Talk About Things Non-Existent*

Paul Dekker

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Yesterday, upon the stair,
I met a man who wasn't there!
He wasn't there again today,
Oh how I wish he'd go away!

[Hughes Mearns, 1899, *Antigonish*]

Abstract
In order to make sense of talk about things non-existent philosophers and linguists tend to construe them as things more than as non-existent. The temptation to do so is obviously strong, but it also appears wrong. This note provides a constructive argument that we can resist this temptation. It presents a simple formal language, with a proof theory, showing that we can make sense of talk about things non-existent without granting them existence.

Introduction

Ontology is a traditional branch of philosophy concerned with the question of what there is. Intrinsically related is the question of what there is not. A first, straightforward, reply to the latter question is that there is nothing that is not. For if there is something, it is. Now it seems that we can disagree and deliberate about the question of what there is not. But how can we disagree about what there is not, if this is, as we just said, nothing? If we disagree about nothing, we don't disagree.

This note is not about the question of what there is or what there is not, neither, a forteriori, about how to approach such questions. It is, rather, about the question whether the previous line of thought shows that there is something wrong with our first intuitions. Is thought and talk of things non-existent, as concluded, impossible, or is it possible after all? My answer will be positive and constructive. It is possible for us to make sense of thought and talk of things non-existent, without, thereby, rendering them existent. This will be argued for in a demonstrative fashion. I will show that we can make sense of such talk, without unwanted existence implications, in a logical framework, that is independently motivated itself.

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* This note is the fifth in a series aiming to contribute to a formal logical framework which allows us to present and study natural language meanings.

I will try and make the above issue somewhat more precise in the remainder of this introduction. Willard Van Orman Quine famously observed:

> [T]*he ontological problem (. . .) "What is there?" (. . .) can be answered (. . .) in a word — "Everything" — and everyone will accept this answer as true. (. . .)* [T]*his is merely to say that there is what there is.* (Quine 1948, p. 21)

It is perhaps somewhat of an overstatement that everyone accepts Quine's answer as true, but it surely has a strong intuitive appeal. On the face of it, *to be*, *to be there*, and, more formally, *to exist* are all one and the same. They are a property belonging to every existing thing and nothing fails it. I would like to formulate the idea in the following way as a first first principle.[1]

First Principle 1 *Everything is, and there is no thing that does not exist.*

People, with all respect, also generally do things like imagining things, making up stories, portraying things as real, construing myths, and belief in delusions. They thereby engage in thought and talk about things that aren't there, things that don't exist. Let me phrase this as a second first principle.

First Principle 2 *There is thought and talk about things non-existent.*

It has appeared to be hard to fully respect both of these first principles. For if we assume that thought and talk about X implies that there is or are X, the second first principle implies that there are things that do not exist, and that, obviously, contradicts the first first principle. We here see two, intuitive, principles, that appear to yield a contradiction, and these are defining characteristics of a *paradox*.

There are many ways to deal with paradoxes, whether or not they are truly paradoxical or only apparently so. Most people simply *ignore the paradoxes* or set them aside as an evil deceit or joke. This is not, however, typical of philosophers and logicians, the kinds of people I'd like to engage with. Some of the latter choose to *embrace the paradoxes*, and admit of in- and para-consistencies. The paradoxes can be taken to show that inconsistencies can be real. Alternatively, some, perhaps most, try and *avoid the paradoxes*, normally by given up some of the principles from which they derive. Perhaps we have to acknowledge that there just is more than what merely exists, but this really means giving up the first first principle. Finally one can try and *deny the paradoxes*. This is the approach taken here. We can try and prevent the paradox by simply not proceeding on the assumption that thought and talk about X implies that there are X. Perhaps, as we will see, it is not easy to do so, but it is possible.

While I cannot fail to see the urge to postulate the existence of things non-existent, and while I am also even not un-willing to respect the urge, what

1. I call it a 'first principle' here not because it is considered a fundamental one, but because, quite generally, it figures as a starting point. It is obvious that the principle can be abandoned, but also then it serves as a point of departure, as a principle one departed from, hence as a principle that came *first*.

I only want to show here is that acknowledging thought and talk about things non-existent constitutes no ground for postulating their existence. Like I already said, the argument will be constructive. For the most part this note consists in the presentation of a formal framework and a demonstration that it allows us to make sense of thought and talk about things non-existent without granting existence to anyone of them. The results, thus, are fairly modest, but they are equally strong. The results are independent of any choice one might wish to make for or against the first principles or for or against accepting paradox and contradictions. My only claim is that, logically speaking, it is possible to keep to the aforementioned first principles without problems or reservations.

I will proceed as follows. The first section provides for a brief, and very incomplete, historical philosophical and linguistic background, mainly for the purpose of illustrating how non-being may perplex the reflective mind. Sections 2 and 3 next present (the outlines of) a required formal framework. This will be a first order quantified modal logic because it has to enable reasoning about individuals in non-actual possibilities. It is easily established that the framework respect the first first principle. Section 4 serves to explain how some idea of intentional, but non-actual, being is already present in our understanding of the proposed formal language. Section 5 finally proposes a, subtle but intuitive, dynamic twist in our understanding of that formal language, that allows us to also fully and explicitly respect our second first principle. Intentional objects, object that only intentionally exist, can be *intentionally reified*, that is, construed as intended to be identical. Intentional being here is, of course, not to be confused with actual being. The concluding section summarizes the results.

1 What Not Is Is Not

Non-being has inspired analytical philosophical reflection at least since antiquity. Plato and Aristotle may have been the first to explicitly mark its paradoxical nature. Plato's Stranger comes to the following conclusion: "Do you see, then, that it is impossible rightly to utter or to say or to think of not-being without any attribute, but it is a thing inconceivable, inexpressible, unspeakable, alogical?"[2] "Don't you see (...) that not-being reduces him who would refute it to such difficulties that when he attempts to refute it he is forced to contradict himself?" (Plato 1921, 238c9–11, 238d4–8) Non-being is here claimed to go beyond what we can think and speak of. For it to be conceivable, or even refutable, it has to be. But if it is, this contradicts that non-being, by definition, is not.

Aristotle appears to have worded a quite like conclusion. While he has, famously, pointed out that truth consists in saying, e.g., of what not is, that it is not, he has noted, not so long before, that non-being somehow is, for it is said

2. τὸ μὴ ὂν (...) ἔστιν ἀδιανόητόν τε καὶ ἄρρητον καὶ αφθεγκτον καὶ ἄλογον.

to be non-being.[3] Even if it looks like just a play with words, Aristotle proposes what seems to be an analytic truth, or close to one, viz., that non-being is non-being. By the same token, however, the truth appears to imply a contradiction, or something close to one, viz. that non-being is being non-being.

It is certainly not quite easy to get a good picture of Plato's and Aristotle's ideas, if any, of what is not, and we also do not need to try and get such a picture here. It may suffice for us to just observe that, prima facie, there is something puzzling and challenging about non-being, conceptually and logically speaking, and also that we may perhaps be tempted to think and speak of it in the wrong way. For one thing, *being*, and equally likely *non-being*, are perhaps not to be thought of as things, but also not as genuine predicates of things.

In the 18-th century Immanuel Kant indeed proposed that a term like '*being*' does not figure as a real predicate or property of things.[4] It is surely problematic to conceive of these terms as predicates that serve to make a distinction among *things*, because then, in accordance with our first first principle above, the predicates would be vacuous, since no thing can fail it. Kant's interest in the concept of existence did not, however, originate from puzzles of non-existence, but from its role in certain proofs of God's existence. He certainly did not deny that one can speak of the existence and non-existence of things considered possible.[5] The fact thus remains that, also according to Kant, there can be things, that are considered possible and that can be properly judged to exist or not, which seems to imply that there are things ("Dinge") that do not exist.[6]

The paradoxical predicament that we find ourselves in has been aptly characterized by Quine. In order to address —raise, state or deny— the existence of something, there has to be something the existence of which is addressed, so it somehow has to be there, i.e., to exist. (Quine 1948, p. 21) For these, and quite similar, reasons Alexius Meinong has, long before, concluded that there are things that do not exist, and actually gave up the first first principle. (Meinong 1904) Meinong distinguished various types of being, such as what we here, for convenience, prefer to just call *being* and, for instance, genuine *existence*. According to Meinong some things are, for instance because they are conceived, which means that they are things that are conceived. They can nevertheless be misconceived, and not actually exist. One can dream of a golden mountain, and

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3. "[T]o say that (...) what is not, is not, is true (...)." [τὸ μὲν γὰρ λέγειν (...) τὸ μὴ ὂν μὴ εἶναι ἀληθές (...). Aristotle 1933, Gamma, VII, 1011b26–28] "Hence we even say that not-being *is* not-being." [διὸ καὶ τὸ μὴ ὂν εἶναι μὴ ὄν φαμεν Aristotle 1933, Gamma, II, 1003b10]

4. "*Sein* ist offenbar kein reales Prädikat, d.i. ein Begriff von irgend etwas, was zu den Begriffe eines Dinges hinzukommen könne." (Kant 1956, B 626)

5. "[D]er Satz: *dieses oder jenes Ding* [welches ich euch als möglich einräume, es mag sein, welches es wolle] *existiert*" "[D]aß ein jeder Existentialsatz synthetisch sei," is something that "jeder Vernünftige" must "billigermaßen" concede. (Kant 1956, B 625/6)

6. Gottlob Frege likewise deemed "*to exist*" not a predicate of things, but, according to him, of concepts. "Weil Existenz Eigenschaft des Begriffes ist." (Frege 1884, § 53) He, however, would not say that *to exist* would be a possible property of a thing conceived, because the thing conceived could not exist.

then there is the golden mountain that one dreams of, but this, of course, does not imply that the golden mountain exists.

Perhaps the fact that he has given up our first first principle, may have triggered most of the occasionally vehement criticisms of Meinongs work. (Russell 1905; Quine 1948) It surely constitutes the cornerstone of many discussions. I believe it is no overstatement to say that a lot of the subsequent analytical philosophical literature tries to find a balance, between, on the one hand, respecting our first first principle, and, on the other, respecting the considerations that motivated Meinong's theory of 'objects', which includes non-existent and even impossible ones. What strikes me most about it is that people often do want to stick to our intuitions that tend to equivocate being with existence, i.e., to our first first principle, but nevertheless do acknowledge other *forms of being* that apparently escape the equivocation.

It would of course be impossible to pay due respect, here, to all of the relevant literature. Even so, I honestly do not know of any existing account that acknowledges anything like intentional or otherwise possible objects that we can reason about, and that, not only nominally, but also substantially, respects the first first principle. Scholars in the respectable philosophical and linguistic literature grant some sort of being to hypothetical entities, mythological and other fictional characters, possible objects, and so on.[7] Since all appear to assume that we can talk about something, even if it is something deemed non-existent, there is the, it seems irresistible, inclination to conclude that there is something that we talk about. Since it doesn't exist, it's *being* must be something which is not being in the ordinary sense, so it implies there *are*, in a certain sense of *are*, things that don't exist, thereby, one way or the other, contradicting the first first principle.

By way of illustration I want to, briefly, mention two, actually three, examples from the recent literature in where the conflicting inclinations appear almost explicitly. Tim Crane and Friederike Moltmann both emphasize that 'intentional objects' do not exist and that they are only there intentionally.

> *Objects of thought are not, as such, entities. An object of thought is just anything which is thought about (. . .). Some objects of thought exist, and some do not. But to say this is not to assume that there is an ontological or quasi-ontological category of 'objects of thought' to which all the things belong. (. . .) When the object of thought does not exist, it is nothing at all* (cf. *Husserl 1900-01: 99*). (Crane 2012, p. 57–8)

7. Whether or not declared Neo- or Non-Meinongians, the following authors all appear to postulate things or stuff that does not 'regularly' exist. Terence Parsons (1980, *Non-Existent Objects*), Richard Routley (1980, *Exploring Meinong's Jungle and Beyond*), Edward Zalta (1983, *Abstract Objects*), Nathan Salmon (1998, *Non-Existence*), Colin McGinn (2000, *Logical Properties*), Peter van Inwagen (2002, *Ontology, Identity, and Modality*), Graham Priest (2005, *Towards Non-Being*), Timothy Williamson (2007, *The Philosophy of Philosophy*), Franz Berto (2012, *Existence as a Real Property*).

> *[I]ntentional objects in general can be made available only by the presence (. . .) of intentional acts on which the intentional objects depend. (. . .) Positing intentional objects (. . .) does not mean taking unsuccessful acts of reference to in fact be successful, referring to intentional objects. (. . .) Intentional objects are not part of the ontology; they are mere projections of intentional acts, which is why they have the status of nonexistent.* (Moltmann 2015, p. 166/145)

Even so, these 'intentional objects' are deemed objects available for reference and quantification. Crane argues that "[W]e can quantify over objects of thought," and that "a domain should be thought of as a universe of discourse, a collection of objects of thought." "[I]f we aim to give a systematic account of our actual thought and language, then we have to make room for quantification over the non-existent." (Crane 2012, p. 58, 64, 65) Likewise, when Moltmann formalizes talk about intentional objects, the objects are existentially quantified, referred to, and also abstracted over. Consider her examples (16b) and (16c).

(16b) John mentioned$_i$ [a woman$_i$]

(16c) $\exists e \exists x(\text{mention}(e,\text{John},x)\ \&\ \text{woman}(x,e))$

"Example (16c) is to be understood as 'There is an event of mentioning an object on the part of John coordinated with an event of attributing the property of being a woman to that object.'" "[F]or a 'non-relational' noun N, '$N(x,e)$' is to understood as 'some act coordinated with e attributes the property expressed by N to x.'" (Moltmann 2015, p. 150) On any regular interpretation of it, (16c) can only be true if there is (exists) some such x, independent of any other events and mentions, apparently counter to the idea that "they are not part of the ontology". Moltmann also discusses example (19a) with formal rendering (21b).

(19a) (. . .) buildings that John could have built (. . .)

(21b) $\lambda x[\Diamond(\text{building}(x)\ \&\ \text{build}(\text{John},x))]$ (Moltmann 2015, p. 152)

The term (21b) is said to give us the denotation of (19a). This denotation is said to contain "possible objects." (p. 152) They must be objects x, such that it is possible that x is a building and that John built x, and it can be claimed that some of these possible objects do not exist.[8]

Like Kant, Crane and Moltmann, credit intentional objects with some sort of possible being, but the latter two emphasize that if their being isn't actual, they are really not there at all. ("[I]t is nothing at all"; "not part of the ontology.") Even so, in their formal languages these objects are, available for reference and quantification. Now this does not need to imply an inconsistency, perhaps it merely shows that the idioms and formalisms that we are used to employ in these discussions are themselves insufficient, or just insufficiently understood. Perhaps the insights and findings of both can be preserved if we have

8. A final example can be found in the even more recent work of Hanoch Ben-Yami. Ben-Yami endorses a *proposition* that reads "There is no need to introduce different kinds of being." and another one that reads " 'There is' statements amount to different things in different contexts. (. . .) the kinds of being—if this is how we should call it—which these statements ascribe to their subjects differ widely." (Ben-Yami 2022, Prop. 1.2, p. 9 and Prop. 3.2.5, p. 12)

available a formally transparent presentation of them that keeps to their main tenets, e.g., our two first principles, while remaining obviously, transparently, consistent. As I already said above, the main motivation of this paper consists in presenting such a formalism. Before I start presenting that, however, it is expedient to establish one more first principle, one that Crane and Moltmann appear to adhere to, and that I want to second here.

Crane and Moltmann agree that intentional objects can be just objects of intentions and nothing more than that. I prefer to use the term "intentional" here as a cover term for any sort of modal notion, such as something possible, conceived, hypothesized, mythological, made up, believed, or 'seen', etc. People may understand and agree that in, or according to, any such kind of intention (possibility, conception, hypothesis, myth, fiction, belief, 'see'-ing, etc.) an object can be intended to exist.[9] We are all able to conceive a possibility with a fat man in the doorway, which there actually isn't, or to locate Vulcan in the solar system according to Le Verrier's hypothesis, or to speculate about Pegasus as portrayed in the myths, to follow an imaginary Ulysses on his imaginary travels, to 'see' roses in a painting of roses, etc. But while we can so to speak understand things like this as if they are real, we know they aren't actually real. It is just not the case that there is some real thing in the doorway, and that we conceive of that as a fat man; there is no thing so that we hypothesize that that thing is plante Vulcan in our solar system; there is no real thing that the myth portrays as Pegasus, and there need be no roses, so that we are seeing them in a painting of roses. Let me phrase this as a third first principle.[10]

First Principle 3 *Intentional objects may only intentionally exist.*

When we talk of 'intentional existence', or 'possible object', or 'hypothetical being', or 'fictional character', etc., one should bear in mind that this only means that there is an intention (possibility, hypothesis, fiction, . . .) that some thing is existing. We do not mean, by these terms, that there is some real thing and that it has the peculiar property of being intended, possible, hypothetical, etc. We can *conceive its existence*, but when we do so it is there only in our conception, in our understanding of a possible, hypothetical, or fictional reality. The natural verbal denominator of an 'intentional' or 'possible' 'object' is in this respect as misleading as the natural, verbal, claim that identity statements state the 'identity' of 'two objects'. The latter, taken literally, is inconsistent, for two objects are never identical. Likewise, no object is intentional, neither is there any object that intentionally exists. We hope the next sections of this note help in making these observation understood also formally.

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9. David Hume: "Whatever we conceive, we conceive to be existent." (Hume 1739, Book I, Part II, § 6). Jaakko Hintikka "It seems to me as obvious as anything in philosophy that there are unrealized *possibilia* (. . .). And it seems eminently plausible that there are unrealized and hence nonexistent objects in some of these unrealized possibilities." (Hintikka 1984, p. 451).
10. This principle is originally a phenomenological one, with roots in the early work of Edmund Husserl. (Husserl 1913, e.g., Vol 2, V, §11 & app. 2).

2 A First Order Modal Logic

In this section I present the outlines of a proof theory for a first order modal language that will eventually demonstrate our first principles to be consistent. For that purpose I assume we have a language that is built up in the usual way from atomic formulas of the form '$Rt_1 \ldots t_j$' and '$t_1{=}t_2$', the usual propositional connectives (e.g., '$\neg$', '$\wedge$', '$\rightarrow$'), indexed quantifiers '$^i\exists x$' and '$^i\forall x$', and the modal operators '$\Diamond^k$' and '$\Box^k$'. (I will come back to these indices '^{i}' and '^{k}' below.) I focus on the proof theory, because there it shows most clearly that we are able to respect our first principles.[11]

The proof-theory consists in a natural deduction system that defines what conclusions can be derived from series of premises. To this purpose I write

$$\phi_1, \ldots, \phi_n \vdash \psi$$

to state that conclusion ψ can be *derived* from a series of premises $\phi_1, \ldots, \phi_n$, thereby validating the conclusion as a logical consequence of these premises. The proofs are presented in a Fitch-style format. The core of the proof system lies in the definition of what a legitimate derivation is. A legitimate derivation is a numbered list of formulas each line in which is either a premise, an axiom or an assumption, or a formula derived from the formulas on previous lines according to explicit deduction rules. I actually assume the reader is already familiar with this type of derivation system, and also with the rules for the introduction and use of propositional logical connectives. In this section I mainly focus on the deduction rules governing the use of the identity sign, the quantifiers and the modal operators.

Our deduction rules for identities are classical, except for the fact that a term 't' must count as declared ('existing') before we can properly state t's self-identity and that we impose a Fregean restriction on Leibniz' substitution rule.

Identity (=)

$\vdots$

n. $t = t$ [=]

The term t must count as declared at line n.

(On the right I use $[t/z]\phi$ to indicate the formula obtained from ϕ by replacing all free occurrences of z in ϕ not in the scope of a modal operator by t.)

Leibniz (L)

$\vdots$

l. $t_1 = t_2$

$\vdots$

m. $[t_1/z]\phi$

$\vdots$

n. $[t_2/z]\phi$ [L, l, m]

The terms t must be free for the variable z in ϕ

We can establish the self-identity of an object if it is declared, which actually means that it counts as existing. We will come back to this point in the next

11. The proof-theory is, of course, backed up with a model-theory. For further details and motivation the reader may consult Dekker 2016; Dekker 2025a. See also the appendix.

section. As said the Leibniz rule is also classical, but for the fact that the actual identity of the referents of two terms is not taken to entail a necessary one. We will also come back to this point in the next section. It suffices here to observe that the actual identity of two terms is not sufficient to license their mutual substitution in modal contexts.

The quantifiers of our language, existential and universal, bind variables in the usual fashion, but they are also equipped with indices, pre-superscripted, that serve to identify and distinguish the possibly distinct domains they are taken to range over. Thus, ${}^{i}\exists x\phi$ says that in domain i there is an x such that ϕ, and ${}^{i}\forall x\phi$ says that in i all things x are ϕ. I also assume that there is a default domain, labeled ^{0}, that is taken to define all there is, in a given possibility.

The proof rules for the quantifiers are classical but for the fact that they keep track of the domains in which individuals are declared to exist.

∃-Introduction (I∃)

$$\vdots$$
$$\text{m. } [z/x]\phi$$
$$\vdots$$
$$\text{n. } {}^{i}\exists x\phi \quad [\text{I}\exists, \text{m}]$$

Variable z must count as in i and declared at line m.

(I use $[z/x]\phi$ to indicate the formula obtained from ϕ by replacing all free occurrences of x in ϕ by z. Variable z must be free for x in ϕ.)

∃-Elimination (E∃)

$$\vdots$$
$$\text{l. } {}^{i}\exists x\phi$$
$$\vdots$$
$$\begin{array}{|ll} \text{m. } [z/x]\phi & [\text{ass.}] \\ \vdots & \\ \text{n-1. } \psi & \end{array}$$
$$\text{n. } \psi \quad [\text{E}\exists, \text{l}]$$

Variable z may not occur free in any assumptions, ${}^{i}\exists x\phi$ or ψ. It counts as in i and declared from line m to n.

If something, z, has been established to be ϕ, and if z is declared in i, then we can conclude that something in i is ϕ. Conversely, if it has been established that something declared in i is ϕ, and if we are able to conclude that ψ holds whenever anything in i is ϕ, then ψ, of course, must hold. These rules are actually the familiar ones, save for the restriction to being in i and declared. Notice that a variable can only come to count as being in i because it used that way in a subderivation induced by a use of the (E∃)-rule — or, cf., below, by that of the (I∀)-rule. A variable may by the use of these rules also come to count as declared, but this is not the only way for a term to come to count as declared, and it also does not imply that the variable counts as declared in subordinate modal contexts. More on this below.

Our quantifiers can, as usual, be defined in terms of one another, so that ${}^{i}\forall x\phi$ equals $\neg{}^{i}\exists x\neg\phi$. This fact validates the following rules for ∀.

∀-Introduction (I∀)

$\vdots$

m. $z = z$ [ass.]

$\vdots$

n-1. $[z/x]\phi$

n. $^{i}\forall x\phi$ [I∀]

Variable z may not occur free in other assumptions, or in ϕ. It counts as in i and declared from line m till n.

∀-Elimination (E∀)

$\vdots$

m. $^{i}\forall x\phi$

$\vdots$

n. $[z/x]\phi$ [E∀, m]

Variable z must count as in i and declared at line n.

If we can conclude that z is ϕ on the mere assumption that it is there in i, then we can conclude that every z in i is ϕ. This is, of course, the standard rule, but now explicitly relativized to domains. Conversely, if everything in i is ϕ, then so is any z, provided that z has been established to be in i.

Like our quantifiers, our modal operators, '$\Diamond$' and '$\Box$', also come with indices. The indices this time serve to distinguish between the various modalities or 'modal bases' that linguists and logicians may identify and distinguish between. (Kratzer 1991; Smets & Velázquez-Quesada 2023) As usual, the two modal operators are considered each other's duals, so that for any index k:

Axiom Scheme 1

$$\vdash \neg\Box^{k}\phi \leftrightarrow \Diamond^{k}\neg\phi \qquad (DUAL)$$

Among the modalities we also postulate a default one, indicated by a zero-index again, that is required to be *as universal as* logically possible. It is assumed to satisfy the S5 axioms and also to include the other modalities. Our derivation system is therefore licensed to draw from all of the following axioms.

Axiom Scheme 2

$$\vdash \phi \rightarrow \Diamond^{0}\phi \qquad (T)$$

$$\vdash \Diamond^{0}\Diamond^{0}\phi \rightarrow \Diamond^{0}\phi \qquad (4)$$

$$\vdash \Diamond^{0}\phi \rightarrow \Box^{0}\Diamond^{0}\phi \qquad (5)$$

$$\vdash \Diamond^{k}\phi \rightarrow \Diamond^{0}\phi \qquad (UP)$$

Finally we assume any modal space to obey logical truths and principles, so that if something is actually derivable, it is derivable in any arbitrary possibility.

Axiom Scheme 3

$$\textit{if } \phi_1, \ldots, \phi_j \vdash \psi, \textit{ then } \Box^{k}\phi_1, \ldots, \Box^{k}\phi_j \vdash \Box^{k}\psi \qquad (DN)$$

This schematic rule of *distributed necessitation* (DN) combines the familiar principle of necessitation with the familiar distribution law.

In the Fitch-style natural deduction format that we employ, the rule of (DN) is most transparently implemented as follows. Suppose our current location in a proof is line m, and that it has been established on previous lines $l_1, \ldots, l_n$ that $\Box^k\phi_1$, ..., $\Box^k\phi_n$, respectively. We then may add the embedded formulas ϕ_1, ..., ϕ_n as assumptions on lines $m{+}1, \ldots, m{+}n$, and try and derive ψ using no formulas on lines before line m. If we succeed on some line $m{+}z$, then we can conclude that $\Box^k\psi$, indicating that it is obtained using (DN), $l_1, \ldots, l_n$, and the derivation on lines $m{+}1$—$m{+}z$. By concluding so, we thereby simultaneously withdraw the assumptions on lines $m{+}1, \ldots, m{+}n$. The rationale behind this procedure is hopefully obvious. In order to derive a conclusion about some arbitrary possibility (of some sort k) we can only draw from what we know that is necessarily true, which is what the $\Box^k$-ed formulas ϕ tell us, and what logically follows from all that. No contingent assumptions about the actual world can be used for that purpose. (Appendix A. presents two sample derivations.)

We need to add to this of (DN) a subtle but significant proviso. If, relative to any premise $\Box^k\phi_l$, a variable counts as in any domain i and as declared there on some line l, it does not necessarily count as declared when we evaluate ϕ_l in the inference supporting the necessitation rule. The fact that it actually exists is no guarantee that it necessarily exists. (Although it still counts as being in i.) In order for the variable to also count there as being declared it may need the support from an additional, explicit, existence assumption, which the final conclusion can be taken to be conditional upon. (In the second sample derivation in the appendix this condition shows up on lines 7 and 15.)

3 Conceptually Grounded Quantification

It is fairly widely acknowledged that natural language quantifiers are almost always, if not universally, relativized to contextually prominent domains. (Westerståhl 1985; von Fintel 1994; Stanley & Szabó 2000) As said, our quantifiers are superscripted with indices to indicate when they quantify over the same or over possibly distinct domains. It has also often been observed that these domains are not just arbitrary subsets of the domains of individuals, but that they are assumed to consist of individuals that are conceived of in a certain way, with certain characteristic properties and standing in certain relations. (Recanati 1996; Reimer 1998; Kratzer 2007) In the current modal setting, our contextually relativized quantifiers actually indeed do so. Our contexts do not just figure as a filter on the domain of individuals, but, rather, as a window on individuals seen. As I hope to make more specific below, intuitively and formally, a context i can be used to present a number of individuals to us in a certain configuration, the way in which they are conceived of. Before I explain how this exactly works, we first have to establish a few subtle, but significant, logical facts concerning cross-possibility existence and identity.

Since we want to model reasoning about individuals in possibilities we have to

respect the possibility that their actual existence, and identity, is not a necessary one. To express this some notation conventions turn to be convenient.

Notation 1 (Existence and Essence)

$$^{i}\exists t := {}^{i}\exists z\, t{=}z \qquad \Box_t \phi := \Box(\exists t \rightarrow \phi)$$

The formula $^{i}\exists t$ is used to express that t exists in context i. If the context is the default one, it just says that t exists, $\exists t$. We can next use such a condition to make necessities conditional upon t's existence. We can thus use $\Box_x Fx$ to express that F is an essential property of x, not because it is necessary that x has that property, which would imply that x necessarily exists, but because x has that property whenever it exists. We can also use this restricted modality to express that we can see, say, Nixon, through some window i, as essentially being Nixon, by means of the formula $^{i}\exists x(n{=}x \wedge \Box_x n{=}x)$. This, of course, does not exclude that we can see the same man, Nixon, through some other window j, and so that we do not see him being. Nixon: $^{j}\exists x(n{=}x \wedge \Diamond n{\neq}x)$. As has already been mentioned above, and as is illustrated in the second sample derivation in the appendix, it is important to make such existence conditions explicit in cross-possibility reasoning.

Existence if regulated by two more logical principles. The first is that intuitive idea that elementary predications can only be true of objects that exist. Thus, if '$AT(t)$' is an atomic formula, possibly an identity statement, that hosts the term 't', we can infer that t exists.

$\exists$-Introduction (I$\exists$)		
	⋮	
m.	$AT(t)$	
	⋮	
n.	$\exists t$	[I$\exists$, m]

Some such is not taken to hold for negated formulas. An atomic formula like Pt may be false if there is no t. We here have, in other words, what is called a *negative free logic*. (Nolt 2020)

It is also assumed every contextual domain i hosts existing things only.

Axiom Scheme 4

$$\vdash {}^{i}\forall x\, {}^{0}\exists x \qquad (E)$$

In the more plastic formulation adopted below this axiom says that window i is *transparent*: whatever we see through it exists.[12]

It may be obvious that in the logic developed so far, *to be self-identical*, *to be something that exists*, and *to exist*, all three turn out to be equivalent.

12. It is without doubt philosophically quite interesting to speculate about dropping this axiom, but this note is surely not the place to do so.

The latter two, '$\exists z\, t{=}z$' and '$\mathsf{E}t$', are equivalent by definition. Both can directly be derived from the first, '$t{=}t$', by the E-introduction rule (IE). And by the $\exists$-elimination rule (E$\exists$) we can derive identity from existence. As an obvious and direct consequence of the last axiom scheme, we also find that our system fully agrees with our first first principle.

Theorem 1

$$\vdash \forall x\, \mathsf{E}x$$

This observation generalizes to all domains. It is unrestrictedly valid that ${}^{i}\forall x\, {}^{i}\mathsf{E}x$. Of course, it is not generally valid that ${}^{i}\forall x\, {}^{j}\mathsf{E}x$, if the indices i and j are distinct and j is not the default one.

As observed above, our contexts, now cast in a modal framework, can be taken to figure as *conceptual windows* on things. Two further constraints delimit what we conceive of as a proper conceptual window. A proper conceptual window on individuals should not only be transparent, as required by (E) above, but it should also be taken to present the individuals distinctly and exhaustively.

Constraint 1 (Distinctiveness) For any conceptual window i

$$\vdash {}^{i}\forall x\, {}^{i}\forall y(x{\neq}y \rightarrow \Box\, x{\neq}y) \qquad (D)$$

The distinctness constraint does not say of any two real objects that are seen through i that they are necessarily distinct, but that they are distinct *in so far as they are seen* through window i. The exhaustiveness constraint next requires that, if we see anything at all through a window, it never hosts any things 'unseen'.

Constraint 2 (Exhaustiveness) For any conceptual window i

$$\vdash {}^{i}\forall x\, \Box({}^{i}\exists z \mathsf{E}z \rightarrow \mathsf{E}x) \qquad (EX)$$

The exhaustiveness constraint has it that if a window supplies a view on any individual in any accessible possibility, then we can see there everything that we can see from any possibility we look from. Since the default modality $\Box^0$ is S5, the constraints (D) and (EX) jointly imply that if in any possibilities anything can be seen through a window i, exactly the same number of things can be seen. It also follows that identities are necessary, as long as the identified individuals are seen through one and the same window, and in so far as they exist.

Theorem 2 For any conceptual window i

$$\vdash {}^{i}\forall x\, {}^{i}\forall y(x{=}y \rightarrow \Box_x\, x{=}y) \qquad (NI)$$

(The proof can be found in the appendix.) This last result is interesting, because it expresses a most famous principle from (Kripke 1981), but with two significant qualifications. Firstly, actual identity is not taken to imply necessary existence. This surely aligns with Kripke's views, but in our formalism it explicitly shows from the formal rendering of the principle. Secondly, the principle

is not conceived of now as a metaphysical one, as it is conceived of by Kripke, who, apparently, deems the principle also logically valid. Here it comes about as a formal consequence of how we prefer to conceive of things in modal reasoning. Obviously, if one, upon reflection, believes (*NI*) to be a valid metaphysical principle, one is of course able and free to award it the status of a logically valid principle. However, failing reasons to do so, one can as well safely assume it is just a contingent constraint on what are proper conceptual windows.

Now that we have some idea of what a proper conceptual window is, we can give further specifications of which individuals are seen through such a window, and also of how they are conceived of. We can do so according to the following general format.[13]

$$^{i}\exists\vec{x}(\phi(\vec{x}) \wedge \Box_{\vec{x}}\,\psi(\vec{x})) \qquad (EW)$$

A stipulation in this format requires i to be a window on some series of individuals that actually stand in the ϕ relation and which are conceived of as standing in the ψ relation. The schematic condition $\phi(\vec{x})$ may include the actual identification of (some of) the $\vec{x}$s seen, and also, for instance, the condition that the $\vec{x}$ are all distinct and, e.g., that there are no other individuals seen through window i. More significant, in most of our applications, is the condition $\psi(\vec{x})$. If the individuals $\vec{x}$ are seen through window i, as schematically defined, they are seen to be ψ. Someone who is said to know or believe something about these individuals, as seen this way, cannot fail to know or believe them to be that way. If agents are reported to have an intentional stance regarding individuals seen through such a window i, the window supplies a view upon what may be called a *resource situation* in the sense of situation theory. For these agents it is a sort of common knowledge that the witnessed individuals are ψ.

Let us briefly consider two examples. In the logico-philosophical literature Willard Van Orman Quine has introduced Ortcutt, presented as a man with a brown hat seen on the beach. Let us formally identify him as *the unique man with the brown hat* that is seen through a window of Quine's and let us label this window w.

$$^{w}\exists x(o{=}x \wedge \Box_{x}\,{}^{w}\imath z(BHz)(x{=}z)) \qquad (EQ)$$

Quine next tells us that some agent, Ralph, believes of Ortcutt, seen this way, that, say, he is a spy: $^{w}\exists x(o{=}x \wedge \Box^{r}\,Sx$. Given the axioms and constraints we have stated above, this entails that Ralph knows that ^{w}the man with the brown hat is a spy: $\Box^{r}\,{}^{w}\imath z(BHz)(Sz)$. Ralph, however, need not realize this is Ortcutt. As Quine also told us, Ortcutt may additionally be given to Ralph as a pillar of the community, through some window w': $^{w'}\exists x(o{=}x \wedge \Box_{x}\,PCz)$. There is no inconsistency, then, also not on behalf of Ralph, if he believes of Ortcutt,

13. Here, $\vec{x}$ must be read as a non-empty sequence of variables $x_1 \ldots x_j$, $^{i}\exists\vec{x}$ must be read as a series of corresponding existential quantifiers $^{i}\exists x_1 \ldots {}^{i}\exists x_j$, $\phi(\vec{x})$ as restricting the actual individuals $\vec{x}$ seen, $\Box_{\vec{x}}$, as restricting the set of accessible possibilities to those where the $\vec{x}$s exist, and $\psi(\vec{x})$ as expressing how the $\vec{x}$ are seen to be related.

seen this way, that he is not a spy: ${}^{w'}\exists x(o{=}x \wedge \Box^r \neg Sx)$. It does follow, of course, that Ralph then believes some pillar of the community not to be a spy: $\Box^r\, {}^{w}\exists z(PCz \wedge \neg Sz)$.

Another example may consists in a sort of textbook presentation of our solar system, with the sun and eight planets, ordered for their relative distances to the sun. Let us label such a window *ss*.

$$^{ss}\exists \vec{x}\ \Box_{\vec{x}}(s{=}x_0 \wedge m{=}x_1 \wedge a{=}x_2 \wedge e{=}x_3 \ldots \wedge x_0{<}x_1{<}x_2{<}x_3 \ldots) \qquad (SS)$$

Observe that the $\vec{x}$ are existentially quantified, so they are stated to exist, and because $\Box$ satisfies (T), they are, respectively, the actual Sun (x_0), Mercury (x_1), Venus (x_2)[14], Earth (x_3), Mars (x_4), etc. Furthermore, as seen through *ss*, the nine objects are also known to be the Sun, Mercury, etc., respectively. They are also conceived of as being order according to their relative distances to the sun. Now if someone is said to know something about the planet Earth the way it is presented through *ss*, this implies that she knows that it is a planet that is further from the sun than Venus is. She may, however, fail to know the very same fact for as far as it concerns the earth that she perceives to be walking on and Venus as she is seeing it in the sky above.

It may have to be noticed that, in the way our solar system is conceived of according to (SS), it is not inconsistent to speculate about there possibly being more planets in our solar system. Even if it were added that no other planet is seen through this window *ss*, there could still actually exist another planet, visible from another window. However, if our window is made to also show that there just is no other planet than those seen, then the window, of course, does exclude the mentioned speculation.

4 The Idea of An Intentional Object

The logical framework presented in the previous two sections is concerned, in the first place, with reasoning about what there is, and about what people may think and say that there is. Understanding what is thought and said that there is, however, involves making sense of things that there are not. If we deny that the world rests on the back of a giant turtle, it seems we have to acknowledge or understand the possibility of there being such a mythical turtle, so as to deny its real or actual existence. And indeed in the very same framework we can consider the possibility of there being certain things, while at the same time being well aware of the possibility that they may not actually be there.

When it comes to such talk and thought about intentional objects, let me first repeat what I already said above. The, not at all original, idea behind the third first principle is that if we speak of there being an intentional object, this

14. I have here employed '*a*', short for the Greek name "Aphrodite", as a name of Venus, because the letter '*v*' is reserved for another use later.

should be understand to mean that there is some intentional act or event, or just '*some intention*', which we understand to have its target object. (Husserl 1913; Crane 2012; Moltmann 2015, a.m.o.) If someone speaks of a 'possible bridge' we are invited to acknowledge a possibility in which there is understood to be a bridge; talk of a 'mythical horse' relates of a myth understood to host some horse; a 'fictional detective' is understood to relate of a fiction accommodating some detective; a 'hypothetical planet' refers to a hypothesis that there be a certain planet; and seeing 'painted roses', in the sense meant here, implies that there is a painting presenting roses, etc. In all of the cases that I intend to cover, here, the idea is that there is not a bridge that is possible, or a detective who is fictional, or a planet that is hypothetical, or roses, that are painted. In none of the cases there is meant to be an object that is intentional. There is just an intention with a target, virtual, object. The idea seems to be fairly intuitive, and it is this intuition that I want to respect in the first place.

In brief, we can be seen to be engaged with possibilities in which there are objects which do not actually exist.[15] We are able to specify, understand, and discuss, possibilities hosting possible individuals that we understand to be there. How are they given to us? They are, as said, not real objects, that now figure in a possibility, but they are presented to us through a possible way of seeing things, a possible window. The possibility of seeing things can be rendered by any sort of modal base — a myth or a fiction perhaps, or a hypothesis, or a plan, or just any 'possibility', characterized using any modal $\Diamond^k$, or just, most generally $\Diamond$. Relative to such a, possibly non-factual, possibility things may be seen through a window, like real things are presented to us in actual reality.

A characterization of intentional objects thus requires the specification of a modal base —because in reality we can see no non-existent things—and of a conceptual window to account for the ways in which possible individuals are conceived of. Intentional objects, or whole bunches of them, can therefore be taken to be '*presented*' according to the following schematic format.

$$\Box^{m\,i}\exists\vec{x}(\phi(\vec{x}) \wedge \Box_{\vec{x}}\,\psi(\vec{x})) \qquad (IW)$$

This is like the specification of what we can see through an extensional window i, as in (EW), but the specification is now given relative to a modal base m, which is possibly non-factual. Notice that we need to assume that the modality m is not inconsistent, so that $\Box^m\phi$ implies $\Diamond\phi$. Otherwise the possible existence of the $\vec{x}$ is not guaranteed.[16]

Since the 'intentional objects' $\vec{x}$ in (IW) are declared under the modal $\Box^m$, they are conceived of as existing there, but not as actually existing. According to that possibility these are $\vec{x}$s that are ϕ, even if not necessarily so. The

15. We have *existence, in fiction*, like we have *truth, in fiction*. (Lewis 1978)
16. We could therefore have employed $\Diamond^m$, in stead of $\Box^m$, but that wouldn't really render the $\vec{x}$s as 'actually ϕ' according to modality m.

objects are, furthermore, in that possibility, conceived of, through i, as being ψ, and this is what they, so conceived, necessarily are.

We can again use Quine's presentation of the man with the brown hat in (Quine 1956) as an example, but now assuming, more reasonably, that the story about Ortcutt is just another myth.

$$\Box^q\, {}^w\exists x(o{=}x \wedge \Box_x\, {}^w\imath z(BHz)(x{=}z)) \qquad (IQ)$$

According to the imaginary possibility of Quine's myth, rendered by $\Box^q$, window w now provides a view on an imaginary unique man with a brown hat. In this possibility it is Ortcutt, and we imagine Otcutt the way in which he is presented to us: as ^{w}the man with a brown hat.

It may need to be emphasized here that, even if the embedded necessity $\Box$ in (IQ) is restricted to what we can possibly see through w, the modality itself is still as universal as possible. Since this $\Box$ is S5, the specification in (IQ), together with the constraints (D) and (EX) on w, entails that if anybody can see anything at all through w, it shows them a unique man with a brown hat. That is to say, it follows that $\Box({}^w\exists z \sqsubseteq z \rightarrow {}^w\imath z(BHz)(Ez))$.

The preceding bits and pieces are built on the assumption that we can imagine (stipulate, make up, etc.) possibilities, in which there are imaginary things, even if the possibilities are not real, and the things in there therefore non-existent. Can they relate to things we assume to be actually there? Since the things do not exist, they simply cannot stand in any relation with them.[17] We are, however, able to speculate about how actual objects would relate to non-actual ones in counterfactual situations. We can appropriately represent some such by correlating intentional objects, in an intentional setting, with objects as they are presented to us in the way we actually see them.

Here is an example. In the 19-th century Urbain Le Verrier hypothesized that there was a planet in an orbit between Mercury and the Sun which he proposed to name 'Vulcan'. Let us assume that we share, with Le Verrier, the view on our solar system as presented to us through window *ss* mentioned above. His hypothesis relates to the real Sun, Mercury, the planet Earth, etc., as they are seen that way. Since we ourselves may doubt, even deny, the existence of Le Verrier's Vulcan, we can properly credit Le Verrier's hypothesis only by taking

17. People do say that they 'are inspired by', say, fictional creatures, but this can therefore be taken to show that 'being inspired by', like 'think and talk about', is an intentional relation, not an extensional one. Likewise, Sherlock Holmes can be said to be more famous than Sir Arthur Conan Doyle, who resented him for that fact. (Graham Moore, 2010, in an interview, `https://www.npr.org/transcripts/131913558`.) This, however, shows that *being famous* and *resent* are construed as intentional relations, too. Observe, finally, that there are also what Manuel García Carpintero calls *meta-textual* uses of names. (García-Carpintero 2022) David Lewis provided the following examples: "Holmes was killed off by Conan Doyle, but later resurrected." (Lewis 1978, p. 39) Like Lewis we will not go into these uses here, because, upon this usage, the names are apparently understood to have a real denotation, in the area of literary studies, say, so they cannot be said to not exist.

it to be a view on the solar system according to an hypothesized possibility. Le Verrier's picture of the solar system can be rendered as what is seen in such a possiblity u, through a window w, in, roughly, the following way.

$$\Box^{u\ w}\exists z\ ^{ss}\exists\vec{x}\ \Box_z\,(s{=}x_0 \wedge v{=}z \wedge m{=}x_1 \wedge a{=}x_2 \wedge e{=}x_3 \ldots \wedge x_0{<}z{<}x_1) \quad (UV)$$

According to this possibility we can see a, hypothetical, Vulcan, and also the Sun and planets as we know them through our own window on the solar system ss. Vulcan (z) is conceived of as being closer to the sun than Mercury ($z{<}x_1$). Of the planets we know, or assume we know, that they actually exist. However, we can consistently deny Le Verrier's hypothesis by simply stating that, while it is conceivably possible that this Vulcan exists ($\Diamond\,^{w}\sqsubseteq v$), it actually does not ($\neg^{w}\sqsubseteq v$). Observe that the possibility claim can only be sustained if our own window ss on the solar system itself does not exclude that possibility.

The, schematic, specification of an intentional window as in (IW) also enables us to render Geach's famous case of intentional identity. (Geach 1967) Geach's case basically involves two medieval villagers, *Hob* and *Nob*, that are said to have some beliefs or other propositional attitudes regarding some, imaginary, local witch. Since we ourselves do not believe in the existence of witches, certainly not imaginary witches, it has been hard to see how we can understand the two villagers as having a joint or coordinated focus on something. However, our intentional windows allow us to precisely characterize them that way. We can stipulate an intentional window w so that:

$$\Diamond\,^{w}\exists x\,\Box_x\,^{w}\imath z(Az)(x{=}z) \qquad (II)$$

Assuming (D) and (EX), (II) renders w an intentional window on an imaginary, in that window unique, local witch (A). The attitudes of Hob and Nob then can be schematically described by $\Box^{h\ w}\exists u(Au \wedge Bu)$ and $\Box^{n\ w}\exists v(Av \wedge Cv)$. Both Hob and Nob are characterized as peeking through the same window w. If their reported attitudes can be possibly jointly true at all, they therefore can only be jointly true of this one and the same individual, the local witch. We will make this formally more precise in the next section.

Notice that (II) does not commit us to there being any actual local witch, it only states the possibility of there being one. Neither do the, schematic, reports of Hob and Nob's beliefs. Notice that, more in general, in none of the examples that we discussed in this section do the existential quantifiers have more than an intentional force only. None of them entails that any window w provides a view on any existing individual at all. None of them actually implies the existence of anything.

Theorem 3

$$(IQ), (UV), (II) \quad \not\vdash \quad \exists x{\sqsubseteq}x$$

This shows that all of the above complies with the third first principle. The

intentional objects that we can understand to see through w are only imagined to be seen through w. They are there only intentionally, not actually.

5 The Method of Intentional Reification

In the previous section I have argued that we can conceive of objects that are only conceived to be, while they are actually non-existent, that we can judge such intentional objects to be related, counterfactually, with real ones, and that different modal bases or intentional states can be understood to be directed at the same object, even if there isn't any. As was one of the purposes of this note, this served to demonstrate that we can make sense if thought and talk about things non-existent, that is, without crediting them any existence. The concept of an *Intentional object*, and of its *intentional existence*, was most generally explicated in terms of a possibility in which such an object is intended and understood to exist.

Even if we have by now some workable idea of an *intentional object*, its *intentional identity* may still remain somewhat perplexing. Friederike Moltmann has proposed the following explanations.

> [*T*]*he identity of intentional objects (. . .) depend(s) on the acts they depend on and on relations of coordination those acts enter with other acts.* [p. 149] *For two intentional acts to share 'the same N' (. . .) the intentional acts (. . .) have to have been coordinated (. . .). [p. 148]*
> [*A*]*n act a is coordinated with an act a′ iff a is meant to refer (or to pretend to refer) to the same object as a′.* (Moltmann 2015, p. 147)

Although I cannot but agree with most of this, it is, as an explanation, not quite successful. The identity of intentional objects is said to depend on the coordination of intentional acts, while the said coordination of these acts is defined in terms of some intended reference to the same object, so in terms of their (intentional) identity. This is actually circular, even if it is not of a vicious kind of circular.

More problematic, I believe, is the idea that Moltmann apparently deems particular referential intentions sufficient for establishing intentional identity. This is dubious. If we think of Geach's case of Hob and Nob, Nob may have the intention to refer (or to pretend to refer) to the same object that Hob intended to refer to. However, no such intention can be sufficient for establishing that Hob had any referential intentions, so no real or even intentional identity can be enforced this way. Moreover, the presence of such an intention also does not appear to be a prerequisite for establishing intentional identity. Hob and Nob may very well participate in a community rumor that some witch is terrorizing the village, and they may have their beliefs and worries as formulated by Geach, even if the two have never communicated themselves and don't know squad about each others beliefs and intentions. It seems that just having two individual

referential intentional acts of a certain sort can neither be a sufficient nor a necessary condition for establishing intentional identity.[18]

Peter Geach, who has originally coined the term *intentional identity*, actually characterizxed it in less demanding terms.

> *We have intentional identity when a number of people, or one person on different occasions, have attitudes with a common focus, whether or not there actually is something at that focus.* (Geach 1967, p. 627)

This is pretty vague, of course, but Geach's characterization does allow us to refer to this 'focus', which apparently can be 'common' to various intentional acts. Such a 'focus' can be understood to serve as a public, non-subjective point in some abstract intentional space, upon which we can coordinate various referential intentions. We can refer to such a 'common focus', also if we do not need to assume that anything is there. Of course, the question then remains what this focus is, and how it allows for the coordination of intentional objects. It turns out that our conceptual windows may do the job.

We can conceive of our windows as providing the sort of focus mentioned by Geach. Windows provide an objective view on a selected part of the world, possibly even an imaginary situation, and they are, thus, 'common' enough to allow us to coordinate the things seen through it, imaginary or not. As we will see, this will not so much consist in establishing some sort of intentional identity of intentional objects, because there aren't any, but in what I will call their '*intentional reification*'. The method of intentional reification consists in the understanding, or construal, of some 'intentional objects' as the same, hereby duly noting that the concept of *intentional object* remains precisely as it is explained in the previous sections.

To make all this more concrete, I will, in what follows. first present an informal, intuitive, description and explanation of this method of intentional reification. I will next sketch how our proof theory can be adapted, easily as a matter of fact, so as to also formally model it.

I hope it is obvious that we can make up intentional objects all the time. One can tell a colleague that someone is after his position, and the gullible colleague may wonder who *it* is. Perhaps there is no good answer to that question, if the competitor was made up, but this doesn't refrain the colleague from worrying and asking others if they perhaps know who *it* is. It is also easy to pretend that one is waving to someone at a distance. People may think *it* is a good friend of yours, and propose to invite *her* for dinner, even if you didn't really wave at anybody at all. There can be a plan for a bridge, and the mayor perhaps already dreams of opening *it*. Macbeth saw a dagger, that wasn't really there, but he saw *it* had blood all over it. It may be that a certain project needs a PhD student, and that *she* will be required to complete here thesis within the

18. There is some exquisite literature that investigates our intuitions about such conditions in specific cases, but it seems it never covers and explains the ease with which intentional identities can be construed in natural language.

project period. As we have seen, Geach discussed a report according to which Hob thinks that a witch has blighted Bob's mare, and Nob wonders whether *she* (...) killed Cob's sow.

In all of these cases we may maintain that there is no real object (competitor, person waved at, bridge, dagger, PhD student, witch, ...) that the persons under discussion should be taken to be concerned with. In all cases, there is however a, retrievable, intention (gossip, pretense, plan, illusion, need, folk-belief, ...) with its own object, an object which we can imagine or understand to be there, without having to judge it to be real. And in all cases we try and understand other people's activities as being concerned with, or directed at, such an object. We may for instance judge their verbal and non-verbal behavior rational, or otherwise appropriate, if we understood it as having been cast, or performed, in the, counterfactual, circumstance that the object would be there. Observe that we ourselves cannot refer to these objects, as they are deemed non-existent. However, in all of the above I have, successfully I hope, counterfactually referred to them, hereby using the italicized pronouns in the scope of another intentional operator. How can we render this, formally?

A quite good first shot at a formalization can be made if we employ free variables and let them take over the role of the pronouns in the description of the cases above. Let us briefly inspect the pretense that I was waving at some person: $\Box^p {}^w\exists x\, \Box_x\, Px$. According to the pretense p, window w supplies a view on someone that is seen as P, short for being the person I waved at. Formally we can, just for the sake of the argument, label this virtual, assumed non-existent, object with a free variable z. The following formula requires z to be essentially identical to the person pretended to be seen through this window. $\Box^p {}^w\exists x(\Box_x(Px \wedge x{=}z) \wedge \Box_z \in x)$. Now we can use this variable, z, to indicate the, only pretended, person as an object of the worried belief of your colleague, c: $\Box^c\, Wz$, where Wz is short for z worrying c. Notice that we hereby do not claim any existence of any intentional object, because there just isn't any. Even so, we have construed the worried belief of your colleague as one that could be true, or satisfied, only if it were related to someone I pretended I have waved at. My pretense and your worried belief are, thus, directed at the same object, but only counterfactually.

Intentional reification in general consists in such a construal, or understanding, of various intentional stances (beliefs, desires, plans, pretenses) as each having a target object and so that they would only be jointly realizable, counterfactually, by one and the same object. In this sense they can be said to target the same thing even if there is no thing that they target. Through this method of intentional reification we can understand the agents who entertain the respective intentions, or at least we can try and understand their verbal and non-verbal behavior, as being directed at such a joint, virtual, target.

The explanation so far may appear somewhat dissatisfying because I have had to resort to the use of free variables that we have no means to bind, neither

existentially, nor universally. Fortunately, this problem can be solved when we resort to a method of dynamic interpretation as advocated in systems of dynamic semantics.[19] In such systems existential quantifiers are taken to declare referents that remain available, in a discourse, for further description and specification. A, quite typical, dynamic entailment runs like this: $\exists xPx, Qx \models \exists z(Pz \wedge Qz)$. Note that once our proof system accounts for this inference, so that $\exists xPx, Qx \vdash \exists z(Pz \wedge Qz)$, we get, by our rule of (DN), that:

$$\Box^k \exists xPx, \Box^k Qx \vdash \Box^k \exists z(Pz \wedge Qz) \qquad (DDN)$$

In a dynamic semantics 'free variables' (anaphoric pronouns) can be understood to relate to declarations (existential quantifiers) figuring earlier in a discourse and this also works when these declarations are made in a modal context, thereby *not* entailing that they relate of an actually existing object.[20]

Let us first briefly re-consider how we think and talk about Quine's myth, thereby assuming that Ortcutt is made up, while Ralph is assumed to be real.

$$\Box^q {}^w\exists x\, \Box_x {}^w\imath z(BHz)(x{=}z) \;\wedge\; \Box^q\, o{=}x \;\wedge\; \Box^r\, Sx \qquad (DIQ)$$

The most important part here is, again, the subformula $\Box_x {}^w\imath z(BHz)(x{=}z))$ characterizing x as the conception of an individual that, if it exists at all, always is the unique man with the brown hat seen through window w. It has been introduced as a possibly non-actual object and according to Quine's myth, rendered by $\Box^q$, it (x) is Ortcutt. Ralph believes him (x) to be a spy. Notice that we can prove that according to Quine's myth, Ortcutt is the man with the brown hat seen in w and according to Ralph the man with the brown hat seen in w is a spy. This is to say that the individual who is Ortcutt according to Quine's myth is the person that Ralph believes to be a spy, even if Ralph does not need to know it is Ortcutt. All of this can be actually true without there being any real Ortcutt. It is not inconsistent to add that x does not exist.

The other examples discussed in the previous section can be dealt with in an analogous fashion. Let us once more consider the case of Vulcan. Some key features of the various speculations can be captured by the following rudimentary narrative, and its formal, schematic, representation.

There is a certain hypothetical planet. It is called Vulcan. It does not exist.

$$\Box^u {}^w\exists z\, \Box_z {}^w\imath x(\phi(x))(z{=}x) \;\wedge\; \Box^u\, z{=}v \;\wedge\; \neg\exists z \qquad (DUV)$$

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19. I hereby refer to the seminal work of Hans Kamp and Irene Heim, on *Discourse Representation Theory and File Change Semantics*, as well as the arguably non-representational semantics of Jeroen Groenendijk and Martin Stokhof's *Dynamic Predicate Logic*, and proper extensions of them. (Geurts, Beaver & Maier 2016; Groenendijk & Stokhof 1991; Dekker 2012)
20. I have spelled out the semantics that validates such dynamic modal entailments in my (Dekker 2012), and I have, independently, presented a (sound and complete) proof system underlying these inferences in (Dekker 2016; Dekker 2025b). A sketch of the basics can be found in the appendix to this note.

The first formula presents a (possibly non-existent) object with characteristic (defining) property ϕ, through (intentional) window w in a (hypothetical) possibility u. The second formula says that, according to the hypothesized possibility, it is Vulcan. We can say that *it* is called Vulcan, because le Verrier has so labelled the ϕ that we possibly see through w. We are justified in intentionally reifying it with the ϕ that he hypothesized according to the first formula. The final formula is the most way of claiming that it does not exist.

Notice that even after Le Verrier's hypothesis has been successfully refuted, it remains there as an hypothesis, which people still can have divergent opinions about. The following observation[21] is not at all unreasonable.

> *"A few, however, remained convinced that not all*
> *the alleged observations of Vulcan were bogus."*

Surely we can make sense of the intentional behavior of these '*few*', who do after believe in the existence of this Vulcan. This observation may help explain why 'the few' engage in making certain calculations, and point their telescopes where, we believe, there is nothing to see. We can, thus, explain, futile, research activities of these colleagues in the way we can understand the digging of a dog somewhere where, we may claim, the dog mistakenly believes a bone is buried. Entertaining a conception of an imaginary bone helps us understand the dog's activities, even if, as said, there is no bone. Likewise, we can explain the investigations of *the few* mentioned above by saying that they inspect the sky precisely where they believe Vulcan is.

With the dynamic amendation in place, we can in principle render the case of Geach's Hob and Nob, even more 'naturally', as follows now.

> According to Hob something conceived of as a witch (A) is such and siuch (B), and according to Nob she is so and so (C).

$$\Box^h({}^w\exists u\, \Box_u\, Au \wedge Bu) \;\wedge\; \Box^n\, Cu \qquad (DII)$$

This says so much that Hob has a view on something that cannot but be a local witch (A) and that he deems to be B, and so that Nob deems it to be C. For this to be true there does not need to be any witch in reality, but for the beliefs attributed to Hob and Nob to be jointly realized, in any possibility, the window w must supply a view on one and the same witch that is both B and C. Observe also that, if Hob's beliefs are assumed to be not inconsistent, then what Nob believes to be C must be a local witch that is seen through w.

The reader may wonder what serves to empirically motivate one to state something like (DII) and to intentionally reify what Hob and Nob's beliefs are understood to be about. This cannot derive from the concepts that Hob and Nob actually employ because that is not what (DII) is saying anything about. If can also not be what the two have a concept of, because that does not exist, or so it has been assumed. It can only be what, we understand, the two have a conception of.

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21. `https://www.chemeurope.com/en/encyclopedia/Vulcan_(hypothetical_planet).html`

We assume we can reasonably understand both Hob and Nob, or the verbal and non-verbal behavior of both, as making sense if it had been directed at some counterfactual object, some presumed local witch. The understanding of the behavior of both as being directed at one and the same non-existent object, can be subsequently backed up by assuming that the two participate in an objective and joint, but actually mistaken, view on reality that presents some one particular local witch. This is to say that there is a non-actual possibility in which some window w provides a view on exactly one witch, a sort of possibility that both deem actual, and one that we can also counterfactually 'see', if we were to assume that possibility to be real.

We can easily specify a window w as providing for such a common focus by requiring that $\Box^{m\,w}\exists x\,\Box_x{}^{w}\imath z(Az)(x{=}z)$. (Here, A is again short for something like being a local witch. The mythical modality $\Box^M$ is, again, assumed to be consistent.) This, actually quite innocent, assumption can be employed to coordinate the two reports about the beliefs of Hob and Nob just by using w as the window we assume they are seeing reality through. A real intentional identity provably follows, as the following theorem states.

Theorem 4 Under the given constraints
$\Box^{m\,w}\exists x\,\Box_x{}^{w}\imath z(Az)(x{=}z),\ \Box^{h\,w}\exists u(Au\wedge Bu),\ \Box^{n\,w}\exists v(Av\wedge Cv)\ \vdash\ \Box_u\,u{=}v$
The derived conclusion can be properly considered to be a *counterfactual identity* because it says that necessarily $u{=}v$, in whatever possibility the two exist, while actually $u{\neq}v$, because the two do not actually exist.

Our last theorem actually serves to show how we now have indeed made sense of talk about intentional objects, in compliance with our second first principle. The premises, and hence the conclusion, can be true, also if u (and v) do not actually exist. Even so, the premises license a necessary, counterfactual, identity and therefore admit of intentional reification. For if u counts as being in w, then $\Box^{n\,w}\exists v(Av\wedge Cv)$ and $\Box_u\,u{=}v$ formally entail that $\Box^n\,(Au\wedge Cu)$.

Conclusion

A dynamic modal logic of the kind sketched in this note, independently motivated itself, allows us to formally render our thought and talk about non-existent objects in a way that respects the three first principles stated above. In accordance with the first first principle, the logic keeps to the innocent and intuitive equivocation of existence and being. When we agree, with the third first principle, that intentional objects intentionally exist, this is not taken to mean that intentional objects are a special kind of objects, and that intentional existence is a special kind of existence. We merely acknowledge the fact that here are certain intentional acts which we understand to be 'directed at objects', which may fail to be real.

We can conceive things, and what we conceive to be real, we are tempted to call a possible object. If the object is not real, it does not exist and there is

only a conception of a thing. In the latter case we conceive a thing that is not there, so there is not a thing that we conceive. Perhaps there is something, in someone's mind or brain, that figures as a place-holder for the thing conceived, a concept or a representation, but that surely cannot be the thing conceived. When Macbeth 'sees' a dagger, which isn't there, he 'sees' a dagger, not a concept or representation of it. Moreover if there is such a concept or representation, then obviously the concept exists, and it surely is not the concept that we deny the existence of when we say that the dagger does not exist.

Regarding our second first principle, how is it then possible to think and talk about things non-existent, when they don't exist? Elaborating on our ability to conceive things, we can conceive of such things as a common target of different intentional acts, as something intended to be present at a common focus. We can intentionally reify the object (target) of one intentional act with that of another, for instance, by thinking of it is the imaginary unique object possibly visible through sone window w. This help us characterize various intentional acts as being directed at the same thing, also if that does not exist, and use it to understand and explain our coordinated behavior.

I think we all can accept, as an acknowledgment of our own regular practices, in daily live, as well as in philosophy, that there are many things we can make up, hypothesize, imagine, and which we can deny the existence of. With this note I have only wanted to present a logical language which is able to respect these practices, and that allows us to properly deny the existence of things said or thought to exist.

There is no doubt that people, after all, credit some sort of being to the non-existent, and even credit them with existence. The literature hosts many bona fide, and interesting, metaphysical systems where such possible objects flourish. I can certainly not prove those people to be mistaken, and I also have no inclination, wish or intention, to do so. The simple reason is that, according to the intuitions expressed in this paper, there is just nothing to prove the existence let alone the non-existence of.

References

Aristotle 1933. *Metaphysics*. Harvard: Harvard University Press.

Ben-Yami, Hanoch 2022. Fictional Characters and Their Names. *Studia Semiotyczne* 36(1), pp. 9–16.

Crane, Tim 2012. Existence and Quantification Reconsidered. In: Tuomas Tahko (ed.) *Aristotelian Metaphysics*, Cambridge: Cambridge University Press. pp. 44–65.

Dekker, Paul 2012. *Dynamic Semantics*. Berlin: Springer.

Dekker, Paul 2016. Exclusively Indexical Deduction. *Review of Symbolic Logic* 9(3), pp. 603–637.

Dekker, Paul 2025a. *Conceptually Grounded Quantification*. Technical Note Series X-2025-02, ILLC/Department of Philosophy, Amsterdam.

Dekker, Paul 2025b. *Discourse Presentation Theory*. Technical Note Series X-2025-??, ILLC/Department of Philosophy, Amsterdam.

Frege, Gottlob 1884. *Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl*. Breslau: Wilhelm Koebner Verlag.

García-Carpintero, Manuel 2022. The Semantics of Fiction. *Mind & Language* 38, pp. 604–618.

Geach, Peter T. 1967. Intentional Identity. *Journal of Philosophy* 64, pp. 627–632.

Geurts, Bart, David I. Beaver & Emar Maier 2016. Discourse Representation Theory. In: Edward N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, Stanford: Stanford University. Spring 2016 edition.

Groenendijk, Jeroen & Martin Stokhof 1991. Dynamic Predicate Logic. *Linguistics and Philosophy* 14(1), pp. 39–100.

Hintikka, Jaakko 1984. Are there nonexistent objects? Why not? But where are they? *Synthese* 60, pp. 451–58.

Hume, David 1739. *A Treatise of Human Nature*. London: John Noon. `www.gutenberg.org`.

Husserl, Edmund 1913. *Logische Untersuchungen. Zweiter Band*. Halle: Max Niemeyer. 2. Umgearbeite Auflage.

Kant, Immanuel (ed.) 1956. *Kritik der Reinen Vernunft*. Hamburg: Felix Meiner Verlag. Originally from1781/87, edited by R.Schmidt.

Kratzer, Angelika 1991. Modality. In: Arnim von Stechow & Dieter Wunderlich (eds.) *Semantik: ein internationales Handbuch der zeitgenössischen Forschung*, Berlin: Walter de Gruyter. pp. 639–650.

Kratzer, Angelika 2007. Situations in Natural Language Semantics. In: Edward N. Zalta & Uri Nodelman (eds.) *The Stanford Encyclopedia of Philosophy*, Stanford: `https://plato.stanford.edu/archives/fall2023/entries/situations-semantics/`. Fall 2023 edition.

Kripke, Saul 1981. *Naming and Necessity*. Oxford: Blackwell. Paperback edition of Kripke, 1980, the revised and enlarged edition of Kripke, 1972, in: Donald Davidson and Gilbert Harman (eds.), *Semantics of Natural Languages*, Reidel, Dordrecht.

Lewis, David 1978. Truth in Fiction. *American Philosophical Quarterly* 15(1), pp. 37–46. Reprinted with postscripts in: Lewis, D., 1983, *Philosophical Papers* V. 1, Oxford University Press, pp. 261–80.

Meinong, Alexius 1904. Über Gegenstandstheorie. In: Alexius Meinong (ed.) *Untersuchungen zur Gegenstandstheorie und Psychologie*, Leipzig: J.A. Barth. pp. 1–51.

Moltmann, Friederike 2015. Quantification with Intentional and with Intensional Verbs. In: Alessandro Torza (ed.) *Quantifiers, Quantifiers and Quantifiers:*

Themes in Logic, Metaphysics, and Language, Dordrecht: Springer. pp. 141–68.

Nolt, John 2020. Free Logic. In: Edward N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, Stanford: Stanford University. FALL 2021 edition.

Plato 1921. *Theaetetus / Sophist*. Loeb Classical Library, London/New York: W. Heinemann and G.P. Putnam's Sons. Translation by Harold North Fowler, source: `https://catalog.perseus.org/`.

Quine, Willard Van Orman 1948. On What There Is. *Review of Metaphysics* 2, pp. 21–38.

Quine, Willard Van Orman 1956. Quantifiers and Propositional Attitudes. *Journal of Philosophy* 53, pp. 177–187. Reprinted in: W.V. Quine, 1966, *The Ways of Paradox and Other Essays*, Random House, New York.

Recanati, François 1996. Domains of Discourse. *Linguistics and Philosophy* 19, pp. 445–475.

Reimer, Marga 1998. Quantification and Context. *Linguistics and Philosophy* 21, pp. 95–115.

Russell, Bertrand 1905. On Denoting. *Mind* XIV, pp. 479–493.

Smets, Sonja & Fernando Velázquez-Quesada 2023. Philosophical Aspects of Multi-Modal Logic. In: Edward N. Zalta & Uri Nodelman (eds.) *The Stanford Encyclopedia of Philosophy*, Stanford: Stanford University. Winter 2023 edition.

Stanley, Jason & Zoltán Szabó 2000. On Quantifier Domain Restriction. *Mind and Language* 15(2&3), pp. 219–261.

von Fintel, Kai 1994. *Restrictions on Quantifier Domains*. Ph.D. thesis, University of Massachusetts, Amherst.

Westerståhl, Dag 1985. Determiners and Context Sets. In: Johan van Benthem & Alice ter Meulen (eds.) *Generalized Quantifiers in Natural Language*, Dordrecht: Foris. pp. 45–71.

Appendix A. Two Sample Derivations

Entailed Possibilities

This derivation illustrates how (*DN*) can be employed. The subderivation on lines 4–8 must be independent and not employ findings from earlier lines.

If $\phi \vdash \psi$	then	$\Diamond\phi \vdash \Diamond\psi$
1.	$\Diamond\phi$	[ass.]
2.	$\neg\Box\neg\phi$	[DUAL,1]
3.	$\Box\neg\psi$	[ass.]
4.	$\neg\psi$	[ass.]
5.	ϕ	[ass.]
	$\vdots$	
6.	ψ	[as assumed]
7.	$\bot$	[PropLog, 4, 6]
8.	$\neg\phi$	[PropLog]
9.	$\Box\neg\phi$	[DN, 3, 4–8]
10.	$\bot$	[PropLog, 2, 9]
11.	$\neg\Box\neg\psi$	[PropLog]
12.	$\Diamond\psi$	[DUAL, 11]

Necessary Identity

The following proof assumes (D) and (EX) and relies on the S5 behavior of $\Box$. This derivation actually illustrates how declarations are used in modal reasoning. Notice that if x and y are in i, and one of them counts as declared, then, by (EX) both of them effectively count as declared.[22] We employ this fact in the derivation at line 7, where the declaration of x entails the declaration of y.

22. In the proof below this can be accounted for by deriving, between the lines 4 and 5, the formula $\Box({}^{i}\exists z \mathrel{E} z \rightarrow \mathrel{E} y)$ from (EX), and using, between lines 7 and 8, line 7′. $({}^{i}\exists z \mathrel{E} z \rightarrow \mathrel{E} y)$ as an additional assumption in the necessitation at line 16. We can then conclude 7″. ${}^{i}\exists z \mathrel{E} z$ from $\mathrel{E} x$ on line 7, and next conclude 7‴. $\mathrel{E} y$ by (E→) from 7′ and 7″.

$^{i}\forall x\,{}^{i}\forall y\,(x{=}y \to \Box_x\, x{=}y)$

1.	$x{=}x$	[ass.]	(x in i & declared)
2.	$y{=}y$	[ass.]	(x and y in i & declared)
3.	$x{=}y$	[ass.]	(x and y in i & declared)
4.	$\Box\Diamond\, x{=}y$	[S5, 3]	(x and y in i & declared)
5.	$\Diamond\, x{=}y$	[ass.]	(x and y in i)
6.	$\neg\Box\, x{\neq}y$	[DUAL, 5]	(x and y in i)
7.	$\exists x$	[ass.]	(x and y in i & declared)
8.	$^{i}\forall x\,{}^{i}\forall y(x{\neq}y \to \Box\, x{\neq}y)$	[D]	(x and y in i & declared)
9.	$(x{\neq}y \to \Box\, x{\neq}y)$	[E$\forall$, 8]	(x and y in i & declared)
10.	$x{\neq}y$	[ass.]	(x and y in i & declared)
11.	$\Box\, x{\neq}y$	[PropLog, 9,10]	(x and y in i & declared)
12.	$\bot$	[PropLog, 6, 11]	(x and y in i & declared)
13.	$\neg x{\neq}y$	PropLog]	(x and y in i & declared)
14.	$x{=}y$	[PropLog, 13]	(x and y in i & declared)
15.	$(\exists x \to x{=}y)$	[PropLog]	(x and y in i)
16.	$\Box(\exists x \to x{=}y)$	[DN, 4, 5–15]	(x and y in i & declared)
17.	$\Box_x\, x{=}y$	[def. $\Box_x$, 16]	(x and y in i & declared)
18.	$(x{=}y \to \Box_x\, x{=}y)$	[PropLog]	(x and y in i & declared)
19.	$^{i}\forall y\,(x{=}y \to \Box_x\, x{=}y)$	[I$\forall$, 18]	(x in i & declared)
20.	$^{i}\forall x\,{}^{i}\forall y\,(x{=}y \to \Box_x\, x{=}y)$	[I$\forall$, 19]	

Appendix B. Model-Theory (Elementary Clauses)

I will here sketch the main and characteristic features of the model-theoretic interpretation of the language discussed in this note. The models that we use are quintuples:

$$\langle \mathcal{W}, \{\mathcal{R}^k\}, \mathcal{U}, \{\mathcal{C}^i\}, \mathcal{I}\rangle$$

Here $\mathcal{W}$ is a set of possibilities and the $\mathcal{R}_k$ are the usual accessibility relations over $\mathcal{W}$ familiar from (multi-)modal logics. $\mathcal{U}$ is a set of points coordinating our talk about possible individuals and we have a family $\mathcal{C}^i$ of methods of individuation, each one consisting of a set of individual conceptions — functions, possibly partial, from $\mathcal{W}$ to $\mathcal{U}$.

We use $\mathcal{C}^0$ to define the domain $D_v = \{c_v \mid c \in \mathcal{C}_0\}$ of a possibility v and any other window i must provide a view on existing individuals only. For any $c \in \mathcal{C}^i : c_v \in D_v$, if defined. Finally $\mathcal{I}$ is an interpretation of the non-logical constants so that for each possibility v $\langle D_v, \mathcal{I}_v\rangle$ is an ordinary extensional predicate logical model. (But for the fact that $\mathcal{I}_v(c)$ may not always be defined, for individual constants c.)

The axioms in this note constrain the accessibility relations and the conceptual windows. In particular, $\mathcal{R}^0$ is required to be an equivalence relation on $\mathcal{W}$, one that subsumes the other $\mathcal{R}^k$s. Windows $\mathcal{C}^i$ that satisfy (EX) are sets of individual conception all defined for precisely the same (sub-) of possibilities, and (D) in addition makes that any two distinct conceptions in one window always have distinct values.

The (static) semantics for our language is defined in (Dekker 2025a). Crucial there is the relation $\mathcal{M}, v, g \models \phi$, the *satisfaction of a formula* ϕ in a model $\mathcal{M}$ relative to a possibility v and under an assignment g assigning individual concepts to the variables of our language. Most clauses are classical, except perhaps for the fact that the satisfaction of atomic formulas requires all of their individual terms to be defined. (Otherwise the formulas are false.) Quantified and modal formulas are interpreted as follows.

$$\begin{array}{rll}
\mathcal{M}, v, g \models {}^{i}\exists x\phi & \text{iff} & \text{there is } c \in \mathcal{C}^i\text{: } c_v \in D_v \text{ and } \mathcal{M}, v, g[x/c] \models \phi \\
\mathcal{M}, v, g \models {}^{i}\forall x\phi & \text{iff} & \text{for all } c \in \mathcal{C}^i\text{: if } c_v \in D_v \text{ then } \mathcal{M}, v, g[x/c] \models \phi \\
\mathcal{M}, v, g \models \Diamond^k\phi & \text{iff} & \text{there is } w\text{: } \mathcal{R}^k vw \text{ and } \mathcal{M}, w, g \models \phi \\
\mathcal{M}, v, g \models \Box^k\phi & \text{iff} & \text{for all } w\text{: if } \mathcal{R}^k vw \text{ then } \mathcal{M}, w, g \models \phi
\end{array}$$

In the dynamic version of the semantics existential quantifiers are modeled by declarations, which require satisfying witnesses in order to be satisfied themselves. (See Dekker 2012; Dekker 2025b for motivation and discussion.) These declarations, as well as the pronouns anaphoric on them, are associated with variables, as they are in the the system of *Dynamic Predicate Logic* (Groenendijk & Stokhof 1991) However, the variables are here used to enhance readability, even though for our logical purposes they are redundant, and actually a stumbling block. In an unfolding discourse the satisfying witnesses c, individual concepts themselves, pile up into sequences e of witnesses.

$$\mathcal{M}, v, g, e \models \Box^k \phi \;\text{ iff }\; \text{for all } w\text{: if } \mathcal{R}^k vw \text{ then } \mathcal{M}, w, g, e \models \phi$$

It must be noticed that the sequence e provides witnesses for declarations made in ϕ, but they do not necessarily have values in v.