A Study of William of Ockham's Logic - From *Suppositio* to Truth Conditions

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Introduction

Avant-propos

This work is the result of my attempts to combine my Philosophy background with the Mathematical Logic inclinations of the institution within which this research was developed. In fact, this twofold character is noticeable in many features thereof; I shall now outline some of them.

The project has two main purposes: the less risky one is to provide an account of William of Ockham's logical thinking, with focus on its aspects which bear a relation with the contemporary issues of intensional logics and possible-world semantics. The more risky one consists of investigating the possibilities of developing a purely extensional treatment of intensional contexts, such as tense and modalities. For the latter, some other extensional/nominalistic systems could have played the role of 'experimental sample'[1], but there seemed to be something intriguing about Ockham, as one wonders whether a philosopher from the XIVth century would have something relevant to add to our present logical issues. Moreover, he is considered to be the founder of nominalism, so the historical interest of such enterprise was self-evident - therefore, the legitimacy of the first purpose.

I shall try to comply with two very distinct kinds of expectations: those which are the desiderata for a History of Philosophy work, and those of logicians, who are interested in the formal correcteness of the system hereby presented. The criteria of excellence of these two lines are almost incompatible, and one wonders if it is not a suicidal enterprise to try to combine them. On the one hand, an Ockham scholar may be discontent with the absence of a few important aspects of Ockham's logic, since I deliberately priorize those related to contemporary logic. On the other hand, a logician may be bothered by the presence of too many 'antiquities', perhaps hindering logical clarity. So, at the risk of displeasing everybody, I nevertheless maintain that such a combination may turn out to be fruitful and informative to both sides.

Chapter 1 will display some fundamental aspects of Ockham's logic and semantics, in a rather historical approach. However, even this part is developed taking into account what I later shall want to establish as my version of an 'ockhamist system'. I consider it to be the flaw of many such reconstructions

[1] Such as Quine's, Goodman's, among others.

that they do not undertake a serious analysis of the underlying concepts; alternatively, some which did rely on such an analysis have reached very interesting results.

Chapter 2 relates some apparently less central (when compared to supposition theory, for example) issues of Ockham's theory to relevant topics of Contemporary Philosophy, such as possible worlds, designation, demonstratives etc... In this chapter I also introduce conceptual tools which I will make use of for the reconstruction undertaken in chapter 3. Finally, Chapter 3 is an attempt to provide truth conditions for quantified, modal and tense propositions[2], based on the truth of singular propositions.

I hereby hope to reach a rather broad account of Ockham's thinking, even though my main target is to build a coherent and correctly structured reconstruction of his theory of propositions.

Preliminary Remarks

I shall now draw some remarks that for the moment may seem unjustified and arbitrary. They will all be re-introduced in due course, but I believe that some simple elements will help the reader to have a better understanding of the present text right from the start.

For reasons which shall become clear (Cf. 1.2 1.3 2.3 3.2), the primitive elements of my reconstruction are propositions of the form 'This is a', while pointing at an object δ, by means of which a term is attributed to the given object. Its transcription is:

$$T\delta \circ a$$

where $T\delta$ denotes the ostension of object δ, $\circ$ the copula and a the term ascribed to δ. The truth value of these propositions is established by valuation (Cf. 3.2.1), thus it is independent of any other element of the model or of the language.

Note that such propositions differ from propositions of the form 'This a is b', which are nevertheless transcribed in a very similar way, namely:

$$Ta \circ b$$

So, for the sake of clarity, the primitive propositions will always display a Greek letter following T, which denotes an *object*, whereas in the last sentence a denotes a *term*.

[2]Henceforth, I will always use 'proposition' for what is often referred to as 'sentence', or 'statement', 'assertion' etc... It is the standard term in Latin, *propositio*, but it is slightly different from our use of the term 'proposition'. In the case of Ockham, 'proposition' is not foremost of a linguistic nature, it is rather of a logical nature, but it is not the 'abstract meaning' of a sentence - such thing does not exist. For the actual linguistic occurrences of a proposition (the tokens), I will simply use 'occurrece of a proposition'.

Also, no variables will be featured in the reconstruction, for the notion of an incomplete term is entirely alien to Ockham; every sign is a sign of something, therefore there is no such thing as a 'vacant sign' (Cf. 1.1 and 1.2).

The quantifiers $\forall$ and $\exists$ will be immediately followed by a term, which is the categorematic term they are supposed to modify (Cf. 3.4). So, a proposition like 'Every man is an animal' receives the following translation:

$$\forall m \circ a$$

Moreover, propositions are for Ockham divided in two kinds, namely categorical and hypothetical.

> A categorical proposition has a subject, predicate, and copula, and it does not include more than one such proposition. A hypothetical proposition is one which is composed of more than one categorical proposition. [Ock80][p.79]

Roughly, this corresponds to our distinction between atomic and non-atomic propositions, that is, those which do not and do feature the logical operators. So, our standard translation for universal propositions,

$$\forall x(M(x) \rightarrow A(x))$$

would be a hypothetical proposition according to Ockham (Cf. 3.4.2). I shall foremost be concerned with *categorical* propositions, but I may occasionally mention a hypothetical one.

Finally, it is obvious by now that the predicative form is **the** logical form for Ockham. I shall not use the standard notation of predicate logic, that is $P(a)$, but rather one where subject and predicate appear as two terms, and the copula is also expressed (by means of the $\circ$ operator). The motivations for this choice will also be rendered along with the philosophical considerations throughout the text.

Acknowledgements

First of all, I would like to thank my advisors, Dick de Jongh and Frank Veltman, for being so brave as to join me for this rather unusual topic. Carlos Areces provided constant technical and theoretical support and valuable friendship, both just as important for the accomplishment of my work. Ernesto Perini Santos, 'the man in Paris', was a crucial expertise on Ockham, and more than once put me on the right track. Also, many people read the numerous drafts of each chapter, and I owe each of them my gratitude.

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Chapter 1

Terms

1.1 Ontology and terms

In this section I shall briefly investigate how Ockham's conceptions on ontology underline and define his concept of term, the basic unit of his semantics. It is meant as a philosophical prologue to his logic, which will be presented more thoroughly in 1.2 and 1.3.

1.1.1 Individuals Vs. Universals

The debate about universals was lively all through the XIVth century[1], and Ockham took an important part therein. Due to the unanimous acceptance of aristotelic principles at the time[2], nobody argued in favor of the autonomous existence of Universals understood as platonic forms, but there were several degrees of realist commitment that one could argue for with regard to this issue. A most relevant exponent of the realist trend would be John Duns-Scotus, who had just died by the time Ockham started his career, and whose ideas were extremely influential at the time. In fact, Ockham uses the critique of Scotus' positions as a privileged means to expose his own, non-realistic ideas.

Some hundred years before, Thomas Aquinas maintained that a common nature belonged to individual beings by distribution, who then actually possessed a fragment of it in themselves[3]. Duns-Scotus' realism is milder, for he argues that properties are only 'formally' distinct from the singulars they inhere to.

> The most striking feature of Scotus' theory of universals, however, is his contention that the common nature of individuating principles

[1] Cf. [DL96]

[2] It is a well-established fact that, while philosophy of the early Middle Ages is deeply marked by neo-platonism, a switch took place at around 1000 or so, when 'new' Aristotle's texts were brought to Europe and translated by the Arabs in Spain and Portugal. Then on, Aristotle became know as **the** Philosopher, with a capital letter, in all medieval texts.

[3] *Or, cette nature* [common nature] *a un être double, l'un dans les singuliers, et l'autre dans l'âme.* [Aqu96] [p.93]

> are neither really distinct - in the sense of being distinct real things - nor distinct only in reason - in the sense of being thought of by means of distinct concepts. Rather Scotus holds that
>
> (T6) The nature and contracting differences are formally distinct, or not formally the same. [Ada82][p.414]

That is, a common nature actually inheres in individuals, in such a way that these cannot be actually, but only formally, brought apart - more than just through concepts. But the notion of 'formally distinct', however appealing it may seem - insofar as it represents a possibly reasonable version of realism - is just too nebulous to be considered satisfactory, and Ockham will turn it into his main target while criticizing Scotus. He holds that no such thing as a common nature really exists in different individuals: it is absurd to say that Socrates has humanity and that Plato has the same kind of 'something' in him, be it formally or in reality.

Actually, the issue is not so much about Ockham's realism, but about which entities he is willing to accept in his ontology. The fact is that he would fiercely defend the idea that nothing but individuals exist, both inside and outside the intellect. His notion of an individual is defined by the following 'test':

> That which cannot pass the test of being conceived *toto mundo destructo* is not a 'thing'(*res*). [Fun86][p.135]

On the epistemic level, it was widely accepted then, due to the predominance of aristotelian positions, that empirical singular things were the first source of knowledge. But Ockham transfers this principle also to the more fundamental level of ontology :

> What is first known by such cognition is an extra-mental thing which is not a sign; but everything outside the mind is singular; therefore, etc. From *Quodlibeta*, VI, q.vi, in [Ock57][p.28]
>
> Therefore, I say there is no such thing that is universal and intrinsic to the things it is common to. From *Ordinatio*, d.2, q.4, in [Spa94][p.132]

That is, the so-called common natures do not convey essence to individuals by means of some kind of individuation process; individuals are what they are integrally, and are not the compound of parts of something common to others, some of which would correspond to their essence, and others to their accidents. Ockham appeals again to the criterion of annihilation for identifying possibly real things: if something can be pictured in a world where nothing else exists but itself, then it is a real individual, hence a real thing. Now, so-called common properties inherent to individuals cannot exist as singulars and as common to many at the time, thus they cannot count as entities of the ontology[4] .

[4] The subject of Ockham's mereology is far more complex than what was analyzed in these lines, so I only hope not to be making any serious mistake except for that of superficiality.

> "Forms" and "natures" cannot be attributed with any ontological status; a "form", even if it existed only as a divine idea, could be annihilated while its presence in that which it informs is conserved. [Fun86][p.139]

This claim, which was later to be defined as a main axiom of (some versions of) nominalism, is of crucial relevance for our purposes in this work, for it indicates Ockham's denial of any kind of classes. Accordingly, Plato and Socrates do not share 'something' in common, namely their humanity, that would justify their membership in the same class defined by a property. This will later lead to my decision not to express Ockham's truth-conditions of propositions in set-theoretical terms, as is common in the literature[5].

If not based on the real or 'formal' state of individuals, how can Ockham account for generality? Or else, does he deny that some terms are predicable of many, and alternatively, maintains that, in the propositions 'Socrates is a man' and 'Plato is a man', the term 'man' could very well be replaced by two distinct terms? Well, Ockham does not deny the existence of universals as **mental concepts**. As such, they are also individuals, but with no extra-mental existence, neither as autonomous entities nor as something present in individuals. A universal thus signifies many things, while still being a singular.

> Therefore, it ought to be said that every universal is one particular thing and that it is not a universal except in its signification, in its signifying many things. [Ock74][p.78]

Hence, Ockham allocates the existence of Universals to the conceptual-linguistic level. Plato and Socrates do not share a common nature in fact, but they both are referred to (to be specified later) by the term 'man', be it a mental term or a spoken or written one (Cf. 1.1.3). That is the reason why Marylin Adams maintains that Ockham's nominalism is also a conceptualism (Cf. [Ada82]).

> ¿From these remarks it is clear that the universal is an intention of the soul capable of being predicated of many. [Ock74][p.81]

To sum up, the entities Ockham is willing to accept in his ontology are all singulars; extra-mental entities, which are of two kinds, things and written and spoken terms, and mental entities, which are concepts, or mental terms. The latter may be concepts of things, concepts of words and concepts which have no bearing in the extra-mental realm of the ontology: universals, as much as species and genus, are of the latter kind. It should be spelled out how mental terms which do have an extra-mental bearing relate to it, in the next section.

[5]It may very well be that both approaches are intertranslatable; in any case the set-theoretical reconstructions of Ockham seem to me formally correct. It is actually with regard to the underlying concepts that I tend to disagree with them. This is also Spade's position, Cf. [Spa96][p.268]

However, one last thing about Universals: they are defined as 'an intention of the soul which can be predicated of many'. But insofar as predication is also a property of written and spoken words, these can also be considered as a subordinated, or conventional, form of Universals (the relation between mental and non-mental terms is to be specified in section 1.1.3).

> ... there are two kinds of universals. Some things are universal by nature... The intention of the soul, of course, is a universal by nature... Other things are universals by convention. [Ock74][p.78]

This is nothing but a brief sketch of Ockham's considerations about Universals, but I hope to have outlined at least two important features, which will be fundamental for my reconstruction: (1) Individuals are the only entities of his ontology; (2) Their division into 'species' and 'genus' - in our terms, into 'sets' or 'classes' - is merely a conceptual-linguistic procedure which has no bearing in their state of being.

1.1.2 How mental terms are formed

It should now be investigated how intentions of the soul can be universals by nature; this is so because of the way mental terms are formed. Given that Ockham lived in 'aristotelian times', no innate or platonic conception of mental terms would possibly be acceptable, and he obviously adheres to a rather empiricist version of them. The most intuitive explanation for the formation of general mental terms, very close to the aristotelian account [6], is that the intellect makes abstraction of the sensitive perception of several individuals sharing a common nature, and thus forms the concept of this common nature. Ockham's account will not follow these lines.

It is of course true for Ockham that any cognition starts with the sensitive apprehension of individuals - what is in conformity with the individual-only ontology presented above. This is in any case the position defended by most authors of the later Middle Ages.

> I maintain that this cognition which is simple, proper to a singular thing, and the first to be acquired, is an intuitive cognition. From *Quodlibeta*, VI, q.vi, in [Ock57][p.28]

However, this intuitive cognition informs only about the existence of the perceived being; Ockham does not make the step to claim that the being's 'nature' or 'essence' can be so apprehended[7]. Intuitive cognition only informs the intellect about some existence, whereas it is by abstractive cognition that mental terms are formed - on the other hand, no contingent truth can be known by abstraction (whether something really is the case outside the soul).

[6] Even though the problematic of mind and its contents is not exactly in Aristotle.

[7] In the Prologue to the *Ordinatio* (in [Ock57], he describes in details the several acts of the intellect, which I shall disregard as it is not so much related to the present purposes.

> Intuitive cognition of a thing is cognition that enables us to know whether the thing exists or does not exist. From prologue to *Ordinatio*, in [Ock57][p.23]

The two kinds of cognition of a thing occur together, so one is led to wonder what point there is in distinguishing them. Well, if common features of individuals could actually be intuitively apprehended, then this would imply that there are indeed the awkward common natures individualized in many, which is plainly false for Ockham. Therefore, the establishment of so called ‘common traits’ between individuals is transferred from the intuitive level to the abstractive one.

But this abstractive cognition is not the ‘inductive’ process of forming a concept only after the apprehension of several singular entities of a certain kind. For Ockham, it is enough for the intellect to encounter one single ‘sample’ of a so-called kind to form the corresponding mental term. So, it is not by abstraction from many that the mental term ’man’ comes to existence in the intellect (which again would lead to undesirable ontological presuppositions with regard to the real existence of a common nature); rather, the apprehension of one single man produces what we could call a ‘print’ in the intellect. In early texts Ockham viewed the relation between mental signs and the thing having produced them in terms of similarity:

> The intellect, seeing a thing outside the mind, forms in the mind a picture resembling it... From *Ordinatio*, D.II, q.viii, in [Ock57][p.41]

He uses the metaphor of a house-builder to explain how a general mental term produced by the perception of an individual will relate to other individuals: it is enough for a house-builder to see one house to produce in him the concept of it. Subsequently, he is able to build other houses in conformity with this concept. However, this metaphor relies heavily on similarity to account for the relation between mental terms and things. He would later (in the *Quodlibeta*, for instance) refine this concept and abandon the notion of resemblance in favor of an analysis of mental terms as **signs**, as shown by Biard in [Bia89][Chap.2][8], but in any case the fact that the apprehension of one single individual is enough to provoke a mental term in the intellect remains; again, the mental term produced by the apprehension of one individual will also be the sign of other individuals (but it is not necessary to posit something common in reality between the individuals which relate to the same mental sign).

> I say that we have a cognition proper to one singular thing, not on account of a greater likeness to one than to another, but because this intuitive cognition is naturally caused only by the one and not by the other, and cannot be caused by the other. ¿From *Quodlibeta*, VI, q.vi, in [Ock57][p.30]

[8] According to other commentators, however, Ockham did not abandon similarity as a criterion for the applicability of a concept.

Instead of resemblance, it is causality that will account for the tight link between a thing and a mental term. That is, the singular entity is the *cause* of the mental term, the latter is formed as a print produced by the former in the intellect, hence the mental term stands for the thing in the intellect because it was caused by it. This account does bear similarities with the modern causal theory of names in that, in both cases, a relation of causality establishes the relation between a thing and a sign of it (its name or the corresponding mental term).

> En revanche, le rapport entre l'objet conçu et le concept est fréquement presenté comme une relation de causalité... le fondement de toute connaissance est le contact direct de l'esprit avec un objet individuel. [Bia89][p.62]

Therefore, some mental terms are entities which, because of the way they are formed, are signs of other entities, namely extra-mental things, and hence lead to the corresponding extra-mental entity. The notion of 'sign' can be traced as far back as Augustine, but Ockham introduces a significant change in it which will actually result in the redefinition of the semiotic triangle between things, terms and words. For Augustine, a sign is only what has a sensitive existence, that it, an extra-mental thing, therefore mental terms could never be signs - actually, it is rather the case that something else (words) are signs of concepts.

But because mental terms bear a causal relation to things, they are indeed signs of these things. The significant change is thus to consider mental terms also as signs, and more generally to view thought as a kind of language, different from spoken language only insofar as it is natural, whereas spoken language is conventional.[9] Ockham defines his mentalistic concept of sign right in the first chapter of the *Summa Logicae*:

> A sign is anything which brings something to mind and can supposit for that thing. [Ock74][p.50]

The reformulation of the concept of sign, and the previous considerations on the status of mental terms, result in a particular account of the relation between (spoken) language and thought, which shall be analyzed in the next section.

1.1.3 Mental and Conventional terms

As indicated above, the semiotic relation between words, things and concepts that Ockham ascribes to is very different from that of his predecessors (and also from the one modern theories most often assume). The aristotelian-boecian scheme (in Biard's terms) goes as follows: words signify concepts, which in their turn 'represent' or 'correspond' to extra-mental things. That is, words relate only in an indirect, mediated way to things; concepts are the significata

[9] There are interesting historical studies about the loss of importance of spoken and written language in Ockham's philosophy, when compared to some of his predecessors, such as the parisian grammarians, for example. Cf. [Jol87].

of words and bear some relation to things (the nature of such relation varies from author to author, and is often quite unclear).

> It is only through this twofold conceptual structure that our simple words are related to what they ultimately signify, the natures, forms, or properties of things existing in reality. [Kli93][p.35]

Now, we know that Ockham does not ascribe to such a view on concepts: mental terms are natural signs (natural insofar as naturally produced) of things, therefore they signify, or replace, things in the intellect. It remains to be established the place occupied by spoken and written terms in his scheme. Words do not signify mental terms, for the semiotic status of mental terms has been modified: they signify, instead of being signified.

> Le concept est un signe; il n'est pas l'objet de la signification du mot. [Bia89][p.61]

Ockham insists on the natural status of mental terms (causally produced) as opposed to the conventional status of written and spoken terms. Unlike mental terms, spoken and written terms only signify by convention, or institution, that is, by means of an intentional decision - and therefore their signification can be altered also by institution, which is plainly impossible for mental terms.

> To sum up: The mind's own intellectual acts are called states of mind. By their nature they stand for the actual things outside the mind or for other things in the mind, just as spoken words stand for them by convention. From *Exposition super librum Perihermenias*, in [Ock57][p.44]

His claim is that the relation between corresponding mental and written or spoken terms is of subordination, which is clearly related to the fact that mental terms signify naturally and written or spoken terms do not. That is, there is indeed some kind of dependence of written or spoken terms with respect to mental terms, as claimed by Ockham's predecessors, but not as a sign/significatum relation. One evidence of mental terms' primary status with regard to written or spoken terms is that, if the signification of a mental term (which for now can be expressed as its extension) changes, then the subordinated written or spoken term's signification will accompany the change without previous deliberation.

Ockham provides himself a very clear explanation of the relation between mental and written or spoken (henceforth *conventional*) terms:

> I say vocal words are signs subordinated to mental concepts or contents. By this I do not mean that if the word 'sign' is taken in its proper meaning, spoken words are properly and primarily signs of mental concepts; I rather mean that words are applied in order to signify the very same things which are signified by mental concepts. Hence the concept signifies something primarily and naturally, whilst

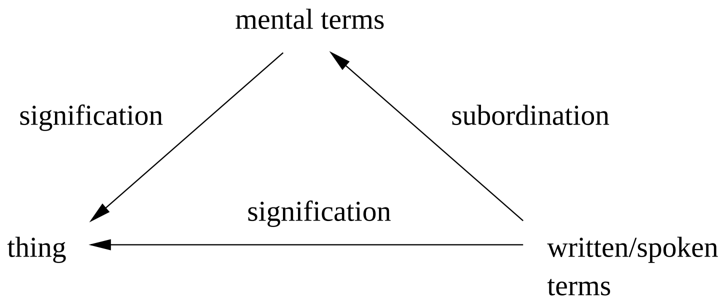

Figure 1.1: Semiotic Triangle.

> the word signifies the same thing secondarily. From *Summa Logicae*, ch.1, in [Ock57][p.48][10]

Now that the relation between mental and conventional terms has been spelled out, another feature of Ockham's semantics emerges, which in fact shall be of major relevance throughout my reconstruction: concepts are signs of things, but words are not signs of concepts. Therefore, concepts will no longer play the mediator role in the allegedly twofold relation between words and things. Rather, what mental and conventional terms signify are entities of the same sort, namely extra-mental entities, more simply described as 'things'. This is displayed in in figure 1.1[11].

The fact that a relation of **direct reference** holds between words and things will be the object of 1.2, but I hope to have shown by now that this claim is closely related to some basic features of his thinking, namely ontological assumptions and the notion of mental terms understood as signs. Henceforth, we shall no longer be very much concerned with mental terms (except when explaining simple supposition, in 1.3.2), but it is important to note that many commentators[12] see Ockham's account of mental terms as one of the most relevant contributions of his philosophy. If, in my reconstruction, they do not occupy a prominent place, this should not be interpreted as a misevaluation of their importance.

Before we leave mental terms aside, there remains one point to be discussed. Some cartesian, modern minds may be wondering how we can be sure that mental terms actually have the 'right' content to be the sign of things in the intellect. This is the modern representation/correspondence problem, and we still feel significant echoes of it in the wide-spread refusal of all mental content-based

[10]For each quotation, I will cite what I consider to be its best translation to my knowledge. In this case, Boehner's translation is better than Loux's.

[11]This scheme bears striking similarities with Kaplan's picture comparing his notion of direct reference with the standard fregean scheme, in [D.77][p.5]

[12]Cf. C. Panaccio, who dedicates the entire second chapter of his book, [Pan92], to this topic.

knowledge in modern philosophy[13]. It basically boils down to the following question: how can I be sure that my concept of A is the same as your concept of A?

But this issue is totally alien to Ockham's philosophy, and actually to all medieval philosophy, as it is a typical modern philosophy (philosophy of the subject) problem. We should content ourselves with his account of mental terms as natural prints left by things in the intellect, which are therefore the same in all intellects.

> Les signes naturels sont des actes ou des dispositions de l'esprit qui sont identiques chez tous les êtres humains. [Bia89][p.61]

Let us now move to the notions of *significatio* and subsequently of *suppositio*, the two key concepts in medieval semantics.

1.2 *Significatio*

As depicted above, the ockhamist concept of signification is much different from the widely-accepted idea that conventional words signify concepts and meanings are abstract entities. Not only does Ockham differ from his predecessors, but also from the present-time standard model, to a large extent based on Frege's notions of *Sinn* and *Bedeutung*. Basically, Ockham sustains that the signification of a word **is** its extension, hence there is no distinction between meaning and reference. One of the consequences of such view is that words refer **directly** to things; this idea shall be of crucial importance throughout this work.

Now, signification is a property of terms *as such* in the framework of medieval semantics. That is, the signification of a term is a property which it has independently of its role in particular propositions; therefore, the relation of signification between a word and the thing it names does not suffer variations according to the context[14], it is somehow a fixed property of the term[15]. Ockham will endorse this view on terms, and will combine it with his extensional version of meaning to develop his own semantic principles. The intuition behind my interpretation of Ockham's semantics is that terms[16] behave as *tags* which apply to things: thus, a term will signify all those items which 'display' this particular 'label'. Note that each object receives multiple tags, namely all those which can be truly predicated of it by means of a demonstrative.

> Le schéma sémiotique aristotelico-boécien subit quelques distorsions. En realité, Guillaume d'Ockham prend place dans la tradition oxonienne de la seconde moitié du XIIIè siècle qui faisait de la chose, et non du concept, l'objet de la signification vocale. [Bia89][p.69]

[13]Wittgenstein and Merleau-Ponty are only but two examples of this trend.

[14]Such variations, we shall see, belong to the realm of *suppositio*.

[15]How fixed this property is, or how much variation it can bear overtime and in different (non-propositional) contexts is a topic of discussion among medieval logicians.

[16]Henceforth, the word 'term' will always refer to what was described above as 'conventional term', unless otherwise stated.

That is, this feature of Ockham's semantics owes a great deal to the Oxford tradition, mainly to Roger Bacon. In fact, most of Ockham's ideas are not of his own creation, but are actually his interpretation of widely-spread concepts and terms of his time - such as *significatio* and *suppositio*. His uniqueness rests rather on the perspicacious way of articulating them all.

1.2.1 Signifying individuals

Being given the ontological and semantic assumptions previously mentioned, it is quite clear that, for Ockham, terms signify things, that is, individuals, since every extra-mental[17] entity is singular. That proper names signify individuals uniquely is not so much of a controversial thesis, even though there are alternative accounts to it. But Ockham holds that other terms signify things very much in the same direct way as proper names signify individuals, the only difference being that most terms display multiple denotation, unlike proper names. Again, the metaphor of terms applying to things as labels comes in handily.

Let us start with terms which have unique denotation. They denote individuals, that is, to the aristotelian 'first substances': 'Socrates' denotes Socrates. For Ockham, not only do they denote such entities, but they also signify them, since they stand for them in discourse[18]; therefore, according to the Oxford principle to which Ockham fully adheres, proper names have individuals as their significata. This can be rendered by the following definition, making use of the nuclear propositions $T\delta \circ a$:

Definition 1.2.1 A term S is a proper name iff it is meant to signify a unique object δ in the domain[19] and $T\delta \circ S$ is true[20].

I have to add the clause 'is meant to signify', instead of only 'signifies', because a common term may also signify only one object, if it happens to be so that only one object signified by this term exists.

What about terms that refer to the so-called 'second substances', that is, to the properties of individuals? At this point, Ockham really distances himself from the usual account of the meaning of general terms. For sure, general terms do not signify their corresponding concept, as this claim has been completely ruled out by the previous considerations. Moreover, according to him, a general term like 'man' does not signify individualized properties in things either, in this particular case the humanity present in all men, as it does for the so called *via antiqua*[21] semantics. In this frame of reference,

> General terms ultimately signify those individualised properties of things that the concept immediately signified by the term represents in a universal, abstract manner in the mind. [Kli93][p.37]

[17] And as a matter of fact, also mental entities are singular.
[18] Here we may draw an interesting comparison with DRT.
[19] As will be specified in 2.1.3.
[20] As was explained in the Preliminary Remarks.
[21] Cf. [Kli93]

But there are no such things as component parts of individuals which are 'individualized' from general properties for Ockham. Now, another possible account for general terms, very much in conformity with extensional principles, would be to claim that the term 'man' signifies the set of all men. But again, that is not what such a term signifies according to Ockham; 'man' signifies every single man, that is, every single object of which the term 'man' can be predicated - and not the set formed by these objects.

> For this noun 'man' does not signify primarily some common nature to all men, as is fancied erroneously by many, but signifies primarily all particular men... For the one who first instituted this noun 'man', seeing some particular man, instituted it to signify that man and whatever substance of the same sort as him. From *Opera Philosophica*, cit. in [Kli93][p.39]

This concept of signifying bears an obvious connection with the way (mental and non-mental) terms are formed, as is explicit in the quotation above. It is clearly very similar to the institution of proper names, the difference being that the denotation (which, in Ockham's semantics, equals signification) of general terms shall not be unique but rather multiple[22]. Hence, general terms also behave like names in that they apply directly to individuals, in fact to many (assuming a model where more than one individual bearing a common term exists, that is, a model in which such terms accomplish their potentiality for multiple denotation).

> Nothing is signified by 'man' except men, and not one man more than another. [Ock74][p.114]

We can thus provide the following definition:

Definition 1.2.2 A common term a signifies an object δ in the domain iff $T\delta \circ a$ is true.

Notice that no ontological commitment concerning the actual 'nature' of objects is made, nor does it have to be made, because generality takes place only on the conceptual/linguistic level. That is, the fact that a collection of individuals are all said to be 'man' does not authorize the inference that they posses 'similar natures'. Hence, a definition which would rely on the real state of things - for instance *a signifies all those things which* **are** *a* - would be committing a serious fallacy. That is why I use propositions of the form 'This is a' - which are in any case the primitive elements of my reconstruction (to be argued for in Chapter 3) - in the definitions: thus, we maintain ourselves within strictly linguistic grounds, but get as close as it gets to the objects.

However, one may be wondering whether it is legitimate at all for terms to have multiple denotation. This seems indeed a bit unusual to our fregean frame of reference, where a general term P denotes a (unique) set composed of all elements whose corresponding simple term a make the sentence $P(a)$ true.

[22] Cf. [Pan92][p.227-30]

> Occam n'admet pas le principe frégeen que Carnap appelle 'principe d'univocité', selon lequel un nom n'est jamais le nom que d'une seule entité. [Pan92][p.26]

Multiple denotation is the price Ockham has to pay to safeguard generality in the context of an economic ontology, strictly composed of individuals. How high this price will turn out to be is still to be established.

1.2.2 Absolute and Connotative terms

Among general, or common, terms, Ockham also makes a distinction between what he calls 'absolute' and 'connotative' names. This distinction roughly corresponds respectively to terms that signify things, such as 'animal' or 'man', and terms which, in a more realist frame of reference, signify properties, such as 'white', or 'good'.

One would expect those two kinds of terms to display different semantic properties, with or without the underlying ontological assumption that they evoke distinct kinds of entities, namely substances or properties. But this is not the case, we shall see that both absolute and connotative terms are of the same 'type' for Ockham, using an anachronistic term, because they signify the same entities, that is, individuals (how could it be any different?).

> But from a semantic point of view, terms connoting such real qualities do not differ essentially from those that would only connote substances: these terms signify what they can supposit for in propositions, namely the subjects of these qualities, in relation to the qualities of which they are subjects. [Kli93][p.40]

That is to say: an absolute term such as 'man' signifies, as previously shown, all particular men. Now, as far as connotative terms go, Ockham maintains a rather striking feature of his semantic system: they also signify individuals, namely those of which they can be truly predicated - or those they can supposit for, as Ockham often says[23]. Hence, the connotative term 'white', Ockham's favorite example, signifies primarily everything it can be predicated of, that is, individuals which, pursuing my metaphor, bear the tag 'white'; the same term signifies secondarily the whiteness in those individuals[24], but it cannot be predicated of whiteness. (I shall further explain the notions of primary and secondary signification just below.)

[23]For the moment, I prefer not to use the notion of supposition - the topic of 1.3 below - to define signification, as it hasn't yet been introduced - although Ockham himself uses this notion before introducing it, probably because it was a rather commonsensical notion at the time. I would rather use the expression 'can be predicated of', as it is implied in the primitive element of my reconstruction, that is, propositions of the form 'This is a'.

[24]Ockham admits qualities inherent to substances in his ontology, for they are also individuals. Thus, the whiteness in Socrates (a substance) is an individual, distinct from the whiteness in Plato - and there is no such thing as a universal whiteness.

Therefore, there is no need to make any addition to Def. 1.2.2 above, for connotative names behave exactly like absolute names as far as (primary) signification goes. For this very reason, they shall be treated equally in the syntax I propose, that is, small letters stand for both connotative and absolute terms.

Where then lies the difference between connotative and absolute terms? Ockham differentiates them by means of the notion of primary/secondary signification. Primary signification is what we have been considering so far as signification *tout court* - that is, in modern terms, the extension of a term - while secondary signification associates terms to things they cannot be predicated of, but to which they are 'obliquely'[25] related.

> Purely absolute terms are those which do not signify something principally and another thing (or the same thing) secondarily. [Ock74][p.69]

Thus, an absolute term like 'man' does not have a secondary signification. Its abstract correspondent, 'humanity', is according to Ockham, its synonym (Cf. [Ock74][ch.5-9]). On the other hand, a connotative term like 'white', signifies secondarily whiteness, and the (absolute) term 'whiteness' does **not** signify primarily white things - in fact, the proposition 'Whiteness is white' is, according to Ockham, plainly false. But the notion of secondary signification is quite broad; for instance, the term 'blind' signifies secondarily sight, as its opposite. (We shall not be strictly interested in those aspects of secondary signification.)

> A connotative name, on the other hand, is one that signifies one thing primarily and another thing secondarily. Connotative names have what is, in the strict sense, called a nominal definition. [Ock74][p.70]

The issue of definitions, more specifically of nominal definitions[26], will be treated in more detail in Chapter 2, but for the moment it is important to mention what the nominal definition of a connotative term would be. It is always the case that what a connotative term signifies secondarily is present in its nominal definition. Thus, the nominal definition of 'white' is 'something informed with whiteness' (Cf. [Ock74][p.70]), and indeed, 'white' signifies secondarily whiteness.

Still, one may be led to wonder why the distinction between absolute and connotative terms is important at all. To start with, I chose to mention it here for the sake of historical fidelity, although it will play no role in the reconstruction. The question is then why Ockham mentions it himself, as one of his contributions is to show exactly that, under his ontological assumptions, absolute and connotative terms signify (primarily) the same kind of entities.

However, the notion of secondary signification renders it possible to deal with nuances of terms that the strictly extensional treatment could not cope

[25] Spade argues, in [Spa96][pp.213-222] that one possible account for secondary signification - which, according to him, is in fact very problematic - is that a (connotative) term signifies secondarily all those things which the terms present in its nominal definition, in *oblique* cases (i.e., not in the nominative case), signify primarily.

[26] Nominal definitions are expressions which can replace the defined term in a proposition.

with. It is somehow comparable, on the level of terms, to what presuppositions are on the level of propositions, that is, what goes beyond a proposition's truth conditions - in this case, what goes beyond primary signification. In fact, Ockham's theory of connotation is complex and extremely relevant in the framework of his system, since it treats numerous semantic phenomena without postulating unnecessary entities. For example, relational terms are treated in terms of their connotation ('pater' signifies fathers, and connotes their children), so that it is not necessary to admit relational entities, such as fatherhood, in the ontology. Another example: 'animal having a heart' and 'animal having kidneys' are not equivalent, as their connotation is not equivalent - one connotes hearts, whereas the other connotes kidneys -, even though their signification (extension) is (presumably) the same, as has been empirically established.

Summing up, the semantic relation of connotation is an essential counterpart of Ockham's economic ontology and extensional definition of signification. Again, I will have to disregard a topic of major importance in Ockham's philosophy, but as connotation does not play a significant role in the truth conditions of propositions (it is foremost a term *qua* term property), it is of lesser relevance in the present reconstruction.[27]

One last remark: it is often discussed by commentators whether it would be possible to provide nominal definitions for all connotative terms, consisting only of absolute terms, and hence to eliminate connotative terms of the list of primitive semantic elements (for the sake of simplicity). Regardless the relevance of such project, it is now rather agreed that it cannot be done (Cf. [Pan92][ch.4]), and that some primitive connotative terms have to be kept in a recursive reconstruction of the lexicon.

1.2.3 Type-lowering

As a consequence of his strictly extensional semantics, all general terms denote for Ockham the same kind of entities, namely individuals[28]. So, all such terms would be of the same 'type', to use an anachronistic term, because they have the same semantic behavior.

In fact, when compared to other semantic systems - we shall here briefly examine what Klima [Kli93] calls the *via antiqua* semantics (that is, the semantics of Ockham's predecessors, such as Thomas Aquinas) and modern type theory -, Ockham's treatment of general terms provokes a type-lowering. That is to say, in Ockham's framework, general terms are of a simpler type than in the above mentioned semantic systems.

According to Klima, in the *via antiqua*, the signification of a general term like 'white' was defined as follows: when applied to a substance, such as Socrates, it would yield the whiteness in Socrates. That is, the signification of 'white' would be, for each individual, their whiteness; the same would hold for other

[27] For a more complete account of Ockham's theory of connotation, see [Pan92][p.241] and [Spa96][Chap.7-B].

[28] The same holds for discrete terms as well - terms with unique denotation -, but this is not the point at the present moment.

general terms, they would signify the correspondent individualized property in individuals.

Alternatively, for Ockham, as we know, a general term signifies all the things it can be truly predicated of. So, if Socrates is white, then the signification of 'white' with respect to Socrates is Socrates himself, and not his whiteness. Thus, general terms signify for Ockham simpler entities than they signify for the so-called *via antiqua* semantics, that is, the individual in question. Notice that Ockham does accept in his ontology the existence of individualized properties (Cf. 1.1.1), but they are not signified by the corresponding general (connotative) term. For Ockham, the whiteness in Socrates is only signified by the term 'whiteness', as are the whitenesses of all other white men indistinctly (but not the common nature whiteness, as it does not exist).

When briefly compared to modern type theory applied to linguistics, Ockham's semantics also represents a type-lowering. On the one hand, insofar as it treats both absolute and connotative terms as of the same type, type theory bears some resemblance to Ockham's account, if compared to the so-called *via antiqua* semantics. In type theory, unlike in the *via antiqua* semantics, all common terms T would be interpreted as the function which yields the truth value 'True' to the elements of the set defined by the common terms (in Ockahm's terms, the individuals of which the term can be predicated), and otherwise 'False'.[29]

Thus, a general term T according to type theory is of type:

$$TYPE(T) = < e, t >,$$

where T is any one-place first-order predicate - Ockham's 'common terms'.

However, for Ockham we would have:

$$TYPE(T) = < e >,$$

just the same as for individual expressions which have unique denotation. Again, we encounter a symmetry between so-called definite terms and general terms: they behave similarly, as they both are of type e, and differ only with regard to unique or multiple denotation. In modern type-theory, on the other hand, proper names and general terms belong to different types altogether, since they are semantically interpreted as two different kinds of entities, namely singular entities and sets (or their characteristic function).

Hence, Ockham's semantics represents a type-lowering for general terms also when compared to modern type-theory. This may be a striking feature of his semantics - most appealing or most repelling, depending on one's perspective.[30]

[29] Or alternatively, a common term is interpreted more simply as just the set defined by it, since

> Sets and their characteristic functions really amount to the same thing. [Gam91][p.83]

[30] One very interesting challenge would be to define a type theory-like modeling of Ockham's semantics.

1.3 *Suppositio*

As much as *significatio* is the property of terms *qua* terms, *suppositio* is the property of terms in a proposition. I stated above that the signification of a term is steady insofar as a term signifies all which it can be predicated of by means of a demonstrative. The supposition of a term, however, varies according to the propositional context, depending on syntactical features such as the presence of syncategorematic terms, but also on semantic and pragmatic aspects of the proposition.

It was rather unanimous among medieval logicians that signification concerned isolated terms and supposition concerned terms in propositions; the notion of supposition is not a creation of Ockham. Some of his predecessors had also developed a very complex theory of supposition, among whom Peter of Spain should be mentioned. But again, Ockham provides his own account of supposition, in particular by abandoning the notion of 'natural supposition' (Cf. [Bia89]).

> Guillaume d'Ockham ... fait du rapport de substitution entre le mot et la chose le caractère foundateur de la supposition. [Bia89][p.76]

Now, the relation between a term and a thing is direct: terms do not represent or mean things in a proposition, they actually **replace** them. Hence, words behave again as signs of things, which reinforces my claim that it is better to view terms as names of things, also on the propositional level.

> Supposition is said to be a sort of taking the place of another. [Ock74][p.189]

In this section, we shall investigate the main aspects of Ockham's supposition theory, namely what it means for a term to supposit for something, the three kinds of supposition and its twofold nature as a theory of reference and a theory of inference.

> Supposition-theory is best viewed as two theories: the theory of supposition proper and the theory of 'descent to singulars'. [Spa82][p.192]

1.3.1 Replacing Individuals

At the risk of being repetitive, we must note that, given Ockham's ontology, the non-linguistic counterparts of a linguistic sign - a term - must be individuals. Hence, if supposition is the relation of substitution of something by a term in a proposition, those entities being substituted can only be individuals. That is why supposition is rightly approximated to the contemporary notion of reference;

> The theory of supposition proper is a theory of reference. [Spa82][p.192]

Now, we know that reference is the chief property of terms for Ockham, whose semantics is highly extensional. In a proposition, terms refer directly to things inasmuch as they replace them; in which sense is this different from the direct relation between terms and what they signify, as explained above? The point is that, due to the propositional context, a term may replace one, some, all or none of its significata, whereas such variation does not occur within the realm of signification. That is, the term 'man' considered as a term signifies all men indistinctly; the same term 'man' in a proposition may replace Socrates, Plato, both or none. Supposition-theory is concerned with clarifying the reference of terms - what they supposit for - in each proposition.

This aspect is obviously connected to the fact that all terms, except proper names, have multiple denotation: so one term may supposit for many different individuals, very likely those which bear the corresponding label (in the case of personal supposition), but not necessarily those. It is thus important to establish what is exactly referred to by a term in a proposition.

However, one of the most interesting features of Ockham's account of supposition is that both the subject **and** the predicate supposit, that is, replace and refer to individuals. The standard account of predication (both that of contemporary predicate logic and that of other medieval philosophers such as Thomas Aquinas) holds that, in a true atomic proposition, the subject term refers to an entity which belongs to the class (set, species, genus) denoted by the predicate. For Ockham, alternatively, predication is not about membership or inherence, but about identity.

> Thus, as I use the term both subject and predicate supposit; and, generally, whatever can be a subject or a predicate of a proposition supposits. [Ock74][p.188]

Just as absolute and connotative terms behave similarly on the level of terms, as far as their semantic properties go, so do subject and predicate on the level of propositions - this is of course not surprising, as the standard predicate proposition has either a proper name as subject and an absolute term as predicate, an absolute term as subject and a connotative term as predicate, or a proper name as subject and a connotative term as predicate. Therefore, just as in the term level, there is a symmetry between subject and predicate also in the propositional level. Not only subject terms name - refer, supposit for - things, predicate terms behave the same way: they do not attribute qualities - or membership to a set - to the subject, they attribute another name to it[31].

It is thus all the more crucial to be able to establish the reference of terms in propositions, since reference concerns both subject and predicate, that is, they both supposit for something - therefore the central role of the notion of supposition in Ockham's semantics. All common terms have multiple denotation,

[31]Indeed, the truth conditions of a simple predication are that a proposition is true iff both subject and predicate supposit for the same thing - which in our interpretation would be equivalent to say that both subject and predicate terms are names that apply to the same individual(s). A more detailed account of truth-conditions of such propositions will be provided in Chap.3.

and only acquire univocity within the proposition[32], which is one of the main claims of the Oxford logicians who preceded Ockham;

> They maintain that no term is univocal but only in context. [DL82][p.177]

Ideally, supposition theory would provide a set of procedures which would make it possible to establish in all cases what terms are suppositing for in a given proposition. To start with, it must be said that a term will most likely supposit for something it can be predicated of by means of a demonstrative, in a proposition of the form 'This is a'.

> A term, at least when it is significatively taken, never supposits in any proposition for a thing unless it can be truly predicated of that thing. [Ock74][p.189]

That is why, according to Karger, the subject term specifies a restricted domain, namely the domain of entities of which the subject-term can be predicated [Kar76][p.23]. But this claim raises problems when the term is **not** significatively taken, and also seems to miss the primary intuition behind Ockham's theory, namely that terms substitute, or supposit, for things, that is, they name individual things instead of establishing sub-sets of entities. Moreover, it seems important to insist on the symmetry between subject and predicate, a feature which is also lost in this account.

Further on, all of supposition theory will be concerned with the various types of supposition; in fact, the definition and identification of such types are the tools to establish what a term supposits for in a given context. In some cases some elements, such as the syncategorematic terms that we call 'quantifiers', provide enough information to clarify the truth conditions of a proposition; in some others, it must be spelled out whether the term is replacing a thing, a concept or a word (see 1.3.2). In others, though, only pragmatic elements will provide enough hints with regard to the supposition of a term, for instance, what is being pointed at by the speaker. That is, there is a strong component of intentionality in Ockham's account (Cf. 2.2.2).

> In the same way, an equivocal term can, in any proposition, supposit for any one of its significata, unless those who use it limit it to one particular significatum. [Ock74][p.192]

We must thus bear in mind that Ockham's theory of supposition is not a full theory as we understand the term, because it does not provide a complete theory of reference for natural language which would be able to disambiguate all cases.

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[32]This brings up a complicated issue about terms as types and terms as tokens. What would seem to be the case for Ockham is that only tokens, in a proposition, would supposit for something. Cf. [Pan92][p.25]

> Appliquée au discours reel, la procedure n'est pas totalement effective puisqu'elle laisse subsister certaines ambivalences systematiques. [Pan92][p.43]

However, what Ockham's supposition theory does show is the wide variety of ways a term may supposit and of entities of different kinds that it may supposit for in a proposition, and that is what we shall examine henceforth.

1.3.2 Types of *suppositio*

Ockham distinguishes three kinds of supposition, namely personal, material and simple supposition. These three categories are again not of his own creation, but were rather common notions among supposition theorists: roughly speaking, a term has personal supposition in a proposition when it replaces its significatum, simple supposition when it replaces the mental term that also signfies its significatum, and material supposition when it replaces the corresponding written or spoken word. Still, as integrated in Ockham's semantics, they receive a new, nominalistic treatment. The most important feature of his theory of supposition is that signification is therein associated to personal supposition, and not to simple supposition, as it was for Thomas Aquinas.

> So we can say that for Aquinas a term has simple supposition when it refers to what it (immediately) signifies, while it has personal supposition when it refers to what falls under its significate. [Kli93][p.38]

Now, the distinction between signification and what falls under it - clearly, a distinction between sense and reference - will of course be entirely alien to Ockham's theory. It was stated above that it did not exist on the term level; it will not be any different on the propositional level. He does retain the rough scheme according to which personal supposition occurs when terms replace things, simple supposition occurs when terms replace concepts and material supposition occurs when terms replace written or spoken words. However, as to be expected, since terms primarily signify things, signification will pertain to personal supposition.

> Universally, personal supposition occurs when a term supposits for the thing it signifies, whether that thing be an entity outside the soul, a spoken word, an intention of the soul, a written word, or any other thing imaginable. [Ock74][p.190]

Thus, the scheme previously described must be refined - Ockham himself warns the reader against the mistake of ascribing to terms having personal supposition the property of suppositing for persons (Cf.[Ock74][p.191]). It is indeed the case that, when suppositing personally, a term is more likely to supposit for extra-mental things, but only because many terms do have extra-mental significata. However, this is not always the case, as many terms only refer to intentions of the soul and have no counterpart in the extra-mental world. It is so for all terms which signify universals, since universals are only mental terms. For instance, the term 'species' does not signify anything but an intention of the soul, therefore it has personal supposition in a proposition such as

$$\text{Man is a species} \tag{1.1}$$

even though it supposits for a concept. But this concept is its primary signification, hence 'species' has personal supposition in this proposition.

We must then introduce into the ontology mental terms which do not signify extra-mental things, either because these do not exist (such as chimaeras and beings alike) or because they exist exclusively in the soul by their very nature, such as universal terms like genus or species. This raises the question on how these mental terms come into existence, given that all mental terms are formed by the intuitive *notitia* of extra-mental things (Cf. 1.1.2). I shall not go into great detail on this issue, however significant it is. But it can be hinted that they are formed in the intellect by abstraction and combination of other mental terms which are indeed formed the 'canonical' way: the mental term 'pegasus' is the combination of the mental term 'horse' with the mental terms 'flying' and 'wings'[33].

Let us now examine the case of terms whose signification is a written or spoken term, and not a thing[34], as for instance the term 'noun' in the proposition

$$\text{Man is a noun} \tag{1.2}$$

'Noun' has therein personal supposition, since its signification *is* a word.

So, there is a certain priority attributed to personal supposition, for, as it is related to signification, it is the most natural one. For a term to have supposition other than personal, this is somehow 'forced' upon it by the speaker.

> Dans toute proposition, un terme peut être pris en supposition personnelle, sauf si l'utilisateur en décide autrement. [Bia89][p.81]

That is why, in a medieval disputation, it was always crucial to stipulate in what sense a certain term was being used - for this is central to the establishment of its truth conditions - but this could only be outlined by the question: 'Which kind of supposition does this term have in this particular proposition?'[35]. The only way to undertake such stipulation is by means of the speaker's intention[36].

[33] I very much regret not being able to deepen this topic of great importance, but its connection with the main subject is only indirect.

[34] Although it does seem intuitive to attribute the same ontological status to the so-called 'real things' and to written or spoken terms, as they both have an extra-mental existence.

[35] There again we have the type/token problem: propositions of the same type can have different suppositions, and thus different truth values, in each of their tokens.

[36] This may sound as rather disappointing to the modern reader, who may see an undesirable trait of pragmatics in Ockham's theory. But there are no markers for the different kinds of supposition in the language, thus there is no other way of spelling it out.

Hence, in propositions (1.1) and (1.2) above, if the term ‘Man’ has personal supposition, and thus is suppositing for one, some or all individual men, both propositions are plainly false, since no man is a species and no man is a noun, properly speaking. From this point of view, a strictly extensional semantics like Ockham’s seems to be doomed to failure, for it cannot account for the truth of such intuitive propositions such as these.

Therefore, the notions of material and simple supposition are necessary conditions for the success of Ockham’s extensionalist enterprise - otherwise, the expressive power of the language would be limited to certain entities and would not account for the reference to concepts and to words. In (1.2), for instance, if the term ‘Man’ has material supposition, then the proposition is true, otherwise not.

> It should be noted that any term which can in any way be a part of a proposition can exhibit material supposition, for every such terms can be the extreme of a proposition and supposit for a spoken or written word. [Ock74][p.197]

That is, material supposition is very classically defined as the supposition for a written or spoken term. This is the case for propositions such as ‘Man has three letters’, ‘Socrates is a proper name’ etc..., where the subject terms supposit for the corresponding words, or ‘strings’, as some contemporary theories name them.

More importantly, Ockham defines the notion of simple supposition as the kind of supposition a term has when it supposits not for something it signifies, but for the mental term that naturally signifies what the term conventionally signifies. This is the case of the term ‘Man’ in (1.1): if it supposits not for one, some or all men, but for the mental term ‘Man’, of which ’species’ can be truly predicated, then the proposition is true, otherwise not.

> Any terms ... can exhibit simple supposition; for every such term, whether mental, vocal, or written, can supposit for a concept of the mind. [Ock74][p.198]

However, although any term can have simple or material supposition,

> A term cannot have simple or material supposition in any proposition, but only in propositions where it is coupled with an extreme which refers to an intention of the soul, a spoken word, or a written word.[37] [Ock74][p. 192]

This clause is related to the truth conditions of propositions, which shall be investigated further on, but for now it is important to outline on the one hand the priority of personal supposition (which a term can have in any proposition) with regard to simple and material supposition - of course, this is an echo on the

[37] It must be said that I am not entirely satisfied with the translation of this quotation, in particular with the terms ‘coupled’ and ‘refers’.

propositional level of how signification is defined for terms. On the other hand, the introduction of simple and material supposition enriches Ockham's semantics tremendously, otherwise it would be restricted to extra-mental entities, and would exclude words and concepts which have no extra-mental bearing - for a nominalist, it is the ontology that must be economic, not the language (it is not a coincidence that there are many more mental terms than things outside the soul).

> Ces deux nouvelles notions, celle de supposition simple surtout, permettent d'interpreter de façon nominaliste de nombreuses propositions apparemment rebarbatives. [Pan92][p.40]

Let us finally provide explicit definitions for the three kinds of supposition:

Definition 1.3.1 A term has personal supposition in a proposition iff it supposits for something that it primarily signifies and can be truly predicated of.

Definition 1.3.2 A term has simple supposition in a proposition iff it supposits for the mental term which also signifies what the term primarily signifies.

Definition 1.3.3 A term has material supposition in a proposition iff it supposits for itself, that is, the written or spoken term.

1.3.3 Types of personal *suppositio*

There is some disagreement among scholars about the place occupied by Ockham's theory of descent to singulars: some[38] hold that it does provide truth-conditions for quantified propositions, the familiar A, E, I and O forms of the aristotelian traditional square of oppositions. Others claim that it is not a theory of truth conditions, but rather a theory of inference[39].

It would seem that the descent to singulars is better viewed as a theory of inference. However, it is still not clear what its purpose was, for it is in a way quite awkward to handle and to place in our contemporary frame of reference.

> One possibility is that it was meant to provide a technique for checking inferences. [Spa82][p.195]

In any case, there are two aspects of this theory which are worth mentioning, namely that it makes explicit the crucial role played by singular propositions of the form 'This is a', and that it presents an interesting and rather unusual account of quantifiers, of a semantic rather than of a syntactic nature.

> According to Ockham, it is only non-logical or categorematic terms that can play genuinely referring roles, so that generality of reference must ultimately be secured by the categorematic terms embedded in general propositions rather than by expressions like 'some' and 'every' which introduce them. [Lou74][p.23]

[38] Cf. [Lou74], [Kar76]
[39] Cf. [Fre80], [Spa82]

That is, since the key concept for terms in propositions is their supposition, generality is not conveyed by means of a syntactic operator that widens or narrows the scope of a term over the domain - which is the common view on quantifiers.

> They do not signify at all, but only determine truth conditions. [Spa82][p.190]

It is rather the whole term, formed by a categorematic term and a syncategorematic one, the so-called 'quantifier', that has the semantical property of suppositing for something. The question is then what it supposits for, or else which of the categorematic term's significata is being replaced.

A first division is between discrete and common supposition: a term has discrete supposition if it is meant to supposit for one or some specific individual. This is the case of proper names, but also of demonstrative pronouns, including those in the plural form. Alternatively, a term has common supposition if it supposits or can supposit for some indistinct individuals, usualy many [40].

There are three kinds of personal common supposition - as opposed to personal discrete supposition, when the suppositing term is a proper name or a demonstrative pronoun taken significatively. Personal common supposition is thus the supposition of general terms, with multiple denotation. In principle, as outlined above, a common term can supposit for any of its significata, unless otherwise indicated; now, the addition of syncategorematic terms plays the role of 'otherwise indicated'.

Personal common supposition is divided into common confused supposition and common determinate supposition. Determinate supposition is thus defined:

> Thus, the following rule can be employed: whenever it is possible to descend to the particulars *[singularia]*[41] under a general term by way of a disjunctive proposition[42] and whenever it is possible to infer such a proposition from a particular *[singulari]*, the term in question has personal determinate supposition. [Ock74][p.200]

Terms with determinate supposition in a proposition are usually of the form 'A man', that is, introduced by an indefinite article in English - however, it displays no article at all in Latin, but only the categorematic term itself, for

[40] However, if there is only one horse, then the term 'horse' supposits for this one individual, but it has common supposition.

[41] The translation of *singularia* into 'particular' is not very felicitous because, in Part II of the *Summa*, particular propositions are what we call 'existential propositions', and not singular propositions.

[42] Ockham defines elsewhere the truth conditions of disjunctive and conjunctive propositions:

> For the truth of a copulative proposition, it is required that each part be true. In *Summa Logicae* II, chap. xxxi. [Ock57][p.80]

> For the truth of a disjunctive proposition it is necessary that one or another part be true. In *Summa Logicae* II, chap. xxxi. [Ock57][p.81]

example 'Homo'. Propositions whose subject are of this form are called in Part II of the *Summa* indefinite propositions, and its truth conditions are similar - actually identical, in the case of personal supposition, that is, the present case - to the truth conditions of the so-called particular propositions, that is, those whose subject-term has the syncategorematic term 'aliquod', which roughly corresponds to our existential quantification.

Therefore, it seems that we are licensed to assume that terms which have determinate supposition in a proposition are, in our translation, of the form:

$$a \circ b \text{ or } \exists a \circ b$$

(We are for now considering the supposition of the subject-terms only.)

Take $\exists m \circ a$ as the transcription of 'Some man is an animal'. So, the descent to singulars would be thus formulated:

Definition 1.3.4 $\exists m \circ a \Longleftrightarrow (Tm1 \circ a) \vee (Tm2 \circ a) \vee \ldots$

where Tm1, Tm2 ... supposit for each individual man.

According to Ockham in Chap.70 of [Ock74], this equivalence holds indeed in both directions, which will not occur for all cases of descent to singulars.

Common confused supposition is in its turn subdivided in confused and distributive supposition and merely confused supposition. Let us first examine confused and distributive supposition: it occurs for subject-terms frequently accompanied by the syncategorematic term 'omnis', in English 'every'; therefore it seems correct to assume the following translation of subject-terms with confused and distributive supposition:

$$\forall a \circ b$$

Ockham provides the following definition of this kind of supposition:

> Confused and distributive supposition occurs when, assuming that the relevant term has many items contained under it, it is possible in some way to descend by a conjunctive proposition and impossible to infer the original proposition from any of the elements in the conjunction. [Ock74][p.201]

That is, it seems that it is possible to descend from the proposition with confused and distributive to a conjunction of singular propositions, but not to ascend from the conjunction to the original proposition. Thus:

Definition 1.3.5 $\forall m \circ a \Longrightarrow (Tm1 \circ a) \wedge (Tm2 \circ a) \wedge \ldots$

However, Karger claims that the other direction of the equivalence can be found in the Part III of the *Summa*, in III-3, 32 (Cf. [Kar76][p.35]), where Ockham deals with induction. In any case, this is less relevant for us now, as I do not intend to establish truth conditions of such propositions based on Ockham's theory of descent and ascent.

Thirdly, the merely confused supposition is less intuitive for us because it would correspond to quantifying on the predicate, at first sight a rather awkward procedure. However, there is actually nothing astonishing about it, considering the symmetry between subject and predicate with regard to supposition. Thus, in a proposition such as

$$\text{Every man is an animal} \tag{1.3}$$

the predicate 'an animal' has merely confused supposition, which means that each man is one of the many animals. So it is not valid to infer from this proposition that

$$(\forall m \circ Ta1) \vee (\forall m \circ Ta2) \vee \dots$$

which would be the case if the predicate 'an animal' had confused and distributive supposition. Alternatively, the following proposition is validly inferred from (1.3):

$$\forall m \circ (Ta1 \vee Ta2 \vee \dots)$$

> The consequent here is a categorical proposition composed of the subject 'man' and the predicate 'this animal or that animal or ... '. It is clear that this predicate is truly predicated of every man and that, consequently, the universal proposition is true. [Ock74][p.201]

We shall refrain from providing a formal transcription of merely confused supposition, for it would demand the introduction of a new operator (similar to the disjunction, but for terms in a categorical proposition) and lead to some other technical difficulties. However, the reader is referred to [Lou74], where Loux provides an elegant formalization of all descents and of all four forms of the square of oppositions based on these three kinds of personal supposition.

A final remark: these definitions are not constituent definitions of my reconstruction, for they do not contain its primitive element, namely singular propositions of the form 'This [pointing at something] is a'. Propositions of the form 'This a is b' are also singular propositions according to Ockham, but they actually correspond to a conjunction of two primitive propositions, namely $T\delta \circ a$ and $T\delta \circ b$.

Hence, the importance of Ockham's theory of descent to singulars is quite restricted in the frame of this reconstruction, among other reasons because I shall not be dealing with inferences and entailments at all. However, it is an interesting aspect of Part I of the *Summa*, which was worth being mentioned.

1.4 Conclusion of Chapter 1

Chapter 1 aimed at outlining that Ockham's semantic theory is strongly embedded in a certain conception of ontology and knowledge, which we can call nominalist. His general strategy is to keep the ontology as economical as possible, strictly composed of individuals, and to allocate to the linguistic level (in the broad sense, including 'mental language') the multiple kinds of entities that some believe exist in reality. Ockham claims that the structure of language does not correspond to the structure of being, and that many features that seem to pertain to the ontological realm are nothing but distinctions of names[43].

Thus, the complexity of his semantics is inversely proportional to the complexity of his ontological assumptions, for he is to account for all kinds of logical facts based on semantical, and not ontological, concepts.

Now, there is one important point that I have disregarded so far, but which is crucial for any reconstruction of Ockham's logic: which notion is to be prior, the notion of *significatio* or the notion of *suppositio*? Differently put, which level should be taken as primitive, the level of terms or the level of propositions? This is an ongoing issue in the literature about Ockham, and there seem to be equally many supporters of both views.

On the one hand, commentators such as Loux [Lou74] and Biard [Bia89] view in Ockham a resemblance with the contemporary tendency of considering the proposition as the most primitive logical and linguistic entity[44]. In this sense, there would be something astonishingly modern about Ockham, as these authors claim. On the other hand, virtually all logicians and philosophers up to Kant considered the term as the most primitive logical element, and there are reasons to believe that it isn't any different for Ockham. So, for historical reasons, many commentators just assumed that Ockham priorized terms, whereas others, such as Panaccio in [Pan92], deliberately deny the 'propositional proposal' maintained by some.

However, when it comes to establishing which of the two notions one considers to be prior, whether signification (the term level) or supposition (the propositional level), one is bound to encounter difficulties. Ockham often states that a term signifies what it can supposit for in a proposition, indicating an priority of propositions with respect to terms. However, the main type of supposition, namely personal supposition, is defined in terms of signification - it occurs when a term replaces what it primarily signifies. There would seem to be a dangerous circularity in such definitions, and hence in Ockham's system altogether.

To escape from this difficulty, I chose neither signification nor supposition as prior, but *predication*, which seems to me to be the true primitive notion behind signification and supposition. This is very much in conformity with Ockham himself, as in fact both signification and supposition can be defined in terms of predication: a term signifies (in the primary sense of signification) everything it can be truly predicated of, and it can supposit for any of the things it can be truly predicated of (personal supposition).

[43] Quine would add that we are actually conveyed to believe in the existence of such features and entities by the use of our very 'platonic' language, which allows an undue multiplication of beings.

[44] Contemporary examples of such tendency are numerous, and range from Wittgenstein to Davidson, from Merleau-Ponty to linguists. Cf. [Pan92][ch.4].

It has been stated previously, but not argued for, that the primitives of my reconstruction are propositions of the kind 'This is a', while pointing at an object $\delta \in$ D, which we transcribe as

$$T\delta \circ a$$

Note that these are very different from propositions of the form 'This a is b', which are the singulars of the descent theory analyzed in 1.3.3.

The point is that propositions like 'This is a', pointing at something, are actually 'in between' propositions and terms; they have the lowest degree of propositionality insofar as they consist of combining a term, 'a', with a thing, which is pointed at by the demonstrative. That is why I often use the metaphor of terms as tags applied to objects. Hence, whether the predication of a term to a thing belongs to the term realm or to the proposition realm is not a straightforward issue, and maybe a not so important one.

The choice of propositions of the form $T\delta \circ a$ as primitive is not incontroversial. For instance, it seems to clash with what Geach calls *Buridan's Law*:

> The reference of an expression E must be specifiable in some way that does not involve first determining whether the proposition in which E occurs is true. [Gea62][p.xi]

Clearly, I do rely on the truth of $T\delta \circ a$ propositions to specify the reference of a. But there lies indeed the danger of circularity in Ockham's system: either the reference of a is defined, or the truth value of $T\delta \circ a$ is defined. Again, the way out for this dilemma is, I think, to view $T\delta \circ a$ propositions as 'semi-propositions', in which truth and reference are actually confounded, since, as argued, they are in between the term level and the propositional level.

This topic will be explored further in chapter 2, but for now it suffices to point out that the issue is not so much about an exclusive choice between a proposition or a term perspective, but rather that Ockham's account establishes an intriguing middle perspective, where terms are directly said of things.

Chapter 2

Problems

In this chapter I try to identify in Ockham's texts echoes of some of the currently discussed topics in philosophy of logic and philosophy of language, and to establish positions on them that are coherent with his thinking. In the course of this I will set up some tools which will be relevant for the definitions of truth conditions in Chapter 3.

2.1 Puzzles of Signification and Supposition

In the previous chapter, I defined the notions of signification and supposition, the key notions of Ockham's semantics, but only treated some simple propositions, which may have led the reader to suspect that Ockham's system can only deal with straightforward and unproblematic cases. Thus, the present section is intended to show that the same apparatus can be used to handle also more complex cases.

2.1.1 Definitions

Since Ockham emphasizes first and foremost the referential aspect of language, it is quite natural to wonder how it would be possible within his framework to perform the crucial operation of referring to other fragments of language *within the language*. That is, if terms bear a primary relation to things, how is one to establish definitions? How is it possible to state that two expressions of the language are equivalent in an informative way? Of course, Ockham admits the phenomenon of synonymity, which is thus defined[1]:

> Expressions are synonymous which simply signify the same thing. [Ock74][p58]

[1]This definition raises the problem of term that have the same signification, that is, the same extension, but nevertheless are not equivalent. Again the notion of connotation comes into play, since by means of their non-equivalent connotation (their secondary signification), terms with the same primary signification can be differentiated.

In fact, expressions are synonymous if they are subordinated to the same mental term (Cf. [Spa96][p.209]). But the point here concerns more directly the problem of definitions. This topic has already been briefly treated in the section about absolute and connotative names (1.2.2), but it deserves further attention.

> But 'definition' has two senses. In one sense of the word we speak of essential definitions (definitions expressing what a thing is) and, in another sense we speak of nominal definitions (definitions expressing what a word means). [Ock74][p.105]

Considering essential and nominal definitions, only the latter is of a purely linguistic nature. Now, it is of course possible to provide an essential definition or a description (in the case of accidental features) of something; however, what is thus referred to is the salient *thing*, and not the term that it is signified by. Hence, the proposition 'Man is a rational animal' is true, as men are indeed rational animals, but this proposition does not provide a definition of the *term* 'Man'.

> Indeed, strictly speaking, absolute terms do not have nominal definitions, for a name with a nominal definition has one such definition. ... in the case of purely absolute terms the meaning of the name can be expressed by different sentences whose constitutive terms do not signify the same thing. [Ock74][p.70]

Therefore, all descriptions and definitions of what an absolute term is and the term itself are not convertible. As a matter of fact, not even the conjunction of all descriptions and the essential definition could replace on the linguistic level the term itself, even though their extension may be equivalent - but each of these expressions is subordinated to a different mental term.

> But, of course, since each definition, description and descriptive definition is an expression, none is identical with the definitum or descriptum; they do, however, signify the same thing. [Ock74][p.109]

The only case in which a term and its definition are interchangeable, or convertible, is the case of nominal definitions. But as we know, only some terms have a nominal definition, namely connotative names.

> Connotative names have what is, in the strict sense, called a nominal definition. [Ock74][p.70]

So, every connotative term has a nominal definition which perfectly corresponds to it not only on the level of their signification (i.e., their extension), but also within the language. So, the difference spotted in chapter 1 between absolute and connotative terms is not so much of a semantic nature (since they both signify the same entities), but rather of a syntactical one, which concerns the possibility of replacing a term by its (nominal) definition *salva veritate*.

For example, consider the beaten case of 'white': any occurrence of 'white' in the language could be replaced by its nominal definition, namely 'something informed by whiteness', so that the new proposition would be strictly equivalent to the former. Note that 'something informed by whiteness' is *not* indeed an essential definition of white, it really is what the term 'white' means; alternatively, the expression 'rational animal' can truly be predicated of all men, but does not define the term 'man' - as a matter of fact, there is no definition for the term 'man', for it is an absolute one.

One may then argue that this view on definitions strongly restricts the expressive power of language: it is not possible to provide definitions of most terms. This is of course related to Ockham's refusal to express the meaning of terms by intensions, for intensions and definitions can get dangerously similar.

Nevertheless, again supposition theory will fulfill its task of making up for Ockham's economical ontology. That is, absolute terms can acquire definitions if taken as the correspondent strings, or, in Ockham's terms, if taken in material supposition. The quotation below makes use of such device:

> 'Chimaera is an animal composed of a goat and a cow', for example, is not true; it carries the false implication that something actually is composed of a goat and a cow. Nevertheless, the proposition in which the terms supposit materially ("Chimaera" and "animal composed of goat and cow" signify the same thing) is true. [Ock74][p.107]

True enough, this is a special case, for the reason why the proposition in question is false is its vacuous subject. In this particular case, the situation is a bit more complicated because 'chimera' is a 'figment-term', that is, a term for an impossible object (which therefore obviously has vacuous supposition in this world).

> Ockham explicitly tells us (*Summa logicae* II.14) that all figment-terms are connotative. Hence, they will all have nominal definitions. [Spa96][p.211]

These are details which I shall not deal with for the moment. The point of the quotation above is to indicate that terms having material supposition may be a possible way to establish within the language what absolute terms are. That is, while the description of what a term means cannot really correspond to the term, but actually to the significatum of the term, the strings corresponding to both term and its definition - both taken in material supposition - may be mutually convertible.

The next natural step - since, while approaching definitions, we were mostly dealing with expressions composed of more than one term - is to explore how Ockham accounts for the signification and supposition of compound expressions, such as complex terms ('white man') and relational expressions.

2.1.2 Compound terms

Since Ockham is, in Geach's terms, 'a resolute champion of the two-name theory' [Gea62][p.35][2], it is legitimate to wonder how he can accomodate propositions with more than strictly two words (plus the copula) in his semantical system. In particular, the case of compound terms - which grammarians call now a 'noun phrase' - and, most of all, the case of relational terms seem to be intriguing. We shall now examine both cases, and try to sketch what an ockhamist 'type theory' would be, to account for how he deals with such expressions.

So far, we have virtually seen nothing but simple two-name propositions, subject and predicate, connected by a copula, as these are the unproblematic cases. But Ockham's logic can also handle more complex terms. Now, the first problem that presents itself concerns the definition of 'term': what a term is is precisely defined with regard to its ability to be the extreme of a proposition.

> In one sense a term is anything which can be the copula or the extreme (i.e., subject or predicate) of a categorical proposition. ... In this sense even a proposition can be a term since it can be a part of a proposition. [Ock74][p.51]

In the proposition "'Socrates is white" is true', the subject is indeed another proposition, hence a proposition can also be a term. Similarly, any word (including syncategorematic words) can be a term, if it has material supposition such as 'Every' in "'Every" has five letters'.

The problem with this approach is that it assumes the well-formedness of propositions to define what a term is (namely, anything which can be an extreme thereof), which hinders the recursive reconstruction of the system. So, if the only criterion of what a term is is its ability to feature in well-formed sentences, it seems impossible to define its semantical nature *as a term*, and hence to provide a procedure to differentiate terms of non-terms - and therefore, *define* well-formed propositions, instead of taking this notion as a primitive. Even though I am aware of this difficulty (which did not seem to be one to Ockham, as his ideas of rigor may have been different), I will assume that there are virtually no constraints on what can be a term, and infelicities are more likely plainly false propositions.

Assume then two basic types of logical entities[3]:

t for simple terms

p for propositions

and one derived type:

---

[2]That is, the idea that all categorical propositions can be transformed into propositions with two terms connected by the copula.

[3]My use of type theory tools here is very modest, and it aims only at clarifying some features of Ockham's sentential parsing.

t* for expressions which can count as terms[4]

Notice that T, the set composed of terms of type t, is a subset of T*, the set of composed of terms of type t*. Indeed, the difference between t and t* is of a syntactic nature (the number of elements in the string), for, as far as semantics goes, they both signify and supposit in the same way - they refer to entities, be it things or mental concepts.

The copula ∘ (with or without modal and tense operators) is then of type:

$$<< t*, t* >, p >$$

Thus, let us content ourselves with the definition of a complex term t* as what is or can be the extreme of a proposition - 'extreme' being understood as the two 'sides' of the copula. One first consequence of this definition of the copula ∘ is that the parsing of a proposition always proceeds by identifying two major constituents therein, the two extremes as a whole, and not their individual parts.

> The expression 'extreme of a proposition' excludes a part of an extreme, even if the part be a categorematic term and a noun. Thus, in 'The white man is an animal', neither 'man' nor 'white' supposit. The whole extreme supposits. [Ock74][p.199]

In this proposition, the whole so-called noun phrase 'The white man'[5] is one extreme thereof, and not only 'man'. As Ockham adds himself, as far as supposition goes, it is the whole compound term that supposits, and not each of its elements, which would yield some kind of strange compositional operation between two suppositions. Naturally, both 'man' and 'white' have distinct *signification*; in fact it is on the level of the signification that the compositional operation takes place, defining thus the signification of the compound term 'white man' - namely, all white men. The supposition of this term will then bear a relation to the signification of the compound term - in this case, 'white man' supposits for some white man.

Moreover, this categorical proposition would be equivalent to the hypothetical proposition[6] 'A man is white and he is an animal', but this formulation would raise anaphorical problems which are not meant to be dealt with here. In any case, it would obviously not be equivalent to the conjunction of two propositions having the same predicate 'animal', and both of them having one

[4] Which can be the extreme of a proposition, as previously discussed. In particular, propositions can also be of type t*, most frequently under the form of that clauses - in Latin, the 'dictum' of a proposition. Cf. 3.6.2

[5] A problem presents itself to the translator, since there are no articles in Latin, so that a term like 'homo albus' is neither definite or indefinite - actually, the question is not even raised. Cf. [Gea62][p.ix].

[6] Ockham provides an extensive analysis of categorical propositions which are equivalent to hypothetical ones, among which are both the case of compound and relative terms [Ock80][ch.11]. But we shall not go into details on the topic, for it actually pertains to the entailment and inference realm, with which I shall not be dealing, as previously stated.

of the parts of the original subject as subject. Alternatively, in the case of a compound term featuring as the predicate of a proposition, such as 'Socrates is a white man', such propositions are equivalent to the simple conjunction of two propositions having the same subject and each one of the parts of the original predicate as predicate. Most importantly, extremes of propositions supposit as a whole, and not each of their parts (if any).

> At any rate, what is a part of an extreme in a proposition does not supposit in that proposition, although it can supposit in some other proposition. [Ock74][p.199]

What seems to be an innocent remark (that terms can be parts of compound terms but also self-standing terms in other propositions) may turn out to be problematic for a type theory-like analysis of Ockham's semantics, as it may challenge the establishment of generative rules. There are no such things as 'adjectives'; rather, some terms such as 'white' can be, in grammar nomenclature, an adjective in some propositions, and a noun in others. This is of course a consequence of how Ockham collapses absolute and connotative terms in one single semantical category (Cf.1.2.2 1.2.3). Hence, all (categorematic) non-compound terms are of type t. When two simple terms (such as 'white' and 'man') are parts of a compound term, their composition yields a term of type t*.

Let us move to the case of relational terms. One of the big myths of the analytical tradition is that, before Frege, it was impossible to account for them, that is, for two-place predicates. (This would be the case of aristotelian and, in particular, medieval logic.) Even though the amplitude of Frege's contribution must not be disregarded, this is simply not true, even if the pre-fregean treatments of relational terms do not feature the same power and generality (Cf. [Som82][Chap.7]). In the case of Ockham, he treats relational terms as 'pater' also within the framework of his two-name theory of propositions.

> 'Adam est père de Cain' apparaît, dans cette analyse, comme constituée d'un sujet ('Adam'), de la copule ('est') et d'un predicat ('père-de-Cain'). [Pan92][p.49]

So, very simply stated, the above quoted sentence is true if 'Adam' and 'the father of Cain' both supposit for the same thing (which is the case). The term 'Cain' has no supposition whatsoever therein, it only contributes to the signification of the compound term 'the father of Cain', and hence to the (personal) supposition of the latter. That is, instead of considering terms like 'father of' as two-place predicates, Ockham's account of such terms can be modeled by means of two successive one-place functions yielding a proposition[7]. From a modern point of view, this procedure is quite familiar in that it is similar to the well known 'skolemization property', that is, the fact that any two-place function

[7] It is not actually accurate to use the nomenclature 'function' to account for Ockham's proposal, as so far I have been doing nothing but arguing that his predicates are **not** functions. Nevertheless, the reader is invited to take this formulation in a 'heuristic way'.

can be converted in the composition of two one-place functions, and also very similar to how most recent formalizations of natural language treat relational terms[8].

Thus, we can ascribe the following type to expressions like 'father of', or 'taller than', but also transitive verbs (provided they receive the canonical translation with the copula)[9]:

$$< t, t* >$$

Notice that, while 'father of' is not really a term (when taken significatively, that is, in personal supposition), 'father' is a perfectly simple term.

> For Ockham, the term 'pater' signifies individual men, in respect to others, their children. [Kli93][p.45]

Note: I do not claim to be providing any interesting formal result; my use of a type-theory-like line of reasoning is only meant to spell out more clearly some features of Ockham's semantics, in particular with regard to the composition of terms[10].

2.1.3 Strict and Large Signification

When dealing with tense and modal predication, one is bound to encounter the issue of scope and domain of terms: for example, in a given proposition such as $\forall x P(x)$, which holds in a world w of the model, does the quantifier range over the whole domain of individuals or only over those in w? Should the predicate P be interpreted locally or globally?

So, in the context of the present work, a reconstruction of Ockham's theory of truth conditions which includes modal and temporal propositions, it must be defined which notion of domains should be considered, taking into account textual evidence.

Among medieval logicians it was frequently held that what a term does, did, will or can signify (that is, past, present, future and possible entities) is its signification, whereas its present and actual significata were named its appellata.

Ockham virtually abolished the notion of appellata, and kept only that of signification, but then twofold:

[8] Admittedly, Ockham does not treat expressions like 'father of', as these are incomplete expressions which are out of the scope of his semantics, and presumably of most if not all Medieval semantics. So what we present here is an extension of his original ideas.

[9] For 'John *loves* Mary', 'John *is lover of* Mary'.

[10] Sommers, in [Som82], argues that the whole pre-fregean tradition approaches language in a naturalistic way, as opposed to the constructionistic approach of the fregean tradition, according to which sentences of natural language must be heavily reconstructed to display their logical form. Still according to Sommers, the 'chomskian turn' established a research program which is closer to the pre-fregean research program, that is, the investigation of logical structures *actually* present in natural language. In this respect, it is no surprise that Ockham's general parsing of propositions is similar to that of some recent systems, even though Ockham himself did not consider conventional language to be the 'natural' one - the mental language was the locus of logic, in which sense there is some similarity between his view on language and the so-called constructive approach.

> First, a sign is said to signify something when it supposits for or is capable[11] of suppositing for that thing in such a way that the name can, with the verb 'to be' intervening, be predicated of a pronoun referring to that thing. [Ock74][P.113]

So, in a more straightforward way, the signification - and accordingly, the personal supposition - of a term is only its present and actual significata. That is, in this sense, signification is a local feature of a term, and so is the domain over which it ranges. We could call it strict signification.

But Ockham also introduces another kind of signification, whose scope is much broader:

> In another sense we say that a sign signifies something when it is capable of suppositing for that thing in a true past, present, or future proposition or in a true modal proposition. Thus 'white' not only signifies what is now white but also what can be white. [Ock74][p.113]

In this second sense, a term has large signification when it breaks the boundaries of time and actuality, and is said to signify all past, present, future and possible entities of which it can be truly predicated by means of a demonstrative. In this sense, terms like 'unicorn' or 'dinosaur' have signification also in the present, actual world. In the first sense of signification, that is, strict signification, there is clearly an existential claim involved - bear in mind the quotation concerning a chimaera in 2.1.1[12]. Alternatively, in the case of large signification, it is only required that the significatum not be impossible, that is, that it could possibly be produced by God (Cf. 2.2.1).

> Thus, it should be understood that a term supposits personally when it supposits for things that are its significata or for things that were, will be, or can be its significata. [Ock74][p.204]

This discussion is all the more important because some terms are by their very nature transworld terms, such as 'remembered', 'extinct', 'predestinate' etc ... : they presuppose the non-existence of the entities they are suppositing for at the moment of utterance, and their existence in some other world of the model. This kind of terms is in fact a major problem for extensional semantic system such as Ockham's: if a term signifies its extension, what about terms with (local) empty extension?

Let us call such terms θ terms. Thus,

Definition 2.1.1 $w \Vdash a \circ \theta \Longrightarrow$ There is no individual δ in w such that $w \Vdash T\delta \circ a$ and $w \Vdash T\delta \circ \theta$ (the last part being trivial, since θ terms had been so defined) .

[11] It may seem that an intensional element is present in the definition of strict signification, but the term 'capable' is added only because a term may signify something and yet never supposit for it in a proposition, if such proposition is never actually uttered, thought of or written.

[12] We shall see that this claim does not apply to the case of proper names, in 2.3.1

This seems to seriously jeopardize Ockham's very notion of signification and supposition, and therefore the reconstruction here proposed. In fact, the truth conditions provided above are the exact opposite of the basic truth conditions for propositions of the form $a \circ b$ (Cf. chapter 3). So it seems that Ockham's semantics cannot handle θ-terms.

That is, as said, a frequent objection to extensionalist semantics, and also the opinion of some Ockham scholars such as Karger [Kar76], who prefers to simply exclude such terms of her reconstruction.

> Only those predicates are to be allowed that have the following property: they are truly predicated of an object if and only if it is the case that the object exists at the time at which the corresponding sentence is uttered. Consequently, no predicates such as 'is remembered' are acceptable. [Kar76][p.77]

I, however, consider it to be possible to handle θ-terms by means of the notion of large signification. It would roughly correspond to what global quantifiers are in some modal logics, that is, terms (or operators) which range globally, and not only locally, as in most modal logics. Thus, maybe by means of the introduction of a different notation - for instance, the index l, for large -, it would be possible to sort whether a term takes strict or large signification in a proposition.

Definition 2.1.2 $w \Vdash a_l \circ \theta \Longleftrightarrow$ There is an individual δ in some w' such that $w' \Vdash T\delta \circ a$ and $w \Vdash T\delta \circ \theta$, even though δ does not exist in w.

The last part of the truth conditions above may seem awkward, but this is due to the very definition of θ terms. If we bear in mind that the existential claim involved in $T\delta \circ a$ propositions is weaker than the claim of non-existence of θ, then this condition is accurate.

However, the global point of view - large signification - is only adopted in some specific cases; in general, the local approach does prevail in Ockham's semantics - that is, the signification of a term is its present and actual significata[13]. In order to refer to past, future or possible individuals, a modal or temporal proposition should be formulated.

Actually, it is luckily so, since the local perspective displays undeniable advantages, and is much more coherent with the general principles of Ockham's logic.

In 'Quantification in Modal Logic' [Gar84][p.250], Garson provides a tree which spells out the various kinds of quantified modal logics in terms of the branching choices one has to make. It is possible now to define which position Ockham's 'predicate modal logic' - that is, the system we can reconstruct based on his texts - would occupy therein.

The first choice is between objectual domain, conceptual domain or substantial domain. Clearly, Ockham's is an objectual domain. Secondly, a branching

[13] With the exception of proper names, since they are rigid designators; in this case, the signification of a proper name is also global, and does not imply the existence of the individual named in the world of enunciation.

takes place between the systems with rigid terms and those with non-rigid terms: again, Ockham's choice is clearly in favor of non-rigid terms, given that terms signify what they can be predicated of, which can vary if the salient individual undergoes changes[14]. Finally, one can opt to have global terms or local terms; as argued, Ockham takes the local perspective (strict signification), but does not exclude the global perspective in special cases (such as the θ terms).

Hence, this is the notion of scope I shall make use of while dealing with modal and temporal propositions. It heavily depends on the concept of locality as opposed to that of globality: however, such concepts only make sense in a Kripke-like frame. In fact, I have been using so far the jargon of 'worlds' to clarify many important issues; I must now provide a justification for my use of Kripke models.

2.2 'Possible Worlds'

This section is intended to license and justify my future use of possible-world semantics in Chapter 3, as it seems that the notion of possible worlds, including its operational features, is not alien to Ockham's theoretical considerations.

2.2.1 Theological considerations

The idea of possible worlds made its entrée in the History of Philosophy during the Middle Ages; for many reasons, such a thing was just unthinkable for ancient philosophers. According to Aristotle, for example, given his physical and metaphysical assumptions, there could only be one world, that is, the actual world, even though its structure was composed of different spheres around the physical world (Cf.*De caelo*). All entities tend to their natural place, that is, physical objects tend to the ground, whereas spiritual entities tend to the celestial spheres above.

Therefore, throughout time, all entities actualize their potentiality, and move closer to their natural place. That is how Aristotle explains change in time, as a linear evolution. Thus, possible is understood as potential as opposed to actual: what is possible will eventually become actual. Hence, every thing is in a different state of its actualization *in this world*, and it could not be any different from what it was, is or will be; that is, our world is unique (which is argued for in *De caelo*), and could not be any different from what it is: there are indeed no possible worlds.

So, there is no conception of modality independent from that of temporality, as modality is explained in terms of strictly non-branching models. A change on the views over modality would only take place as a side consequence of another medieval discussion, namely that concerning God's omnipotence. In this sense, the idea of possible worlds came to existence embedded in a theological context which had broken radically with the ancient cosmology and metaphysics.

[14]Again, this does not hold for proper names, which are rigid designators. Cf. 2.3.1

> Almost from the outset, the Scholastic discussion about the extent of God's power abandoned implicitly the strictly temporal understanding of modalities. [Fun86][p.146]

Roughly, the argument goes as follows: if God is omnipotent, as the holy texts claim Him to be, then He could have made things different, that is, there could have been alternative worlds He could have created. Moreover, He can also interfere at any moment in His own creation, and thus alter the path of its evolution (which means that His power is able to provoke 'branching in time', to say so).

The discussion is subtle and complex[15], and even though there was agreement at Ockham's time that God could indeed have created different worlds, each author would present their own account of to what extent and how God could interfere in the present state of affairs. In particular, Ockham's account establishes what we could call *logical* limits to God's omnipotence:

> All things are possible for God, save such as involve a contradiction. [Boe57][p.xix]

God can, among other things, create infinitely many possible worlds, as long as none of them contains a contradiction. This claim is remarkably similar to Leibniz's views on possible worlds, according to which possible worlds are all the different possible combinations of beings in one world that do not involve contradiction. Also for Leibniz, God could have created any of them as the actual world, but once He has done so, He will most likely not interfere in it, for it would be the best world possible (this topic shall be further explored below).

Thus, Ockham's 'possible worlds' resemble very much the present-time conception of possible worlds in that it is a construction clause that a world cannot contain a contradiction, that is:

$$w \Vdash \phi \wedge w \Vdash \neg\phi$$

is impossible.

> The omnipotent can nonetheless do everything doable[16] that does not include a contradiction. In Ockham, *Comments on the Sentences*, cit. in [Fun86][p.149]

Moreover, the so-called[17] 'ockhamist frames' also have the interesting feature of incorporating a non-linear time, since God can interfere and modify the state of things any time. In other terms, ockhamist temporal frames are said to be 'branching': there may be more than one possible world accessible from a world. However, one of the threads is the sequence of actual 'worlds' through time, which is depicted in figure 2.1 by the bold lines.

[15] Funkenstein presents a very complete analysis thereof in [Fun86].

[16] Ockham adds the clause 'doable' because God cannot make another God, even though this does not include a contradiction per se.

[17] Prior seems to be responsible for introducing this terminology, as it is a well-known fact that he displayed unusual (for a logician) interest in medieval philosophy

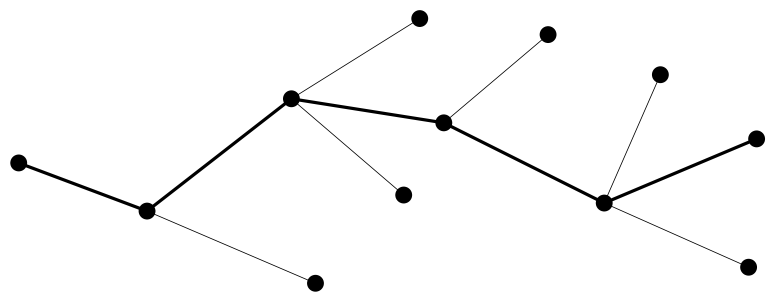

Figure 2.1: Ockhamist frame.

One interesting related topic, which however shall not be treated, is the physics system that must be developed to incorporate the possibility of God's interference upon beings; obviously, the traditional aristotelian physics, according to which objects follow one strict trajectory that cannot be altered, would be insufficient to account for God's interference[18].

As previously said, for Leibniz, God could but would not interfere in His own creation: the reason for this is that He chose to create the most perfect world in the first place, that is, the one where the greatest number of creatures can coexist without contradiction. Therefore, it would be against His nature to modify His creation into something less perfect. Similarly, for Ockham, even though it is conceivable that God would interfere in His own creation throughout time, there are also better and worse worlds among the possible worlds - a point to which his predecessors, such as Duns Scotus, gave no attention.

> For Ockham the question of plurality [of worlds] does bear relevance to the question of better worlds. [Fun86][p.142]

That is, there seems to be a clear relation between Ockham's account on possible worlds and Leibniz's; if, moreover, one accepts the claim that Leibniz's philosophy is the main inspiration for our conception of possible worlds, it would seem quite justified to apply possible-world models to Ockham's semantics, on the basis of these theological considerations. I shall now attempt to show that the idea of possible worlds can be applied to Ockham's logical conceptions also on an operative level.

A final remark: strictly for logical purposes, Ockham often makes use of mental experiments of counterfactual situations to analyze properties of terms or the validity of assertions. For instance, when arguing against those that claim that the sign 'every' (*omnis*) requires three appellata (that is, three elements in its domain of application), he says:

[18] There are interesting historical studies on how Ockham's physics played an important role in the later development of physics, for example on the possibility of objects' effect on each other at a distance.

> If there are only two men and they are white, then 'Every man is white' is true, since it has no false singular - and yet it does not have three appellata. [Ock80][p.97]

(Note that Ockham mentions that two men would be enough, but presumably only one man would also be enough, as is stated in other examples.) That is, the notion of models and of different possible ontological configurations is present also in analytic devices. Therefore, it does seem justified to assume that Ockham accepted the possibility of different worlds, that is, of different states of affairs.

2.2.2 Context and Intentionality

As previously argued, Ockham's semantics heavily depends on the notion of multiple denotation - otherwise, it could not account for generality. Thus, terms signify all those individuals they can be predicated of indistinctly. It is only on the propositional level, by means of their supposition, that terms can refer to some of their significata and not to all.

> ... a term may supposit either for all or for some of the objects that constitute its possible supposition. [Kar76][p.22]

What provokes the fact that, in a proposition, a term supposits for some particular individuals, and not for others it could supposit for? There are of course some elements within the language which modify a term's supposition, such as syncategorematic expressions like 'Every' or 'Some'. Still, these do not help determining to which specific individuals a term is referring.

Indeed, it is above all by means of pragmatic elements that the supposition of a term is clarified. Again, we are not dealing with the problem of definite/indefinite reference, as the standard markers for (in)definiteness, articles, are simply not present in the object-language (Latin). Even though Ockham would like his system to be a logic, whose object is the mental, pure language, and not a grammar, he cannot escape some language dependency such as in the case of absence of articles.

However, we know that indefinite expressions are referential (cf. DRT), and that definite descriptions can lack reference[19]. The fact is that, given the emphasis on multiple denotation, the terms of a proposition may have different supposition in its particular occurrences (in fact, indeed they do in most cases). This is what I would call the 'horizontal flexibility' of a term's supposition: it is only by means of the circumstance and, most of all, of the speaker's intentions that supposition of a term is fixed upon one or some individuals.

Consider a proposition like 'Homo est albus'. The supposition of the term 'homo' (and of 'albus', if the proposition is true) can be Socrates, in one particular occurrence of it, or Plato, in another occurrence. Since both 'homo' and

[19]The example is, I think, Donnellan's: in the funeral of a very rich man, whom I did not know very well, I may utter the (distasteful) sentence 'Well, the butler will have to find himself a new job!', even though there may not be a butler, and in any case I don't know the butler.

'albus' have multiple denotation, only contextual and pragmatical elements will indicate which individual the terms are really suppositing for[20].

> In the same way an equivocal name can, in any proposition, supposit for any one of its significata, unless those who use it limit it to one particular significatum. [Ock74][p.192]

That is, just as much as proper names are equivocal in that, in conventional language, a proper name usually names more than one individual(Cf. 2.3.1), and then pragmatic elements are necessary to disambiguate the names' reference, a term with multiple denotation depends on pragmatics (such as pointing, among others) to have a unique supposition. So, we are now clearly in the territory of pragmatics and context-dependence.

To make matters more complicated, because of supposition theory, terms also have what I would call 'vertical flexibility', that is: they can have personal, simple or material supposition. Again, the kind of supposition a term actually has in the occurrence of a proposition is only defined by the speaker's intention - for, in the case of terms that, say, 'should' have a different kind of supposition in a proposition, if they have the 'wrong' kind of supposition, the proposition is false, and not undefined (for example, if 'man' has personal supposition in 'Man is a species'.)[21]. This means that, again, singular occurrences of propositions may indicate different kinds of suppositions of the terms therein - and accordingly, different truth values for each singular occurrence.

> ... even if sentences are abstract, utterances of them are not, and it is an ontological safeguard to make sure that, in principle, sentences are not considered as independent entities. [Kar76][p.11]

To sum up, we are here dealing with the classical opposition type/token. Clearly, for Ockham, only tokens are to be taken as the elements of his semantics, since what terms in a given proposition refer to can vary horizontally and vertically. So, a type has no logical relevance, for it cannot be analyzed in terms of its logical features (among which supposition is the most important).

> Le nominalisme radical d'Ockham épouse ce qu'on appelle aujourd'hui l'inscriptionnalisme: les seules realités linguistiques ou conceptuelles sont les occurrences singulières; les types linguistiques n'existent pas. ... Le terme n'existe que dans ses inscriptions singulières. [Pan][p.20]

That is, there are also ontological reasons, related to the ontological status of propositions, which indicate that only tokens should be considered, and not types. Types would be some sort of abstract universal of which tokens would be manifesting individuals, which is definitely unacceptable in Ockham's ontology.

[20] Notice that we are not dealing with truth conditions now; in that case, it is not relevant which specific individual the terms are suppositing for.

[21] Under a different interpretation, there would also be sentential constraints determining the kind of supposition of a term in a proposition, and not only pragmatic elements. This is an unsettled issue for me. Cf. Conclusion of the Thesis.

Hence, for Ockham, a proposition only acquires a logical structure *in a context*. The point is thus that both intentionality and tokenness can be dealt with in possible-world models, since such models display the feature of *local validity* (i.e., satisfiability). which is essential to handle context-dependent approaches to propositions. Most importantly, Ockham's notion of truth is obviously of a local nature, thanks to supposition theory and multiple denotation.

2.2.3 Modal and temporal contexts: local truth

Tense and modal propositions will be more thoroughly analyzed in chapter 3, along with the stipulation of their truth conditions in Ockham's system. However, now is the moment to argue in favor of the kind of models I intend to use for such enterprise, namely possible-world models.

Firstly, with regard to tense propositions, the role of the copula 'est' is crucial. That is, it is by means of the tensed copulas 'fuit' (past-tense) and 'erit' (future-tense) that tense propositions are generated. More interestingly, the truth conditions of such propositions can always be expressed in terms of the truth of some simpler proposition where the copula is the present-tense 'est', but then the truth of the latter is relative to a different time - one would say, a different world. The most obvious case is when the temporal 'effect' (I do not want to use the term 'operator' yet) ranges over the whole proposition, for example in 'Homo fuit albus'. Under this interpretation, the truth of this proposition only requires that 'Homo est albus' was true at some time in the past.

But along with 'tense propositions with a dictum'[22], there are also tense propositions in which the temporal 'effect' ranges only over the predicate. In these cases, it is obviously not enough to simply replace the tensed copula by the present-tense copula and locate the truth of the new proposition at a different time. Nevertheless, the procedure Ockham proposes also requires that some other present-tense proposition (namely, one of our $T\delta \circ a$ propositions) be true at a different time.

> Thus, it is required that a proposition in which the predicate is predicated of a pronoun referring precisely to that for which the subject supposits was true at some time (if the proposition is past-tense) or will be true at some time (if the proposition is future-tense). [Ock80][p.105]

That is, the truth conditions of tense propositions as defined by Ockham are very easily expressed in terms of the local truth of other present-tense propositions; in other words, Kripke-models, with their feature of locality, seem to come in handily to model Ockham's analysis of such propositions.

[22]Ockham does not use the terms 'with a dictum' and 'without a dictum' - corresponding to our notions of modality 'de dicto' and 'de re', respectively - for tense propositions, only for modal propositions. I am relying on the standard way nowadays to treat tense and modal propositions as being of the same nature in committing such a historical mistake.

> In view of the use made by Ockham of tensed verbs, it is required that truth should be made relative to time. [Kar76][p.11]

Accordingly, the tense of the copula will also define the domain of a given proposition, which is related to the discussion in 2.1.3.

> 'Chevaux' dans 'les chevaux sont des animaux domestiques' ne tient lieu, en vertu du fait que le verbe est au présent, que des chevaux qui existent au moment de l'énonciation. [Pan][p.8]

Ockham's treatment of modal propositions follows a different strategy from the one for tense propositions (in fact, modal propositions have just become 'autonomous' from their previous exclusively temporal status, Cf. **??**). In the case of modal propositions, the copula does not play as important a role, even though there exists the modal copula 'potest essere' ('can be'). Alternatively, Ockham views modes as *predicates* which are predicated of entire propositions - in this sense, modes are also of t type, and anything that can be predicated of a proposition, such as 'known', 'true', 'possible' etc..., is a mode for Ockham. Therefore, the truth conditions of modal propositions can be defined along the lines of the general truth conditions of simple two-term propositions (Cf. [Pan92][p.49-52]).

> For example, by means of 'That every man is an animal is necessary', it is asserted that the mode 'necessary' is truly predicated of the proposition 'Every man is an animal', the dictum of which is 'That every man is an animal'. [Ock80][p.109]

But as it turns out, the truth of modal propositions is also defined in terms of the truth of *de inesse* propositions (i.e., present-tense non-modal propositions), which indicates that modes could be viewed as operators even within Ockham's framework - the advantage of it being that we can then apply some well-known properties of recent modal logic. Moreover, he does not provide an extensive analysis of the modes themselves, of what it means for a proposition to be necessary or possible. Hence, it seems we are licensed to apply the usual semantics for the $\Box$ and $\Diamond$ operators, be it for convenience, if not for complete historical faithfulness.

> Modes must be what we would call operators. Ockham's theory concerning modes could be faithfully rendered as follows: modes function as modal operators, but it is always possible to explicate the modal operator away, by replacing it by a modal predicate corresponding to the modal operator, and by extracting from the original sentence an appropriate sentence of which the modal predicate is asserted to hold. [Kar76][p.182]

Just as much as in the case or tense propositions, if the scope of the mode is only the predicate term - what Ockham calls a modal proposition *in sensu divisio* - then the truth conditions of such a proposition will not be defined in terms of the 'dictum' proposition, but in terms of the appropriate demonstrative proposition of the form $T\delta \circ a$. In any case, there occurs some sort of 'truth jump' as well, which again I believe can be felicitously modeled in a Kripke model.

Of course, the use of Kripke models does not impose itself, as at any rate such a notion is not explicitly formulated by Ockham. Indeed, some reconstructions of Ockham's logic were based on different models, for instance Montague's pragmatic model with indexes, by Karger in [Kar76].

> However, in the case of Ockham's conception of necessity and possibility, such an addition to our model is, fortunately, superfluous. A pragmatic model including moments of time is sufficient to define the truth of Ockhamist modal sentences of possibility and of necessity. [Kar76][p.185]

My intention here is not to create a polemic with such a remarkable work, especially since I can rely on results posterior to 1976 - for example, proposals which overtook Montague's initial model (Cf.[Sta80][p.447]) - which were unknown at the time it was written. In any case, I believe that Kripke models can be successfully applied to Ockham's system, and I can provide one argument to sustain my choice: Karger claims that, for Ockham, past and future are indivisible, that is, each is one single element of the model, which is thus composed only of three elements, namely past, present and future. Now, there is textual evidence that Ockham did not consider past and future to be indivisible, for instance the following quotation:

> Thus, given the force of that expression, 'Socrates was twice white' has a negative exponent; for the proposition is equivalent to 'Socrates was first white and at a later time he was not white and at a still later time he was white. [Ock74][p.211]

If the model for time were indeed composed only of three parts, then indexicals should in principle be enough to account for it. But that is not the case, and I claim again that possible worlds can be used to account for the semantics of the truth conditions that Ockham establishes for modal and tense propositions. In any case, I hope to have been able to show that the idea of possible worlds is not entirely alien to Ockham's considerations, and that the notion of local truth is a key element in his semantics, as much as it is in Kripke models.

2.3 Designation

I shall foremost investigate the mechanisms of reference and designation for singular propositions. However, since the truth conditions of more complex propositions will be defined in terms of the truth value of singular propositions, it is reasonable to assume that what is here described holds for designation and reference in general, that is, for all terms.

Singular sentences are those that feature either a proper name, or a demonstrative pronoun, or a demonstrative pronoun and a common term as their subject. In these cases, the subject has discrete supposition and is meant to supposit for only one determinate object.

> A term has discrete suppositio if it is either a proper name or a demonstrative. In either case, it is clear that the term can supposit for one object. [Kar76][p.19]

But the mechanism of reference that holds for such terms will not be much different from the general mechanism of reference, since the whole system is permeated by the central notion of supposition.

2.3.1 Names

Thus, proper names have discrete supposition, since they supposit for only one individual, namely the one they name. Ockham does acknowledge the fact that in most cases there is more than one individual named by each name, for instance there are more people who are called Socrates, in addition to the famous philosopher. But the point is that proper names are what Ockham calls equivocal terms, that is, conventional terms which are subordinated to more than one mental concept.

> A word is equivocal if, in signifying different things, it is a sign subordinated to several rather than to one concept or intention of the soul. ... An example is the name 'Socrates', which is assigned to several men. [Ock74][p.75]

Hence, to each of the Socrates one may know corresponds one single mental term, and if the conventional term 'Socrates' does not account for this univocity, this is only due to the imperfection of conventional language as opposed to mental language.

In any case, in spite of their being equivocal, proper names do have the semantic behavior of suppositing for one, determinate individual; proper names 'pick' immediately their significatum out of the domain of individuals, that is, they do behave as tags or labels applied to individuals, which have no further content than to designate one object. In this sense, Ockham's account is very similar, if not identical, to wide-spread views on proper names of the recent decades, mostly those of Kripke.

> The paradigm of the assigning of proper names is tagging. [Kri77][p.73]

The issue concerning proper names became particularly prominent in the work of Bertrand Russell, and since then has been the object of frequent analysis. In Russell's views, a proper name behaves as a tag only in the case of immediate acquaintance with its bearer - what Russell called 'knowledge by acquaintance'. Otherwise, without the empirical perception of the individual in question, proper names would be abbreviations of definite descriptions. That is, since I never met Socrates, who died more than 2000 years before I was born, when I use the name Socrates to refer to the famous philosopher, I am making use of a series of elements that I know about him - that he never wrote a line, that he held endless public conversations on philosophical topics, that he was sentenced to death -, such that there is uniqueness of reference in my use of the name.

That is, in both cases - knowledge by acquaintance or by description - a proper name would provide a single referent out of the domain, be it immediately by empirical perception or by means of an associated intension, that is, a definite description. This model is highly embedded in epistemological considerations, which we may disregard[23], but its main problem is that, in intensional contexts, it leads to the well-known paradoxes about identity and reference, which indicates that, also with regard to non-perceived individuals, proper names do not behave as descriptions.

As far as Ockham is concerned, proper names are never associated with descriptions, since he emphatically rejects the use of any intensional entity to account for reference and meaning. Just as some XXth century opponents of Russell's theory of names as definite descriptions, he would claim that proper names refer directly to individuals. Moreover, given his notion of an individual as a self-standing entity (Cf. 1.1.1), it is clear that the reference to an individual by its name does not depend on the individual's characteristics or essence, to use the adequate parlance. That is, a proper name always names the same individual, no matter the changes it may undergo through time or in different possible worlds.

> For example, if Socrates is now white for the first time, then 'A white thing was Socrates' is true, and yet no present-tense proposition in which the predicate 'Socrates' - or that for which it supposits - is predicated of the subject 'white thing', was ever true before. [Ock80][p.107]

That is, the notion of self-identity for Ockham is to be understood as primitive, and not as defined in terms of equality of properties. So, Socrates may change color, or any other feature, and still remain numerically identical to himself.

Hence, it seems correct to claim that, also for Ockham, proper names are *rigid designators*, as they supposit univocally for the same object in all possible circumstances - that is, the variations of a term's supposition analyzed in 2.2.2 do not apply to proper names, they are the only exception in that their denotation is steady and unique.

> 'Socrates was white' cannot bear two readings as molecular sentences can, namely one in which the subject-term supposits for what was,

[23]Even though some commentators see similarities between Ockham and Russell in this respect: some authors (Cf. Panaccio in [Pan80]) identify in Ockham a similar distinction between knowledge by acquaintance and by description, with regard to the formation of mental terms.

> the other where it supposits for what is. [Kar76][p.136]

This is quite straightforward, for when a proper name is a subject-term, what it supposits for is the same individual regardless of whether it was, is, will or may be. This topic will be thoroughly explored in chap.3, along with a systematic presentation of Ockham's temporal and modal logic, so in a way it is inadequate to introduce it now. Nevertheless, I will save the more technical discussion to chap.3, and for the moment I only hope to conceptually establish that proper names refer to the same individual in al worlds.

Karger (in [Kar76][p.136]) claims that Ockham is mistaken in that a proposition such as 'Socrates was white' does bear two readings, namely that Socrates now exists and was white in the past[24] , and that Socrates was white in the past, therefore existed in the past, but does not exist anymore[25]. However, she seems to confuse reference with existence, which is by the way a frequent objection to Kripke's account of names as rigid designators, that is, that the notion of rigid designation would imply that the individual named by a proper name must exist in all worlds in which the name is uttered.

But it is not so; rigid designation is a claim concerning the reference of a *term*, and not a claim concerning the existence of an *individual*. The language realm must be distinguished from the ontological realm. In the next section, we shall see that the supposition of demonstratives does involve the presupposition of the existence of the individual supposed for. However, given the very nature of proper names as tags, their power to refer to one single individual is not affected by the individual's existence or non-existence.

> All I mean is that in any possible world where the object in question does exist, in any situation where the object would exist, we use the designator in question to designate that object. [Kri77][p.79]

So, Socrates is known as Socrates whenever he exists, but whenever he does not exist as well. In the case of propositions with an operator between the subject and the predicate (later to be defined as modal or temporal propositions in 'sensu diviso'), it is only the predicate that involves an existential claim (related to the predicate's appellation of the form, also to be later explained. Cf [Pan99][26]). Thus, it is possible to provide the following truth conditions[27]:

Definition 2.3.1 $t \Vdash S(P\circ)w \Leftrightarrow$ What is signified by S in t, regardless of its existence in t, exists in some $t' < t$, is signified by S in t' (trivial, since S is a rigid designator), and $t' \Vdash T\varsigma \circ w$ (ς denotes the salient object).

Definition 2.3.2 $t \Vdash P(S \circ w) \Leftrightarrow$ There is some $t' < t$ such that $t' \Vdash T\varsigma \circ S$ and $t' \Vdash T\varsigma \circ w$.

[24] To be transcribed as $t \Vdash S(P\circ)w$. Cf. 3

[25] To be transcribed as $t \Vdash P(S \circ w)$.

[26] I am very grateful to Prof. Panaccio for sending me some of his articles, including unpublished material.

[27] These definitions are out of place, they actually belong to Chapter 3. Still, it is informative to mention them now as well, in the context of a discussion about proper names.

It is not a surprise that these two definitions seem to be equivalent: I will prove in 3.5.3 that, in the case of proper names as subject terms (only), temporal (and presumably modal) propositions 'de dicto' and and their corresponding 'de re' (to be defined in 3.5) are equivalent.

The fact of being able to refer even if the referred object doesn't exist in the domain of reference holds only for proper names. For all the other terms, either what they supposit for must exist, or else the notion of large signification must come into play.

2.3.2 Demonstratives

On the one hand, it seems unjustified to explore Ockham's views on demonstratives simply because there are none. Nowhere in his writings does he undertake a theoretical analysis thereof, let alone a theory of demonstratives. It was commonly assumed at the time that the semantics of a demonstrative was the same as that of a proper name, of which it was a mere substitute, and therefore that the former would only be a derived semantic element. On the other hand, they are just about everywhere in the *Summa Logicae*; virtually all definitions of crucial concepts such as signification and supposition encompass demonstratives.

> Thus, when a term stands for something in a proposition in such a way that we use the term for the thing and the term or its nominative case is truly predicated of the thing (*or the pronoun referring to the thing*), the term supposits for that thing; or this, at least, is true when the term is taken significatively. [Ock74][p.189] My emphasis.

It has been previously outlined that a term cannot be predicated of a thing, that this would constitute an infelicitous confusion between the domain of words and the domain of things. However, as the above quotation shows, the intuition behind Ockham's view on the relation between words and things is that terms behave as if they were in fact predicated of things - hence, my metaphor that terms are like tags bearing on things. So, demonstratives are vital, since they play an intermediate role from terms to things so that, in a singular proposition of the form $T\delta \circ a$, the term a is actually predicated of another term, namely the demonstrative.

> Demonstratives are not self-sufficient words - they rely on an act of pointing that cannot be present in the sentence - but it follows therefrom that whatever is predicated of a demonstrative can be readily taken as being predicated directly of the object itself. [Kar76][p.15]

So the act of ostension has the power of bringing a thing directly to the realm of terms, and hence ties tightly the discourse to individuals of the ontology. It is a pure act of reference which says nothing about the individual pointed at, except that it exists and that it is thus also part of the discourse domain[28].

[28] The act of pointing and ascribing linguistic existence to an individual bears an interesting resemblance with the *notitia intuitiva* of an object, which just indicates that it exists, but says nothing else about it. Cf. ??

Once this is accomplished, all propositions built up from these demonstrative propositions will presumably have a clear bearing on things, since the basic act of reference is contained in them.

Therefore, the usual scheme according to which complex propositions have a bearing on reality by means of their intensions, which in turn determine their extensions, is emphatically refused by Ockham. Alternatively, it is because complex propositions are related to demonstrative propositions that language actually refers to things. Hence, Ockham's scheme defines the referential power of language in terms of its recursive structure, whose basic elements are propositions the subject of which, a demonstrative, bears a relation of direct reference to things.

What is peculiar about demonstratives, as opposed to names, is that they require the existence of the designated object. Just like names, they select one single element of the domain, but names have this durative feature (related to their causal nature) that allows them to perform reference even though the referendum may not then be present. Demonstratives, on the contrary, can only bring existing things to the realm of terms - the salient object must exist and be present *in the context of the demonstration*, that is, in the world w where the $T\delta \circ a$ proposition holds.

> A demonstrative 'should' designate a presently existing object. [Kar76]-[p.138]

In fact, a (positive) demonstrative proposition in which nothing is demonstrated is plainly false, for the subject supposits for something that does not exist[29]. That is, a demonstrative is not a semantically independent term, for it does not signify anything if not accompanied by the demonstration of something. In this respect, it is rather an operator than a term, for which reason it probably seemed less interesting to the eyes of a semanticist like Ockham. In my reconstruction, demonstratives are hybrid elements of the language: they can be interpreted as terms, since they supposit for what they designate in propositions, but they are also a 'choice' operator, which picks one single element out of the domain, and only thereby acquires semantical relevance.

> A demonstrative without an associated demonstration is incomplete. [D.77][p.9]

In fact, demonstratives have for Ockham the purely referential power that Kaplan ascribes to them, in particular to his 'Dthat' device, which emphasizes the referential use of terms, especially that of demonstratives, as opposed to their descriptive use. (Cf. [Kap78]).

As presented in 1.3.3, demonstratives also play a major role in Ockham's theory of ascent and descent, in the form of propositions of the kind 'This a is b'. It has been previously argued that these propositions, even though they are

[29] Truth conditions of propositions with vacuous subject shall be explored in details in chap. 3.

also singular, have very distinct semantic features, when compared to $T\delta \circ a$ propositions - hence, it seems all the more sensible to treat the theory of ascent and descent as a theory of inference, and not as truth conditions of quantified propositions. Nonetheless, it is remarkable that in both cases demonstratives are positioned between ontology and language, between individuals and the generality of propositions.

> Soit qu'ils fournissent un critère opératoire pour reconnaître l'applicabilité des termes généraux aux choses individuelles, comme dans les definitions de genus, de signification et de supposition, soit encore qu'ils marquent l'aboutissement de la descente logique qui conduit des propositions générales aux propositions singulières ... toujours ils apparaissent comme la limite, interne à l'ordre des signes, de la décomposition analytique du général. [Pan80][p.196]

One last important remark must be made: even though demonstratives presuppose the existence of the corresponding demonstratum, this requirement is not of an empirical nature: it is not so that the object so to say 'pointed at' has to be physically present at the moment of the demonstration. In this respect, Ockham's ontology is very close to our notion of models, in that the domain of entities is not composed of actually perceived beings, as in a more physically oriented conception of ontology. Thus, the demonstration of empirically inaccessible objects, such as in 'This is God', or 'This is [the mental term] species' is unproblematic.

Moreover, the use of demonstratives made by Ockham is different from that by Kaplan, for instance. For Kaplan, on the one hand, one needs both the context of reference *and* the context of utterance to determine the truth conditions of a proposition (in particular, those with indexicals). For Ockham, on the other hand, propositions have local validity - determined by the context of reference, to say so -, but, as far as 'utterance' goes, the perspective is global. So, the demonstration does not presuppose one speaker who is in a specific world of the model, and therefore can only perform demonstrations in *that* world; in fact, such actions can 'occur' anywhere. The intuition behind it is of a God who has the whole model at display, and can 'point at' any element therein, in any world.

Of course, if it wasn't so, it would be impossible to claim that propositions of the form 'This is a' are the atomic elements of Ockham's logical system - otherwise the language would be restricted to physically perceived objects, and hence too limited, which is not the case. So, given the features of demonstratives just outlined, in particular their intermediate position between things and terms, I hope to have provided more elements to support my choice of $T\delta\circ a$ propositions as my atomic elements.

2.3.3 Designation and Reference

Singular propositions - that is, those whose subject is a proper name or a demonstrative - are crucial elements of Ockham's logic because they clearly present a general feature of all terms with regard to things, that is, that there is a relation of direct reference between terms and things. Both in the case of names and of demonstratives, one single element of the domain is designated in a direct way, thanks to their semantical behavior.

> Une grande partie de la semantique ockhamiste a precisément pour fonction d'asseoir la legitimité du discours général dans ce monde d'individus. Les propositions singulières, il est facile de le concevoir, jouent là un rôle privilegié; ce sont elles, au bout du compte, qui branchent le plus directement les sujets parlants sur la concrétude du réel. [Pan][p.4]

The property of direct designation that discrete terms bear with regard to their referent is in some respects very similar to Russell's idea of logical proper names. Indeed, Ockham's claims that the referential power of language - and accordingly, the apprehensive power of knowledge - is due to its generation from some simple (singular) propositions display an interesting resemblance with Russell's ontology and epistemology[30]. Also Kaplan notices the similarity between his ideas on direct reference and some of Russell's ideas [D.77][p.3].

> By this I mean theories of meaning according to which certain singular terms refer directly without the mediation of a fregean Sinn as meaning. [D.77][p.1]

These views on singular propositions may be summed up as follows:

(1) Direct designators have denotation, but no intension. (2) Their correct use implies the real existence of what is designated.[31] (3) Unicity of reference. (4) Immediate sensitive contact with the designated object. [Pan80][p.196] (This is Panaccio's formulation, approximating Ockham to Russell.)

The most interesting of these conditions, which will hold for all terms, and not only for singular terms, is condition (1): terms do not relate to their extensions by means of intensions as mediators. Thus, what for Kaplan and Russell holds for some terms (direct designators for the former, logical proper names for the latter), holds for **all** terms in Ockham's semantics, not only for uniquely referential terms. In such cases, the property of direct reference is more evident, but also common terms with multiple denotation were shown in Chapter 1 to bear a direct relation to things: for instance, the extension of a a term like 'man' is not defined in terms of an intension (namely, the formal definition of the property man).

Again we encounter the disturbing fact that objects that in predicate logic are differentiated in 'terms' and 'properties' behave analogously in Ockham's semantics, or are of the same type. Just as a (discrete) term does not need an intension to meet its reference, a property (a common term) is able to meet

[30] That is, with the positions he sustained during one particular period, concerning the primary nature of atomic, concrete propositions.

[31] This condition does not hold for 'real' proper names, as previously shown.

its many denotata without the 'help' of an intension, that is, a membership function which defines what is and what is not among its extension.

The distinction between terms and properties seems to be unproblematic for cases such as 'Socrates', which is 'clearly' a term, or 'white', which is 'clearly' a property. But what to say of 'winner', 'friend' or even 'man'? Donnnellan (Cf. [Don66]) very sharply perceived that general terms - or alternatively, 'properties' - do not always behave as descriptions, but very often also as referential terms, a property which, in the analytic tradition inaugurated by Frege, was thought to pertain exclusively to individual terms.

Some 600 years before Donnellan, Ockham already explicitly emphasized the referential feature of all terms, not only of discrete terms, but also of common terms. However, because they have multiple denotation, this feature is more clearly visible in discrete terms; these refer directly to one individual, which they 'pick', whereas common terms need more apparatus to determine their supposita in a proposition.

In this respect, there is a sound intuition behind Russell's idea that proper names are equivalent to definite descriptions, that is, that the relation established between names and things was similar to that between descriptions and things. However, he inverted what Ockham would consider to be the real order of primacy: it is not that names behave as descriptions, it is actually that so-called descriptions, or general terms, can actually behave in a referential way.

> If pointing can be taken as a form of describing, why not take describing as a form of pointing? [Kap78][p.234]

Something similar holds for Quine's proposal to treat proper names as predicates which denote a singleton composed of the one bearer of the name. Quine perceived that terms and properties actually have very similar semantical behaviors, and do not denote different sorts of entities in the model (namely, individuals and sets). However, instead of proposing that predicates be viewed as names, and thus be interpreted as individuals (which would be more coherent with regard to his 'nominalistic' inclinations), he proposed that names be viewed as predicates, and hence that they be interpreted as sets (namely singletons). But this proposal misses the intuition that terms, and more clearly individual terms, refer directly, and not through membership in the set defined by the predicate.

Clearly, Ockham lines up behind the first proposal stated above, that all terms should be treated as names, and therefore focuses on the referential feature of language, in detriment of its so-called descriptive feature. Thus, by this device Ockham guarantees that all propositions are embedded in the ontology of individuals: complex propositions, such as quantified, tense or modal propositions, have their truth conditions defined in terms of singular propositions (the topic of Chapter 3); furthermore, propositions whose subject is a common term have the same referential power as singular propositions, as argued, so in all cases the discourse is firmly tied to the ontology.

2.4 Conclusion of Chapter 2

Chapter 2 drew comparisons and established similarities between some aspects of Ockham's logic and recent topics in philosophy of logic and philosophy of language. I think I should make some remarks on the purpose of doing so.

First of all, I do not think that the relevance of Ockham's logic depends on the possibility to relate it to recent topics and results; it is autonomously interesting, and not exclusively from a historical perspective. The positivistic enterprise of looking back in the past to check what a given author already 'knew' of the marvelous things later discovered inevitably points in the direction of what he *failed* to know. The underlying ideology of such a project is that our present knowledge is of course the best, so that an author from the past is only significant insofar as he or (unfortunately less often) she announced the present state of affairs.

A better perspective on a comparative study is the one that seeks in the past new ideas on how to handle present problems, or rather, a fresh point of view on things. This is for instance the perspective of classic comparative studies on medieval logic such as Geach's *Reference and Generality* [Gea62] and Sommers's *The Logic of Natural Language* [Som82][32]. Geach's seminal work was one of the first studies to propose the use of logic - fregean quantified theory, then the 'only' logic - to formalize natural language and investigate some of its phenomena, such as anaphoric relations, pronouns etc ... At the time, most philosophers of language objected to the idea of such formal studies, so Geach sought inspiration in medieval logic, which had built powerful formal accounts of language, to introduce a whole new (in our times) research program.

Some twenty years later, Sommers started with the basic intuition of Geach's work - namely, that medieval logic provides an interesting perspective on how to deal formally with natural language -, but disagreed with Geach's assumption that fregean quantified theory is the adequate tool to analyze it. Instead, he proposed the traditional subject/predicate form, underlying the whole of pre-fregean logic, as the actual logical principle of natural language, rather than the functional approach introduced by Frege[33]. I agree with Sommers that medieval logic simply cannot be analyzed in terms of the usual concepts of fregean predicate logic, for which reason I proposed a different notation.

However, Sommers's claim has a much larger scope than mine, namely that natural language in general is successfully analyzed by means of medieval logic notions. I do not intend to provide a general account of natural language, not even of a fragment of it: more modestly, I am only presenting Ockham's logic and semantics, even though I do believe that it can provide some fresh perspective on predicate modal and tense logic[34]

[32] I owe this last reference, and some of the ideas in this chapter, to Robert van Rooy.

[33] *The negative verdict on the prospects for a viable traditional logic [i.e., pre-fregean logic] has had the effect of deflecting twentieth-century logic from its traditional task of characterizing the canonical fragment of natural language to the quite different task of constructing powerful but artificial logical languages.* [Som82][p.11]

[34] Basically, predicate modal logic is a 'monster' created by the simple addition of modal

So, the aim of this chapter was actually to clarify some aspects of Ockham's system, which, for the modern reader, may become clearer through a comparison with some well-known recent results and ideas. It is not a surprise that such similarities exist, after all (that is how things go) ideas come and go, but each time dressed in different clothes.

In addition, in this chapter I spelled out some notions and concepts that will be of great importance for the reconstruction in Chapter 3, in particular those related to domains (2.1.3), to the frames to be adopted (2.2) and to some important semantic facts about designation and reference (2.3). I thus hope that all tools are wetted for Chapter 3.

operators to quantified predicate logic, as it had successfully been done for propositional logic; but predicate modal logic ended up raising many unexpected problems (even though there were hints that it would not be as simple as in propositional logic, starting with the basic fact that propositional logic is decidable, whereas first-order predicate logic is not). Hence, I do think it is misleading to consider it to be self-evident that a modal language with referential power should just be an extension of predicate logic with the addition of modal operators. Alternatively, it seems that interesting results on 'referential modal logic' are being produced in hybrid logic [Bla00], which abandoned the standard quantificational perspective altogether.

Chapter 3

Truth Conditions

In this last chapter I will attempt to provide truth conditions of quantified, modal and temporal propositions, proceeding by increasing complexity. I will try to remain as faithful as possible to Ockham, even if what he proposes may seem occasionally awkward (otherwise, this would not be a study of Ockham's logic anymore).

Introduction: Syntax

As this chapter is almost exclusively about the semantics in Ockham's logic, first I must provide the syntax of the language whose propositions' truth conditions I will define.

Alphabet

a, b, c ... ; A, B, C ... ; $T\alpha$, $T\beta$ ... ; $\bar{a}$, $\bar{b}$ ... ; $\forall$, $\exists$, $\Box$, $\Diamond$, $\circ$, $\neg$, P, F.

Terms

- Small letters: a, b, c ...

  They denote general terms.

- Capital letters: S, J, O ...

  They denote proper names.

- T followed by a Greek letter (demonstrative terms): $T\delta$, $T\varsigma$ ...

  They denote the demonstration of the object denoted by the Greek letter in the meta-language.

- For each small letter, the same letter with a bar (negative terms): $\bar{a}, \bar{b}$ ...

  They denote the complementary terms 'non-a'.

- $\forall$ and $\exists$ followed by a small letter (quantified terms): $\forall a$, $\exists b$ ...
- Modes: $\Box$ and $\Diamond$.

Copula

- Basic copula: $\circ$
- Negative copula: $\neg\circ$
- Tense copulas: $(P\circ)$ (past) and $(F\circ)$ (future)
- Modal copulas: $(\Box\circ)$ (necessary) and $(\Diamond\circ)$ (possible)

Basic propositions

- Subject terms: Small letters, capital letters, demonstrative terms $(T\delta)$, quantified terms $(\forall a, \exists b)$.
- Predicate terms: Small letters, capital letters (provided the subject term is of the form $T\delta$), negative terms $(\bar{a})$.

Let s be a subject term, $*\circ$ any copula and p a predicate term. A basic proposition is of the form:

$$s * \circ p$$

Intensional propositions with dictum

- Tense operators: P and F
- Modal operators: $\Box$ and $\Diamond$

Let μ be any intensional operator. Then an intensional proposition *with dictum* is of the form:

$$\mu(s * \circ p)$$

For the modal operators only, a modal proposition with dictum is also of the form:

$$q \circ \mu$$

where q is a basic proposition.

Well-formed propositions

Well-formed propositions are either basic propositions or intensional propositions *with a dictum*, as defined.

3.1 Truth

One of the most interesting features of Ockham's logic is his notion of truth: it is surprisingly close to some XXth century theories of truth in the sense that it is foremost of a *formal* nature, related to the notion of reference, and not to the notion of description and correspondence with reality.

The standard notion of truth at the time, still widely commonsensical, is Aristotle's definition: a proposition is true if it asserts that what is the case *is* the case, or that what is not case *is not* the case; a proposition is false if it asserts that what is not the case is the case, or that what is the case is not the case. Such a definition is realistic in that it relies on the descriptive power of language to depict reality, on its representative content. The state of affairs define which propositions are true and which are false, according to their corresponding or not corresponding to it.

Ockham's definition of truth abandons much of this dependency with respect to reality, and posits truth as a semantical and formal property based on the reference of terms (their supposition, of course). The quotation below outlines Ockham's rejection of more realistic conceptions of truth:

> On this point it should be noted that for the truth of such a singular proposition which is not equivalent to several propositions it is not required that the subject and the predicate be really identical, or that the predicate be in reality in the subject or that it really inhere in the subject, or that the predicate be united to the subject outside the mind. ... Rather, it is sufficient and necessary that the subject and predicate supposit for the same thing. [Ock80][p.86]

So, in conformity with the two-term theory of propositions, truth is defined in terms of the semantics of each one of the two terms, subject and predicate - namely, that they both should supposit for the same thing so that a proposition be a true proposition. Again, nothing is said about what the subject or the predicate actually are, because they have hardly any descriptive content: they are only terms which refer to, or supposit for, something, with no ontological commitment. So, it is not all the way accurate to describe Ockham's theory of truth as 'truth as identity', as opposed to the 'truth as inherence' theory advocated by some[1]. The point is that the 'identity' which guarantees the truth of a proposition takes place only on the level of the semantic relation between terms and thing, and not on the level of the entities properly speaking.

However, it would be incorrect to say that Ockham's theory of truth breaks completely with all descriptive content: terms do not supposit arbitrarily for whichever things one may choose, like pure labels. As stressed many times before, terms can primarily supposit for the things they can be predicated of by means of a demonstrative - the familiar $T\delta \circ a$ propositions -, and this relation of

[1] Be it in terms of inherence, which is an ontologically embedded notion, or more simply in terms of membership. In terms of inherence, it is for instance the notion of predication sustained by many medieval philosophers, for example Thomas Aquinas.

predication is not arbitrary. Of course, it would be possible to provide a realistic account of it - terms can be predicated of things insofar as thing truly *are* what is predicated of them -, but one may also choose to disregard this ontological commitment and focus on the referential power of terms.

In any case, Ockham's notion of truth is primarily based on the concept of supposition, namely, of identity of supposition: a proposition is true iff its two extremes supposit for the same individual(s). Why not, then, choose supposition as primitive notion, instead of predication? Again, because supposition can be defined in terms of predication, and most of all, because real supposition theory involves pragmatic elements, along with sentential constraints, which do not easily fit in a reconstruction[2].

> First, a sign is said to signify something when it supposits for or is capable of suppositing for that thing in such a way that the name can, with the verb 'to be' intervening, be predicated of a pronoun referring to that thing. [Ock74][p.113]

As far as notation goes, since truth is a local property of propositions (Cf. 2.2), I shall use the standard notation for satisfiability in a world w - $w \Vdash \Phi$ - to express that a proposition Φ is true in w. This notation will be used both in the object-language and in the meta-language.

3.2 Present-tense propositions with two terms

In this section, 'term' should be understood as defined in 2.1.2, that is, as t*. So, 'two terms' means a concatenation of any number of categorematic terms plus the copula, including compound expressions and so-called two-place predicates. Excluded for the moment are quantifying signs, tensed copulas and modal expressions.

3.2.1 Basic case

As mentioned in the conclusion of Chapter 1, the choice of propositions of the form $T\delta \circ a$ as primitive elements for a reconstruction of Ockham's logic is controversial: it can be argued that the truth of a proposition cannot be defined prior to the reference of its terms, and that propositions of this kind are not different from any other proposition, its truth being equally defined in terms of the terms' supposition. This is certainly the case, according to Ockham himself:

> Thus, for the truth of 'This is an angel', it is not required that the common term 'angel' be really identical with what is posited as the subject, or that it be really in that subject, or anything of the sort. ... And, therefore, if in 'This is an angel' the subject and the

[2]This is one possible path to expand the analysis of Ockham's logic, a formalization of supposition theory.

predicate supposit for the same thing, the proposition will be true. [Ock80][p.86]

Again, notice that the demonstrated object does not have to actually *be* an angel, on the essential level: it is enough that the term 'angel' supposits for that object just as much as the demonstrative 'This' does. Now, when can the term 'angel' supposit for something? Well, if it can be truly predicated of it[3], by means of a demonstrative... This is the dangerous circularity underlying Ockham's system, which can only be avoided by the choice of one primitive notion, be it supposition or predication.

True enough, the choice of predication as primitive raises the problem of focusing on the truth of some specific propositions $T\delta \circ a$, instead of on the reference of terms: actually, the truth of such propositions *defines* the (possible) reference of a term a. Nevertheless, such propositions are not real propositions, they are *semi-propositions*, since they relate a term to a thing. This is the reason why I insist on the tag metaphor: it is as if $T\delta \circ a$ propositions ascribed tags to things, not arbitrary tags, but tags corresponding to terms that can be truly predicated of things[4]. In this sense, reference and truth pertain to the same basic level, both defined by the tags that individuals bear.

So, the valuation of this system operates on the truth of $T\delta \circ a$ propositions, what actually corresponds to defining the (possible) reference of terms. This fact is related to the hybrid feature of Ockham's logic, namely that, unlike standard predicate logic, the valuation concerns *propositions*, and not the status of individuals (their membership to certain sets).

Hence, for each world w, each term a and each individual δ, we have that:

Definition 3.2.1 [Valuation] For each individual in each world, the valuation defines which terms can be truly predicated of it - that is, which 'tags' it bears - by means of the (defined) truth value of $T\delta \circ a$ propositions.

For example, consider individual δ in world w: $w \Vdash T\delta \circ a, w \nVdash T\delta \circ b, w \Vdash T\delta \circ S \ldots$, for all terms.

As displayed in figure 3.1, individuals bear 'tags' in each world, corresponding to the terms that are truly predicated of them. These terms may change from world to world for each individual, except for their proper name (denoted by a capital letter). ('I', in the figure, stands for the relation of transworld identity.)

3.2.2 Singular and indefinite propositions

By means of the valuation above specified, it is possible to provide the truth conditions of basic two-term propositions, be they discrete or common proposi-

[3] Here we are considering personal supposition, but as simple and material supposition are defined in terms of personal supposition, there does not seem to be a loss of generality.

[4] Think of a collection of cans in a kitchen cupboard, each of them containing some kind of spice, and of the act of tagging each of these cans with the appropriate term - namely the spice inside. The difference here is that this procedure is extended to all terms that could be predicated of something, such as color, size, but also non-concrete features.

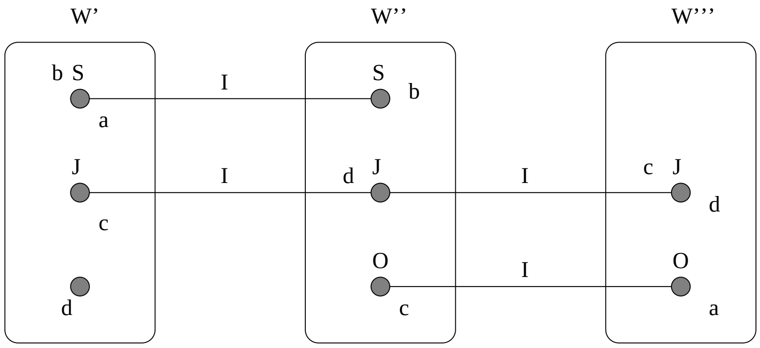

Figure 3.1: A model.

tions - that is, whether the subject has unique supposition or not. As a matter of fact, as far as truth conditions go, nothing differentiates a discrete from a common term, just as they had symmetrical definitions in 1.2.1.

Again, Ockham insists on the formal nature of the truth of a given proposition, instead of its truth depending on the state of affairs, as can be seen in the quotation below:

> Similarly, by means of propositions like ‘Socrates is a man’ and ‘Socrates is an animal’ it is not asserted that Socrates has humanity or animality. ... Rather, it is asserted that Socrates is truly a man and is truly an animal. Nor, indeed, is it asserted that Socrates is the predicate ‘man’ or the predicate ‘animal’. Rather, it is asserted that he is a thing for which the predicate ‘man’ or the predicate ‘animal’ stands or supposits. For both of these predicates stand for Socrates. [Ock80][p.87]

Therefore, the truth conditions of a singular proposition whose subject is a proper name are the following:

Definition 3.2.2 $w \Vdash S \circ a \iff$ There is one individual ς such that $w \Vdash T\varsigma \circ S$ and $w \Vdash T\varsigma \circ a$

What is peculiar about proper names is that their meaningful use in a given world w does not imply their existence in this world, as argued in 2.3.1. However, since the predicate also supposits, the thing for which the predicate supposits must exist in that particular world[5] - which, in the case of a true proposition, will imply that the subject also exists in this world (as both subject and predicate then supposit for the same thing). But the existential claim rests upon the predicate (which is related to the ‘form appellation’ to be discussed in 3.5.1).

[5]With the exception of θ predicates discussed in 2.1.3

Let us now analyze the case of singular propositions whose subject is a demonstrative followed by a proper name. They are, as previously argued, very different from $T\delta \circ a$ propositions, because they do not relate a term to a thing, but rather 'pick' one single element out of a collection of possible supposita for a common term. Hence, here the demonstrative is more of a choice operator, conveying singularity to a proposition whose subject is a common term.

Definition 3.2.3 $w \Vdash Ta \circ b \Longleftrightarrow$ There is one individual δ in w such that $w \Vdash T\delta \circ a$ and $w \Vdash T\delta \circ b$

In particular, this kind of singular propositions also defines the truth of indefinite propositions, that is, propositions whose subject is a common term alone[6]. This is indeed what Ockham does in the following quotation:

> For example, the truth of 'This animal is a man' or 'That animal is a man' is sufficient for the truth of 'An animal is a man'. [Ock80][p.92]

However, as we know, such singular propositions are better viewed as part of Ockham's inference theory, and not of his truth-condition theory. Moreover, since the truth of these singular propositions can be defined in terms of the truth of simple $T\delta \circ a$ propositions, both formulations are equivalent. Hence, for the sake of simplicity, it seems more suitable to provide the truth conditions of indefinite propositions in terms of the usual primitive element, which is by the way Ockham's own strategy on many occasions.

Definition 3.2.4 $w \Vdash a \circ b \Longleftrightarrow$ There is an individual δ in w such that $w \Vdash T\delta \circ a$ and $w \Vdash T\delta \circ b$.

Notice that we lose the uniqueness featuring in singular propositions, but not the existential import - in fact, indefinite propositions are just as 'referential' as singular propositions. Furthermore, their truth conditions are virtually identical to the truth conditions of singular propositions. This was to be expected, and is in conformity with the intuition that, for Ockham, all terms behave as names, the difference being that some have unique denotation and some have multiple denotation (and proper names have an exclusive durative effect, hence they do not imply the present existence of their significata).

> But what is sufficient for the truth of such an indefinite proposition - if, indeed, it is indefinite? It should be said that it is sufficient for the truth of such a proposition that the subject and predicate supposit for the same thing if the proposition is affirmative, or that they not supposit for the same thing if the proposition is negative. Thus, what suffices for the truth of a singular proposition suffices here, for our understanding with respect to such a proposition is the same as that with respect to a singular proposition. [Ock80][p.94]

[6]Bear in mind that there are not articles in Latin, neither definite nor indefinite.

Hence, terms in indefinite propositions have just as much referential power as the subject of singular propositions - of course, since their denotation is also individuals, and not sets of individuals, as in the standard semantics for predicate logic. Further analysis of indefinite propositions (with respect to simple and material supposition, and not only personal supposition) is to be provided in 3.4.1.

3.3 Negation and vacuous supposition

Among those who do not agree that Ockham's theory of truth is defined in terms of identity of supposition in a proposition, such as Freddoso in [Fre80] - and hence maintain that it is actually quite aristotelian, and not formal, as I claim -, one common argument is to sustain that identity of supposition cannot account for negative propositions. But this is not true, and I shall attempt to show in 3.3.1 that Ockham himself provides the truth conditions for negative propositions within the framework of his truth theory.

It will come out of his definitions that, as suspected, affirmative propositions do presuppose the present existence of their subject, with the exception of proper names as subjects, which will lead to the issue of vacuous supposition in 3.3.2 below.

3.3.1 Negation

Firstly, it must be said that Ockham's treatment of the negation is quite different from the present-time standard treatment in that it is not *sentential*. To my knowledge, there are no elements in Ockham's text which would point towards our notion of sentential negation, that is, 'It is not the case that ... '. Alternatively, he deals with *copula* negation, that is 'a is not b', which is not at all equivalent to 'It is not the case that a is b'. The negative counterpart of an affirmative proposition is not its contradictory (as in the case of sentential negation), but rather its sub-contrary (the two propositions can be true simultaneously), since they are both existential propositions in what they claim (Cf. the truth conditions just below).

Still, with the appropriate definitions, it is possible to account for the semantics of most if not all propositions based on the notion of copula negation. I shall first be concerned with indefinite affirmative and negative propositions; particular and universal (quantified) negative propositions shall be analyzed in due course.

The truth conditions for a negative proposition as defined by Ockham are rather intuitive insofar as they are symmetrically opposite to the truth conditions for affirmative propositions. A two-term affirmative proposition is true iff its two terms supposit for the same thing; accordingly, a negative proposition is true if its two terms do **not** supposit for the same thing, including the case when the subject does not supposit for anything - in which case they obviously do not supposit for the same thing.

> On the other hand, if such a proposition is negative, then it is required that the subject and the predicate not supposit for all the same things. In fact, it is required either that the subject supposit for nothing or that it supposit for something for which the predicate does not supposit. [Ock80][p.92]

So, the truth of a negative proposition is twofold. In a way, the truth of an affirmative proposition is also twofold in that it implies the existence of the suppositum. That is why, according to Karger in [Kar76], there are no such things are pure categorical propositions in Ockham's logic: an affirmative proposition such as $a \circ b$ would have to be transcribed as:

$$\exists a \wedge a \circ b$$

However, I prefer to consider the existence of the subject as a presupposition related to the very notion of supposition, which defines Ockham's truth theory - a term supposits for something that must exist. (Therefore, I include the presupposition of existence in the meta-language, and not in the object-language.)

Thus, an (indefinite) affirmative proposition is false iff its two terms do not supposit for the same thing or if the subject does not supposit for anything. Accordingly, this is the very definition of a (indefinite) negative proposition's truth conditions: the opposite of one of the two necessary and sufficient conditions for the truth of an affirmative proposition must hold for the truth of a negative proposition. So, either it is vacuously true, or it is existentially true - there is one individual of which the subject of the given negative proposition can be truly predicated and of which the predicate of such proposition cannot be truly predicated. In the context of sentential negation, the negative counterpart of an indefinite affirmative proposition would be a universal proposition of the form 'No a is b'.

> Thus, a negative proposition has two causes of truth. 'A man is not white', for example, has two causes of truth; for either there are no men, so that no man can be white; or there is a man but he is not white. [Ock74][p.206]

We can thus accept the following definition of the truth conditions of such a negative proposition:

Definition 3.3.1 $w \Vdash a \neg \circ b \iff$ (There is no individual δ in w such that $w \Vdash T\delta \circ a$) $\vee$ (There is an individual δ in w such that $w \Vdash T\delta \circ a$ and $w \nVdash T\delta \circ b$)

So, even though the copula negation does not range over the whole proposition, its scope is wide enough to force the possibility of vacuous truth of a negative proposition.

Furthermore, Ockham analyzes a case of narrow scope of the negation, namely when it ranges only over the predicate - what, in set-theoretical terms, would define the set complementary to the set associated with a given predicate. For Ockham, this is not a real negative proposition, but actually an affirmative proposition whose predicate is of the form 'non-a'.

> Hence, 'A donkey is a non-man' ['Asinus est non-homo'] is equivalent to 'A donkey is something and a donkey is not a man'. [Ock80][p.119]

That is to say, the truth conditions of such a proposition include the existential presupposition of an affirmative proposition, plus the non-identical supposition of terms such as in one of the clauses for the truth of a negative proposition.

Definition 3.3.2 $w \Vdash a \circ \bar{b} \Longleftrightarrow$ There is an individual δ in w such that $w \Vdash T\delta \circ a$ and $w \nVdash T\delta \circ b$.

It might seem that the lack of sentential negation is a limitation in the system here presented. This may very well be true, but in fact Ockham did not ignore the notion of contradiction - two propositions that cannot simultaneously be true -, as it was a well-established feature of the classical square of opposites. The real difference between his account and ours is that a contradiction was not straightfowardly obtained by the introduction of a negation in front of an affirmative proposition, as the negation was merely a matter of copula negation. But the copula negation is strong enough to account for all four cases of quantification, as we shall see below.

Besides, it is not impossible to introduce *ad hoc* sentential negation in this system; it is indeed not present in Ockham's texts, but there are no indications that it would be a mistake to do so either. In this case, the present system would feature more possibilities of negation than standard fregean predicate logic, which is related to Ockham's term-based approach, as opposed to an (exclusively) sentential approach.

3.3.2 Vacuous supposition

As argued, Ockham does consider the possibility of a term suppositing for something that does not exist, hence not suppositing for anything[7]. In other terms, a term can have vacuous supposition.

Ockham simply deals with vacuous supposition within the framework of his two-valued logic, and not by introducing a third value 'undefined', which is a rather common procedure in quantified modal logic (Cf.[Gam91][ch.3]). Instead, given the existential presupposition of affirmative propositions, if the subject of such propositions has vacuous supposition, then the proposition is plainly false.

> In affirmative propositions a term is asserted to supposit for something. Thus, if it supposits for nothing the proposition is false. [Ock74][p.206]

Conversely, as previously argued, a negative proposition whose subject has vacuous supposition is by definition true, which leads sometimes to awkward conclusions such as in the following quotation:

[7] It is sometimes said about his logic that it does not and cannot cope with such cases, given its explicit extensionalist orientation. This is in fact not true, as we see.

> For example, if no man is white, then 'A white man is not a man' is true. [Ock80][p.93]

It can be argued that this view on vacuous supposition leads to even more counter-intuitive results such as this: if all terms have strictly personal supposition, then 'Chimaera is composed of goat and cow' is false, whereas 'Chimaera is not composed of goat and cow' is true (for a solution to this paradox, see 2.1.1). This is indeed the price to pay for the choice of an extensional semantics and of a two-valued logic, but since Ockham also introduces other semantic 'tools', such as other kinds of supposition, connotation etc..., it is usually possible, case by case, to solve such paradoxes and at the same time to hold on to extensionality and to two truth values.

Thus, we can provide the following definition:

Definition 3.3.3 If there is no individual δ in w such that $w \Vdash T\delta \circ a$, then the term a has *vacuous supposition.*

If a has vacuous supposition in a proposition $a \circ b$ in a world w, then $w \nVdash a \circ b$ and $w \Vdash a\neg \circ b$ for any term b.

What appears to be clear from this definition is that we obviously have to approach the domain issue again, with respect to vacuous supposition. Again, the domains Ockham considers are *local domains*, composed of individuals pertaining to one world. A term in a proposition is said to have vacuous supposition *in a world*, that is, if its suppositum does not exist in the world where the proposition holds.

However, as a consequence of previous results, we know that proper names have transworld domains, that is, they signify and supposit for someone even if the individual does not exist in that particular world (they are durative). Hence, a proper name will have vacuous supposition only if its suppositum does not exist in *any* of the worlds in the model. The same property holds for terms taken in large signification, as argued in 2.1.3. So:

Definition 3.3.4 A term S or a_l has vacuous supposition iff there is no individual δ in any world w of the model such that $w \Vdash T\delta \circ a$.

3.4 'Quantified' propositions

I use the term 'quantified' between inverted commas because, as extensively argued, Ockham's views on what we call 'quantification' are actually quite far off from our standard procedures; to start with, Ockham views quantification as a semantic procedure more than as a syntactic one. Particular and universal 'signs' (syncategorematic terms such as 'some', 'every', 'no' etc...) are added to the extreme of a proposition to define the supposition of the *whole term*; that is, in 'Every man is an animal', it is not 'man' that supposits, but rather 'Every man'.

Moreover, quantifiers do not effect variables, as in standard quantified theory, but actually terms, which are complete without the addition of the quantifying sign.

Still, the truth conditions for particular and universal propositions can be successfully rendered, and are in fact not very different from the usual truth conditions for quantified sentences in standard predicate logic.

3.4.1 Particular propositions

As hinted in 3.2.2, particular and indefinite propositions have the feature of sharing the same truth conditions when they supposit personally. This is related to the existential pressupposition of affirmative indefinite propositions, which is then made explicit in particular propositions by the addition of the 'quantifier'. Hence, it seems that the addition of the particular sign 'aliquod' - which can fairly well be translated as 'some' - to a common term in a proposition is absolutely superfluous, since it does not alter the truth conditions of the given proposition, if it is taken in personal supposition.

> It should first be noted that if a proposition is not called indefinite or particular except when its subject term supposits personally, then an indefinite and a particular are always interchangeable. [Ock80][p.91]

One wonders why then Ockham simply did not collapse the two kinds of propositions in only one. Now, firstly it must be said that, even though this distinction does not pertain to the issue on truth conditions, the referential mechanism of an expression such as 'Aliquis homo' is different from that of 'Homo', both taken in a proposition. Just as much as, in English, 'Some man', 'A man' and 'The man' perform different tasks in the discourse, without the particular sign 'aliquod', a general term has a more determinate supposition than if the sign is added.

However, if we consider propositions by themselves, outside the discourse or an entailment chain, particular and indefinite propositions will have the same truth conditions provided their subjects supposit personally. Thus:

Definition 3.4.1 $w \Vdash \exists a \circ b \iff$ There is an individual δ in w such that $w \Vdash T\delta \circ a$ and $w \Vdash T\delta \circ b$.

However, if we do not restrict ourselves to personal supposition, then differences between particular and indefinite propositions do rise, and they are no longer interchangeable. As to be expected, if the supposition of a term undergoes what I called vertical variation (Cf. 2.2.2), the truth conditions of the proposition also change, and then the equivalence does not hold anymore.

> Second, it should be noted that if anyone claims that every proposition is indefinite in which the subject is a common term without a sign - whether it supposits personally or simply or materially -

> then it follows that he should say that a particular and an indefinite are not always interchangeable. This happens when the subject of the indefinite supposits simply and the subject of the particular supposits personally. For example, these two propositions are not interchangeable: 'Man is a species' and 'Some man is a species'. [Ock80][p.93]

On the one hand, if 'Man' has simple supposition in the proposition 'Man is a species', then the proposition is true, because both terms supposit for the same thing, namely the mental term 'man', of which 'species' is truly predicated. On the other hand, 'Some man' has personal supposition in 'Some man is a species', it supposits for a man, hence the proposition is false - since 'species' can only supposit for a mental term, not for a man, the two terms do not supposit for the same thing. So, under this reading, particular and indefinite propositions are not interchangeable, as they do not have the same truth values.

One may wonder: what then if both propositions have simple supposition? Would they be interchangeable then? In fact, Ockham does not allow for the subject of a particular propositions to have other supposition than personal, which in our terms would mean that he allows for quantification only on the domain of extra-mental things.

> And so 'Man is a species' must be distinguished, since 'man' can supposit either simply or personally. However, 'Some man is a species' should not be distinguished, since in it the term 'man' can supposit only personally. The reason is that a particular sign is added to it. [Ock80][p.94]

It is understandable from an ontological point of view why Ockham chose to restrict the range of particular signs such as 'some' (and presumably, also of universal signs such as 'every' and 'no') to the domain of extra-mental things, for this is the well established domain of perceived individuals. However, it is none the less intriguing to imagine the possibility of quantification also over concepts and written and spoken words, maybe in the context of a sorted logic. In any case, this would be an extension of Ockham's ideas which is not present in his own texts, so we should drop it for the moment.

One important point to be settled is the effect of the negation in a particular proposition. According to the famous square of opposites, a proposition such as 'Some a is b' is the contradictory of 'No a is b', but is the *sub-contrary* of 'Some a is not be'. In fact, the square of opposites allows for more possibilities of negation than fregean predicate logic, in which *all negativity is sentential or definable by sentential negation.* [Som82][p.5]. So, in the square of opposites propositions have a contradictory but also a contrary or a sub-contrary - two contrary propositions can both be false at the time, and two sub-contrary propositions can both be true at the same time[8] . For what matters to us now, the sub-contrary of a particular proposition is also a particular proposition.

[8] Although Ockham does not explicitly mention the traditional square, we can provide the following 'ockhamist' formulation of it: Consider (1) $\forall a \circ b$; (2) $\forall a(\neg\circ)b$; (3) $\exists a \circ b$ and (4)

> The truth of 'This animal is not a man', referring to any animal, is sufficient for the truth of 'Some animal is not a man'. [Ock80][p.92]

Hence, the following truth condition would seem to be accurate:

Definition 3.4.2 $w \Vdash \exists a(\neg\circ)b \iff$ (There are no individuals δ in w such that $w \Vdash T\delta \circ a$) or (There is an individual δ in w such that $w \Vdash T\delta a$ and $w \nVdash T\delta b$).

It may seem extremely awkward that an existential proposition such as this can be vacuously true if there are no supposita for its subject term - it is in any case not so according to standard predicate logic. But Ockham stresses himself that the truth conditions of indefinite and particular propositions are the same provided their terms supposit personally, so this must also hold for negative propositions. In fact, these are the truth conditions for indefinite negative propositions defined in 3.3.1.

Again, I shall not treat sentential negation of quantified propositions, as Ockham does not consider this possibility himself, but it would seem that the usual equivalence,

$$w \Vdash \neg\forall(a \circ b) \iff w \Vdash \exists(a \neg \circ b)$$

would hold.

3.4.2 Universal propositions

Universal propositions are, according to Ockham, those which feature one of what he calls universal signs; the list thereof is more extensive than what standard predicate logic usually considers to be universal quantifiers. He also introduces the notion of universal sign ranging over parts of individuals, which I shall not treat.

> It should first be noted that there are several types of universal propositions, corresponding to the diversity of universal signs. ... e.g. 'every', 'any', 'each one', 'no', 'both', 'neither', and so on. [Ock80][p.95]

In this chapter, I shall only treat the two 'standard' universal signs, namely 'Every' and 'No' - to be respectively transcribed as $\forall$ and $\neg\exists$. It would be an interesting challenge to analyze also the truth conditions of the other universal signs (something similar to the semantics of determiners established by means of generalized quantifiers), but I will not do it for now.

As to be expected, the truth conditions of universal propositions which feature the sign 'Every' can be provided in terms of the truth value of the usual nuclear element, $T\delta \circ a$ propositions. Again, Ockham criticizes a realistic notion of truth, and proposes his own formal version of it, based on the concepts of supposition and predication.

$\exists a(\neg\circ)b$. (1) and (4), as well as (2) and (3) are contradictories; (1) and (2) are contraries; (3) and (4) are sub-contraries.

> It should first be noted that for the truth of such a universal proposition it is not required that the subject and predicate be in reality the same thing. Rather, it is required that the predicate supposit for all those things that the subject supposits for, so that it is truly predicated of them. [Ock80][p.97]

In fact, when it comes to personal supposition (which is the case here, as previously argued), the notions of supposition and predication are interchangeable in the definition of truth conditions. Ockham often makes use of both, perhaps to stress the role played by the salient singular propositions. Moreover, in the case of a subject introduced by the universal sign 'Every' in a proposition, the supposition of a term is equivalent to its signification (in a specific domain, hence strict signification), as a term signifies all which it can be truly predicated of. So, if we take the proposition 'Every man is an animal', it is true for the following reasons:

> Similarly, the predicate truly supposits for each thing that the subject supposits for and is truly predicated of each thing that the subject is truly predicated of. Therefore, the proposition is simply true. [Ock80][p.97]

Even though, in the first part of *Summa Logicae*, Ockham hints that a universal proposition of the kind 'Every a is b' could not be inferred from the conjunction of its singulars (Cf. 1.3.3)- namely, 'This a is b', 'That a is b' etc... -, singular propositions can in fact account for its truth conditions, since, as stressed before, inference and truth conditions are two different notions in Ockham's system. In particular, the truth of a universal proposition is not defined as a conjunction of singulars; the wide range of such propositions is evoked in the meta-language only.

Definition 3.4.3 $w \Vdash \forall a \circ b \Longleftrightarrow$ (There is at least one individual δ in w such that $w \Vdash T\delta \circ a$) and (For all individuals δ in w such that $w \Vdash T\delta \circ a$, $w \Vdash T\delta \circ b$).

One of the interesting features (from a modern perspective) of Ockham's account of universal propositions of the kind 'Every a is b' is that they cannot be vacuously true[9]. In standard predicate logic, the transcription of such propositions is:

$$\forall x(A(x) \rightarrow B(x))$$

So, if there are no x which belong to A - alternatively, if there is no element in the set defined by the property A -, then the proposition is said to be vacuously true, for an implication whose antecedent is false is always true. The same does not hold in Ockham's account of 'Every a is b' propositions: they are affirmative

[9] Some modern systems also ascribe existential import to the universal quantifier, Cf. Free Logic.

propositions, thus they are false if the subject has vacuous supposition, as argued above[10].

Can we forgive Ockham for such a deviation from what we accept to be correct? Are we willing to give up on the possibility of universal propositions being vacuously true? The good news is that we do not have to, as this familiar principle can be re-established with the aid of another distinction, namely between categorical and hypothetical propositions. The standard translation into predicate logic of 'Every a is b' propositions is actually, from an ockhamist point of view, a hypothetical proposition, as it makes use of the material implication, and is not equivalent to the categorical proposition $\forall a \circ b$. Admittedly, he does not discuss the specific case of universal propositions, but he does discuss other cases of categorical and hypothetical propositions which seem to be equivalent but are not:

> Hence, these propositions are distinct: 'A donkey is an animal' and 'If a donkey exists, an animal exists'. For the one is categorical and the other is conditional and hypothetical - and they are not interchangeable. [Ock80][p.99]

Moreover, Ockham does admit that an implication whose antecedent is false is true (Cf. [Ock57][p.84]). So, the usual truth conditions of universal propositions can also be found in Ockham's system, provided we question the uniqueness of their translation and accept that there are two similar but not equivalent translations for them.

The case of 'No a is b' universal propositions meets the usual clauses for negative propositions: they are true iff there are no individuals of which 'a' is truly predicated (vacuous supposition) or there are no individuals of which both 'a' and 'b' are truly predicated. This can be more precisely stated as follows:

Definition 3.4.4 $w \Vdash \forall(a \neg \circ b) \iff$ (There are no individuals δ in w such that $w \Vdash T\delta \circ a$) or (For all individuals δ in w such that $w \Vdash T\delta \circ a$, $w \nVdash T\delta \circ b$)

Notice that, while $w \Vdash \exists a \neg b$ does not imply that there is an individual δ in w such that $w \Vdash T\delta \circ a$, this does not hold if the negation only applies to the term (term negation). That is, if $w \Vdash a \circ \bar{b}$, then there is at least one individual δ in w such that $w \Vdash T\delta \circ a$ (and by the way, $w \nVdash T\delta \circ b$).

That is, given the existential import of affirmative propositions, and accordingly the possibility of vacuous truth for negative propositions, a proposition of the form 'No a is b' is successfully transcribed as 'Every a is not be'. Again, the equivalence with a sentential negation of the form $\neg\exists(a \circ b)$ would certainly hold, but such cases are simply not treated by Ockham.

[10]It is the counterpart of the fact that existential negative propositions can be vacuously true.

3.5 Temporal propositions

According to Ockham, temporal propositions - whose tense is assigned by the copula, as argued in 2.2.3 - suffer from a systematic ambiguity in written and spoken language: the same temporal proposition always bears two readings, namely one according to which the subject supposits for something that was or will be (in conformity with the tense of the copula), and another according to which the subject supposits for something that is at the moment the proposition holds.

> For the subject can supposit either for that which is such-and-such or for that which was such-and-such, if the proposition is past-tense; or it can supposit either for that which is such-and-such or for that which will be such and such, if the proposition is future-tense. [Ock80][p.105]

On the other hand, in both cases the supposition of the predicate is unambiguous: it supposits for what was or will be.

> In either case, if the proposition is affirmative, it is required that the predicate under its proper form, i.e. the very same predicate, be truly predicated of that for which the subject supposits. [Ock80][p.105]

That the predicate always supposits for what was or will be, depending on the copula's tense, is what is meant by the expression 'the predicate appellates its form', which I shall explore in 3.5.1 below.

Moreover, the ambiguity evoked above which affects written and spoken tense propositions presumably does not affect mental language, since two distinct logical structures are certainly differentiated in the intellect. So, I shall introduce two tense *operators*, P for past and F for future, which may seem illegitimate, since in Ockham's account the 'temporal burden' is born by the copula only. But again, this is the case only in conventional language[11], where the ambiguity exists, so, in terms of logical structure, it seems justified to introduce these operators.

3.5.1 Predicate appellates its form

So far, I have emphatically insisted on the symmetry between subject and predicate, as predicates do *not* denote a set or a property in Ockham's account, but rather individuals, just as much as subject terms.

But this holds only for present-tense non-modal propositions. We shall see now that, in temporal and modal propositions, subject and predicate do not have identical semantic behavior - not that the latter denotes something other than individuals, but what it can supposit for is not the same as what the former can supposit for.

[11] Bear in mind that what Ockham means by 'conventional language' is what we now understand as 'natural language', that is, written and spoken language. Cf.1.1.3.

As I said, the subject of a temporal proposition can supposit for something that existed or will exist at the time corresponding to the proposition's tense or for something that exists at the moment the proposition holds[12]. The predicate, alternatively, can only supposit for something that existed or will exist, and that is what Ockham means by the predicate appellating it form.

As for the very expression 'appellation of the form', it is quite unclear what 'appellation' and 'form' mean within Ockham's framework. Panaccio provides a brief explanation on the history and meaning of this expression ([Pan99][p.9-10], but, most significantly, it was a common expression at the time, frequently mentioned in connection with the issue of temporal and modal propositions, and according to Ockham himself, it is an 'ancient dictum' which he adopted out of respect for the tradition. I shall not be concerned with the implications and connotations of this expression, and shall only try to explain its relation to the truth conditions of temporal (and later modal) propositions.

Let us do that by means of an example provided by Ockham: given a certain state of affairs, two propositions which seem to be equivalent - as the subject of one is the predicate of the other and conversely - have in fact different truth values, hence are not equivalent.

> Likewise, if Socrates is now white for the first time, then 'Socrates was white' is false - and yet Socrates was that of which 'white' is now truly predicated. But since 'Socrates is white' was not true, it follows that 'Socrates was white' is now false. [Ock80][p.107]

The term 'white', as the predicate of this proposition, must appellate its form by suppositing for something which *in the past* was white. But since Socrates is now white for the first time, the proposition 'This is white', pointing at Socrates, is not true in the past. Of course, we must rely on transworld identity for this account, as its core idea is that there is a relation of self-identity between Socrates of the past and Socrates of now[13]. In any case, the term 'white' must supposit for something existing in the past (according to the tense of the copula), and whatever it may supposit for then (namely all white things then), it cannot be Socrates, since Socrates was not white then. Hence, the proposition is false.

Alternatively, if 'white' is the subject and 'Socrates' the predicate, then the past-tense proposition is (surprisingly) true, under the same circumstances:

> For example, if Socrates is now white for the first time, then 'A white thing was Socrates' is true, and yet no present-tense proposition in which the predicate 'Socrates' - or that for which it supposits - is predicated of the subject 'white thing' was ever true before. ... For in 'A white thing was Socrates' the term 'white thing' supposits for

[12] I do not want to say 'is uttered', because this would raise issues concerning the ontological status of propositions which I am not going to deal with.

[13] All things are self-identical to themselves in different worlds (where they exist), but, as terms have local extension (Cf. 2.1.3), only proper names are able to express in the language this relation of transworld self-identity.

> Socrates, who is now white. And 'This is Socrates', referring to Socrates, was true before. [Ock80][p.107]

That is, 'a white thing' supposits for something which is now (i.e. the time when the proposition holds) white, namely Socrates. But this white thing which is Socrates was in the past also Socrates (as names are rigid designators), so that 'Socrates' supposits for something which existed in the past, of which 'Socrates' could be truly predicated, and of this same thing 'white' can be *now* truly predicated. So this proposition is true.

These distinctions, which for now may seem quite awkward, shall become clear once we provide formal truth conditions for temporal propositions just below. What is essential for now is that the asymmetry between subject and predicate be evident.

In particular, if a term a is the subject of a temporal proposition, it can supposit for one or more individuals δ, namely all those such that $T\delta \circ a$ is or was/will be true; if a term b is the predicate of a temporal proposition, it can supposit for one or more individuals δ such that $T\delta \circ b$ was/will be true, but not for those such that $T\delta \circ b$ *is* true.

One obvious conclusion is that the tense 'effect' - or maybe by now we can call it 'operator' - always ranges over the predicate, but not always over the subject. Based on this fact, I shall introduce the P and F operators either ranging over the whole proposition - which, however imprecise, I shall call temporal propositions 'de dicto' - or over the predicate only - again, imprecisely stated, temporal propositions 'de re'.

Karger, in [Kar76], carefully notices that tense operators (as much as modal ones) cannot range only over predicates, that they must range over sentences, be they the original ones or 'extracted' ones[14]. The reason for this is an undesirable ontological implication of operators ranging over predicates: it is as if there were 'past or future entities', for which the predicate plus the operator would supposit.

Indeed, it is against Ockham's nominalistic spirit to admit such ethereal entities, but in fact he himself states that the subject term of a temporal proposition can supposit for what is or what was/will be and that its predicate can only supposit for what was/will be. So, I do not see the problem with operators 'ranging' over predicates: it is true that the combination of an operator and a term should not produce a different term, signifying past/future entities, in the manner that other signs can be added to a term to form another term (universal/particular signs, another term etc ...). That is the reason why I place the operator in front of the copula, as in $a(P\circ)b$, and not just before the term, to avoid such connotations. Hence, be it only for convenience, it seems reasonable to say that an operator 'ranges' over a predicate, provided it be so understood. In any case, it is a legitimate heuristic device to explain the difference between the two possible readings of a temporal proposition.

[14] 'Extracted' sentences would be the relevant demonstrative sentences, to be seen in the truth conditions of 'de re' temporal propositions.

Besides, I do adopt a possible-world model, in which there are many worlds, each of them 'inhabited' by individuals. These are not exactly 'past or future entities', for there is not one world which is the milestone dividing past and future. In such a model, the interpretation of a predicate over which an operator ranges is quite intuitive. So I will use the term 'to range over a predicate', which in any case is operationally equivalent to say that an operator ranges over an extracted proposition.

3.5.2 'De dicto'

I reluctantly use the term 'de dicto' for temporal propositions whose tense operator ranges over the whole proposition. But Ockham did not provide a name himself for such propositions, and, for the modern reader, the term 'de dicto' univocally means intensional (modal and temporal) propositions whose operator is 'outside' a non-intensional proposition.

However, in standard modal logic, the term 'de dicto' - and its counterpart 'de re' - only applies to quantified propositions or propositions with variables, for such a distinction does not exist for atomic cases[15]. This is again a consequence of Frege's sentential approach, the echoes of which we have already encountered while dealing with the negation. Once more, that a proposition is 'split' in two terms creates more possibilities for introducing the operators, thereby increasing the expressive power of the language.[16]

Let me provide a brief explanation for the non-familiarized reader. Let μ be any intensional operator. In standard predicate modal logic[17], a proposition is said to have 'de dicto' modality if the operator is outside the scope of the quantifier. Example:

$$\mu\exists x(B(x))$$

Alternatively, intensional 'de re' propositions have the following shape:

$$\exists x(\mu B(x))$$

That is, the operator is inside the scope of the quantifier. However, if the variable x is assigned a term, then there is only one corresponding atomic proposition, namely

$$\mu B(a)$$

For Ockham, however, the distinction between 'de dicto' and 'de re' also holds for atomic propositions, as a consequence of the term point of view that he adopts - which, again, made it essential to introduce the two-term 'split' notation here in use. So, it is the case here that the operator may range over whole atomic propositions **or** only over their predicate.

[15] Disregarding propositions with descriptions.
[16] It is possible that it increases its complexity too, but it is not necessarily so.
[17] Generated by the mere additions of modal operators to standard predicate logic.

In the case presently analyzed, I call temporal propositions 'de dicto' those in which both predicate and subject supposit for something that was/will be, and therefore the temporal operator ranges over the whole proposition. Ockham does not provide examples of this sort, exactly because they are more intuitive than the 'de re' cases, but let us consider an example nevertheless:

$$\text{Dinosaurs were big} \tag{3.1}$$

If this past-tense proposition is to receive a 'de re' reading, then it is false, for then the subject supposits for something that does not exist now. However, if it is to receive the 'de dicto' reading, then it is true.

Let us transcribe such propositions as follows: $P(a \circ b)$. I acknowledge the awkwardness of ascribing an operator to it, whereas the tense is in fact rendered by the copula, but it does seem to be an accurate transcription in terms of its logical structure. Then, its truth conditions are:

Definition 3.5.1 $t \Vdash P(a \circ b) \iff t' \Vdash a \circ b$, for some $t' < t \iff$ For some $t' < t$, there is an individual δ in t' such that $t' \Vdash T\delta \circ a$ and $t' \Vdash T\delta \circ b$.

The same holds for singular, particular and universal propositions: under this reading of temporal propositions, it is stated that the whole proposition is true in a past or future time. Hence, it suffices to apply the usual truth conditions for singular, particular and universal propositions, as defined in 3.2 and 3.4, to a past or future time. The same goes for negative propositions: their truth conditions can be adapted *mutatis mutandi*.

The general scheme for temporal 'de dicto' propositions is:

Definition 3.5.2 $t \Vdash P\Phi \iff t' \Vdash \Phi$, for some $t' < t$, for past-tense propositions.

Definition 3.5.3 $t \Vdash F\Phi \iff t' \Vdash \Phi$, for some $t' > t$, for future-tense propositions.

We shall see that the case of temporal 'de re' propositions is more complex, hence more cases will have to be analyzed.

Moreover, a further discussion on the case of proper names should be undertaken, as a complement to 2.3.1. It shall be undertaken once I have also introduced the definition of 'de re' temporal propositions.

3.5.3 'De re'

As we know, the subject of temporal propositions can supposit for something that exists in the past/future, but also for something that exists in the present. In these cases, the tense operator ranges only over the predicate, which in all cases 'appellates its form'; that is to say, the tense operator is 'inside' the proposition, which is the reason why I chose to call them 'de re' temporal propositions.

In such cases, their truth conditions cannot be expressed in terms of the corresponding present-tense proposition *simpliciter*, obtained by replacing the tensed copula by the present-tense copula. It seems that the whole recursive structure of Ockham's system would be seriously compromised. However, once more the always present nuclear propositions $T\delta \circ a$ will come in handily; in fact, it is by means of such demonstrative propositions that Ockham explains the truth conditions of the so-called 'de re' temporal propositions.

> For example, for the truth of 'The white was black', it is not required that it was ever true that the white is black. What is required is that 'This is black' was once true, where the reference is to something for which the subject supposits in 'The white was black'. [Ock74][p.205]

Thus, Ockham clarifies that the truth conditions of temporal propositions under this reading are not the same as those of 'de dicto' temporal propositions - it is not required that 'The white is black' was true, in which case this temporal proposition would never be true. Rather, the truth of this proposition depends on the past truth of *another* proposition, namely 'This is black', pointing at the suppositum of 'white' in the original proposition.

Definition 3.5.4 $t \Vdash w(P\circ)b \iff$ There is an individual δ in t such that $t \Vdash T\delta \circ w$ and $t' \Vdash T\delta \circ b$, for some $t' < t$.

The same applies to similar future-tense propositions:[18]

> Similarly, 'A boy will be an old man' is true, and yet 'A boy is an old man' will never be true. Rather, 'This is an old man' will be true, referring to that person who now is a boy. [Ock80][p.106]

Definition 3.5.5 $t \Vdash b(F\circ)o \iff$ There is an individual δ in t such that $t \Vdash T\delta \circ b$ and $t' \Vdash T\delta \circ o$, for some $t' > t$.

In fact, the truth of 'de re' temporal propositions can also be defined in terms of the truth of 'de dicto' temporal demonstrative propositions. It is of course the case that, if $t < t'$ and $t' \Vdash T\delta \circ a$, then $t \Vdash F(T\delta \circ a)$. The same holds if $t > t'$, then with the P-operator.

In the cases here analyzed, two terms which are opposite (white-black and boy-old man) are associated in temporal propositions, which means that only under a 'de re' reading can they be true. However, the same holds also for any temporal proposition, it is always possible to give it a 'de re' or a 'de dicto' interpretation. So, again the disambiguation will not take place within written and spoken language: it is always necessary to stipulate which logical structure is intended by the speaker. That is, the supposition of terms can undergo not only horizontal and vertical variation (Cf. 2.2.2), but also a temporal variation (more precisely, only the subject can undergo temporal variation).

[18] It is important to stress the symmetry between past- and future-tense propositions in the *Summa Logicae*, because the picture presented in *Tractatus de Futuris Contingentibus* is a bit different.

There is one exception to the rule: proper names. As previously stated (2.3.1), not only are they rigid designators - they designate the same individual in all worlds where it exists -, but they also have the power to refer to the same individual even in the worlds where it does *not* exist. Due to their durative effect, proper names are the only terms which can escape the locality and the presupposition of existence of every term in a proposition. For instance, affirmative propositions whose subject is a proper name are the only affirmative ones which can be true even though the suppositum of the subject does not exist in the world where they hold.

Let me next present a theorem related to the discussion in 2.3.1 (μ is any modal or temporal operator):

Theorem 3.5.6 $w \vDash S(\mu\circ)w \iff w \Vdash \mu(S \circ w)$

Proof: $\Rightarrow$ Assume $\Vdash S(\mu\circ)w$. So, in some world w' (according to the semantics of μ) there is an individual ς such that $w' \Vdash T\varsigma \circ w$. ¿From the definition of proper names (as rigid designators): $w' \Vdash T\varsigma \circ S$. So $w' \Vdash S \circ w$. So $w \Vdash \mu(S \circ w)$.

$\Leftarrow$ Assume $w \nVdash S(\mu\circ)w$. Case 1: (from def. of proper names) there is no ς in any world w' such that $w' \Vdash T\varsigma \circ S$, so in all w', $w' \nVdash S \circ w$. So, $w \nVdash \mu(S \circ w)$. Case 2: for some world w' (according to the semantics of μ), $w' \nVdash T\varsigma \circ w$. So $w' \nVdash S \circ w$. So $w \nVdash \mu(S \circ w)$.

That is, only in the case of propositions with proper names as subjects are the 'de dicto' and the 'de re' readings of them equivalent. This is clearly because there is no difference between the subject suppositing for what is or for what was/will be in the case of proper names: it is always question of the same individual, and the use of a proper name does not imply that its bearer exists in that world where the proposition it occurs in holds.

Before concluding the section on 'de re' temporal propositions, let me provide one more truth definition of such a proposition, for if there are still doubts concerning the viability of providing recursive truth conditions for all propositions of the language in terms of $T\delta \circ a$ propositions. Let us consider the case of 'every' universal propositions:

Definition 3.5.7 $t \Vdash \forall a(P\circ)b \iff$ For all δ in t such that $t \Vdash T\delta \circ a$, $t' \Vdash T\delta \circ b$ for some $t' < t$.

The other cases - particular, singular and negative proposition - can also easily be rendered.

3.6 Modal propositions

As shown by Panaccio in [Pan99], there is a significant kinship between Ockahm's treatment of temporal propositions and that of modal propositions[19].

[19] Interestingly, Prior, the creator of tense logic, was a great reader of Ockham (what does not seem to be a coincidence).

According to Spade, not only Ockham treated tense and modality along the same lines: this was the case of most authors at that time (Cf. [Spa96][p.318]. In particular, according to Panaccio, the idea that the predicate appellates its form in temporal propositions has a very similar counterpart in modal propositions.

> In Ockham's *Summa Logicae*, the doctrine that the predicate appellates its form is first introduced in chapter 72 of part I, in the course of discussing the supposition of predicate terms within sentences with past-tense or future-tense verbs, but it is attributed from the start a much wider relevance: although briefly expounded, it is also expected to shed light on some crucial aspects of the semantics of modal sentences generally, including epistemic sentences. [Pan99][p.3]

Indeed, we shall encounter in modal propositions something very similar to the distinction between so-called 'de re' and 'de dicto' temporal propositions. Also in modal propositions, the mode can be predicated of - or can 'range over' - the dictum of an entire proposition, or can be introduced modifying only the predicate. These notions will be explored in details just below, and the reader should not be surprised to encounter truth conditions which are similar to those of temporal propositions.

3.6.1 What modes are

Any discussion about modalities is very likely to be metaphysically laden, for the metaphysical questions about what it means for something to be necessary, or possible, or contingent have to be addressed at some point. Much has been written in this respect about how medieval philosophers viewed modalities, one of the most prominent examples thereof being Knuuttila's *Modalities in Medieval Philosophy* [Knu93]. However, the scope of the present work does not leave room for an extensive analysis of all the underlying topics about modalities; as I adopt a foremost logical perspective, in particular with respect to the truth conditions of modal propositions, a broader discussion cannot take place.

However, I would like to add a few relevant remarks on Ockham's views on modalities, insofar as they can shed some light into the reconstruction proposed below.

First of all, what are modes for Ockham? What is a modal proposition? We have seen before that the standard case of a modal proposition is a proposition whose subject is another proposition of which something is predicated; accordingly, a mode is anything that can be predicated of a proposition (or actually, of its dictum, which corresponds in English to 'that-clauses'). Moreover, propositions in which a mode is connected to the copula (most frequently, under the form of an adverb) are also modal.[20]

[20] Again, we must assume the well-formedness of these sentences to be able to define what is a mode, what can truly be predicated of another proposition.

> Only those sentences were counted as modal, however, which included modal terms connected to the copula (divided or *de re* modals) or connected by the copula to sentences or to *dicta* (composite or *de dicto* modals). [Knu93][p.167]

So, the first kind of modal propositions are called *de dicto* propositions. Further clarifications must be made (they can be taken in the sense of composition or in the sense of division, which will be explained soon), but the general scheme is that such propositions have the usual predicative structure, whereas the second kind of modal propositions are generated by the addition of a modal adverb to the copula.

> In the sense of composition it is always asserted that such a mode is truly predicated of the proposition corresponding to the dictum in question. [Ock80][p.109]

Once the shape of modal propositions has been defined, it must be said that, among the many modes, I shall only deal with necessity and possibility, and their respective opposites, contingency and impossibility (these are the four 'classical' - aristotelian - modalities). I shall not treat epistemic modalities, such as 'known' and 'believed', or any other kind thereof, even though there exist medieval discussions on these topics, in particular in Ockham's writings.

As for what possibilities are, as has been argued in 2.2.1, the idea of possibility suffered a major change during the Middle Ages, under the influence of theological considerations which were alien to previous discussions on modalities (in particular in Aristotle). Possible became then understood as the domain of what in intelligible, or in other words, what God could make real, but was nevertheless *not* real.

> A historically important part of Scotus' theory is the view that the domain of possibility is an a priori area of what is intelligible and as such does not have any kind of existence. ... I argue that this transcendental theory of modality and conceivability also occurs in William of Ockham. [Knu93][p.138]

So the domain of possibility is an intensional, intellectual construct, namely everything conceivable which does not involve contradiction - that is, all that which could exist, by means of God's omnipotence, but does not necessarily exist (assuming that the actual also belongs to the domain of possibilities). Thus, although Ockham does not discuss the notion of logical possibility explicitly (to my knowledge), it is possible to establish from the metaphysical considerations what logical possibility must be - that is, what it means for a proposition to be possible: it means that the particular set of circumstances which make it true could be produced by God. This notion is in fact quite close to our notion of possibility as truth in some possible world, that is, the semantics of the $\Diamond$-operator.

Accordingly, necessity is (logically) defined as truth under all circumstances[21]:

> However, with respect to a necessary proposition it should be noted that a proposition is called necessary not because it is always true, but because it is true if it exists and cannot be false. [Ock80][p.111]

Here Ockham introduces something similar to Kaplan's context of utterance, but it only concerns the existence or non-existence of a given proposition[22]. As we know, propositions must be understood as tokens, that is, they must acquire a singular occurrence (either mental, written or spoken) to be part of the ontology, otherwise they are not even propositions, let alone are they true or false . So, a necessary proposition is a proposition which is true every time it exists, that is, every time it is uttered or written or thought of[23] . That is, the semantics of the necessity modality as defined by Ockham are the same as the semantics of the $\Box$-operator in standard modal logic[24], with the difference that he introduces a contextual clause concerning the local existence of the given proposition[25]

> Universal and particular affirmative composite sentences maintain that their *dicta* are true in all possible state of affairs (necessary sentences) or in some possible states of affairs (possibility sentences). [Knu93][p.171]

As argued, what is said about possibility and necessity should hold *mutatis mutandis* for impossibility and contingency, since impossibility is the opposite of possibility[26] - and hence can be defined as truth in no world - and contingency is the opposite of necessity - that is, a contingent proposition is not true in all worlds. But epistemic and deontic modalities are more delicate and should be treated as a special case, and not inferred from these general remarks on the classical modalities.

> Something analogous should be said of an impossible proposition ... And the same holds, *mutatis mutandis*, for a contingent proposition. But in order for a proposition to be known or believed etc., more is required. [Ock80][p.111]

I shall not analyze other kinds of modalities, but this has in fact been done by some scholars, for example Boh in 'Elements of Epistemic Logic in Later Middle

[21] Notice that a metaphysical discussion about necessity would be much more complex than that about possibility; it would have to be argued why it can be so that something is always the case.

[22] Kaplan's contexts of utterance concern many more elements.

[23] Spade, in [Spa96][p.313], explores all the difficulties of this definition of necessity; in particular, it leads to some paradoxes related to the demand that a proposition must be a token associated with the notion of necessity as truth in all worlds. That is, there is much more to be said on this issue than what I say here.

[24] Assuming a universal accessibility relation.

[25] The ontological status of propositions is, as I have said before, a topic which should be more thoroughly discussed than it is here.

[26] There are problems concerning the definition of possibility, Cf. [Spa96][p.313], which I shall disregard here.

Ages'[27], or Panaccio in 'Belief-sentences: outline of a nominalistic approach'[28], and [Pan99]. In fact, the remarks in this section are only meant to support my interpretation in the next two sections of the truth conditions Ockham provides for modal propositions, and do not exhaust the topic. I will indeed apply the usual semantics of the $\Box$- and $\Diamond$-operators, even though Ockham's analysis of modes does not follow the same approach (namely, it is formost based on the idea of predicating a mode from a proposition). But I believe that the operator interpretation is not very distant from many of Ockham's remarks, as I have argued here.

3.6.2 With dictum

First of all, what does the expression 'with dictum' mean? The 'dictum' of a proposition in Latin is its transposition to the accusative case[29], which is equivalent in English to the introduction of 'That' in front of the proposition - a 'that-clause'. Example: 'Socrates is white'. The 'dictum' of this English proposition would be 'That Socrates is white'. To be the extreme of a proposition (the subject), a proposition must be in the 'dictum' form.

Thus, it is evident also from his jargon that Ockham deals with modal propositions as regular predicative propositions, in which the subject is a proposition in the 'dictum' form and the predicate is a mode. In this section, I shall formulate the truth conditions of modal propositions in Ockham's way, that is, as predicative propositions, but will also make a step further and apply the usual semantics for modal operators, as it provides some interesting insights. But it is clear that this is only a possible extension of Ockham's system, since he himself only treats modes as predicates.

> And it should first be noted that sometimes a proposition is called modal because the dictum of the proposition is taken with such a mode. This is clear in the following cases: 'That every man is an animal is necessary', 'That every man is running is contingent' ... [Ock80][p.109]

Such as temporal propositions, modal propositions 'with dictum' are systematically ambiguous: they can be read either in the sense of composition, that is, as the mode being predicated of the whole proposition, or in the sense of division. In the latter, according to Ockham, they are equivalent to modal propositions 'without dictum', that is, when the mode becomes an adverb inside the proposition, which then does not have the predicative structure (dictum - subject; mode - predicate) present in propositions 'with dictum'.

[27] In C. Wenin (ed.) *L'homme et son univers au moyen âge II.* 1986, Louvain-la-Nouve: Editions de l'Institut Supérieur de Philosophie.

[28] In M. Marion and R. S. Cohen (eds.), *Quebec Studies in the Philosophy of Science II* 1996.

[29] For something is called the dictum of a proposition when the terms of the proposition are taken in the accusative case and the verb in the infinitive mood. [Ock80][p.109]

> A modal proposition of the first type must always be distinguished with respect to composition and division. In the sense of composition it is always asserted that such a mode is truly predicated of the proposition corresponding to the dictum in question. [Ock80][p.109]

Hence, in this case - i.e., modal propositions with dictum in the sense of composition -, as the mode is predicated of the whole proposition, we could say that the mode 'ranges' over the whole proposition. In the sense of division, however, a proposition has the same truth conditions as modal propositions without a dictum, so I should treat this case in the section below.

> However, the sense of division of such a proposition is always equipollent to a proposition taken with a mode and without such a dictum. [Ock80][p.109]

In any case, we again encounter the imperfections of written or spoken language, in that they cannot differentiate the two readings of the same modal proposition. Once more, a logical analysis is necessary to render the two readings, which reinforces the interest of their formal transcriptions.

Let us first provide the truth conditions of modal propositions with dictum in the sense of composition in a predicative fashion, as we encounter in Ockham. The advantage therein is that no further definition of truth is necessary, as such propositions are just regular predicative propositions whose truth conditions are the same as defined above - a proposition is true iff its two terms supposit for the same thing. So, a modal proposition of this type is true iff the salient mode is truly predicated of that which the proposition supposits for.

> When read in the composite sense, the *dictum* in modal propositions of the first kind [with a dictum] is a term in material supposition, suppositing for the proposition from which the *dictum* was formed. [Spa96][p.310]

Let p denote any proposition in the appropriate 'dictum' form, and $\Box$ the predicate 'necessary' (the same holds for any other mode).

Definition 3.6.1 [Predicative fashion] $w \Vdash p \circ \Box \Longleftrightarrow$ There is a proposition ϕ in w such that $w \Vdash T\phi \circ p$ and $w \Vdash T\phi \circ \Box$.

However, since it is the case that the mode ranges over the whole proposition, it seems reasonable to propose another possible transcription for it: $\Box(a \circ b)$. Moreover, if we apply the usual semantics of the $\Box$ operator, the following truth conditions rise:

Definition 3.6.2 [Operator fashion 1] $w \Vdash \Box\Phi \Longleftrightarrow$ In all worlds w', $w' \Vdash \Phi$.

More specifically:

Definition 3.6.3 [Operator fashion 2] $w \Vdash \Box(a \circ b) \Longleftrightarrow$ In all worlds w' where there is a suppositum for a , $w' \Vdash a \circ b$. So, in all w' where there is an object δ such that $w' \Vdash T\delta \circ a$, $w' \Vdash T\delta \circ b$.

We encounter here the very serious problem of expansive domains[30] that any predicate modal logic has to deal with. I chose to introduce the clause of existence of the object, instead of accepting expansive domains, because it seems to me to be in conformity with Ockham's definition of necessity mentioned in the previous section, even though the latter is about the existence of propositions, and not of any object. In any case, I am not alone in sustaining this clause:

> It is reasonable to suppose that the possible truth of 'This is black' means that it is true in at least one of the possible states of affairs in which the possible being referred to by 'this' occurs, and that the necessary truth of 'This is black' means that it is true in all worlds in which the thing referred to by 'this' occurs. [Knu93][p.171]

Let us now analyze a rather complex example given by Ockham, which becomes surprisingly clearer under the operator perspective here introduced.

> For instance, in the sense of composition 'That every true contingent proposition is true is necessary' is true and necessary, and yet each of its singulars is false. For 'That this true contingent proposition is true is necessary', referring to any of them, is false, since this true contingent proposition can be false. [Ock80][p.109]

The singulars of such a proposition would be true if it was understood in the sense of division, as we shall see below. Let tcp denote the term 'true contingent proposition' (for one I will use a 'compound' transcription for a compound term, for the sake of clarity) and t the predicate 'true'.

$$w \Vdash \Box(\forall tcp \circ t) \Longleftrightarrow \text{For all } w' \text{ where there is an individual } phi \text{ such that } w' \Vdash T\phi \circ tcp \text{ , } w' \Vdash \forall tcp \circ t.$$

So, for all propositions ϕ in any w' such that $w' \Vdash T\phi \circ tcp$, $w' \Vdash T\phi \circ t$. This is intuitive, since in all worlds, what is a true contingent proposition in a given world w is also a true proposition *in that world*.

However, in the sense of division, this proposition is not true, for this would imply the following: for all propositions ϕ in w such that $w \Vdash T\phi \circ tcp$, $w \Vdash \Box(T\phi \circ t)$, and this would mean that, for all ϕ in w such that $w \Vdash T\phi \circ tcp$, in all w' where ϕ exists, the very same proposition would always be true ($w' \Vdash T\phi \circ t$), which is not true of a contingent true proposition.

That is, the operator analysis can shed some light on some modal propositions, of which Ockham's analysis of modes as predicates, however elegant in the framework of his system, provides a less clear account.

[30] That is, that every world must contain all individuals that the worlds it stems from contain. So, if wRw', then for all objects δ such that $\delta \in w$, $\delta \in w'$.

3.6.3 Without dictum

As previously stated, modal propositions with dictum in the sense of division are equivalent to modal propositions without a dictum, so I shall treat these two propositional forms as only one case, since they are indeed the same in their logical structure.

> For example, 'That every man is an animal is necessary' in the sense of division is equipollent to 'Every man is of necessity (or necessarily) an animal. [Ock80][p.109]

Ockham very clearly states that these two types of modal propositions (with a dictum in the sense of composition and without a dictum/with a dictum in the sense of division) are not equivalent ('convertible', in his terms). The same proposition can be true if read as belonging to the first type, and false otherwise. For example, 'That every man is an animal is necessary' is true, while 'Every man is necessarily an animal' is false, according to Ockham.

> With respect to modal propositions without a dictum of a proposition, which are altogether equipollent to propositions with a dictum in the sense of division, it should be noted that such propositions are not interchangeable with the first type discussed. In fact, one can be true even if the other is false, and conversely. For example, in Aristotle's opinion, 'That every man is an animal is necessary' is true in the sense of composition, while 'Every man is necessarily an animal' is false. [Ock80][p.111]

It is evident thus that the two logical types of modal propositions have different truth conditions. But where lies exactly the difference? A good way to approach this distinction is via the truth conditions of temporal propositions; as argued, in temporal propositions, the predicate appellates its form, that is, the temporal effect always affects the predicate, but may or may not affect the subject. Well, something similar seems to take place in the case of modal propositions.

> For this reason it should be noted that for the truth of such [modal] propositions it is required that the predicate under its *proper form* belongs to that for which the subject supposits, or to a pronoun referring to that for which the subject supposits. . . . , just as it was explained in the case of past-tense and future-tense propositions. [Ock80][p.112]

That is, in the case of modal propositions without a dictum, the mode is not predicated of (does not range over) the whole proposition, but only affects the predicate, since it 'appellates its form'[31]. So, such propositions also receive

[31] Notice however that Ockham does not use the expression 'appellates its form' in modal contexts.

truth conditions in terms of the truth of non modal propositions, but not with respect to the original proposition. Rather, the convenient mode is predicated of a demonstrative proposition where the original predicate is predicated of a pronoun which demonstrates something that the original subject supposits for.

> For example, for the truth of 'Every truth is necessarily true' it is required that each proposition be necessary in which the predicate 'true' is predicated of anything for which the subject 'truth' supposits. That is, it is required that each proposition like the following be necessary: 'This is true', 'That is true', referring to something for which the subject supposits. And since it is not the case that each such proposition is necessary, 'Every truth is necessarily true' is simply false. [Ock80][p.112]

Let us transcribe a modal non-quantified proposition without a dictum[32] as $a(\Box\circ)b$, where the operator is placed before the copula as if it was an adverb, just as in the corresponding written or spoken proposition[33]. We have then:

Definition 3.6.4 [Predicative fashion] $w \Vdash a(\Box\circ)b \Longleftrightarrow$ There is an individual δ in w such that $w \Vdash T\delta \circ a$ and $w \Vdash (T\delta \circ b) \circ \Box$.

Then, for the truth of $(T\delta \circ b) \circ \Box$, the conditions defined for modal propositions with a dictum would apply (that both proposition and mode should supposit for the same thing). However, just as much as in the previous section, it is possible to adopt an operator perspective over the truth conditions of modal propositions without a dictum, in which case it would be licensed to argue that the operator ranges only over the predicate, as follows:

Definition 3.6.5 [Operator fashion] $w \Vdash a(\Box\circ)b \Longleftrightarrow$ There is an individual δ in w such that $w \Vdash T\delta \circ a$, and in all worlds w' where δ exists, $w' \Vdash T\delta \circ b$.

The classical problem of expansive domains reappears, and again I prefer to add a clause concerning the existence of the relevant object δ. This is of course a problem which is not present in Ockham's account, as it does not arise if one contents oneself with the predicative fashion of handling modal propositions. I am thus obliged to supply an *ad hoc* solution to the issue, as close as possible to Ockham's spirit.

Thus, the basic principle of modal propositions without a dictum is that their truth depends on the truth of some relevant demonstrative proposition. Following this principle, any modal proposition without a dictum (negative, quantified etc ...) can receive satisfactory truth conditions, even though a general scheme cannot be provided, as it was possible for modal propositions with a dictum (Cf. Definition 4.6.2). So, it would be interesting to examine another case of modal propositions without a dictum, the case of a universal proposition, since most of Ockham's examples are of this kind.

[32] Needless to say that the same holds for modal propositions with a dictum in the sense of division.

[33] That is, so far I have made no commitments with regard to the logical status of the operator.

Definition 3.6.6 $w \Vdash \forall a(\Box\circ)b \Longleftrightarrow$ For all individuals δ in w such that $w \Vdash T\delta \circ a$, in all worlds w' where δ exists, $w' \Vdash T\delta \circ b$.

Notice that I have only treated cases of necessary propositions in the previous two sections, and did not deal with the other modalities (possibility, impossibility, contingency), which is by the way also Ockham's procedure - most probably because necessity is more problematic than the other modalities. But it is evident that the same truth conditions apply to the other modalities, with due modifications.

3.7 Conclusion of Chapter 3

I hope the reader is convinced by now that the split two-name notation was indeed a convenient vehicle for the reconstruction of Ockham's logic. It is clear that standard predicate logic simply cannot handle all the logical cases analyzed by him. In Chapter 1, we had seen that there were philosophical reasons to adopt an alternative notation, since, for Ockham, all terms - subject and predicate - behave semantically as names, and therefore have the same semantic and syntactical properties. So it was crucial to introduce a notation which would respect these principles, and that is not the case for predicate logic standard notation.

However, Chapter 3 presented more elements to support the split notation: from a very 'technical' point of view, standard predicate logic does not have the expressive power to account for Ockham's logical system. This was already felt with respect to affirmative and negative propositions, and the existential import of the former: it became all the more evident in the context of his temporal and modal logical systems. As we have seen, the distinction between 'de dicto' and 'de re' modalities (using the modern jargon) does not take place only on the level of quantified propositions, as in standard predicate modal logic, but also affects atomic propositions. The notion of supposition was a powerful tool to distinguish the two cases, namely the case in which both subject and predicate refer to individuals in the same world, and that in which subject and predicate refer to individuals in different worlds. Most of the domain problems raised by standard predicate modal logic are smoothly dealt with within Ockham's framework, since the reference of terms is the major feature of this system.

However, its expressive power is not equivalent to that of standard predicate modal and temporal logic[34]. Standard predicate logic can count on the strength of variables, which are extremely powerful to treat cases of incomplete terms, quantification over variables etc ..., which Ockham's system cannot cope with, since the notion of incomplete terms cannot appear in it. But Ockham's system, at least under the form presented here, is able to treat cases which standard predicate logic cannot: with the split notation, all kinds of operators - negation, quantifiers, modal and temporal operators - can range either over the whole proposition or only over the predicate, as shown. In fact, I have not treated the

[34] This could certainly be proved.

case of quantification over predicates, but it is possible, and Ockham himself explores this notion, in the context of his ascent and descent theory, in *Summa Logicae* Part I.

As for the other operators, in this chapter I have shown the many possibilities created by their introduction either before the whole proposition or just before the predicate. I consider it to be one of the most interesting features of Ockham's logic, and I hope the reader agrees.

Chapter 4

Conclusions

Let me draw a brief sketch of my itinerary in this work. In Chapter 1, I started with an analysis of some aspects of Ockham's philosophical thinking: his nominalist refusal to include any entities which are not individuals in his ontology, his views on how mental terms are causally formed by the perception of individuals, his distinction between mental and conventional (written and spoken) language. Concerning the last topic, the modern reader must always beware while dealing with medieval philosophy, for what the authors of that time - in particular Ockham - named 'natural language' is the *mental language*; what we now call natural language was for them *conventional language*.

I then had the basic tools to move to Ockham's semantics properly speaking. First, I explored the notion of *significatio*, as the property of terms *qua* terms, stressing that the signification of a term is what we now call its extension, that is, the things it stands for, and not an intension. In fact, not only subject terms signify individuals, but also those terms that we usually view as typical predicates - and usually denote functions or sets. Such terms, which Ockham refers to as connotative terms, also signify individuals, namely all those each of them can be truly predicated of. This was perhaps the first time that a significant reason was provided to justify my introduction of a different notation for predicative two-term propositions, that is, $a \circ b$ instead of $B(a)$, to investigate Ockham's logic.

Once signification had been defined, I then moved to *suppositio*, that is, the property of terms in propositions. Terms in a proposition refer to things directly, it is as if they replaced things in propositions: this is the relation of supposition, that is, terms supposit for individuals in propositions. Supposition theory is actually twofold, it comprises a theory of inference - briefly analyzed in 1.3.3 - and a theory of reference. According to Ockham, a term can supposit either for what it signifies, or for the concept in the mental language that also signifies what the term signifies, or for itself as written or spoken word. These three cases define three different types of supposition - personal, simple and material.

These considerations seemed to be enough to allow for some further steps.

So, in Chapter 2 I approached some contemporary issues and tried to establish positions which would be in general coherent with Ockham's thinking: first I dealt with some extensions of the semantic framework defined in Chapter 1, exploring the problems of definition, complex terms and domains of signification and reference. Then I proceeded to justify my claim that a possible-world semantics could be used to analyze Ockham's logic without too much anachronism, as the notions of possible worlds and local truth are indeed present in his writings.

I then moved to the issues of proper names and designation. My claim was that all terms have direct reference for Ockham - direct reference as Kaplan understands it, but then holding for all terms. They refer directly to all things they can be truly predicated of, and not by the mediation of an intension. Proper names also have direct reference, but furthermore they are rigid designators, that is, they refer to the same individual in all possible worlds where it exists.

In Chapter 3 I undertook the reconstruction of Ockham's truth conditions for propositions. I proceeded by increasing complexity, so I started explaining my choice of atomic element, $T\delta \circ a$ propositions, where the evaluation operates. It was then possible to define the truth of the standard two-name predicative proposition of the form $a \circ b$: it is true iff a and b supposit for the same individual. Based on this notion of truth, I was able to define the truth conditions of negative and 'quantified' propositions.

I then explored Ockham's temporal and modal logic, introducing a bit of possible-world semantics. In both cases, the most remarkable feature of his system is that an operator (tense or modal) can range either over the whole proposition or only over the predicate. With the help of the notation I had previously introduced, $a \circ b$ for predicative propositions, it was possible to show that Ockham's account of modal and temporal propositions displays the advantage of being referentially transparent. It is always clear to which domain the suppositum of a term belongs, depending on the range of the operator.

Thus, Ockham's radical emphasis on the referential aspect of terms provides interesting insights to referentially opaque contexts, such as temporal and modal predicate logic. This was for me the main motivation for this reconstruction - to investigate the results of applying an extensional and purely referential semantic framework like Ockham's to intensional contexts. I believe this has successfully been achieved, not only from a historical point of view, but also as a possibly fruitful way to deal with modal and temporal contexts in general.

These are the things which I think were accomplished. However, throughout this work I have also encountered a number of problems and issues that I could not spell out as much as I would have liked to, due to time and space constraints, but which now have become possible future projects. I will present some of them.

One topic for which I still haven't got the answer concerns supposition theory. After all, for Ockham, the kind of supposition a term is to have in a proposition (personal, simple, material), is it a matter of pragmatics or of semantics? In other words, I still don't know whether the strongest component to define the supposition of a term in a proposition is the intentionality of the speaker, or if sentential constraints also come into play in an essential manner.

On the one hand, in some places Ockham states that terms can have personal supposition in all propositions, but that only when the other extreme of the proposition supposits for a concept or for a written or spoken word can a term have simple or material supposition. On the other hand, he also says clearly that, in the proposition 'Man is a species', if 'Man' has simple supposition, then the proposition is true, but if it has personal supposition, then the proposition is *false*, and not undefined. That is, I do not think there are three truth values in Ockham's account, so the propositions with the 'wrong' supposition are plainly false. But this topic deserves further investigation, which I would like to undertake some day.

Another limitation in this work - this time, a deliberate one - is the absence of an analysis of entailment and inference (it was only briefly treated, with respect to ascent and descent theory). Ockham did treat numerous kinds of syllogisms, including temporal, modal and 'mixed'[1] syllogisms, but his analysis thereof is in Part III of *Summa logicae*, which hasn't been translated from Latin to any modern language yet; moreover, the topic is huge, much bigger than the scope of a Master's thesis[2]. Still, it is very important to one day check whether my modeling and transcription of Ockham's logic also works for his considerations on entailment and inference, and my plans are to either wait for Part III to be translated or to work very hard on my Latin to be able to use the text as a research object.

There is one more topic I had planned to treat as an appendix, but I eventually realized it was much larger than I had thought it was. It is not only in the *Summa logicae* that Ockham analyzes temporal propositions: in fact, his account of tense in the *Treatise on Future Contingents* [Ock83] is very complex and follows a different direction. It is based on his considerations in that book that Prior introduced the expression 'ockhamist' to qualify some properties of a tense logic which he claimed is to be found in the *Treatise* [Pri67], [Pri68]. The line of research introduced by Prior received quite some attention, and now there is a considerable number of articles about ockhamist frames, formulas, semantics, temporal logic etc ... , as a simple search in the *Philosopher's Index* with the term 'ockhamist' shows.

So the interest of studying the *Treatise on Future Contingents* is obvious, as it indirectly gave rise to some ideas which are at the present moment being explored. One interesting topic would be to check whether what these authors claim to be 'ockhamist' can indeed be found in Ockham's texts properly speaking. This is another project I would like to pursue one day.

Finally, also related to Prior's work, emerges the idea that Ockham's logic has a hybrid component, to use an anachronistic term, in that it lies in between a propositional and a predicative approach. The basic idea of hybrid logic is the following:

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[1] Syllogisms with both temporal and non-temporal or modal and non-modal propositions.

[2] This is indeed the topic of Elisabeth Karger's PhD dissertation, where she analyzes in detail all kinds of syllogisms mentioned by Ockham, which allows her to draw the rules of inference of his system.

> That by exploiting the notion of *formulas as terms* to the full, we will be able to define systems combining the best of modal and classical techniques. [Are00][p.137] My emphasis.

My interpretation of Ockham follows these lines in that I think he treats *terms as formulas*: terms are true of individuals, just as much as formulas are true in worlds, in standard possible-world semantics. Since he emphasizes the referential power of terms, they behave a lot like the *nominals* defined in hybrid logic, that is, formulas that hold in only one world each, so that each world can be referred to in the object-language by their corresponding nominal which becomes the world's 'name' - hence, nominals (Cf. [Bla00]). Now terms for Ockham are also names for individuals if they 'are true' of them, or in them.

It is not a coincidence to find this 'hybrid' perspective in Ockham. The reason for that, as I said before, is that Prior, who came up with ideas that later inspired the development of hybrid logic, was indeed very familiar with Ockham's writings.

My final point is then that turning to medieval logic is not only historically relevant, it is also fruitful for present-time logic in general in that it presents some points of view which may be surprisingly fresh and novel. Standard predicate logic is now less and less considered to be the 'one' logic, and the inspection of 'old-fashioned' logical systems which are based in different principles may provide insights for new lines of research. This procedure is not new, in fact some authors have done it before - I would name Prior and Geach -, but I do not think that all has been said in this respect. Therefore, I hope to have added something interesting to this panorama with this Master's thesis, and maybe to continue the investigation, with one of the four topics mentioned in this conclusion or yet something else.

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