Games as Acceptors

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Every systematic effective procedure which constructs a (two person) game from some input data characterizes a formal language. This language consists of those input strings for which the resulting game is won by the first player. By this procedure we can ascribe to games a role which they so-far have rarely played in the literature: *Games as Acceptors*.

Truth is that this idea is neither new nor unknown. In the late seventies and early eighties researchers in Computation and Complexity Theory have established a group of characaterizations of the class PSPACE where games play an essential role. This very robust class can be characterized in terms of space complexity on reasonable sequential models, as the polynomial time bounded class on Ultra Parallel but Uniform Machines of various sorts (as expressed by the *Parallel Computation Thesis*), as the polynomial time bounded class on alternating squential devices, which establishes a link with machine computation games played by two opponents both controling the machine, and by the expressive power of Quantified Propositional Logic. These characterizations and their proofs represent one of the nicest achievements in Computation Theory around 1980.

However, it is quite possible that the strength of these results is also responsible for the fact that the connection between games and other computational models has not been investigated further. The general known fact that (finite Combinatorial Complete Information two-peron games in general show up a PSPACE complete problem to characterize the winning instances) has discouraged people from looking further.

In fact there are many other modes of computation which can be related to games. A prime example is the realm of nondeterministic computation, where in fact games have appeared in the literature which capture NP by having an NP-complete decision problem of being a winning game or not. However, most of these games are single player solitaire games, like the famous tiling game (in fact one of the first problems shown to be NP-complete by Levin). Even Deterministic computation can be modelled by a game, be it one which no human ever will be interested to play: like filling out your I-1040 tax form the game offers no choice to the player and already at the start it has been decided whether you win or loose.

Aside from an extension in this downward direction (towards more simple games) there are other topics which ask for being investigated from the perspective of games as acceptors.

It is well known in the realm of machine based computation there is nothing like the Church-Turing Paradise we experience in recursion Theory. We have a stable complexity theory centered on the Turing Machine model, but extending this stability (as expressed by the Invariance Thesis) towards alternative models like register machines, von-Neuman like computer models like the RAM and pointer based models like the SMM has turned out to be non trivial; several less suitable definitions have found a strong foothold in the literature.

In the realm of Parallel models the lack of stability is even larger. There is a class of parallel models based on exponential growth potential and uniformity which obeys the parallel computation thesis in a rather strict way; outside this realm chaos prevails, and each model seems to acquire its own complexity characterization (either above or below PSPACE).

On the side of the games the lack of formal foundations has its consequences. Games have been investigated for 50 years by economists but their emphasis is on the strategic form which eliminates all structural information available in the extensive form. The games corresponding to computational activities in general will be given in an extensive form. There seems to be a lack of understanding about the correct model for extensive games in general (the analogue of the Turing Machine for extensive games still must be invented), and moreover the question of equivalence between games (when are two games the same) seems far from settled.

The purpose of the proposed program to investigate the relation between games and computational models in computer science is two-fold. It should increase insight on both sides of the border.

In computer science, particularly in the area of interactive protocols and zero-knowledge exchange of information, models of computational interaction have been described which by nature are games, where features like probabilistic moves and incomplete information become relevant. Yet these games exhibit some strange feature. The winning conditions in these games are rather extraordinary: a winning starting position supports a winning strategy in a standard way, but a lost position must remain lost even if the player has the possibility of performing a number of cheats and illegal moves (up to full Byzantine behavior). This yields games where the nature of the reduction turning the game into an acceptor has obtained a *promise problem* character. It would be interesting to investigate to what extent these interactive protocols can be expressed in terms of more regular games, but also whether this promise feature can play a role in game theory as well.

Then there are acceptance modes investigated in computer science where it is far from evident whether a game is involved or not, notwitstanding the fact that these acceptance notions are defined in terns of structural properties of the full computation tree of a machine. The *Leaf languages* as investigated by the Würzburg school represent a prime example. It would be interesting to investigate whether in general a game can be invented corresponding to this leaf language recognition mechanism. If so what is the game; if not, does this fact shed some light on this acceptance mechanism in the sense that the limitation of the use of this mechanism representable by games correlates with the degree of naturalness of this mechanism.

On the side of the games the challenge is to investigate whether concepts from computation theory can be invoked to provide for a more robust and invariant concept of extensive games supporting structural equivalence notions and closely tight in with complexity theory.

At this stage this problem area has generated more questions than answers. In my presentation I will exhibit the landscape which was uncovered during the 1980-ies and indicate where the loose ends are located.