‘Games That Make Sense’: logic, language, and multi-agent interaction

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Gabriel Sandu tells an appealing story of natural language viewed in terms of games and game theory, bringing together several strands from the philosophical, logical, and even computational literature. In this short invited note, I will take the cruising altitude a few levels up from his, and show you a panoramic picture where the clamour of the raw facts on the ground has become just soothing, but wholly negligeable background noise.

1 Meaning is a many-mind notion

What do games have to do with natural language? On the traditional view of linguists and logicians, *syntax* is about grammatical code, *semantics* is about mathematical relationships between syntactic code and structures in reality, while the rest of language use is the bustling but unsystematic world of *pragmatics*. In particular, on this view, meaning does not involve agency of any kind: it is a ‘0-agent notion’. But starting from the 1970s, another view emerged placing actions of language users at centre stage, making meaning the ‘information change’, or more general ‘context change potential’ of linguistic expressions. Speakers or writers change the information states of their hearers or readers, and semantics should describe these changes. This action and update-oriented ‘1-agent view’ of meaning is the basis of the well-known Amsterdam paradigm of ‘dynamic semantics’ developed by Groenendijk, Stokhof and Veltman and their students, and it also underlies the well-known ‘discourse representation theory’ of Hans Kamp and Irene Heim.[1] Of course, this move also involves shifting the agenda. In particular, it relocates the traditional boundary line between semantics and

[1] See the *Handbook of Logic and Language* [14] for a survey of paradigms and sources in dynamic semantics broadly conceived, which also run over into computer science.

pragmatics in the study of language, and entire philosophical conferences have been devoted to that tectonic movement.[2]

But once on this road, it seems strange to stop here. Psychologists of language like Herb Clark [18] have shown convincingly that much of language use is directed toward the hearer's rather than the speaker's perspective, it is the hearer's uptake which determines the success of communication. And once you start thinking all this through, you wind up in a 'hermeneutical circle' of speakers taking into account how their hearers will interpret what they are saying, and hearers taking into account how speakers will phrase what they are saying, and level after level of stacked mutual information unfolds, leading to the iterative 'theory of mind' and mutual expectations that keep human behaviour stable according to philosophers and psychologists. It also leads naturally to *game theory*, since that is where these circles find their resting place in reflective and action equilibria.

2 Games have a history with natural language

Indeed, the idea that natural language has an intimate relationship with games has recurred through the 20th century. In the 1950s, the later Wittgenstein famously moved away from the crystalline logical structure of the *Tractatus* to a paradigm of rule-generating 'language games', and as Gabriel Sandu shows, authors like Stenius tried to put more substance into the game metaphor. Also in the 1950s, maybe under the influence of the then nascent game theory,[3] various proposals were made for analyzing logic in terms of 'logic games', casting basic logical activities like argumentation (Lorenzen [34]) or model comparison (Ehrenfeucht-Fraïssé; cf. [21]) as two-player games, with winning strategies encoding proofs, models, or invariance relations, as the case might be.[4] In particular, Gabriel Sandu discusses one of these, Hintikka's evaluation games for first-order logic [27], which later made its way into the study of natural language under the name of 'Game-Theoretical Semantics' (*GTS*). We will return to these games later, which mainly analyze the 'logical skeleton' of *sentence construction*: connectives, quantifiers, and anaphoric referential relationships. Thus, logic is still the driver of the analysis here – and the expression 'game-theoretic' does not suggest any deep contacts with game theory.[5]

Also in the same 1960s, another, logic-free, style of game-theoretic analysis for natural language came up in Lewis' work (cf. [32]), going back to Schelling [45] on *signaling games*. In this way of looking at language, Nash

[2] Viewed in this way, natural language is no longer a descriptive medium, but rather a *programming* language for bringing about cognitive changes.

[3] Much of the modern history of logic and its interfaces remains to be written, since authors usually stay with the aftermath of the foundational era in the 1930s.

[4] Van Benthem [10] is an extensive survey and discussion of logic games today.

[5] But see below for some mathematical contacts between logic games and game theory.

equilibria establish stable meanings for *lexical items*, the smallest atoms of sentence construction. While this new view remained largely a small undercurrent,[6] it has now become a major contender, with the authors discussed by Gabriel Sandu: Parikh [37], Dekker & van Rooij [19], van Rooij [42], Jaeger & van Rooij [31]. While logic games are largely about winning and losing only, these modern signaling games involve real preferences that communicating linguistic agents have about matching up intended and perceived meaning, grammatical structure,[7] as well as computational costs in doing so. Thus, they involve more serious connections with game theory, and at the same time, with the topological and metric structure of human perceptual and conceptual spaces (cf. Gärdenfors and Warglien [23]). This may well be the most serious encounter between linguistics and game theory today,[8] and there are many interesting questions about its connection to the earlier logic-game based approaches like *GTS*. Sandu is quite right in putting this link on the map in his piece, though much still remains to be clarified.

3 Evaluation games, language, and interactive logic

The basic idea of Hintikka-style evaluation games is that two players, **Verifier** and **Falsifier**, disagree about whether a given first-order formula φ is true in a given model $\mathcal{M}$, under some assignment s of objects to variables.[9] The rules of the game reflect this scenario - and they may be seen as describing dynamic mechanisms of evaluation or investigation of facts about the world. With disjunctions $\varphi \vee \psi$, **Verifier** must *choose* a disjunct to defend (**Falsifier** is opposed to both), with conjunctions $\varphi \wedge \psi$, the choice is **Falsifier**'s. A negation $\neg\varphi$ triggers a *role switch*, where players change roles in the game for φ. Moreover, quantifiers let players choose an object from the domain: $\exists x\varphi$ lets **Verifier** choose a 'witness', $\forall x\varphi$ lets **Falsifier** choose a 'challenge', after which play continues with the game for the formula φ. These moves change assignments of objects to variables, because the new value of x now becomes the chosen object d. When the game reaches an atomic formula, it is checked against the current assignment, and **Verifier** wins if it is true, and loses otherwise. In all, this produces a two-agent scenario of changing assignments which has the following basic property.

[6] Lewis himself did add interesting thoughts on 'Score-Keeping in a Language Game'. Also, the stream of work on common knowledge in epistemic logic relates to Lewis' study of conventions, though there are even some earlier sources in the social sciences.

[7] This scenario comes partly from linguistic Optimality Theory and its 'rule-free' paradigm which casts language users as optimizing syntactic and semantic analysis of assertions along a set of constraint-based preferences.

[8] Economics and cognitive science are other natural partners in this mix, as in the newly established interdisciplinary Bielefeld Heisenberg Center in 'Games and Cognition'.

[9] It has often been Fruitful – e.g., in situation theory and in dynamic semantics – to use first-order logic, not as a literal translation medium for natural language, but as a methodological 'test lab' for investigating basic features of actual usage.

A formula φ is true at $(\mathcal{M}, s)$ iff **Verifier** has a *winning strategy* in the evaluation game $Game(\varphi, \mathcal{M}, s)$.

Much can be said about this simple game. For instance, the dynamic view of logical constants as moves in a game is intriguing, and so is the multi-agent 'pulling apart' of basic logical notions into different roles for different players. In this setting, players' strategies become logical objects in their own right now, expressing 'dependencies' in interactive behaviour. This powerful and appealing viewpoint also underlies other logic games, and its many repercussions are still not fully developed today, where we seem to be witnessing the birth pangs of an 'interactive logic'. [10] Van Benthem [6] also points out surprising connections with the early foundations of game theory. In particular, the law of Excluded Middle for first-order logic says that **Verifier** can always win games of the form $\varphi \vee \neg\varphi$. Unpacking this by the above rules, the law says that either **Verifier** or **Falsifier** has a winning strategy in the evaluation game for any formula φ. This 'determinacy' can be proven via Zermelo's Theorem about zero-sum two-player games of finite depth, which in its turn also follows from Excluded Middle plus some logically valid game transformations.[11] Thus, semantical evaluation, and hence also linguistic meaning in a procedural sense, meets with classical game theory – a connection elaborated in [6].

In particular, viewed in this way, major issues in natural language semantics meet in interesting ways with basic questions about games. Here is one. As we said, applying logical operations in formal languages serves as a model for sentence construction in natural language. And the most famous semantic issue arising then is Frege's Principle of *compositionality*: which says that the meaning of any linguistic expression can be determined stepwise, in tandem with its construction out of grammatical parts. Here, too, games offer a fresh perspective. As we saw, logical operations correspond to moves in an evaluation game – but we can also state the above scenario differently, since it has nothing to do with the specific games involved. Disjunction and conjunction are really quite general *game operations*, taking two games G, H to a *choice game* $G \vee H$ or $G \wedge H$ starting with a choice by one of the players. Likewise, negation forms the obvious *dual game* to any given game. Thus, issues of linguistic compositionality become questions about *game algebra*, and the laws satisfied by natural game operations. For instance, van Benthem [8] shows how the complete game algebra underlying first-order logic is a decidable mixture of principles from Boolean Algebra plus laws for a left-, though not right-distributive operation $G; H$ of *sequential composition* of games. Thus, if we take the evaluation game perspective

[10] The recent strategic Eurocores Project 'LogiCCC: Modeling Intelligent Interaction in the humanities, computational and social sciences' is an effort to put this on the map.

[11] Evaluation games for other logical languages can be much more complex, involving infinite histories – as happens, e.g., with the modal μ-calculus: cf. [17].

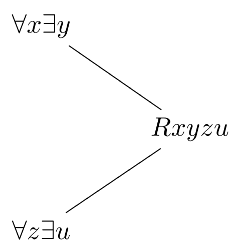
on natural language seriously as a view of multi-agent processes, we must understand the algebraic structure of the natural operations creating complex games for compound linguistic expressions out of simple ones.

4 Imperfect information and dependence

But logical evaluation games in *GTS* have further interesting features from realistic game theory, viz. *imperfect information.* Standard logic games, with the above evaluation games as a prime example, assume perfect information: players can observe each move that is played, and their only uncertainties are about future moves yet to be played. Gabriel Sandu has been one of the prime movers in a generalization, however, where the perfect information is abandoned in the process of semantic evaluation. Quantifier sequences in natural language sometimes show patterns of dependence and independence where it seems very natural to assume that access is blocked to objects chosen earlier. In the ‘slash notation’ of ‘independence-friendly logic’ (’*IF* logic’), a sequence like

$$\forall x \exists y \forall z \exists u/x\ Rxyzu$$

represents a ‘branching quantifier’ that can be written two-dimensionally as



This is true iff **Verifier** has a winning strategy consisting of responses to objects chosen by **Falsifier**, where the choice for u only depends on the object chosen for z. In this scenario, evaluation games are no longer determined, and they may even have only mixed equilibria in random strategies, letting probability into the inner sanctum of logic. There is a large technical literature on this generalization of classical evaluation games, but its game content is under debate, and Hintikka has been downplaying the original game motivation. Indeed, *IF* logic has inspired a mathematical analysis as generalized predicate logic by Hodges [30], while Väänänen [48] extracts the abstract logic of *dependence* at stake here without special game models.

But the jury is still out. For instance, Van Benthem [8] analyzes branching quantifiers in terms of a new game operation of the *parallel product* $G \times H$ of two games being played simultaneously without intermediate communication. [12] One reason why this move is of interest to natural language is as follows. It has been claimed that *IF* logic is deeply non-compositional, a difficulty related to the absence of natural 'sub-games' in games with imperfect information (Osborne & Rubinstein [36]). But introducing parallel product operations makes the underlying game algebra compositional again. Sandu's article actually discusses another recent game-theoretic take on *IF*, stemming more from the game semantics of programming languages. Abramsky [1] makes connections between *IF* logic and fragments of linear logic, whose parallel products do allow for intermediate communication, copying moves from one sub-game to another. In all then, the question of the complete multi-agent game algebra behind evaluation processes for natural language seems open, although by this stage, we have an incomparably deeper mathematical take on 'language games' than that of the 1950s.

5 Which games 'make sense' for natural language?

Our story so far does not exhaust the varieties of games that have been, or can be, brought to bear on natural language. There is a throng of further candidates, reflecting the many levels at which language can be studied.

5.1 Logic games

For a start, there are many logic games, and some fit natural language just as well as evaluation games for sentences φ against models $\mathcal{M}$. In much ordinary communication, there is no model at all of the described situation to evaluate against. What seems much more realistic then is 'consistency management'. We take in what the speaker says, and try to integrate this into consistent 'discourse representation structures' or more abstract semantic information states, unless the pressures on the successive updates become too high, and a conversational collapse takes place. But for this consistency management, a much more appropriate scenario might be *logic games of model construction*, which build models for sets of formulas (Hodges [30], Van Benthem [8]). In the semantics of natural language, the relevant distinction is 'dynamics of evaluation' (as in systems like *DPL*) versus '*dynamics of interpretation*', viewed as constructing a model or 'discourse representation' that makes sense of the current linguistic utterances.[13]

Interestingly, from a logical point of view, model building games are closely related to *dialogue games for proof.* As we said earlier, these were already introduced by Lorenzen [34], who wanted to explain logical validity of inferences $\mathbf{P} \Rightarrow C$ as the existence of a winning strategy in argumentation or debate for the **Proponent** of the conclusion C against any **Opponent**

[12] Van Benthem, Ghosh & Liu [13] provide its complete game logic and algebra.

[13] Indeed, van Benthem & van Eijck [4] already proposed that the proper take on Hintikka's view of natural language would be model building games associated with the method of '*semantic tableaux* rather than with semantic model checking.

granting the premises **P**. This raises the whole issue of *inferential* views of language and communication, which we will not pursue here. Historically, through the intermediate stage of Blass [16], Lorenzen dialogue games eventually led to game semantics for linear logic and programming languages in Abramsky's style. Thus, the games that Sandu tries to connect up with *IF* logic seem really quite different in spirit – but a link may be made through 'proof-theoretic' or category-theoretic semantics (Abramsky [2]).[14]

5.2 Signaling games

Now add the signaling games from the recent work by Parikh, van Rooij, and others, mentioned above. Sandu makes a simple and prima facie seamless connection, but I wonder about the consistency of scenarios. Signaling games really represent a very different scenario of language use, prior to the level of logic games. A logical evaluation game can only work when two things have already been settled: (a) the meaning of the logical operations, and (b) the denotations of the basic lexical items such as predicates and object names. But signaling games are about establishing the latter, and maybe even the former, connections in the first place!

Now in standard communication scenarios, we may assume that this initial phase has been achieved already, so that a global or at least a local, 'linguistic convention' has arisen. In that case, we can focus on the higher tasks of making claims, and convincing others. But there can be cases where the two tasks meet, as in the creation of the right anaphoric links, which do not have fixed conventional meanings. It is here where Sandu focuses his discussion, and I have nothing to add to that.[15] Even so, it seems fair to say that we have no integrated technical theory of logic games and signaling games, and I wonder what would be a good way of combining them. Do we need a game algebra for natural language which allows for composition of heterogeneous games of quite different sorts?

Finally, from the viewpoint of natural language, we have not even reached the complete picture of what goes on in ordinary conversation. There may be games that fix meanings for lexical items and for truth or falsity of expressions whose meaning is understood. But having achieved all that, the 'game of conversation' only starts, since we must now convey information, try to persuade others, and generally, further our goals – and maybe a bit of the others' as well. This is the area where Dutch-style logicians have been developing a broad family of 'dynamic-epistemic logics' for analyzing information update, belief revision (cf. Baltag, Moss & Solecki [3], Gerbrandy [24], Van Ditmarsch, Van der Hoek & Kooi [20], Van Benthem, Van

[14] This take on natural language interpretation seems closer to Categorial Grammar and its semantics in the lambda calculus, cf. Van Benthem [5], Moortgat [35].

[15] Other natural examples arise in the semantic scenarios of 'bi-directional Optimality Theory', many of which go beyond anaphora.

Eijck & Kooi [12]). These systems have already been given game-theoretic interpretations (Van Benthem [7], [11]), and recent twists toward dealing with rational agency include dynamic logics for preference change (cf. the dissertations of Liu [33], Girard [25] and Roy [43]).

But conversation and communication is also an arena where game theorists have entered independently, witness the earlier references in Van Rooij [42], and the recent signaling games for conversation proposed in Feinberg [22]. Again, there is an interface between logic and game theory to be developed here, and it has not happened yet.

5.3 The long term: language communities

Finally, there is one more level where games meet with natural language. We have talked about lexical meaning assignment, compositional semantics for single expressions, about checking for truth, argumentation, or information flow. But these are all short-term processes that run against the backdrop of a much larger, and potentially infinite process, viz. natural language use in communities with its *conventions over time*. In terms of computer science, the former are terminating special-purpose processes for concrete tasks, while the latter are about the never-ending 'operating system' of natural language. Here, again, signaling games are relevant, and they have been applied to such diverse issues as the emergence of Gricean norms in pragmatics (Van Rooij [41]) or of warning signals, or argumentative strategies (Rubinstein [44]).

In these scenarios, a significant move takes place, from single games to iterated games with *infinite runs*. Scenarios often involve thought experiments in terms of biological fitness and evolutionary stability against 'invaders' deviating from equilibrium. This is still about games and natural language, but with a very different agenda of explaining global, rather than local features of linguistic behaviour. And it is a far cry from logic games, involving rather dynamical systems theory for computing equilibria. Even so, it makes sense to ask for contacts after all. Infinite games like repeated Prisoner's Dilemma are iterated game constructions out of simple base games, so a discrete algebra of game constructions still makes sense in this extended setting. Moreover, logic games are often infinite, most clearly in the game semantics for linear logic and associated programming languages. And even from a narrowly logical point of view, questions about stability of reasoning practices make just as much sense as they do for linguistic conventions in language.

Thus, despite some conceptual and technical differences of emphasis and style in the literature right now, the encounter between logic and game theory in the arena of natural language seems far from concluded.

5.4 Natural language as a circus: a carroussel of games

I started by saying that natural language has three main aspects of syntax, semantics and pragmatics. By now it will be clear that ‘linguistics’ can ask questions about many levels of language use, asking for explanations of simple word meanings to successful discourse, and eventually the existence of broad norms and conventions that hold linguistic communities together. It also seems clear that games, whether from inside logic or directly from game theory, have an attractive role to play here, as an explicit way of bringing out the interactive multi-agent character of language use.

But what is the total picture? I have described natural language as a carroussel of games, where you can walk from one activity to another, and line up for the associated game. Is there a unifying principle, perhaps, one ‘super-game’? Should we find clues in mathematics, at some level of ‘deep game algebra’, or rather in the communicative character of home sapiens? I do not know, but I think that these questions are worth asking, if ’games and language’ is to be more than a bunch of separate clever techniques.

6 Coda: but what about ‘logic of games’?

Many people have heard of fruitful, and even Nobel-prize winning connections between logic and game theory – but the above story would probably leave them bewildered. What we have discussed in this note are game-theoretic models for basic linguistic and logical activities. But there is a quite different interface, too, where logic and language play their traditional role, viz. the description and analysis of game forms, strategies, information and reasoning of agents. This involves epistemic, doxastic and dynamic logics, providing analyses of notions such as rationality and its associated game solution procedures. In this descriptive guise, logic plays the same role toward game theory as it does toward multi-agent systems or process theories in computer science. Indeed, this more traditional use of logical techniques constitutes the main thrust of work in my own ILLC environment in Amsterdam, where games serve as one rich and intuitively appealing model of intelligent interaction that we want to capture by logical means. [16] This is also the sense in which computer scientists have embraced game theory as a richer model for computation (Grädel [26]), and philosophical logicians as a concrete model for rationality (Stalnaker [46]). All these contacts can take place while logic keeps its standard semantic and proof-theoretic face. Of course, game-theoretic ideas can reach logic in this way, and they do – but there is no need for logic to ‘let’ game theory ‘under its skin’, and recast itself as a family of games, as we have suggested in the above.

[16] Cf. Van Benthem [5], [9]; as well as the bundle of European projects constituting the recent LogiCCC team ‘Logics for Interaction’.

Nevertheless, the latter more radical view, too, has its basis in the history of logic, and it constitutes what Abramsky [2] calls *logic as embodying process* rather than logic as external process description.[17] Indeed, the two directions are related. We can use standard logical languages to describe games, and then go on to use games to reinterpret what these logical languages are. The result is a wonderful circle – carroussel? – where the two fields spin happily together on each other's backs. I find that interactive view well in line with the spirit of the present Book.

References

[1] S. Abramsky, 'Socially Responsive, Environmentally Friendly Logic', in T. Aho and A. Pietarinen (eds.), *Truth and Games*, Acta Philosophica Fennica 78, 2006, pp. 17-46.

[2] S. Abramsky, 'Information, Processes, and Games', Computing Lab, Oxford University. To appear in: P. Adriaans & J. van Benthem, eds. *Handbook of the Philosophy of Information*, Amsterdam; Elsevier

[3] A. Baltag, L. Moss & S. Solecki, 'The Logic of Public Announcements, Common Knowledge, and Private Suspicions', in:*Proceedings TARK*, 1998, pp. 43-56

[4] J. van Benthem & J. van Eijck, 'The Dynamics of Interpretation', *Journal of Semantics* 1:1, pp. 3-20, 1982

[5] J. van Benthem *Language in Action: categories, lambdas, and dynamic logic*, Amsterdam: Elsevier, 1991

[6] J. van Benthem, *Logic in Games*, lecture notes Institute for Logic, Language and Computation ILLC, Amsterdam, 1999

[7] J. van Benthem, 'Games in Dynamic Epistemic Logic', in: G. Bonanno & W. van der Hoek, eds. *Bulletin of Economic Research* 53:4, 2001, pp. 219-248

[8] J. van Benthem, 'Logic Games are Complete for Game Logics', *Studia Logica* 75, 2003, pp. 183-203

[9] J. van Benthem, 'Logical Construction Games', *Acta Philosophica Fennica* 78, in T. Aho and A. Pietarinen (eds.), *Truth and Games*, Acta Philosophica Fennica 78, 2006, pp. 123-138.

[17] The lecture notes *Logic in Games* [6] call this 'logic games' versus 'game logics'.

[10] J. van Benthem, ‘Logic Games: From Tools to Models of Interaction’, ILLC Research Report, Amsterdam. To appear in: A. Gupta, R. Parikh & J. van Benthem, eds. *Logic at a Cross-Roads*, Allied Publishers, Mumbai, *forthcoming*

[11] J. van Benthem, ‘Logic, Rational Agency, and Intelligent Interaction’, ILLC Research report. To appear in: D. Westerstahl et al. (eds.), *Proceedings of Beijing Congress on Logic, Methodology, and Philosophy of Science*, London: College Publications, 2007

[12] J. van Benthem, J. van Eijck & B. Kooi, ‘Logics of Communication and Change’, in: *Information and Computation* 204, 2006, 99. 1620-1666

[13] J. van Benthem, S. Gosh & F. Liu, ‘Modeling Simultaneous Games with Concurrent Dynamic Logic’, in: J. van Benthem, S. Ju & F. Veltman (eds.), *A Meeting of the Minds*, Proceedings LORI Beijing 2007, College Publications, pp. 243-258

[14] J. van Benthem & A. ter Meulen, eds., *Handbook of Logic and Language*, Amsterdam: Elsevier, 1997

[15] P. Blackburn, J. van Benthem & F. Wolter, eds., *Handbook of Modal Logic*, Elsevier Science Publishers, Amsterdam, 2006

[16] A. Blass, ‘A Game Semantics for Linear Logic’, *Annals of Pure and Applied Logic* 56, 1992, pp. 183-220

[17] J. Bradfield & C. Stirling, ‘*PDL* and Modal μ-Calculi’, in: P. Blackburn, J. van Benthem & F. Wolter (eds.), *Handbook of Modal Logic*, Elsevier Science Publishers, Amsterdam, 2006

[18] H. Clark, *Using Language*, Cambridge: Cambridge University Press, 1996

[19] P. Dekker and R. van Rooij, ’Bi-Directional Optimality Theory: An Application of Game Theory’, *Journal of Semantics*, 17, 2000, pp. 217-242

[20] H. van Ditmarsch, W. van der Hoek & B. Kooi, *Dynamic Epistemic Logic*, Synthese Library Series, Volume 337, Springer

[21] R. Fraïssé, *Sur quelques classifications de systèmes de relations*, Thesis, University of Paris, 1953

[22] Y. Feinberg, ’Meaningful Talk’, 2007, *Present volume*

[23] P. Gärdenfors & M. Warglien, 'Cooperation, Conceptual Spaces, and the Evolution of Semantics', in: P. Vogt et al. (eds.), *Symbol Grounding and Beyond*, Springer, 2006

[24] J. Gerbrandy, *Bisimulations on Planet Kripke*, dissertation, Universiteit van Amsterdam, 1999

[25] P. Girard, *Modal Logic for Preference and Belief Change*, dissertation, Universiteit van Amsterdam and Stanford University, 2008

[26] E. Grädel, 'Games and Automata for Synthesis and Validation', ERCIM News No. 57, April 2004

[27] J. Hintikka, *Logic, Language Games and Information*, Clarendon, Oxford, 1973

[28] J. Hintikka & G. Sandu, 1997, 'Game-Theoretical Semantics', in: J. van Benthem & A. ter Meulen (eds.), *Handbook of Logic and Language*, Elsevier, Amsterdam, pp. 361-410.

[29] W. Hodges, *Building Models by Games*, Cambridge University Press, Cambridge, 1985

[30] W. Hodges, 'Compositional semantics for a language of imperfect information', *Logic Journal of the IPGL* 5, 1997, pp. 539-563

[31] G. Jaeger & R. van Rooij, 'Language Structure: Psychological and Social Constraints', *Synthese* 159: pp. 99-130, 2007

[32] D. Lewis, *Convention*, Harvard University Press, 1969

[33] F. Liu, *Changing for the Better: Preference Dynamics and Agent Diversity*, dissertation, Universiteit van Amsterdam, 2008

[34] P. Lorenzen, *Einführung in die Operative Logik und Mathematik*, 1955, Berlin: Springer

[35] M. Moortgat, 'Categorial Type Logics', in: J. van Benthem and A. ter Meulen (eds.), *Handbook of Logic and Language*, Elsevier, 1997, pp. 93-177

[36] M. Osborne & A. Rubinstein, *A Course in Game Theory*, Cambridge (Mass.): MIT Press, 1994

[37] P. Parikh, 'Communication and Strategic Inference', *Linguistics and Philosophy* 14, 1991, pp. 473-513.

[38] P. Parikh, *The Use of Language*, CSLI Publications, 2000

[39] R. Parikh, ‘The Logic of Games and its Applications’, in: *Annals of Discrete Mathematics* 24, 1985, pp. 111-140

[40] M. Pauly & W. van der Hoek, ‘Modal Logic of Information and Games’, in: P. Blackburn, J. van Benthem & F. Wolter, eds., *Handbook of Modal Logic*, Elsevier Science Publishers, Amsterdam, 2006

[41] R. van Rooij, ‘Optimality-Theoretic and Game-Theoretic Approaches to Implicature’, *Stanford Encyclopedia of Philosophy*, 2002

[42] R. van Rooij, ‘Signalling Games Select Horn Strategies’, in G. Katz, S. Reinhard, and Ph. Reuter (eds.) *Sinn und Bedeutung VI*, Proceedings of the Sixth Annual Meeting of the Geselschaft für Semantik, University of Osnabrück, 2002, pp. 289-310

[43] O. Roy, *Intentions and Plans in Decision and Game Theory*, dissertation, Universiteit van Amsterdam, 2008

[44] A. Rubinstein, *Economics and Language*, Cambridge University Press, 2000

[45] T. Schelling, *The Strategy of Conflict*, Harvard University Press, 1960

[46] R. Stalnaker, ‘Reference and Necessity’. In: B. Hale & C. Wright (eds.) *A Companion to the Philosophy of Language*, Oxford: Blackwell, 1997

[47] R. Stalnaker, ‘Extensive and Strategic Form: Games and Models for Games’, *Research in Economics* 53:2, 1999, pp. 93-291

[48] J. Väänänen, *Dependence Logic: A New Approach to Independence Friendly Logic*, Cambridge: Cambridge University Press, 2007