The semantics of actions in knowledge games

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1 Knowledge games

A *knowledge game* is defined by a dealing of cards over players, a set of possible actions (or moves), an order protocol to determine who is next to move, and a procedure to determine if a player has won. Cards do not change hands during a play of the game. A *state of the game* is defined by a given dealing of cards and an action sequence, initially empty.

A *dealing* of cards is a function from the set of cards C to the set of players P. The *initial state of the game* can be represented by the dealing of cards. The initial state space S_s corresponding to a dealing s, is the subset of P^C where all players have the same number of cards as in s. Dealings s and s' are indistinguishable from each other for player p if they agree on his cards. This induces a structure $(\sim_p)_{p \in P}$ on S_s. $\sim$ is the transitive closure of all $\sim_p$. $\sim$ is the universal relation on S_s.

Let c_p be the proposition corresponding to player p holding card c. The initial state of the game given a dealing s corresponds to the pointed Kripke model $(\langle S_s, (\sim_p)_{p \in P}, V \rangle, s)$ with $\forall s' \in S_s : V_{s'}(c_p) = 1 \Leftrightarrow s'(c) = p$.

A pointed multimodal $S5$ model is called a *knowledge state*. A knowledge state for a set of agents B is called a *B knowledge state*. Every state of a knowledge game is a knowledge state. A game action is a knowledge state transforming operation.

2 Uniform knowledge programs

An action transforming a knowledge state can be described by a knowledge program. We define the class of uniform knowledge programs (UKP). Many actions in knowledge games [Ditmarsch 99] can be modelled as *deterministic* uniform knowledge programs. This includes all game actions in Cluedo, such as showing a card (only) to another player given a request for one of three cards. The class of uniform knowledge programs (UKP) is a subclass of the class of uniform partial knowledge programs (UPKP). Uniform partial knowledge programs are formed by tests, sequential execution, nondeterministic choice, and the 'learning with choice' operation '!': $\pi \, !_B \, \pi'$ stands for 'π, and group B learns (π or π')' – a group learns a nondeterministic choice between two programs, of which the first one is executed.

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We define: $\pi\ !_B\ \pi' \in \mathsf{UPKP}$, if $\pi, \pi' \in \mathsf{UPKP}$, and $|\pi| = |\pi'|$, and $|\pi| \subset B$. Here $|\pi|$ is the *size* of a program: the group of agents 'involved' in it, inductively defined such that $|\pi\ !_B\ \pi'| = B$. So we require that group B can *only* learn programs concerning subgroups *strictly* contained in it. The constraint $|\pi| = |\pi'|$ on program constructs is why we have called our programs *uniform*. The class of uniform knowledge programs (UKP) given a set of agents P is the subclass of UPKP with size P.

The interpretation of a UKP program π is a relation $[\![\pi]\!]$ between knowledge states. As π is composed of partial uniform knowledge programs, we introduce the concept of *partial interpretation* $[\![\pi]\!]^{|\pi|}$ of a program. The partial interpretation of a program is its interpretation in a *restricted knowledge state*. The restriction of a P knowledge state $s = (\langle S, (\sim_a)_{a \in P}, V\rangle, s)$ to the set of agents $B \subset P$ is the B knowledge state $s{\restriction}B := (\langle S, (\sim_a)_{a \in B}, V\rangle, s)$.

The idea behind the interpretation of 'public learning with choice' $\pi\ !\ \pi'$ (if $B = P$ in $!_B$ we delete the index) is the following: 'First' we *partially* interpret program $\pi \cup \pi'$. This results in a finite set S of alternative (nonsimilar) $|\pi|$ knowledge states. 'Then' group $|\pi|$ 'chooses' one of S from those that can be reached by executing π. 'Last', we construct a P knowledge state from S. As players in $|\pi|$ know what they have chosen, there is no $|\pi|$ access between worlds from different alternatives in S. For the players in $P \backslash |\pi|$ however, we add $P \backslash |\pi|$ access between *any* previously (to the execution of $\pi\ !\ \pi'$) indistinguishable worlds, whether or not from the same state in S!

We will present the semantics of UKP programs, and several examples.

Our approach is strongly motivated by [Baltag et al. 99] and [Gerbrandy 99]. A related approach is found in [Baltag 99]. A simple translation $\triangle$ embeds UKP programs into Gerbrandy's DEL programs. Its essential clause is $(\pi\ !_B\ \pi')^{\triangle} := \pi\ ; U_B(\pi \cup \pi')$. We now have found a much desired class of DEL programs under whose execution the class of $S5$ models is closed.

3 Knowledge programs

Some actions in knowledge games are not 'uniform': a group might learn about *different* subgroups. Also, some actions in knowledge games have parts that had best be seen to take place *simultaneously*. An example of both is the action where a player (simultaneously) shows a card to a player and one of his other cards to another player. We are currently extending UKP with parallel execution of programs and with non-uniform learning.

References

[Baltag 99] Alexandru Baltag, *A Logic of Epistemic Actions.* Manuscript.

[Baltag et al. 99] Alexandru Baltag, L. S. Moss & S. Solecki, *The Logic of Public Announcements and Common Knowledge.* Proceedings of TARK 1998 conference.

[Ditmarsch 99] Hans van Ditmarsch, *The logic of knowledge games: showing a card.* Proceedings of BNAIC 1999 conference.

[Gerbrandy 99] Jelle Gerbrandy, *Bisimulations on Planet Kripke.* ILLC Dissertation Series DS-1999-01. Amsterdam 1999.