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# WHAT IS MODAL LOGIC?

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## 1 INTRODUCTION

This paper contains no results. Instead, it deals with methodological issues in (propositional) modal logic. More precisely, this paper is concerned with methodological issues in what may be called *extended (propositional) modal logic*, a rapidly expanding and active field that comprises of modal formalisms that differ in important aspects from the traditional format by extending or restricting it in a variety of ways. The paper surveys the parameters along which extensions of the standard modal format have been carried out, it proposes a unifying framework for modal logic, and identifies several research topics that arise naturally in this setting.

It has been a long time since modal logic (ML) dealt with just two operators $\Diamond$ and $\Box$. Nowadays every possible way of deviating from the syntactic, semantic and algebraic notions pertaining to this familiar duo seems to be explored. The creation of such new, or extended modal logics is largely application driven. In many applications ML is used as a formalism to reason about certain aspects of relational structures. This connection with relational structures makes ML into a powerful tool — besides ML they occur naturally in many parts of linguistics, mathematics, computer science and artificial intelligence. As new (aspects of) structures become important because of new applications, the need arises to go beyond existing modal formalisms to more powerful ones. Or, on the other hand, it may be necessary to consider languages that are somehow weaker or restricted versions of earlier ones because of computational considerations. Moreover, it may be necessary to add new features to existing modal languages, like, for example, extra operators, extra sorts, or allowing for novel modes of evaluation.

This diversity of the field, and the ensuing wealth of formalisms, notions and techniques has a number of less agreeable side-effects. To start off with a superficial one, the question often arises whether or why a particular system still is ... a *modal* logic. This lack of identity also shows up in the phenomena that many specific small or local results are being proved using roughly the same arguments over and over again, while there is some more general result subsuming these instances 'waiting just around the corner.' A unifying approach to ML should reveal these results with the right amount of generality. Another unsatisfactory

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- $\mathcal{C}_s$ $(s \in \mathcal{S})$ is a set of constants;
- $\mathcal{O}_s$ $(s \in \mathcal{S})$ is a set of connectives;
- $\mathcal{F}$ is a set of function symbols.

We think of the elements of $\mathcal{F}$ as *modal operators*; via the semantics these will encode simple patterns in the structures in which the modal language will be interpreted. Each (propositional) variable and each constant is assumed to be equipped with a sort symbol as are the argument places of the modal operators and connectives.

Then, the formulas $Form_s$ of sort $s$ $(s \in \mathcal{S})$ are built up as follows.

| | |
|---|---|
| Connectives: | $\bullet$, |
| Modal operators: | $\#$, |
| Atomic formulas: | $p_s \in \mathcal{V}_s \cup \mathcal{C}_s$, |
| Formulas: | $\phi \in Form_s$, |

$$\phi ::= p_s \mid \bullet(\phi_{1,s}, \ldots, \phi_{n,s}) \mid \#(\phi_{s_1}, \ldots, \phi_{s_n}),$$

where it is assumed that $\bullet$ and $\#$ return values of sort $s$. (One side remark: according to the above set-up there may seem to be little difference between connectives and modal operators; §4 contains some remarks on this issue.)

**2.2.** EXAMPLE. In the basic modal language we have two sorts: $p$ (for propositions) and $r$ (for relations), the usual set of propositional variables $(p_0, p_1, \ldots)$ and constants $(\bot, \top)$, and only one constant but no variables of the relational sort $(R)$; the connectives are the usual ones, while there is only one modal operator, $\langle\cdot\rangle\cdot$, whose first argument should be of the relational sort, and whose second argument should be of the propositional sort.

## SEMANTICS

A system of modal logic not only specifies the syntax of legal formulas, it also provides a semantics to interpret these formulas. As will become clear when I present examples in §3, the semantic desiderata include multiple domains, a uniform approach to dealing with the semantics of the modal operators, and a flexible way of incorporating side-conditions on the interpretations of the symbols in our language.

Generalizing our intuitions from the basic modal format, our modal operators will be interpreted as describing certain simple patterns in the relational structures underlying our modal languages. Such patterns are given as formulas of a classical logic $L$. Here we will take a classical logic to be any logic in the sense of (Barwise & Feferman 1985, Chapter 2), but often one can think of first-order logic when we write classical logic.

**2.3.** DEFINITION. Let $\mathcal{F}$ be as in 2.1, and let $\# \in \mathcal{F}$. A *pattern* or *L-pattern* $\delta_\#$ for $\#$ is a formula in some classical logic $L$ that specifies the semantic definition of $\#$.

A pattern will typically have the form $\lambda x_{s_1} \ldots \lambda x_{s_n} . \phi(x_{s_1}, \ldots, x_{s_n}; x_{s_{n+1}}, \ldots, x_{s_m})$, where $x_{s_1}$, $\ldots, x_{s_n}, x_{s_{n+1}}, \ldots, x_{s_m}$ are variables of sort $s_1, \ldots, s_n, s_{n+1}, \ldots, s_m$, respectively, the variables $x_{s_{n+1}}, \ldots, x_{s_m}$ are free variables, and all non-logical symbols occurring in $\phi$ are either among these variables or constants from $\mathcal{C}$.

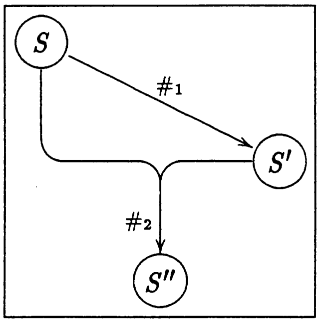


Figure 1: Impression of an extended modal logic.

## 3 Examples

The examples of modal logics fitting the framework of §2 are divided somewhat roughly according to the way they modify the basic modal format.

The basic modal format. In the basic modal format there are two sort symbols $p$ (for propositions) and $r$ (for relations). The variables of sort $p$ are $\mathcal{V}_p = \{ p_0, p_1, \dots \}$; the constants $\mathcal{C}_p$ are $\{ \bot, \top \}$; the connectives $\mathcal{O}_p$ of sort $p$ are $\{ \neg, \wedge \}$. For the relational sort we have $\mathcal{V}_r = \emptyset$, $\mathcal{C}_r = \{ R \}$, $\mathcal{O}_r = \emptyset$. Also, $\mathcal{F} = \{ \langle\cdot\rangle\cdot \}$, where the first argument place is marked for symbols of sort $r$, and the second one for symbols of sort $p$, and the 'result sort' is $p$.

The sole constraint here is that $W_r \subseteq (W_p)^2$ (and hence $V(R) \subseteq (W_p)^2$). The pattern for $\langle\cdot\rangle\cdot$ is $\lambda x_r . \lambda x_p . \exists y((x, y) \in V(x_r) \wedge y \in V(x_p))$. As there is only one relational symbol that can be used as input for this pattern we might as well use the 'old' notation $\Diamond\cdot$ whenever this is convenient; the pattern then becomes $\lambda x_p . \exists y((x, y) \in V(R) \wedge y \in V(x_p))$.

Below I present a number of examples of modifications of the basic modal format. I will usually assume that the example formalisms extend the basic format.

More operators. A common motivation for extending a modal formalism is the need to capture more or new aspects of relational structures. The most obvious way to go about things is to add extra operators; such additions can easily be accounted for in the framework of §2.

One possibility is to simply add an extra operator to the existing stock, and give its pattern in terms of the 'material' already present. This is exactly what happens in *tense logic*, where the basic modal pattern is complemented with its backward-looking version through an operator $P\cdot$ with pattern $\lambda x_p . \exists y\, ((y, x) \in V(R) \wedge y \in V(x_p))$.

An addition motivated by the wish to overcome some of the deficiencies in the expressive power of BML, concerns the $D$-operator. The pattern of this unary operator reads $\lambda x_p . \exists y\, (y \neq x \wedge y \in V(x_p))$. This simple addition ('move to a *different* point, and check for a proposition there') to the basic modal language with $\mathcal{C}_r = \{ R \}$ makes all universal first-order patterns involving $R$ definable in the modal language (cf. (Koymans 1989, De Rijke 1992*a*)).

Alternatively, one may have to add new relations, and consider patterns defined in terms of them. Provability logic, for example, where the dual $[\cdot]\cdot$ of $\langle\cdot\rangle\cdot$ is used to simulate provability in an arithmetical theory, has been expanded with modal operators simulating (rel-

To get rid of the high computational complexity that results from the set-theoretical assumptions underlying *DML*, Van Benthem (1991*b*) proposes a system of *arrow logic*. In (at least one mutation of) arrow logic there are two kinds of propositions: one ranging over (sets of) arrows, and one ranging over (sets of) states, as usual. The important point is that arrows are not treated as pairs of states but as unanalyzed objects. Amongst others the set of modal operators $\mathcal{F}$ in this system contains an operator $L$ taking a relation and a property of states, and returning a property of arrows: $\lambda x_s.\exists y\,((x,y) \in V(1) \wedge y \in V(x_s))$, where, intuitively, $(x,y) \in V(1)$ says that the state $y$ is a left endpoint of the arrow $x$. (Cf. (Van Benthem 1992, Marx et al. 1992, Vakarelov 1992).)

MORE STRUCTURE, 1. We've encountered several ways now of extending an existing modal format: through the addition of operators, by turning to more complex modes of evaluation, and thirdly, also, by adding sorts. A fourth mode of extension concerns the need to in some way add more structure to a sort already present.

*PDL* provides an example. Clearly, the big difference between the basic modal format and *PDL* is not just that *PDL* has a 'larger' stock of relation symbols, but that *PDL* provides means to add structure in the relational component. In most mutations of *PDL*, whenever $\alpha$ and $\beta$ are relations, one is able to form relations $\alpha;\beta$, the sequential composition of $\alpha$ and $\beta$, $\alpha \cup \beta$, the union of $\alpha$ and $\beta$, and $\alpha^*$, the transitive closure of $\alpha$. In terms of the framework of §2 this means that for $\mathcal{C}_r$, the set of connectives of the relational sort, we have $\mathcal{C}_r = \{\,;, \cup, ^*\,\}$. A variety of modifications is available of this set of relational connectives; for instance, intersection, complementation and/or converse have been added, and restrictions allowing only special combinations of intersection and composition have also been studied (cf. (Harel 1984, Blackburn 1993*a*)).

MORE STRUCTURE, 2. Instead of adding more structure *amongst* elements of a sort, it has also been proposed to add *internal* structure to elements of a sort. The point is this. In BML, for example, we are interested in the patterns of transitions between states, and in the way the truth value of formulas is affected by these patterns. In this setting we are *not* interested in the nature of these states. But in many applications it may be necessary or at least convenient to be more specific about the nature of the states.

An elegant way to accommodate this need is to amalgamate two modal languages into one: a *global* one to reason about the global transitions between the states (or points of time, or ...), and a *local* one to handle the internal aspects of the states. Finger & Gabbay (1992) present a canonical way to amalgamate two logical systems $L_1$ and $L_2$ into one system $L_1(L_2)$ while preserving several properties of $L_1$ and $L_2$, to be able to reason about the underlying "2-level structure" as a whole. Specifically: assume $L_1$, $L_2$ are both presented in a manner similar to BML (with superscripts $i$ indicating the language of $L_i$); and for simplicity assume that $\mathcal{V}_p^1 = \emptyset$, $\mathcal{F}^i = \{\,\Diamond_i\,\}$. The language $L_1(L_2)$ then has $\mathcal{V}_p = \mathcal{V}_p^2$, $\mathcal{O}_p$ is the usual set of Boolean connectives, $\mathcal{F} = \{\,\Diamond_1, \Diamond_2\,\}$, where $\Diamond_1$ is not allowed to occur in the scope of $\Diamond_2$; also, $W_p$ is a collection of $L_2$-structures. Then, evaluation of formulas is handled in the obvious way: $\Diamond_1$ is handled globally, while $\Diamond_2$ takes you 'inside' an element of $W_p$. (A side remark: this is an instance of a much more general phenomenon, called *zooming in*, *zooming out* in Blackburn & De Rijke (1993), of adding and forgetting structure in so-called layered relational models.)

**Q:** Isn't the framework presented in §2 so general that virtually any formal system counts as a modal logic?
**A:** An ML has multiple sorts and functions between sorts that describe simple patterns of the underlying relational structures. In their most natural formulations many formal systems don't enjoy those properties (I would not call propositional logic a modal logic), yet a lot of them have formulations that do enjoy those properties. I don't think that this is all that important. What is important is that the framework captures and clarifies our intuitions about ML, that it offers a unifying approach to ML, and that it pays off in terms of new insights and questions — and I think it does.

**Q:** On a related note: how does the framework of §2 relate to other general approaches to logic, like, for example, *abstract model-theoretic logic*?
**A:** Two important aspects of abstract model-theoretic logic are (1) the isolation and study of specific logics for the analysis of various (mathematical) properties, and (2) the investigation into the relations between such logics. As a tool for reasoning about relational structures ML is subsumed by (1); so far little attention has been paid to examining the relations between systems of ML. Thus, ML may be viewed as being part of abstract model-theoretic logic; but it has a very special status of its own, being many-sorted by nature, and paying special attention to functions between sorts that describe simple relational patterns.

**Q:** What is the difference between a connective and a modal operator?
**A:** By comparing their patterns modal operators can be organized in hierarchies, using a variety of ways to measure the complexity of the patterns. Modal operators whose patterns are 'simple' according to one way of measuring, would then be called connectives. E.g., one naive way of measuring the complexity of patterns is to simply count the number of sorts it contains; at the lower end of this hierarchy one finds homogeneous patterns, that is: patterns relating only objects of the same sort — the way I see it this is where the connectives reside.

**Q:** Which patterns count as 'good' or 'nice' patterns?
**A:** This is related to the above point, and a question that is still largely unanswered. Patterns may be organized according to their quantificational properties. First-order patterns $\phi(x)$ in one free variable with both restricted quantification and quantifier rank 1 yield 'nice' modal operators (cf. (Van Benthem 1993)). The corresponding modal logics admit a decent sequent-style axiomatization, are decidable, and enjoy interpolation.[2] These results may fail miserably for modal operators whose patterns have a more complex quantificational structure.

Alternatively, modal operators and their patterns may be classified according to their behavior with respect to relations between models and operations on models. In this respect modal operators with a first-order pattern $\phi(x)$ with restricted quantifiers only and quantifier rank 1 again qualify as 'nice' operators: they are characterized (in a sense which can be made precise) by their invariance under appropriate bisimulations (again, cf. (Van Benthem 1993)).

More generally, broad (semantic) criteria for classifying modal operators and their patterns have yet to be invented, although certain case studies have been carried out.[3] One desideratum

[2] I am assuming here that no additional axioms beyond the 'basic' system have been added.

[3] An example: in (Van Benthem 1991*a*) it is shown that there is only one mapping from relations to propositions that is a homomorphism, and that only two mappings in the opposite direction are homomorphisms.

formulas (with first-order patterns) are equivalent to a special kind of second-order formulas $\forall \ldots ST(\phi)$, where the universal prefix binds all the (transcribed) propositional variables of $\phi$. By Sahlqvist's Theorem a large class of modal formulas in BML may be shown to correspond to first-order conditions on frames after all. Now, by moving on to richer modal formalisms like the system of *D*-logic, or *DML* from §3, a larger class of second-order conditions of the form described above is obtained. Still, for those particular systems a version of Sahlqvist's correspondence result can be established.

What is needed here is a *general* result encompassing those individual ones that stretches Sahlqvist's Theorem to its limits, so to say. Without going into details here, with the syntactic and semantic framework given in §2 there is a clear direction in which to generalize the old result: the idea is to take an arbitrary ML-vocabulary $\tau$ together with (first-order) patterns for the modal operators in $\tau$ — and extend the old result to this 'arbitrary' language (cf. (De Rijke 1993*a*)). The benefit of striving towards such generalizations may not just be achieving greater generality, but also gaining a better understanding of what made the 'old' result work in the first place.

PROOF THEORY. Although this paper belongs to the Amsterdam school of modal logic which has traditionally emphasized the semantic aspects of the enterprise, I do feel that a framework for modal logic should also address the issue of *proof theory*. The proof theory of ML has not kept pace with its model theory, mainly due to the fact that much of the innovating motivation in ML arises from its semantic use, where proof theory may not be the most obvious research topic.

One general issue to be dealt with is this. Should one demand that an ML have a complete proof procedure? In order to answer this, one should keep in mind the role ML is supposed to play. The first is, as a 'deductive machine', and the second as an instrument for reasoning about and characterizing structures. As many systems of ML are designed with applications in mind, a complete proof theory seems desirable. But completeness is not just another property a system might have or not have. It may be that completeness is too stringent a requirement; even when using ML as a deductive machine completeness might be sacrificed for other advantages, such as greater expressive power.

And even when one does strive for and obtain completeness results in ML a lot still remains to be done. Most existing proof systems for ML are presented as Hilbert style calculi, but these "are not suited for the purpose of actual deduction" (Bull & Segerberg 1984, p. 28). Rather, sequent calculi seem to be needed, but in order to be tractable a sequent calculus needs to enjoy properties like the subformula property. In this respect the approach advocated by Wansing (1992) seems promising, as it appears to allow generalizations to arbitrary modal languages.

## 5 Concluding remarks

At the risk of overdoing it, let me repeat once more the picture of modal logic that I have outlined in this note. A system of modal logic is a many-sorted formalism in which the modal operators emerge as functions from sorts to sorts that describe simple patterns in the underlying relational structures. And although the examples of 'truly' many-sorted systems of modal logic given in this note are still quite traditional, I think that we will see the development of lots of many-sorted modal formalisms in the near future, especially with the

van der Hoek, W. & de Rijke, M. (1992), Counting objects in generalized quantifier theory, modal logic and knowledge representation, Technical Report IR-307, Free University, Amsterdam.

Jónsson, B. & Tarski, A. (1952), 'Boolean algebras with operators, Part I', *American Journal of Mathematics* **73**, 891–939.

Koymans, R. (1989), Specifying Message Passing and Time-Critical Systems with Temporal Logic, PhD thesis, Eindhoven University of Technology.

Kracht, M. & Wolter, F. (1991), 'Properties of independently axiomatizable bimodal Logics', *Journal of Symbolic Logic* **56**, 1469–1485.

Manna, Z. & Pnueli, A. (1992), *The Temporal Logic of Reactive and Concurrent Systems. Vol. 1 Specification*, Springer-Verlag, New York.

Marx, M. et al. (1992), 'Arrow logic'. Manuscript, CCSOM, University of Amsterdam.

Reynolds, M. (1992), 'Axiomatization and decidability of $F$ and $P$ in cyclical time', *Journal of Philosophical Logic.* To appear.

de Rijke, M. (1992*a*), 'The modal logic of inequality', *Journal of Symbolic Logic* **57**, 566–584.

de Rijke, M. (1992*b*), A system of dynamic modal logic, Technical report # CSLI-92-170, Stanford University. Also appeared as ILLC Report LP-92-08, University of Amsterdam.

de Rijke, M. (1992*c*), 'Unary interpretability logic', *Notre Dame Journal of Formal Logic* **33**, 249–272.

de Rijke, M. (1993*a*), 'Correspondence theory for extended modal logic'. Manuscript, ILLC, University of Amsterdam.

de Rijke, M. (1993*b*), 'Axioms for Peirce algebras'. Manuscript, ILLC, University of Amsterdam.

Thijsse, E. (1992), Partial Logic and Knowledge Representation, PhD thesis, Tilburg University.

Vakarelov, D. (1992), A modal theory of arrows. Arrow logics I, Technical Report ML-92-04, ILLC, University of Amsterdam.

Venema, Y. (1990), 'Expressiveness and completeness of an interval tense logic', *Notre Dame Journal of Formal Logic* **31**, 529–547.

Wansing, H. (1992), Sequent calculi for normal modal propositional logics, Technical Report LP-92-12, ILLC, University of Amsterdam.

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