Three 13th-century views of quantified modal logic

SARA L. UCKELMAN
INSTITUTE FOR LOGIC, LANGUAGE, AND COMPUTATION

ABSTRACT. There are two reasons why medieval logic is of interest to modern logician: One is to see how similar it is to modern logic and the other is to see how different it is. We study three 13th-century works on modal logic and give two examples their views of modal logic differ from modern views of the same: the nature of modality and the truth conditions for modal sentences. Because of the different goals of the medieval logicians, modern logicians must take care in arguing for or against the correctness of the medieval logical theories.

Keywords: 13th century, modal proposition, modal syllogism, modal square of opposition, William of Sherwood.

1 Two reasons to study medieval logic

There are two reasons why the study of medieval logic is of interest to the modern logician. The first is to see how closely logical theories in different branches (modal logic, temporal logic, quantifier logic, etc.) resemble modern logical theories in these same branches. The second is to see how much they differ. Investigating a topic in medieval logic for either of these reasons will result in something informative and illuminating. If the medieval theory is similar to the modern theory, one can ask to what extent we can shed new light on the medieval theory by modeling it with modern formal tools. If the medieval theory differs from the modern theory, one can ask what the causes of these differences are, whether they are purely historical, accidental, or whether they reflect conscious differences in goals and aims, and, if the latter, what we can learn from these differences.

In this paper we compare contemporary philosophical (as opposed to mathematical) modal logic with three 13th-century views of modal logic. What we discover falls under the heading of the second reason: The comparison demonstrates that there is a fundamental difference between how these 13th-century logicians approached and used modal logic and how philosophical logicians of the 21st-century approach and use modal logic. This gives us cause to be careful that we do not discount medieval modal logic as being narrow or unfruitful: Because its aims are different from ours, we should not expect it to be applicable in the same circumstances.

Paper submitted to Advances in Modal Logic 2008

The three 13th-century texts that we consider are William of Sherwood's *Introductiones in logicam* [11] (translated into English with commentary in [10]), the short text *De propositionibus modalibus* [9], and Pseudo-Aquinas's *Summa totius logicae Aristotelis* [8].[1] Of the three, the provenance of the *Introductiones* is best known; the author can be ascribed with confidence, and while a definitive date of the text is not known, it is quite likely that the text was compiled between 1240 and 1248, a period in which Sherwood was a master in the Arts Faculty at the University of Paris [10, p. 8]. The other two texts are both connected to St. Thomas Aquinas. Aquinas was long considered to be the author of the *Summa*, though current thought is that this is highly unlikely. Conversely, the authorship of the *De modalibus* text was considered doubtful until the early 20th century when Grabmann attributed it to Aquinas; if he is the author of the *De modalibus*, it is a juvenile and early work [7, p. 13]. We shall follow Grabmann in attributing *De modalibus* to Aquinas, but reflecting our uncertainty about the authorship of the *Summa*, we shall refer to the author of that text as Pseudo-Aquinas. Despite questions about the authorship of the two text, it is clear from their content that they date from the same period as Sherwood's *Introductiones* or slightly later [2].

Before we can discuss the views of these three authors, and how they compare to modern approaches to modal logic, we must first address the question of what we mean by the phrase "modal logic". Broadly speaking, the phrase can apply to any type of logic to which new operators expressing different modes, such as modes of belief, knowledge, time, necessity, agency, etc., are added. It is not uncommon, today, to speak of deontic logic, epistemic logic, temporal logic, etc. all as "modal" logics. However, we take the phrase in a more narrow sense, using it to refer only to the logic of necessity and possibility. The term "modal" comes from Latin *modus* 'mode, mood', and when medieval authors speak of adding a *modus* to a sentence, they generally specify that it is one of the following six modes: *verum*, *falsum*, *necessarium*, *impossibile*, *possibile*, *contingens*.[2] By restricting our attention to just statements of necessity and possibility in this paper, we are following their customary usage of the phrase "modal proposition".

A note about references: Citations from William of Sherwood refer to page number unless a section number is explicitly indicated. The Aquinas text is referenced by sentence number, and Pseudo-Aquinas by tract, chapter, and sentence number.

2 Modes and modal propositions

All three of the 13th-century authors define modal propositions as being constructed from categorical propositions (recall that a categorical proposition or statement is, à la Sherwood, *cuius substantia consistit ex subiecto et praedicato* [11, p. 12][3]). The class of modal propositions is defined in

[1] Translations of quotes from these two sources are by the present author.
[2] True, false, necessary, impossible, possible, and contingent, respectively.
[3] "one whose substance consists of a subject and a predicate" [10, p. 27].

a jointly semantic-syntactic fashion. First, on the syntactic side, a modal proposition is a categorical proposition to which a mode has been added. The three authors all give slightly different definitions of *modus* ‘mode’. Aquinas says that a mode is a *determinatio adiacens rei, quae quidem fit per adiectionem nominis adiectivi, quod determinat substantivum... vel per adverbium, quod determinat verbum* [9, 2][4], that is, both adverbs and adjectives are modes. Pseudo-Aquinas says a mode is an *adjacens rei determinatio; idest, determinatio facta per adjectivum* [8, tract. 6, cap. 11, 2][5], that is, modes are adjectives. And Sherwood takes the other route; his definition of mode includes only adverbs: *Modus igitur dicitur communiter et proprie. Communiter sic: Modus est determinatio alicuius actus, et secundum hoc convenit omni adverbio* [11, p. 32].[6]

But not all categorical sentences to which adverbs or adjectives have been added are, strictly speaking, modal. The second part of the definition, which the three authors all include, is the semantic side: It is only those categorical statements where the adverb determines or modifies the composition of the subject and the predicate that are correctly called modal.[7] This *determinatio* is a semantic concept, as it modifies the *significatio* (‘signification’, roughly, the meaning) of the sentence. The six modes which can determine the inherence expressed in a categorical sentence are *verum*, *falsum*, *necessarium*, *impossibile*, *possibile*, and *contingens*. However, because the addition of “true” and “false” to a categorical proposition does not change its signification (because *nihil addunt supra significationes propositionum de inesse* [9, 9][8]) these two modes will be omitted from consideration and the focus will be on the four modes *necessarium*, *impossibile*, *possibile*, and *contingens*.

At this point in his presentation of modality, Sherwood makes a distinction which the other two authors do not. He notes that there are two ways that *impossibile* and *necessarium* can be used. Both ways can be expressed in terms of temporal notions:

> *uno modo, quod non potest nec poterit nec potuit esse verum, et est impossible per se... alio modo, quod non potest nec poterit esse verum, potuit tamen ... et est impossibile per accidens. Et*

[4] “a determining attribute of a thing, which is made by an addition of an adjective word, which determines a substantive... or by an adverb, which determines a verb”.

[5] “an adjoining determination of a thing; that is, a determination made through an adjective”.

[6] “The word ‘mode’ is used both broadly and strictly. Broadly speaking, a mode is the determination of an act, and in this respect it goes together with every adverb” [10, p. 40].

[7] *Quidam determinat compositionem ipsam praedicati ad subiectum... et ab hoc solo modo dicitur propositio modalis*, “some determine the composition itself of the predicate with the subject... and by this mode alone is a proposition called modal” [9, 6]; *modalis vero in qua inhaerentia praedicati ad subjectum modificatur*, “a modal [is that] in which the inherence of the predicate to the subject is modified” [8, tract. 6, cap. 7, 4]; *proprie sic: modus est determinatio praedicati in subiecto* [11, p. 32], “strictly speaking, a mode is the determination of [the inherence of] the predicate in the subject” [10, p. 40].

[8] “they attach nothing above the significations of the assertoric propositions”.

$$\Box_{ps}\varphi \quad := \quad \varphi \wedge G\varphi \wedge H\varphi$$
$$\Box_{pa}\varphi \quad := \quad \varphi \wedge G\varphi \wedge \Diamond\neg H\varphi$$

Figure 1. Sherwood's necessity operators

> *similiter dicitur necessarium per se, quod non potest nec potuit nec poterit esse falsum... Necessarium autem per accidens est, quod non potest nec poterit esse falsum, potuit tamen* [11, p. 34].[9]

Essentially, Sherwood is defining the necessity operators found in the table in Figure 1, where we list them translated the familiar notation of temporal logic. (We discuss the correct interpretation of the $\Diamond$ in the definition of necessity *per accidens* in §4.) As we'll see in §3, we can define the impossibility operators from the necessity operators by negation, so we do not need to list them separately. According to Sherwood, *possibile* and *contingens* also have twofold usage. On the one hand, they can be used of statements which can both be true and be false, and so are neither impossible or necessary; this is the sense which is generally ascribed to "contingent" in modern usage. On the other hand, they can be used of statements which can be true, even if they cannot be false; this is the sense which is generally ascribed to "possibility" in modern usage, under the assumption that the axiom $\Box\varphi \rightarrow \Diamond\varphi$ is valid. While some medieval authors follow this distinction, using *possibile* for things which can be true, even if they cannot be false, and *contingens* in the stricter fashion for things which can be true or false, the two terms were regularly conflated, and as they were in the three texts we're considering, we'll follow their lead.

2.1 Construction

Once the relevant modes have been identified, the syntactic ways that they can be added to a categorical proposition must be distinguished. There are two ways that a mode can determine the composition of a categorical proposition. The three authors each make the distinction, but in slightly different ways and with different labels.

Aquinas's text divides modal propositions into those which are modal *de dicto* and those which are modal *de re*. This text is generally credited as being the source of the use of this distinction in modern philosophy and modal logic.[10] He makes the distinction this way:

[9] [impossible] is used in one way of whatever cannot be true now or in the future or in the past; and this is "impossible *per se*"... It is used in the other way of whatever cannot be true now or in the future although it could have been true in the past... and this is "impossible *per accidens*". Similarly, in the case something cannot be false now or in the future or in the past it is said to be "necessary *per se*"... But it is "necessary *per accidens*" in case something cannot be false now or in the future although it could have been [false] in the past [10, p. 41].

[10] See [12, p. 1], where the terms are first introduced in modern contexts. Von Wright credits Aquinas with this distinction, probably in reference to *De modalibus*, as this was

> *Modalis de dicto est, in qua totum dictum subiicitur et modus praedicatur, ut Socrates currere est possibile; modalis de re est, in qua modus interponitur dicto, ut Socratem possibile est currere* [9, 16].[11]

The *dictum* of a sentence is what the sentence expresses; a categorical proposition's *dictum* can be formed, as the Aquinas tells us, by substituting the infinitive form for the indicative verb, and the accusative case for the nominative subject.[12] This same distinction is found in Pseudo-Aquinas but in a more elaborate fashion:

> *Ad sciendum autem earum quantitatem, notandum quod quaedam sunt propositiones modales de dicto, ut, Socratem currere est necesse; in quibus scilicet dictum subjicitur, et modus praedicatur: et istae sunt vere modales, quia modus hic determinat verbum ratione compositionis, ut supra dictum est. Quaedam autem sunt modales de re, in quibus videlicet modus interponitur dicto, ut, Socratem necesse est currere: non enim modo est sensus, quod hoc dictum sit necessarium, scilicet Socratem currere; sed hujus sensus est, quod in Socrate sit necessitas ad currendum* [8, tract. 6, cap. 11, 14–15].[13]

Sherwood makes the same distinction but does not use the *de dicto*/*de re* terminology. Instead he distinguishes between adverbial modes and nominal modes; categorical propositions with adverbial modes correspond to the class of *de re* modal sentences, and those with nominal modes correspond to the class of *de dicto* modal sentences [11, pp. 34–38].

2.2 Quantity

The type of modal sentence (that is, whether it is *de re* (or adverbial) or *de dicto* (or nominal)) must be established before the further properties of the sentence can be determined. Modal propositions, like categorical propositions, have both quantity and quality, and the authors give rules by

attributed to Aquinas by the 1950's. Dutilh Novaes in [1, fn. 9] notes that von Wright was introduced to the distinction by Peter Geach.

[11] "Modality is *de dicto* in which the whole *dictum* is made the subject and the mode is predicated, as in 'that Socrates runs is possible'; modality is *de re* in which the mode is inserted into the *dictum*, as in 'Socrates is possibly running'."

[12] *quod quidem fit si pro verbo indicativo propositionis sumatur infinitivus, et pro nominativo accusativus*, "because indeed it happens if an infinitive verb is assumed for the indicative verb of the proposition, and the accusative case [is assumed] for the nominative the accusative case" [9, 12].

[13] "However for knowing the quantity of them, it must be noted that certain ones are modal propositions *de dicto*, as in 'that Socrates runs is necessary', in which clearly the *dictum* is made subject, and a mode is predicated: and these are truly modals, because a mode here determines the verb by reason of composition, as in what's said above. However certain others are modal [propositions] *de re*, in which a mode is interposed in the dictum, as in 'Socrates is necessarily running': indeed by this mode the sense is not that this dictum is necessary, namely 'that Socrates runs', but of this the sense is that in Socrates is necessity for running."

which the quantity and quality of modal propositions can be recognized. The quantity of a categorical proposition can be one of four types: singular, particular, universal, or indefinite. A categorical proposition is singular when the subject term picks out only one object, e.g., because it is either a proper name or because it is modified by a definite article such as *hoc* or *illud*. It is particular when the subject term picks out more than one object, because it is modified by a particular quantifier such as *quoddam* or *aliquid*. It is universal when the subject term picks out all objects of which the term can be truly predicated, because is modified by a universal quantifier such as *omnem* or *nullum*. Finally, a categorical is indefinite; this is when the subject term refers to some object or objects, but no particular object or objects, because no quantifier or definite article is used, and the subject is not a proper name.

The division into modal statements *de dicto* and *de re* is motivated partly by the differences in how the quantity of the two types of statements is determined. Modal *de re* statements have the same quantity as their underlying categorical sentences. But this is not the case for modal *de dicto* statements. According to both Aquinas and Pseudo-Aquinas, *de dicto* statements always have singular quantity, even though they may contain universal or particular quantifiers within them.[14] This is because the subject of a *de dictum* sentence is a *dictum*, and a *dictum* is essentially a proper name; it has a unique referent. Because Sherwood doesn't use the *de dicto*/*de re* distinction, his identification of the quantity is phrased somewhat differently, but with the same end result: When a categorical statement with a nominal mode is interpreted as if it had an adverbial mode, then the quantity of the sentence is determined by the quantity of the underlying categorical claim. But when it is not interpreted this way, then the *dictum* of the sentence is the subject, and this is singular.

2.3 Quality

The quality of a proposition (categorical or otherwise) is determined by the presence or absence of a negation: For categorical sentences, it is the negation of the composition between the subject and the predicate, for modal sentences it is the negation of the mode. If the composition or the mode is affirmed, then the sentence is affirmative, and if it is denied, then it is negative. In this way, a categorical proposition which is negative can become positive when made into a modal proposition, and similarly a positive categorical proposition can become negative when made modal, because, as Aquinas notes, *propositio modalis dicitur affirmativa vel negativa secundum*

[14] *Sciendum quod omnes enunciationes modales de dicto sunt singulares, quantumcumque sit in eis signum universale*, "it must be known that all modal *de dicto* assertions are singulars, although in it may be a universal sign" [8, tract. 6, cap. 11, 21]; *sciendum est autem quod omnes modales de dicto sunt singulares, eo quod modus praedicatur de hoc vel de illo sicut de quodam singulari*, "However it must be understand that all modals *de dicto* are singulars, because a mode is predicated of this or that in the same way as of a certain singular" [9, 17].

affirmationem vel negationem modi, et non dicti [9, 19].[15] As an example, *Socrates non currit* is a negative categorical proposition, but *Socrates non currere est possibile* is an affirmative modal proposition. Note that the quantity of a proposition is a syntactic property, because it depends on the presence or absence of the term *non*, whereas the quality of a categorical proposition is semantic, because it does not depend on the addition of a specific term but rather on the truth conditions of various predications of the subject term on different objects.

The importance of being able to determine the quality and quantity of a modal proposition is grounded in the importance which is ascribed to the inferential relations of modal propositions, as it is the quality and the quantity that determines which propositions can be inferred from which others. We discuss these next.

3 Inferential relations

The inferential relations which are discussed in the three treatises can be divided into two groups: implications and conversions. The implications considered are the relations of contradiction, contrariety, subcontrariety, subalternation, and superalternation (the relations which make up the square of opposition). The conversions considered are the traditional Aristotelian ones, conversion *per accidens* and conversion *simplex*, along with equivalences which can be generated through the square of opposition. These implications and conversions are used to develop a modal syllogistic.

3.1 Implications

Sherwood notes that modes can be combined with negation in one of the following four ways [10, p. 48]:

A without negation

B with more than one negation

C with one negation, before the mode

D with one negation, after the mode

Since we have four modes, and four ways that a mode can be combined with negation, this gives us sixteen syntactically different modes; these modes can occur both adverbially and nominally (or, to say the same thing, in *de dicto* and in *de re* statements). The question is whether these sixteen syntactically different modes are all semantically distinct, or whether there are any pairs which are equipollent (to call them by their standard medieval name *equipollens*). The answer is that each of the sixteen can be placed into one of four groups, called *ordines* 'orders' or 'series' (see Figure 2). An order is essentially an equivalence class, since *omnes propositiones, quae sunt in eodem ordine, aequipollent* [9, 23].[16] The four orders make up the corners of

[15] "a modal proposition is called affirmative or negative according to the affirmation or negation of the mode, and not of the *dictum*".

[16] "[A]ll propositions which are in the same order are equipollent".

ordo 1	*possibile* *contingens* *non impossible* *non necessarium non*	*non possibile* *non contingens* *impossibile* *necessarium non*	*ordo* 3
ordo 2	*possibile non* *contingens non* *non impossibile non* *non necessarium*	*non possibile non* *non contingens non* *impossibile non* *necessarium*	*ordo* 4

Figure 2. The four *ordines*

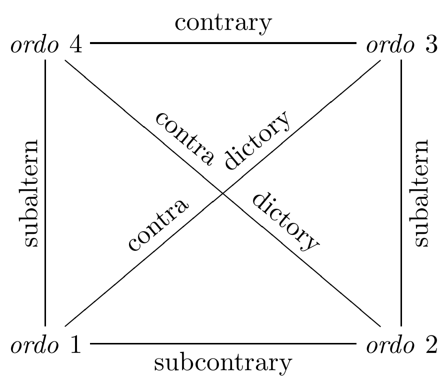

Figure 3. Modal square of opposition

a square of opposition illustrating the inferential relationships (see Figure 3). This square of opposition can be found in the manuscripts of each of the three treatises. After the square of opposition is presented, Aquinas's treatise makes reference to the mnemonic poem for constructing the square, whereupon the text ends.[17]

[17] The mnemonic poem shows up in various forms in 13th-century texts. Sherwood gives the text as follows: *Sit tibi linea subcontraria prima secunde. / Tertius est quarto semper contrarius ordo. / Tertius est primo contradictorius ordo. / Pugnat cum quarto contradicendo secundus. / Prima subest quarte vice particularis habens se. / Hac habet ad seriem se lege secunda sequentem* [10, fn. 91]. Aquinas rearranges the lines, adds a few of his own, and omit some of Sherwood's: *Tertius est quarto semper contrarius ordo. Pugnat cum quarto contradicendo secundus. Sit subcontraria linea tibi prima secundae. Tertius est primo contradictorius ordo. Prima subest quartae vicem particularis habens. Sed habet ad seriem se lege secunda sequentem. Vel ordo subalternus sit primus sive secundus. Primus amabimus, edentulique secundus. Tertius illiace, purpurea reliquus. Destruit u totum sed a confirmat utrumque, destruit e dictum, destruit i que modum* [9, 35–4]. Pseudo-Aquinas reduces the poem to just the names for each of the corners of the square: *Amabimus, edentuli, illiace, purpurea* [8, tract. 6, cap. 13, 22].

3.2 Conversions

Both Sherwood and Pseudo-Aquinas discuss how modal propositions can be converted from one form to another. By "conversion" both authors mean the two types of conversion which Aristotle presents in giving rules for the proving of syllogisms, conversion *simplex* or *per se* and conversion *per accidens*. Simple conversion of a categorical exchanges the subject and predicate terms, leaving the quality and the quantity of the sentence unchanged; accidental conversion swaps the subject and predicate, but also changes the quantity, from universal to particular or vice versa.[18] Sherwood also mentions a third type of conversion, conversion *per contrapositionem*, where the subject and predicate are swapped and replaced with their infinite counterparts (e.g., 'man' is replaced with 'non-man'; *infinitum* is the standard medieval name for such terms.)

In tract. 7, cap. 3, Pseudo-Aquinas tells us that *propositiones de necessario et impossibili eodem modo convertuntur sicut propositiones de inesse, et per idem principium probantur* [8, 2]. Though he does not say so explicitly, it is clear from all of his examples that he is discussing conversion principles for *de dicto* statements; all of his examples use nominal modes, not adverbial ones. Because it is not obvious that necessary and impossible propositions can be converted in the same way that assertoric (that is, categorical) propositions can be, he gives proofs for various conversions. We give the first, because it exemplifies the techniques used in the rest. It is a proof that

necesse est nullum b esse a (1)

can be simply converted into

necesse est nullum a esse b (2)

First, Pseudo-Aquinas notes that the opposite of (2) implies the opposite of (1). But the opposite of (2),

non necesse est nullum a esse b (3)

is equipollent to

possibile est aliquod a esse b (4)

The equipollence between (3) and (4) holds because *impossibile* and *non necessarium non* are equipollent (as we saw in the previous section), and this latter equipollence holds because *non nullus* and *aliquis* are equipollent. Next, he notes that from (4) the following can be proved through an expository syllogism (an expository syllogism is one which one premise is a singular proposition. Pseudo-Aquinas discusses these in tract. 7, cap. 2.):

possibile est aliquod b esse a (5)

[18] *Dicitur autem conversio simplex, quando de praedicato fit subjectum, et de subjecto praedicatum, manente secunda propositione in eadem qualitate et quantitate cum prima. Per accidens vero dicitur, quando de subjecto fit praedicatum, et e converso, manente eadem qualitate propositionis, sed mutata quantitate* [8, tract. 7, cap. 2, 4–5]; a similar definition can be found in [11, cap. 3, §2].

But (5) is the contradictory of (1). Since we were able to prove the contradictory of the antecedent from the contradictory of the consequent, we can conclude that (1) can be converted into (2). That (2) can be converted back into (1) by similar reasoning is obvious.

The other proofs are similar and so will not be discussed further here.

3.3 Modal syllogisms

Sherwood tells the reader, before he even gives the definition of a mode, that the reason it is important to separate modal propositions from assertoric ones is that

> *Cum intentio sit de enuntiatione propter syllogismum, consideranda est sub differentiis, in quibus differentiam facit in syllogismo. Quales sunt haec: . . . modale, de inesse et aliae huiusmodi. Differt enim syllogismus a syllogismo per has differentias* [11, p. 30].[19]

Kretzmann points out that "in spite of this remark, which seems to promise a consideration of the modal/assertoric difference as it relates to the syllogism, there is no treatment of modal syllogisms in any of the works that have been ascribed to Sherwood" [10, fn. 58].

This leaves us with the *Summa*. Pseudo-Aquinas discusses modal syllogisms in tract. 7, caps. 13–15. Unfortunately, in many cases, his presentation is less than clear. The three chapters are devoted to the different combinations of necessary, impossible, and contingent premises with assertoric premises in syllogisms. Each combination is considered, and if it is valid, no argument is given, and if it is invalid, a counterexample is given. The result is an unfortunate tangle of case-by-case examples and rules with limited applicability.

Additionally, in giving the various examples of valid and invalid syllogisms, Pseudo-Aquinas moves between *de dicto* and *de re* formulations indiscriminately. For example, when he says that a syllogism in any mood or figure (for the technical details and terminology of Aristotelian syllogisms, see the Appendix) which has two necessary premises will have a necessary conclusion, the example that he gives is the following [tract. 7, cap. 13, 7–9]:

Necesse est omnem hominem esse animal.
Necesse est omne risibile esse hominem.
Ergo necesse est omne risibile esse animal.

But when he gives an example to show that a necessary conclusion does not follow from an assertoric major and a necessary minor, he uses *de re* modalities [tract. 7, cap. 13, 21–23]:

[19] [s]ince our treatment is oriented toward syllogism, we have to consider them under those differences that make a difference in syllogism. These are such differences as. . . modal, assertoric; and others of that sort. For one syllogism differs from another as a result of those differences [10, p. 39].

Omnis homo est albus.
Omne risibile necessario est homo.
Ergo omne risibile necessario est album.

The unclarity which results from his indiscriminate use of *de dicto* and *de re* statements in his examples is compounded by the fact that very few explicit rules for resolving the validity of classes of syllogisms are given. In assertoric syllogisms, the two rules commonly discussed are the *dici de omni* and the *dici de nullo*:

> *Est autem dici de omni, quando nihil est sumere sub subjecto, de quo non dicatur praedicatum; dici vero de nullo est, quando nihil est sumere sub subjecto, a quo non removeatur praedicatum* [8, tract. 7, cap. 1, 36].

Pseudo-Aquinas often appeals to these two rules when he gives arguments for the invalidity of certain syllogisms with one modal and one assertoric premise. It is only when he considers syllogisms which have one necessary premise and one contingent or possible premise that he formulates a new rule. The rule is:

> *si aliquod subjectum sit essentialiter sub aliquo praedicato, quicquid contingit sub subjecto, contingit sub praedicato* [8, tract. 7, cap. 15, 10].

Clearly this rule is an attempt to make a modal variant of the *dici de omni*.

It is at this point in the treatise that the modern logician could be forgiven for finding himself frustrated. The lack of both precision and perspicuity make one wonder whether there is anything to be gained in further study. If one is interested solely in developing a reliable modal syllogistic, there are other authors where this material is more easily accessible. But if one is interested in understanding the parts of the modal theory which are difficult not just because they are unclear but because they are fundamentally different from modern modal theories, then there are a number of things that can be said; we turn to these in the next section.

4 Contrasts with modern views of modal logic

We are now in the position to note two places where the medieval conception of modality and modal reasoning diverge from the modern conception of the same, with interesting consequences for our understanding of medieval modal logic.

4.1 The nature of modality

The first is that in modern propositional modal logic, the modality being expressed is the *de dicto* modality. A modal operator is an operator at the level of *formulas*. A formula of the form $\Box\varphi$ is read "it is necessary that φ", where the addition of "that" before "φ" is the syntactic construct in English for forming the *dictum* of a sentence. It isn't even clear that *de re* modality, with its emphasis on the inherence of the subject in the predicate, can be interpreted in a propositional context in a coherent fashion. Because of the subject-predicate nature of the *de re* sentences, it is clear that we are working with some type of first-order logic, not a propositional logic. But in the context of predicate logic, there is some temptation to say that *de re* statements aren't *really* about modality; they're just about a (perhaps special) type of predicates which we could call, e.g., possibly-P. But syntactically, these are just like any other predicate, and semantically, we would be perfectly within our bounds to give the truth conditions to predicates like possibly-P in the same way that we do predicates like P, through an assignment function. Then we could use $\Box$ and $\Diamond$ to express *real* modality, modality applying at the level of entire formulas.

This approach to modality is in direct contrast with that of William of Sherwood. Sherwood is reluctant to accept categorical statements with nominal modes (that is, *de dicto* modals) as modal statements [11, p. 36]. Recall that in §2 when we presented the different definitions of 'mode', all three authors agreed that under the most strict interpretation, only those categorical sentences where the mode determines the inherence of the subject and predicate are really modal. Both Aquinas and Pseudo-Aquinas are willing to let sentences such as *possibile est aliquod a esse b* to count as being determinations of the subject a in the predicate b, without really spelling out how we are to understand this determination, but Sherwood will only call such sentences modal when they are interpreted in the *de re* fashion. Under this interpretation:

> *Si enim dicam 'Socratem currere est contingens', idem est secundum rem ac si dicerem 'Socrates contingenter currit'* [11, p. 38].[20]

Can modifications in the inherence of a subject in a predicate even be represented in first-order modal logic? If the underlying categorical statement is universal or particular, then the distinction between the nominal and adverbial modes is easy: It is just the distinction between, e.g., $\Box\forall xF(x)$ and $\forall x\Box F(x)$ (see, e.g., [4, §4.3]). But this will not work for singular or indefinite statements, where there is no quantifier.

In [3, p. 108], Fitting gives two different ways that the formula $\Diamond P(c)$ could be read. If we take as models 5-tuples $\mathcal{M} = \langle W, R, D, I, V\rangle$, where W is the set of worlds, R the accessibility relation, D the domain function assigning a non-empty set of objects to each world, I the interpretation function which assigns each constant to an object in each world and each n-ary predicate to a set of n-tuples of objects in each world, and V is a valuation function assigning values to free variables, and we stipulate that every object in a world has a constant which is interpreted as that object, then the two possibilities for $\mathcal{M}, w \models \Diamond P(c)$ can be represented as:

[20] if I say 'that Socrates is running is contingent', it is just the same, with respect to what is signified, as if I were to say 'Socrates is contingently running' [10, p. 45].

1 There is a world x such that wRx and $\mathcal{M}, x \models_V Py$ where $V(y) = I(c, x)$.

2 There is a world x such that wRx and $\mathcal{M}, x \models_V Py$ where $V(y) = I(c, w)$.

The first reading can be interpreted as modality *de dicto*: The most natural reading of "it is possible that c is P" is "there is a possible world where the interpretation of c at that world is in the interpretation of P". The second is a plausible reading of modality *de re*, namely that what c actually is in the current world, that very thing is in the interpretation of P in another possible world.

This means that sentences of the form $\Diamond P(c)$ are essentially ambiguous: Their syntactic structure gives no clues as to whether they should be interpreted in the first or the second way. But from the point of view of the medieval logicians, this is precisely what they want: Natural language sentences such as *Socrates est possibile currere* are ambiguous, and we, as users of natural language, must make a choice in the interpretation of the sentence (perhaps based on context) when we wish to reason about it in a formal setting. The choice of interpretation will, naturally, affect the validity of the syllogisms in which these premises are found.[21]

This distinction is given in terms of simple predications, but its analysis easily extends to more complicated sentences such as *Omnis homo est possibile currere*. If we formalize this as $\forall y(Hy \rightarrow \Diamond_{dr} Cy)$[22] to show that we are interested in the *de re* analysis, then $\mathcal{M}, w \models \forall y(Hy \rightarrow \Diamond_{dr} Cy)$ is true if and only if for arbitrary m

$$\text{if } I(m) \in I(H, w), \text{ then } \exists x, wRx \text{ and } \mathcal{M}, x \models_v C(y) \text{ where } y \in I(m, w)$$

Note that x can be different for different m; this is exactly what we want, for if we required that it be the same world where all the currently existing men are running, then the sentence would collapse into the *de dicto* reading.

4.2 The truth conditions of modal sentences

The second discrepancy between modern modal logic and medieval logic as presented in these three texts comes from the emphasis. In modern modal logic, emphasis is placed on the truth conditions of the modal propositions considered in and of themselves; when working with Kripke semantics, this emphasis manifests itself in the choice of the R relation or a restriction on the valuation functions for the propositions. This is in contrast to the three texts that we've seen, where the emphasis is placed on the inferential relations between modal propositions, e.g., the relations which form the Square of Opposition, conversions and of modal propositions, and classes of valid syllogisms. (Speaking anachronistically, we could say that the medieval logicians were more interested in proof theory than in model theory.) Pseudo-Aquinas does not provide any explicit truth conditions for modal

[21] Since this is not an acceptable solution for many contemporary logicians, Fitting in [3, §3], Fitting and Mendelsohn in [4, ch. 9], and Garson in [5, ch. 19] all introduce lambda abstraction to solve the problem.

[22] For present purposes it does no harm to omit consideration of existential import.

propositions *considered in themselves* (as opposed to *considered with respect to other modal propositions*). This is most surprising when considered in conjunction with the stated goal of the entire treatise. *Omnes homines natura scire desiderant*, the text opens [8, Prologue, 1]. But, he goes on to say, knowledge only comes as a result of demonstration, and a demonstration is a valid syllogism with necessarily true premises. Because this is the only route to knowledge (valid syllogisms which have merely, but not necessarily, true premises can only lead to probable knowledge; these syllogisms are subsumed under 'dialectic', which our author says he will not consider in this treatise [8, Prologue, 11]), it is quite surprising that nothing is said about how to determine whether a premise is necessarily true, or (a slightly different question) whether a necessary premise is true.

Aquinas devotes two sentences to the truth conditions of modal propositions, when he draws a conceptual parallel between the four modes and the four combinations of quality and quantity in categorical propositions. He says:

> *Attendendum est autem quod necessarium habet similitudinem cum signo universali affirmativo, quia quod necesse est esse, semper est; impossibile cum signo universali negativo, quia quod est impossibile esse, nunquam est. Contingens vero et possibile similitudinem habent cum signo particulari: quia quod est contingens et possibile, quandoque est, quandoque non est* [9, 20–21].[23]

This interpretation of necessity and impossibility corresponds to Sherwood's definition of necessity and impossibility *per se* that we saw in §2. And as we saw in §3 that impossibility can be defined from necessity using negation, so too can possibility; so the type of possibility that Aquinas is discussing here is possibility *per se*, meaning that we can also formalize it with temporal notions, as

$$\Diamond_{ps}\varphi := (\varphi \vee F\varphi \vee P\varphi) \wedge (\neg\varphi \vee F\neg\varphi \vee P\neg\varphi) \tag{6}$$

But if this temporal formula expresses the truth conditions of sentences of possibility and contingency, and there little reason to think that Sherwood would reject this definition while accepting the other, then we are left with the question of what exactly Sherwood means when he says that a statement which is necessary *per accidens* "could have been false in the past". That is, we must ask what type of possibility is being expressed by the $\Diamond$ in

$$\Box_{pa}\varphi := \varphi \wedge G\varphi \wedge \Diamond\neg H\varphi \tag{7}$$

[23] "However, it must be understood that 'necessary' has a likeness with a universal affirmative sign, because what is necessary, always is; [and] 'impossible' [has a likeness] with a universal negative sign, because what is impossible, never is. But 'contingent' and 'possible' have a likeness with a particular sign: because what is contingent and possible, sometimes is, sometimes isn't."

We can prove easily that $\Diamond\varphi$ here cannot be a short-hand for $(\varphi \vee F\varphi \vee P\varphi) \wedge (\neg\varphi \vee F\neg\varphi \vee P\neg\varphi)$. Let w be an arbitrary point where $\Box_{pa}\varphi$ is true. We know then that $w \models \varphi$ and $w \models G\varphi$, and

$$w \models (\neg H\varphi \vee F\neg H\varphi \vee P\neg H\varphi) \wedge (H\varphi \vee FH\varphi \vee PH\varphi) \tag{8}$$

The problem is the second conjunct. If $\Box_{pa}\varphi$ is to be distinguished from $\Box_{ps}\varphi$, we know that neither $H\varphi$ nor $FH\varphi$ can be true, for then the two would be equivalent. Thus there is some $t \ll w$ such that $t \models H\varphi$; t cannot be an immediate predecessor of w or otherwise $PH\varphi$ would be equivalent with $H\varphi$, if we assume reflexivity. But this with the first conjunct forces there to be some t', $t < t' < w$ where $t' \models \neg\varphi$. And in this case, the interpretation of $\Box_{pa}\varphi$ would be that φ is true now and always in the future, but was false at some point in the past, and not that φ is true now and always in the future but *could have been* false in the past (even if it never was). If we take seriously Sherwood's counterfactual truth conditions for necessity *per accidens*, then the possibility involved cannot be temporal possibility.

There is a natural solution to the problem, though it is not one explicitly endorsed by Sherwood. If we remember that φ is not just a simple propositional construct, but a subject-predicate sentence like *Socrates est necessario currere*, then we can solve the question of the interpretation of $\Diamond$ by using the formal distinction between the *de re* and *de dicto* readings that we presented in the previous section. Then if *Socrates est necessario currere* is interpreted with necessity *per accidens*, it can be rewritten as

$$C(s) \wedge \Box_{ps}FC(s) \wedge \Diamond_{dr}\neg C(s) \tag{9}$$

that is, Socrates is running now, it is necessary *per se* that he is running in the future, but he is possibly (*de re*) not running. The reason that this explication doesn't collapse the same way that the other one did is that the *de re* possibility here is not defined with respect to past, present, or future times, but to possible worlds; i.e., this type of possibility is in a sense perpendicular to the temporal notion of possibility (see Figure 4). And thus we see how Sherwood's insistence that it is the adverbial modal sentences which are the real modal sentences, and not the nominal ones, can be used to explain how, under a temporal notion of modality, the distinction between necessity *per accidens* and necessity *per se* can be maintained in the way that he has defined them.

5 Concluding remarks

Section 4.4 of [4] addresses the question "is quantified modal logic possible?" Fitting and Mendelsohn note that

> for much of the latter half of the twentieth century, there has been considerable antipathy toward the development of modal logic in certain quarters. Many of the philosophical objectors

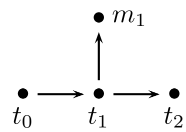

Figure 4. $\Box_{ps}$ is evaluated w.r.t. t_n, $\Diamond_{dr}$ w.r.t m_1

> find their inspiration in the work of W.V.O. Quine, who as early as (Quine, 1943), expressed doubts about the coherence of the project... Quine does not believe that quantified modal logic can be done coherently... [4, p. 89]

These philosophical doubts are cited as the cause for the lack of development of quantified modal logic in modern times; Garson in his introduction says

> The problem is that quantified modal logic is not as well developed... Philosophical worries about whether quantification is coherent or advisable in certain modal settings partly explains this lack of attention [5, p. xiii]

This suspicion of quantified modal logic is deep-seated and pervasive among contemporary philosophical logicians; skim through any article which discusses quantified modal logic from a philosophical (as opposed to mathematical) point of view, and you will find at least one disparaging remark about it. In this paper we have demonstrated a lesson worth learning from the medieval logicians: quantified modal logic does not have to be a scary, intractable field of study, but in fact can be developed in a systematic fashion from the logic of simple categorical statements. Not only is this development conceptually quite natural, it is in some sense more natural than a modal logic for unanalyzed propositions.

Appendix

This appendix is a brief refresher course on basic (non-modal) Aristotelian syllogisms. Aristotelian syllogisms can be divided into four figures; the figure determines the order of the terms in the premises and conclusion (see Figure 5). The predicate term of the conclusion is called the *major term*;

	1st figure			2nd figure	
A__C,	B__C:	A__C	B__A,	B__C:	A__C
	3rd figure			4th figure	
A__B,	C__B:	A__C	B__A,	C__B:	A__C

Figure 5. The four figures

the subject term of the conclusion is called the *minor term*. The term which occurs in both premises but not in the conclusion is the *middle term*.

The premise containing the major term is called the *major premise* and the premise containing the minor term is called the *minor premise*. Moods are created from the figures by inserting one of the four copulae

a "Every __ is a __" (universal affirmative)

e "No __ is a __" (universal negative)

i "Some __ is a __" (particular affirmative)

o "Some __ is not a __" (particular negative)

Since each figure has three slots and there are four different copulae, this means there are 64 moods. Only 24 of these moods are valid. The medievals gave mnemonic names to 19 of the 24 valid moods, where the vowels indicate the copulae of the major premise, the minor premise, and the conclusion (in that order), and the consonants indicate which of the four basic syllogism moods it is to be converted into, and by which conversion methods. This list has been extended in modern times to include names for all 24 of the valid moods. These are [6, §1]:

1st figure Barbara, Celarent, Darii, Ferio, Barbari, Celaront

2nd figure Cesare, Camestres, Festino, Baroco, Cesaro, Camestrop

3rd figure Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison

4th figure Bramantip, Camenes, Dimaris, Fesapo, Fresison, Camenop

BIBLIOGRAPHY

[1] Dutilh Novaes, C. 2004. "A medieval reformulation of the de dicto / de re distinction", in Libor Běhounek, *Logica Yearbook 2003*, (Prague: Filosofia): 111–124.

[2] Eschmann, I.T. 1956. "A catalogue of St. Thomas' works: bibliographical notes" appendix to E. Gilson, *The Christian philosophy of Saint Thomas Aquinas* (New York: Random House): 381–439.

[3] Fitting, M. 1999. "On quantified modal logic", *Fundamenta informaticae* 39, no. 1: 105–121.

[4] Fitting, M. & R.L. Mendelsohn. 1998. *First-order modal logic*, Synthese Historical Library vol. 277 (Dordrecht: Kluwer Academic Publishers).

[5] Garson, J.W. 2006. *Modal logic for philosophers* (New York: Cambridge University Press).

[6] Lagerlund, H. 2004. "Medieval theories of the syllogism", in Edward N. Zalta, ed., *The Stanford encyclopedia of philosophy* (Spring 2004 edition). `http://plato.stanford.edu/archives/spr2004/entries/medieval-syllogism/`.

[7] O'Grady, P. 1997. "Aquinas on modal propositions: introduction, text, and translation", *International philosophical quarterly* 37, no. 1, issue no. 145: 13–27.

[8] Pseudo-Thomas Aquinas. 2006. *Summa totius Logicae Aristotelis* (WWW: Fundación Tomás de Aquino quoad hanc editionem Iura omnia asservantur). `http://www.corpusthomisticum.org/xpl.html`

[9] Thomas Aquinas. 2006. *De propositionibus modalibus* (WWW: Fundación Tomás de Aquino quoad hanc editionem Iura omnia asservantur). `http://www.corpusthomisticum.org/dpp.html`

[10] William of Sherwood. 1966. *William of Sherwood's introduction to logic*, trans. by N. Kretzmann, (Minneapolis, MN: University of Minnesota Press).

[11] William of Sherwood. 1995. *Introductiones in logicam / Einführung in die Logik*, ed. and trans. by H. Brands & Ch. Kann (Hamburg: Felix Meiner Verlag).

[12] von Wright, G.H. 1951. *An essay in modal logic* (Amsterdam: North-Holland).

Sara L. Uckelman
Institute for Logic, Language, and Computation, Plantage Muidergracht 24, 1018TV Amsterdam, The Netherlands
S.L.Uckelman@uva.nl