) takes the schema (ii) to be the weakest form of hereditary principle of practical reasoning:

> "If X believes (knows) that (p implies q), then that X intends p implies that X intends q."

However, by Kenny's criterion inference scheme (ii) is not valid since '$(q \wedge (p \rightarrow q)) \rightarrow p$ ' is not a tautology (note that 'assertoric sentences must be replaced by corresponding imperative sentences' [26] p.75). As usual, conflicting syntactic intuitions provoke semantic resolution. The identification of two types of $fiat$-sentences is the source of conflicting intuitions . Kenny is right in a way: inference schemata '$Fiat(p)$, therefore $Fiat(p \vee q)$' and '$Fiat(p)$,

[7] Kenny uses Polish notation:"CQP" is "Q→P". Fraktur letters $\mathfrak{F}$ and $\mathfrak{E}$ stand for "Fiat" and "Est" respectively.

$Est(p \rightarrow q)$, therefore $Fiat(q)$' are not valid, but Cross and Castañeda are right too, since in the modified form '$Fiat(p)$, therefore $Fiat\,(\Diamond(p \vee q))$' and '$Fiat(p)$, $Est\,(\Box(p \rightarrow q))$, therefore $Fiat(\Diamond q)$' are valid.

The criterion of validity of practical inference must account for non-monotonicity, *i.e.* the possibility of defeating the conclusion by additional premises, and un-detachment, *i.e.* the impossibility of detaching the conclusion from its inferential basis. Above mentioned tests do not seem to cope adequately with these features of practical inference since both kinds employ assertoric logic. The 'direct translation' of practical logic to assertoric logic does not allow for defeasibility, in any way. On the other hand, Kenny's 'reversed translation' is more permissive since additional $fiat$ premises of practical inference are transferred into the conclusion of assertoric inference, which in that way may become invalid. Although Kenny's criterion is more congenial to the nature of practical inference in so far non-monotonicity is concerned, still it presents them as mere enthymems.

In modeling the evolution of motivational states we cannot exclude the possibility that an extended motivational state becomes reduced to a pure one as the effect of accepting of a new goal or a prohibition. In such a case a sentence stating a possibility for derived goal or a motivated desire may be defeated. Modal $fiats$ have no independent standing, they can not be detached from their justification: that fact is reflected in their dynamic interpretation, as tests, and intuitively interpreted as possible sub-goals. Lacking independence modal $fiats$ qualify themselves as suitable for the role of conclusion of practical inference: they exhibit important properties recognized in philosophical analysis (un-detachment, defeasibility, possibility for becoming a goal). The elements in the definition of a modal $Fiat\,(\Diamond\varphi)$exhibit above mentioned features as follows: acceptability of a non-modal $Fiat\,(\varphi)$and non-acceptability of $Fiat\,(\neg\varphi)$ indicates the possibility of and the inclination towards φ becoming a goal, mention of the class $\mathcal{M}$ indicates that possible goal is a subgoal, and test-semantics indicates defeasibility and undetachment..

$$[Fiat\,(\Diamond\varphi)]\,\sigma = \begin{cases} \sigma, \text{ if } \sigma \in \mathcal{M} \wedge [Fiat\,(\varphi)]\,\sigma \notin \mathcal{F} \wedge [Fiat\,(\neg\varphi)]\,\sigma \in \mathcal{F} \\ \langle P, \emptyset \rangle \text{ , otherwise} \end{cases}$$

4.1 Non-monotonicity, relevance and containment

The dynamic notion of logical consequence that is most closely related to classical conception of 'information containment' is 'Update Test Consequence'.

Being one among the other 'dynamic styles of inference' (seebvan Benthem [4]), 'Update Test Consequence' style itself allows for different variants. Veltman (in [33], p.224) gives three notions of validity in the framework of that 'style of inference': an argument is valid$_1$ (valid$_2$) iff updating the minimal state (any information state) with premises $\psi_1, ..., \psi_n$ in that order yields an information state in which the conclusion φ is accepted. The third notion of validity that Veltman discusses in [33] is 'Test Consequence': an argument is valid$_3$ if all states that accept all premises also accept the conclusion. Validity$_1$ is not monotonic in the strong sense, validity$_2$ is left monotonic, validity$_3$ is monotonic. Simple informal proof for left monotonicity of valid$_2$ inference would proceed as follows: call C-states those that after sequential update of premise result in a state that accepts the conclusion; anterior accepting of an additional premise would yield either one of C-states and then inference remains valid, or additional premise yields a state which is not a C-state, in which case validity$_2$ is not affected. The practical inference is non-monotonic in a strong sense since a relative goal may be defeated by an additional premise no matter where it is positioned (in front, in the middle or behind premises). Therefore, the definition of validity of practical inference will have the form of validity$_1$ or, (using Groenenveld's term in [21]) of 'Ignorant Update Test Consequence' relation.

The additional condition of validity of practical inference requires that the resulting state is not a final one. The evolution of cognitive-motivational states begins with an initial state in which no goal, no factual or regularity information are accepted. The initial state 0 (Definition7) represents an idealized state of an agent with no goals, facts or rules accepted. A final state is a state with no possible development, a fixed point for any sentence. Unlike classical logic of theoretical reasoning where static principle *ex falso quodlibet* holds, in practical reasoning we may not allow for similar principle since a final state lacks the possibility of development. The relevance requirement pertains to the fact that typical conclusion of instrumental practical inference is a modal fiat, while a $fiat$ that corresponds to an intention is unrelativized . Therefore, recognition of a relativized $fiat$ as a conclusion requires that it may acquire the status of an unrelativized fiat, and therefore $Fiat(\varphi)$ should be acceptable, but in a final state no sentence is acceptable. Extended motivational states allow for forming an intention. The stating of conditions for forming an intention belong to a theory of a rational agent which connects a logic with a of an agent's capabilities and dispositions. The 'Ignorant update-to-test consequence' incorporates the non-monotonicity by taking the initial

state as the one whose 'processing with premises' must lead to the state in which conclusion is accepted. The definition of validity of practical inference includes three elements: 1.*non-monotonicity* by recurrence to initial state, 2.*relevance* by exclusion of final states, and 3.*containment*.by identifying a state resulting from successive accepting of premises with the state in which conclusion is accepted.

Definition 29 *Validity of practical inference:*

$$((\varphi_1 \wedge ... \wedge \varphi_n)\text{, therefore } \psi) \text{ iff } ([\phi_1] ... [\phi_n] 0 = \sigma \wedge \sigma \notin \mathcal{F} \wedge [\psi] \sigma = \sigma)$$

or
$((\varphi_1 \wedge ... \wedge \varphi_n)$, therefore $\psi)$ iff
$([\phi_1] ... [\phi_n] 0 \notin \mathcal{F} \ \wedge \ [\psi] ([\phi_1] ... [\phi_n] \sigma) = [\phi_1] ... [\phi_n] 0)$

Example 30 *Validity of instrumental inferences. Instrumental inferences are usually considered essential in explaining action. The underlying idea is that desirability of an end is transferred to sufficient or necessary means, but the transferred desirability of a means is of weaker kind. Modal-free instrumental practical reasoning concerning sufficient means is of the form:* $Fiat(p), Est(q \rightarrow p)$, *therefore,* $Fiat(q)$*.Here we encounter apparently invalid inference if our attention is paid only to statement letters - it is a common mistake known under the name "affirmation of consequent". By Kenny's criterion of 'inverted assertoric logic' it is a valid schema of practical logic, while by Cross' criterion of 'direct assertoric logic' it is invalid. By our criterion it is invalid, too, but its modally refined form is valid:* $Fiat(p), Est(\Box(q \rightarrow p))$, *therefore,* $Fiat(\Diamond q)$*. Modified Aristotel's example may illustrate the point: This woman is to be healed. If she is rubbed, she will be healed. So I'll rub her. The conclusion must be weakened (like 'Maybe I should rub her') if inference is to be valid. On the other hand, non-modal instrumental inference concerning 'necessary means' is usually classified as valid, since its descriptive content has the form of modus ponendo ponens:* $Fiat(p), Est(p \rightarrow q)$, *therefore,* $Fiat(q)$*.By Kenny's criterion this schema is not valid. Again, modal translation reconciles conflicting intuitions. Non-modal version is invalid, but its modally refined version is a valid one:* $Fiat(p), Est(\Box(p \rightarrow q))$, *therefore,* $Fiat(\Diamond q)$.

Proof. Validity of *practical modus ponens* will be proved if updating the initial state with premises leads to a state in which conclusion is accepted .

- Initial model $0 = \langle W, W \times W \rangle$
- Updating with $(P1)$ introduces R-variation on initial model

$[Fiat\,(p)]\,0 = \langle W, \{(w,v) \mid p \in w \wedge p \notin v\}\rangle = \sigma^1$

- Updating with $(P2)$ introduces P-variation, which, in turn, introduces R-variation

$[Est\,(\Box\,(p \rightarrow q))]\,\sigma^1 =$

$= \langle \{w \mid p \notin w \vee q \in w\}\,, \{(w,v) \mid (p \notin w \vee q \in w) \wedge (p \notin v \vee q \in v) \wedge p \in w \wedge p \notin v\}\rangle =$

$= \langle \{w \mid p \notin w \vee q \in w\}\,, \{(w,v) \mid p \in w \wedge q \in w \wedge p \notin v\}\rangle = \sigma^2$

- Conclusion test:
  - $\sigma^2 \in \mathcal{M}$
  - $Fiat\,(q)$ is acceptable, since

$[Fiat\,(q)]\,\sigma^2 =$
$= \langle \{w \mid p \notin w \vee q \in w\}\,, \{(w,v) \mid p \in w \wedge q \in w \wedge p \notin v \wedge q \notin v\}\rangle = \sigma'$
and $\sigma' \notin \mathcal{F}$

  - $Fiat\,(\neg q)$ is not acceptable, since

$[Fiat\,(\neg q)]\,\sigma^2 =$
$= \langle \{w \mid p \notin w \vee q \in w\}\,, \{(w,v) \mid p \in w \wedge q \in w \wedge q \notin w \wedge q \notin v\}\rangle =$
$= \langle \{w \mid p \notin w \vee q \in w\}\,, \emptyset\rangle = \sigma''$
and $\sigma'' \in \mathcal{F}$
therefore,
$[Fiat\,(\Diamond q)]\,\sigma^2 = \sigma^2$

4.2 Semantic tables

The validity of practical inference is established by a simple decision procedure. The decision procedure consists in creating two 'possibility elimination tables' in which possibilities are eliminated according to semantics of sentences, and in testing whether conclusion is accepted. The two tables may be understood as follows: left table records preferred possibilities, right table records epistemic possibilities, and both tables record practical possibilities.

1. For elementary $fiat$ sentences eliminate all open possibilities in left table in which propositional content is not true and eliminate all open possibilities in right table in which it is true

2. For elementary est sentences eliminate all open possibilities in right table in which its propositional content is not true

3. For sentences of the form $Est(\Box\varphi)$ eliminate all open possibilities in both tables in which φ is not true

4. For sentences of the form $Est(\Diamond\varphi)$ test whether is there an open possibility with φ true in any table, if there is at least one proceed, otherwise close all.

5. For each elimination write the number index of eliminating premise

6. For sentences of the form $Fiat(\Diamond\varphi)$ perform test on open possibilities:

   (a) Check are there any identical open possibilities in both tables. If yes, close all, otherwise continue.

   (b) In the left table write a in the column where φ is true, in the right table write a in the column where φ is not true (test of acceptability for $Fiat(\varphi)$). If there is an a in both tables, continue, otherwise close all open possibilities)

   (c) In the left table write b in the column where $\neg\varphi$ is true, in the right table write b in the column where $\neg\varphi$ is not true (test of acceptability for $Fiat(\neg\varphi)$). If there is b on both sides, close all open possibilities, otherwise continue.

7. Apply the rules 1.-6. in the conclusion row, write x for each eliminated possibility. If there is no x in conclusion row and if there are open possibilities (unmarked columns) on both sides of the conclusion row of the table, the inference is valid, otherwise invalid.

Example 31 *Desirability of sufficient means*

$$\frac{(1)Fiat\,(p)\qquad (2)Est\Box\,(q \rightarrow p)}{Fiat\,(\Diamond q)}$$

possibility	p,q	p	q	-	p,q	p	q	-
eliminated by			1	1	1	1	2	
conclusion test	a	b						a

Valid!

Example 32 *Desirability of necessary means*

$$\frac{(1)Fiat\,(p)\qquad (2)Est\Box\,(p \rightarrow q)}{Fiat\,(\Diamond q)}$$

possibility	p,q	p	q	-	p,q	p	q	-
eliminated by		2	1	1	1	1		
conclusion test	a							a

Valid!

Example 33 *Disjunctive command*

$$\frac{(1)Fiat(p \vee q)}{Fiat\,(\Diamond q)}$$

possibility	p,q	p	q	-	p,q	p	q	-
eliminated by				1	1	1	1	
conclusion test	a	b	a					a

Valid!

Example 34 *Weakening of conjuncts.*

$$\frac{(1)Fiat(p \wedge q)}{Fiat\,(\Diamond q)}$$

possibility	p,q	p	q	-	p,q	p	q	-
eliminated by		1	1	1	1			
conclusion test	a					a	b	a

Valid!

Example 35 *Conjunctive inference.*

$$\frac{(1) Fiat(p \wedge q)}{Fiat\,(q)}$$

possibility eliminated by conclusion test

p,q	p	q	-
	1	1	1

p,q	p	q	-
1			
		x	

Invalid!

4.3 Further research

The topics for further research include treatment of deontic logic and the logic of preference in the framework of $Fiat/Est$ logic. The expressive power of $Fiat/Est$ language is strong enough to capture the meaning of obligations and requests as original goals (non-modal $fiats$), while rules that permit, forbid or do not allow may be treated as negations of $fiats$. The accessibility relation should be refined in such a way to allow for representability of preference relations between goals.

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Berislav Žarnić

University of Split

Croatia

E-mail: berislav@mapmf.pmfst.hr