Conditional questions and which-interrogatives

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Abstract

Analyses developed so far, failed to provide an adequate semantics for the interpretation of which-interrogatives of the form “Which P are Q?”. In this paper a new logical language is developed, where which-interrogatives are expressed by universally quantifying over conditional interrogatives. Furthermore, a new semantic notion is introduced, that of an alternative. By means of the latter an adequate interpretation is given and shortcomings concerning answerhood and trivial interrogatives are sufficiently dealt with.

Contents

1 Introduction

In natural language there are, among others, two particular types of sentences: *indicatives* and *interrogatives*. In English, the latter are syntactically characterized by word order and the occurrence of interrogative pronouns. Semantically, indicatives are characterized by their truth conditions and interrogatives by their answerhood conditions.

The purpose of this paper is to develop an adequate semantics for the interpretation of which-interrogatives of the form "Which P are Q?". We will investigate whether the semantics gives rise to an adequate logical notion of answerhood. Moreover, some other logical properties will be presented and we will discuss whether these properties are in accordance with one's intuitions about which-interrogatives.

In the logical language of our presentation the following interrogatives can be adequately represented:

- Simple yes/no interrogatives like "Is your father at home?".
- Conditional interrogatives like "If Mary reads this book then will she recommend it to Peter?".
- Who-interrogatives like "Who is walking in the park?"
- Which-interrogatives like "Which students passed the test?".

Other approaches developed an adequate semantics for the interpretation of interrogatives in general but they failed to provide an adequate analysis for conditional questions (Hulstijn, 1997), which-interrogatives (Groenendijk & Stokhof, 1984) and their possible answers. In this paper an attempt will be made to consider a logical framework in which interrogatives can be adequately interpreted and to eliminate, if possible, shortcomings of the analysis. The semantic framework used is possible world semantics. Indicatives and interrogatives are part of the new logical language. The terms *proposition* and *question* are used to describe the semantic objects that correspond to the indicatives and interrogatives respectively.

The thesis is organised as follows: Initially a simple query language will be introduced: the language of propositional logic enriched with the question operator. By means of that language, the syntax and semantics of simple yes/no-interrogatives will be defined and some logical properties will be discussed (section 2.1). Next, the syntax and semantics of conditional interrogatives like "If p, is it the case that q?" will be defined and an alternative approach concerning the interpretation of those interrogatives will be presented and compared with our approach (section 2.2).

In section 3.1, we develop a new query language by extending a predicate logical language with the addition of the question operator and universal quantification into interrogatives. In the new language who-interrogatives can be defined. In section 3.2, the kernel of this thesis, an interpretation of which-interrogatives of the form "Which P are Q?" is analysed. Issues like trivial interrogatives and logical answerhood are discussed. In section 3.3 two alternative approaches are presented and compared with our own proposal. In the last section we return to the issue of answerhood where a new type of answer, the descriptive answer, is presented and discussed.

2 The propositional logical case

2.1 Simple yes/no-interrogatives

In this section we will investigate how the language of classical propositional logic can be extended to express simple yes/no-interrogatives. The syntax and semantics of the new system will be defined and a uniform interpretation for indicatives and interrogatives will be adopted, which will have as a result the use of a single notion of entailment (notation: $\models$). This new notion can uniformly apply to indicatives and interrogatives: in $\phi \models \psi$, if ϕ and ψ are both indicatives the definition gives rise to the standard entailment relation between indicatives. If ϕ and ψ are both interrogatives we have entailment between interrogatives. Finally, if ϕ is an indicative and ψ an interrogative we obtain the notion of logical answerhood that determines which indicatives are answers to a particular question.

Moreover in this section, we will present the analysis of simple yes/no-interrogatives according to Groenendijk & Stokhof (1990, 1997). Essentially the same things hold as in our proposal, the only difference is that entailment and logical answerhood are not uniformly defined for indicatives and interrogatives.

2.1.1 The syntax of simple yes/no-interrogatives

The language of propositional logic L is extended with the question operator which will be denoted with the ?-symbol. In the new query language QL, ϕ is an *indicative* iff $\phi \in L$, and ϕ is an *interrogative* iff $\phi \in QL$ and $\phi \notin L$. For the formulation of the syntax, ϕ, ψ etc. are used as meta-variables ranging over all sentences, $\phi!$ as a meta-variable ranging over indicatives and $\phi?$ as a meta-variable ranging over interrogatives. Simple yes/no-interrogatives are formed syntactically by prefixing a question mark to the formulae of L (the language of propositional logic). In the extended language there are now two distinct categories: indicatives and interrogatives. The question mark can only occur as a left-outermost operator; compound interrogatives do not occur in the language under consideration. Formally:

Definition 1 (Syntax)

Let L be a language of propositional logic. QL is the smallest set such that

1. If $\phi \in L$ then $\phi \in QL$
2. If $\phi \in L$ then $?\phi \in QL$

Clause 1 states that for the formulation of simple yes/no-interrogatives the ?-operator is always placed in front of indicative sentences. Clause 2 is a non-iterative rule and it depicts the natural language fact that an interrogative of the form "Is it the case that is it the case that p?" ($??p$) cannot be defined.

Here are some question/answer sequences and their English glosses.

- $?p$; $\neg p$
  "Is your mother at home?"; "No, (she is not at home)."
- $?(p \to q)$; $p \to q$
  "If Mary reads this book, will she recommend it to Peter?"; "Yes, (if Mary reads this book then she will recommend it to Peter)"

In the system of Groenendijk & Stokhof, (1997), simple yes/no-interrogatives are defined in the same way "by prefixing a question mark to the formulae of a standard language of propositional logic" resulting also in the existence of "two distinct sentential categories in the extended language: indicatives and interrogatives". The previous examples apply in this case as well.

Next, we will consider the interpretation of simple yes/no-interrogatives and it will be shown that such interrogatives correspond to a partition of the logical space. Moreover, the notions of entailment and logical answerhood will be defined and we will indicate that entailment is uniform for indicatives and interrogatives.

2.1.2 The semantics of simple yes/no-interrogatives

An indicative can be true or false but this is not the case for interrogatives. We wish to define the semantics in a way that there could be similarities between the interpretation of indicatives and interrogatives since they are not independent of each other: interrogatives express questions; the notion of a question is related to that of an answer; and indicatives express answers.

In formulating the semantics for QL we consider possible world semantics and a standard truth definition for L. The set W corresponds to the whole logical space that consists of all possible worlds. In the propositional case, a possible world can be identified with a valuation function which assigns the values *true* or *false* to the atomic formulas. $|\phi|$ denotes the set of worlds where ϕ is true. The interpretation of the sentences of QL (indicatives and interrogatives) is uniformly defined in terms of relations between possible worlds.

In the interpretation of an indicative, two worlds are related in case the indicative is true in both these worlds. Two worlds are not related in case the indicative is false in at least one of them. In the interpretation of an interrogative, two worlds w and v are related in case we are indifferent as to whether the actual world is more like w or more like the v. Two worlds are disconnected (there is no relation between them) when we are interested in whether the actual world is more like the one or more like the other.

The interpretation of an indicative ϕ is called a proposition and it is denoted by $\|\phi\|$; the interpretation of an interrogative $?\phi$ is called a question and it is denoted $\|?\phi\|$. As we will see in more detail later in this paper, both $\|\phi\|$ and $\|?\phi\|$ are sets of pairs of worlds, subsets of W^2, but they correspond to different relations. A proposition corresponds to a total relation on a subset of the logical space W, whereas a question corresponds to an equivalence relation[1] on the whole logical space. Despite

[1] An equivalence relation is defined as a reflexive, symmetric and transitive relation.

of that, the fact remains that indicatives and interrogatives are uniformly interpreted as sets of pairs of worlds, worlds that are elements of the whole logical space.

An equivalence relation on a set generates equivalence classes which correspond to totally connected subsets of that set. The non-empty equivalence classes correspond to a partition of the set. A partition of a set A is a set of non-empty subsets of A such that the union of those subsets equals A and no two of these subsets overlap (Groenendijk & Stokhof, 1984, 1990). Formally:

Definition of a partition

If A is a set of elements then $\mathcal{A}$ is a partition of A iff

- For all X in $\mathcal{A}$ it holds that $X \neq \emptyset$
- For all X, Y in $\mathcal{A}$ it holds that if $X \neq Y$ then $X \cap Y = \emptyset$
- $\bigcup \mathcal{A} = A$

The interpretation of simple yes/no-interrogatives is in essence the same as in the system developed in Groenendijk & Stokhof (1984, 1997) except that in our presentation we make use of a single notion of entailment for indicatives and interrogatives alike. The semantics for the propositional query language is defined as follows:

Definition 2 (Semantics)

For $\phi \in L$, let $|\phi| \subseteq W$ be the set of worlds where ϕ holds.

1. $\langle w, v\rangle \in \|\phi\|$ iff ($w \in |\phi|$ and $v \in |\phi|$)
2. $\langle w, v\rangle \in \|?\phi\|$ iff ($w \in |\phi| \Leftrightarrow v \in |\phi|$).

Indicatives are assigned a proposition as semantic value where a proposition is taken to be the pairs of those worlds in which the indicative is true. For example, the proposition expressed by the indicative "John walks" consists of those pairs of worlds where John actually walks. A proposition is a total relation on $|\phi|$. The second clause gives the interpretation of simple yes/no-interrogatives formulated as $?\phi$. The question $\|?\phi\|$ consists of all those pairs of worlds $\langle w, v\rangle$ such that they are either both in or both not in the set of worlds where ϕ is true. The question $\|?\phi\|$ is an equivalence relation on the whole logical space W.

To this equivalence relation corresponds a partition of the set of possible worlds which consists of the equivalence classes of W under this relation. Therefore, questions are semantic objects that correspond to partitions of the logical space. The elements of the partition are called blocks and they correspond to propositions. These propositions are the possible semantic answers to the question; they are mutually exclusive and exhaust the logical space. The view that questions correspond to partitions of the logical space embodies the view that a semantic interpretation of an interrogative determines what its (complete semantic) answers are.

A simple yes/no non-tautological question[2] corresponds to a bipartition of the logical space, hence it has two possible semantic answers. For example, there are two complete answers to the question "Does John walk?" since it divides the whole logical space into two blocks. One block consists of all the worlds where John actually walks and it corresponds to the affirmative answer. The other block consists of the worlds where John does not walk and it corresponds to the negative answer.

An advantage of the uniform interpretation of indicatives and interrogatives is that we can determine a single notion of entailment.

Definition 3 (Entailment)

$\phi_1, \ldots, \phi_n \models \psi$ iff $\|\phi_1\| \cap \ldots \cap \|\phi_n\| \subseteq \|\psi\|$.

Entailment is a semantic notion and it is defined by means of meaning inclusion. If $\phi_1, \ldots \phi_n, \psi$ are all indicatives we have a special notion of entailment, entailment between indicatives. A sequence of indicatives $\phi!_1, \ldots, \phi!_n$ entails the indicative $\psi!$ iff the proposition$\|\phi!_1\| \cap \ldots \cap \|\phi!_n\|$ is included in the proposition expressed by $\psi!$.

We consider a simple case: $\phi! \models \psi!$ iff $\|\phi!\| \subseteq \|\psi!\|$ which means that the indicative $\phi!$ entails the indicative $\psi!$ iff all worlds where $\phi!$ is true are in the set of worlds where $\psi!$ is true. For the entailment between indicatives the following holds:

$$\begin{aligned} \phi! \models \psi! \quad &\Leftrightarrow \quad \|\phi!\| \subseteq \|\psi!\| \\ &\Leftrightarrow \quad |\phi!| \subseteq |\psi!| \end{aligned}$$

So, $\phi! \models \psi! \Leftrightarrow \|\phi!\| \subseteq \|\psi!\|$ is equivalent to: $\phi!$ entails $\psi!$ iff the truth of the first guarantees the truth of the second. For example, "John walks and John talks" entails "John walks" because if the first indicative is true then it is guaranteed that the second is also true. This can be written in the logical language as $\phi! \wedge \psi! \models \phi!$. But "John walks or John sleeps" does not entail that "John sleeps" because the truth of the first does not guarantee the truth of the second. The corresponding expression in the logical language is $\phi! \vee \psi! \not\models \phi!$. The latter definition of entailment is actually already known and this fact shows that in the indicative case it does not matter whether we use the notion of proposition ($\|\phi!\|$) or truth ($|\phi!|$) to describe entailment.

Answerhood is captured by the special instance $\phi! \models \psi?$ where $\phi!$ is an indicative.

Answerhood

$\phi! \models \psi?$ iff $\|\phi!\| \subseteq \|\psi?\|$

The indicative $\phi!$ is an answer to the interrogative $\psi?$ iff the proposition expressed by $\phi!$ corresponds to a set of worlds which is included in

[2]A question is called tautological when the partition it makes consists of only one block: the whole logical space. It is always answered either positively or negatively. Two examples of tautological questions are $\|?\top\|$ or $\|?\bot\|$.

one block of the partition made by $\psi?$. In other words: an indicative is an answer to an interrogative iff it expresses a proposition which entails one of the complete semantic answers to the question expressed by the interrogative. For example, the proposition expressed by the indicative "John walks" is an answer to the question "Does John walk?" since it corresponds to one of the two blocks of the partition made by the question. Under the notion of answerhood, the indicative "John walks and talks" expresses an answer to the previous question since it entails one of its complete semantic answers, namely "John walks".

Another special instance of the notion of entailment is entailment between interrogatives:

$$\phi? \models \psi? \text{ iff } \|\phi?\| \subseteq \|\psi?\|.$$

An interrogative $\phi?$ entails an interrogative $\psi?$ iff every block of the partition made by $\phi?$ is included in a block of the partition made by $\psi?$. This actually indicates that every complete answer to $\phi?$ is a complete answer to $\psi?$. Entailment between simple non-tautological yes/no-interrogatives is rather poor. It cannot hold of two different bipartitions of the same logical space that every block in one is part of some block in the other.

Things are slightly different when we have a sequence of yes/no-interrogatives as premises[3].

$$?\phi_1, \ldots, ?\phi_n \models ?\psi \text{ iff } \|?\phi_1\| \cap \ldots \cap \|?\phi_n\| \subseteq \|?\psi\|.$$

A sequence of interrogatives $?\phi_1 \ldots ?\phi_n$ entails the interrogative $?\psi$ iff the partition expressed by $\|?\phi_1\| \cap \ldots \cap \|?\phi_n\|$ is included in one of the blocks of the bipartition made by $\|?\psi\|$. The sequence of interrogatives "Is your father at home? Is your mother at home?" entails the interrogative "Is your father at home?" because every complete answer to the sequence of interrogatives is a complete answer to the latter interrogative. For example, the proposition "My father is not at home. My mother is at home." is a complete answer to "Is your father at home? Is your mother at home?" and it is included in one of the blocks of the partition made by the question "Is your father at home?"

Entailment between interrogatives does not mirror entailment between the corresponding indicatives. For instance $\phi \wedge \psi \models \phi$ but $?(\phi \wedge \psi) \not\models ?\phi$ since not every answer to $?(\phi \wedge \psi)$ is an answer to $?\phi$. The indicative $\phi \wedge \psi$ is an answer to both. The indicative $\neg(\phi \wedge \psi)$, however, is an answer to $?(\phi \wedge \psi)$ but it is not an answer to $?\phi$.

Any simple yes/no non-tautological interrogative entails the tautological interrogative. Every block of the bipartition is included in the only block of the partition the tautological interrogative corresponds to. This actually asserts that every possible answer to any interrogative is an answer to the tautological interrogative.

[3]This time the entailment $?\phi_1 \ldots ?\phi_n \models ?\psi$ is not so poor because the intersection of bipartitions can be a partition of the logical space with more than two blocks. Let $\mathcal{A}$, $\mathcal{B}$ be two partitions of the same set. Their intersection $\mathcal{A} \sqcap \mathcal{B}$ is defined as follows:
$\mathcal{A} \sqcap \mathcal{B} = \{X \cap Y | (X \in \mathcal{A}) \& (Y \in \mathcal{B}) \& (X \cap Y \neq \emptyset)\}$.

2.2 Adding conditional interrogatives

So far, only simple yes/no-interrogatives such as "Will John come to the party?" or "Is your father at home?" had a corresponding semantic object in the system under consideration. We will extend the syntax and semantics of the system in such a way that conditional interrogatives of the form "If Mary reads this book, will she recommend it to Peter?" can be adequately represented. This interrogative can be most characteristically answered like a simple yes/no-interrogative, for example, "Yes, (if Mary reads this book she will recommend it to Peter)" or "No, (if Mary reads this book she will not recommend it to Peter)". In our approach, however, conditional questions are not regarded the same as simple yes/no questions because the interpretation of the former does not correspond to a partition of the logical space as it does to the interpretation of the latter.

Moreover, we will study an alternative interpretation where conditional questions correspond to partitions of the logical space (Hulstijn, 1997) and we will compare it to our own proposal.

2.2.1 The syntax of conditional interrogatives

In order to enrich the expressive power of the logical language, the syntax of the propositional query language will be extended with the addition of a third clause which formulates conditional interrogatives.

Definition 4 (Conditional interrogatives, syntax)

Let L be a language of propositional logic. QL is the smallest set such that

1. If $\phi \in L$ then $\phi \in QL$
2. If $\phi \in L$ then $?\phi \in QL$
3. If $\phi \in L$ and $\psi? \in QL$ then $\phi \rightarrow \psi? \in QL$.

Conditional interrogatives of the form $p \rightarrow ?q$ as well as of the form $p \rightarrow (q \rightarrow ?r)$ can be expressed in the language. In general, clause 3 of definition 5 adds to the syntax of the query language (QL) interrogatives of the form $(\phi_1 \rightarrow (...(\phi_n \rightarrow ?\psi)...))$ where ϕ_i are always indicatives and $?\psi$ is always an interrogative[4].

Although we formulate conditional interrogatives using the same symbol $\rightarrow$ (implication arrow) as in the underlying indicative language, the formula $\phi \rightarrow \psi$ can never be ambiguous. When ϕ is an indicative and ψ an interrogative we obtain a conditional interrogative and when both ϕ and ψ are indicatives an ordinary indicative implication is the result. In other words, the expressive power of the logical language is enriched not by adding a new arrow to the language but by introducing a new rule to the syntax which uses the same connective in a different way.

[4] In the third clause instead of $\psi? \in QL$ we could write $\psi \in QL$. The question operator is actually redundant. We use it here to stress the fact the in a conditional interrogative the meta-variable ψ, the consequent, can never range over indicatives

With the following examples we will emphasize that $?(\phi \to \psi)$ and $\phi \to ?\psi$ are not the same interrogatives.

1. $?(\phi \to \psi)$; $\phi \to \psi$

   "Is it the case that if Mary has money she will go on vacation?";"Yes, (if Mary has money she will go on vacation)".

2. $?(\phi \to \psi)$; $\neg(\phi \to \psi)$

   "Is it the case that if John comes to the party Mary will also come?"; "No,(it is not the case that if John comes to the party then Mary will also come".

3. $\phi \to ?\psi$; $\phi \to \psi$

   "If Mary has money will she go on vacation?"; "Yes, (if Mary has money she will go on vacation)"

4. $\phi \to ?\psi$; $\phi \to \neg\psi$

   "If John comes to the party will Mary also come?"; "No, (if John comes to the party then Mary will not come)"

The first two clauses are examples of simple yes/no-interrogatives and their characteristic answers. Notice that in the formula $?(\phi \to \psi)$ the question mark occurs as a left outermost operator. In the first clause the answer to the question is affirmative and it is expressed by the indicative $\phi \to \psi$ whereas in the second clause the answer is negative and it is represented by the negation of the implication, i.e. $\neg(\phi \to \psi)$.

Clauses 3 and 4 are examples of conditional interrogatives and their characteristic answers. The question in $\phi \to ?\psi$ is about the consequent. Note that the answer in the third clause is affirmative and is expressed by the same indicative as in clause 1, namely $\phi \to \psi$. The answer in the fourth clause is negative and it is expressed by the indicative $\phi \to \neg\psi$ which is the negation of the consequent and it is not the same as in clause 2.

2.2.2 The semantics of conditional interrogatives

In this section the semantics of conditional interrogatives of the form $\phi \to \psi?$ will be discussed. We will propose an interpretation which does not correspond to a partition of the logical space. So, in our system conditional questions are not considered to be a special case of simple yes/no-questions. Furthermore an alternative interpretation of conditional interrogatives based on partition semantics will be presented.

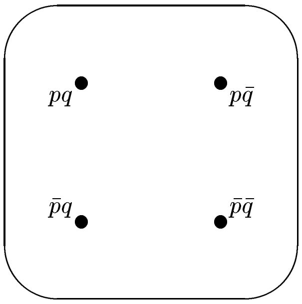
The semantics of the propositional query language is extended with the addition of a third clause.

Definition 5 (Conditional interrogatives, semantics)

For $\phi \in L$, let $|\phi| \subseteq W$ be the set of worlds where ϕ holds.

1. $\langle w, v\rangle \in \|\phi\|$ iff $w \in |\phi|$ and $v \in |\phi|$
2. $\langle w, v\rangle \in \|?\phi\|$ iff $w \in |\phi| \Leftrightarrow v \in |\phi|$.
3. $\langle w, v\rangle \in \|\phi \to \psi?\|$ iff $\langle w, v\rangle \in \|\phi\| \Rightarrow \langle w, v\rangle \in \|\psi?\|$.

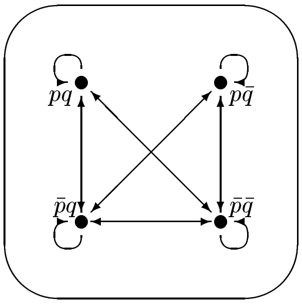
The interpretation of the conditional interrogative $\phi \to \psi?$ amounts to those pairs of worlds such if they are in $||\phi||$ then they are also in $||\psi?||$.



W

Figure 1: The logical space W consisting of four worlds.

We consider a simple example where the logical space W is a particular set of four possible worlds as we can see in figure 1. In the first world p and q are the case, in the second p and $\neg q$, in the third $\neg p$ and q and in the last one $\neg p$ and $\neg q$. According to the semantics, the interpretation of the interrogative $p \to ?q$ consists of those pairs of (possible) worlds such that if they are in $||p||$, the proposition expressed by p, then they are also in $||?q||$, the question expressed by $?q$ (figure 2).



$||p \to ?q||$

Figure 2: The interpretation of the conditional interrogative $p \to ?q$

The conditional question $||p \to ?q||$ excludes those pairs of worlds where p is true in both worlds and q is true in one world and false in the other (see figure 2).

From this simple example we observe that the conditional question $||p \to ?q||$ does not correspond to a partition of the set of possible worlds W. The latter notion is not of much help in case we want to detect the possible (complete) semantic answers to the interrogative $p \to ?q$. In partition semantics the possible complete semantic answers (propositions) correspond to the blocks and this is not the case in this example (see figure 2). Consequently, we will consider replacing the notion of partition in order to properly analyse conditional questions.

To sum up: In a yes/no question the relation between the worlds is an equivalence relation which corresponds to a partition of the logical space. Each block of the partition corresponds to a proposition which is a possible complete semantic answer to that question. On the other hand, in a conditional question the relation between the worlds is symmetric, reflexive but not transitive (see figure 2). We want this relation to correspond to a new notion which somehow structures the logical space in such a way that every maximal subset is a possible semantic answer to that question.

Let Q be the interpretation of an interrogative, conditional or not. An alternative in Q is a maximal subset of W in which all worlds are connected. Each one of the alternatives corresponds to a possible complete semantic answer. For every two alternatives one cannot be a subset of the other but they can overlap. Formally:

Definition of an alternative

Let Q be the interpretation of an interrogative. An alternative in Q is a non-empty subset A of W such that the following holds:

1. $\forall w, v \in A$, $\langle w, v\rangle \in Q$
2. $\forall w \in A$, $\forall v$ $\langle w, v\rangle \in Q \rightarrow v \in A$

In association with our simple example, we suppose that $\|p \rightarrow ?q\|$ is the question "If Mary comes to the party is it the case that Tom will also come to the party?". The interpretation of $p \rightarrow ?q$ corresponds to a cover of the logical space (figure 2), and the alternatives correspond to the propositions $\|p \rightarrow q\|$ and $\|p \rightarrow \neg q\|$[5] (see figure 3). According to the analysis these alternatives are the possible complete semantic answers. We see that $\|p \rightarrow q\|$ and $\|p \rightarrow \neg q\|$ are the propositions "Yes, (if Mary comes to the party then Tom will also come)" and "No, (if Mary comes to the party then Tom won't come)" respectively, which are actually the possible answers to this question. Notice that this logical analysis corresponds very well to one's intuitions concerning the semantic interpretation and the logical answerhood of conditional interrogatives.

It is interesting to see that a partition of the logical space is a special case of a cover of this space and the blocks of the partition are special cases of alternatives. In other words, the way conditional interrogatives are interpreted in the system under consideration does not contradict the already existing semantic notions. Such an interpretation is rather an extension of the partition semantics so that conditional interrogatives can be adequately represented.

As a consequence, the notions of entailment and logical answerhood are defined as in the case of simple yes/no-interrogatives (see definitions 3 and 4 respectively). Therefore we can determine the possible complete answers to $\|p \rightarrow ?q\|$ by means of the answerhood condition as well (see definition 4). The proposition expressed by the indicative $p \rightarrow q$ is an answer to the question expressed by the interrogative $p \rightarrow ?q$ (formally:

[5] Note that $\|p \rightarrow q\|$ is not a subset of $\|p \rightarrow \neg q\|$ and vice versa but they overlap.

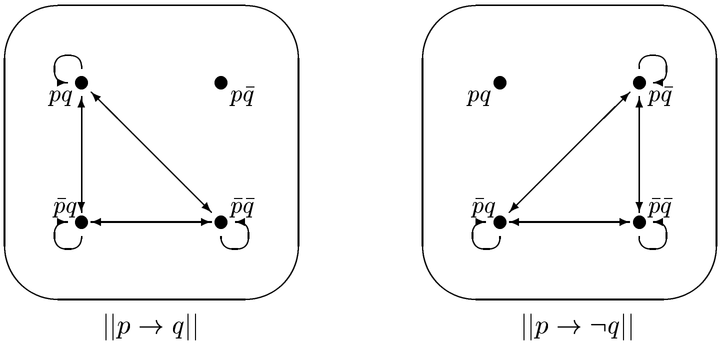


Figure 3: The two possible complete semantic answers to the question $||p \to ?q||$.

$p \to q \models p \to ?q$) since $||p \to q|| \subseteq ||p \to ?q||$. Similarly, we see that $p \to \neg q \models p \to ?q$.

With the answerhood condition more propositions that correspond to answers for $||p \to ?q||$ can be determined. For example:

- $p \wedge q \models p \to ?q$
- $\neg(p \wedge q) \models p \to ?q$
- $\neg p \models p \to ?q$

The proposition expressed by the indicative $p \wedge q$ is an answer to the interrogative $p \to ?q$ but it is not a complete one. Such an answer is considered overinformative since it reveals more information that the question asks for. Apparently, to the question "If Mary has money, will she go on vacation?" the answer "Mary has money and she will go on vacation" asserts something stronger that the proposition "If Mary has money she will go on vacation". Similarly for the second clause. As for the third clause, it implies that an answer like "Mary does not have money" is possible answer but not a complete one.

In what follows we will discuss some facts resulting from the answerhood condition and the interpretation of conditional interrogatives. First, recall that $\models \phi?$ iff $\phi?$ is a trivial (or tautological) interrogative. Formally, $\models \phi?$ iff $\forall w, v \in W$ $\langle w, v\rangle \in ||\phi?||$. A trivial question consists of only one alternative: the logical space as a whole. We use $\phi!$ as a meta-variable ranging over indicatives.

Fact 1

$\phi! \models \psi?$ iff $\models \phi! \to \psi?$

Fact 2 (Triviality)

1. $\models p \to ?p$
2. $\models \neg p \to ?p$
3. $\models p \to ?\top$

4. $\models \bot \rightarrow ?p$

Fact 1 is easy to show:

$$\begin{aligned} \phi! \models \psi? \quad &\Leftrightarrow \quad \|\phi!\| \subseteq \|\psi?\| \\ &\Leftrightarrow \quad \forall w, v : \langle w, v\rangle \in \|\phi!\| \Rightarrow \langle w, v\rangle \in \|\psi?\| \\ &\Leftrightarrow \quad \models \phi! \rightarrow \psi? \end{aligned}$$

The indicatives p and $\neg p$ are the possible complete answers to the question $\|?p\|$. From definition 4 we get $p \models ?p$ and $\neg p \models ?p$ which, according to **Fact 1**, are equivalent to clauses 1 and 2 of **Fact 2** respectively. The answer expressed by the indicative p is a possible complete answer to the tautological question expressed by $?\top$, i.e $p \models ?\top$. From **Fact 1** we conclude that $\models p \rightarrow ?\top$ (clause 3). In a similar way, since contradiction is trivially an answer to any question, we deduce clause 4.

2.2.3 Another approach to conditional questions

In this section we will consider the interpretation of conditional interrogatives developed in Hulstijn (1997) which is in accordance with partition theory. We assume that the syntax is basically similar to the syntax of our approach which implies that conditional interrogatives are expressed by means of the implication arrow ($\rightarrow$). What has changed is the interpretation of such interrogatives. In Hulstijn's interpretation, conditional questions correspond to an equivalence relation on the logical space. We will argue that such an approach represents unsatisfactory certain logical properties like logical answerhood and trivial questions.

The semantics for yes/no interrogatives and conditional interrogatives is defined as follows:

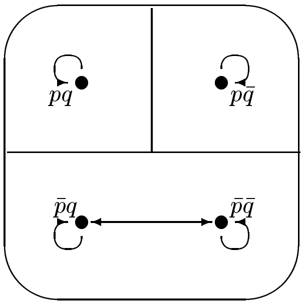
Definition II (Alternative semantics of the propositional query language)

Let $\phi \in L$ and let $|\phi|$ be the set of worlds where ϕ is true.

1. $\langle w, v\rangle \in \|\phi\|$ iff $w \in |\phi|$ and $v \in |\phi|$.
2. $\langle w, v\rangle \in \|?\phi\|$ iff $w \in |\phi| \Leftrightarrow v \in |\phi|$
3. $\langle w, v\rangle \in \|\phi \rightarrow \psi?\|$ iff $\langle w, v\rangle \in \|?\phi\|$ and $\langle w, v\rangle \in \|\phi\| \Rightarrow \langle w, v\rangle \in \|\psi?\|$

A proposition $\|\phi\|$ consists of those pairs of worlds where ϕ is true (clause 1). The interpretation of the simple yes/no-interrogative $?\phi$ amounts to those pairs of worlds such that they are either both in or both not in the set of worlds where ϕ is true (clause 2). By means of clause 3 we can interpret conditional interrogatives under Hulstijn's approach. The only difference with our system is the addition of the condition $\langle w, v\rangle \in \|?\phi\|$. In this way the question $\|\phi \rightarrow \psi?\|$ is an equivalence relation on W and it corresponds to a partition of the logical space.

We consider again our simple example where the logical space W consists of four worlds like in figure 1. The interpretation of $p \rightarrow ?q$ is given in figure 4. In comparison with our interpretation of $p \rightarrow ?q$ (see figure



$||p \rightarrow ?q||$

Figure 4: Alternative interpretation of the conditional interrogative $p \rightarrow q$ (Hulstijn, 1997)

3) we observe that some pairs of worlds have been disconnected. The question $||p \rightarrow ?q||$ is a partition of the logical space consisting of three blocks. According to this analysis its possible complete semantic answers are the blocks of the partition which correspond to the propositions $||p \wedge q||$, $||p \wedge \neg q||$ and $||\neg p||$. From figure 4 we see that the question "If Mary reads this book will she recommend it to Peter?" has three possible answers: the propositions "Mary reads this book and she recommends it to Peter", "Mary reads this book and she does not recommend it to Peter" and "Mary does not read this book".

The propositions $||p \rightarrow q||$ and $||p \rightarrow \neg q||$ do not correspond to any of the three blocks of the partition made by $||p \rightarrow ?q||$. So, the indicatives "Yes, (if Mary reads this book then she will recommend it to Peter)" and "No, (if Mary reads this book then she will not recommend it to Peter)" are not among the possible complete answers to the question. Also, by means of the answerhood condition, we have the following: $||p \rightarrow q||$ is not included in the set of pairs of worlds $||p \rightarrow ?q||$ because the pair $\langle pq, \neg pq \rangle \in ||p \rightarrow q||$ but $\langle pq, \neg pq \rangle \notin ||p \rightarrow ?q||$. Similarly for the answer $||p \rightarrow \neg q||$. Therefore:

- $p \rightarrow q \not\models p \rightarrow ?q$
- $p \rightarrow \neg q \not\models p \rightarrow ?q$.

Hulstijn's interpretation of conditional interrogatives is not in accordance with one's intuitions concerning answerhood since even in our simple example propositions like "Yes, (if Mary reads book then she will recommend it to Peter)" and "No, (if Mary reads this book then she will not recommend it to Peter)", expressed by $||p \rightarrow q||$ and $||p \rightarrow \neg q||$ respectively, are not considered logical answers. Instead, we get answers like $||p \wedge q||$ and $||p \wedge \neg q||$ that reveal more information than one asks for. Moreover, if one allows $||\neg p||$ to be an answer to $||p \rightarrow ?q||$, we have the strange aspect that a proposition which cancels the reason the question was posed in the first place, is an answer. Recall that in our presentation $||p \wedge q||$, $||p \wedge \neg q||$ and $||\neg p||$ are considered possible but not complete answers.

In the approach of Hulstijn (1997) we do not obtain facts similar to **Fact 1** and **Fact 2**. Formally, it is easy to see that $\phi! \models \psi?$ iff $\models \phi! \to \psi?$ is not the case. If $\models \phi! \to \psi?$, then we have that $\phi! \models \psi?$. On the other hand, if $\phi! \models \psi?$, then not necessarily $\models \phi! \to psi?$.

For example, $p \models ?p$ but the conditional interrogative $p \to ?p$is not trivial in Hulstijn's system. Similarly for $\neg p \to ?p$. Furthermore, trivial conditional interrogatives like $p \to ?\top$ or $\bot \to ?p$ cannot be represented in Hulstijn's system.

- $\not\models p \to ?p$
- $\not\models \neg p \to ?p$
- $\not\models p \to ?\top$
- $\not\models \bot \to ?p$

With the use of semantics we see that both $p \to ?p$ and $\neg p \to ?p$ are equivalent to $?p$ which is incorrect. The conditional interrogative $p \to ?p$ corresponds to the question "If it is raining then is it raining?" whereas $?p$ corresponds to the question "Is it raining?".

To conclude, the alternative approach is in accordance with the partition semantics but it falls short to adequately represent logical answers and triviality of conditional interrogatives.

3 The predicate logical case

3.1 Who-interrogatives

In this section the syntax and semantics of the predicate logical case is extended so that simple constituent interrogatives (wh-interrogatives) such as "Who is walking in the park?" or "Who plays with whom?" can be adequately represented. One can look upon such interrogatives as asking for the actual denotation of a particular property, in case of one-constituent, or for a relation in the case of multiple constituent interrogatives.

In the new query language QL, who-interrogatives are formulated by universal quantification over interrogatives and their interpretation forms a partition of the logical space. The semantics gives rise to some new logical facts concerning entailment between interrogatives and logical answerhood.

We will compare our proposal with the approach of Groenendijk and Stokhof (1984, 1997) where who-interrogatives are formed by means of a non-iterative rule. The semantics is similar to the thesis approach. The only difference is that the notion of entailment is not the same for indicatives and interrogatives.

3.1.1 Syntax of who-interrogatives

For the formulation of the syntax, we start from the language of predicate logic L and we arrive at a new query language QL by adding the question operator and universal quantification over questions. As in the propositional case, ϕ, ψ etc. are used as meta-variables ranging over all sentences, $\phi!$ as a meta-variable ranging over indicatives and $\phi?$ as a meta-variable ranging over interrogatives. ϕ is an *indicative* iff $\phi \in L$ and ϕ is an *interrogative* iff $\phi \in QL$ and $\phi \notin L$. In the extended language there is now a new type of interrogatives: who-interrogatives. Formally:

Definition 6

Let L be the language of predicate logic. QL is the smallest set such that

1. If $\phi \in L$, then $\phi \in QL$.
2. If $\phi \in L$, then $?\phi \in QL$.
3. If $\phi? \in QL$, then $\forall x\phi? \in QL$.

In the current query language who-interrogatives like for example "Who is walking in the park?" and "Who loves whom?" are expressed by $\forall x?Px$ and $\forall x\forall y?Rxy$ respectively. In general, who-interrogatives of the form $\forall x_1 ... \forall x_n \phi?$ can be represented in the new query language. Clause 3 is an iterative rule.

In what follows we will present some question/answer sequences.

- $?\exists xPx$; $\exists xPx$
  "Is someone walking in the park?"; "Yes."
- $?\forall xPx$; $\neg\forall xPx$
  "Is everyone walking in the park?"; "No."

- $\forall x?Px$; $\forall x(Px \leftrightarrow (x = a))$
  "Who is walking in the park?"; "(Only) Alfred."
- $\forall x \forall y?Rxy$; $\forall x \forall y(Rxy \leftrightarrow (x = a \wedge y = m))$
  "Who loves whom?"; "Alfred loves Mary."

The first two interrogatives are examples of simple yes/no-interrogatives. We see that there is no universal quantification over questions and the query operator occurs at the left outermost place of the interrogative. The last two interrogatives are examples of who-interrogatives. The interrogative $\forall x?Px$ asks for the specification of the property P and the interrogative $\forall x \forall y?Rxy$ asks for the specification of the actual denotation of the property R. Obviously, when $\forall x \forall y?Rxy$ is answered, $\forall y \forall x?Ryx$ is answered too, and vice versa. Knowing who loves whom and knowing who is loved by whom amounts to the same thing. A fixed domain of discourse is assumed and proper names are treated as rigid designators. So, questions like "Who is Alfred?" expressed by the interrogative $\forall x?(x = \alpha)$ are tautological.

In the following paragraphs we will consider the approach of Groenendijk & Stokhof (1997) and define its syntax.

Definition III (Who-interrogatives, syntax)

Let PL be a language of predicate logic. We extend this to the query language QL such that:

1. If $\phi \in PL$, then $\phi \in QL$
2. If $\phi \in PL$, $\vec{x}$ a sequence of n variables $(0 \leq n)$, then $?\vec{x}\phi \in QL$.

In case the query operator binds no variables we have a yes/no interrogative. For example $?\exists xPx$ asks whether there is an object that has the property P. If the query-operator binds a single variable, a who-question results, e.g. $?xPx$ asks who walks in the park. When two variables are bound, as in $?xyRxy$, we get an interrogative asking for the denotation of a two place relation i.e. for the specification of which pairs of objects stand in the relation R. In general an interrogative $?x_1 \ldots x_n\phi$ asks for a specification of the actual denotation of an n-place relation.

3.1.2 The semantics of who-interrogatives

For the analysis of the semantics we start again from a standard interpretation for the indicative part L. For QL the interpretation is defined as a set of pairs of worlds relative to an assignment g. The set $|\phi|_g$ is the set of worlds where ϕ is true relative to an assignment g. A single domain is assumed for all possible worlds and individual constants are interpreted as rigid designators meaning that they refer to the same object in every world.

In the predicate logical language, indicatives and yes/no-interrogatives are interpreted basically in the same way as in the propositional logical case but this time assignments are taken into consideration. A new clause is added for the interpretation of who-interrogatives.

Definition 7

For $\phi \in L$, let $|\phi|_g \subseteq W$ be the set of worlds where ϕ holds under the assignment g. We have

1. $\langle w, v\rangle \in \|\phi\|_g$ iff $w \in |\phi|_g$ and $v \in |\phi|_g$.
2. $\langle w, v\rangle \in \|?\phi\|_g$ iff $w \in |\phi|_g \Leftrightarrow v \in |\phi|_g$.
3. $\langle w, v\rangle \in \|\forall x \phi?\|_g$ iff $\forall d \in D : \langle w, v\rangle \in \|\phi?\|_{g[x/d]}$
   $g[x/d]$ is the assignment g' that is exactly like g except that it maps x to d.

We will sketch a simple example to investigate the interpretation of who-interrogatives. Consider the question "Who is walking in the park?" expressed by the interrogative $\forall x ?Px$. This question asks for the specification of the individuals who are walking in the park. A pair of worlds $\langle w, v\rangle$ is in the set $\|\forall x?Px\|_g$ iff for all elements d in the domain of discourse D (in this case for all individuals), it is in the partition made by $\|?Px\|_{g[x/d]}$. The question $\|\forall x?Px\|$ is the set of mutually exclusive propositions each of which characterises a possible denotation of the property P (the property of walking in the park) relative to an assignment g. Their union exhausts the logical space W.

Hence, the question "Who is walking in the park?" expressed by $\forall x?Px$ is a partition of W. There are as many blocks in the partition as there are possible denotations of the property of walking in the park. Each one of the blocks is a possible answer. For example "Bill and Mary (and only them)" corresponds to a block of the partition made by the who-question and therefore to a complete semantic answer. Indicatives like "Bill *or* Mary" cannot be represented in our system.

In general, constituent questions partition the logical space in as many blocks as there are possible denotations of the property or relation in question. What is considered a complete answer to any constituent question is the exhaustive characterization of properties or relations between first order objects.

Since the semantics has not basically changed, the notions of entailment and answerhood remain the same but this time we also take assignments into consideration. In what follows we will define entailment for the predicate logical case and we will discuss two special instances: entailment between interrogatives and answerhood. Entailment between indicatives is basically the same as in the propositional logical case.

Definition 8 (Entailment)

$\phi_1, \ldots, \phi_n \models \psi$ iff $\forall g$: $\|\phi_1\|_g \cap \ldots \cap \|\phi_n\|_g \subseteq \|\psi\|_g$.

Answerhood

$\phi! \models \psi?$ iff $\forall g$: $\|\phi!\|_g \subseteq \|\psi?\|_g$

We consider some examples of entailment between interrogatives:

1. $\forall x?Px \models ?\exists x Px$
2. $\forall x?Px \models ?Pa$

3. $\forall x?Px \models \forall x?\neg Px$

Whenever the question posed by $\forall x?Px$ is answered, the question put by $?\exists xPx$ is answered, too. That there are no objects that have property P is one possible answer to the question "Who has property P?", which at the same time provides a negative answer to the question whether someone has that property. Any other possible answer to the first question, specifying some non-empty set of objects having property P, would at the same time imply a positive answer to the second question (clause 1). Since a single domain of discourse is assumed, any complete answer to the question "Who has property P?" will always provide an answer to the question whether Alfred has the same property, too (clause 2). For the same reason, knowing the answers to the question "Who has property P?" entails knowing the answers to the question "Who doesn't have property P?" and vice versa (clause 3).

In what follows we will study logical answerhood for who-interrogatives. Note that in the predicate logical case, nothing has changed for yes/no-interrogatives except that this time assignments are taken into consideration.

1. $\neg\exists xPx \models \forall x?Px$
2. $\forall xPx \models \forall x?Px$
3. $\forall x(Px \leftrightarrow (x = a \vee x = b)) \models \forall x?Px$

That there are no individuals who walk in the park is a possible answer to the question "Who is walking in the park?"(clause 1). "Everyone is walking in the park" is a possible answer to the same question (clause 2). We don't need to examine which particular objects we are talking about since we assumed a single domain for all possible worlds. Note that even with a non-fixed domain such an answer is adequate, although not complete. Such an answer gives the information that all individuals in the domain have the property of walking in the park. In the case of the non-fixed domain those individuals may not be known to the person who asks the question.

A complete semantic answer to the question "Who is walking in the park?" is for example "Alfred and Bill" and it is expressed by the indicative $\forall x(Px \leftrightarrow (x = a \vee x = b))$ (clause 3) which informs us that "Alfred and Bill are the only ones who are walking in the park". The proposition denoted by $Pa \wedge Pb$ states that "(*At least*) Alfred and Bill are walking in the park" and it is not a complete semantic answer.

Notice that the property $(x = a) \vee (x = b)$ is rigid which means that it denotes the same set of objects in every world in the logical space W[6]. The answer scheme of clause 3 can be generalised as follows:

$\forall x(Px \leftrightarrow Ax) \models \forall x?Px$ where A is a rigid property.

In that way descriptive answers can be expressed in the system under consideration. Here is an example:

- "Who walks in the park?"; "The students".
  $\forall x?\text{walk}(x)$; $\forall x(\text{walk}(x) \leftrightarrow \text{stud}(x))$

[6]**Definition (Rigidity)** Let ϕ be an open formula and g an assignment. ϕ is a rigid formula iff $\forall g : \forall w, w_1 \in W : w \in |\phi|_g \Leftrightarrow w_1 \in |\phi|_g$. Equivalently ϕ is a rigid formula iff $\models ?\phi$.

Here the property in question does not have to be rigid in the strict semantic sense. It is sufficient to be rigid with respect to the knowledge of the person who asks the question. The property of being a student is not a rigid one. But as long as the person who asks the question knows who the students are, such an answer is not obscure and it can be expressed in the system under consideration.

The interpretation of wh-interrogatives (single case) as defined in Groenendijk and Stokhof (1997) is as follows.

Definition IV (Who-interrogatives, semantics)

Let D be the domain of discourse and g an assignment.

$\langle w, v\rangle \in \|?x\phi\|_g$ iff $\forall d \in D$: $w \in |\phi|_{g[x/d]}$ iff $v \in |\phi|_{g[x/d]}$

We notice that this interpretation is the same as in our system. We consider a simple example: the interrogative "Who is walking in the park?" is represented by $?xPx$. The question $\|?xPx\|$ partitions the logical space in as many blocks as there are possible denotations of the property of walking in the park (1-place relation) and every block of the partition corresponds to a complete semantic answer.

Nothing has basically changed in this approach concerning entailment between interrogatives provided that a fixed domain of discourse is assumed and proper names are treated like rigid designators. The only difference is that the definition of entailment is not uniform for indicatives and interrogatives, but this fact does not influence things in this case. Answerhood for who-interrogatives is essentially the same as in our presentation: An indicative $\phi!$ is an answer to an interrogative $?x\psi$ iff the proposition $\|\phi!\|$ is included in one block of the partition made by $\|?x\psi\|$. If a single domain is assumed and proper names are treated like rigid designators, we have the same observations as in the examples for the answerhood condition of our own approach.

3.2 Which-interrogatives

The goal of this thesis is to provide an adequate interpretation for which-interrogatives of the form "Which P are Q?". In this section, it will also be investigated whether the semantics is adequate to analyse logical answerhood for such interrogatives.

Which-interrogatives ask which objects with property P have property Q as well. In this approach interrogatives of the form "Which P are Q?" will be presented by the formula $\forall x(Px \rightarrow ?Qx)$ and they result from universal quantification over conditional interrogatives.

Moreover, in this section two alternative approaches will be presented and compared with our proposal. In the first approach, the formulation of which-interrogatives remains the same, namely $\forall x(Px \rightarrow ?Qx)$, but the interpretation is different. The conditional part of the which-interrogative is interpreted as in Hulstijn (1997). In the second approach, which-interrogatives are presented as interrogative formulas of the form $?x(Px \wedge Qx)$ and they are interpreted according to the semantics of the approach by Groenendijk & Stokhof (1984, 1990). Both these approaches are based on partition theory, whereas ours adopts the interpretation of conditional interrogatives as developed in section 2.2.2., where possible answers correspond to mutually overlapping subsets of the logical space.

3.2.1 The syntax of which-interrogatives

The syntax of the extended language is formulated by adding the clause that forms conditional interrogatives (see clause 3 of definition 6) to the clauses of definition 9.

Definition 9, syntax

Let L be the language of predicate logic. QL is the smallest set such that

1. If $\phi \in L$, then $\phi \in QL$.
2. If $\phi \in L$, then $?\phi \in QL$.
3. If $\phi \in L$ and $\psi? \in QL$ then $\phi \to \psi? \in QL$.
4. If $\phi? \in QL$ then $\forall x\phi? \in QL$.

In the new query language which-interrogatives of the form "Which P are Q?" can be syntactically formulated as $\forall x(Px \to ?Qx)$ and they result from the combination of clauses 3 and 4, i.e. quantification over conditional interrogatives. With the combination of clauses 3 and 4, interrogatives like "Which boy talked to which girl?" can be expressed with the formula $\forall x(Px \to \forall y(Qy \to ?Rxy))$.

3.2.2 The semantics of which-interrogatives

The interpretation of the new query language can be obtained by combining our semantics for conditional interrogatives and who-questions.

Definition 10, semantics

For $\phi \in L$, let $|\phi|_g \subseteq W$ be the set of worlds where ϕ holds under the assignment g. We have

1. $\langle w, v\rangle \in \|\phi\|_g$ iff $w \in |\phi|_g$ and $v \in |\phi|_g$.
2. $\langle w, v\rangle \in \|?\phi\|_g$ iff $w \in |\phi|_g \Leftrightarrow v \in |\phi|_g$.
3. $\langle w, v\rangle \in \|\phi \to \psi?\|_g$ iff $\langle w, v\rangle \in \|\phi\|_g \Rightarrow \langle w, v\rangle \in \|\psi?\|_g$
4. $\langle w, v\rangle \in \|\forall x\phi?\|_g$ iff $\forall d \in D : \langle w, v\rangle \in \|\phi?\|_{g[x/d]}$
   $g[x/d]$ is the assignment g' that is exactly like g except that it maps x to d.

We consider the interpretation of simple which-interrogatives of the form $\forall x(Px \to ?Qx)$. We have:

$\langle w, v\rangle \in \|\forall x(Px \to ?Qx)\|_g$ iff
$\forall d \in D$: $\langle w, v\rangle \in \|Px \to ?Qx\|_{g[x/d]}$ iff
$\forall d \in D$: $\langle w, v\rangle \in \|Px\|_{g[x/d]} \Rightarrow \langle w, v\rangle \in \|?Qx\|_{g[x/d]}$ iff
$\forall d \in D$: $w, v \in |Px|_{g[x/d]} \Rightarrow (w \in |Qx|_{g[x/d]} \Leftrightarrow v \in |Qx|_{g[x/d]})$

The question $\|\forall x(Px \rightarrow ?Qx)\|$ consists of all those pairs of worlds $\langle w, v\rangle$ such that for every element d in the domain of discourse D, the following holds: if d is in the denotation of the property P in both worlds w and v, then d is in the denotation of the property Q in w iff it is in the denotation of Q in v. The question $\|\forall x(Px \rightarrow ?Qx)\|$ excludes those pairs of worlds such that for some $d \in D$, d is in the denotation of P in both worlds and d is in the denotation of Q in one world but not in the other.

We consider the following simple example. Assume that the question "Which students passed the test?" is expressed by the interrogative $\forall x(Px \rightarrow ?Qx)$. Suppose also that the domain of discourse consists of only two individuals, D={Alfred, Bill}. Table 1 consists of all the possible worlds and it shows the relation between them according to the interpretation of the interrogative. The possible worlds correspond to different possible denotations of the property P, being a student and the property Q, having passed the test. The notation $ab\backslash a$ indicates the world where both Alfred and Bill are students but only Alfred passed the test. We use $+$ to show that two worlds are related and $-$ to show that they are not. The relation on W corresponding to the which-question is symmetric, that's why only half the table is filled. Notice furthermore that only nine instead of sixteen worlds are depicted. For example $-\backslash ab$, $-\backslash a$, $-\backslash b$ which express the fact that there are no students all collapse to $-\backslash-$.

	$ab\backslash ab$	$ab\backslash a$	$ab\backslash b$	$ab\backslash -$	$a\backslash a$	$a\backslash -$	$b\backslash b$	$b\backslash -$	$-\backslash -$
$ab\backslash ab$	+								
$ab\backslash a$	-	+							
$ab\backslash b$	-	-	+						
$ab\backslash -$	-	-	-	+					
$a\backslash a$	+	+	-	-	+				
$a\backslash -$	-	-	+	+	-	+			
$b\backslash b$	+	-	+	-	+	+	+		
$b\backslash -$	-	+	-	+	+	+	-	+	
$-\backslash -$	+	+	+	+	+	+	+	+	+

Table 1: All possible worlds and the relations between them. A + means that two worlds are related.

By means of table 1, we arrive at figure 5. Which-interrogatives are formulated by universal quantification over conditional questions. Since conditional questions do not correspond to a partition of the logical space, the same is expected for $\|\forall x(Px \rightarrow ?Qx)\|$. This question corresponds to a cover of W. Its alternatives are the subsets of W where all worlds are related to each other.

We see from figure 5, that the elements of the cover are four alternatives which are mutually overlapping subsets of W and exhaust the logical space. Each one of the alternatives corresponds to a proposition which, according to the analysis, is a complete semantic answer for $\|\forall x(Px \rightarrow ?Qx)\|$. The four alternatives and therefore the possible answers are as follows:

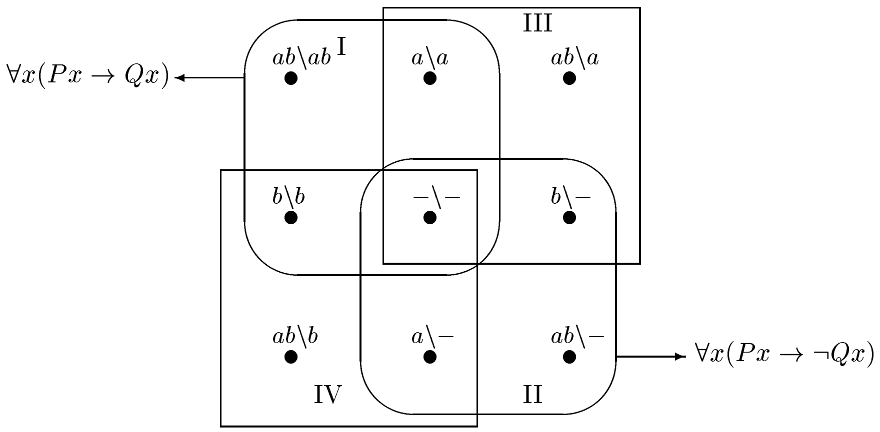


Figure 5: The interpretation of $\forall x(Px \rightarrow ?Qx)$ in the simple case where D={Alfred, Bill}

Alternative I corresponds to the answer "All students (passed the test)" expressed by the indicative $\forall x(Px \rightarrow Qx)$. Note that although the proposition $\|\forall x(Px \rightarrow Qx)\|$ informs us that all the students in the domain of discourse passed the test, it does not inform us about the particular individuals who are students who passed the test. We can get that information only if we know who the students are.

Alternative II corresponds to the answer "No students (passed the test)" expressed by the indicative $\forall x(Px \rightarrow \neg Qx)$. Note that this answer is expressed by the indicative which is the universal closure of the conditional $Px \rightarrow \neg Qx$.

Alternative III corresponds to the answer $\|\forall x(Px \rightarrow (Qx \leftrightarrow x = a))\|$ which in this simple example is equivalent to $\|(Pa \rightarrow Qa) \wedge (Pb \rightarrow \neg Qb)\|$ and expresses the fact that if both Alfred and Bill are students, Alfred is the only one who passed the test. As was the case for the first alternative, alternative III does not reveal that Alfred or Bill are students but it indicates that if there is a student who passed the test then it was Alfred.

We obtain similar facts for alternative IV which corresponds to the answer $\|\forall x(Px \rightarrow (Qx \leftrightarrow x = b))\|$ and it expresses the fact that if Bill is a student, then he is the only student who passed the test.

In natural language the question "Which students passed the test?" has four possible characteristic answers in the simple situation under discussion: "All the students", "No students", "Only Alfred (is a student who passed the test)" and "Only Bill (is a student who passed the test)". The first two answers are represented in the logical language by the indicatives $\forall x(Px \rightarrow Qx)$ and $\forall x(Px \rightarrow \neg Qx)$ respectively. These are the indicatives by which alternatives I and II are expressed. In other words, we observe that for these two answers our logical analysis corresponds very well to our intuitions.

The answer "Only Alfred" is expressed by $\forall x((Px \wedge Qx) \leftrightarrow x = a)$ and the answer "Only Bill" by $\forall x(Px \wedge Qx) \leftrightarrow x = b)$. These propositions are logically overinformative answers since they express the fact that Alfred (or Bill) *is* a student and he passed the test. The last two propositions express something stronger than the answers to which the last two alternatives correspond. This is because when the question "Which students passed the test?" is answered in natural language, one (usually) knows who the students are. In our proposal, alternatives III and IV state that if those individuals are students then they are the only students who passed the test.

The previous observation does not imply that the answers which correspond to the last two alternatives of figure 5 are inappropriate. They may express something weaker but they still considered semantic answers. After all, the indicative $\forall x((Px \wedge Qx) \leftrightarrow x = a)$ entails $\forall x(\mathrm{P}x \rightarrow (Qx \leftrightarrow x = a))$ so under the answerhood condition both propositions count as answers to the question $\|\forall x(Px \rightarrow ?Qx)\|$. Our approach accepts both propositions as answers and the stronger one provides more information than the question asks for.

We will investigate some interesting results concerning entailment between interrogatives for which-questions. We consider the following examples:

- $\forall x?Qx \models \forall x(Px \rightarrow ?Qx)$
- $\forall x(Px \rightarrow ?Qx) \models \forall x((Px \wedge Rx) \rightarrow ?Qx)$

The interrogative "Who passed the test?" entails the interrogative "Which students passed the test?" since every alternative of the first interrogative is included in one of the alternatives of the second. In other words, every complete answer to the first interrogative provides an answer to the second. According to the first clause, if we know who passed the test then we are in a position to know which students passed the test. If we have an answer in the generalised domain then we have an answer in the restricted domain as well. The same holds for the second clause. The interrogative "Which students passed the test?" entails the interrogative "Which female students passed the test?" since every complete answer to the former interrogative is a complete answer to the latter. We see that this logical property corresponds very well to one's intuitions. In natural language when it is known who passed the test, the information which students passed the test is also known and this is exactly what this logical property mirrors.

Some other logical facts concerning our approach for conditional questions are considered.

Fact 2 (Triviality)

1. $\models \forall x(Px \rightarrow ?Px)$
2. $\models \forall x(\neg Px \rightarrow ?Px)$
3. $\models \forall x(Px \rightarrow ?(x = x))$
4. $\models \forall x((x \neq x) \rightarrow ?Px)$

An interrogative like "Which men are men?" is represented by $\forall x(Px \rightarrow ?Px)$ and it is always answered by the indicative "All men" expressed in the logical language by $\forall x(Px \rightarrow Px)$. An interrogative like "Which women are men?" is expressed by $\forall x(\neg Px \rightarrow ?Px)$ So, trivial questions and their answers have a corresponding semantic object in the logical language.

3.3 Other approaches to which-questions

In what follows we will present two alternative approaches concerning the expression and interpretation of which-interrogatives. Both these presentations are based on partition theory but they give different interpretations. It will be investigated what answerhood relation both approaches give rise to and also some other logical properties.

3.3.1 The first approach

Under this alternative approach the question "Which P are Q?" is expressed by the interrogative $\forall x(Px \rightarrow ?Qx)$, like in our presentation, only this time the interpretation of the conditional part is Hulstijn's interpretation of conditional interrogatives as discussed in section 2.2.3. We obtain the following interpretation for $\forall x(Px \rightarrow ?Qx)$:

$\langle w, v\rangle \in \|\forall x(Px \rightarrow ?Qx\|_g$ iff
$\forall d \in D$: $\langle w, v\rangle \in \|(Px \rightarrow ?Qx)\|_{g[x/d]}$ iff
$\forall d \in D$: $\langle w, v\rangle \in \|?Px\|_{g[x/d]}$ & $\langle w, v\rangle \in \|\mathrm{Px}\|_{g[x/d]} \Rightarrow \langle w, v\rangle \in \|?Qx\|_{g[x/d]}$.

This interpretation differs from ours in the addition of the condition $\langle w, v\rangle \in \|?Px\|$. By this, a question like "Which students passed the test?" first asks for the individuals who are students and further it asks which of those students passed the test.

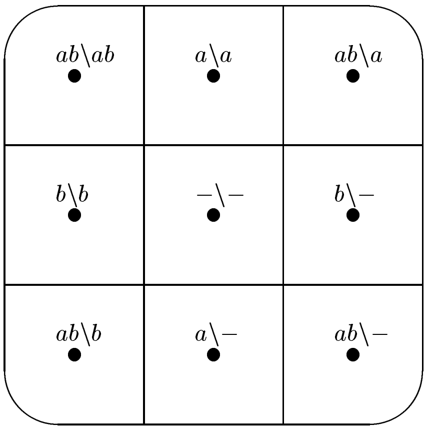


Figure 6: The interpretation of $(Px \rightarrow ?Qx)$, Hulstijn (1997)

Consider again our simple example, where the domain of discourse is $D=$ {Alfred, Bill}. The logical space consists of nine worlds[7]. The interpretation of this interrogative corresponds to the partition shown in figure 6. The partition consists of nine blocks as there are nine possible denotations of the properties P and Q. According to partition theory each block corresponds to a complete semantic answer.

Hulstijn's approach does not correspond to one's intuitions about the possible answers of which-interrogatives. First, this interpretation gives nine possible answers whereas in natural language there are only four possible characteristic answers for this particular example: "All the students", "No students", "Only Alfred (is a student who passed the test)" and "Only Bill (is a student who passed the test)". Second, none of the blocks in the partition corresponds to any of these answers. The latter answers do not say explicitly who the students are, whereas the blocks of the partition correspond to propositions which contain this kind of information. In addition we have:

- $\forall x(Px \to Qx) \not\models \forall x(Px \to ?Qx)$
- $\forall x(Px \to \neg Qx) \not\models \forall x(Px \to ?Qx)$

Propositions like "All students (passed the test)" and "No students (passed the test)", expressed by the indicatives $\forall x(Px \to Qx)$ and $\forall x(Px \to \neg Qx)$ respectively, are not considered complete semantic answers according to this approach.

There are also some other logical properties that do not hold in Hulstijn's presentation. The interrogative "Who passed the test?" does not entail the interrogative "Which students passed the test?" since not every block in the partition made by the first interrogative is included in one block of the partition made by the second. Formally $\forall x?Qx \not\models \forall x(Px \to ?Qx)$. In this case as well this approach fails to provide adequate representation of one's intuitions.

Also, there is the following.

- $\not\models \forall x(Px \to ?Px)$
- $\not\models \forall x(\neg Px \to ?Px)$
- $\not\models \forall x(Px \to ?(x = x))$
- $\not\models \forall x((x \neq x) \to ?Px)$

Questions like "Which men are men?" or "Which women are men?" are not considered trivial. In fact, under this interpretation, such expressions are equivalent to who-interrogatives. For example, the question "Which men are men?" expressed by the interrogative $\forall x(Px \to ?Px)$ boils down to $\forall x?Px$ which is the question "Who are the men?" a completely different question[8].

[7] As it was the case in our own approach some worlds are equivalent to already existing ones. For example $-\backslash ab$, $-\backslash a$, $-\backslash b$, which express the fact that there are no students, in our example all collapse to $-\backslash -$. That's why there is no need to depict all sixteen worlds

[8] For the interpretation of $\forall x(Px \to ?Px)$ we have:
$\langle w,v\rangle \in ||\forall x(Px \to ?Px)||_g$ iff
$\forall d \in D\ \langle w,v\rangle \in ||?Px||_{g[x/d]}\ \&\ \langle w,v\rangle \in ||Px||_{g[x/d]} \Rightarrow \langle w,v\rangle \in ||?Px||_{g[x/d]}$ iff
$\forall d \in D\ \langle w,v\rangle \in ||?Px||_{g[x/d]}$ iff
$\langle w,v\rangle \in ||\forall x?Px||_g$

To conclude, Hulstijn's approach does not provide a correct semantic interpretation for which-interrogatives. In addition, there are shortcomings in the analysis of logical answers and some other logical properties like entailment between interrogatives and triviality.

3.3.2 The second approach

In this case which-interrogatives like "Which P are Q?" are syntactically formulated as $?x(Px \wedge Qx)$[9]. Such interrogatives ask for the objects in the domain of discourse that have both properties P and Q. The interpretation is based on the analysis of Groenendijk & Stokhof, (1984, 1997). According to that approach, any interrogative of the form $?x(Px \wedge Qx)$ is interpreted as follows:

$\langle w, v\rangle \in \|?x(Px \wedge Qx)\|_g$ iff
$\forall d \in D$: $w \in |(Px \wedge Qx)|_{g[x/d]}$ iff $v \in |(Px \wedge Qx)|_{g[x/d]}$

The question $\|?x(Px \wedge Qx)\|$ structures the logical space in as many subsets as there are possible denotations of the complex property of being both P and Q. Each subset corresponds to a proposition which (exhaustively) specifies those objects which have both the properties P and Q. Their union exhausts the logical space. Hence the question expressed by the interrogative $?x(Px \wedge Qx)$ corresponds to a partition of the logical space which consists of as many blocks as there are possible denotations of the conjunctive property $P \wedge Q$.

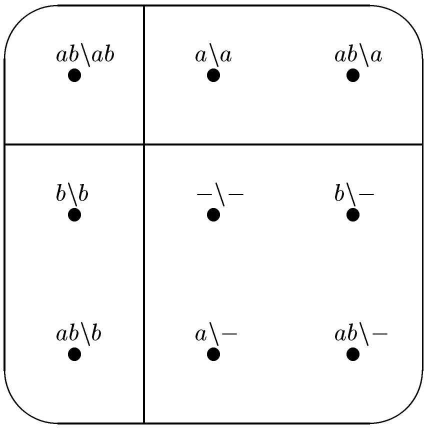


Figure 7: The interpretation of $?x(Px \wedge Qx)$ where D={Alfred, Bill} according to Groenendijk & Stokhof (1984, 1997)

Consider again our simple example with domain D={Alfred, Bill}. The question "Which students passed the test?" is expressed by the interrogative $?x(Px \wedge Qx)$. The interpretation of $?x(Px \wedge Qx)$ partitions the

[9] If a single domain of discourse is assumed and if individual constants are treated like rigid designators, then it is easy to see that the expression $?x(Px \wedge Qx)$ is equivalent to $\forall x?(Px \wedge Qx)$ of our system.

logical space consisting of nine worlds, into four blocks as shown in figure 7. Every block corresponds to a complete semantic answer.

Under this interpretation the complete answers are expressed by the indicatives $\forall x(Px \wedge Qx)$, $\forall x((Px \wedge Qx) \leftrightarrow x = a)$, $\forall x((Px \wedge Qx) \leftrightarrow x = b)$ and $\forall x(Px \rightarrow \neg Qx)$. We notice that only the last indicative corresponds to one of the characteristic answers to the question "Which students passed the test?". The other blocks correspond to answers which give more information than the question asks for. For example the proposition expressed by the indicative $\forall x(Px \wedge Qx)$ is not the answer "All the students". That indicative represents something stronger, the fact that all the objects in the domain *are* students and they passed the test.

For other logical properties one observes the following. Trivial questions such as "Which students are students?" cannot be expressed in this approach. Such interrogatives are represented in the logical language by formulas of the form $?x(Px \wedge Px)$. It is easy to show that the previous interrogative is equivalent to $?xPx$ which is the who-interrogative "Who are the students?". In connection with the previous paragraph, concerning answerhood, if $\forall x(Px \wedge Qx)$ is an answer to the question $?x(Px \wedge Qx)$ then one also accepts that $\forall x(Px \wedge Px)$ is an answer to $?x(Px \wedge Px)$ which is wrong.

Furthermore, if we consider the interrogatives "Which doctors are vegetarians?" and "Which vegetarians are doctors?" we notice that they are represented by an equivalent formula, namely $?x(\mathrm{doc}(x) \wedge \mathrm{veg}(x))$. Their interpretation corresponds to the same partition of the logical space. Therefore these two questions have the same answers. But this is wrong because in natural language these are different questions and they have different answers. For example, that all doctors are vegetarians does not mean that all vegetarians are doctors.

To conclude, the approach of Groenendijk & Stokhof is not appropriate for the analysis of which-interrogatives. Under this approach, trivial which-interrogatives and $\forall$-answers cannot be properly represented as well.

3.4 Another type of answers

Until now the only answers we considered for which-interrogatives of the form "Which P are Q?" are list-answers and the two borderline cases "All P" and "No P". List-answers are the propositions which express a list of individuals such that if they have the property P then they have the property Q. For example, for the question "Which students passed the test?" a list-answer is given by the proposition "Alfred, Bill and Mary (and only them)". The borderline answers for the same question are "All students" and "No students".

In this section we will express another type of answers which will be called descriptive. For these kind of answers it is adequate to say that they are propositions that answer the question by means of a property. An example of a descriptive answer to the previous question is "The students with a grade higher that 5."

We have the following fact:

Fact 3

Let A be a rigid property. The following holds:
$\forall x(Px \rightarrow (Ax \leftrightarrow Qx)) \models \forall x(Px \rightarrow ?Qx)$

Fact 3 states that the propositions expressed by the indicatives $\forall x(Px \rightarrow (Ax \leftrightarrow Qx))$ are answers to the interrogative $\forall x(Px \rightarrow ?Qx)$ if A is a rigid property. These propositions are complete answers since they correspond to alternatives of the logical space.

In the strict semantic sense, that A is a rigid property to some sets of worlds W means that it denotes the same set of objects in every world in W. For descriptive answers of the form $\forall x(Px \rightarrow (Ax \leftrightarrow Qx))$ this is not necessarily the case. It suffices that A is rigid or better yet, known with respect to the knowledge of the person who asks the question. List-answers are a special case of descriptive answers where the rigid property A is of the form $x = a$ or $(x = a \vee x = b)$ and so on.

In what follows we will consider some examples of which-interrogatives. We will see how they can be typically answered either by a list-answer or by a descriptive answer. Let's consider our first example

1. *Which numbers written on the blackboard are even?*;
$\forall x(\text{num}(x) \rightarrow ?\text{even}(x))$

This question can be answered by listing all the even numbers that are written on the blackboard provided one has access to the blackboard and knows what the even numbers are.

We can also reply with a descriptive answer. We consider the proposition "The numbers that can be divided by 2" expressed by $\forall x(\text{num}(x) \rightarrow (Ax \leftrightarrow \text{even}(x)))$ where A is the rigid property "dividability by 2". According to **Fact 3** we have
$\forall x(\text{num}(x) \rightarrow (Ax \leftrightarrow \text{even}(x))) \models \forall x(\text{num}(x) \rightarrow ?\text{even}(x))$

Property A is rigid in the strict semantic sense because in every world it denotes the same set of objects, i.e even numbers.

This proposition can be an answer to our question even when the person who answers the question does not have access to the blackboard. In this case, the answer can also be expressed by the proposition "(I can't see the blackboard but) the numbers that can be divided by 2". If the person who asks the question is aware of the rigid property "dividability by 2" then he is in a position to know which numbers written on the blackboard are even. This proposition provides an adequate answer also in the case where the person who asks the question does not have access to the blackboard or in the case he/she is not familiar with the notion of even numbers. In the latter case the descriptive answer implies "Find the numbers written on the blackboard which can be divided by 2. Then your question is answered.".

Things are different in the following example:

2. *Which even numbers are written on the blackboard?*;
$\forall x(\text{even}(x) \rightarrow ?\text{written}(x))$

This interrogative may seem equivalent to the previous one but it is not. A list-answer (the listing of all the even numbers written on the blackboard) for this question is an appropriate one to the extent that the person who answers has access to the blackboard and the person who asks the question knows what the even numbers are. We notice that the previous rigid property "dividability by 2" is not suitable in this case. To this question, it is difficult to form a descriptive answer such that $(\text{even}(x) \to (A(x) \leftrightarrow \text{written}(x))$ where A is rigid. Also in the case where the person who answers the question cannot see the blackboard, it is difficult to think of a descriptive answer that could give the information the question asks for.

3. *Which students passed the test?*;
$\forall x(\text{stud}(x) \to ?\text{pass}(x))$

This interrogative can be typically answered by listing all the students who actually passed the test. If the answer is expressed by $\forall x(\text{stud}(x) \to ((x = a) \leftrightarrow \text{pass}(x)))$ then one does not know whether Alfred is a student but if he is then he is the only student who passed the test. If the answer is $\forall x((\text{stud}(x) \wedge \text{pass}(x) \leftrightarrow (x = a)))$ then one knows that Alfred is the only student who passed the test.

A descriptive answer like "The students with a grade higher than 5" can be represented by the indicative $\forall x(\text{stud}(x) \to (Ax \leftrightarrow \text{pass}(x)))$ where A is the rigid property "having a grade higher than 5". This answer expresses the fact that if there is an individual who is a student, then he/she passed the test if and only if he/she has a grade higher than 5 and there is no other student who passed the test. Formally we have

$\forall x(\text{stud}(x) \to (Ax \leftrightarrow \text{pass}(x))) \models \forall x(\text{stud}(x) \to ?\text{pass}(x))$

Note that in general the property of having a grade higher than 5 is not rigid. But if the property in question is rigid with respect to the property of being a student and for the person who asks the question, the answer works completely. The property A is not necessarily rigid for the person who gives the answer. The answer is a sensible one to the extent that it is reasonable for the answerer to assume that the property is rigid for the person who asks the question. Such an answer does not reveal who the students are but this issue as well is not under the responsibility of the person who answers the question.

To conclude, in our approach we considered another type of answerhood, the descriptive answers (see **Fact 3**). From the previous examples, we observe that there is no particular way of determining which of the answers (descriptive or list-answers) are best to use. The choice depends on the question and the kind of information it asks.

4 Conclusion

The primary purpose of this paper was to provide an adequate representation for which-interrogatives. Initially we proposed a new interpretation for conditional interrogatives of the form $\phi \rightarrow \psi?$ which structured the logical space into a cover instead of a partition. The elements of the cover are alternatives instead of blocks. Compared to Hulstijn's approach for conditional interrogatives, ours offered a better insight to such questions and it gave rise to a notion of logical answerhood which is in accordance with our intuitions.

In this way we prepared the path for the analysis of which-interrogatives of the form "Which P are Q?". Our proposal was to express such interrogatives by means of quantification over conditional interrogatives namely $\forall x(Px \rightarrow ?Qx)$. The interpretation structured the logical space into a cover; its alternatives are the possible complete answers.

In our approach we dealt with the following issues:

Adequate semantic interpretation. We chose an analysis such that the logical space can be structured in a way that it gives rise to complete answers which do not give more information than it is asked for and are close enough to one's intuitions about such answers. For the interpretation of $\forall x(Px \rightarrow ?Qx)$ we sketched a simple example where the domain consisted of only two individuals. In that case the question structured the logical space in four alternatives which correspond to four possible answers, the same number of ways one could characteristically answer such a question in this simple case. Such propositions are for example $\|\forall x(Px \rightarrow Qx)\|$ or $\|\forall x(Px \rightarrow (Qx \leftrightarrow x = a))\|$. We observed that such propositions did not inform us about those individuals who have both properties P and Q but about those individuals such that if they have the property P then they have property Q.

The kind of answers that can be expressed in the logical language. We considered two types of answers that can be expressed in our system, list-answers and descriptive answers. For example, for the question "Which students passed the test?" we can obtain list answers, (i.e. the list of students who passed the test) and descriptive answers like "The students with a grade higher than 5". Borderline cases like "All students" and "No students" can also be represented by means of universal closure over the conditionals $(Px \rightarrow Qx)$ and $(\mathrm{P}x \rightarrow \neg Qx)$ respectively.

Trivial questions like "Which men are men?". In general questions like "Which men are men?" can be expressed by $\forall x(\mathrm{P}x \rightarrow ?\mathrm{P}x)$ and their interpretation is in accordance with our intuitions. Trivial questions structure the logical space in one single block indicating the fact that there is only one way to reply. In the case of the interrogative "Which men are men?" the answer is "All men" expressed by $\forall x(Px \rightarrow Px)$.

In this paper we considered two alternative approaches for which- interrogatives, one based on Hulstijn (1997) and the other on Groenendijk & Stokhof (1984). We argued that in comparison with those approaches our presentation formed better semantics.

Our proposal dealt sufficiently with the semantics of interrogatives and more specifically with trivial interrogatives, and logical answerhood. In addition, our analysis made it possible for descriptive answers to be expressed (see section 3.4). However, our proposal has a weak point. The alternatives of the question $\|\forall x(Px \rightarrow ?Qx)\|$ (excluding the borderline cases) correspond to answers that express something weaker than one would normally expect. For example, the alternatives of the question "Which students passed the test?" correspond to propositions which assert that *if* certain individuals are students then they passed the test. Normally, one would characteristically answer with a proposition which states that those individuals *are* students and that they passed the test. Further investigation may suggest a more comprehensive analysis of such interrogatives and consider a way to overcome this inadequacy.

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