Dialogue Games and Connexive Logic

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Extended Abstract:

In the first part of this talk we show that, because of the flexibility of such frameworks and their ability to capture dynamical aspects of argumentations, approaches to logic based on dialogue games[1] are fruitful tools for the study of non-classical logics. In the second part dialogue games for so-called connexive logic will be presented in more detail.

1 Non-Classical Logics and Dialogues[2]

The set of rules defining the games of *Dialogical Logic* is devided into structural rules, regulating the general course of the games, and particle rules, determining for each logical particle how the corresponding formulas might be attacked and defended. Several non-classical logics can be obtained by slightly changing the set of structural rules against a fixed background of particle rules.[3] For example, the formulation of dialogue games for free logics requires the addition of a structural rule restricting the Proponent's use of constants for attacking or defending quantifiers, while the formulation of paraconsistent logics asks for the addition of a structural rule restricting the Proponent's possibilities of attacking negations. As the changes of the set of rules for different non-classical logics are independent of each other, it is easy to combine these logics. For example, we obtain a combination of paraconsistent and free logics by adding both structural rules mentioned above[4].
Another possibility of formulating non-classical logics within a dialogue games framework is to add appropriate rules that allow to check metalogical constraints also on the object language level of the games. For example, considerations on relevance could be treated as follows: If in a dialogue game one player suspects that one of his opponent's moves is redundant, he might show this by taking over his opponent's position in a subsection and winning this position without making the move in question. We follow a similar strategy (of building metalogical constraints into the object language level of the games) when presenting dialogue games for connexive logic in the next part.

[1]We will focus on the so-called *Dialogical Logic* inaugurated by Paul Lorenzen and Kuno Lorenz, but most of the arguments presented should also be valid for similar approaches.

[2]Most of the arguments presented in this part are discussed at greater length in H. Rückert, 'Why Dialogical Logic?' (to appear in H. Wansing (ed.), *Essays on Non-Classical Logic*, Oxford University Press, in preparation).

[3]This strategy follows the so-called *Došen's Principle* which is well known in *Display Logic* (cf. K. Došen, 'Sequent Systems and Groupoid Models I', *Studia Logica*, 47 (1988), 353-389).

[4]For a detailed examination of combinations of paraconsistent, free and intuitionistic logics see S. Rahman, 'On Frege's Nightmare' (to appear in H. Wansing (ed.), *Essays on Non-Classical Logic*, Oxford University Press, in preparation).

2 Dialogue Games for Connexive Logic[5]

Many of the discussions about conditionals can best be put as follows: can those conditionals that involve an entailment relation be formulated with in a formal system? The grounds for the failure of the classical approach to entailment have usually been that they ignore the *meaning connection* between antecedent and consequent in a valid entailment. One of the first theories in the historyof logic about meaning connection resulted from the stoic discussions on tightening the relation between the If- and the Then-part of conditionals, which in this context was called $\sigma\upsilon\nu\alpha\rho\tau\eta\sigma\iota\zeta$(connection). This theory gave a justification for the validity of what we today express through the formulae $\neg(a \Rightarrow \neg a)$ and $\neg(\neg a \Rightarrow a)$, which we call the first Boethian and the first Aristotelian thesis of connexivity, respectively. Hugh MacColl and, more recently, Storrs McCall searched for a formal system in which the validity of these formulas could be expressed. Unfortunately neither system is very satisfactory. Connexive logic is not an extension of classical logic: $(a \rightarrow \neg a) \vee (\neg a \rightarrow a)$ is a classical tautology. In this part we introduce dialogue games including the formulation of a new connexive If-Then ("$\Rightarrow$"), which allow the Proponent to develop (formal) winning strategies not only for the above mentioned connexive theses but also for $(a \Rightarrow b) \Rightarrow \neg(a \Rightarrow \neg b)$ and $(a \Rightarrow b) \Rightarrow \neg(\neg a \Rightarrow b)$, which we call the second Boethian and the second Aristotelian thesis of connexivity, respectively.
As we understand it, MacColl's reformulation of the meaning connections implicit in traditional hypotheticals comprises the following conditions for the connexive If-Then:

1. The If-part should be contingent or not inconsistent. In other words, the If-part should not yield a redundant Then-part by producing an inconsistency.

2. The Then-part should not yield a redundant If-part. That is, the Then-part should not be tautological.

These two (metalogical) conditions can be readily expressed by means of the two new dialogical operators $\mathbf{V}$ and $\mathbf{F}$ and thus be build into the object language level. Stating $\mathbf{V}A$ commits to defend A in a subsection of the dialogue game, whereas stating $\mathbf{F}A$ commits to defend $\neg A$. In both cases the defender is allowed to state atomic formulas whenever he needs them in the subsection; thus, his partner has to play *formally* (the relations between the different sections of a dialogue game are regulated by appropriate structural rules). The new connexive If-Then then reads:

$$A \Rightarrow B := (A \rightarrow B) \wedge (\mathbf{V}A) \wedge (\mathbf{F}B)$$

The formulation of the corresponding strategy tableaux systems has to take care of the possible changes of the formal duty by refining the tableaux rules as well as the closing rules.
Our dialogical approach to connexive logic has as an immediate consequence that uniform substitution does not hold anymore ($a \Rightarrow a$ is connexively valid, but $(b \Rightarrow b) \Rightarrow (b \Rightarrow b)$ is not). But by defining the new concept of singular logical form a very restricted form of uniform substitution can still be maintained, allowing in fact only the renaming of the atomic formulas. By losing uniform substitution it also seems as if the formulation of a calculus becomes impossible, nevertheless a tableaux system can be formulated.

[5]The dialogical connexive logic presented in this part has been developed jointly with PD Dr. Shahid Rahman (Universität des Saarlandes, Germany). See S. Rahman / H. Rückert, 'Dialogical Connexive Logic' in S. Rahman / H. Rückert (ed.), *New Perspectives in Dialogical Logic* (special issue of *Synthese*, in preparation).