An inquisitive dynamic epistemic logic

Floris Roelofsen*

December 2, 2011

Abstract

This paper develops a logic that combines the main features of dynamic epistemic logic with those of inquisitive semantics. We argue that this merge helps both traditions a step further. From the viewpoint of dynamic epistemic logic, the main benefit lies in the fact that inquisitiveness does not only enter the picture at the level of speech acts, but already at the level of semantic content, which means in particular that it becomes possible to deal with embedded questions. From the viewpoint of inquisitive semantics, the main vantage point is that we inherit from dynamic epistemic logic a perspicuous way of representing the epistemic states of the conversational participants, and a way to specify explicitly how utterances and other speech acts affect these epistemic states.

1 Introduction

The aim of this paper is to merge two logical frameworks which have both been developed in recent years to analyze the exchange of information through linguistic communication. The first framework is that of *dynamic epistemic logic* (DEL) (see van Ditmarsch, van der Hoek, and Kooi, 2007; van Benthem, 2011, for recent overviews). This framework allows us to formally specify how certain speech acts change the epistemic state of the participants in a conversation. Importantly, an agent's epistemic state is represented in DEL in such a way that it does not only embody the agent's knowledge about the configuration of the world, but also her knowledge about other agents' knowledge, and about those other agents' knowledge about yet other agents' knowledge, etcetera. In short, epistemic states in DEL embody *higher-order knowledge*.[1] Moreover, DEL is not only concerned with the epistemic states of individual agents, but also with various notions of group knowledge. Most prominently, it allows us to represent

*This paper was presented at the 22^{nd} *Workshop on Games, Logic, Language, and Computation* in Amsterdam, December 1, 2011. I am very grateful for inspiring conversations with Yanjing Wang at the *International workshop on Logic and Rational Interaction* in Guangzhou, China, October 10-13, 2011, which triggered me to develop the ideas presented here.

[1] The term *knowledge* is used here as a placeholder; many subtly different notions of knowledge and belief can be modeled in DEL by varying the underlying epistemic logic.

the *common knowledge* of a group of agents, and to model how this common knowledge changes as a conversation proceeds.

Most work in the DEL tradition so far focusses on one particular type of speech act: namely that of making an *assertion*. Evidently, assertions play an important role in the process of exchanging information. However, an equally important role is played by *questions*. Information exchange can be seen as a process of raising and resolving issues. Participants provide information by making assertions, and request information by asking questions.

The importance of questions has been recognized in recent work within the DEL tradition, in particular by Van Benthem and Minică (2011). A characteristic feature of the approach taken by Van Benthem and Minică is that it leaves the semantics of the basic static fragment of the logical language untouched. In order to bring questions into the picture, it enriches the dynamic part of the language—the part that describes the *speech acts* that may be performed by agents in uttering a sentence. More specifically, besides the familiar assertion operator $[!\varphi]$, the enriched system also has a question operator $[?\varphi]$. Intuitively, $[!\varphi]\psi$ means that 'asserting φ leads to a state where ψ holds,' while $[?\varphi]\psi$ means that 'asking whether φ leads to a state where ψ holds.' Thus, on this approach the basic static language does not contain any sentences that are 'interrogative' in any syntactic sense, or sentences that are 'inquisitive' in any semantic sense. A question is seen as a *speech act* that may be performed by an agent in uttering a certain sentence. But in terms of syntactic form and semantic content, sentences that are used in asking questions are not taken to be any different from sentences that are used in making assertions. In particular, they are not taken to be interrogative or inquisitive in any sense.

An alternative, more radical approach would be to actually enrich the semantics of the basic static fragment of the language, in such a way that the proposition expressed by every sentence in this fragment already captures both its informative and its inquisitive content. On such an approach, it would be natural for the static fragment of the language to contain interrogative sentences of the form $?\varphi$, and for such sentences to express a proposition embodying the issue of whether φ is the case. The dynamic part of the language could then be simplified: instead of an assertion operator $[!\varphi]$ and a separate question operator $[?\varphi]$, we could have a single *utterance operator* $[\varphi]$, where φ could be syntactically indicative or interrogative, and semantically informative and/or inquisitive. Intuitively, $[\varphi]\psi$ would then mean that 'uttering φ leads to a state where ψ holds.' Thus, on this approach, inquisitiveness does not enter the picture at the speech act level, but rather already at the level of the syntax and semantics of the basic static language.

We believe that this alternative approach has some crucial advantages. Most importantly, it would allow us to deal with *embedded questions*. For instance, it would become possible to deal with conditional questions (e.g., *If John goes to the party, will Mary go as well?*) and questions embedded under knowledge operators (e.g. *John knows whether Mary will go to the party*). This is impossible if questions only enter the picture at the speech act level, because in such a setup our logical language does not contain sentences of the form $p \to ?q$ or

$K_a?q$.

This brings us to the second framework, namely that of *inquisitive semantics* (INQ) (Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011, among others). In this framework, the meaning of a sentence does not only embody its informative content, but also its inquisitive content. Therefore, it is ideally suited to deal with questions, in particular embedded questions. On the other hand, whereas much work in the INQ tradition has been devoted to developing a richer notion of semantic content, relatively little attention has been paid so far to the level of speech acts. Of course, there are ideas about the effects that the utterance of a sentence may have on the epistemic states of the conversational participants (see, for instance, Groenendijk, 2008; Farkas and Roelofsen, 2011). However, the way in which these ideas have been formalized so far is rather preliminary. In particular, it assumes a very simple representation of the participants' epistemic states, abstracting away, for instance, from higher-order information.

One of the strengths of the DEL framework is exactly that it allows us to represent the participants' epistemic states in a more sophisticated way, and that it allows us to specify in a very precise way how utterances affect these epistemic states. Thus, merging DEL with INQ will bring both traditions a step further. The DEL framework will be enriched in such a way that it can deal with inquisitiveness, not only at the level of speech acts, but also at the level of sentences and their semantic content. On the other hand, the INQ framework will be enriched in such a way that it will be possible to specify in a more sophisticated way how the utterance of a sentence changes the epistemic states of the conversational participants.

The paper is organized as follows. In section 2, we briefly review the main features of a simple DEL system with questions as speech acts. In section 3, we review the main features of INQ, and again specify a simple system. Then, in section 4, we merge the two systems, and discuss in some more detail how this merge advances both traditions. Section 5 concludes with some suggestions for further work.

2 Dynamic epistemic logic with questions

In this section, we will provide a brief overview of the dynamic epistemic logic with questions, DELQ, developed by Van Benthem and Minică (2011). For more background on dynamic epistemic logic in general we refer to van Ditmarsch *et al.* (2007). We will follow van Benthem and Minică here in presenting DELQ in two steps: first, we will consider a *static* system with information and issues, and then we will *dynamify* this system by adding assertions and questions as speech act operators.

2.1 A static system with information and issues

Our exposition of DELQ starts with a definition of *epistemic issue models*, which are used by van Benthem and Minică to represent the information that has been established at a certain point in a conversation and the issues that have been raised up until that point. Throughout the paper, we will assume a fixed set of agents $\mathcal{A}$, and a fixed set of atomic sentences $\mathcal{P}$.

Definition 1 (Epistemic issue models)**.**
An epistemic issue model M is a quadruple $\langle W, \sim_{\mathcal{A}}, \approx, V\rangle$, where:

- W is a set of primitive objects
- $\sim_{\mathcal{A}} = \{\sim_a \mid a \in \mathcal{A}\}$ is a set of equivalence relations on W
- $\approx$ is an equivalence relation on W
- V is a function that assigns a truth value to every atomic sentence in $\mathcal{P}$, relative to every $w \in W$

The objects in W are usually referred to as *possible worlds*. For every $a \in \mathcal{A}$, the equivalence relation $\sim_a$ encodes the information that is available to agent a. Two worlds $w, v \in W$ are related by $\sim_a$ if and only if they are *indistinguishable* based on the agent a's information. The other equivalence relation, $\approx$, encodes the *issue* that has been raised so far.[2] Like any other equivalence relation, $\approx$ induces a *partition* on W. Two worlds $w, v \in W$ are in the same block of the partition if and only if they are related by $\approx$. And a partition can be thought of as representing an issue: which block of the partition contains the actual world? (see, for instance, Groenendijk and Stokhof, 1984).

Example 1. Consider the model depicted in figure 1. The model consists of four worlds: 11 is a world where both p and q are true, 10 is a world where p is true and q is false, etcetera. The blocks in the partition represent the current issue: we want to know whether we are in a world where q is true (11 or 01) or in a world where q is false (10 or 00). The dotted lines indicate that, based on the currently available information, we cannot distinguish world 11 from 10 and we cannot distinguish world 01 from 00. That is, we know whether p holds, but we don't know whether q holds.

To describe epistemic issue models, van Benthem and Minică introduce a logical language with four modal operators: a universal modality U, a knowledge modality K_a for every agent $a \in \mathcal{A}$, a question modality Q, describing the current issue, and a resolution modality R_a, for every agent $a \in \mathcal{A}$, describing the information that would be available to a if the current issue were resolved.

[2] Van Benthem and Minică (2011) show that $\approx$ can be relativized to agents, just like $\sim$, such that for every agent $a \in \mathcal{A}$, $\approx_a$ encodes the issues that have been raised so far by agent a. For simplicity, we present DELQ here without this possible refinement.

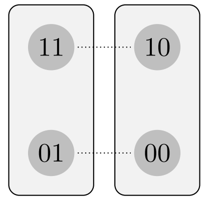

Figure 1: A simple epistemic issue model.

Definition 2 (Static fragment of $\mathcal{L}_{\text{DELQ}}$)**.**
The static fragment of the language of DELQ, $\mathcal{L}_{\text{DELQ}}$, is defined as follows:

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid U\varphi \mid K_a\varphi \mid Q\varphi \mid R_a\varphi$$

As usual, disjunction and implication are defined as abbreviations: $\varphi \vee \psi := \neg(\neg\varphi \wedge \neg\psi)$ and $\varphi \rightarrow \psi := \neg\varphi \vee \psi$. The language is interpreted as follows.

Definition 3 (Semantics for the static fragment of $\mathcal{L}_{\text{DELQ}}$)**.**

1. $M, w \models p$ iff $V(w, p) = 1$
2. $M, w \models \neg\varphi$ iff $M, w \not\models \varphi$
3. $M, w \models \varphi \wedge \psi$ iff $M, w \models \varphi$ and $M, w \models \psi$
4. $M, w \models U\varphi$ iff $M, v \models \varphi$ for all $v \in W$
5. $M, w \models K_a\varphi$ iff $M, v \models \varphi$ for all $v \in W$ such that $v \sim_a w$
6. $M, w \models Q\varphi$ iff $M, v \models \varphi$ for all $v \in W$ such that $v \approx w$
7. $M, w \models R_a\varphi$ iff $M, v \models \varphi$ for all $v \in W$ such that $v \sim_a w$ and $v \approx w$

The universal modality U and the knowledge modalities K_a are treated as usual, but the question modality Q and the resolution modalities R_a are new. Intuitively, $M, w \models Q\varphi$ means that φ holds in all worlds in the block of the current partition/issue that contains w, and $M, w \models R_a\varphi$ means that after resolution of the current issue, agents a knows φ in w.

2.2 Questions and assertions

The central idea in dynamic epistemic logic is that speech acts can be interpreted as operators that *transform* the model of evaluation. In DELQ, there are two speech act operators: a question operator $[?\varphi]$, and an assertion operator $[!\varphi]$. Thus, the basic static language is extended as follows.[3]

[3]Besides the question and assertion operators, Van Benthem and Minică (2011) introduce three additional dynamic operators as well. These additional operators are left out of consideration here, since they do not seem relevant for our present purposes.

Definition 4 (Full language of DELQ)**.**
The full language of DELQ, $\mathcal{L}_{\text{DELQ}}$, is defined as follows:

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid U\varphi \mid K_a\varphi \mid Q\varphi \mid R_a\varphi \mid [!\varphi]\psi \mid [?\varphi]\psi$$

Intuitively, $[!\varphi]\psi$ means that 'asserting φ leads to a state where ψ holds' and $[?\varphi]\psi$ means that 'asking whether φ leads to a state where ψ holds.'

Technically, speech act operators are taken to transform the model of evaluation in certain ways. These transformations are defined as follows.[4]

Definition 5 (Model transformations)**.**
Let $M = \langle W, \approx, \{\sim_a \mid a \in \mathcal{A}\}, V\rangle$ be an epistemic issue model, w and v two worlds in W, and φ a sentence in $\mathcal{L}_{\text{DELQ}}$. Then:

1. $w =_\varphi v$ iff φ has the same truth value in M, w as in M, v
2. $M^{!\varphi} = \langle W, \approx, \{\sim_{a,\varphi} \mid a \in \mathcal{A}\}, V\rangle$, where $w \sim_{a,\varphi} v$ iff $w \sim_a v$ and $w =_\varphi v$
3. $M^{?\varphi} = \langle W, \approx_\varphi, \{\sim_a \mid a \in \mathcal{A}\}, V\rangle$, where $w \approx_\varphi v$ iff $w \approx v$ and $w =_\varphi v$

Given these model transformations, we are now ready to specify the semantics for the full language of DELQ.

Definition 6 (Semantics for DELQ)**.**
The first seven clauses are specified in definition 3. The additional clauses are:[5]

8. $M, w \models [!\varphi]\psi \quad \text{iff} \quad M^{!\varphi}, w \models \psi$
9. $M, w \models [?\varphi]\psi \quad \text{iff} \quad M^{?\varphi}, w \models \psi$

This completes our overview of DELQ. We will discuss its vantage points and limitations in more detail below, but first we will turn to a brief overview of inquisitive semantics.

3 Inquisitive semantics

We will provide an overview here of the most basic system of inquisitive semantics, INQB, which has been developed and investigated in Ciardelli (2009); Groenendijk and Roelofsen (2009); Ciardelli and Roelofsen (2011); Roelofsen (2011).[6] This basic system has been extended in two directions in recent work.

[4] This definition presupposes a definition of the truth-conditions or sentences in $\mathcal{L}_{\text{DELQ}}$, which will be given right below.

[5] As pointed out by Van Benthem and Minică (2011), both assertions and questions can be taken to have certain *preconditions*. For instance, in certain conversational settings a precondition for asserting φ is that the speaker believes φ to be true, and a precondition for asking whether φ is that the speaker does not know whether φ is true or not. Such preconditions can be implemented in DELQ, but are left out of consideration here for simplicity.

[6] An earlier version of inquisitive semantics, which differs significantly from the one adopted here, can be found in Groenendijk (2009) and Mascarenhas (2009). See Ciardelli and Roelofsen (2011) for a detailed argument in favor of the current system.

On the one hand, it has been extended to the first-order setting (see Ciardelli, 2009, 2010; Roelofsen, 2011; Groenendijk and Roelofsen, 2011), and on the other hand it has been refined in order to capture more than just informative and inquisitive content (see Ciardelli *et al.*, 2009; Roelofsen and van Gool, 2010; Pruitt and Roelofsen, 2011). We will focus here on the basic system, but DELQ could also be merged with any of the extended systems, in essentially the same way.

Definition 7 ($\mathcal{L}_{\text{INQB}}$)**.** The language of INQB, $\mathcal{L}_{\text{INQB}}$, is defined as follows:

$$p \mid \bot \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \to \psi$$

We will use $\neg\varphi$ as an abbreviation for $\varphi \to \bot$, $!\varphi$ as an abbreviation for $\neg\neg\varphi$, and $?\varphi$ as an abbreviation for $\varphi \vee \neg\varphi$. The first is standard, the other two will become clear momentarily.

Sentences will be evaluated relative to *states*, which are defined as sets of possible worlds. In INQB, possible worlds are taken to be functions assigning truth values to atomic sentences. Notice that this is slightly different from the notion of possible worlds in DEL: there, possible worlds are primitive objects, and truth values are assigned to atomic sentences *relative to* possible worlds, by valuation functions.

Definition 8 (Possible worlds and states)**.**

- A possible world is a function that assigns a truth value to every atomic sentence in $\mathcal{P}$. The set of all possible worlds is denoted by $\mathcal{W}$.
- A state is a set of possible worlds. The set of all states is denoted by $\mathcal{S}$.

The central notion in the semantics is not that of truth, but rather that of *support*. Support is a relation between states and sentences, defined recursively as follows.

Definition 9 (Support)**.**

1. $s \models p \quad \text{iff} \quad \forall w \in s : w(p) = 1$
2. $s \models \bot \quad \text{iff} \quad s = \emptyset$
3. $s \models \varphi \wedge \psi \quad \text{iff} \quad s \models \varphi \text{ and } s \models \psi$
4. $s \models \varphi \vee \psi \quad \text{iff} \quad s \models \varphi \text{ or } s \models \psi$
5. $s \models \varphi \to \psi \quad \text{iff} \quad \forall t \subseteq s : \text{if } t \models \varphi \text{ then } t \models \psi$

It follows from the above definition that the empty state supports any formula φ. Thus, we may think of $\emptyset$ as the *inconsistent* state.

Fact 1 (Persistence)**.** If $s \models \varphi$ then for every $t \subseteq s$: $t \models \varphi$

Fact 2 (Singleton states behave classically)**.** For any world w and formula φ:

$$\{w\} \models \varphi \iff \varphi \text{ is classically true under the valuation } w$$

It follows from definition 9 that the support-conditions for $\neg\varphi$ and $!\varphi$ are as follows.

Fact 3 (Support for negation)**.**

1. $s \models \neg\varphi \quad \text{iff} \quad \forall w \in s : w \models \neg\varphi$
2. $s \models\, !\varphi \quad \text{iff} \quad \forall w \in s : w \models \varphi$

In terms of support, we define the *proposition* expressed by a sentence, and the *possibilities* for a sentence. We also define the *truth-set* of a sentence φ as the meaning that would be associated with φ in a classical setting.

Definition 10 (Truth sets, propositions, possibilities)**.**

1. The *truth set* of φ, $|\varphi|$, is the set of all worlds w such that $\{w\} \models \varphi$.
2. The *proposition* expressed by φ, $[\varphi]$, is the set of all states supporting φ.
3. A *possibility* for φ is a maximal state supporting φ, that is, a state that supports φ and is not properly included in any other state supporting φ.

The following result guarantees that the possibilities for a sentence completely determine the proposition expressed by that sentence, and vice versa.[7]

Fact 4 (Propositions and possibilities)**.** For any state s and any sentence φ:

$$s \in [\varphi] \iff s \text{ is contained in a possibility for } \varphi$$

Fact 5 (Characteristic properties of propositions)**.**

- For every sentence φ, $[\varphi]$ is a non-empty and persistent set of states, i.e. if $s \in [\varphi]$ and $t \subseteq s$, then also $t \in [\varphi]$.
- If S is a non-empty, persistent set of states, then there is a sentence φ such that $[\varphi] = S$.

Example 2 (Disjunction)**.** Inquisitive semantics crucially differs from classical semantics in its treatment of disjunction. To see this, consider figures 2(a) and 2(b). In these figures, it is assumed that $\mathcal{P} = \{p, q\}$; world 11 makes both p and q true, world 10 makes p true and q false, etcetera. Figure 2(a) depicts the truth set—that is, the classical meaning—of $p \vee q$: the set of all indices that make either p or q, or both, true. Figure 2(b) depicts the possibilities for $p \vee q$ in INQB. One possibility is made up of all worlds that make p true, and the other of all worlds that make q true.

[7] In Groenendijk and Roelofsen (2009); Ciardelli and Roelofsen (2011), the proposition expressed by φ is not defined as the set of all states supporting φ, but rather as the set of all possibilities for φ, i.e., the set of all maximal states supporting φ. Fact 4 below guarantees that in the present setting, the two definitions are interchangeable. However, while the definition adopted here extends naturally to the first-order setting, the definition of propositions in terms of maximal supporting states is problematic in the first-order setting, since certain first-order sentences do not have maximal supporting states. Thus, fact 4 does not hold in the first-order setting. See Ciardelli (2009, 2010) and Roelofsen (2011) for discussion.

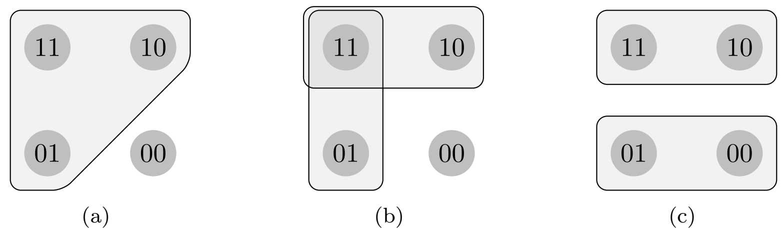

Figure 2: (a) classical picture of $p \vee q$, (b) inquisitive picture of $p \vee q$, and (c) polar question $?p$.

In INQB, we think of a sentence φ as expressing a proposal to update the common ground of a conversation in such a way that the new common ground comes to support φ. Worlds that are not contained in any state supporting φ will therefore not survive any of the updates proposed by φ. In other words, if any of the updates proposed by φ is executed, all worlds that are not contained in any state supporting φ will be eliminated. Thus, it is natural to think of $\bigcup[\varphi]$ as the *informative content* of φ.

Definition 11 (Informative content). $\mathsf{info}(\varphi) = \bigcup[\varphi]$

In classical propositional logic, CPL, the informative content of a sentence φ is embodied by its truth set, $|\varphi|$. The following result says that, as far as informative content goes, INQB does not diverge from CPL. In this sense, INQB is a 'conservative extension' of CPL.

Fact 6. For any $\varphi \in \mathcal{L}_{\text{INQB}}$: $\mathsf{info}(\varphi) = |\varphi|$

A sentence φ is informative in a state s iff it proposes to eliminate at least one world in s, i.e., iff $s \cap \mathsf{info}(\varphi) \neq s$. On the other hand, φ is inquisitive in s iff in order to reach a state $s' \subseteq s$ that supports φ it is not enough to incorporate the informative content of φ itself into s, i.e., iff $s \cap \mathsf{info}(\varphi) \not\models \varphi$.

Definition 12 (Inquisitiveness and informativeness relative to a state).

- φ is *informative* in s iff $s \cap \mathsf{info}(\varphi) \neq s$
- φ is *inquisitive* in s iff $s \cap \mathsf{info}(\varphi) \not\models \varphi$

These relative notions of inquisitiveness and informativeness also have natural absolute counterparts.

Definition 13 (Absolute notions of inquisitiveness and informativeness).

- φ is *informative* iff it is informative in $\mathcal{W}$, i.e., iff $\mathsf{info}(\varphi) \neq \mathcal{W}$
- φ is *inquisitive* iff it is informative in $\mathcal{W}$, i.e., iff $\mathsf{info}(\varphi) \not\models \varphi$

Fact 7 (Alternative characterization of inquisitiveness).

- φ is *inquisitive* iff there are at least two possibilities for φ.

Example 3 (Disjunction continued)**.** As in the classical setting, $p \vee q$ is *informative*, in that it proposes to eliminate the world where both p and q are false. But it is also *inquisitive*, in that it proposes to move to a state that supports either p or q, while merely eliminating the world where both p and q are false is not sufficient to reach such a state. Thus, it requests a response that provides additional information. This inquisitive aspect of meaning is not captured in the classical setting.

Definition 14 (Questions, assertions, and hybrids)**.**

- φ is a *question* iff it is not informative;
- φ is an *assertion* iff it is not inquisitive;
- φ is a *hybrid* iff it is both informative and inquisitive.

Example 4 (Questions, assertions, and hybrids)**.** We saw above that $p \vee q$ is both informative and inquisitive, i.e., hybrid. $!(p \vee q)$ is an example of an assertion; the unique possibility for $!(p \vee q)$ is depicted in figure 2(a). Finally, $?p$ is an example of a question; the two possibilities for $?p$ are depicted in figure 2(c). These two possibilities together cover the entire logical space, so $?p$ does not propose to eliminate any world. Therefore, $?p$ is a question.

INQB naturally gives rise to two notions of entailment. The first notion is the classical notion of *informative* entailment. In INQB, this notion is defined as follows.

Definition 15 (Informative entailment)**.** $\varphi \models_{\mathsf{info}} \psi$ iff $\mathsf{info}(\varphi) \subseteq \mathsf{info}(\psi)$

It follows from fact 6 that informative entailment in INQB coincides with entailment in CPL and can therefore be axiomatized in exactly the same way.

Besides informative entailment, INQB also gives rise to a notion of *inquisitive* entailment. This notion is defined as follows.

Definition 16 (Inquisitive entailment)**.** $\varphi \models_{\mathsf{inq}} \psi$ iff $[\varphi] \subseteq [\psi]$

One sentence φ inquisitively entails another sentence ψ just in case every state that support φ also supports ψ. This means that whenever we carry out the proposal that is made in uttering φ in one way or another (by moving to a state that supports φ), we automatically also carry out the proposal that is made in uttering ψ.

Inquisitive entailment is strictly stronger than informative content: whenever $\varphi \models_{\mathsf{inq}} \psi$ we also have that $\varphi \models_{\mathsf{info}} \psi$, but not necessarily the other way around. Inquisitive entailment has been axiomatized in Ciardelli and Roelofsen (2011). In Roelofsen (2011) it has been shown that inquisitive entailment induces a Heyting algebra on the set of all propositions in INQB, and that disjunction, conjunction, negation, and implication behave semantically as *join*, *meet*, *(relative) pseudo-complement* operators w.r.t. inquisitive entailment.

4 Merging the two frameworks

In this section, we will first spell out in some detail how, in our view, dynamic epistemic logic could benefit from incorporating some of the main features of inquisitive semantics, and how, vice versa, inquisitive semantics could benefit from incorporating some of the main features of dynamic epistemic logic. After that we will present a system that brings the main ingredients of DELQ together with those of INQB.

4.1 Motivation

One striking feature of DELQ and other systems in the DEL traditions is that they build on classical logic in a very conservative way. In particular, the basic propositional fragment of the language is interpreted exactly as in CPL. Moreover, the knowledge operators K_a are interpreted exactly as in classical epistemic logic. The novelty of the system lies in extending this basic language with new operators. In particular, questions are brought into the picture as *speech act* operators. There are no sentences in the language of DELQ that are *interrogative* in any syntactic sense, or *inquisitive* in any semantic sense. There are no sentences of the form $?\varphi$, which we could take to be syntactically interrogative. And the proposition expressed by a sentence is always a set of pointed models: those pointed models where the sentence is true. Thus, the proposition expressed by a sentence can be taken to capture the *informative* content of that sentence, as usual, but not its *inquisitive* content.

In inquisitive semantics, on the other hand, the language of propositional logic gets a more fine-grained interpretation: the proposition expressed by a sentence is intended to embody both its informative and its inquisitive content. From the proposition expressed by a sentence in INQB, we can still straightforwardly derive its classical meaning (we saw that for every φ, $|\varphi| = \bigcup[\varphi]$). However, the proposition expressed by φ in INQB tells us *more* than just what its classical meaning (i.e., its informative content) is; it also embodies the sentence's inquisitive content. Moreover, the language of INQB contains sentences of the form $?\varphi$, which are naturally classified as *interrogative* sentences in a syntactic sense. Thus, in INQB inquisitiveness already enters the picture at the level of the syntax and semantics of the basic static language, rather than only at the level of speech acts.

One important advantage of this setup is that is allows us to deal with *embedded questions*. For instance, the language contains sentences like $p \to ?q$, which naturally correspond to *conditional questions* in natural language (e.g., *If John goes to the party, will Mary go as well?*). The language of DELQ does not contain such sentences. By merging the two systems this limitation would be overcome. The logical language that would result from this merge would also contain sentences like $K_a?p$, which naturally correspond to sentences in natural language like *John knows whether Mary will come to the party.* In a first-order extension of the system, we would even have sentences like $K_a(?x.Px)$, which would correspond to constructions like *John knows who will come to the* *party*. Notice that these type of sentences are currently not dealt with in either DELQ or INQB. Thus, by merging the two systems we will be able to deal with constructions that are beyond the reach of each individual system.

Another reason to merge the two systems, especially from the viewpoint of inquisitive semantics, is that the multi-agent Kripke models that are used in DEL make it possible to describe the epistemic states of a set of agents at a certain point in a conversation in a precise and perspicuous way, capturing both the information that is available to each of the individual agents about the configuration of the world, as well as *higher-order information* that the agents may have about one another's epistemic states. Most work on inquisitive semantics has focused so far on developing a suitable notion of meaning, embodying both informative and inquisitive content, while less progress has been made in spelling out what the effects are of uttering a sentence with informative and/or inquisitive content on the epistemic states of the agents involved in a conversation (see Groenendijk, 2008; Balogh, 2009; Farkas and Roelofsen, 2011, for preliminary proposals). In particular, in modeling such effects, attention has so far been restricted to the *common ground* of the conversation—the information that all the agents in the conversation have publicly committed to. The multi-agent Kripke models used in DEL make it possible to develop a more fine-grained account, which does not only reflect changes of the common ground, but also changes of the individual epistemic states.

Thus, summarizing, there are at least two good reasons to develop a system that combines the main features of DELQ with those of INQB. First, inquisitiveness will not only enter the picture at the speech act level, as in DELQ, but also at the level of the syntax and semantics of the basic static language. As a result, it will be possible to deal with embedded questions. Second, the multi-agent Kripke models used in DEL will allow us to model the effects that an utterance has on the epistemic states of the participants of a conversation in a much more sophisticated way than has so far been done in the inquisitive semantics tradition.

4.2 Inquisitive dynamic epistemic logic

In this section we will develop an inquisitive dynamic epistemic logic, IDEL, which combines the main features of DELQ and INQB. We will first present a static system, and then move on to the full dynamic system.

4.2.1 Inquisitive epistemic logic

We will start by presenting an inquisitive semantics for the language of epistemic logic. We will refer to this system as IEL. As before, we will assume a fixed set of agents $\mathcal{A}$ and a fixed set of atomic sentences $\mathcal{P}$.

Definition 17 ($\mathcal{L}_{\text{IEL}}$)**.** The language of IEL, $\mathcal{L}_{\text{IEL}}$, is defined as follows:

$$p \mid \bot \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \to \psi \mid K_a\varphi$$

As in INQB, we will take $\neg\varphi$ to be an abbreviation for $\varphi \to \bot$, $!\varphi$ an abbreviation for $\neg\neg\varphi$, and $?\varphi$ an abbreviation for $\varphi \vee \neg\varphi$.

As in INQB, sentences will be evaluated relative to states. However, besides information about the configuration of the world, in IEL states should also embody information about the epistemic states of the agents involved in the conversation. To this end, we will not define states simply as sets of worlds in the sense of INQB, i.e., sets of valuation functions, but rather as sets of worlds in a multi-agent epistemic Kripke model.

We will work with a specific Kripke model, namely the canonical model for the epistemic logic S5.[8] Every world in this model corresponds to a certain 'state of affairs,' where by a state of affairs we mean a certain configuration of the world together with a certain epistemic state for each of the agents involved. Moreover, because the model is canonical, every possible state of affairs (that is consistent with the axioms of S5) corresponds to some world in the model. In other words, the canonical model determines the space of all logically possible states of affairs.

Definition 18 (The canonical model for S5)**.**
The canonical model for S5 is a triple $M^c = \langle W^c, {\sim_{\mathcal{A}}}^c, V^c\rangle$, where:

- W^c is the set of all maximal S5-consistent sets.[9]
- $\sim_{\mathcal{A}} = \{\sim_a \mid a \in \mathcal{A}\}$ is a set of equivalence relations on W^c such that $w \sim_a v$ if and only if for every sentence φ, if $K_a\varphi \in w$ then $\varphi \in v$.
- V^c is a function that assigns a truth value to every $p \in \mathcal{P}$ relative to every $w \in W^c$, in such a way that $V^c(p, w) = 1$ iff $p \in w$.

The elements of W^c are referred to as the *worlds* in M^c. States are defined as sets of such worlds.

Definition 19 (States)**.**

- A state is a set of worlds in M^c. The set of all states is denoted by $\mathcal{S}$.

As in INQB, the central notion in the semantics is not that of *truth*, but rather that of *support*. As far as atomic sentences and propositional connectives are concerned, the support relation is defined exactly as in INQB. The only clause that is new is the one for knowledge operators. In this clause, we will use $\sigma_{a,w}$ to denote the epistemic state of agent a in w, i.e., the set of worlds that are indistinguishable from w for a, $\{v \in W^c \mid v \sim_a w\}$.

[8] Our choice for S5 here is rather arbitrary; other epistemic logics could be used just as well. In fact, it may be most appropriate in the present setting to use a logic for *belief*, like KD4 or KD45, rather than a logic for *knowledge*. But it is not necessary to take a definitive stance on this issue here.

[9] A set of sentences is S5-consistent iff no contradiction can be derived from them and the axioms of S5 using Modus Ponens. A maximal S5-consistent set is an S5-consistent set that is not contained in any other S5-consistent set. The axioms of S5 are **K**: $K_a(\varphi \to \psi) \to (K_a\varphi \to K_a\psi)$, **T**: $K_a\varphi \to \varphi$, **4**: $K_a\varphi \to K_aK_a\varphi$, and **5**: $\neg K_a\varphi \to K_a\neg K_a\varphi$. For more background information on modal logics, see Blackburn *et al.* (2002).

Definition 20 (Support in IEL).
Let $s \in \mathcal{S}$, $p \in \mathcal{P}$, and $\varphi, \psi \in \mathcal{L}_{\text{IEL}}$. Then:

1. $s \models p$ iff $\forall w \in s : V^c(p, w) = 1$
2. $s \models \bot$ iff $s = \emptyset$
3. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$
4. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$
5. $s \models \varphi \to \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$
6. $s \models K_a\varphi$ iff $\forall w \in s : \sigma_{a,w} \models \varphi$

The clause for knowledge operators says that $K_a\varphi$ is supported in a state s just in case, for every world $w \in s$, the epistemic state of agent a in w supports φ. This gives us a unified treatment of knowledge-*that* and knowledge-*whether* constructions, assuming that *that* is translated into our logical language as !, and *whether* as ?.

Example 5. Consider the following sentences:

(1)	a.	Alex knows that Peter is coming.	$K_a!p$
	b.	Alex knows that Peter or Quinten is coming.	$K_a!(p \vee q)$
	c.	Alex knows whether Peter is coming.	$K_a?p$

For a state s to support the first sentence, every w in s must be such that $\sigma_{a,w}$ supports $!p$, which means that every world in $\sigma_{a,w}$ must be one where p holds. Similarly, for s to support the second sentence, every w in s must be such that $\sigma_{a,w}$ supports $!(p \vee q)$, which means that every world in $\sigma_{a,w}$ must be one where either p or q holds. Finally, for s to support the third sentence, every w in s must be such that $\sigma_{a,w}$ supports $?p$, which means that we must either have that every world in $\sigma_{a,w}$ is one where p holds, or that every world in $\sigma_{a,w}$ is one where $\neg p$ holds. These are precisely the desired predictions for these sentences.[10]

Based on the support relation, we define the truth set of a sentence, the proposition expressed by a sentence and the possibilities for a sentence, just as we did in INQB.

Definition 21 (Truth sets, propositions, possibilities).

1. The *truth set* of φ, $|\varphi|$, is the set of all worlds w such that $\{w\} \models \varphi$.
2. The *proposition* expressed by φ, $[\varphi]$, is the set of all states supporting φ.

[10] The present system can be further refined in order to account for embedded disjunctive questions (see Roelofsen and van Gool, 2010; Pruitt and Roelofsen, 2011) and to deal with Gettier's objections against the notion of knowledge as justified true belief (see Uegaki, 2011).

3. A *possibility* for φ is a maximal state supporting φ, that is, a state that supports φ and is not properly included in any other state supporting φ.

As in INQB, propositions in IEL are non-empty, persistent sets of states. That is, fact 5 directly carries over from INQB to IEL. Also as in INQB, the informative content of a sentence can still be defined as the union of all states that support that sentence.

Definition 22 (Informative content). $\mathsf{info}(\varphi) = \bigcup[\varphi]$

We can define informative and inquisitive sentences, as well as questions, assertions, and hybrids, exactly as we did in INQB (see definitions 12, 13, and 14). Finally, the notions of informative and inquisitive entailment also directly carry over from INQB to IEL (see definitions 15 and 16).

Thus, the extension of INQB to the language of epistemic logic is rather straightforward. The next step is to add dynamic speech act operators to the system.

4.2.2 Inquisitive dynamic epistemic logic

Recall that in DELQ there were two speech act operators, one for assertions and one for questions. This was necessary because in DELQ the proposition expressed a sentence only embodied the informative content of that sentence. In IEL, just like in INQB, the proposition expressed by a sentence captures both its informative and its inquisitive content. This means that we no longer need to introduce two distinct speech act operators for questions and assertions. Instead we will have a single operator for utterances more generally. In addition to this, we will introduce an *acceptance* operator, which is used to model the speech act of accepting the informative content of a previously uttered sentence.

Definition 23 ($\mathcal{L}_{\text{IDEL}}$). The language of IDEL, $\mathcal{L}_{\text{IDEL}}$, is defined as follows:

$$p \mid \bot \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \to \psi \mid K_a\varphi \mid [\varphi]_a\psi \mid [ok]_a\psi$$

The new constructions are $[\varphi]_a\psi$ and $[ok]_a\psi$. Intuitively, $[\varphi]_a\psi$ is intended to mean that 'an utterance of φ by agent a would lead to a state that supports ψ,' while $[ok]_a\psi$ is intended to mean that 'acceptance by agent a of the informative content of the previously uttered sentence would lead to a state that supports ψ.' For simplicity, we will assume here that expressions of the form $[\varphi]_a\psi$ are only well-formed if φ itself does not contain any speech act operators. That is, φ must always be in $\mathcal{L}_{\text{IEL}}$.

As in DELQ, speech acts will be taken to change the discourse context. Thus, in order to describe the effect of speech acts more precisely, we first have to specify what we take discourse contexts to be. We will build here on work by Farkas and Bruce (2010); Farkas and Roelofsen (2011). We take a discourse context to be a pair $\langle s, T\rangle$, where s is a state, representing the information that has become available in the conversation so far, and T is a stack of IEL-propositions, representing the proposals that have been made so far. We will refer to T as the Table, and to its elements as propositions that have been put on the Table.

Definition 24 (Stacks)**.**

- For any $n \in \mathbb{N}$, a stack of length n is a tuple with n elements.
- If T is a stack of length $n \geq 1$, then for every $0 \leq m \leq n$, T_m denotes the mth element of T.
- If T is a stack of length $n \geq 1$, then $\mathsf{top}(\mathrm{T})$ denotes the nth element of T.
- If T is a stack of length n, and x an object, then $T + x$ is a stack T' of length $n+1$, such that $T'_m = T_m$ for all $1 \leq m \leq n$, and $T'_{n+1} = x$.

Definition 25 (Discourse contexts)**.**

- A discourse context is a pair $\langle s, T\rangle$, where s is a state and T a stack of IEL-propositions. The set of all discourse contexts is denoted by $\mathcal{C}$.

Now we are ready to specify the effect of a speech act on the discourse context. Again, we will build here on the analysis of Farkas and Bruce (2010); Farkas and Roelofsen (2011). We take the effect of an utterance of φ by an agent a to be twofold: first, the proposition expressed by φ is put on the Table, and second, worlds where a's information state does not support the informative content of φ are eliminated from s. Thus, in uttering φ, a speaker (i) publicly commits to the informative content of φ, and (ii) puts the proposition expressed by φ on the Table.

Definition 26 (The effect of an utterance on the discourse context)**.**
Let $\langle s, T\rangle \in \mathcal{C}$, $a \in \mathcal{A}$, and $\varphi \in \mathcal{L}_{\text{IEL}}$. Then $\langle s, T\rangle^{\varphi_a} = \langle s^{\varphi_a}, T^{\varphi_a}\rangle$, where:

1. $s^{\varphi_a} = \{w \in s \mid \sigma_{a,w} \subseteq \mathsf{info}(\varphi)\}$
2. $T^{\varphi_a} = T + [\varphi]$

It is perhaps good to emphasize that T is always a stack of IEL-propositions. Thus, in the second clause of definition 26, $[\varphi]$ is the proposition expressed by φ in IEL.

The speech act of acceptance has a simpler effect than that of uttering a sentence: it does not put a new proposal on the Table, but only eliminates worlds in which the epistemic state of the agent of the speech act does not support the informative content of the proposition that is on top of the Table. Thus, in making an acceptance move, a speaker publicly commits to the informative content of the previously uttered sentence.

Definition 27 (The effect of acceptance on the discourse context)**.**
Let $\langle s, T\rangle \in \mathcal{C}$, $a \in \mathcal{A}$, and $\varphi \in \mathcal{L}_{\text{IDEL}}$. Then $\langle s, T\rangle^{ok_a} = \langle s^{ok_a}, T^{ok_a}\rangle$, where:

1. $s^{ok_a} = \{w \in s \mid \sigma_{a,w} \subseteq \mathsf{info}(\mathsf{top}(T))\}$

2. $T^{ok_a} = T$

Notice that $\mathsf{top}(T)$ is only defined if T contains at least one element, which means that $\langle s, T\rangle^{ok_a}$ is only well-defined if T contains at least one element. This reflects the anaphoric nature of acceptance: an acceptance move is appropriate only if there is at least one proposal on the Table.

Having specified how utterances and acceptance moves change the discourse context, we are now ready to define when a discourse context supports a sentence in $\mathcal{L}_{\text{IDEL}}$. The first six clauses are essentially the same as those for IEL, only now support is determined relative to discourse contexts rather than states. The two additional clauses deal with constructions involving speech act operators.

Definition 28 (Support in IDEL)**.**
Let $\langle s, T\rangle \in \mathcal{C}$, $p \in \mathcal{P}$, $a \in \mathcal{A}$, and $\varphi, \psi \in \mathcal{L}_{\text{IDEL}}$. Then:

1. $\langle s, T\rangle \models p \quad \text{iff} \quad \forall w \in s : V^c(p, w) = 1$
2. $\langle s, T\rangle \models \bot \quad \text{iff} \quad s = \emptyset$
3. $\langle s, T\rangle \models \varphi \wedge \psi \quad \text{iff} \quad \langle s, T\rangle \models \varphi$ and $\langle s, T\rangle \models \psi$
4. $\langle s, T\rangle \models \varphi \vee \psi \quad \text{iff} \quad \langle s, T\rangle \models \varphi$ or $\langle s, T\rangle \models \psi$
5. $\langle s, T\rangle \models \varphi \rightarrow \psi \quad \text{iff} \quad \forall s' \subseteq s :$ if $\langle s', T\rangle \models \varphi$ then $\langle s', T\rangle \models \psi$
6. $\langle s, T\rangle \models K_a\varphi \quad \text{iff} \quad \forall w \in s : \langle \sigma_{a,w}, T\rangle \models \varphi$
7. $\langle s, T\rangle \models [\varphi]_a\psi \quad \text{iff} \quad \langle s, T\rangle^{\varphi_a} \models \psi$
8. $\langle s, T\rangle \models [ok]_a\psi \quad \text{iff} \quad \langle s, T\rangle^{ok_a} \models \psi$

In this system, several discourse related notions can be defined. For instance, we could say that a discourse context $\langle s, T\rangle$ is *stable* if and only if s is contained in every proposition in T. This means that we have reached a state that supports all the sentences that were uttered so far. In other words, all the proposals that were made so far have been carried out satisfactorily. Similarly, we could say that a sentence φ has the potential to *resolve* a discourse context $\langle s, T\rangle$ just in case an utterance of φ by one of the agents, and subsequent acceptance by all the other agents, would lead to a stable discourse context. We could also add operators corresponding to these notions to the object language. For instance, we could add an operator R to the language, and say that a sentence $R\varphi$ is supported by a discourse context $\langle s, T\rangle$ if and only if φ has the potential to resolve $\langle s, T\rangle$. A detailed exploration of such notions will be left for another occasion.

5 Conclusion and outlook

In this paper, we developed an inquisitive dynamic epistemic logic that combines the main features of DELQ and INQB, and we argued that this merge helps both traditions a step further. From the viewpoint of DEL, the main benefit lies in the fact that inquisitiveness now does not only enter the picture at the level of speech acts, but already at the level of semantic content, which means in particular that it becomes possible to deal with embedded questions. From the viewpoint of INQ, the main vantage points are (i) that we now have a perspicuous and well-understood way of representing the information of the conversational participants, including their higher-order information, and (ii) that the system allows us specify explicitly how utterances and other speech acts, such as acceptance, affect the epistemic states of the conversational participants.

Of course, the work done here is only a beginning, and opens up several avenues for further research. First of all, the system developed here is a merge of the most basic systems in the dynamic epistemic logic tradition and the inquisitive semantics tradition. In both traditions, these basic systems have been extended in several ways, and those extended systems could of course be combined to obtain richer versions of IDEL. Second, the logical properties of IEL and IDEL should be investigated in detail. In particular, it would be interesting from a logical point of view to establish an axiomatization of informative and inquisitive entailment in IEL. But other logical notions, such as notions of answerhood and subquestionhood, could be defined and investigated as well. And finally, we hope that the system will be helpful in linguistic analyses of discourse-related phenomena. Initial work in this direction has been pursued in Farkas and Roelofsen (2011).

References

Balogh, K. (2009). *Theme with variations: a context-based analysis of focus*. Ph.D. thesis, University of Amsterdam.

Blackburn, P., De Rijke, M., and Venema, Y. (2002). *Modal Logic*. Cambridge University Press.

Ciardelli, I. (2009). Inquisitive semantics and intermediate logics. Master Thesis, ILLC University of Amsterdam.

Ciardelli, I. (2010). A first-order inquisitive semantics. In M. Aloni, H. Bastiaanse, T. de Jager, and K. Schulz, editors, *Logic, Language, and Meaning: Selected Papers from the Seventeenth Amsterdam Colloquium*. Springer.

Ciardelli, I. and Roelofsen, F. (2011). Inquisitive logic. *Journal of Philosophical Logic*, **40**(1), 55–94.

Ciardelli, I., Groenendijk, J., and Roelofsen, F. (2009). Attention! *Might* in inquisitive semantics. In S. Ito and E. Cormany, editors, *Proceedings of Semantics and Linguistic Theory (SALT XIX)*.

Farkas, D. and Bruce, K. (2010). On reacting to assertions and polar questions. *Journal of semantics*, **27**, 81–118.

Farkas, D. and Roelofsen, F. (2011). Polarity particles in an inquisitive discourse model. Manuscript, University of California at Santa Cruz and ILLC, University of Amsterdam.

Groenendijk, J. (2008). Inquisitive semantics and dialogue management. ESSLLI course notes, www.illc.uva.nl/inquisitive-semantics.

Groenendijk, J. (2009). Inquisitive semantics: Two possibilities for disjunction. In P. Bosch, D. Gabelaia, and J. Lang, editors, *Seventh International Tbilisi Symposium on Language, Logic, and Computation*. Springer-Verlag.

Groenendijk, J. and Roelofsen, F. (2009). Inquisitive semantics and pragmatics. Presented at the Workshop on Language, Communication, and Rational Agency at Stanford, available via www.illc.uva.nl/inquisitive-semantics.

Groenendijk, J. and Roelofsen, F. (2011). An inquisitive witness semantics. Manuscript, ILLC University of Amsterdam.

Groenendijk, J. and Stokhof, M. (1984). *Studies on the Semantics of Questions and the Pragmatics of Answers*. Ph.D. thesis, University of Amsterdam.

Mascarenhas, S. (2009). Inquisitive semantics and logic. Master Thesis, University of Amsterdam.

Pruitt, K. and Roelofsen, F. (2011). Prosody, syntax, and semantics of disjunctive questions. Manuscript, University of Massachusetts Amherst and University of Amsterdam.

Roelofsen, F. (2011). Algebraic foundations for inquisitive semantics. In H. van Ditmarsch and J. Lang, editors, *Proceedings of the Third International Conference on Logic, Rationality, and Interaction*, pages 233–243. Springer-Verlag.

Roelofsen, F. and van Gool, S. (2010). Disjunctive questions, intonation, and highlighting. In M. Aloni, H. Bastiaanse, T. de Jager, and K. Schulz, editors, *Logic, Language, and Meaning: Selected Papers from the Seventeenth Amsterdam Colloquium*, pages 384–394. Springer.

Uegaki, W. (2011). Inquisitive knowledge attribution and the Gettier problem. In M. Aloni, F. Roelofsen, K. Schulz, G. Weidmann-Sassoon, and M. Westera, editors, *Pre-proceedings of the 18th Amsterdam Colloquium*, pages 00–00.

van Benthem, J. (2011). *Logical dynamics of information and interaction*. Cambridge University Press.

Van Benthem, J. and Minică, Ş. (2011). Toward a dynamic logic of questions. *To appear in the Journal of Philosophical Logic*.

van Ditmarsch, H., van der Hoek, W., and Kooi, B. (2007). *Dynamic Epistemic Logic*. Springer.